Ballroom Intersection Protocol (BRIP) Efficient Autonomous Driving at Intersections

Reza Azimi, Gaurav Bhatia and Ragunathan (Raj) Rajkumar
Carnegie Mellon Universtiy
Emails: rezaazimi@cmu.edu, gnb@ece.cmu.edu, raj@ece.cmu.edu

***Abstract*—In order to prevent vehicles from colliding with each other at an intersection, some type of traffic control mechanism, such as stop signs and traffic lights, are provided so that traffic can navigate safely through the intersection. However, many collisions occur at intersections. In fact up to 44% of car crashes happen inside and around intersections [5]. These intersection related accidents cause approximately 8500 fatalities and over one million injuries a year in the USA. Further, because traffic traveling in one direction is generally stopped at busy intersections to allow traffic to flow in another direction, an intersection creates traffic congestion and frustration. According to 2011 Urban Mobility Report, the delay endured by the average commuter was 34 hours which costs more than $100 billion each year [1].**

Autonomous vehicles are expected to revolutionize transportation. Multiple driver-assist systems have been implemented in vehicles, and various autonomous vehicles have been demonstrated at the DARPA Urban Challenge [2]. General Motors' Electrical-Networked Vehicles (EN-V), CMU's autonomous vehicle and Google's car are just a few recently unveiled examples. Autonomously driven vehicles and controlled intersections offer an opportunity to safely and efficiently allow vehicles traveling in perpendicular or cross directions to safely navigate an intersection. In this paper, we propose a method called Ballroom Intersection Protocol (BRIP) to manage the safe and efficient passage of autonomous vehicles through intersections. To achieve high throughput at intersections, BRIP enhances the utilization of the capacity of the intersection area. In other words, it maximizes the intersection area occupied by vehicles at the same time and allows vehicles to efficiently and continuously cross without stopping behind or inside the intersection area. Our simulation results show that we are able to avoid collisions and also increase the throughput of the intersections up to 96.24% compared to common traffic light signalized intersections.

I. Introduction

The operation of modern vehicles is becoming more autonomous, i.e., being able to provide driving control with less and less driver intervention. Cruise control systems have been on vehicles for a number of years where the vehicle operator can set a particular speed of the vehicle, and the vehicle will maintain that speed without the driver operating the throttle. In adaptive cruise control systems, not only does the system maintain the set speed, but also will automatically slow the vehicle down in the event that a slower-moving vehicle ahead is detected using various sensors, such as radar and cameras. Some modern vehicles also provide autonomous parking where the vehicle will automatically provide the steering control for parking the vehicle. Some vehicle systems intervene if the driver makes harsh steering changes that may affect the vehicle stability. A few vehicle systems attempt to maintain the vehicle near the center of a lane. Further, fully autonomous vehicles have been demonstrated that can drive in urban traffic, observing all of the rules of the road. As vehicle systems improve, they will become more autonomous with the goal being a completely autonomous vehicle.

Safe and enhanced traffic throughput at busy intersections with autonomous driving is technically challenging and an unresolved problem. One approach to solve this problem has been investigating the use of vehicular ad-hoc networks. The Federal Communications Commission (FCC) has allocated 75 MHz of spectrum in the 5.9 GHz band for the use of safety applications in Intelligent Transportation Systems (ITS). Dedicated Short Range Communications (DSRC) and Wireless Access in a Vehicular Environment (WAVE) [3] have enabled the development of various safety applications based on vehicle-to-infrastructure (V2I) and vehicle-to-vehicle (V2V) communications.

Known techniques to accomplish the goal of safe and efficient intersection crossing, using vehicle-to-infrastructure(V2I) communications, typically employ a central arbiter module, such as an intersection manager, to resolve space-time conflicts between vehicles and dispatch space-time reservations to communication-equipped vehicles approaching the intersection [6], [10]. However, the practicality of such an arbiter module to resolve conflicts and reserve space-time slots in a timely manner is still unknown due to the maintenance and operation issues of a computationally complex arbiter at traffic intersections.

In previous work, we have introduced a family of vehicular network protocols to manage the safe passage of traffic across intersections [8], [9]. These completely distributed protocols rely on vehicle-to-vehicle (V2V) communications and localization to control and navigate vehicles within the intersection area. Autonomous vehicles approaching an intersection use DSRC radios to periodically broadcast information such as position, heading and intersection crossing intentions to other vehicles. The vehicles then decide among themselves regarding such questions as who crosses first, who goes next and who waits.

In this paper, we present a new method to manage traffic flow safely through the intersection while increasing the throughput and decreasing the delay at intersections. In this approach, vehicles do not stop at an intersection, and the synchronous movement of approaching vehicles permits a continuous traffic flow, in which all vehicles can cross the intersection area with a constant speed. This method includes broadcasting a synchronization signal to all vehicles approaching the intersection and broadcasting an intersection flow time to all of the approaching vehicles that identifies which travel lanes travel in which direction. The method also includes identifying an arrival synchronization pattern for all of these vehicles and controlling the speed of the vehicles traveling through the intersection and the time slots for the vehicles entering the intersection so that vehicles traveling in perpendicular or cross directions to the intersection will simultaneously travel through the intersection without colliding with each other. The name is derived from the fact that this is similar to a group of ballroom dancers all moving across a dance floor without colliding into one another.

The rest of this paper is organized as follows. Section II describes the assumptions and requirements of our proposed system. Section III introduces our Ballroom Intersection Protocol (BRIP) and some examples to clarify its efficiency and usage. In Section IV, we evaluate our protocol and demonstrate comparisons between BRIP and common intersection management systems. Section V includes the conclusion and provisioned future work.

II. Assumptions and Requirements

Road intersections have various shapes and geometries. To illustrate the core aspects of our protocol, we define an intersection as a square box which has pre-defined entry and exit points for each lane. Please note that this assumption is not a limitation to our protocol and implementation on any type of intersection is straightforward. Figure 1 shows a perfect-cross intersection with 2 lanes arriving at the intersection from each direction. The intersection area is considered to be a big grid, which is divided into smaller cells. Each cell is big enough to fit an average size vehicle in it. Cells can be virtually merged to fit in bigger-size vehicles in them.

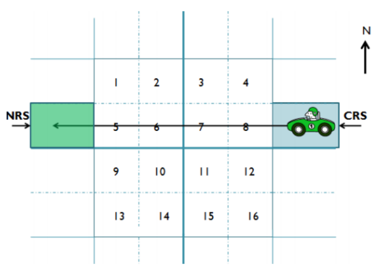

Fig. 1: Illustration of Intersection Grid, CRS and NRS

Vehicles are equipped with high-accuracy Global Positioning System (GPS) devices and have access to a digital road-map database. Based on a vehicle's intention at the intersection and the information obtained from the GPS device and map, it can figure out the following parameters.

- *Current Road Segment (CRS)*: it is the road used by the vehicle when entering the intersection area.
- *Next Road Segment (NRS)*: it is the road used by the vehicle when exiting the intersection area.

Each intersection has its own restrictions for entering lanes. For example, in some intersections, left lanes can only be used to go straight or only to turn left or a combination of both. We define the *intersection type* based on the intersection's turn restrictions. Here are three types of intersections that we analyze as examples throughout this paper.

- Type I: Vehicles on all lanes are restricted to only go straight and no turns are allowed.
- Type II: Vehicles on the left lane must go straight and vehicles on the right must turn right.
- Type III: Vehicles on the left lane must turn left and vehicles on the right lane must go straight.

As we mentioned earlier, vehicles can obtain their *CRS* and *NRS* based on their turning intentions and the information from their GPS and road-map database. However, due to intersection turn restrictions, a vehicle has to be in the appropriate lane before entering the intersection box. The information regarding the *intersection type* should be available to each vehicle. This vital information can be obtained from road signs, or road markings using the autonomous vehicle's on-board sensors. It can also be also sent using vehicle-to-infrastructure (V2I) communications from a Road Side Unit (RSU) located at the intersection to an On-Board Unit (OBU) in the vehicle. The *intersection type* could be also available using the road-map database, in which every intersection in a road-map database is marked with its *intersection type*.

Throughout this paper, we assign unique identifiers to each road segment and lane attached to the intersection area. Figure 2 shows an intersection scenario in which vehicle A is approaching an intersection from the west and going to the north. Therefore, its *CRS* is R_{west} and the *NRS* is R_{north}. Using the map database, the vehicle knows that the approaching intersection is of *type III*. Based on its *CRS*, *NRS* and the *intersection type*, it should be entering the intersection on the left lane L_{left}, in order to perform the appropriate left turn and go to the intended destination. Therefore, the vehicle is going to enter the intersection area from (R_{west}, L_{left}) and exit to (R_{north}, L_{left}) as illustrated in Figure 2.

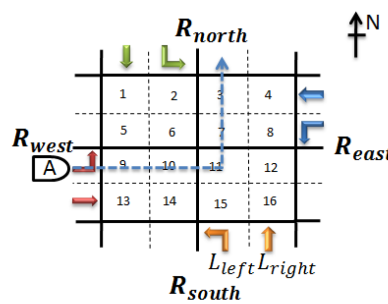

Fig. 2: Road and Lane restrictions of intersection *type III*.

III. The Ballroom Intersection Protocol (BRIP)

In this section we propose a method which controls and synchronizes the arrival of the vehicles at an intersection, and allows them to cross the intersection area efficiently and continually. The Ballroom Intersection Protocol (BRIP) is designed to enhance the capacity utilization of the intersection area. In other words, we want to maximize the intersection area occupied by vehicles at the same time without causing any accidents or deadlock situations. This will increase the throughput of the intersection as multiple vehicles are able to cross the intersection simultaneously. Additionally, the BRIP protocol allows vehicles to continuously cross the intersection area with a constant speed, without stopping before or inside the intersection box. This behavior reduces the delays faced by each vehicle and also increases passenger comfort.

As the protocol's name indicates, BRIP is built based on the main idea of the synchronized movements of participants in ballroom dancing. In this context, vehicles replace people and perform the synchronized movements in order to cross the intersection area safely and efficiently.

A. Synchronized and Staggered Arrival Pattern (SSAP)

Each intersection is assigned a predefined traffic pattern based on its turn restrictions for each entering lane. As we mentioned earlier, turn restriction information of an intersection is reported as the *intersection type*. Each vehicle approaching an intersection will receive information from a suitable source as to the particular traffic pattern for the intersection and a synchronization signal that defines how the vehicle will enter the intersection and at what speed. Using this information, vehicles will arrive at the intersection at permitted time slots with a specific speed, and will maintain that speed throughout the intersection. We refer to this information as the Synchronized and Staggered Arrival Pattern (SSAP) of an intersection.

The SSAP information can be provided to the vehicles in any suitable manner, such as through the Internet, satellite, V2V or V2I wireless communications, an intersection wireless device, radio service, cellular signals from a remote server. Alternatively, GPS timing signals can be used for time synchronization, and travel lanes/intersection entering time pattern can be residing on-board the vehicle for a given intersection.

For autonomously controlled vehicles, this information will be automatically provided to the vehicle controller where the vehicle follows the planned route through the intersection. In other cases, it is possible for the vehicle to be operated manually through the intersection as long as the vehicle driver maintains the necessary synchronization and speed of the vehicle. By providing such a system where vehicles traveling through an intersection in cross directions can simultaneously navigate the intersection, an optimal throughput and capacity utilization of a traffic intersection can be provided.

Figure 3 illustrates an example of a vehicle communications system showing a vehicle transmitting and receiving information wirelessly from various sources. The communications system includes a map database, a navigation system and an autonomous vehicle controller. The map database stores map information at any level of detail that is available, including specific information about intersections, such as the number of lanes, the lane travel patterns, etc. The map database operates in association with the navigation system to display the various maps and other information that is available, and allow a user to input, plan and display a route. Also, the map database can store the information concerning which intersection allows travel in which directions, as will become apparent from the discussion below. The controller controls the operation of the vehicle, including steering, brake, throttle, etc., if the vehicle is autonomous or semi-autonomous. The controller includes a clock that can be synchronized to vehicle SSAP at a particular intersection. The vehicle also includes a wireless port that allows the vehicle to wirelessly transmit information and receive information from many sources, such as Internet, satellite, a wireless infrastructure, etc. The wireless port also allows the vehicle to provide V2I and V2V communications.

We posit that the synchronized arrival of vehicles from different directions allows them to cross the intersection

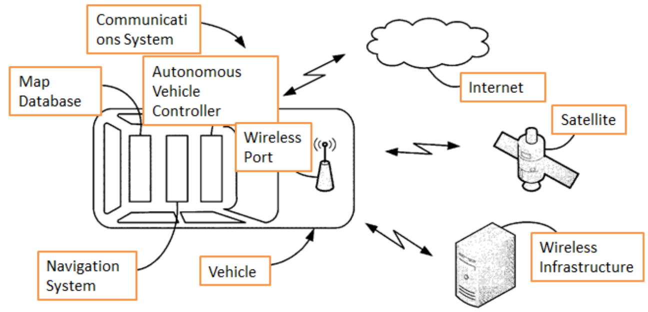

Fig. 3: illustration of a vehicle communications system showing a vehicle transmitting and receiving information wirelessly from various sources;

simultaneously. Figure 4 demonstrates an example scenario of a *type I* intersection, with 1-lane per direction. Starting at time step t_0 when all the vehicles have arrived at the intersection synchronously and have not entered the intersection boundaries yet. In the next few time steps, vehicles enter and progress inside the intersection box with at a constant speed. As we can see in Figure 4, the transition between states at time slots t_1 and t_2 is similar to the synchronized ballroom dance movements. The information regarding the required arrival time and desired speed, is provided to each vehicle as a part of the intersection's SSAP. This synchronized and continuous movement of vehicles allows them to enter the next cells along their trajectories without colliding with each other and continue at a constant speed to the point that all vehicles have safely crossed and exited the intersection area.

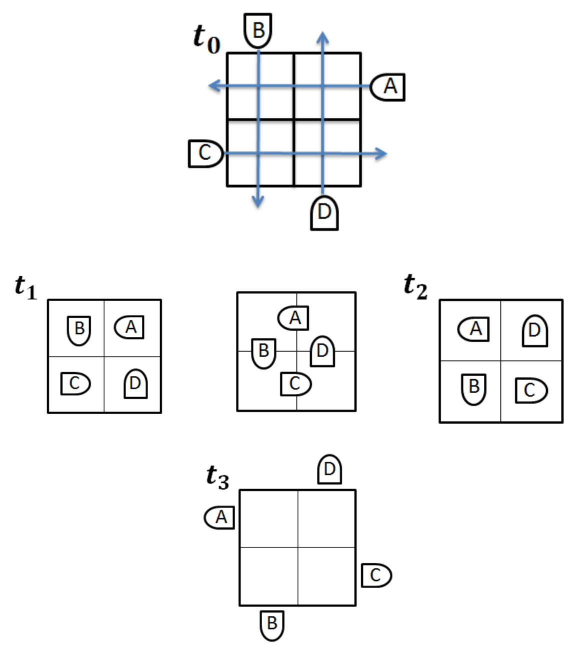

Fig. 4: An example scenario of BRIP for type I intersection

Now, we define the terms that will be used in this section.

- (r, r): Intersection cell size.
- (l, w): Vehicle's size, respectively length and width.
- $D_{x,y}$: Distance between points x and y.

- $D_{X,Y}$: Distance between the centers of vehicle X and vehicle Y.
- Γ_X: Arrival Time of vehicle X at the intersection.
- ν_X: Speed of vehicle X.
- L_X: Lane number of vehicle X.
- T: SSAP's time slot duration.

T is equivalent to the time required for the vehicle to cross an intersection cell.

$$T = \frac{r}{\nu} \tag{1}$$

Based on the BRIP, the following rules are applied to vehicle X, when it approaches an intersection.

Algorithm 1 BRIP, Receiver Vehicle

Input: *Intersection type* and SSAP
Output: Vehicle X's movement at the intersection
use turn info to be on the correct lane
use SSAP to set the speed in order to arrive at the eligible time slot

To achieve the goals of the BRIP of allowing the safe and efficient passage of vehicles through intersections, we identify the physical requirements such as the intersection size and vehicle's physical parameters, and distance constraints among following vehicles. As we mentioned earlier, the intersection grid is divided into smaller cells, each of which can fit an average-size vehicle in it. Therefore, it is logical to assume that no more than one entire vehicle fits in one cell. This can be stated as $l < r < 2 * l$.

An accident occurs if two or more vehicles occupy the same space at the same time. Figure 5 shows an example scenario of an *type I* intersection. The potential space conflict zones are the squares with size (w, w) at the center of each intersection cell, which are enumerated as $1, 2, 3, 4$. We define these potential space conflict zones as the *critical sections*. There is a potential conflict among two vehicles if there is a common *critical section* along their trajectories through the intersection.

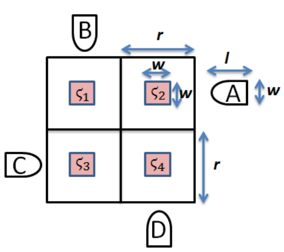

Fig. 5: *Critical Sections* for type I intersection

The following terms will be utilized in our BRIP rules and proofs:

- S_X: Set of *critical sections* along vehicle X's trajectory through the intersection.
- C_X: Current *critical section* occupied by vehicle X.

Rule 1. ***BRIP Accident Freedom Rule:***

For any vehicles A and B, if there is a common critical section along their trajectories, they cannot be inside the same critical section at the same time.

$$\forall A, B : if(S_A \bigcap S_B \neq \phi) \Rightarrow C_A \neq C_B$$

Figure 6 shows the scenario in which, vehicles A and B have entered an intersection *type I* and are now in the middle of the first cell along their trajectories. Point p_1 denotes the left-down corner of the *critical section*, which is indeed the last possible point of collision where point *a* of vehicle A might collide with point *b* of vehicle B.

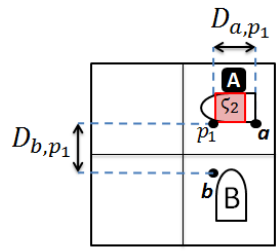

Fig. 6: Distance constraints for type I intersection

Based on the intersection's SSAP, vehicles A and B have to enter the intersection box at the same time $\Gamma_A = \Gamma_B$, and with the same speed: $\nu_A = \nu_B$, and maintain that speed through their trajectory inside the intersection region. Therefore, to avoid an accident at the time that vehicles are at point p_1 or any time before that:

$$D_{b,p_1} > D_{a,p_1} \Rightarrow \frac{r-l}{2} + \frac{r-w}{2} > w + \frac{l-w}{2}$$
$$\Rightarrow r - \frac{l+w}{2} > \frac{l+w}{2}$$

So,

$$r > l + w \tag{2}$$

We now calculate the distance and timing requirements for the vehicle arrivals which enter the intersection area following vehicles A and B. Figure 7 shows the same scenario as Figure 6, in which vehicle C is arriving at the intersection from the same direction as vehicle A. Point p_2 denotes the last possible point of collision where point *b* of vehicle B will collide with point *c* of vehicle c. To prevent

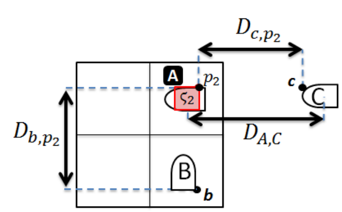

Fig. 7: Car following distance constraints for type I intersection

a collision at the time that both vehicles are at point p_2 or any time before that:

$$D_{c,p_2} > D_{b,p_2}$$
$$D_{b,p_2} = l + w + \frac{r-w}{2} + \frac{r-l}{2} = r + \frac{l+w}{2}$$

So,

$$D_{c,p_2} > r + \frac{l+w}{2} \tag{3}$$

We can now calculate the distance constraint between vehicle A and its following vehicle C:

$$D_{A,C} > D_{c,p_2} + l - \frac{l-w}{2}$$

Therefore,

$$D_{A,C} > D_{c,p_2} + \frac{l+w}{2} \tag{4}$$

From Equations 2 and 3:

$$D_{A,C} > r + \frac{l+w}{2} + \frac{l+w}{2} = r + l + w \tag{5}$$

From Equation 1, we have $r > l + w$, therefore by replacing it in Equation 5:

$$D_{A,C} > 2 * (l + W) \tag{6}$$

This means that the distance between two consecutive vehicles in the same lane and direction should be bigger than $2 * (l + w)$ to avoid any accidents with crossing vehicles in perpendicular directions. As r is bigger than $l + w$, we set the distance between two consecutive vehicles in this arrival pattern to be:

$$\Delta = 2 * r \tag{7}$$

Therefore, the inter-arrival time between those vehicles is as follows:

$$\lambda = \frac{\Delta}{\nu} = \frac{2r}{\nu} = 2T \tag{8}$$

Since the intersection is assumed to be symmetric, these constraints are applied to vehicles arriving from all four directions.[1] Using the above distance and timing constraints, we now define the arrival pattern for intersections.

Considering a 2*2 lanes intersection of *type I*, with 1-lane per direction, the arrival pattern in the intersection's SSAP should be an integer multiple of the calculated inter-arrival time:

$$\mathit{\Gamma}_X = n * \lambda = 2nT \tag{9}$$

where $n = 1, 2, ...$

Please note that not every allowed slots in the arrival pattern needs to be occupied by a vehicle, and can be left empty without causing any disruption in the system. For example, when dealing with low-volume traffic, some of the slots in the SSAP will be unoccupied and therefore, less number of vehicles will be present at the intersection area at any given time and not all the intersection cells will be occupied. This will not in any way affect the entrance flow of the vehicles and their safe passage through the intersection grid.

In the extreme case illustrated in Figure 8, where the traffic flow entering the intersection uses every assigned slot, there is always a vehicle in the dedicated arrival pattern and therefore no spot is empty in the SSAP. We refer to this situation as the *maximum entrance flow*. When dealing with a *maximum entrance flow*, we enter a steady state in which all intersection cells are occupied by crossing vehicles at any given time slot T.

[1]We are relaxing this assumption in the ongoing work.

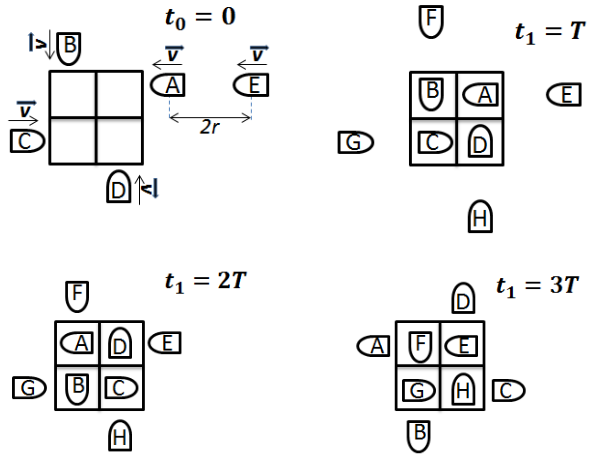

Fig. 8: Maximum entrance flow for type I intersection

As illustrated in Figure 8, from time step t_1, all intersection cells are occupied by vehicles and this steady state will last while there are vehicles arriving at all available time slots in the arrival pattern. We can also observe that the pattern is repeated after every two time slots. for example, the patterns at t_1 and t_3 are isomorphic to each other. It is straightforward to conclude that the arrival pattern's of the *type I* intersection, is periodic with the period value of $2T$. The SSAP for intersection types II and III will be provided in section III.C.

B. Deadlock Freedom

A deadlock is a situation in which two or more competing actions are each waiting for another to finish, and thus none ever does. A deadlock situation can occur inside the intersection area among the vehicles which are trying to cross the intersection at the same time. To represent these scenarios, we use *wait-for* graphs. A *wait-for* graph is a directed graph used for deadlock detection in operating systems and relational database systems. A deadlock exists if the graph contains any cycles.

We now investigate a possible deadlock scenario, in which all vehicles progress inside the intersection area as much as possible without getting to an accident with other vehicles. As we can see in Figure 9, vehicle A's next *critical section* ς_1 is occupied by vehicle B, vehicle B's next *critical section* ς_3 is occupied by vehicle C, vehicle C's next *critical section* ς_4 is occupied by vehicle D, and finally vehicles D's next *critical section* ς_2 is occupied by vehicle A. This means that none of these vehicles can progress inside the intersection grid as each of their next *critical sections* is occupied by other vehicles.

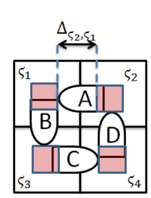

Fig. 9: Deadlock scenario

For the purpose of this paper, we define the elements of our intersection *wait-for* graph as follows. Vehicles are represented as the nodes of our *wait-for* graph, and an edge from vehicle A to vehicle B implies the vehicle B is holding a critical section that vehicle A needs to complete its trajectory through the intersection grid. Thus, vehicle A is waiting for vehicle B to release (leave) that specific *critical section*. It can be seen clearly in Figure 10 that the corresponding *wait-for* graph contains a cycle and therefore it is a deadlock situation.

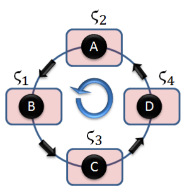

Fig. 10: Wait-for graph

We will use the following terms in our proofs:

- $\Phi_X = \varsigma_n$: Vehicle X is at the border of *critical section* ς_n, and waiting to enter ς_n.
- $\Delta_{\varsigma_n,\varsigma_m}$: Distance between two following *critical sections* ς_n and ς_m

Theorem 1. *Without loss of generality, the BRIP is deadlock-free.*

Proof: The deadlock condition is as follows:

$C_A = \varsigma_2$ and $C_B = \varsigma_1$ and $C_C = \varsigma_3$ and $C_D = \varsigma_4$

And, at the same time:

$\Phi_A = \varsigma_1$ and $\Phi_B = \varsigma_3$ and $\Phi_C = \varsigma_4$ and $\Phi_D = \varsigma_2$

The distance between *critical sections* occupied by vehicles A, B, C and D is:

$$\Delta_{\varsigma_2,\varsigma_1} = \Delta_{\varsigma_1,\varsigma_3} = \Delta_{\varsigma_3,\varsigma_4} = \Delta_{\varsigma_4,\varsigma_2} = r - w \quad (10)$$

From the deadlock condition, we have:

$C_A = \varsigma_1$ and $\Phi_A = \varsigma_2$ This means that vehicle A is partially in *critical section* ς_2 and also at the border of *critical section* ς_1. Therefore:

$$l > \Delta_{\varsigma_1,\varsigma_2} \quad (11)$$

From Equation (1), we had:

$$r > l + w$$

Therefore, using Equations (1) and (11):

$$l > \Delta_{\varsigma_1,\varsigma_2} \Rightarrow l_A > (r - w) > (l + w - w) \Rightarrow l > l \quad (12)$$

Equation (12) is a contradiction. Therefore, the deadlock conditions of $C_A = \varsigma_2$ and $\Phi_A = \varsigma_1$ cannot be true at the same time. Similar arguments apply to vehicles B, C and D. ∎

C. Multi-Lane Intersections

In this subsection, we look at multi-lane intersections. We first consider an intersection of *type I*, with 2-lanes per direction. Despite the larger number of lanes arriving at the intersections, the same distance constraints are applied to all approaching vehicles. Figure 12 shows a scenario in which two vehicles are approaching an intersection from all four directions. To avoid the collision between the vehicles arriving from perpendicular roads and different lanes, such as vehicles A approaching from (R_{east}, L_{left}) and F approaching from (R_{north}, L_{right}), we need to insert an appropriate offset in the arrival pattern of these vehicles. This offset will result in the staggered arrival of vehicles depending on their current lane information.

The minimum staggering offset required is equal to the SSAP's time slot duration, T, which is the time required for a vehicle to cross an entire cell. This offset guarantees a safe distance to avoid vehicles A and C to occupy their common *critical section* ς_5 at the same time. Therefore, the arrival pattern in the intersection's SSAP is as follows:

$$\Gamma_X = \begin{cases} 2nT, & \text{if } L_X = L_{left}. \\ 2nT + T, & \text{if } L_X = L_{right}. \end{cases} \quad (13)$$

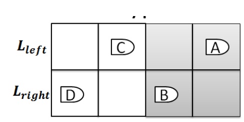

Fig. 11: Arrival pattern for type I intersections

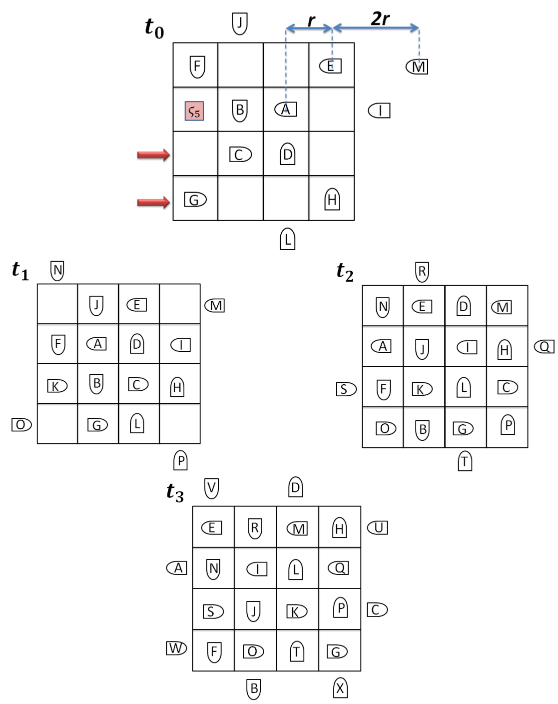

Fig. 12: Example scenario for type I intersections

It is straightforward to conclude that the arrival pattern's of the *type I* intersection, is periodic with the period value of $2T$. A simulation video of *type I* intersection can be seen at: https://www.youtube.com/watch?v=shJZZGSXW8o

We now generalize the arrival pattern for *type I* intersections. To avoid collisions between crossing vehicles, a staggering offset is inserted for each additional entering lane.

Algorithm 2 SSAP for *multi-lane type I intersections*

Input: $\Delta = 2r$ and $T = \dfrac{r}{\nu}$ and *current lane* L_X
Output: Vehicle X's arrival pattern

$$\Gamma_X = 2nT + \beta T$$

where β is the *staggering offset coefficient*.

The value of β depends on the current lane of the vehicle. β is equal to zero for the leftmost lane and its value increases with the increments of 1 toward the rightmost lane. The table below is an example of β values for a *type I* intersection with 3 lanes.

Current Lane	L_{left}	L_{middle}	L_{right}
β	0	1	2

We believe that the BRIP is the optimal method to manage the traffic through *type I* intersections. The following terms will be used:

- O_i: Number of occupied cells at time slot i
- N: Total number of intersection cells
- Ψ: Steady state's period. $O_i = O_{i+\Psi}$
- ICU: Intersection capacity utilization

We define the Intersection Capacity Utilization (ICU) as the fraction of occupied intersection cells to the total number of cells when there is a *maximum entrance flow* for the duration of the steady state period. This can be also written as follows:

$$ICU = \frac{\sum_{i=j}^{j+\Psi-1} O_i}{\Psi * N} \tag{14}$$

where j indicates any time slot during the steady state.

In the example scenario of Figure 12, if there is a *maximum entrance flow*, we will reach the steady state at time slot t_2. All the 16 intersection cells are occupied at t_2, t_3 and the next time slots. This situation holds for all future time slots if no malfunctioning occurs. Since the number of occupied cells is equal to the total number of intersection cells during the steady state period, the ICU is equal to 100%. This is indeed the maximum intersection capacity utilization.

We conclude that by following the intersection's SSAP rules, the BRIP fro *type I* intersection, provides the optimal intersection management by allowing the continuous entrance flow of the vehicles, and intersection crossing with constant speed while utilizing the maximum capacity of the intersection space.

We now study *type II* intersections with 2-lanes per direction. Figure 14 shows an example of such an intersection. In this case, vehicles on the left lane must go straight, and vehicles on the right lane must turn right. Vehicles on the left lane have the same constraints as in *type I* intersections, therefore they have a similar arrival pattern. On the other hand, as vehicles on the right lane do not have any trajectory conflicts with other vehicles at the intersection, they can cross the intersection at any time slot and their arrival pattern can be back-to-back. The SSAP of intersection *type II* is as follows:

$$\Gamma_X = \begin{cases} 2nT, & \text{if } L_X = L_{left}. \\ nT, & \text{if } L_X = L_{right}. \end{cases} \tag{15}$$

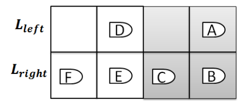

Fig. 13: Arrival pattern for type II intersections

It is straightforward to conclude that the arrival pattern's of the *type II* intersection, is periodic with the period value of $2T$. A simulation video of *type II* intersection can be seen at: https://www.youtube.com/watch?v=LKj2YscDk8s

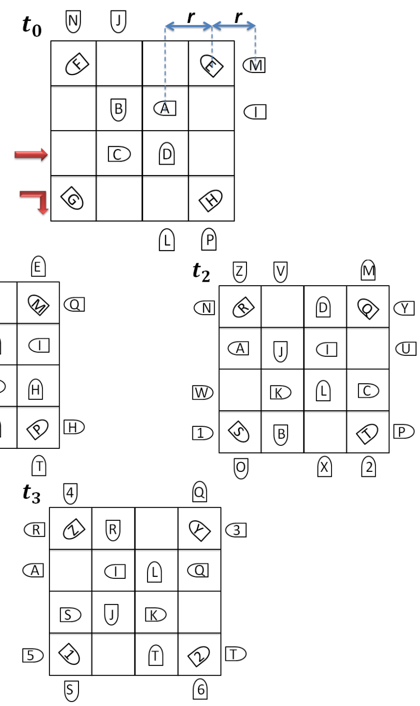

Fig. 14: Example scenario for type II intersections

Next, we study *type III* intersections with 2-lanes per direction. Figure 16 shows an example of such an intersection. Vehicles on the right lanes must go straight. In contrast, vehicle on the left lanes must turn left, and the left turn is longer in distance and each vehicle occupies more *critical sections* to complete its trajectory along the intersection grid. This affects the arrival pattern, since the following vehicles on the left lanes should arrive only when there is no potential collision with currently crossing vehicles. On the other hand, vehicles on the right lane are going straight and can have the same arrival pattern to avoid collisions with vehicles on the right lanes of perpendicular roads. Additionally they might collide with vehicles on the left lane from the opposite direction. These conflicts can also be resolved by inserting a staggering offset of T, so that no common *critical section* is occupied by more than one vehicle at any given time slot. The SSAP of intersection *type III* is as follows:

$$\Gamma_X = \begin{cases} 3nT, & \text{if } L_X = L_{left}. \\ 3nT + T, & \text{if } L_X = L_{right}. \end{cases} \tag{16}$$

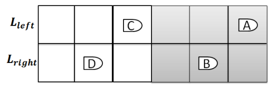

Fig. 15: Arrival pattern for type III intersections

It is straightforward to conclude that the arrival pattern's of the *type III* intersection, is periodic with the period value of $3T$. A simulation video of *type III* intersection can be seen at: https://www.youtube.com/watch?v=dQM0e5ilE_c and https://www.youtube.com/watch?v=pPojYhVkGXI

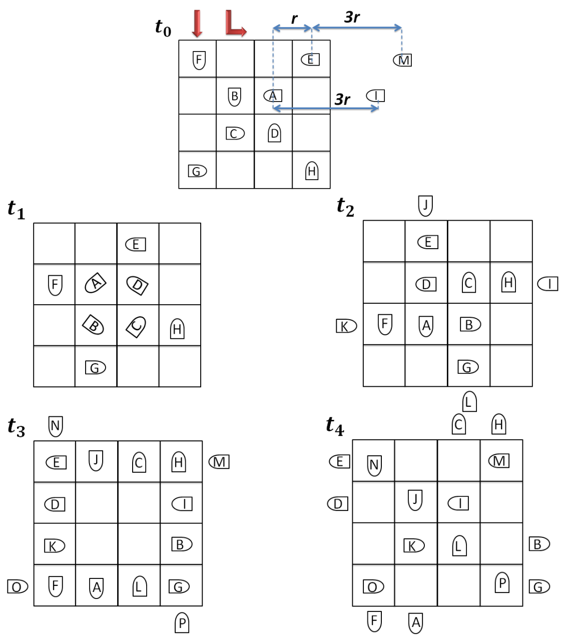

Fig. 16: Example scenario for type III intersections

We ca now generalize the SSAP for 2-lanes intersections of any type.

$$\Gamma_X = \alpha nT + \beta T \tag{17}$$

where α and β are respectively the *periodic pattern coefficient* and the *staggering offset coefficient*.

The value of α depends on the number of central critical sections needed for the traversal of vehicles on the left lane. For intersections *type I and II*, as can be seen in Figures 12 and 14, left-lane vehicles occupy two central critical sections along their trajectories inside the intersection grid. For *type III* intersections, as can be seen in Figures 16, three central critical sections will be occupied to perform a left turn by each vehicle arriving on the left lane of any direction. Table I shows the value of α for intersection types I, II and III.

Intersection Type	I	II	III
α	2	2	3

TABLE I: *Periodic Pattern Coefficient*

We conclude that each vehicle can figure out its eligible arrival slots using its current lane and the approaching intersection type. The vehicle then adjusts its velocity, to arrive at the intersection at the allowed time slot and also with the desired speed which is enforced by the intersection's SSAP and should maintains that speed throughout its intersection traversal.

D. Intersection Cascades

In this subsection instead of an isolated intersection, we consider intersection cascades in which multiple intersections are connected in a row. By looking at previous examples of different intersection types in Figures 12,14 and 16, one can see that the exit pattern of the vehicles from each direction of the intersection is exactly similar to the arrival pattern of that direction. Therefore, in the case that intersection cascades are of the same type, vehicles will have the same arrival pattern from the first intersection to enter the next intersections. To achieve a higher throughput and eliminate the need for speed adjustments when traveling between these intersections, the desired arrival speed to the first intersection should be calculated based on the distance between two consecutive intersections. In this case, the vehicle can maintain the same speed and arrive to the second intersection at an eligible time slot, without any speed adjustments.

In other cases, when the connected intersections are not of the same type, the exit pattern of one is not similar to the arrival pattern of the next intersection. Thus, each vehicle is required to adjust its velocity to adopt with the arrival pattern of the upcoming intersection and enter each intersection at that intersection's eligible time slot and desired speed. Please note that the value of the desired speed is tightly dependent of the distance between intersection cascades and vehicle's physical characteristics such as acceleration and deceleration parameters. This might be problematic when the road length between two consecutive intersections is too short. To solve this problem, the desired speed for vehicles to enter the first intersection should be compatible with its value for the second intersection. This allows vehicles to adjust their velocity before entering the first intersection and enter the following intersection cascades without drastic changes in its velocity.

E. Anomalies and Mishaps at the Intersection

V2I and/or V2V wireless communications can be employed to deal with anomalies such as vehicle breakdowns, flat tires or other mishaps. Anomalies can also include passage of emergency vehicles such as police cars, ambulances and fire trucks. V2I and/or V2V wireless communications can be used for recovery and collision avoidance in case of such anomalies. Once the transient has passed, normal operation resumes. For autonomously controlled vehicles, this information will be automatically provided to the vehicle controller where the vehicle follows the planned route through the intersection. In other cases, it is possible for the vehicle to be operated manually through the intersection as long as the vehicle driver maintains the necessary synchronization and speed of the vehicle. To deal with error margins, we suggest increasing the required distance between following vehicles by having more empty slots in the arrival pattern. By providing such a system where vehicles traveling through an intersection in cross directions can safely and simultaneously navigate the intersection, maximal throughput and capacity utilization of a traffic intersection can be provided.

F. Non-Vehicular Application

Despite the fact that our Ballroom Intersection Protocol (BRIP) is designed to manage the traffic through intersections, it is applicable to a variety of intelligent applications. There is a significant need for more efficient setup and management of manufacturing lines and distribution centers. We believe that BRIP can be easily deployed in manufacturing lines of companies to reduce the required physical space, avoid any potential collisions of products and significantly increase the throughput of these lines. Additionally, it is beneficial for distribution systems. An example of these methods is the Kiva systems of Amazon.com [4]. Kiva robots have been deployed to reduce labor requirement and increase efficiency by smart transportation of boxes in distribution facility centers. BRIP is suitable for managing the movement of Kiva robots. Our intersection management protocol can be implemented to increase the safety and efficiency of these small and low-speed robots.

IV. Implementation and Evaluation

In this section, we briefly describe the implementation of our intersection protocols in our hybrid simulator-emulator, called AutoSim. We also analyze our work by comparing the performance of BRIP rules intersections to current intersection management technologies such as stop signs and traffic lights.

A. AutoSim

AutoSim simulator-emulator is an extension to GrooveNet [7] with 3-D graphics and other capabilities. AutoSim is a modular-based hybrid simulator-emulator for vehicular networks. The hybrid capability of AutoSim enables the interaction among simulated vehicles and real vehicles equipped with DSRC radios. It also benefits from real street-map-based topography, which enables city-wide simulations. AutoSim consists of various core individual models such as mobility, communication, control and pose estimation for each simulated vehicle.

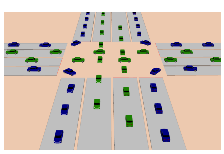

Fig. 17: Snapshot from AutoSim

B. Metric

The *maximum entrance rate* φ is defined as the factor of the maximum number of vehicles allowed to enter the intersection at each slot of the intersection SSAP. This is calculated for each direction entering the intersection grid as follows:

$$\varphi = \frac{C_{\alpha T}}{\alpha} \tag{18}$$

, where C is the number of vehicles entering during the intersection's arrival pattern period αT.

Our other metric is the trip time for a vehicle, and is defined as the time taken by that vehicle to go from a fixed start-point before the intersection to a fixed end-point after the intersection. We calculate the trip time for each simulated car under each model and compare that against the trip time taken by the car assuming that it stays at a constant street speed and does not stop at the intersection. The difference between these two trip times is considered to be the Trip Delay due to the intersection. We take the average trip delays across all cars in a simulation sequence as our metric of comparison.

C. Scenarios

Since there is a large variation in intersection types, we restrict our attention to Four-way Perfect-Cross Intersections, in which the intersection legs are at perfect right angles to the neighboring leg. In our simulations, the traffic generation follows the Poisson random distribution. Our simulations range from very sparse traffic rural intersections to very busy urban intersections with the mean value of 3000 cars per hour in each direction. Each vehicle is removed from simulation when it reaches its destination. Each simulation run uses 1000 vehicles, and each run is terminated when the last vehicle reaches its destination.

D. Experimental Results

Table II shows the *maximum entrance rate* for intersection types I, II and III. The approaching vehicles follow the BRIP rules, and the Intersection Capacity Utilization (ICU) values calculated for those intersection types in Table II. For *type I* intersections, as can be seen in Figures 11 and 12, allowing more than 2 vehicles per direction in the 4 existing slots violates the BRIP accident freedom rule and distance constraints. Since this leads to potential collisions with other crossing vehicles at future time slots inside the intersection grid, the calculated value indicates the *maximum entrance rate* of the *type I* intersection.

Intersection *type II* has the highest *maximum entrance rate* among the considered intersection types. Since the vehicles on the rightmost lane turn right and have no potential conflict with other vehicles at the intersection, there is no extra gap between any two following vehicles on these lanes. Therefore, they can enter the intersection area back-to-back at every SSAP time slot. On the other hand, the lowest value of *maximum entrance rate* belongs to *type III* intersection, where the left turn trajectory occupies more *critical sections* and more potential trajectory conflicts with other crossing vehicles. Therefore, the safe distance between following vehicles is increased to avoid any collisions inside the intersection grid, and this decreases the *maximum entrance rate* of this type of intersections.

Intersection Type	I	II	III
φ	$8/2 = 4$	$12/2 = 6$	$8/3 = 2.66$
$ICU(\%)$	100	75	56.25

TABLE II: *Maximum entrance rate and Intersection capacity utilization*

We have compared the common traffic light models to our proposed Ballroom Intersection Protocols for three different intersection types. Figures 18 and 19 show this comparison for perfect-cross intersections of *types I, II and III* respectively. The traffic is assumed to be symmetric, meaning equal amount of traffic volume in every direction and an equal amount of turn ratios. The X-axis is the traffic volume determined as cars per second and the Y-axis is the delay in seconds.

As we expected, traffic light models face significantly higher delays than our BRIP models. In the case of the traffic light models, vehicles arriving at the intersection when there is a red light, have to stop for the other directions with a green light. These vehicles must wait for the green light even if there are no other vehicles on perpendicular roads. In contrast, BRIP models increase the parallelism and allow the continuous passage of vehicles from all entering directions.

Under the BRIP rules, there is no delay at the entrance of the intersection and vehicles can cross with the constant desired speed based on the intersection's Staggered and Synchronized Arrival Pattern. The only source of delay is from vehicle decelerations in order to adjust their speed to the desired speed, and to obey the car following distance constraints, so that the vehicle enters the intersection grid at allowed time slots.

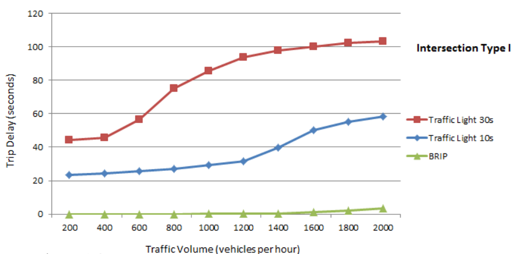

Fig. 18: Delay comparison for type I intersections

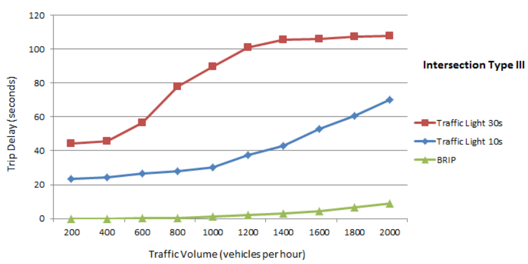

Fig. 19: Delay comparison for type III intersections

As can be seen in Figures 18 and 19, BRIP model outperforms the traffic light models significantly. For example, in the case of a *type I* intersection, BRIP has no delay for low-volume traffic and a negligible delay for medium and high-volume traffic. BRIP improves the performance by 98.03%, 96.24%, respectively comparing to the traffic light models with the green light durations of 30 seconds and 10 seconds.

Next, we look at asymmetric traffic where major and minor roads intersect. In this case, the traffic volume is significantly different on the intersection roads. Therefore, under BRIP rules most of the allowed slots in the major road will be occupied. On the other hand, for minor roads of the same intersection many slots are left empty without causing any disruptions. A simulation video of this type of intersections can be seen at: https://www.youtube.com/watch?v=CH76UH5Z014.

Figures 20 shows this comparison for a *type I* intersection. As can be seen in this Figure, our BRIP model is significantly outperforming the traffic light models. BRIP decreases the delays by approximately 95.4% over the traffic light model with green light duration of 10 seconds. Our results show that in contrast to the traffic light models, BRIP is suitable for asymmetric intersections and manages the traffic with a very high throughput and similar to symmetric intersections.

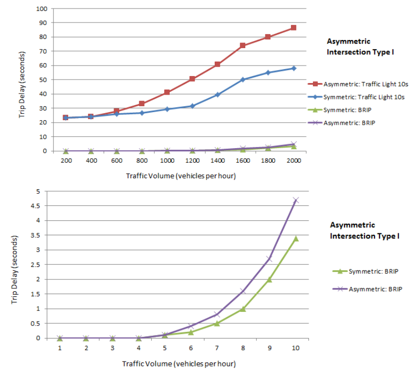

Fig. 20: Delay comparisons for symmetric and asymmetric type I intersections

V. Conclusion

The current technologies for managing the traffic through intersections, such as stop signs and traffic lights, are not very safe and efficient. High number of vehicle crashes and significant delays faced by commuters at intersections has motivated this work. In this paper, our goal was to design new intersection management methods which significantly increase the throughput of the intersections by 96.24%, and avoid collisions. Our results indicate that our Ballroom Intersection Protocols (BRIP) benefit from properties such as freedom from deadlock and allows all vehicles to cross the intersection grid with a constant speed without stopping before the intersection area. The synchronous movement of approaching vehicles permits a continuous traffic flow and significantly increase the traffic throughput.

In our ongoing work, we are addressing the following limitations. As mentioned in this paper, we have currently assumed the same size for all the vehicles. We want to deal with larger-size vehicles and study their effects on the throughput of our protocols. Additionally, we are looking at the speed and arrival pattern requirements for various *intersection types*, as well as general solution to deal with short distances between intersection cascades in order to maximize the overall throughput for all the consecutive intersections.

References

[1] The 2011 urban mobility report, published by the texas transportation institute. available: http://mobility.tamu.edu/ums/.

[2] The darpa urban challenge, 2007. available: http://www.darpa.mil/grandchallenge.

[3] Draft sae j2735-200911 dedicated short range (dsrc) message set dictionary.

[4] Kiva systems. available: http://www.darpa.mil/grandchallenge.

[5] Us department of transportation-federal highway administration publication, national agenda for intersection safety. available:http://safety.fhwa.dot.gov/intersection/resources/intersafagenda/.

[6] K. Dresner, P. Stone. A multiagent approach to autonomous intersection management. 2008.

[7] R. Mangharam, D. S. Weller, R. Rajkumar,P. Mudalige and Fan Bai. Groovenet: A hybrid simulator for vehicle-to-vehicle networks.

[8] R.Azimi, G. Bhatia, R. Rajkumar, P. Mudalige. Interesection management using vehicular networks, society for automotive engineers (sae) world congress. April 2012.

[9] R.Azimi, G. Bhatia, R. Rajkumar, P. Mudalige. Stip: Spatio-temporal intersection protocols for autonomous vehicles , acm/ieee 5th international conference on cyber-physical systems. April 2014.

[10] Richard K. Shulman, Mike; Deering. Crash avoidance metrics partnership (camp) - second annual report,sponsored by nhtsa, washington d.c., april 2002 - march 2003, dot hs 809 663.