Distributed flow shop scheduling problem

The distributed flow shop scheduling problem (D-FSP) generalizes the FSP to multiple independent factories, each of which has the same number of machines in series. Each job is to be assigned to one of these factories and processed at different stages in the factory. Once assigned, a job cannot be transferred to another line.

MIP

Objective:

$$\min \quad C_{max} \tag{MIP-DFSP}$$

Subject to:

$$c_{j1} \geq P_{j1}, \quad \forall j \in \mathcal{J} \tag{1}$$

$$c_{ji} \geq c_{ji-1} + P_{ji}, \quad \forall j \in \mathcal{J}, i \in \mathcal{I} \setminus \{1\} \tag{2}$$

$$\sum_{f \in \mathcal{F}} q_{jf} = 1, \quad \forall j \in \mathcal{J} \tag{3}$$

$$c_{ji} \geq c_{j'i} + P_{ji} - M(3 - x_{ijj'} - q_{jf} - q_{j'f}), \quad \forall i \in \mathcal{I}, f \in \mathcal{F}, j, j' \in \mathcal{J} : j > j' \tag{4}$$

$$c_{j'i} \geq c_{ji} + P_{j'i} - M(2 + x_{ijj'} - q_{jf} - q_{j'f}), \quad \forall i \in \mathcal{I}, f \in \mathcal{F}, j, j' \in \mathcal{J} : j > j' \tag{5}$$

$$C_{max} \geq c_{ji}, \quad \forall j \in \mathcal{J}, i \in \mathcal{I} \tag{6}$$

$$c_{ji} \geq 0, \quad \forall j \in \mathcal{J}, i \in \mathcal{I} \tag{7}$$

$$x_{ijj'} \in \{0, 1\}, \quad \forall j, j' \in \mathcal{J} : j > j', i \in \mathcal{I} \tag{8}$$

$$q_{jf} \in \{0, 1\}, \quad \forall j \in \mathcal{J}, f \in \mathcal{F} \tag{9}$$

We use the binary assignment variable $q_{jf} \in \{0, 1\}$ to assign each job j to factory f. Constraint (1) ensures that the completion time of each job j on the first machine is greater than its processing time on that machine. Constraint (2) ensures that the difference between the completion times of job j at stages i and $i - 1$ is at least as large as its processing time on machine i. Constraint (3) ensures that each job is assigned exactly one factory. Constraints (4) and (5) ensure no overlap on each machine. Constraint (6) bounds the makespan. Constraints (7) - (9) define the nature of decision variables.

CP Model

Objective:

$$\min \quad C_{max} \tag{CP-DFSP}$$

Subject to:

$$Task_{ji} = \text{IntervalVar}(P_{ji}), \quad \forall j \in \mathcal{J}, i \in \mathcal{I} \tag{1}$$

$$Task^*_{jif} = \text{IntervalVar}(P_{ji}, \text{Optional}), \quad \forall j \in \mathcal{J}, i \in \mathcal{I}, f \in \mathcal{F} \tag{2}$$

$$\text{PresenceOf}(Task^*_{j1f}) = \text{PresenceOf}(Task^*_{jif}), \quad \forall j \in \mathcal{J}, i \in \mathcal{I} \setminus \{1\}, f \in \mathcal{F} \tag{3}$$

$$\text{Alternative}(Task_{ji}, \{Task^*_{jif} : f \in \mathcal{F}\}), \quad \forall j \in \mathcal{J}, i \in \mathcal{I} \tag{4}$$

$$\text{NoOverlap}(Task^{*}_{jif} : j \in \mathcal{J}), \quad \forall i \in \mathcal{I}, f \in \mathcal{F} \tag{5}$$

$$\text{EndBeforeStart}(Task_{ji-1}, Task_{ji}), \quad \forall j \in \mathcal{J}, i \in \mathcal{I} \setminus \{1\} \tag{6}$$

$$C_{max} = \max_{j}(\text{EndOf}(Task_{j|\mathcal{I}|})) \tag{7}$$

Constraint (1) defines an interval variable for each job at each stage. Constraint (2) defines an interval variable for each operation at each stage. Constraint (3) ensures that if a job is assigned to a factory, all its operations are processed in this factory. Constraint (4) assigns one factory to each job. Constraint (5) ensures that no two operations overlap on each machine. Constraint (6) ensures that stage i of job j is after stage $i-1$. Constraint (7) defines the makespan.