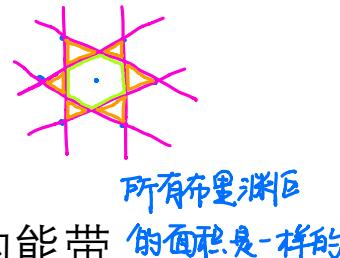


第三章 能带论作业

1. 对原子间距为 a 的由同种原子构成的二维密排结构

(1) 画出第一和第二布里渊区； $\pi k_F^2 = n^{-1} = \frac{8}{4} a^2$



(2) 求出每个原子有一个自由电子时的费米波矢；

(3) 利用紧束缚近似法，推导s态原子波函数形成的能带

$$E = E_0 - J_0 - J \sum e^{i\vec{k} \cdot \vec{r}}$$

$$= E_0 - J_0 - J (\cos + \cos + \cos)$$

2. 由相同原子组成的一维原子链，每个原胞中有两个原子，原胞长度为 a ，原胞内两个原子相对距离为 b

单原子多轨道
多反子多轨道类型

(1) 根据紧束缚近似，只计入近邻相互作用，写出原子s态相对应的晶体波函数的形式

$$\psi(\vec{r}) = \sum_m e^{ikx} (C_1 \varphi_1 + e^{ikb} \varphi_2)$$

(2) 求出相应能带的 $E(k)$ 函数

$$(如果 b=0 就是一群电子的 2J 轨道) \\ = \sum e^{ikx} (C_1 \varphi_1(r-ma) + C_2 \varphi_2(r-ma-b))$$

(3) 当 $b = 1/2a$ 时，求出相应能带的 $E(k)$ 函数，有什么发现？

$$(H_{at} + \Delta V) \psi = E \psi \Rightarrow \sum_m (H_{at} + \Delta V - E) e^{ikma} (C_1 \varphi_1(r-ma) + C_2 \varphi_2(r-ma-b)) = 0$$

$$\Rightarrow \text{左} \left[\varphi_1(r) [\varepsilon_0 - E(k)] c_1 + \sum e^{ikma} c_1 \langle \varphi_1(r) | \Delta V | \varphi_1(r-ma) \rangle + \sum e^{-ikma} c_2 \langle \varphi_1(r) | \Delta V | \varphi_2(r-ma-b) \rangle \right] = 0$$

只考虑最近邻.



$$\Rightarrow (\varepsilon_0 - E(k)) c_1 + c_1 \underbrace{\langle \varphi_1(r) | \Delta V | \varphi_1(r) \rangle}_{-\beta}$$

$$+ c_2 \underbrace{\langle \varphi_1(r) | \Delta V | \varphi_2(r-b) \rangle}_{-J_1} - J_1$$

$$+ e^{-ika} c_2 \underbrace{\langle \varphi_1(r) | \Delta V | \varphi_2(r-b+a) \rangle}_{-J_2} - J_2 = 0$$

$$(\varepsilon_0 - E(k)) c_2 + c_1 \underbrace{\langle \varphi_2(r-b) | \Delta V | \varphi_1(r) \rangle}_{-J_1^*}$$

$$+ c_2 \underbrace{\langle \varphi_2(r-b) | \Delta V | \varphi_2(r-b) \rangle}_{-\beta} - \beta$$

$$+ c_1 e^{ika} \underbrace{\langle \varphi_2(r-b) | \Delta V | \varphi_1(r-a) \rangle}_{-J_2^*} - J_2^* = 0$$

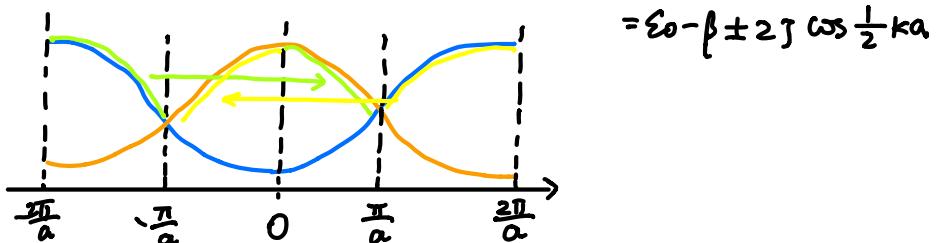
$$\Rightarrow \begin{pmatrix} \varepsilon_0 - E(k) - \beta & -J_1 - J_2 e^{-ika} \\ -J_1^* - J_2^* e^{ika} & \varepsilon_0 - E(k) - \beta \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = 0$$

$$\Rightarrow (\varepsilon_0 - E(k) - \beta)^2 = |J_1 + J_2 e^{ika}|^2$$

$$\Rightarrow E(k) = \varepsilon_0 - \beta \pm |J_1 + J_2 e^{ika}|$$

$$a \rightarrow \infty \text{ 时 } J_2 \rightarrow 0 \quad E(k) = \varepsilon_0 - \beta \pm J_1$$

$$b \rightarrow \frac{a}{2} \text{ 时 } J_1 = J_2 \quad E(k) = \varepsilon_0 - \beta \pm J \sqrt{2 + 2 \cos ka}$$



在 k 空间是不随 k 变而产生的
变化周期只是态的半移.

3.(Ashcroft&Mermin 10.2) 在处理立方晶体时，最方便的线性组合是使用三个简并原子**p能级**，分别具有 $x\phi(r)$, $y\phi(r)$ 和 $z\phi(r)$ 波函数形式，其中函数 ϕ 只取决于 **r** 矢量的大小。通过将(10.12)行列式设置为零，可以求解出三个相应**p能带**的能量

$$|(\varepsilon(\mathbf{k}) - E_p)\delta_{ij} + \beta_{ij} + \widetilde{\gamma_{ij}}(\mathbf{k})| = 0 \quad (10.30)$$

其中 : $\widetilde{\gamma_{ij}}(\mathbf{k}) = \sum_R e^{i\mathbf{k}\cdot\mathbf{R}} \gamma_{ij}(\mathbf{R})$

$$\gamma_{ij}(\mathbf{R}) = - \int d\mathbf{r} \psi_i^*(\mathbf{r}) \varphi_j(\mathbf{r} - \mathbf{R}) \Delta U(\mathbf{r})$$

$$\beta_{ij} = \gamma_{ij}(\mathbf{R} = 0)$$

(10.30)中省略了一个乘 $\varepsilon(\mathbf{k}) - E_p$ 的项，它产生的非常小的修正，类似于s能带情况下的(10.15)分母给出的修正。

$$\begin{aligned}
(\varepsilon(\mathbf{k}) - E_m)b_m &= -(\varepsilon(\mathbf{k}) - E_m) \sum_n \left(\sum_{R \neq 0} \int \psi_m^*(\mathbf{r}) \psi_n(\mathbf{r} - \mathbf{R}) e^{i\mathbf{k}\cdot\mathbf{R}} dr \right) b_n \\
&\quad + \sum_n \left(\int \psi_m^*(\mathbf{r}) \Delta U(\mathbf{r}) \psi_n(\mathbf{r}) d\mathbf{r} \right) b_n \\
&\quad + \sum_n \left(\sum_{R \neq 0} \int \psi_m^*(\mathbf{r}) \Delta U(\mathbf{r}) \psi_n(\mathbf{r} - \mathbf{R}) e^{i\mathbf{k}\cdot\mathbf{R}} d\mathbf{r} \right) b_n \quad (10.12)
\end{aligned}$$

$$\varepsilon(\mathbf{k}) = E_s - \frac{\beta + \sum \gamma(\mathbf{R}) e^{i\mathbf{k}\cdot\mathbf{R}}}{1 + \sum \alpha(\mathbf{R}) e^{i\mathbf{k}\cdot\mathbf{R}}} \quad (10.15)$$

- (a) 由于立方对称, 证明 $\beta_{xx} = \beta_{yy} = \beta_{zz} = \beta$; $\beta_{xy} = 0$
- (b) 假设 $\gamma_{ij}(\mathbf{R})$ 只考虑最近邻 \mathbf{R} , 证明对于简立方布拉维晶格来说, $\tilde{\gamma}_{ij}(\mathbf{k})$ 是对角的, $x\phi(r), y\phi(r)$ 和 $z\phi(r)$ 每个都独立产生能带。(如果 $\gamma_{ij}(\mathbf{R})$ 考虑到次近邻 \mathbf{R} , 则这种情况不再适用)

(c) 对于面心立方，只考虑最近邻 γ_{ij} ，证明能带由下列方程的解给出：

$$0 = \begin{vmatrix} \varepsilon(\mathbf{k}) - \varepsilon^0(\mathbf{k}) + & & \\ 4\gamma_0 \cos \frac{1}{2}k_y a \cos \frac{1}{2}k_z a & -4\gamma_1 \sin \frac{1}{2}k_x a \sin \frac{1}{2}k_y a & -4\gamma_1 \sin \frac{1}{2}k_x a \sin \frac{1}{2}k_z a \\ & & \\ -4\gamma_1 \sin \frac{1}{2}k_y a \sin \frac{1}{2}k_x a & \varepsilon(\mathbf{k}) - \varepsilon^0(\mathbf{k}) + & -4\gamma_1 \sin \frac{1}{2}k_y a \sin \frac{1}{2}k_z a \\ & 4\gamma_0 \cos \frac{1}{2}k_z a \cos \frac{1}{2}k_x a & & \\ & & & \\ -4\gamma_1 \sin \frac{1}{2}k_z a \sin \frac{1}{2}k_x a & -4\gamma_1 \sin \frac{1}{2}k_z a \sin \frac{1}{2}k_y a & \varepsilon(\mathbf{k}) - \varepsilon^0(\mathbf{k}) + \\ & & 4\gamma_0 \cos \frac{1}{2}k_x a \cos \frac{1}{2}k_y a & \end{vmatrix}$$

其中：

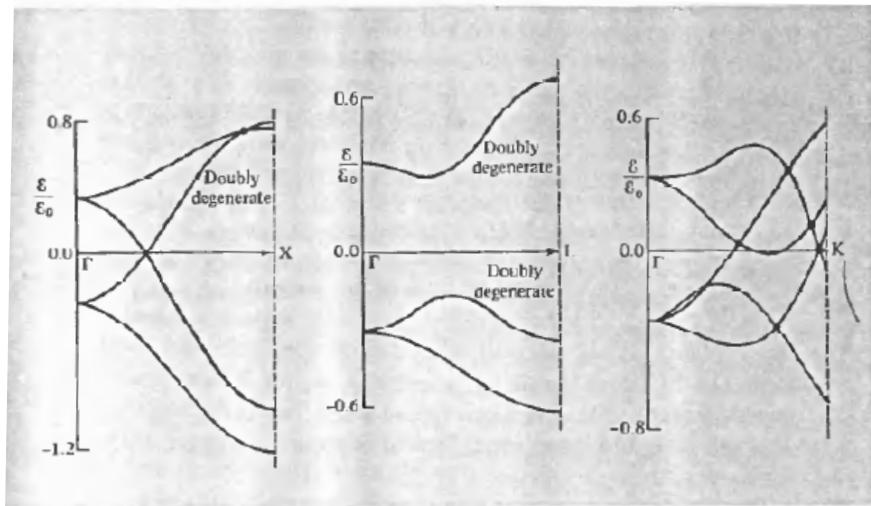
$$\varepsilon^0(\mathbf{k}) = E_P - \beta - 4\gamma_2 (\cos \frac{1}{2}k_x a \cos \frac{1}{2}k_z a + \cos \frac{1}{2}k_x a \cos \frac{1}{2}k_y a + \cos \frac{1}{2}k_y a \cos \frac{1}{2}k_z a)$$

$$\gamma_0 = - \int d\mathbf{r} [x^2 - y(y - 1/2a)] \phi(r) \phi([x^2 + (y - 1/2a)^2 + (z - 1/2a)^2]^{1/2}) \Delta U(\mathbf{r})$$

$$\gamma_1 = - \int d\mathbf{r} x(y - 1/2a) \phi(r) \phi([(x - 1/2a)^2 + (y - 1/2a)^2 + z^2]^{1/2}) \Delta U(\mathbf{r})$$

$$\gamma_2 = - \int d\mathbf{r} x(x - 1/2a) \phi(r) \phi([(x - 1/2a)^2 + (y - 1/2a)^2 + z^2]^{1/2}) \Delta U(\mathbf{r})$$

(d) 证明三个能带在 $\mathbf{k} = \mathbf{0}$ 时简并，并且当 \mathbf{k} 沿着立方轴 (ΓX) 或 立方体对角线 (ΓL) 方向时，存在双重简并。沿着上述方向画能带图 (类似于图 10.6)



4月8日交

上讲回顾

Bloch电子半经典动力学:

- 电子动力学的半经典模型
 满带不导电, 空穴, 有效质量
- 恒定电场、磁场下电子的运动 (回旋运动)
- 费米面与能带的实验测量

朗道能级, De Haas-Van Alphen振荡

$$\dot{\mathbf{r}} = \mathbf{v}_n(\mathbf{k}) = \frac{1}{\hbar} \nabla_{\mathbf{k}} \epsilon_n(\mathbf{k})$$

$$\hbar \dot{\mathbf{k}} = -e[\mathbf{E}(\mathbf{r}, t) + \mathbf{v}_n(\mathbf{k}) \times \mathbf{B}(\mathbf{r}, t)]$$

本讲主要内容

输运现象：

- 非平衡分布函数与玻尔兹曼方程
弛豫时间近似，仍然是Drude公式？如何
超越弛豫时间近似？
- 热输运与热电耦合

第五章 输运现象

非平衡分布函数 $f_n(\mathbf{r}, \mathbf{k}, t)$

定义： $f_n(\mathbf{r}, \mathbf{k}, t) \frac{d^3 r d^3 k}{4\pi^3}$ 表示在 t 时刻、相空间区域 $d^3 r d^3 k$ 中、
第 n 条能带上的电子数

平衡情况下 $f_n(\mathbf{r}, \mathbf{k}, t) \rightarrow f^{(0)}[\mathbf{r}, \varepsilon_n(\mathbf{k})] = \boxed{\frac{1}{e^{[\varepsilon_n(\mathbf{k}) - \mu(\mathbf{r})]/k_B T(\mathbf{r})} + 1}}$

- 与时间无关
- 化学势和温度处于局域平衡态 可以有空间的不均匀
- 动量和能带依赖关系仅通过能带上相应电子态的能量
- Fermi-Dirac 分布 (满足 Pauli 不相容原理) 不考虑带间跃迁

第五章 输运现象

非平衡分布函数随时间演化——Boltzmann方程

$$\frac{df_n}{dt} = \frac{\partial f_n}{\partial t} + \dot{\mathbf{r}} \cdot \nabla_{\mathbf{r}} f_n + \underline{\dot{\mathbf{k}} \cdot \nabla_{\mathbf{k}} f_n} + \left(\frac{\partial f_n}{\partial t} \right)_{\text{collisions}}$$

漂移项 (drifting terms) 包含粒子运动以及外力的影响

遵从半经典电子动力学方程

碰撞项 (collision terms) 描述所有其它过程

在没有碰撞的理想系统 (一切皆可预测)

$$f_n(\mathbf{r}, \mathbf{k}, t) = f_n(\mathbf{r} - \dot{\mathbf{r}} dt, \mathbf{k} - \dot{\mathbf{k}} dt, t - dt)$$

否则有

$$\begin{aligned} f_n(\mathbf{r}, \mathbf{k}, t) &= \underline{f_n(\mathbf{r} - \dot{\mathbf{r}} dt, \mathbf{k} - \dot{\mathbf{k}} dt, t - dt)} + \left(\frac{\partial f_n}{\partial t} \right)_{\text{collisions}} dt \\ &= f_n(\vec{\mathbf{r}}, \vec{\mathbf{k}}, \vec{t}) + (\vec{\mathbf{r}} \cdot \nabla_{\mathbf{r}} f_n + \vec{\mathbf{k}} \cdot \nabla_{\mathbf{k}} f_n + \frac{\partial f_n}{\partial t}) dt \end{aligned}$$

第五章 输运现象

弛豫时间近似

$$\left(\frac{\partial f_n}{\partial t} \right)_{\text{collisions}} = -\frac{1}{\tau} [f_n(\mathbf{r}, \mathbf{k}, t) - f_n^{(0)}(\mathbf{r}, \mathbf{k})]$$

不处于热平衡态
的电子会被带走

$$f_n^{(0)}(\mathbf{r}, \mathbf{k}) = f^{(0)}[\mathbf{r}, \varepsilon_n(\mathbf{k})] = \frac{1}{e^{[\varepsilon_n(\mathbf{k}) - \mu(\mathbf{r})]/k_B T(\mathbf{r})} + 1}$$

- 弛豫时间近似是最简单近似且满足碰撞把系统带向（局部）平衡态但不会让系统从平衡态偏离
- 弛豫时间可以依赖位置 \mathbf{r} , 动量 \mathbf{k} , 通常 $\tau = \tau[\varepsilon_n(\mathbf{k})]$ 。

碰撞不能把处于平衡态
的系统带偏离平衡态

除非系统
非常不均匀

对于不同的碰撞，弛
豫时间很可能是不一样的

本讲小结及下讲内容

本讲要点：

分布函数与Boltzmann方程，物理图像与形式解

下讲内容：

电输运、热输运现象描述，热电耦合。教材6.1-6.3。

本讲主要内容

输运现象：

- 非平衡分布函数与玻尔兹曼方程

弛豫时间近似，仍然是Drude公式？如何
超越弛豫时间近似？

- 热输运与热电耦合
- 霍尔系数与磁电阻

第五章 输运现象

形式解，使用 $\tau = \tau[\varepsilon_n(\mathbf{k})]$

$$\frac{df_n}{dt} = \left(\frac{\partial f_n}{\partial t} \right)_{\text{collisions}} = -\frac{1}{\tau[\varepsilon_n(\mathbf{k})]} [f_n - f_n^{(0)}]$$

$$f_n[\mathbf{r}(t), \mathbf{k}(t), t] = \int_{-\infty}^t dt' \frac{f_n^{(0)}[\mathbf{r}(t'), \mathbf{k}(t')]}{\tau\{\varepsilon_n[\mathbf{k}(t')]\}} e^{-\int_{t'}^t \frac{dt''}{\tau\{\varepsilon_n[\mathbf{k}(t'')]\}}}$$

特别远的过去时间在影响很小。

分部积分

$$f_n[\mathbf{r}(t), \mathbf{k}(t), t] = f_n^{(0)}[\mathbf{r}(t), \varepsilon_n[\mathbf{k}(t)]] - \int_{-\infty}^t dt' e^{-\int_{t'}^t \frac{dt''}{\tau\{\varepsilon_n[\mathbf{k}(t'')]\}}} \frac{df_n^{(0)}}{dt'}$$

$$f^{(0)}[\mathbf{r}, \varepsilon_n(\mathbf{k})] = \frac{1}{e^{[\varepsilon_n(\mathbf{k}) - \mu(\mathbf{r})]/k_B T(\mathbf{r})} + 1}$$

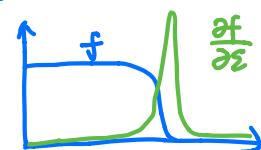
第五章 输运现象

形式解，使用 $\tau = \tau[\varepsilon_n(\mathbf{k})]$

$$\frac{df_n}{dt} = \left(\frac{\partial f_n}{\partial t} \right)_{\text{collisions}} = -\frac{1}{\tau[\varepsilon_n(\mathbf{k})]} [f_n - f_n^{(0)}]$$

$$f_n(\mathbf{k}, t) = f_n^{(0)} - \int_{-\infty}^t dt' e^{-\int_{t'}^t \frac{dt''}{\tau[\varepsilon_n(\mathbf{k}(t''))]} \times \left(-\frac{\partial f_n^{(0)}}{\partial \varepsilon} \right)}$$

所有核心的变化
都在费米面附近
且发生
类似于δ函数


$$\times \left\{ \dot{\mathbf{v}_n} \cdot \left[\nabla \mu(\mathbf{r}) + \frac{\varepsilon_n - \mu(\mathbf{r})}{T(\mathbf{r})} \nabla T(\mathbf{r}) \right] + \mathbf{v}_n \cdot [e\mathbf{E} + e\mathbf{v}_n(\mathbf{k}) \times \mathbf{B}] \right\}$$

$\frac{\partial f}{\partial \varepsilon}$

$$e^{-\int_{t'}^t \frac{dt''}{\tau[\varepsilon_n(\mathbf{k}(t''))]}} \approx e^{-\frac{t-t'}{\tau[\varepsilon_n(\mathbf{k})]}}$$

弛豫时间为常数 $e^{-\frac{t-t'}{\tau}}$

第五章 输运现象

能量缓慢变化, $\tau\{\varepsilon_n[\mathbf{k}(t)]\}$ 也变化缓慢, 近似为常数

所有的变化都在表面附近

$$1\text{eV} \sim 10000 \text{ K}$$

$$300 \text{ K} \sim 26 \text{ meV}$$

$$\int_{-\infty}^t dt' e^{-\int_{t'}^t dt''/\tau} = \tau$$

常数弛豫时间近似下的分布函数演化方程

$$f_n(\mathbf{k}, t) = f_n^{(0)} - \tau \left(-\frac{\partial f_n^{(0)}}{\partial \varepsilon} \right)$$

$$\times \mathbf{v}_n \cdot \left[\nabla \mu(\mathbf{r}) + \frac{\varepsilon_n - \mu(\mathbf{r})}{T(\mathbf{r})} \nabla T(\mathbf{r}) + \textcircled{eE} + e\mathbf{v}_n(\mathbf{k}) \times \mathbf{B} \right]$$

第五章 输运现象

直流电导率

$$f_n(\mathbf{k}, t) = f_n^{(0)}(\mathbf{k}) - \tau \left(-\frac{\partial f_n^{(0)}}{\partial \varepsilon} \right) \mathbf{v}_n \cdot (\mathbf{eE})$$

$-nev$

$$J = -e \sum_n \int \frac{d^3 k}{4\pi^3} \mathbf{v}_n f_n(\mathbf{k})$$

$$= -e \sum_n \int \frac{d^3 k}{4\pi^3} \mathbf{v}_n f_n^{(0)}(\mathbf{k}) + e^2 \tau \sum_n \int \frac{d^3 k}{4\pi^3} \left(-\frac{\partial f_n^{(0)}}{\partial \varepsilon} \right) \mathbf{v}_n \mathbf{v}_n \cdot \mathbf{E}$$

$$\equiv \sum_n \sigma_n \cdot \mathbf{E} \quad \text{不是 Hall 效应} \quad \text{晶体场(能带)带来的.}$$

$$\sigma_n^{\alpha\beta} = \cancel{e^2 \tau} \int \frac{d^3 k}{4\pi^3} \left(-\frac{\partial f_n^{(0)}}{\partial \varepsilon} \right) v_n^\alpha v_n^\beta$$

Drude $\frac{ne^2 \tau}{m}$

第五章 输运现象

直流电导率

$$\sigma_n^{\alpha\beta} = e^2 \tau \int \frac{d^3 k}{4\pi^3} \left(-\frac{\partial f_n^{(0)}}{\partial \varepsilon} \right) v_n^\alpha v_n^\beta$$

$$\begin{aligned}\sigma^{\alpha\beta} &= e^2 \tau \int \frac{d^3 k}{4\pi^3} \left(-\frac{\partial f_n^{(0)}}{\partial \varepsilon} \right) v^\alpha \frac{1}{\hbar} \frac{\partial \varepsilon}{\partial k_\beta} \\ &= \frac{e^2 \tau}{\hbar} \int \frac{d^3 k}{4\pi^3} \left(-\frac{\partial f_n^{(0)}}{\partial k_\beta} \right) v^\alpha \\ &= \frac{e^2 \tau}{\hbar} \int \frac{d^3 k}{4\pi^3} \left[-\frac{\partial}{\partial k_\beta} \left(f_n^{(0)} v^\alpha \right) \right] + \left\{ f_n^{(0)} \frac{\partial v^\alpha}{\partial k_\beta} \right\}\end{aligned}$$

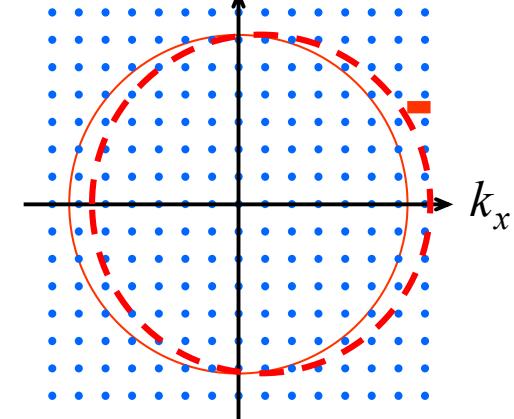
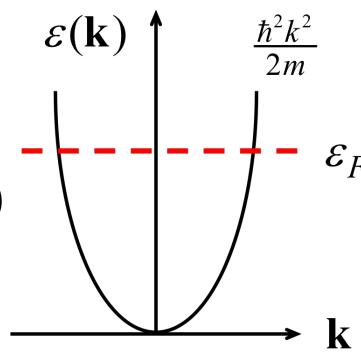
(1) 与Drude模型的对比

$$\sigma_n^{\alpha\beta} = e^2 \tau \int \frac{d^3 k}{4\pi^3} f_n^{(0)} \frac{1}{\hbar^2} \frac{\partial^2 \varepsilon_n(\mathbf{k})}{\partial k_\alpha \partial k_\beta}$$

$$\sigma^{\text{Drude}} = \frac{ne^2 \tau}{m}$$

(2) Fermi面的贡献

$$-\frac{\partial f_n^{(0)}}{\partial \varepsilon} = \delta (\varepsilon_n - \mu)$$



$$\vec{J} = e^2 \int \frac{d^3 K}{4\pi^3} \vec{v}_n \cdot e\vec{E} \cdot \underline{\underline{\vec{v}_n \tau}} \left(-\frac{\partial f^*}{\partial \varepsilon} \right)$$

mean free path

$$= e^2 \int \frac{d^3 K}{4\pi^3} \vec{v}_n \cdot \underline{e\vec{E}\tau} \cdot \vec{v}_n \left(-\frac{\partial f^*}{\partial \varepsilon} \right)$$

两次碰撞间能跑的方程

$\tau \uparrow, \Delta E \uparrow$, 越偏离平衡态

$$= e^2 \int \frac{d^3 K}{4\pi^3} \vec{v}_n \cdot \underline{\underline{e\vec{E}\tau}} \cdot \vec{v}_n \left(-\frac{\partial f^*}{\partial \varepsilon} \right)$$

$$= e^2 \int \frac{d^3 K}{4\pi^3} \vec{v}_n \cdot \underline{\underline{(-df)}} \quad \text{对平衡的偏移} \cdot \text{速度} = \text{电流}$$

第五章 输运现象

直流电导率

$$\sigma_n^{\alpha\beta} = e^2 \tau \int \frac{d^3 k}{4\pi^3} \left(-\frac{\partial f_n^{(0)}}{\partial \varepsilon} \right) v_n^\alpha v_n^\beta$$

(3) 张量，对称性（以立方晶体为例）

- 三个轴对称，因此 $\sigma_{xx} = \sigma_{yy} = \sigma_{zz}$
- 非对角元为0

第五章 输运现象

超越弛豫时间近似

考虑更加具体的散射（碰撞）几率

前提条件：带内散射，自旋守恒，散射在相空间局域

散射几率定义：在 dt 时间内，一个电子从 \mathbf{k} 态散射到 \mathbf{k}' 附近 d^3k 范围内（假设这些态全部未占据）的几率为

$$\frac{w_{\mathbf{kk}'} dt d^3 k}{(2\pi)^3}$$

$$\frac{1}{\tau(\mathbf{k})} = \int \frac{d^3 k'}{(2\pi)^3} w_{\mathbf{kk}'} [1 - f(\mathbf{k}')]$$

弛豫时间依赖于分布函数（与弛豫时间近似的差别！）

第五章 输运现象

超越弛豫时间近似——散射导致分布函数的改变
一个电子被散射的几率

$$\frac{1}{\tau(\mathbf{k})} = \int \frac{d^3 k'}{(2\pi)^3} w_{\mathbf{k}\mathbf{k}'} [1 - f(\mathbf{k}')]$$

在 dt 时间内从 \mathbf{k} 附近 $d^3 k$ 散射出去的电子数 $- \left(\frac{df(\mathbf{k})}{dt} \right)_{\text{out}} \frac{d^3 k dt}{(2\pi)^3}$

$$\left(\frac{df(\mathbf{k})}{dt} \right)_{\text{out}} = - \frac{f(\mathbf{k})}{\tau(\mathbf{k})} = -f(\mathbf{k}) \int \frac{d^3 k'}{(2\pi)^3} w_{\mathbf{k}\mathbf{k}'} [1 - f(\mathbf{k}')]$$

在 dt 时间内散射进入 \mathbf{k} 附近 $d^3 k$ 的电子数 $\left(\frac{df(\mathbf{k})}{dt} \right)_{\text{in}} \frac{d^3 k dt}{(2\pi)^3}$

$$\left(\frac{df(\mathbf{k})}{dt} \right)_{\text{in}} = [1 - f(\mathbf{k})] \int \frac{d^3 k'}{(2\pi)^3} w_{\mathbf{k}'\mathbf{k}} f(\mathbf{k}')$$

详细讨论参见Ashcroft&Mermin P.316-319

第五章 输运现象

超越弛豫时间近似——散射导致分布函数的改变

$$\left(\frac{df(\mathbf{k})}{dt} \right)_{\text{collision}} = - \int \frac{d^3 k'}{(2\pi)^3} \{ w_{\mathbf{k}\mathbf{k}'} f(\mathbf{k}) [1 - f(\mathbf{k}')] - w_{\mathbf{k}'\mathbf{k}} f(\mathbf{k}') [1 - f(\mathbf{k})] \}$$

弛豫时间近似 $\left(\frac{df(\mathbf{k})}{dt} \right)_{\text{collision}} = - \frac{f(\mathbf{k}) - f^{(0)}(\mathbf{k})}{\tau(\mathbf{k})}$

	RELAXATION-TIME APPROXIMATION	GENERAL
$\left(\frac{dg(\mathbf{k})}{dt} \right)_{\text{coll}}^{\text{out}}$	$-\frac{g(\mathbf{k})}{\tau(\mathbf{k})}$	$- \int \frac{d\mathbf{k}'}{(2\pi)^3} W_{\mathbf{k}\mathbf{k}'} [1 - g(\mathbf{k}')] g(\mathbf{k})$
$\left(\frac{dg(\mathbf{k})}{dt} \right)_{\text{coll}}^{\text{in}}$	$\frac{g^0(\mathbf{k})}{\tau(\mathbf{k})}$	$\int \frac{d\mathbf{k}'}{(2\pi)^3} W_{\mathbf{k}'\mathbf{k}} g(\mathbf{k}') [1 - g(\mathbf{k})]$

散射（跃迁）几率 $w_{\mathbf{k}\mathbf{k}'} = \frac{2\pi}{\hbar} |\langle \mathbf{k} | U | \mathbf{k}' \rangle|^2 \delta[\varepsilon(\mathbf{k}) - \varepsilon(\mathbf{k}')]$

Fermi 黄金定则

弹性过程

本讲小结及下讲内容

本讲要点：

分布函数与Boltzmann方程，物理图像与形式解，弛豫时间近似
的相关讨论，直流电导率的计算，超越弛豫时间近似

下讲内容：

热输运现象描述，热电耦合，霍尔系数与磁阻。教材6.3-6.4。

上讲回顾

输运现象：

- 非平衡分布函数与玻尔兹曼方程
弛豫时间近似，仍然是Drude公式？如何超越弛豫时间近似？
- 热输运与热电耦合
- 霍尔系数与磁电阻

本讲主要内容

输运现象：

- 非平衡分布函数与玻尔兹曼方程
弛豫时间近似，仍然是Drude公式？如何超越弛豫时间近似？
- 热输运与热电耦合
- 霍尔系数与磁电阻

第五章 输运现象

常数弛豫时间近似下的分布函数演化方程

$$f_n(\mathbf{k}, t) = f_n^{(0)} - \tau \left(-\frac{\partial f_n^{(0)}}{\partial \varepsilon} \right) \times \mathbf{v}_n \cdot \left[\underbrace{\nabla \mu(\mathbf{r}) + \frac{\varepsilon_n - \mu(\mathbf{r})}{T(\mathbf{r})} \nabla T(\mathbf{r})}_{\text{某种势能的驱动力}} + e\mathbf{E} + e\mathbf{v}_n(\mathbf{k}) \times \mathbf{B} \right]$$

直流电导率

某种势能的驱动力 电化势(?)

$$\mathbf{J} = -e \sum_n \int \frac{d^3 k}{4\pi^3} \mathbf{v}_n f_n(\mathbf{k}) \equiv \sum_n \boldsymbol{\sigma}_n \cdot \mathbf{E}$$

$$\sigma_n^{\alpha\beta} = e^2 \tau \int \frac{d^3 k}{4\pi^3} \left(-\frac{\partial f_n^{(0)}}{\partial \varepsilon} \right) v_n^\alpha v_n^\beta$$

第五章 输运现象

热输运过程

$$dU = TdS - pdV + \mu dN$$

热流的定义 : $dQ = TdS = dU - \underline{\mu dN}$

$$J^q = J^u - \mu J^n$$

类比电流的公式 $J = \sum_n \int \frac{d^3 k}{4\pi^3} (-e) v_n f_n(\mathbf{k})$

能量流 $J^u = \sum_n \int \frac{d^3 k}{4\pi^3} \varepsilon_n(\mathbf{k}) v_n f_n(\mathbf{k})$

粒子流 $J^n = \sum_n \int \frac{d^3 k}{4\pi^3} \mathbf{1} v_n f_n(\mathbf{k})$

热流 $J^q = \sum_n \int \frac{d^3 k}{4\pi^3} [\varepsilon_n(\mathbf{k}) - \mu] v_n f_n(\mathbf{k})$

第五章 输运现象

$$f(\mathbf{k}) = f^{(0)}(\mathbf{k}) - \tau \left(-\frac{\partial f^{(0)}}{\partial \varepsilon} \right) \mathbf{v}(\mathbf{k}) \cdot \left[e\mathbf{E} + \nabla\mu(\mathbf{r}) + \frac{\varepsilon(\mathbf{k}) - \mu(\mathbf{r})}{T(\mathbf{r})} \nabla T(\mathbf{r}) \right]$$

$$f(\mathbf{k}) = f^{(0)}(\mathbf{k}) + \tau \left(-\frac{\partial f^{(0)}}{\partial \varepsilon} \right) \mathbf{v}(\mathbf{k}) \cdot \left\{ -e\mathcal{E} + \frac{\varepsilon(\mathbf{k}) - \mu(\mathbf{r})}{T(\mathbf{r})} [-\nabla T(\mathbf{r})] \right\}$$

$$\mathcal{E} \equiv \mathbf{E} + \frac{\nabla\mu(\mathbf{r})}{e}$$

$$J = \cancel{-e} \int \frac{d^3k}{4\pi^3} \tau \left(-\frac{\partial f^{(0)}}{\partial \varepsilon} \right) \mathbf{v}(\mathbf{k}) \mathbf{v}(\mathbf{k}) \cdot \left\{ -e\mathcal{E} + \frac{\varepsilon(\mathbf{k}) - \mu(\mathbf{r})}{T(\mathbf{r})} [-\nabla T(\mathbf{r})] \right\}$$

与 Σ 和 Ω 都是线性关系

$$\mathbf{L}^{11}\mathcal{E} + \mathbf{L}^{12}(-\nabla T)$$

第五章 输运现象

$$f(\mathbf{k}) = f^{(0)}(\mathbf{k}) - \tau \left(-\frac{\partial f^{(0)}}{\partial \varepsilon} \right) \mathbf{v}(\mathbf{k}) \cdot \left[e\mathbf{E} + \nabla\mu(\mathbf{r}) + \frac{\varepsilon(\mathbf{k}) - \mu(\mathbf{r})}{T(\mathbf{r})} \nabla T(\mathbf{r}) \right]$$

$$f(\mathbf{k}) = f^{(0)}(\mathbf{k}) + \tau \left(-\frac{\partial f^{(0)}}{\partial \varepsilon} \right) \mathbf{v}(\mathbf{k}) \cdot \left\{ -e\mathcal{E} + \frac{\varepsilon(\mathbf{k}) - \mu(\mathbf{r})}{T(\mathbf{r})} [-\nabla T(\mathbf{r})] \right\}$$

$$\mathcal{E} \equiv \mathbf{E} + \frac{\nabla\mu(\mathbf{r})}{e}$$

$$J^q = \int \frac{d^3k}{4\pi^3} \tau \left(-\frac{\partial f^{(0)}}{\partial \varepsilon} \right) [\varepsilon(\mathbf{k}) - \mu] \mathbf{v}(\mathbf{k}) \mathbf{v}(\mathbf{k}) \cdot \left\{ -e\mathcal{E} + \frac{\varepsilon(\mathbf{k}) - \mu(\mathbf{r})}{T(\mathbf{r})} [-\nabla T(\mathbf{r})] \right\}$$

$$\text{L}^{21}\mathcal{E} + \text{L}^{22}(-\nabla T)$$

丁^q与ε和μ都是线性关系

第五章 输运现象

$$J = -e \int \frac{d^3k}{4\pi^3} \tau \left(-\frac{\partial f^{(0)}}{\partial \varepsilon} \right) v(k) v(k) \cdot \left\{ -e\varepsilon + \frac{\varepsilon(k) - \mu(r)}{T(r)} [-\nabla T(r)] \right\}$$

$$J^q = \int \frac{d^3k}{4\pi^3} \tau \left(-\frac{\partial f^{(0)}}{\partial \varepsilon} \right) [\varepsilon(k) - \mu] v(k) v(k) \cdot \left\{ -e\varepsilon + \frac{\varepsilon(k) - \mu(r)}{T(r)} [-\nabla T(r)] \right\}$$

非对称元表示魏方

L. Onsager

$$TdS = \sum_i F_i \cdot J_i$$

不满足

$$\begin{pmatrix} J \\ J^q \end{pmatrix} = \begin{pmatrix} L \\ L^q \end{pmatrix} \begin{pmatrix} F_i \\ F_i \end{pmatrix} \Rightarrow L \bar{J} = L \bar{J} \bar{i}$$

$$\mathcal{L}^{(\alpha)} = e^2 \int \frac{d^3k}{4\pi^3} \left(-\frac{\partial f^{(0)}}{\partial \varepsilon} \right) \tau v(k) v(k) (\varepsilon(k) - \mu)^{\alpha}$$

$$\bar{J} = \frac{-e}{T} \int \frac{d^3k}{4\pi^3} \left(-\frac{\partial f^{(0)}}{\partial \varepsilon} \right) \vec{v}(E) \vec{v}(E) [\varepsilon(E) \mu(P)]$$

$$\bar{J} = \frac{1}{T} \int \frac{d^3k}{4\pi^3} \tau \left(-\frac{\partial f^{(0)}}{\partial \varepsilon} \right) (\varepsilon(k) - \mu)^2 \vec{v}(E) \vec{v}(E)$$

$$L^{11} = \mathcal{L}^{(0)} \quad L^{12} = -\frac{1}{eT} \mathcal{L}^{(1)} \quad L^{21} = -\frac{1}{e} \mathcal{L}^{(1)} \quad L^{22} = \frac{1}{e^2 T} \mathcal{L}^{(2)}$$

$$\begin{aligned} \downarrow &= -e \int \frac{d^3k}{4\pi^3} \tau \left(-\frac{\partial f^{(0)}}{\partial \varepsilon} \right) \vec{v}(E) \vec{v}(E) (-e) \\ &= e^2 \int \frac{d^3k}{4\pi^3} \tau \left(-\frac{\partial f^{(0)}}{\partial \varepsilon} \right) \vec{v}(E) \vec{v}(E) \end{aligned}$$

$$\begin{aligned} \downarrow &= e \int \frac{d^3k}{4\pi^3} \tau \left(-\frac{\partial f^{(0)}}{\partial \varepsilon} \right) (\varepsilon(k) - \mu) \vec{v}(E) \vec{v}(E) \\ &= T L^{12} \end{aligned}$$

第五章 输运现象

计算输运系数 $L^{(\alpha)} = e^2 \int \frac{d^3 k}{4\pi^3} \tau \left(-\frac{\partial f^{(0)}}{\partial \varepsilon} \right) v(k) v(k) [\varepsilon(k) - \mu]^\alpha$

$$\sigma(\varepsilon) \equiv e^2 \int \frac{d^3 k}{4\pi^3} \delta[\varepsilon - \varepsilon(k)] \tau v(k) v(k)$$

对等能面积分

$$= \int d\varepsilon \left(-\frac{\partial f}{\partial \varepsilon} \right) \sigma(\varepsilon) (\varepsilon - \mu)^\alpha$$

$$\mathcal{L}^{(\alpha)} = \int d\varepsilon \left(-\frac{\partial f^{(0)}}{\partial \varepsilon} \right) (\varepsilon - \mu)^\alpha \sigma(\varepsilon)$$

$$\mathcal{L}^{(0)} = \sigma(\mu) = \sigma(\varepsilon_F)$$

如何计算 $\mathcal{L}^{(1)}$ 和 $\mathcal{L}^{(2)}$? —— 回到第一章中的 Sommerfeld 展开

$$\mathcal{L}^{(1)} = \frac{\pi^2}{3} (k_B T)^2 \frac{\partial}{\partial \varepsilon} \sigma(\varepsilon) \Big|_{\varepsilon_F}$$

$$\mathcal{L}^{(2)} = \frac{\pi^2}{3} (k_B T)^2 \sigma(\varepsilon_F)$$

第五章 输运现象

热导率定义：在没有电流情况下，热流与温度梯度的比例系数

$$\begin{pmatrix} J \\ J^q \end{pmatrix} = \begin{pmatrix} L^{11} & L^{12} \\ L^{21} & L^{22} \end{pmatrix} \begin{pmatrix} \mathcal{E} \\ -\nabla T \end{pmatrix}$$

$$J = L^{11} \mathcal{E} + L^{12} (-\nabla T) = 0$$

$$\mathcal{E} = -(L^{11})^{-1} L^{12} (-\nabla T)$$

$$J^q = [-L^{21}(L^{11})^{-1} L^{12} + L^{22}](-\nabla T) \equiv \kappa(-\nabla T)$$

$$\kappa = \underbrace{-L^{21}(L^{11})^{-1} L^{12}}_{\approx 0} + L^{22} \approx L^{22} = \frac{1}{e^2 T} \mathcal{L}^{(2)} = \frac{\pi^2}{3e^2} k_B^2 T \sigma(\varepsilon_F)$$

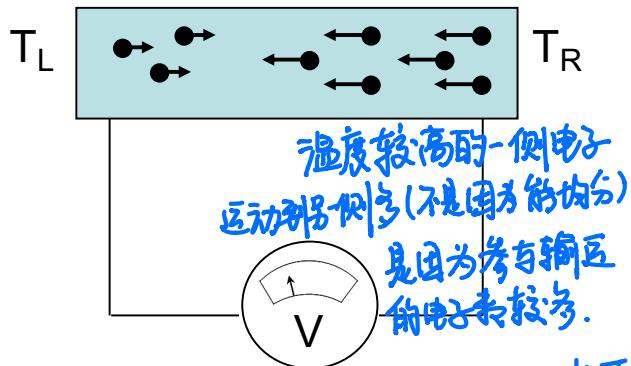
Wiedeman-Franz Law：热导率与电导率的比值除以温度是个常数

成因：电子同时携带热流与电流

$$\frac{\kappa}{\sigma T} = \frac{\pi^2 k_B^2}{3e^2}$$

第五章 输运现象

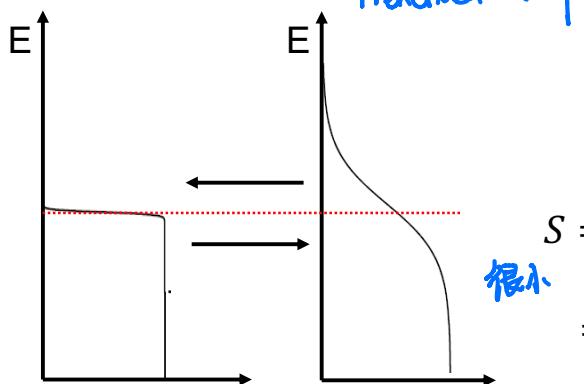
热电动势与Seebeck效应



$$\begin{pmatrix} J \\ J^q \end{pmatrix} = \begin{pmatrix} L^{11} & L^{12} \\ L^{21} & L^{22} \end{pmatrix} \begin{pmatrix} \mathcal{E} \\ -\nabla T \end{pmatrix}$$

$$J = L^{11} \mathcal{E} + L^{12} (-\nabla T) = 0$$

$$\mathcal{E} = E + \frac{\cancel{\nabla u(r)}}{e} = \frac{L^{12}}{L^{11}} \nabla T$$



$$E = \frac{L^{12}}{L^{11}} \nabla T = S \nabla T$$

$$\begin{aligned} S &= \frac{L^{12}}{L^{11}} = -\frac{1}{eT\sigma} \mathcal{L}^{(1)} \\ &= -\frac{\pi^2 k_B^2 T}{3e\sigma} \frac{\partial}{\partial \varepsilon} \sigma(\varepsilon) \Big|_{\varepsilon_F} = -\frac{\pi^2 k_B^2 T}{3e} \frac{\partial \ln \sigma(\varepsilon)}{\partial \varepsilon} \Big|_{\varepsilon_F} \end{aligned}$$

第五章 输运现象

$$S = -\frac{\pi^2 k_B^2 T}{3e} \frac{\partial \ln \sigma(\varepsilon)}{\partial \varepsilon} \Big|_{\varepsilon_F}$$

如果考虑弛豫时间与能量的关系

$$\sigma(\varepsilon) = e^2 \int \frac{d^3 k}{4\pi^3} \delta[\varepsilon - \varepsilon(\mathbf{k})] \tau(\varepsilon) \mathbf{v}(\mathbf{k}) \mathbf{v}(\mathbf{k}) \propto \int_{\varepsilon_F} \tau(\varepsilon) v(\varepsilon) dS$$

$$S = -\frac{\pi^2}{3} \frac{k_B^2 T}{e} \left(\frac{\partial \ln \langle \tau(\varepsilon) \rangle}{\partial \varepsilon} + \frac{\partial \ln \langle v(\varepsilon) \rangle}{\partial \varepsilon} + \frac{\partial \ln A(\varepsilon)}{\partial \varepsilon} \right)_{\varepsilon=\varepsilon_F}$$

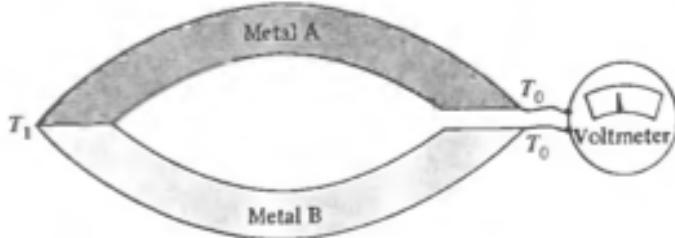
自由电子解 $S = \boxed{-\frac{\pi^2}{3} \frac{k_B^2 T}{e} \left(\frac{3}{2\varepsilon_F} + \frac{\partial \ln \langle \tau(\varepsilon) \rangle}{\partial \varepsilon} \Big|_{\varepsilon=\varepsilon_F} \right)}$

真实材料中还需要考虑费米面的形状、费米速度的分布等因素

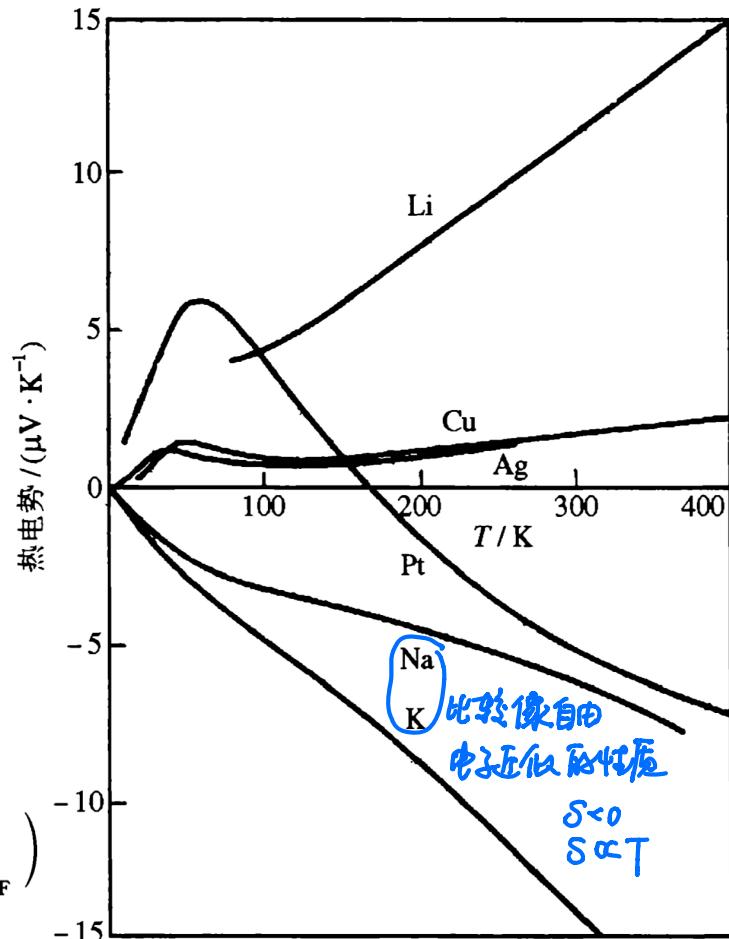
第五章 输运现象

热电动势的实验测量

- 声子曳引 (phonon drag)
- 磁振子曳引
-

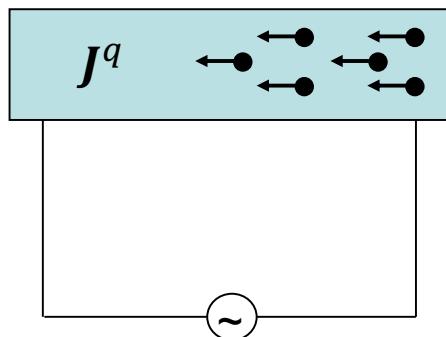


$$S = -\frac{\pi^2}{3} \frac{k_B^2 T}{e} \left(\frac{3}{2\epsilon_F} + \frac{\partial \ln \langle \tau(\epsilon) \rangle}{\partial \epsilon} \Big|_{\epsilon=\epsilon_F} \right)$$



第五章 输运现象

Seebeck效应的逆效应：Peltier效应，电流导致温差



$$J^q = \Pi J$$

$$= L^{21} \mathcal{E} + L^{22} (\cancel{-\nabla T})$$

$$= \frac{L^{21}}{L^{11}} J$$



$$\Pi = TS \quad ? \quad S = \frac{L^{12}}{L^{11}}$$

Kelvin relation

第五章 输运现象

霍耳效应和磁致电阻

$$m \frac{d\vec{v}}{dt} = \vec{F} - \frac{m\vec{v}}{\tau}$$

胡安 4.1

自由电子气漂移理论：

$$m \left(\frac{d}{dt} + \frac{1}{\tau} \right) \vec{v} = -e(\vec{E} + \vec{v} \times \vec{B})$$

设 \vec{B} 沿 \hat{z} 轴方向，电场 \vec{E} 在 $\hat{x} - \hat{y}$ 平面内： $\vec{B} = B\hat{z}$, $\vec{E} = E_x\hat{x} + E_y\hat{y}$

$$m \left(\frac{d}{dt} + \frac{1}{\tau} \right) v_x = -e(E_x + Bv_y)$$

$$v_x = -\frac{e\tau}{m} E_x - \frac{eB}{m} \tau v_y = -\frac{e\tau}{m} E_x - \omega_c \tau v_y$$

$$m \left(\frac{d}{dt} + \frac{1}{\tau} \right) v_y = -e(E_y - Bv_x)$$

稳定情况下

$$v_y = -\frac{e\tau}{m} E_y + \omega_c \tau v_x$$

$$m \left(\frac{d}{dt} + \frac{1}{\tau} \right) v_z = 0$$

$$v_z = 0$$

$$\bar{J} = -ne\vec{v} \quad \sigma_0 = \frac{ne^2\tau}{m}$$

$$v_x = -\frac{e\tau}{m} E_x - \omega_c \tau \left(-\frac{e\tau}{m} E_y + \omega_c \tau v_x \right)$$

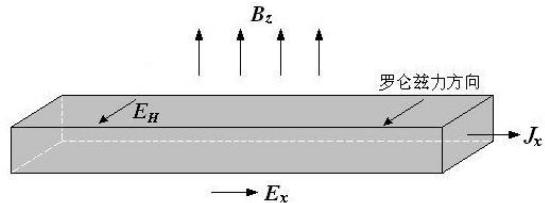
$$v_x = \frac{1}{1 + (\omega_c \tau)^2} \cdot \frac{e\tau}{m} (-E_x + \omega_c \tau E_y)$$

$$\begin{pmatrix} J_x \\ J_y \end{pmatrix} = \frac{\sigma_0}{1 + (\omega_c \tau)^2} \begin{pmatrix} 1 & -\omega_c \tau \\ \omega_c \tau & 1 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \frac{1}{\sigma_0} & \frac{\omega_c \tau}{\sigma_0} \\ -\frac{\omega_c \tau}{\sigma_0} & \frac{1}{\sigma_0} \end{pmatrix} \begin{pmatrix} J_x \\ J_y \end{pmatrix}$$

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \frac{1}{\sigma_0} & \frac{\omega_c \tau}{\sigma_0} \\ -\frac{\omega_c \tau}{\sigma_0} & \frac{1}{\sigma_0} \end{pmatrix} \begin{pmatrix} J_x \\ J_y \end{pmatrix}$$

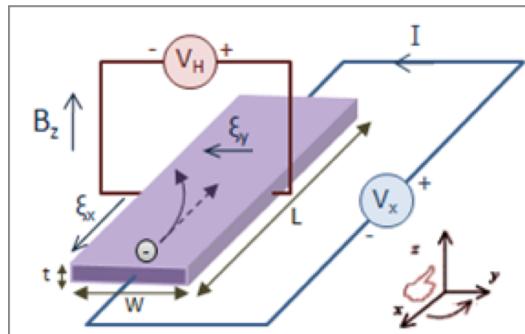
沿 \hat{x} 方向测电流, $J_y = 0$



y 方向电场 : $E_y = -\frac{\omega_c \tau}{\sigma_0} J_x = -\frac{B}{ne} J_x$ 霍尔场 E_H

霍尔角 : θ_H $\tan \theta_H \equiv \frac{E_H}{E_{||}} = \frac{-BJ_x / (ne)}{J_x / \sigma_0} = -\frac{e\tau}{m} B$ $\sigma_0 = \frac{ne^2 \tau}{m}$

霍尔系数 : R_H $R_H \equiv \frac{E_y}{BJ_x} = \frac{-BJ_x / (ne)}{BJ_x} = -\frac{1}{ne}$

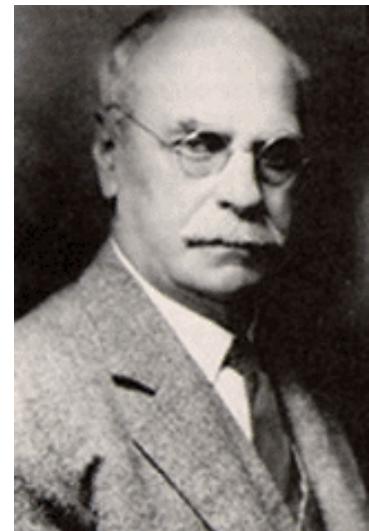


1879: 霍尔效应

1880: 反常霍尔效应

1980: 量子霍尔效应

2013: 量子反常霍尔效应 Edwin Hall



"On a New Action of the Magnet on Electric Currents".
American Journal of Mathematics 2 (3): 287–92 (1879).

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} 1 & \frac{\omega_c \tau}{\sigma_0} \\ -\frac{\omega_c \tau}{\sigma_0} & 1 \end{pmatrix} \begin{pmatrix} J_x \\ J_y \end{pmatrix}$$

x 方向电场 : $E_x = \frac{1}{\sigma_0} J_x = \rho_0 J_x$ 与磁场无关, 无磁致电阻

实验上, 所有金属在磁场下都表现出电阻率的增加。 *Magnetoresistance*

自由电子气漂移理论忽略能带的贡献, 过于简单。

双能带模型下的霍耳效应和磁致电阻

两种类型的载流子

有效质量: m_1, m_2 平均弛豫时间: τ_1, τ_2

载流子浓度: n_1, n_2

回旋共振频率: $\omega_i = \frac{eB}{m_i}$ 零磁场下电导率: $\sigma_i = \frac{ne^2\tau_i}{m_i}$

$$m_1 \left(\frac{d}{dt} + \frac{1}{\tau_1} \right) \vec{v}_1 = -e(\vec{E} + \vec{v}_1 \times \vec{B})$$
$$\begin{pmatrix} J_{1x} \\ J_{1y} \end{pmatrix} = \frac{\sigma_1}{1 + (\omega_1 \tau_1)^2} \begin{pmatrix} 1 & -\omega_1 \tau_1 \\ \omega_1 \tau_1 & 1 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

$$m_2 \left(\frac{d}{dt} + \frac{1}{\tau_2} \right) \vec{v}_2 = -e(\vec{E} + \vec{v}_2 \times \vec{B})$$
$$\begin{pmatrix} J_{2x} \\ J_{2y} \end{pmatrix} = \frac{\sigma_2}{1 + (\omega_2 \tau_2)^2} \begin{pmatrix} 1 & -\omega_2 \tau_2 \\ \omega_2 \tau_2 & 1 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

$$\vec{J} = \vec{J}_1 + \vec{J}_2$$

$$\begin{pmatrix} J_x \\ J_y \end{pmatrix} = \begin{pmatrix} \frac{\sigma_1}{1 + (\omega_1 \tau_1)^2} + \frac{\sigma_2}{1 + (\omega_2 \tau_2)^2}, & -\frac{\sigma_1 \omega_1 \tau_1}{1 + (\omega_1 \tau_1)^2} - \frac{\sigma_2 \omega_2 \tau_2}{1 + (\omega_2 \tau_2)^2} \\ \frac{\sigma_1 \omega_1 \tau_1}{1 + (\omega_1 \tau_1)^2} + \frac{\sigma_2 \omega_2 \tau_2}{1 + (\omega_2 \tau_2)^2}, & \frac{\sigma_1}{1 + (\omega_1 \tau_1)^2} + \frac{\sigma_2}{1 + (\omega_2 \tau_2)^2} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

$$\begin{pmatrix} J_x \\ J_y \end{pmatrix} = \begin{pmatrix} \frac{\sigma_1}{1+(\omega_1\tau_1)^2} + \frac{\sigma_2}{1+(\omega_2\tau_2)^2}, & -\frac{\sigma_1\omega_1\tau_1}{1+(\omega_1\tau_1)^2} - \frac{\sigma_2\omega_2\tau_2}{1+(\omega_2\tau_2)^2} \\ \frac{\sigma_1\omega_1\tau_1}{1+(\omega_1\tau_1)^2} + \frac{\sigma_2\omega_2\tau_2}{1+(\omega_2\tau_2)^2}, & \frac{\sigma_1}{1+(\omega_1\tau_1)^2} + \frac{\sigma_2}{1+(\omega_2\tau_2)^2} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

低磁场情况下： $\omega_1\tau_1 = \frac{eB}{m_1}\tau_1 \ll 1, \omega_2\tau_2 = \frac{eB}{m_2}\tau_2 \ll 1$, 令 $J_y = 0$

$$J_x = (\sigma_1 + \sigma_2)E_x - (\sigma_1\omega_1\tau_1 + \sigma_2\omega_2\tau_2)E_y$$

$$J_y = (\sigma_1\omega_1\tau_1 + \sigma_2\omega_2\tau_2)E_x + (\sigma_1 + \sigma_2)E_y = 0 \quad E_x = -\frac{\sigma_1 + \sigma_2}{\sigma_1\omega_1\tau_1 + \sigma_2\omega_2\tau_2}E_y$$

$$J_x = -\frac{(\sigma_1 + \sigma_2)^2 + (\sigma_1\omega_1\tau_1 + \sigma_2\omega_2\tau_2)^2}{\sigma_1\omega_1\tau_1 + \sigma_2\omega_2\tau_2}E_y$$

$$R_H = \frac{E_y}{BJ_x} = -\frac{\sigma_1\omega_1\tau_1 + \sigma_2\omega_2\tau_2}{B[(\sigma_1 + \sigma_2)^2 + (\sigma_1\omega_1\tau_1 + \sigma_2\omega_2\tau_2)^2]} \approx -\frac{\sigma_1\omega_1\tau_1 + \sigma_2\omega_2\tau_2}{B(\sigma_1 + \sigma_2)^2} = \frac{\sigma_1^2 R_1 + \sigma_2^2 R_2}{(\sigma_1 + \sigma_2)^2}$$

$$R_i = -\frac{\omega_i\tau_i}{B\sigma_i} = -\frac{\frac{eB}{m_i}\tau_i}{B\frac{n_i e^2 \tau_i}{m_i}} = -\frac{1}{n_i e}$$

$$\text{高磁场情况下 : } w_1 t_1 = \frac{eB}{m_1} t_1 \gg 1, w_2 t_2 = \frac{eB}{m_2} t_2 \gg 1,$$

$$\begin{pmatrix} J_x \\ J_y \end{pmatrix} = \begin{pmatrix} \frac{\sigma_1}{1+(\omega_1\tau_1)^2} + \frac{\sigma_2}{1+(\omega_2\tau_2)^2}, & -\frac{\sigma_1\omega_1\tau_1}{1+(\omega_1\tau_1)^2} - \frac{\sigma_2\omega_2\tau_2}{1+(\omega_2\tau_2)^2} \\ \frac{\sigma_1\omega_1\tau_1}{1+(\omega_1\tau_1)^2} + \frac{\sigma_2\omega_2\tau_2}{1+(\omega_2\tau_2)^2}, & \frac{\sigma_1}{1+(\omega_1\tau_1)^2} + \frac{\sigma_2}{1+(\omega_2\tau_2)^2} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

$$J_x = \left[\frac{\sigma_1}{(\omega_1\tau_1)^2} + \frac{\sigma_2}{(\omega_2\tau_2)^2} \right] E_x - \left(\frac{\sigma_1}{\omega_1\tau_1} + \frac{\sigma_2}{\omega_2\tau_2} \right) E_y$$

$$J_y = \left(\frac{\sigma_1}{\omega_1\tau_1} + \frac{\sigma_2}{\omega_2\tau_2} \right) E_x + \left[\frac{\sigma_1}{(\omega_1\tau_1)^2} + \frac{\sigma_2}{(\omega_2\tau_2)^2} \right] E_y = 0$$

$$J_x = \frac{\left[\frac{\sigma_1}{(\omega_1\tau_1)^2} + \frac{\sigma_2}{(\omega_2\tau_2)^2} \right]^2 - \left(\frac{\sigma_1}{\omega_1\tau_1} + \frac{\sigma_2}{\omega_2\tau_2} \right)^2}{\left(\frac{\sigma_1}{\omega_1\tau_1} + \frac{\sigma_2}{\omega_2\tau_2} \right)} E_y \approx - \left(\frac{\sigma_1}{\omega_1\tau_1} + \frac{\sigma_2}{\omega_2\tau_2} \right) E_y$$

$$\sigma_i = \frac{n_i e^2 \tau_i}{m_i}$$

$$R_H = \frac{E_y}{BJ_x} \approx - \frac{1}{\left(\frac{\sigma_1}{\omega_1\tau_1} + \frac{\sigma_2}{\omega_2\tau_2} \right) B} = - \frac{1}{(n_1 + n_2)e} = - \frac{1}{n_{eff} e}$$

有效载流子浓度

$$\omega_i = \frac{eB}{m_i}$$

$$\begin{pmatrix} J_x \\ J_y \end{pmatrix} = \begin{pmatrix} \frac{\sigma_1}{1+(\omega_1\tau_1)^2} + \frac{\sigma_2}{1+(\omega_2\tau_2)^2}, & -\frac{\sigma_1\omega_1\tau_1}{1+(\omega_1\tau_1)^2} - \frac{\sigma_2\omega_2\tau_2}{1+(\omega_2\tau_2)^2} \\ \frac{\sigma_1\omega_1\tau_1}{1+(\omega_1\tau_1)^2} + \frac{\sigma_2\omega_2\tau_2}{1+(\omega_2\tau_2)^2}, & \frac{\sigma_1}{1+(\omega_1\tau_1)^2} + \frac{\sigma_2}{1+(\omega_2\tau_2)^2} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix} \quad \text{令 } J_y = 0$$

$$J_x = \left[\frac{\sigma_1}{1+(\omega_1\tau_1)^2} + \frac{\sigma_2}{1+(\omega_2\tau_2)^2} \right] E_x - \left[\frac{\sigma_1\omega_1\tau_1}{1+(\omega_1\tau_1)^2} + \frac{\sigma_2\omega_2\tau_2}{1+(\omega_2\tau_2)^2} \right] E_y$$

$$J_y = \left[\frac{\sigma_1\omega_1\tau_1}{1+(\omega_1\tau_1)^2} + \frac{\sigma_2\omega_2\tau_2}{1+(\omega_2\tau_2)^2} \right] E_x + \left[\frac{\sigma_1}{1+(\omega_1\tau_1)^2} + \frac{\sigma_2}{1+(\omega_2\tau_2)^2} \right] E_y = 0$$

$$\rho = \frac{E_x}{J_x} = \frac{\frac{\sigma_1}{1+(\omega_1\tau_1)^2} + \frac{\sigma_2}{1+(\omega_2\tau_2)^2}}{\left[\frac{\sigma_1}{1+(\omega_1\tau_1)^2} + \frac{\sigma_2}{1+(\omega_2\tau_2)^2} \right]^2 + \left[\frac{\sigma_1\omega_1\tau_1}{1+(\omega_1\tau_1)^2} + \frac{\sigma_2\omega_2\tau_2}{1+(\omega_2\tau_2)^2} \right]^2}$$

$$= \frac{\frac{\sigma_1}{1+\mu_1^2 B^2} + \frac{\sigma_2}{1+\mu_2^2 B^2}}{\left[\frac{\sigma_1}{1+\mu_1^2 B^2} + \frac{\sigma_2}{1+\mu_2^2 B^2} \right]^2 + \left[\frac{\sigma_1\mu_1 B}{1+\mu_1^2 B^2} + \frac{\sigma_2\mu_2 B}{1+\mu_2^2 B^2} \right]^2}$$

$$\frac{\Delta\rho}{\rho_0} = \frac{\rho - \rho_0}{\rho_0} = \frac{\sigma_1\sigma_2(\mu_1 - \mu_2)^2 B^2}{(\sigma_1 + \sigma_2)^2 + (\mu_1\sigma_2 + \mu_2\sigma_1)^2 B^2}$$

$$\omega_i = \frac{eB}{m_i}$$

$$\mu_1 = -\frac{e\tau_1}{m_1}, \mu_2 = -\frac{e\tau_2}{m_2}$$

迁移率

$$\rho_0 = \frac{1}{\sigma_1 + \sigma_2}$$

$$\frac{\Delta\rho}{\rho_0} = \frac{\rho - \rho_0}{\rho_0} = \frac{\sigma_1\sigma_2(\mu_1 - \mu_2)^2 B^2}{(\sigma_1 + \sigma_2)^2 + (\mu_1\sigma_2 + \mu_2\sigma_1)^2 B^2}$$

低磁场情况下：

$$\frac{\Delta\rho}{\rho_0} \approx \frac{\sigma_1\sigma_2(\mu_1 - \mu_2)^2}{(\sigma_1 + \sigma_2)^2} B^2$$

高磁场情况下：

$$\frac{\Delta\rho}{\rho_0} \approx \frac{\sigma_1\sigma_2(\mu_1 - \mu_2)^2}{(\mu_1\sigma_2 + \mu_2\sigma_1)^2}$$

上述讨论是建立在闭合轨道基础之上的。如果存在一些电子或空穴的开轨道，上述高场磁电阻的结果会产生戏剧性的改变。

$$\begin{pmatrix} J_x \\ J_y \end{pmatrix} = \begin{pmatrix} \frac{\sigma_1}{1+(\omega_1\tau_1)^2} + \frac{\sigma_2}{1+(\omega_2\tau_2)^2}, & -\frac{\sigma_1\omega_1\tau_1}{1+(\omega_1\tau_1)^2} - \frac{\sigma_2\omega_2\tau_2}{1+(\omega_2\tau_2)^2} \\ \frac{\sigma_1\omega_1\tau_1}{1+(\omega_1\tau_1)^2} + \frac{\sigma_2\omega_2\tau_2}{1+(\omega_2\tau_2)^2}, & \frac{\sigma_1}{1+(\omega_1\tau_1)^2} + \frac{\sigma_2}{1+(\omega_2\tau_2)^2} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

高场下：

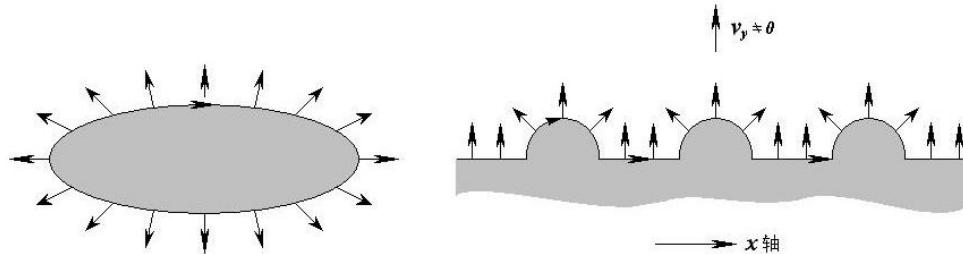
$$\begin{pmatrix} J_x \\ J_y \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \frac{1}{R_H B} \\ -\frac{1}{R_H B} & \sigma_{yy} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

$$R_H = -\frac{1}{(n_1 + n_2)e} = -\frac{1}{n_{eff}e}$$

$$\rho = \frac{E_x}{J_x} = \frac{1}{\sigma_{xx} + 1/(R_H^2 B^2 \sigma_{yy})}$$

$$\rho = \frac{E_x}{J_x} = \frac{1}{\sigma_{xx} + 1/(R_H^2 B^2 \sigma_{yy})}$$

$$\sigma_{xx} = \sigma_{yy} = \frac{\sigma_1}{(\omega_1 \tau_1)^2} + \frac{\sigma_2}{(\omega_2 \tau_2)^2} = \left(\frac{n_1 m_1}{\tau_1} + \frac{n_2 m_2}{\tau_2} \right) / B^2 = \frac{A}{B^2}$$



闭合轨道高场情况下: $\sigma_{xx} = \sigma_{yy} \rightarrow 0$ $\rho \sim AR_H^2$

开轨道高场情况下: $\sigma_{yy} \neq 0$ $\sigma_{yy} \rightarrow S$

$$\rho = \frac{1}{\frac{A}{B^2} + \frac{1}{R_H^2 B^2 S}} \rightarrow \frac{B^2}{A + \frac{1}{R_H^2 S}}$$

高场磁电阻效应的测量对实验上确定金属等能面的形状十分重要!

本讲小结及下讲内容

本讲要点：

热输运的分布函数描述，热电耦合，霍尔系数与磁电阻

两节

下讲内容：

复习，以及习题讨论与答疑。

13.3 13.5 16.2