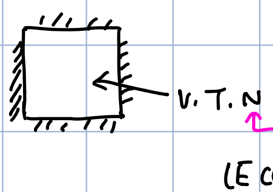
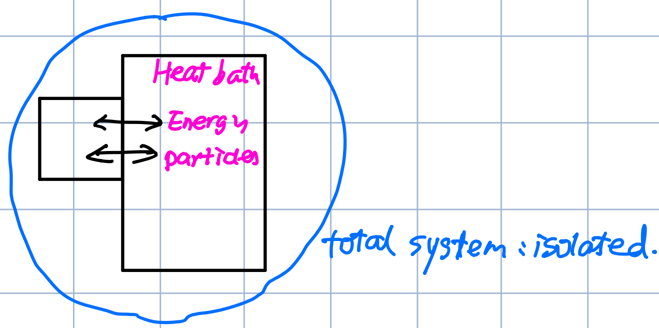
Ensembles	variables	System
Microcanonical	E.N.V	isolated system
Canonical	T.N.V	heat bath (for maintaining the temperature a constant)
Grand canonical	T.μ.V	heat bath & particle bath (maintain T & μ)

Chapter 10 Grand Canonical Ensemble

1.





Assumptions: 1° $E_{system} + E_{bath} = E_{total}$, $E_{sys} \ll E_{total} \sim E_{bath}$

2° $N_{system} + N_{bath} = N_{total}$. $N_{sys} \ll N_{total} \sim N_{bath}$

3° The total system follows microcanonical distribution.

2. Derivation

$$\underset{\text{total}}{P_{``i"}} = \begin{cases} a, & \text{if } E_{``i"tot} = E_L \\ 0, & \text{otherwise} \end{cases}$$

$``i"_{tot} \rightarrow ``i"_{bath} + ``i"_{sys}$

$$P_{``i"sys} = \sum_{``i"bath} P_{``i"bath} = \sum_{``i"bath} \begin{cases} a, & \text{if } E_{``i"total} = E_L = E_{``i"sys} + E_{``i"bath} \\ 0, & \text{otherwise} \end{cases}$$

$= a \cdot$ # of "i" bath having $E_{bath} = E_{Total} - E_{``sys"}$

$= a\Omega(E_{total} - E_{sys}, N_{total} - N_{sys})$

$k_B \ln\Omega \rightarrow$ entropy

$$\ln\Omega = \ln\Omega(E_{tot} - \underline{E_{sys}}, N_{tot} - \underline{N_{sys}}) \approx \ln\Omega(E_{tot}, N_{tot}) - E_{sys}\frac{\partial \ln\Omega}{\partial E} - N_{sys}\frac{\partial \ln\Omega}{\partial N} - \cdots$$

$1^{st} + 2^{nd}$ laws of thermodynamics

$$T^{-1} = \frac{\partial S}{\partial E}. \quad \frac{1}{k_B T} = \frac{1}{k_B}\frac{\partial S}{\partial E} = \frac{\partial \ln \Omega_{tot}}{\partial E}$$

$$-\mu T^{-1} = \frac{\partial S}{\partial N} \quad \frac{-\mu}{k_B T} = \frac{\partial \ln \Omega_{tot}}{\partial N}$$

$\frac{\partial \ln \Omega}{\partial E} \equiv \beta$ $\quad$ $\frac{\partial \ln \Omega}{\partial N} \equiv \alpha = \mu\beta$

$$\ln P_{\text{"}i\text{"}sys} = \ln a + \ln \Omega = \ln a + \ln \Omega(E_t, N_t) - E_s \frac{1}{k_B T} + N_s \frac{\mu}{k_B T} + \cdots$$

$$\Rightarrow P_{\text{"}i\text{"}sys, N_{sys}} \propto \exp(-\beta E_{\text{"}i\text{"}sys} + \mu\beta N_{\text{"}i\text{"}sys})$$

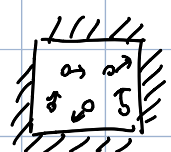
3. Normalization

$$1 = \sum_{\text{"}i\text{"}, N} P_{\text{"}i\text{"}, N} = A \sum_{\text{"}i\text{"}, N} \exp(-\beta E_{\text{"}i\text{"}s} + \alpha N)$$

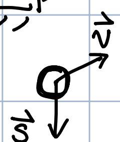
$$A = \left(\sum \exp(-\beta E_{\text{"}i\text{"}sys} + \alpha N)\right)^{-1} \quad P_{\text{"}i\text{"}, N} = \frac{\exp(-\beta E_{\text{"}i\text{"}sys} + \alpha N)}{\sum_{\text{"}i\text{"}, N} \exp(-\beta E_{\text{"}i\text{"}sys} + \alpha N)}$$

$$Z \equiv \sum_{\text{"}i\text{"}, N} \exp(-\beta E_{\text{"}i\text{"}sys} + \mu N)$$ Grand Canonical Partition Function.

Chapter 11 Quantum Ideal gas

1. 

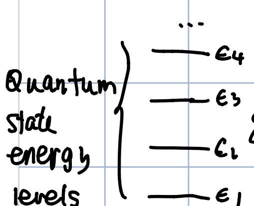
Ideal gas → No interaction between particles.
weak



particles → Quantum particles : Fermions: spin half integer · $\hbar$ $\quad e^-$
↑ carry spins. intrinsic
Bosons: spin integer · $\hbar$ $\quad \gamma$

- Pauli Exclusion Principle: No two fermions can occupy same Quantum State.
- Indistinguishability: cannot label the particles.

2. What do we determine?



$\varepsilon_{\text{"}i\text{"}}$ → energy states of a single particle
↑ i_1, i_2, i_3. $\quad$ ↑ ideal gas

On average $\langle n_{"i"} \rangle \to$ average occupation number (can be non-integer)
as a function of "i"

3. Derivation

consider a distribution of occupation number

$$\begin{array}{cccc} \epsilon_1, & \epsilon_2, & \epsilon_3, & \epsilon_4 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ n_1 & n_2 & n_3 & n_4 \end{array} \quad \text{integers}$$

Note: No labels.

we know: $N = n_1 + n_2 + n_3 + \cdots$ # of particles in the system.

label of quantum states

system energy of quantum state "i" $E_{"i"} = n_1 \epsilon_1 + n_2 \epsilon_2 + \cdots$

Gibbs Distribution of Grand Canonical Ensemble.

$$P_{"i",N} = P(n_1, n_2, n_3, \cdots)$$

$$\propto \exp(-\beta E_{"i"sys} + \alpha N) = \exp(-\beta(n_1\epsilon_1 + n_2\epsilon_2 + \cdots) + \alpha(n_1 + n_2 + \cdots))$$

ideal gas

Grand canonical partition function

$$Z(\alpha, \beta) = \sum_{\{n_i\}} \exp(-\beta(n_1\epsilon_1 + n_2\epsilon_2 + \cdots) + \alpha(n_1 + n_2 + \cdots))$$

system

$$= \prod_{"i"} \sum_{n_i = 0}^{\infty} \exp(-\beta n_i \epsilon_i + \alpha n_i) = \prod_{"i"} Z_{"i"}(\alpha, \beta, \epsilon_{"i"})$$

Fermions (Fermi-Dirac Theory) $n = 0, 1$. No double/triple ... occupation.

$$Z^{FD}_{"i"} = 1 + \exp(-\beta\epsilon_i + \alpha) \quad \text{Fermi-Dirac partial partition function.}$$

Bosons (Bose-Einstein Theory) $n = 0, 1, 2, \cdots$

$$Z^{BE}_{"i"} = \sum_{n=0}^{\infty} \exp(-\beta n \epsilon_{"i"} + \alpha n) = \sum_{n=0}^{\infty} \exp[(-\beta\epsilon_{"i"} + \alpha) \cdot n] = \frac{1}{1 - \exp(-\beta\epsilon_{"i"} + \alpha)}$$

$$\langle n_{"k"} \rangle = \sum n_k P(n_k)$$

$$P(n_k) = \sum_{\{n_i\}}^{n_k} P(n_1, n_2, \cdots, n_k, \cdots)$$

keep n_k unsummed
sum over all other n_i's

Probability Reduction.