Parallel machine scheduling problem

The parallel machine scheduling problem (PMSP) schedules a set of single-operation jobs on multiple machines that are disposed in parallel. Unlike other scheduling models, the PMSP is a pure assignment problem.

MIP

Objective:

$$\min \quad C_{max} \tag{MIP-PMSP}$$

Subject to:

$$\sum_{i \in \mathcal{M}} y_{ji} = 1, \quad \forall j \in \mathcal{J} \tag{1}$$

$$C_{max} \geq \sum_{j \in \mathcal{J}} P_{ji} y_{ji}, \quad \forall i \in \mathcal{M} \tag{2}$$

$$y_{ji} \in \{0, 1\}, \quad \forall j \in \mathcal{J}, i \in \mathcal{M} \tag{3}$$

Constraint (1) ensures each job is assigned to exactly one machine. Constraint (2) calculates the makespan. Constraint (3) defines the nature of the binary variable y_{ji}, which equals 1 if job j is assigned to machine i.

CP Model 1

Objective:

$$\min \quad C_{max} \tag{CP-PMSP 1}$$

Subject to:

$$X_j = \text{IntegerVar}(1, |\mathcal{M}|), \quad \forall j \in \mathcal{J} \tag{1}$$

$$C_{max} = \max_j \left(\sum_{i \in \mathcal{M}} P_{ji} (X_j == i) \right) \tag{2}$$

$$|\mathcal{M}| C_{max} \geq \sum_{j \in \mathcal{J}} \text{Element}([P_{j1}, \cdots, P_{j|\mathcal{M}|}], X_j) \tag{3}$$

Constraint (1) defines an integer variable for each job. Constraint (2) calculates the makespan. Constraint (3) is a lower bound for the makespan. "Element" is a function that returns an element of a given indexed array.

CP Model 2

Objective:

$$\min \quad C_{max} \tag{CP-PMSP 2}$$

Subject to:

$$Task^*_{ji} = \text{IntervalVar}(P_{ji}, \text{Optional}), \quad \forall j \in \mathcal{J}, i \in \mathcal{M} \tag{1}$$

$$\text{Alternative}(Task_j, \{Task^*_{ji} : i \in \mathcal{M}\}), \quad \forall j \in \mathcal{J} \tag{2}$$

$$\text{NoOverlap}(Task^*_{ji} : j \in \mathcal{J}), \quad \forall i \in \mathcal{M} \tag{3}$$

$$C_{max} = \max_j(\text{EndOf}(Task_j)) \tag{4}$$

Constraint (1) defines an operation interval variable for each operation on each machine. Constraint (2) selects one interval variable for each job that determines the assignment. Constraint (3) ensures no overlap of operations on each machine. Constraint (4) calculates the makespan.

CP Model 3

Objective:

$$\min \quad C_{max} \tag{CP-PMSP 3}$$

Subject to:

$$Y_{ji} = \text{NewBoolVar}(), \quad \forall j \in \mathcal{J}, i \in \mathcal{M} \tag{1}$$

$$\text{AddMaxEquality}\left(C_{max}, \left[\sum_{j\in\mathcal{J}} P_{j1}Y_{j1}, \ldots, \sum_{j\in\mathcal{J}} P_{j|\mathcal{M}|}Y_{j|\mathcal{M}|}\right]\right) \tag{2}$$

$$\text{AddExactlyOne}(Y_{j1}, \cdots, Y_{j|\mathcal{M}|}), \quad \forall j \in \mathcal{J} \tag{3}$$

Constraint (1) creates a binary variable for each operation on each machine. Constraint (2) calculates the makespan, which is the maximum of total processing times of machines. Constraint (3) enforces exactly one binary variable for each job takes the value of one.