

Mathematical Proof of Effectiveness of Platoon-based Traffic Control at Intersections

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Abstract—It was found that the platoon-based algorithm provides better performance at intersections than the conventional signal timing algorithms. To the best of our knowledge, researches have not proved formally the effects of platoon-based algorithm on the traffic control. In this paper, a Petri Nets model is proposed to describe the traffic behavior at intersections. Then based on the structure analysis of the model, it is proved that in order to minimize the total queue length at any time, the platoon should not be interrupted as far as possible. According to the result, a simple traffic control algorithm is proposed for cooperative driving at intersections. Further, through computer simulations, it is shown that the platoon-based algorithm outperforms other traffic control strategies.

I. INTRODUCTION

A vehicle platoon is defined as a group of vehicles traveling together [1]. Its effects on traffic control at highway and intersections have been studied for a long time. Since 1950s, researchers have started the study of characteristics of platoon. Mathematical models for describing vehicle platoon movements have developed [2-4]. The most famous project PATH (Partners for Advanced Transit and Highways) has demonstrated that the operation of AHS (Automated Highway Systems) vehicles in automated platoons could increase the capacity per lane of passenger cars by factors of 2-3 above the conventional lane capacity [5]. Authors in [6] used a platoon identification system to optimize traffic flow at intersections. Jiang [7] used platoons as a design parameter of traffic signal timing to minimize traffic delay at major-minor type of intersections. All these researches have shown that the platoon-based algorithm provides better performance than the conventional signal timing algorithms. However, the inherent reason why the platoon-based traffic control strategy can improve traffic efficiency has not been studied.

In this paper, we aim to prove formally the effects of platoon-based traffic control at intersections by a Petri Nets model. The reminder of this paper is organized as follows. First, a Petri Nets model is proposed to describe vehicles behavior at intersections. Next, based on this model, the analysis is carried out to prove that in order to minimize the

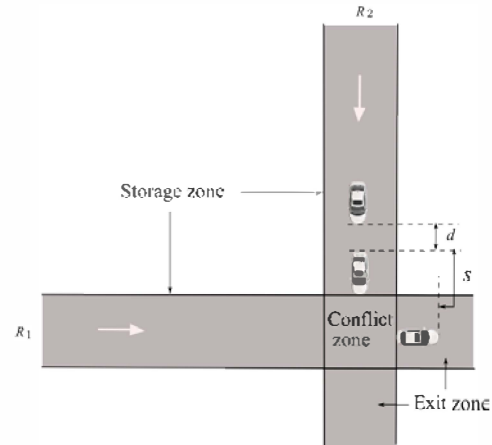


Figure 1. Configuration of an intersection with two lanes

total queue length at intersections, the switching of the right-of-way (r.o.w) between conflicting movements must be avoided as far as possible. In other words, platoons should not be interrupted as possible as we can. A simple platoon-based control strategy is proposed according to the result. Finally we evaluate the proposed algorithm through simulations and conclude with a few remarks.

II. PROBLEM DESCRIPTION

Before introduce the Petri Nets model, let us consider vehicles' behaviors at a 2-way intersection (Fig.1). The intersection can be divided into three zones: the storage zone, the conflict zone and the exit zone. To ensure the safety in the conflict zone, r.o.w.s are assigned to vehicles of non-conflicting (compatible) movements.

1. d : the minimum safe headway between two successive vehicles on the same lane;
2. S : the minimum safe headway between two successive vehicles from conflicting movements.

S takes into account the start-up lost time [8] and the time for clearance of the conflict zone. In the reminder, we will call d and S safe headways.

We assume that when the headway of two successive vehicles on the same lane is not greater than d , they constitute a platoon.

III. MODELING WITH PETRI NETS

Petri Nets (PN) is a promising tool for describing and studying discrete event systems. The idea of applying the Petri Nets (PN) to model traffic networks is not new. It dates back at least to 1986, a colored PN is applied to model the traffic light control at intersections by Jensen [9]. In [10], a hybrid model

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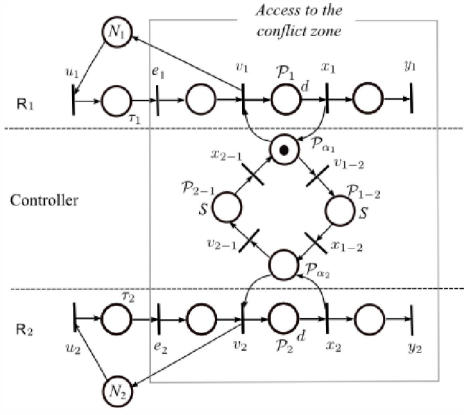


Figure 2. Timed PN model of 2-way intersection with traffic control

is proposed to model an urban network of signalized intersections, where traffic state is represented by a continuous model and the traffic light control is modeled by a discrete model. More recently, models based on dioid algebra [11] are used to model the presence of vehicles individually, while respecting certain macroscopic properties of traffic. The model presented in [12] can describe three state of traffic conditions (fluid, saturated, and congested) and consider each vehicle individually.

A. Preliminaries

Petri Nets are a graphical and mathematical modeling tool for describing and studying discrete event systems. A PN is a directed graph consisting of two kinds of nodes, called places and transitions. Places are drawn as circles, transitions as bars or boxes. In modeling, places represent conditions or states of the system and transitions represent events or actions which cause the change of state. A formal definition of a Petri nets is a 5-tuple, $PN = (P, T, F, W, M_0)$, where:

- $P = \{p_1, p_2, \dots, p_m\}$ is a finite set of places;
- $T = \{t_1, t_2, \dots, t_n\}$ is a finite set of transitions;
- $F \subseteq (P \times T) \cup (T \times P)$ is a set of arcs; Each arc simply connects a place with a transition or a transition with a place;
- $W : F \rightarrow \{1, 2, 3, \dots\}$ is a weight function;
- $M_0 : P \rightarrow \{0, 1, 2, 3, \dots\}$ is the initial marking.

Furthermore, $P \cap T = \emptyset$ and $P \cup T \neq \emptyset$.

The presence of tokens in a place indicates that resources are available or the truth of the condition is held. Pictorially, tokens are represented as black dots. The state or marking of a PN is defined by the number of tokens in each place. A state is changed according to the transition firing rule. A transition t is said to be enabled if each input place p of t is marked with at least $w(p, t)$ tokens, where $w(p, t)$ is the weight of the arc from p to t ; a firing of an enable transition t removes $w(p, t)$ tokens from each input place p to t , and adds $w(t, p')$ tokens to each output place p' of t , where $w(t, p')$ is the weight of the arc from t to p' . The firing represents an occurrence of an event or an action taken.

TABLE I. TRANSITION MEANING

Transitions	Meaning
$e_i(t)$	Number of vehicles ready to cross the conflict zone from R_i until the time t
$e_i(k)$	Time when the k th vehicle of R_i is ready to cross the conflict zone
$v_i(t)$	Number of vehicles that have accessed to the conflict zone from R_i until the time t
$v_i(k)$	Time when the k th vehicle from R_i accesses to the conflict zone
$y_i(t)$	Number of vehicles from R_i that have left the conflict zone until the time t
$y_i(k)$	Time when the k th vehicle of R_i leaves the conflict zone
v_{i-j}	Beginning of the safe headway S
x_{i-j}	End of the safe headway S

B. PN model of a 2-way intersection

A Timed PN model is presented in Fig.2 to model vehicles' behaviors at a 2-way intersection. A timed PN model is a PN model where time delays are associated with transitions and/or places. The model is composed of three sub-models. From top to bottom, the sub-model R_1 , the sub-model controller, the sub-model R_2 . They describe events related to observations of the vehicles' behaviors at intersection. The meaning of transitions is given in Table I.

First, let us focus on the sub-models R_1 and R_2 . The two models are symmetric. They describe vehicles' behaviors in the road R_i , $i \in \{1, 2\}$. The presence of a token in each place means that a vehicle crosses the concerned zone. τ_i denotes the lower bound of time taken to cover the road R_i . N_i is the capacity of the road R_i . Place P_i is timed to hold the minimum safe headway d between two successive vehicles when they cross the conflict zone.

Next, we concentrate on the sub-model of the control system. This model prevents two conflicting vehicles from crossing the intersection at the same moment. In addition, this model describes the state of the switching of the r.o.w. Place P_{i-} is an immediate place which denotes the state of r.o.w. Transition v_{i-j} decides the distribution of r.o.w. Place P_{i-j} is timed to hold the safe headway S when the r.o.w switches from one conflicting movement to another.

IV. PROOF OF THE EFFECTIVENESS OF PLATOON-BASED CONTROL

As described before, the objective of this paper is to prove that the platoon-based traffic control strategy will increase the traffic efficiency at intersections. We consider the sum of queue lengths as the metric for evaluating the effectiveness of the traffic control strategy. Indeed, there is a strong relation between intersection throughput, vehicle delays and queue length [13].

Based on the model in Fig.2, we can write the behaviors in the part of controller (places P_{i-}) as follows:

$$\begin{cases} v_1(t) + v_{1-2}(t) \leq 1 + x_1(t) + x_{2-1}(t) \\ v_2(t) + v_{2-1}(t) \leq x_2(t) + x_{1-2}(t) \end{cases} \quad (1)$$

subject to the constraints:

$$\begin{cases} v_i(t), x_i(t), v_{i-j}(t), x_{j-i}(t) \geq 0, i, j \in \{1, 2\}, i \neq j \\ v_i(t) \leq e_i(t) \\ 0 \leq v_{1-2}(t) - v_{2-1}(t - S) \leq 1 \end{cases}$$

ALGORITHM I. PLATOON-BASED CONTROL STRATEGY

- 1 A vehicle has the r.o.w if:
 - (a) It is the nearest one at the intersection;
 - (b) If its lead vehicle possesses r.o.w and the headway is less than d ;
- 2 Otherwise, it stops in front of the conflict zone and waits for r.o.w.

Suppose $Q(t)$ denotes the sum of queue lengths at time t . Let $q_1(t)$ and $q_2(t)$ denote the queue length in the roads R_1 and R_2 at time t , respectively. It is easy to infer that $q_i(t) = e_i(t) - v_i(t)$.

Recall that the objective is to minimize $Q(t) = q_1(t) + q_2(t)$ for any t . The following proposition helps us to prove the effectiveness of the platoon-based control.

Proposition 1 The optimal solution of $\min_t Q(t)$ minimizes the times of switching of the r.o.w between conflicting movements.

Proof. Let us consider that the PN model presented in Fig.2 functions at maximum speed. In other words, if a road is empty, a switching of r.o.w is enforced. Thus, inequalities in (1) become equalities:

$$v_1(t) - x_1(t) = 1 + x_{2-1}(t) - v_{1-2}(t) \quad (2)$$

Since $x_1(t) = v_1(t - d)$ and $x_{2-1}(t) = v_{2-1}(t - S)$, (2) can be rewritten as:

$$v_1(t) - v_1(t - d) = 1 + v_{2-1}(t - S) - v_{1-2}(t) \quad (3)$$

For simplicity's sake, let d be the unit of time and $S = s' \times d$, (3) turns to :

$$v_1(t) - v_1(t - 1) = 1 + v_{2-1}(t - s') - v_{1-2}(t) \quad (4)$$

Suppose $\Delta v_1(t) = v_1(t) - v_1(t - 1)$, replace it in $v_1(t)$ and have:

$$\begin{aligned} v_1(t) &= v_1(t) - v_1(t - 1) + v_1(t - 1) - v_1(t - 2) + \dots + v_1(0) \\ &= \Delta v_1(t) + \Delta v_1(t - 1) + \dots + \Delta v_1(0) = \sum_{i=0}^t \Delta v_1(i) \end{aligned} \quad (5)$$

Replace (4) in (5), we obtain:

$$v_1(t) = \sum_{i=0}^{t-s'} v_{2-1}(i) - \sum_{i=0}^t v_{1-2}(i) + t + 1$$

Apply the same analysis process for the road R_2 and we have:

$$v_2(t) = \sum_{i=0}^{t-s'} v_{1-2}(i) - \sum_{i=0}^t v_{2-1}(i)$$

Hence, the objective function $Q(t)$ becomes:

$$Q(t) = \sum_{i=1}^t e_i(t) + \sum_{i=t-s'+1}^t v_{1-2}(i) + \sum_{i=t-s'+1}^t v_{2-1}(i) - t - 1 \quad (6)$$

From equation (6), we can see that since $v_{1-2}(t)$ and $v_{2-1}(t)$ are monotonic increasing function, each firing of transitions v_{1-2} and v_{2-1} increases the objective function $Q(t)$. Hence, the switching of the r.o.w must be avoided as far as possible. In other words, when the headway between two successive vehicles from the same road is not greater than d , it is better to constitute the platoons. And a platoon should not be interrupted while traveling through the intersection.

From this proposition, we can give a simple control strategy for cooperative vehicles (Algorithm I). Cooperative vehicles [14] are equipped with an on-board unite which is allowed to communicate with other vehicles or infrastructure through wireless communication. In Algorithm I, vehicles communicate with its neighbor. If each vehicle whose distance to the lead vehicle is not greater than d , it constitute a platoon with the lead vehicle. Vehicles in the same platoon pass the intersection without interruption. It is noted that for the simplicity's sake, we assume that the PN model functions at maximum speed. It implies that there is at least one vehicle in both roads ready to cross the conflict zone. In this case, we propose a simple rule to distribute the r.o.w, i.e., the nearest one to the conflict zone obtains the r.o.w.

V. EXTENSIONS

Proposition 1 can be easily applied to isolated intersections with different layouts. For example, let us consider a typical intersection with 4 ways as illustrated in Fig.3. A Timed PN with Multipliers model (TPNMs) is presented in Fig.4 to describe the traffic control at this intersection. A TPNMs is a Timed PN with natural numbers (multipliers) associated to arcs.

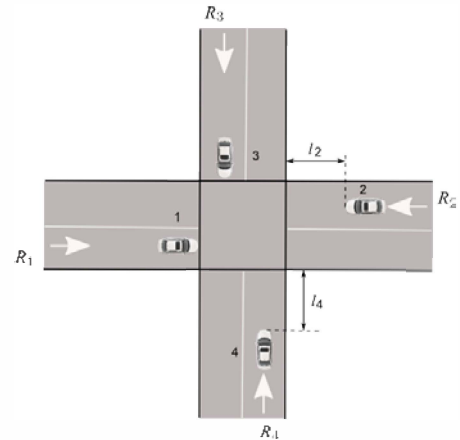


Figure 3. 4-way intersection

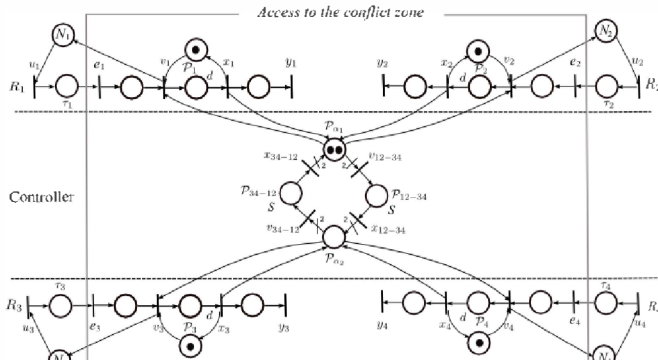


Figure 4. TPNMs model of traffic control at a 4-way intersection

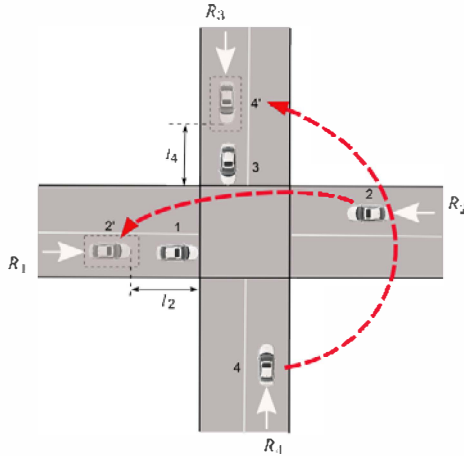


Figure 5. Virtual vehicles generated by mapping technology

The conflicting part of the model (places \mathcal{P}_i) can be described as follows:

$$\begin{cases} v_1(t) + v_2(t) + 2 \times v_{12-34}(t) \leq 2 + x_1(t) + x_2(t) + 2 \times x_{34-12}(t) \\ v_3(t) + v_4(t) + 2 \times v_{34-12}(t) \leq x_3(t) + x_4(t) + 2 \times x_{12-34}(t) \end{cases} \quad (7)$$

subject to the constraints:

$$\begin{cases} v_i(t), x_i(t) \geq 0, i, j \in \{1, 2, 3, 4\} \\ v_{12-34}(t), x_{12-34}(t), v_{34-12}(t), x_{34-12}(t) \geq 0 \\ v_i(t) \leq e_i(t) \\ 0 \leq v_{12-34}(t) - v_{34-12}(t - S) \leq 1 \end{cases}$$

Recall that in section IV, in order to hold the equality of (1) we have assumed that the PN model functions at the maximum speed. For inequalities (7), in order to hold the equality, we must assume that each road in non-conflicting movement has at least one vehicle prepared to cross intersection. However, this supposition is contradictory to most of real scenarios. For example, in a non-conflicting movement, there is at least one vehicle ready to cross the intersection in one road while in another road, no vehicle has prepared (as illustrated in Fig. 3). In this case, the equality of (7) cannot be held.

To solve the problem, we use the concept of “virtual vehicle” which is introduced by Uno [15] for the merging control of platoon in the highway. In this concept, virtual vehicles are generated and mapped onto another lane. Hence, it is possible to deduce the precedence relationship between vehicles according to their distances. Take an example in

Fig.5, since the movement of the vehicle 2 does not conflict with that of the vehicle 1, a virtual vehicle 2' can be generated in the road R_1 with the same distance l_2 to the conflict zone. A virtual vehicle 4' is generated in the road R_3 in the same way.

A new PN model is presented (on the left of Fig.6) to describe this new system after mapping vehicles. The model is similar to that introduced in Fig.2, which is also composed of three sub-models. From top to bottom, a sub-model R_{12} for the roads R_1 and R_2 , a sub-model controller and a sub-model R_{34} for the roads R_3 and R_4 . To simply this model, suppose the time distance between two vehicles $p'' = d/2$. Now, the simplified model (on the right of Fig.6) is exactly the same as the model

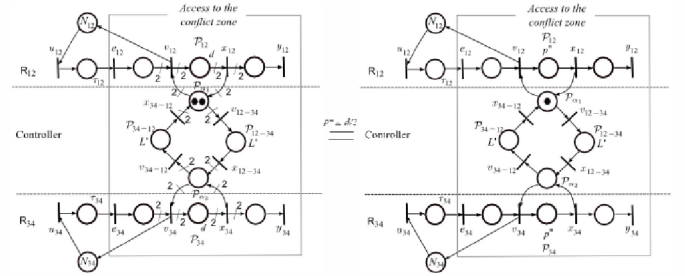


Figure 6. PN model of traffic control at 4-way intersection after mapping vehicles

in Fig.2. Hence, we can obtain the same results deduced before. The control strategy for a 4-way intersection is the same as Algorithm I: After mapping vehicle on the same lane, vehicle from the non-conflicting movements communicates with its neighbor. Each vehicle whose headway to the lead vehicle is less than or equal to d constitutes a platoon with the lead vehicle.

VI. SIMULATIONS

This section presents the simulation results. We assume the arrivals of vehicles obey the Poisson distribution which represents a real-life traffic system with sufficient accuracy [16-18]. Since the overtaking is not considered as vehicles are close to the intersection, the car-following model [19-21] is used to simulate the vehicles' behaviors. The simulation is implemented in a typical 4-way intersection as shown in Fig.3. The characteristics of vehicles and the intersection are detailed in Table II. Comparisons are made among the following four traffic control systems:

- 1) Adaptive control system: a traditional traffic control method which is proven efficient in the current traffic system. Here, the method presented in [22] is used for comparison;
- 2) New traffic control system based on wireless communication proposed by Gradinescu [23]: the controller keeps tracking vehicles by the wireless communication, thus the estimation of the traffic volume is more precise. The timing plan is updated during each cycle. The famous Webster's formula [17] is applied to calculate the cycle length and the green time according to the estimated demand;

TABLE II

CHARACTERISTIC OF VEHICLES AND THE INTERSECTION

Vehicle		Intersection	
Length	5 m	Length of the storage zone	105 m
Reaction time	1 s	Lane width	3.5 m
Acceleration	2 m/s ²	Weather condition	Dry road
Deceleration	-2 m/s ²	Coefficient of friction	0.8
Maximal speed	15 m/s	Saturation flow rate	0.5 veh/s

- 3) New traffic control system proposed in [24]: the control strategy is based on the Little's formula [25]. The average delay experienced by the vehicles in the network is directly proportional to the average queue size. Hence, the controller tries to minimize the queue size by giving a green light to the approach with the maximal weighted queue length. The queue lengths are measured precisely by the advanced wireless communication and positioning technologies.
- 4) Autonomous intersection management proposed in [26, 27] based on FCFS "First Come, First Served" control policy.

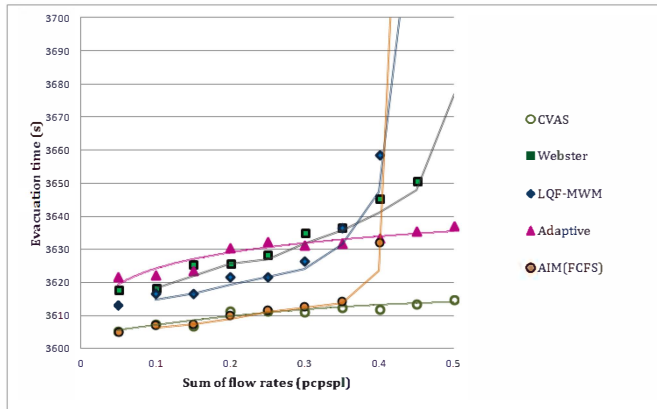


Figure 7. Evacuation time

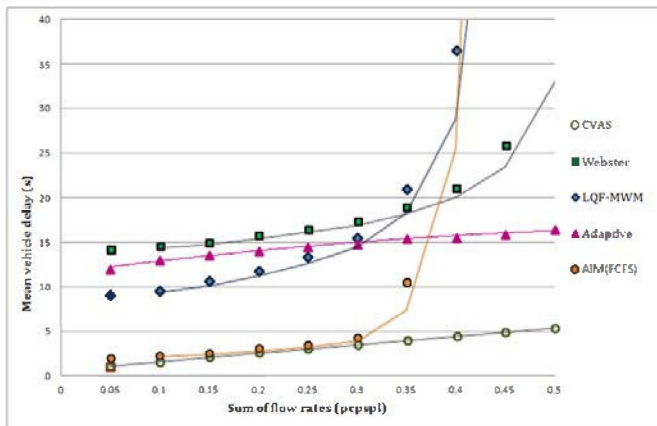


Figure 8. Mean vehicle delay

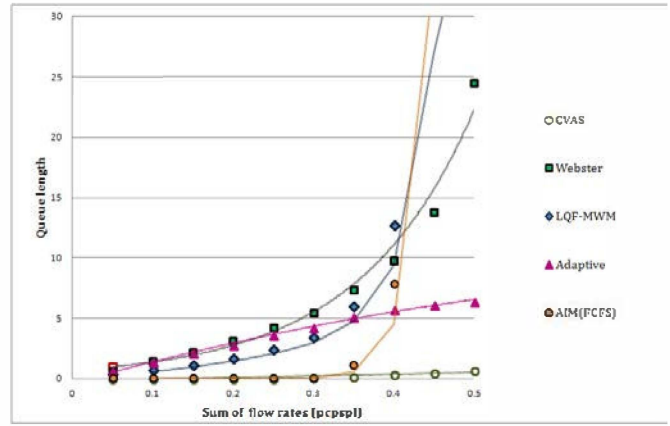


Figure 9. Mean queue length

It is noted that although the new traffic systems (2) and (3) adopt the advanced technologies of wireless communication and positioning systems, the traffic control is still based on traffic lights. In the following, we use the abbreviations "Adaptive", "Webster", "LQF-MWM", and "AIM (FCFS)" to represent the above systems, respectively. "CVAS" represents the proposed platoon-based traffic control system for cooperative metrics.

The performances of all the systems are evaluated by the following criteria:

- 1) Evacuation time: the time to evacuate all the vehicles present in one hour;
- 2) Mean vehicle delay: this quantity is calculated by the difference between the estimated travel time in the absence of the intersection control and the travel time in the presence of the intersection control;
- 3) Mean queue length: it is defined as the cumulative number of vehicles stopped before the conflict zone divided by the simulation time.

The simulation step is 1 s. Each simulation runs sixty minutes and each point in the following figures (Fig.7~Fig.9) represents an average of ten runs. The unit of flow rate "pcpspl" denotes passenger cars per second per lane.

First, let us compare CVAS with the traditional traffic control system. The results show that the proposed algorithm outperforms the system "Adaptive". Besides, its performance is as stable as the system "Adaptive". Turning to the comparison with other new traffic control systems, CVAS also presents the best performance. We observe that "Webster" exhibits the poorest performance at low traffic load since its inherent limitation of the control strategy: although the green time is updated every cycle, it cannot react quickly to the traffic. For "LQF-MWM", it shows good performance at low traffic load (≤ 0.3 pcpspl), however as the traffic load increases, its performance grows worse. That is due to the frequent change of r.o.w when each approach accumulates too many vehicles waiting for the r.o.w. The same conclusion can be drawn from "AIM (FCFS)". When the traffic load is low, i.e. ≤ 0.35 pcpspl, it has the compatible performance with CVAS; as the traffic load increases, it shows a sharp decline in its performance. These comparisons prove that the

platoon-based traffic control strategy will improve the traffic efficiency at intersections.

VII. CONCLUSION

This paper uses a timed Petri Nets to model the traffic behavior at intersections. Based on this model, it proves that a platoon-based traffic control strategy will improve traffic efficiency at intersections. A simple traffic control strategy is proposed accordingly. In addition, these results have the potential to be applied to any intersection with more complicated layout. Simulation results show that the platoon-based traffic control strategy outperforms the conventional controller which is popularly used at present and even better than the innovative systems based on the new technologies. In the future, we will enrich the results and applies them to connected intersections.

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