

# Mathematics Foundation of Computer Science Formula Booklet

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# 1 Ring Laws

## 1.1 Ring Laws

$$a + 0 = a \quad a \times 1 = a \quad (\text{Neutral elements})$$

$$a + b = b + a \quad a \times b = b \times a \quad (\text{Commutativity})$$

$$\begin{aligned} a + (b + c) &= (a + b) + c \\ a \times (b \times c) &= (a \times b) \times c \end{aligned} \quad (\text{Distributivity})$$

$$a \times 0 = 0 \quad (\text{Annihilation})$$

## 1.2 Cancellation Laws

$$a + c = b + c \Rightarrow a = b$$

$$a \times c = b \times c \Rightarrow a = b, \quad c \neq 0$$

## 2 Sets

$[A \setminus B]$  : Set A without elements shared with B

$A = \{a, b\}$ ,  $B = \{1, 2\}$ ,  $A \times B = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$

$|A|$  : Cardinality of A

## 3 Relations

### 3.1 Relations

Graph: Binary relation  $R \subseteq A^2$  on a single set

Vertices: Elements of  $A$

Edges: Elements of  $R$

$A^2 : A \times A$

Reflexivity:  $(x, x) \in R$

Irreflexive:  $\forall x \in A. (x, x) \notin R$

Reflexive-closure:  $R \cup \{(x, y) \in A^2 \mid x = y\} \subseteq A^2$

Symmetry:  $\forall (x, y) \in A^2. (x, y) \in R \Rightarrow (y, x) \in R$

Anti-symmetry:  $\forall x \in A. (x, y) \in R \wedge (y, x) \in R \Rightarrow x = y$

Symmetric-enclosure:  $\{(a, b), (b, a), (a, c), (c, a), \dots\}$

Transitivity:  $\forall x, y, z \in A. (x, y) \in R \wedge (y, z) \in R \Rightarrow (x, z) \in R$

Transitive-closure:  $R \cup R; R \cup R; R; R \cup R; R; R \cup R \dots$

Transitive-closure: "All  $R$  Paths"

Order Relation: When  $R \subseteq A^2$  is Reflexive, Anti-Symmetric and Transitive

Equivalence Relation: Reflexive, Symmetric, Transitive

Equivalence Class: Set of  $\in A$  which are all  $\in$  equivalence closure

## 4 Functions

### 4.1 Function Requirements

Definedness:  $\forall a \in A \exists b \in B. (a, b) \in R$

Single-valuedness:  $\forall a \in A \forall b, b' \in B. (a, b) \in R \wedge (a, b') \in R \Rightarrow b = b'$

### 4.2 Properties

Range / Image:  $\{b \in B \mid \exists a \in A. (a, b) \in R\} \subseteq B$

Injectivity:  $\forall a, a' \in A. a \neq a' \Rightarrow f(a) \neq f(b)$

Surjectivity:  $\forall b \in B \exists a \in A. f(a) = b$

Bijectivity:  $|F^{-1}[\{b\}]| = 1$

Bijectivity: Injective And Surjective Simultaneously

Forward Image:  $F[X] = \{b \in B \mid \exists a \in X. f(a) = b\}$

Backward Image:  $F^{-1}[Y] = \{a \in A \mid f(a) \in Y\}$

$F[\bar{X}]$  : Compliment of pre-image

$F[\overline{X}]$  : Compliment of forward-image

Everywhere defined: All of A is the pre-image

## 5 The Inner Product

### 5.1 Conversions

Linear Equation To Parametric:

$$x_1 = d + bx_2 + cx_3 \dots \Rightarrow \begin{pmatrix} d \\ 0 \\ 0 \\ \dots \\ \dots \end{pmatrix} + x_2 \cdot \begin{pmatrix} b \\ 1 \\ 0 \\ \dots \\ \dots \end{pmatrix} + x_3 \cdot \begin{pmatrix} c \\ 0 \\ 1 \\ \dots \\ \dots \end{pmatrix} + \dots + x_n \cdot \begin{pmatrix} n^{th} \text{ co-eff} \\ \dots \\ \dots \\ \dots \\ 1 \end{pmatrix}$$

Parametric to linear (line):

$$ax_1 + bx_2 = d$$

$$a = -v_2$$

$$b = v_1$$

$$d = -v_2p_1 + v_1p_2$$

Parametric to linear (plane):

$$ax_1 + bx_2 + cx_3 = d$$

$$a = v_2w_3 - v_3w_2$$

$$b = v_3w_1 - v_1w_3$$

$$c = v_1w_2 - v_2w_1$$

$$d = ap_1 + bp_2 + cp_3$$

## 5.2 Inner Product

$$\langle \vec{v}, \vec{w} \rangle : v_1 \times w_1 + v_2 \times w_2 + \dots + v_n \times w_n$$

$$\langle \vec{v} + \vec{w}, \vec{u} \rangle : \langle \vec{v}, \vec{u} \rangle + \langle \vec{w}, \vec{u} \rangle$$

$$\langle s \cdot \vec{v}, \vec{w} \rangle : s \cdot \langle \vec{v}, \vec{w} \rangle$$

$$\langle \vec{v}, \vec{w} \rangle : |\vec{v}| \times |\vec{w}| \times \cos a$$

$$\text{Orthogonal Test} : \langle \vec{v}, \vec{w} \rangle = 0$$

$$\langle \vec{v}, \vec{v} \rangle : |\vec{v}|^2$$

$$|\vec{v}| : \sqrt{\langle \vec{v}, \vec{v} \rangle}$$

## 5.3 Geometry

$$\vec{n} : \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$\frac{d}{|\vec{n}|} : \text{Distance from origin}$$

$$\langle \vec{n}, X \rangle = d = \langle \vec{n}, P \rangle \text{ where } X: \text{Arbitrary Point}, P: \text{Point on the line}$$

$$\text{Projection of line } \vec{v} \text{ on line } \vec{n}: \frac{\langle \vec{n}, \vec{v} \rangle}{\langle \vec{n}, \vec{n} \rangle} \times \vec{n}$$

Distance of Point  $Q$  to line  $P$ :

$$\frac{\langle P, \vec{n} \rangle - \langle Q, \vec{n} \rangle}{|\vec{n}|} = \frac{d - \langle \vec{n}, Q \rangle}{|\vec{n}|} = \frac{\langle \vec{n}, \vec{Q}P \rangle}{|\vec{n}|}$$

Projection of Point  $Q$  on a Plane:

$$Q' : Q + \frac{d - \langle \vec{n}, Q \rangle}{\langle \vec{n}, \vec{n} \rangle} \cdot \vec{n}$$



Reflection of Point Q in a Plane

$$Q'' : Q + 2 \times \frac{d - \langle \vec{n}, Q \rangle}{\langle \vec{n}, \vec{n} \rangle} \cdot \vec{n}$$

## 6 Bases

### 6.1 Bases

Linear Combinations:

$$(\sum_{i=1}^n a_i \cdot \vec{v}_i) + (\sum_{i=1}^n b_i \cdot \vec{v}_i) = \sum_{i=1}^n (a_i + b_i) \cdot \vec{v}_i$$

$$s \cdot (\sum_{i=1}^n a_i \cdot \vec{v}_i) = \sum_{i=1}^n (s \times a_i) \cdot \vec{v}_i$$

Theorem 8 for linear independence:

$$\sum_{i=1}^n a_i \cdot \vec{v}_i = \vec{0} \Rightarrow a_1 = a_2 = a_3 = \dots = 0$$

Value of particular co-efficient (coordinates):

$$a_k = \frac{\langle \vec{w}_k, \vec{v}_k \rangle}{\langle \vec{v}_k, \vec{v}_k \rangle}$$

Orthonormal:  $\langle \vec{v}, \vec{v} \rangle = 1$

Positive definite:

$$\langle \vec{v}, \vec{v} \rangle \geq 0$$

$$\langle \vec{v}, \vec{v} \rangle = 0 \Rightarrow \vec{v} = \vec{0}$$

Computing Orthogonal bases from bases:

$$\vec{w}_1 = \vec{v}_1$$

$$\vec{w}_2 = \vec{v}_2 - \frac{\langle \vec{v}_2, \vec{w}_1 \rangle}{\langle \vec{w}_1, \vec{w}_1 \rangle} \cdot \vec{w}_1$$

$$\vec{w}_3 = \vec{v}_3 - \frac{\langle \vec{v}_3, \vec{w}_1 \rangle}{\langle \vec{w}_1, \vec{w}_1 \rangle} \cdot \vec{w}_1 - \frac{\langle \vec{v}_3, \vec{w}_2 \rangle}{\langle \vec{w}_2, \vec{w}_2 \rangle} \cdot \vec{w}_2$$

and so on...