Sequence-dependent setup time flow shop scheduling problem

The sequence-dependent setup time flow shop scheduling problem (SDST-FSP) extends the classical flow shop scheduling problem by introducing setup operations before processing each job on a machine. The setup time depends on the sequence of jobs, meaning that the time required to prepare a machine for a job varies based on the previously processed job. This variation significantly impacts scheduling decisions.

MIP

Objective:

$$\min \quad C_{max} \tag{MIP-SDST-FSP}$$

Subject to:

$$\sum_{j' \in \{0, \mathcal{J}\} \setminus \{j\}} z_{jj'} = 1, \quad \forall j \in \mathcal{J} \tag{1}$$

$$\sum_{j \in \mathcal{J} \setminus \{j'\}} z_{jj'} \leq 1, \quad \forall j' \in \mathcal{J} \tag{2}$$

$$\sum_{j \in \mathcal{J}} z_{j0} = 1 \tag{3}$$

$$c_{ji} \geq c_{ji-1} + P_{ji}, \quad \forall j \in \mathcal{J}, i \in \mathcal{I} \setminus \{1\} \tag{4}$$

$$c_{ji} \geq c_{j'i} + P_{ji} + S_{ijj'} - M(1 - z_{jj'}), \quad \forall i \in \mathcal{I}, j, j' \in \mathcal{J} : j \neq j' \tag{5}$$

$$C_{max} \geq c_{ji}, \quad \forall j \in \mathcal{J}, i \in \mathcal{I} \tag{6}$$

$$z_{jj'} \in \{0, 1\}, \quad \forall j, j' \in \mathcal{J} : j > j' \tag{7}$$

$$c_{ji} \geq 0, \quad \forall j \in \mathcal{J}, i \in \mathcal{I} \tag{8}$$

We use sequence-based approach to formulate the problem. The binary variable $z_{jj'} = 1$ indicates that job j is processed immediately after job j'. Using such binary variables requires the definition of a dummy job 0 as the first job in the sequence. The number of binary sequencing variables is $|\mathcal{J}|^2$. Constraints (1), (2), and (3) determine the sequence of jobs where each job follows exactly one job by constraint (1) and precedes at most one job by constraint (2). The last job in the sequence does not precede any job. The dummy job is the only job that definitely precedes one job by constraint (3). Constraint (4) ensures that the completion time of job i on machine i is no less than that on machine $i - 1$ plus the processing time. Constraint (5) ensures that the completion time of job j on machine i is no less than that of the incumbent job j', plus the setup time that is performed after job j' for job j on machine i (i.e., $S_{ijj'}$), plus the processing time of job j on machine i (i.e., P_{ji}). Constraints (6) bounds the makespan. Constraints (7) and (8) define the nature of decision variables.

CP Model

Objective:

$$\min \quad C_{max} \tag{CP-SDST-FSP}$$

Subject to:

$$Task_{ji} = \text{IntervalVar}(P_{ji}), \quad \forall j \in \mathcal{J}, i \in \mathcal{I} \tag{1}$$

$$\text{EndBeforeStart}(Task_{ji}, Task_{ji-1}), \quad \forall j \in \mathcal{J}, i \in \mathcal{I} \setminus \{1\} \tag{2}$$

$$SV_i = \text{SequenceVar}(Task_{ji} : j \in \mathcal{J}), \quad \forall i \in \mathcal{I} \tag{3}$$

$$\text{NoOverlap}(SV_i, S_i), \quad \forall i \in \mathcal{I} \tag{4}$$

$$\text{SameSequence}(SV_i, SV_{i-1}), \quad \forall i \in \mathcal{I} \setminus \{1\} \tag{5}$$

$$C_{max} = \max_j(\text{EndOf}(Task_{j|\mathcal{I}|})) \tag{6}$$

Constraint (1) defines one interval variable for each job at each stage. Constraint (2) ensures the stage i of job j is after stage $i-1$. Constraint (3) defines the sequence variable for each machine. Constraint (4) ensures no overlap between operations on each machine. Constraint (5) ensures the execution of jobs on each machine is the same. Constraint (6) calculates the makespan.