

Distilling Multi-Step Reasoning Capabilities of Large Language Models into Smaller Models via Semantic Decompositions

Kumar Shridhar * Alessandro Stolfo * Mrinmaya Sachan

Department of Computer Science, ETH Zürich

{shkumar, stolfoa}@ethz.ch

Abstract

Step-by-step reasoning approaches like chain-of-thought (CoT) have proved to be a very effective technique to induce reasoning capabilities in large language models. However, the success of the CoT approach depends primarily on model size, and often billion parameter-scale models are needed to get CoT to work. In this paper, we propose a knowledge distillation approach, that leverages the step-by-step CoT reasoning capabilities of larger models and distills these reasoning abilities into smaller models. Our approach DECOMPOSITIONAL DISTILLATION learns a semantic decomposition of the original problem into a sequence of subproblems and uses it to train two models: a) a *problem decomposer* that learns to decompose the complex reasoning problem into a sequence of simpler sub-problems and b) a *problem solver* that uses the intermediate subproblems to solve the overall problem. On a multi-step math word problem dataset (GSM8K), we boost the performance of GPT-2 variants up to 35% when distilled with our approach compared to CoT. We show that using our approach, it is possible to train a GPT-2-large model (775M) that can outperform a 10X larger GPT-3 (6B) model trained using CoT reasoning. Finally, we also demonstrate that our approach of problem decomposition can also be used as an alternative to CoT prompting, which boosts the GPT-3 performance by 40% compared to CoT prompts.

1 Introduction

Large language models (LLMs) have demonstrated strong performance on a wide range of reasoning tasks (Brown et al., 2020; Hoffmann et al., 2022; Chowdhery et al., 2022, *inter alia*).

One particularly interesting strategy is chain-of-thought (CoT) prompting, which has been shown to elicit strong reasoning abilities in LLMs by asking the model to include intermediate reasoning

Chain of Thought

Q: A robe takes 2 bolts of blue fiber and half that much white fiber. How many bolts in total does it take?

A: It takes $2/2 = 1$ bolt of white fiber. So the total amount of fabric is $2 + 1 = 3$ bolts of fabric. The answer is 3.

Reasoning by Asking

Q: A robe takes 2 bolts of blue fiber and half that much white fiber. How many bolts in total does it take?

A: How many bolts of white fiber does it take? It takes $2/2 = 1$ bolt of white fiber. How many bolts in total does it take? So the total amount of fabric is $2 + 1 = 3$ bolts of fabric. The answer is 3.

Figure 1: As opposed to the chain of thought reasoning (CoT) that enables LLM to generate a series of natural language-based intermediate steps to solve a reasoning task, the DECOMPOSITIONAL DISTILLATION approach asks a question at each intermediate step corresponding to the various CoT reasoning steps. Here, Q refers to the input problem, and A denoted the LLMs reasoning processes after prompting.

steps while solving a problem (Wei et al., 2022b; Wang et al., 2022). However, the CoT prompting has only been made possible due to the scaling of these models to hundreds of billions of parameters, making it one of the emergent properties of LLMs (Wei et al., 2022a).

For this reason, significant compute resources or expensive API calls are required for CoT reasoning. Therefore, we deem it desirable to transfer such reasoning capabilities into smaller models. One way to attempt this would be to induce CoT-like reasoning abilities in smaller models by *knowledge distillation*, i.e., training small models with the CoT outputs from the larger model. However,

* Equal contribution;

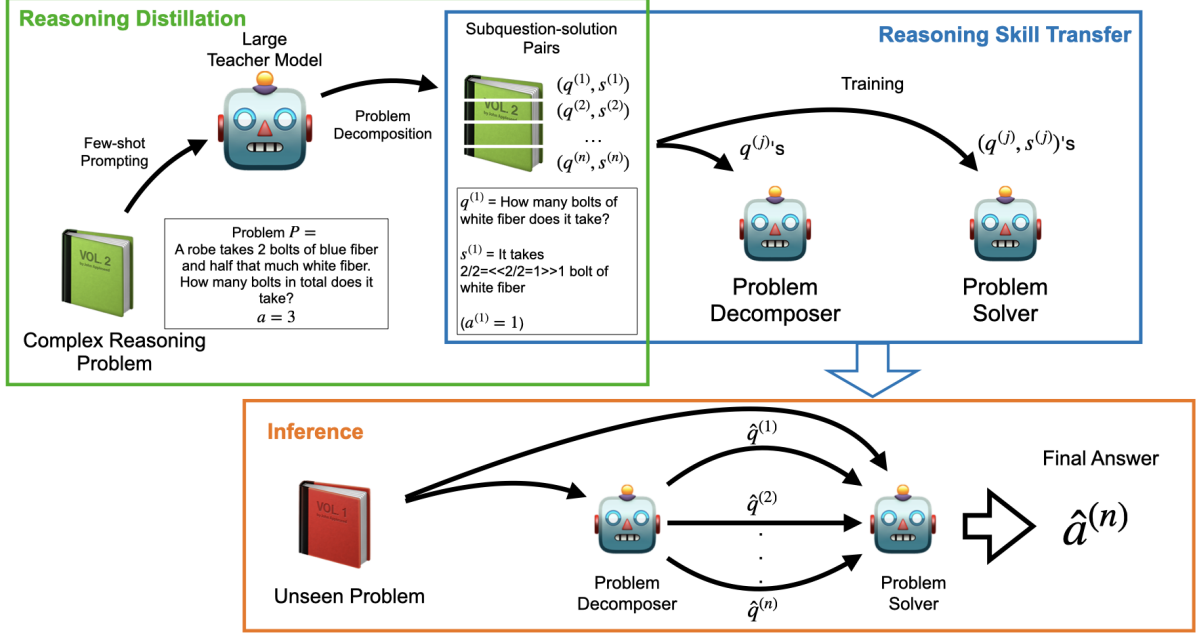


Figure 2: Detailed explanation of our approach DECOMPOSITIONAL DISTILLATION. On top, an LLM is prompted to decompose the input problem P into a series of subquestion-solution pairs $(q_i^{(j)}, s_i^{(j)}, j \in \{1, \dots, n_i\})$ with an answer at each step $a_i^{(j)}$. The generated subquestions-solutions are used to train two student models: a) the *Problem Decomposer* that learns to mimic the subproblem generator of the LLM and b) the *Problem Solver*, which learns to solve each subquestion. At the bottom, the inference process is depicted for an unseen problem and no LLM is present. The *Problem Decomposer* breaks the unseen problem into simpler subquestions and the *Problem Solver* solves each one of them eventually leading to the final answer $a_i^{(n_i)}$.

small-size language models are known to be poor reasoners (Stolfo et al., 2022), and the CoT reasoning abilities cannot be transferred directly to the smaller models. In our work, we justify that training smaller models¹ directly with CoT outputs to implicitly learn to generate a sequence of reasoning steps to answer a given problem is a suboptimal strategy (details in Section 5).

In this paper, we introduce DECOMPOSITIONAL DISTILLATION, a general framework that takes advantage of the reasoning abilities of LLMs and transfers these capabilities into smaller models. Our methodology is motivated by the Knowledge distillation framework (Hinton et al., 2015), where a larger teacher model transfers knowledge to a smaller student model.

In contrast to standard knowledge distillation, DECOMPOSITIONAL DISTILLATION distills the teacher model’s reasoning abilities using its generated solutions as a proxy. Our approach consists

of two steps (illustrated by the two upper boxes in Figure 2): first, we prompt an LLM with examples of problems decomposed in intermediate subquestion-solution pairs. This way, the model, using the same question-answer structure, generates the intermediate steps to obtain the solution for a new problem. (We report an example of our question-answer decomposition and a comparison to a traditional CoT prompt in Figure 1.) Second, we transfer the decomposed intermediate reasoning into a student structured as a pair of smaller models: a *problem decomposer* and a *problem solver*. The former acts as a decomposition unit that learns to break down a complex reasoning problem into smaller subproblems, while the latter solves the intermediate subproblems produced by the decomposition unit leading to a solution of the overall problem.

To demonstrate the effectiveness of our approach, we consider a multistep math word problems dataset, GSM8K (Cobbe et al., 2021) due to its popularity and presence of step-wise reasoning solution steps. Our distilled models outperform our baseline models of the same size that were trained to reproduce the standard chain of thought

¹As stated in Li et al. (2022), we also argue that small and large models are very relative terms and context dependant. We consider GPT-3 like models with billions of parameters as large models, while GPT-2 models with millions of parameters as small.

reasoning without decomposed sub-questions by over 35%. While the results provide a proof of concept of our proposed methodology, it is essential to note that the approach is not specifically tailored to the given domain and could, in the future, be applied to other reasoning tasks.

2 Related Work

Questions as supervision The idea of inquiring or asking information-seeking questions for discovery learning has been studied well in the past (Bruner, 1961). Rao and Daumé III generated clarification questions based on Stack Exchange questions as supervision, Klein and Nabi (2019) used a joint question answering model to ask questions from a given span of text and later answer it, and (Rajani et al., 2019; Schwartz et al., 2020) asked questions to improve the common sense QA models. However, our work is focused on multistep reasoning tasks, where the availability of intermediate clarifying questions and reasoning steps are often not available, and is provided by a teacher model.

Decomposing Math Word Problems In our work, we use a math word problem (MWP) dataset. Solving MWPs has been an interesting area of research for the last couple of years (Kushman et al., 2014; Hosseini et al., 2014; Roy et al., 2015; Seo et al., 2015; Sachan and Xing, 2017; Amini et al., 2019; Xie and Sun, 2019; Zhang et al., 2020). However, the majority of the modern approaches are shifting towards the use of LLMs, often relying on approaches involving prompting or in-context learning (Cobbe et al., 2021; Kojima et al., 2022a; Wei et al., 2022b; Chowdhery et al., 2022; Lewkowycz et al., 2022; Srivastava et al., 2022). The most popular approach is the chain of thought prompting (Wei et al., 2022b), which prompts the language model to generate a series of natural language-based intermediate steps that improve the reasoning capabilities in LLMs. Wang et al. (2022) took another step forward and sampled multiple reasoning paths and selected the most relevant output using majority voting. Huang et al. (2022) used the most voted outputs to further fine-tune the model for better performance. Kojima et al. (2022b) further improved the reasoning of LLM in a zero-shot manner by appending “Let’s think step by step” to the prompt. Our work, however, not only trains the model rather than just prompting but also explicitly guides the reasoning path

using sub-questions at each step which is missing in all the above reasoning chains. One work that is close to our work is by Zhou et al. (2022) which decomposes the questions into multiple sub-questions and asks the language model to solve each sub-question sequentially. However, this work is also restricted to prompting and only works with LLMs with billions of parameters and cannot be directly transferred to smaller models.

Knowledge Distillation We use a student network that imitates the teacher and this goes in the direction of knowledge distillation (Ba and Caruana, 2014; Hinton et al., 2015). Snell et al. (2022) demonstrated the usefulness of providing instruction that can assist the models in achieving better reasoning abilities. Similar to our hypothesis, Eisenstein et al. (2022) argued that question-answering systems should not only focus on the final answer but also rationales that justify their reasoning to help reason better. We go beyond this in our work and work on not just the question-answering system but also on what questions need to be asked at each step that can assist in learning that reasoning step better. Finally, similar to our hypothesis of injecting reasoning capabilities in smaller models, Li et al. (2022) used CoT-like reasoning from LLMs to train smaller models on a joint task of generating the solution and explaining the solution generated. We, on the other hand, use the LLM to generate subquestions and solution pairs and use them together to inject reasoning capabilities into smaller models.

Motivation from Learning Sciences Explicit guidance through sub-questioning at each step helps in better reasoning and has a strong basis in learning science literature (Wood et al., 1976; Fantuzzo et al., 1989; Wood, 1994). We see our proposed learning strategy DECOMPOSITIONAL DISTILLATION as a form of *collaboration script*. A collaboration script is a set of instructions which aims to guide and support two or more learners to interact and behave during collaborative learning in a way that all learning partners benefit from collaboration. In our case, an LLM (acting as a teacher) decomposes the complex problem into simpler subquestion-solutions, and two smaller models act as learners and learn to mimic the reasoning steps and the guiding questions while benefiting from each other’s learning.

3 Methodology

We describe our DECOMPOSITIONAL DISTILLATION approach in the setting of math word problems (MWP). We consider a dataset \mathcal{D} in which each problem P_i is paired with a solution S_i that contains a numerical answer a_i . The task of solving the problem using a model f consists of predicting a solution $\hat{a} = f(P)$ such that $\hat{a} = a$. In this section, we describe the two steps that constitute our distillation method and the inference procedure.

3.1 Extracting the Reasoning Capability from the Teacher

For a small subset of \mathcal{D} , we break down each problem P_i into n_i intermediate reasoning steps. Each intermediate step is a subquestion-solution pair, denoted $q_i^{(j)}, s_i^{(j)}, j \in \{1, \dots, n_i\}$. We refer to the ordered list of subquestion-solution pairs for an example as $(q_i^{(1)}, s_i^{(1)}), \dots, (q_i^{(n_i)}, s_i^{(n_i)})$. We construct these intermediate reasoning steps manually as we only need a couple of examples as prompts. Each intermediate solution $s_i^{(j)}$ ($j \in \{1, \dots, n_i\}$) contains an intermediate numerical answer $a_i^{(j)}$, with $a_i^{(n_i)} = a_i$. By combining this intermediate annotation, we construct a few-shot prompt (example provided in the Appendix Section A).

For each remaining problem $P \in \mathcal{D}$ that was not manually decomposed, we then prompt a large language model \mathcal{M} to generate the intermediate reasoning steps for the remaining problems in \mathcal{D} in the form of subquestion-solution pairs (process illustrated by the top left box in Figure 2).

We make sure that the chain of intermediate question-solution pairs is meaningful by checking whether the last solution matches with the ground-truth numerical answer, i.e., whether $a_i^{(n_i)} = a_i$. If that is not the case, we discard the problem and re-sample a new chain by prompting the model again (for a maximum of 3 times). This way, we obtain an augmented dataset \mathcal{D}^* in which a portion of the problems is paired with a sequence of subquestion-solution pairs leading to the correct result.

3.2 Transferring the Reasoning Capability into the Student

We break the student models into two separate models: a *problem decomposer* that is trained to produce meaningful subquestions that can assist in reaching the correct solution, and a *problem solver*,

that solves the problem in a step-by-step manner given the subquestions produced by the *problem decomposer*.

Problem Decomposer Using the problems in \mathcal{D}^* that contain the augmented chain of intermediate questions and solutions, we train a *problem decomposer* model \mathcal{M}_{PD} that learns to produce the sequence of subquestions $\{q^{(1)}, q^{(2)}, \dots\}$ needed to solve a given problem. We use a pre-trained transformer-based model (Vaswani et al., 2017), and train it on the given sequence of subquestions. Given a subquestion $q^{(j)}$ of problem P , consisting of a sequence of m_j tokens $\{x_j^{(1)}, \dots, x_j^{(m_j)}\}$, we use a typical autoregressive language modelling loss, \mathcal{L} :

$$\mathcal{L}(P, q^{(j)}) = - \sum_{k=1}^{m_j} \log \mathbb{P}_{\mathcal{PD}}(x_j^{(k)} | x_j^{(k-1)}, q^{(j-1)}, P), \quad (1)$$

where $\mathbb{P}_{\mathcal{PD}}(x|c)$ is the probability assigned by \mathcal{M}_{PD} to token x given context c , and $x^{(y)}$ indicates the sequence $\{x^{(1)}, \dots, x^{(y)}\}$. The learning objective for problem P is then modelled as:

$$\mathcal{L}_{PD}(P) = \sum_{j=1}^{n_i} \mathcal{L}(P, q^{(j)}). \quad (2)$$

However, training the *problem decomposer* directly on the sequence of subproblems leads to worse performance due to no control over the generated sequence of subquestions (more details in the Ablation Section 6).

We propose a guiding mechanism that constrains the question-generation process to follow a strict set of constraints.

Following Shridhar et al. (2022), we consider a second sequence-to-sequence model that is trained to generate the equations $e_q^{(j)}$, later used as a guiding mechanism for the sequence and the number of questions to be generated. The equations are a task-specific example (for the MWP case) of a guiding mechanism for \mathcal{M}_{PD} , and they can be possibly replaced with any set of constraints for other tasks.

The equation generation follows the same objective defined in Equation 1 with questions replaced with the equations from the solutions at each step. The generated equations² are later used to guide

²An equation $e_q^{(j)}$ has the form $\langle\langle x \times y = a^{(j)} \rangle\rangle$.

the \mathcal{M}_{PD} , where the generated equations are appended to the encoder to be conditioned on, and the updated learning objective is defined as:

$$\mathcal{L}_{PD}(P) = \sum_{j=1}^{n_i} \mathcal{L}(P \oplus \text{eq}, q^{(j)}), \quad (3)$$

where \oplus depicts the concatenation operation and $\text{eq} = [\text{eq}^{(1)}, \text{eq}^{(2)}, \dots]$ is the list of the equations leading to the intermediate numerical results $a^{(j)}$'s.

Problem Solver The *problem solver* component consists of a pre-trained language model \mathcal{M}_{PS} , trained independently of the *problem decomposer*, using the teacher-generated chain of subquestion-solution pairs. Similarly to the PD model, the learning objective of the *problem solver* is computed at a token level for each intermediate solution:

$$\begin{aligned} \mathcal{L}(P, s^{(j)}) = & \\ - \sum_{k=1}^{l_j} \log \mathbb{P}_{\mathcal{PS}}(y_j^{(k)} | y_j^{:(k-1)}, q^{(j)}, s^{:(j-1)}, P), & \end{aligned} \quad (4)$$

where l_j and the y_j 's represent, respectively, the length and the tokens of the intermediate solution $s^{(j)}$. Then, the objective is combined for all the substeps of problem P:

$$\mathcal{L}_{\mathcal{PS}}(P) = \sum_{j=1}^{n_i} \mathcal{L}(P, s^{(j)}). \quad (5)$$

During training, the previous intermediate solutions generated by the *problem decomposer* are replaced with the teacher-generated solutions using teacher forcing (Cho et al., 2014). However, the solutions generated by the *problem solver* are used at inference time.

3.3 Inference-time Predictions

When predicting an unseen problem P , the *problem decomposer* is queried to produce the sequence of subquestions $\{\hat{q}^{(1)}, \dots, \hat{q}^{(n)}\}$ meant to guide the reasoning of the problem solver model. After these questions are generated, they are shown to the *problem solver* iteratively, decoding the intermediate solution $\hat{s}^{(j)}$ at step j token by token according to the model's probability distribution over its vocabulary:

$$\mathbb{P}_{\mathcal{PS}}(y_j^{(k)} | y_j^{:(k-1)}, \hat{q}^{(j)}, \hat{s}^{:(j-1)}, P), \quad (6)$$

where $y_j^{(k)}$ is the k -th token being decoded in greedy fashion.

After the last solution $\hat{s}^{(n)}$ has been generated, the numerical prediction $\hat{a}^{(n)}$ is parsed from the text using simple heuristics.

4 Empirical Analysis

4.1 Dataset

We study how DECOMPOSITIONAL DISTILLATION can assist smaller models to learn better reasoning on a multi-hop math word problems dataset, GSM8K³ (Cobbe et al., 2021). GSM8K consists of 8.5K grade school math word problems with each problem requiring 2 to 8 reasoning steps to solve. The solutions primarily involve a sequence of elementary calculations using basic arithmetic operations (+ − ∗ /). The dataset is partitioned into 7.5K training problems and 1K test problems. Since the GSM8K dataset has problem solutions in a step-by-step format leading to the final answer, we used the ground truth as a proof of concept for our experiments. However, for other datasets with just the final answer, the step-by-step decomposition will be provided by the teacher model.

4.2 Implementation Details

We used GPT-2 variants (Radford et al., 2019) as the main student question-answering model in our work. GPT-3 (Brown et al., 2020) served as the teacher model for decomposing the complex problem into a series of simpler substeps. We used T5-large (Raffel et al., 2020) as the backbone of our question generator module with the main problem as input and sub-questions generated as output.

All models were trained using the Huggingface library (Wolf et al., 2020) on an NVIDIA Tesla A100 GPU with 40GB of GPU memory. Each experiment ran for the same number of iterations to ensure fairness with a periodic evaluation over the validation set. Teacher forcing was used during training to replace the generated answers with true answers from the training dataset.

4.3 Evaluation Metrics

For the evaluation of the question-answering performance on the GSM8K dataset, we compute the accuracy based on the final answer provided by the *problem solver*. For the question generation part, we report automatic evaluation using SacreBLEU

³<https://github.com/openai/grade-school-math>

(Post, 2018) which is based on exact word overlap, BERT F1 score (Zhang et al., 2019) which is based on DeBERTa (He et al., 2020) as the similarity model. We also report #Q, the number of questions generated compared to the number of reasoning steps needed to solve a problem.

5 Results and Discussion

Does DECOMPOSITIONAL DISTILLATION help in achieving better reasoning performance?

The GSM8K dataset consists of a Socratic version where subquestion-solution pairs are provided for each MWP. We assumed this as a substitution for a teacher model as this is the best result that a teacher model can achieve (we interpret this as an upper bound). We also experimented with generating these subquestions using a GPT-3 model with prompts, obtaining similar results (BERT F_1 score of 95%). Since the end goal is to induce the reasoning capabilities into smaller models, we use CoT reasoning as our baseline. We used these sub-questions to train variants of the GPT-2 model (small, medium, and large) and compared our results with the CoT baseline. Table 1 demonstrates the effectiveness of our method with all models achieving higher accuracy when trained with our proposed methodology DECOMPOSITIONAL DISTILLATION (up to 38%). The approach favours the larger model as larger models have a better capacity to encode the reasoning capabilities in their parameters. We also demonstrate that training with DECOMPOSITIONAL DISTILLATION can help a much smaller model (GPT-2 large with 774M parameters) reason equally well to an almost 10X larger model (GPT-3 with 6B parameters).

Models	Methodology	Accuracy
GPT-2 Small (124M)	CoT	5.05
	Sub-ques	6.44 (↑ 20%)
GPT-2 Medium (355M)	CoT	7.88
	Sub-ques	12.74 (↑ 38%)
GPT-2 Large (774M)	CoT	14.10
	Sub-ques	21.08 (↑ 33%)
GPT-3 (6B)	CoT	21.0

Table 1: Accuracy comparison (in %) of using CoT vs DECOMPOSITIONAL DISTILLATION (Sub-ques) on the GSM8K dataset.

Our approach can also be extended to prompting-based models, where DECOMPOSITIONAL DISTILLATION can improve the performance of GPT-3 with 1-shot prompting by over 40% (Table 2). Here, LLM like GPT-3 is used to decompose the main

problem into simpler problems and then GPT-3 is used to solve it in a 1-shot setting with decomposed sub-questions and solutions as prompts.

Models	Methodology	Accuracy
GPT-3 (1-shot) (175B)	CoT	27.5
	Sub-ques	47.1 (↑ 41%)

Table 2: Accuracy comparison (in %) of using CoT vs DECOMPOSITIONAL DISTILLATION (Sub-ques) on the GSM8K dataset for GPT-3 model with prompting.

Can we model the sub-questioning capabilities into a smaller model?

Using a large language model like GPT-3 can be an obvious choice for decomposing a complex problem into step-by-step reasoning steps.

However, relying on a large model at test time is undesirable (e.g., for resource constraints). Therefore, we study whether a smaller model can learn to decompose a problem into subquestions required to guide the reasoning process. This can again be thought of as a knowledge distillation between a teacher and student model with the major difference being this time the student learns to ask a series of suitable questions needed to solve the problem.

We use a T5 model that learns to ask a series of questions needed to answer the problem in a chain-of-thought style. We calculate the BLEU and BERT F_1 scores alongside the match in the number of questions generated (#Q) between the generated sub-questions and the ground truth (for other datasets where we don’t have access to sub-questions, this will be generated by a prompt-based model like GPT-3) on the GSM8K dataset. Directly fine-tuning a T5 model without any guidance towards what kind of questions needs to be generated was not effective in our case as the model was generating sometimes more, sometimes fewer questions that were confusing the question answering model (GPT-2 variants). This led to a sharp performance drop (more discussion in the ablation studies in section 6). We used a guidance-based strategy where we used the equations from the ground truth to assist the models in generating the right number of questions and the performance increased on all three metrics - BLEU, BERT F_1 , and the number of questions. The results are reported in Table 3. Note that here we additionally train another model for generating equations at test time to assist the *problem decomposer* to condition the sub-questions

from it.

Models	Methodology	BLEU	BERT F_1	# Q
T5-large (770M)	No-guidance	51.5	0.78	0.42
	Guidance	58.8	0.81	0.80

Table 3: BLEU, BERT F_1 and the number of questions (# Q) comparison between the question generator model and the Socratic subquestions present in the GSM8K dataset.

Finally, we compare the reasoning capabilities of the models when a smaller *problem decomposer* is used instead of ground truth (or even prompt-based models like GPT-3) in Table 4. Even with a much smaller question generator model (200X smaller than GPT-3), DECOMPOSITIONAL DISTILLATION achieved better reasoning performance than the CoT approach for all different GPT-2 models.

GPT-2	CoT	Sub-ques QG	Oracle Ques
Small	5.05	5.98	6.44
Medium	7.88	11.57	12.74
Large	14.10	17.89	21.08

Table 4: Accuracy comparison (in %) of using CoT vs DECOMPOSITIONAL DISTILLATION generated by a QG model (Sub-ques QG) and the Oracle Ques (Socratic sub-ques from the ground truth) on the GSM8K dataset.

6 Ablation Studies

Impact of non-guided question generator on question answering performance. When a *problem decomposer* is not guided (with equations, in our case) to know how many questions to generate, the model asks a lot of redundant questions that sometimes confuse the model, leading to a decrease in accuracy. Figure 3 shows a comparison between the non-guided *problem decomposer* effects on the reasoning accuracy compared to the equation-guided model. It is worth noting that using subquestions that are not ordered or well structured can harm the reasoning capabilities of the model, just like students in the real world.

Eliminating the need for a *problem decomposer* model. We studied the impact of the *problem decomposer* by removing it completely from the training and testing process. We trained a student to answer sub-questions based on the intermediate steps directly meaning depending on what step question is asked, the *problem solver* learns to answer that step. If sub-question $q^{(1)}$ is asked, the

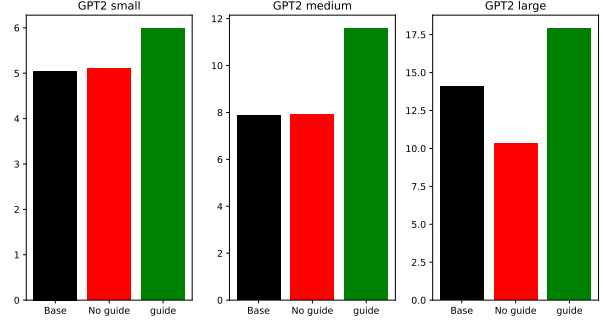


Figure 3: Accuracy comparison when the question generation was guided using equations (guide) vs the non-guided greedy generation scheme (No guide).

model learns to answer $s^{(1)}$, and when $q^{(2)}$ is asked, the model learns to build the steps $\{s^{(1)}, s^{(2)}\}$ and not just $s^{(2)}$. This means that during the test time, a multistep question can range from 2 to 8 steps and the model builds the needed steps to reach the final answer a . This phenomenon can be thought of as curriculum learning (Bengio et al., 2009) where simple to more complex concepts are taught to the model and the learning happens gradually. However, in our case, the model did not learn to map solution steps iteratively; leading to a much worse performance during test time. The results demonstrating the need for the *problem decomposer* are summarised in Table 5.

GPT-2	No sub-ques	sub-ques QG
Small	2.70	5.98
Medium	7.20	11.57
Large	8.18	17.89

Table 5: Accuracy comparison (in %) of using no student questioning module (No sub-ques) vs using one (Sub-ques QG) on the GSM8K dataset.

7 Conclusion and Future Work

Chain of thought-style step-by-step reasoning methods has proven to be very effective for reasoning in LLMs. In this work, we propose ways to further improve it by explicitly asking questions at each step, thus making it possible to train smaller models with this approach. We demonstrate the effectiveness of our proposed methodology on a popular multi-step math word problem dataset.

However, we have used the provided ground truth solution in our experiments, which is very specific to the GSM8K dataset. In many real-world scenarios, such intermediate solutions are not available, and in such cases, we rely on LLMs to gen-

erate those steps leading to additional noise in the training data. Recent works like STaR (Zelikman et al., 2022) can prove very useful for such cases, where if the generated rationale leading to the final answer is incorrect, the rationale is regenerated given the correct answer (ground truth). This idea can also be applied to the student models where the *problem decomposer* and the *problem solver* are trained jointly to improve each other’s performance.

Finally, we only use one solution from the LLM to distil information in the student model and following Wang et al. (2022) multiple subquestion-solution pairs can be sampled and using majority voting, all the correct pairs can be used to distil knowledge in the student models. We leave this for future work.

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A GPT-3 prompting

We used one-shot prompting for GPT-3 meaning we provide one example (Q,A) alongside the Problem to the model and let it predict the solution (S) for the next sub-question (Q) provided iteratively. The input prompt is provided in **bold** and the GPT-3 output in *italics*.

GPT-3 API CALL 1:

Problem: John has 10 hectares of a pineapple field. There are 100 pineapples per hectare. John can harvest his pineapples every 3 months.

Q: How many pineapples does John have? S:
John has $100 \times 10 = <<100*10=1000>>1000$ pineapples on his field.

Q: How many times can John harvest his pineapples? S: *John can harvest his Pineapple $12 / 3 = <<12/3=4>>4$ times per year.*

GPT-3 API CALL 2:

Problem: John has 10 hectares of a pineapple field. There are 100 pineapples per hectare. John can harvest his pineapples every 3 months.

Q: How many pineapples does John have? S:

John has $100 \times 10 = <<100*10=1000>>1000$ pineapples on his field.

Q: How many times can John harvest his pineapples? S: *John can harvest his Pineapple $12 / 3 = <<12/3=4>>4$ times per year.*

Q: How many pineapples can John harvest within a year? S: *Therefore John can harvest $1000 \times 4 = <<1000*4=4000>>4000$ pineapples per year. ##### 4000*