







High Energy String Theory and the Celestial Sphere

Connecting worldsheet CFT and celestial CFT

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Introduction

- String theory is equipped with an intrinsic parameter, α' . In the field theory limit ($\alpha' \to 0$), strings become point-like and describe a QFT. The opposite limit, $\alpha' \to \infty$, leads to the appearance of an infinite number of massless higher-spin states in the string spectrum [1]. Moreover, most of the powerful properties of string amplitudes are known to stem from the inherent 2d CFT structure on the string worldsheet: soft UV behaviour, modular invariance and loop corrections, etc.
- A 2d CFT also arises in the celestial holography program, where 4d scattering amplitudes in asymptotically flat spacetimes are recast as conformal correlators on the celestial sphere at null infinity, \mathcal{I} . The resulting *celestial amplitudes* are tightly constrained by the symmetries of the underlying celestial CFT (CCFT), which is expected to eventually allow for first-principles computations of bulk physics from boundary data, similar to the well-known AdS/CFT correspondence.
- Since celestial amplitudes are obtained via Mellin transforms that integrate out energy, they are typically UV divergent in field theory—unlike in string theory, where UV-softness ensures well-defined celestial string amplitudes [2]. We exploit this feature in our paper to relate the free worldsheet CFT to the CCFT in the high-energy limit of string theory.
- Specifically, we relate the saddle-point approximation of string amplitudes as $\alpha' \to \infty$ with a stationary phase expansion of celestial string amplitudes for large conformal weight, at all orders. This approach points to an intrinsic construction of CCFT by relating it to a (free) worldsheet CFT.

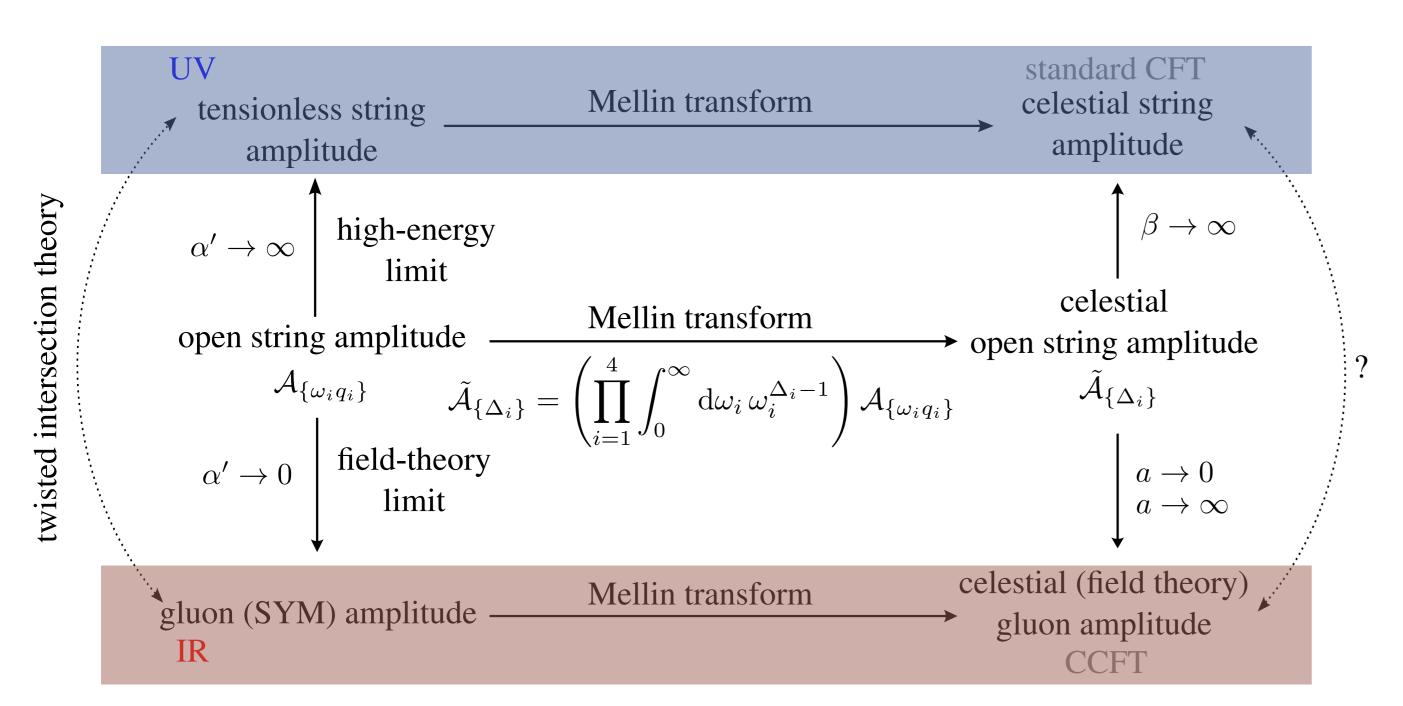


Figure 1: Various limits of the 4-point open string gluon amplitude considered in this work.

Representations of string amplitudes

For canonical colour ordering, the 4-point tree-level open string (gluon) amplitude is given by

$$\mathcal{A}(1,2,3,4) = \frac{\Gamma(1-s)\Gamma(1-u)}{\Gamma(1+t)} A_{YM}(1,2,3,4) = -A_{YM}(1,2,3,4) s \int_0^1 dx \, x^{-s-1} (1-x)^{-u}, \quad (1)$$

with the Mandelstam invariants $s = \alpha'(p_1 + p_2)^2$, $t = \alpha'(p_1 - p_3)^2$ (s + t + u = 0). For closed strings,

$$\mathcal{M} = \pi \frac{su}{t} \frac{\Gamma(-s)\Gamma(-u)\Gamma(-t)}{\Gamma(s)\Gamma(u)\Gamma(t)} A_{\mathbf{YM}}(1,2,3,4) \tilde{A}_{\mathbf{YM}}(1,2,3,4).$$

In the field theory ($\alpha' \to 0$) limit, one has [3]

$$\mathcal{A}_{0} = \exp \left\{ \sum_{n=1}^{\infty} \frac{\zeta(2n)}{(2n)} \left(s^{2n} + u^{2n} - t^{2n} \right) \right\} \exp \left\{ \sum_{k=1}^{\infty} \frac{\zeta(2k+1)}{(2k+1)} \left(s^{2k+1} + u^{2k+1} + t^{2k+1} \right) \right\} A_{YM}.$$

When $\alpha' \to \infty$ on the other hand, we find

$$\mathcal{A}_{+\infty} = \sqrt{2\pi \frac{\sin(\pi t)}{t} \frac{\sin(\pi t)}{\sin(\pi s)}} s^{-s} u^{-u} t^{-t} (-1)^{-u-t} \exp \left\{ \sum_{k=1}^{\infty} \frac{\zeta(1-2k)}{(2k-1)} \left(\frac{1}{s^{2k-1}} + \frac{1}{u^{2k-1}} + \frac{1}{t^{2k-1}} \right) \right\} A_{YM}.$$

This expression may be understood as quantum fluctuations around the classical solution A_c to the path integral

$$\mathcal{A} \sim \int \mathcal{D}g \,\mathcal{D}X \, \exp\left\{-\frac{1}{4\pi\alpha'}\int d\sigma^1 d\sigma^2 \sqrt{g} g^{\alpha\beta} \partial_{\alpha} X^{\mu} \partial_{\beta} X_{\mu}\right\} \prod_{i=1}^4 V_o(p_i) \overset{\alpha' \to \infty}{\sim} \mathcal{A}_c,$$

with the positions z_i of the four open string vertices V_0 subject to the condition

$$x_0 \equiv \frac{(z_1 - z_2)(z_3 - z_4)}{(z_1 - z_3)(z_2 - z_4)} = -\frac{s}{t}.$$
 (2)

This amounts to performing the saddle-point approximation of (1) around x_0 ,

$$-s \int_{0}^{1} dx \, x^{-s-1} (1-x)^{-u} = \sqrt{\frac{2\pi as}{1-a}} B^{s} \left[1 + \sum_{n=1}^{\infty} \frac{C_{2n}}{(-s)^{n}} \right], \quad C_{2l} = (\bigstar)|_{\frac{1}{s^{l}}}, \quad a = -\frac{u}{s}, \quad (3)$$

where $B = (1-a)^{1-a}(-a)^a$. Subleading terms are suppressed in powers of $\frac{1}{\alpha'}$ and encode the effects of integrating out massive higher-spin states at high energies. In our paper, we repeat those steps for the Virasoro-Shapiro amplitude of closed strings and find that the single-value map [4]

$$\operatorname{sv}\zeta(2k) = 0$$
, $\operatorname{sv}\zeta(2k+1) = 2\zeta(2k+1)$, $k \ge 1$

is only valid in the low-energy regime ($\alpha' \to 0$), where one has

$$\mathcal{M}_0 = \pi \operatorname{sv} \mathcal{A}_0(1, 2, 3, 4) \tilde{A}_{YM}(1, 2, 3, 4).$$

String worldsheet and celestial sphere

Parametrizing massless momenta as $p^{\mu} = \omega q^{\mu}$, $q^{\mu} = \frac{1}{2}(1+|z|^2,z+\bar{z},-i(z-\bar{z}),1-|z|^2)$, with z_i,\bar{z}_i coordinates at \mathcal{I} , the celestial string amplitude is obtained as

$$\tilde{\mathcal{A}}_{\{\Delta_l\}}(\{z_l, \bar{z}_l\}) = \left(\prod_{k=1}^n \int_0^\infty d\omega_k \, \omega_k^{\Delta_k - 1}\right) \, \delta^{(4)} \left(\omega_1 q_1 + \omega_2 q_2 - \sum_{m=3}^n \omega_m q_m\right) \, \mathcal{A}(\{\omega_l, z_l, \bar{z}_l\}).$$

For (1) this yields, with \mathcal{A}'_{FT} the field-theoretic celestial gluon amplitude [2]

$$\tilde{\mathcal{A}}_{\{\Delta_i\}} = (2\pi)^{-1} (\alpha')^{\beta} \tilde{\mathcal{A}}'_{FT}(\{\Delta_i\}) a^{-\frac{\beta}{3}} (1-a)^{-\frac{\beta}{3}} I(a,\beta), \quad \beta \equiv -\frac{1}{2} \sum_{k=1}^{4} (\Delta_k - 1), \tag{4}$$

with
$$I(a,\beta) = -\Gamma(1-\beta) \int_0^1 \frac{\mathrm{d}x}{x} \left[\ln x - a \ln(1-x) \right]^{\beta-1},$$
 (5)

which has poles for $\beta \in 1, 2, 3, ...$ As $a \to 0$ or ∞ , $\tilde{\mathcal{A}} \to \tilde{\mathcal{A}}'_{FT}$, as one may write [2]

$$I(a,\beta) = \pi \delta(\beta) + \frac{i\pi}{1 - e^{-2\pi i\beta}} \sum_{n=1}^{\infty} \operatorname{Res}_{s=n} \left[(as)^{-\beta} B\left(-s, 1 + \frac{s}{r}\right) \right].$$

Thus, $\beta = n \in \mathbb{N}^*$ is a soft pole of (5) associated with the exchange of massive strings at level n. Here we perform a stationary phase approximation of $I(a,\beta)$ as $\beta \to \pm i\infty$ instead, keeping a fixed. This also localizes on the saddle (2), meaning that the worldsheet pins onto the celestial sphere as $\alpha' \to \infty$. We then match this expansion in $\frac{1}{\beta}$ with the Mellin transform of (3), and find

$$B^{\alpha'} \frac{C_{2k}}{(\alpha')^k} \leftrightarrow (\alpha')^{\beta} \frac{(-1)^k C_{2k} (\ln B)^{\beta - \frac{1}{2} + k}}{\cos(\pi \beta) \Gamma\left(\beta + \frac{1}{2} + k\right)} \sim (\alpha')^k \frac{(-1)^k C_{2k} (\ln B)^{\beta - \frac{1}{2} + k}}{\left(\beta + \frac{1}{2}\right) \cdot \dots \cdot \left(\beta - \frac{1}{2} + k\right)}.$$
 (6)

This shows that the poles at $\beta = -\frac{1}{2}, -\frac{3}{2}, \dots$ of (5) are in 1:1 correspondence with the subleading corrections in $\frac{1}{(\alpha')^k}$ in (3). In the β -plane these effects correspond to operators with $\Delta_k \geq 2$ and account for higher-spin modes in the ultra-high energy regime of string theory. From (3) we see that these are tied to combinations of $\zeta(1-2k) \in \mathbb{Q}$. Finally, $\Gamma(1-\beta) \propto |\beta|^{|\beta|}$ as $\Re(\beta) \to -\infty$, which is a manifestation of black hole dominance [5].

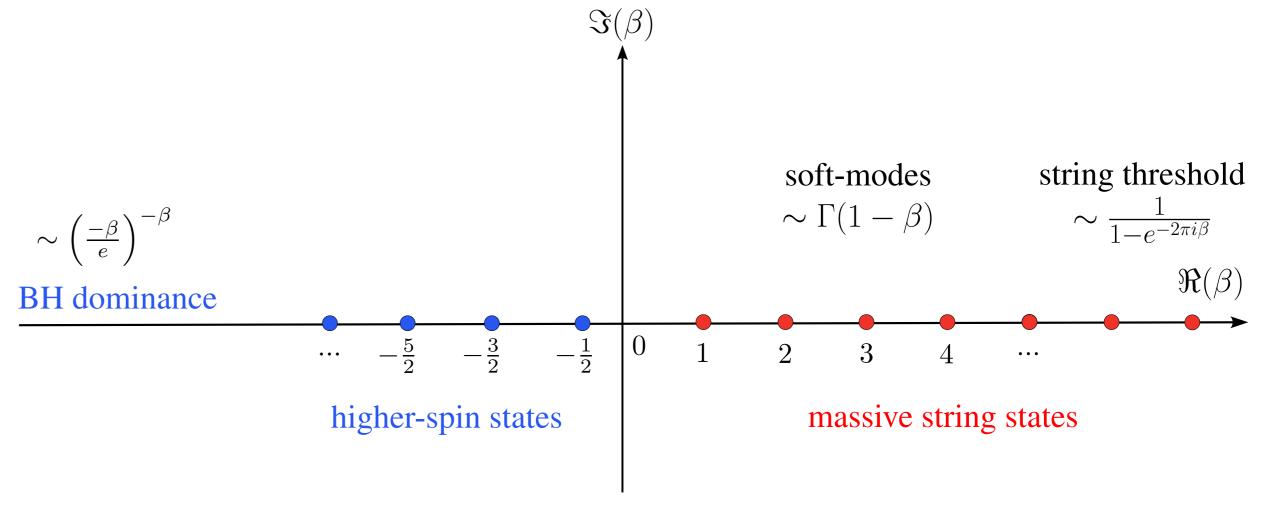


Figure 2: Complex β plane with UV (blue) and IR (red) regions and corresponding string threshold.

Conclusion

We have presented expressions for string amplitudes in the high-energy limit at tree level in flat backgrounds, highlighting their number-theoretic properties, and showing that the sv-map is only valid as $\alpha' \to 0$. We have also established the correspondence (6) between the celestial open string amplitude and the string amplitude as $\alpha' \to \infty$, order by order in $\frac{1}{\alpha'}$ and $\frac{1}{\beta}$. This is a first step towards a string realization of flat-space holography, relating the worldsheet CFT to the (less understood) CCFT.

Open questions, in no particular order:

- Generalization to $n \ge 5$? 1-loop? Define generic conditions for bijective map with CCFT?
- Could studying the $\alpha' \to \infty$ and $\beta \to -\infty$ limits shed light on properties of CCFT in the IR?
- $\alpha' \to \infty$ ~ tensionless strings, with CCS₂ \cong BMS₃ symmetry [6]. Lessons from CCFT?

• Twisted intersection theory relates $\alpha' \to 0$ and $\alpha' \to \infty$. What about the celestial side?

- Closed strings and KLT/double-copy relations? String monodromy on the celestial sphere?
- Relation to results in AdS? Cf, e.g., the work of Alday et al. [7]
- What is the Carrollian analogue of this picture?

References

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