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New Queueing Approach to the Vehicle Platoon Analysis

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***Abstract*—Influence of the leading vehicle to the follower in a single lane traffic flow and stochastic variations of headways lead to the formation of the vehicle platoons. In this paper we use the analogy between single lane traffic flows and single server queueing systems for the analysis of the vehicle platoons. We derive formulas for the mean number of vehicles in a platoon, mean number of subsequent vehicles in a free flow state, as well as the mean length of a platoon and inter-platoon distance.**

Keywords - platoon, time headway, distance headway, single server queue, busy period.

I. INTRODUCTION

Car following models is an integral part of microscopic traffic simulation tools. They model interaction between a leader-follower pair of vehicles travelling in the same lane. One of the interesting phenomena in such traffic flow is the sudden formation of the vehicle platoons. Classical traffic flow theories do not explain these phenomena since the traffic is assumed to be deterministic [1]. Recently several efforts have been made for development stochastic theory of traffic flow [2]–[4]. In stochastic models vehicle positions are typically described by complicated equations, which take into account the vehicle speeds and distances between vehicles.

In [5], we introduce the notion of desired position and desired arrival time of the vehicle. We used it for demonstration that distance and time delays of the vehicles relative to their desired position and arrival time satisfy simple equation similar to the Lindley equation for the waiting time in the single server queue [6].

In this paper, we make a close look at the analogy between single lane traffic flows and single server queueing systems. In the next section we study a traffic flow in the time headway mode and the distance headway mode. In Section 3 we analyze vehicle platoons i.e., a group of vehicles where each vehicles travels at the safe following distance/time from the lead vehicle. For the vehicles between two platoons headway to the lead vehicle is larger than safe following distance/time (free flow state). We derive formulas for the mean number of vehicles in a platoon, mean number of subsequent vehicles in a free flow state, as well as the mean length of a platoon and inter-platoon distance.

II. LINDLEY-TYPE RECURSION

Desired position of the vehicle indicates position that would be reached when the influence of the leading vehicle is negligible. Similarly, desired arrival time at a position is arrival time in the free driving state. We assume that each driver, excepting the driver of the first vehicle, the vehicle 0, maintains at least as large distance/time headway to a leader vehicle as the safe following distance/time. We take the position of the vehicle to be position of its rear bumper and define several variables to be used throughout this paper:

$d_k^t(s)$ - the desired arrival time of k-th vehicle to the position s, with $d_0^t(s) < d_1^t(s) < d_2^t(s) < d_3^t(s) < \ldots$;

$a_k^t(s)$ - the actual arrival time of k-th vehicle to the position s;

$\delta_k^t(s) = a_k^t(s) - d_k^t(s)$ - the time delay of k-th vehicle at the position s;

$f_k^t(s)$ - the safe following time of (k+1)-th vehicle at the position s;

$d_k^s(t)$ - the desired position of k-th vehicle at time t, with $d_0^s(t) < d_1^s(t) < d_2^s(t) < d_3^s(t) < \ldots$;

$a_k^s(t)$ - the actual position of k-th vehicle at time t;

$\delta_k^s(t) = d_k^s(t) - a_k^s(t)$ - the space delay of k-th vehicle at time t;

$f_k^s(t)$ - the safe following distance of (k+1)-th vehicle at time t.

Here superscripts t and s stand for "time" and "space" respectively.

In the context of this work, a platoon is defined as a group of vehicles where each vehicles travels at the safe following distance/time from the lead vehicle. Distance/time between first and last vehicles in a platoon will be called as congestion period. Vehicles between platoons are in free driving state i.e., distance/time headway between vehicles are larger than safe following distance/time.

A. Time Headway Mode

We say that a traffic flow is in *the time headway mode* at a position s if the following conditions hold

$$a_0^t(s) = d_0^t(s),$$

$$a_k^t(s) = \max\{d_k^t(s), a_{k-1}^t(s) + f_{k-1}^t(s)\}, \ k \ge 1. \qquad (1)$$

In the time headway mode the time headway between any two subsequent vehicles is always greater or equal to the safe following time of the follower vehicle. The time headway

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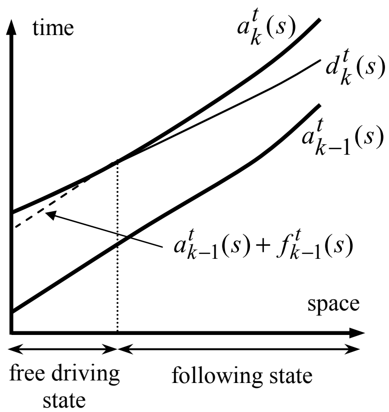

Figure1. Driving states of the *k*-th vehicle in the time headway mode.

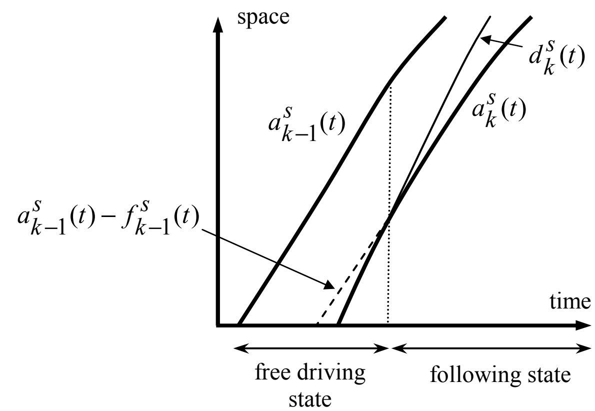

Figure 2. Driving states of *k*-th vehicle in the distance headway mode.

$a^t_k(s) - a^t_{k-1}(s)$ may exceed the safe following time $f^t_{k-1}(s)$ only if the desired arrival time of the follower vehicle to the position coincides with its actual arrival time.

In the time headway mode the moment $A^t_{k-1}(s) = a^t_{k-1}(s) + f^t_{k-1}(s)$ can be used to distinguish between states of the vehicle k at the position s. We say that k-th vehicle s in *the free driving state* if

$$d^t_k(s) > a^t_{k-1}(s) + f^t_{k-1}(s) \tag{2}$$

and it is in *the following state* if

$$d^t_k(s) \le a^t_{k-1}(s) + f^t_{k-1}(s) \tag{3}$$

In the free driving state, according to (1), actual arrival time of k-th vehicle to the position s coincides with its desired arrival time. In the following state k-th vehicle is delayed with respect to its desired arrival time. The states of k-th vehicle in the time headway mode are shown in Figure 1, where the bold solid lines represent the arrival time of the rear bumper of the vehicle as a function of its position.

Figure 3 illustrates a traffic flow in the distance headway mode on a single-lane road at a certain position s. The upper part of the figure shows desired arrival times of the vehicles, the lower part shows actual arrival times, and the time delays are shown in the middle of the figure.

It follows from (1) that for all $k \ge 1$ the time delay $\delta^t_k(s) = a^t_k(s) - d^t_k(s)$ can be calculated as

$$\delta^t_k(s) = 0,$$

if $d^t_k(s) > a^t_{k-1}(s) + f^t_{k-1}(s)$, and

$$\delta^t_k(s) = a^t_{k-1}(s) + f^t_{k-1}(s) - d^t_k(s)$$

if $d^t_k(s) \le a^t_{k-1}(s) + f^t_{k-1}(s)$. Therefore the time delays satisfy the following recursion

$$\delta^t_0(s) = 0,$$

$$\delta^t_k(s) = (\delta^t_{k-1}(s) + f^t_{k-1}(s) - \tau^t_k(s))^+,\ k \ge 1. \tag{4}$$

Here $\tau^t_k(s) = d^t_k(s) - d^t_{k-1}(s)$ is the interval between desired arrival times of k-th and (k-1)-th vehicles to the position s, and th $a^s_{k-1}(t) - f^s_{k-1}(t)$ e positive part z^+ of z is defined by

$$z^+ = \begin{cases} 0, & z \le 0, \\ z, & z > 0. \end{cases}$$

Consider a single server queueing system $\mathrm{Q}^t(s)$ defined by intervals between customer arrivals $T^t_k(s) = \tau^t_k(s),\ k \ge 1$, and service times $S^t_k(s) = f^t_k(s),\ k \ge 0$. Then the waiting time of the customers, $W^t_k(s)$, satisfy the Lindley recursion [6]:

$$W^t_0(s) = 0,$$

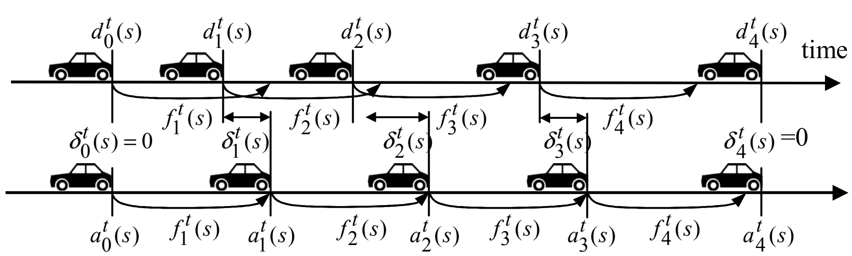

Figure 3. Traffic flow in the time headway mode at a certain position *s*.

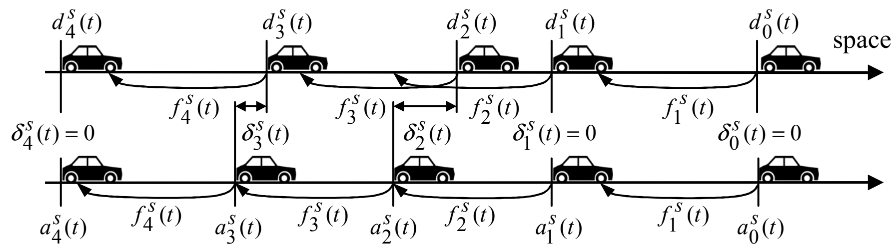

Figure 4. Traffic flow in the distance headway mode at a certain time *t*.

$$W_k^t(s) = (W_{k-1}^t(s) + S_{k-1}^t(s) - T_k^t(s))^+,\ k \ge 1, \qquad (5)$$

which is exactly the same as recursion (4) for the time delays of the vehicles.

B. Distance Headway Mode

We say that a traffic flow is in *the distance headway mode* at time t if the following conditions hold:

$$a_0^s(t) = d_0^s(t),$$

$$a_k^s(t) = \max\{d_k^s(t), a_{k-1}^s(t) - f_{k-1}^s(t)\},\ k \ge 1. \qquad (6)$$

In the distance headway mode the distance headway between any two subsequent vehicles is never less than the safe following distance of the follower vehicle. The distance headway $a_{k-1}^s(t) - a_k^s(t)$ may exceed the safe following distance $f_{k-1}^s(t)\}$ only if the desired position of the follower vehicle at time *t* coincides with its actual position.

The position of the safety point $A_{k-1}^s(t) = a_{k-1}^s(t) - f_{k-1}^s(t)$ behind (*k*-1)-th vehicle can be used to distinguish between states of *k*-th vehicle at time *t* in the distance headway mode. We say that *k*-th vehicle is in *the free driving state* at time *t* if we have

$$d_k^s(t) < a_{k-1}^s(t) - f_{k-1}^s(t). \qquad (7)$$

In this case, according to (1), actual position of *k*-th vehicle at time *t* coincides with its desired position. At time *t* *k*-th vehicle is in *the following state* if we have

$$d_k^s(t) \ge a_{k-1}^s(t) - f_{k-1}^s(t). \qquad (8)$$

In the following state *k*-th vehicle is delayed with respect to its desired position due to the obstacles from the leader vehicle. The states of *k*-th vehicle in the distance headway mode are shown in Figure 4, where the bold solid lines represent the position of the rear bumper of the vehicle as a function of the time.

Figure 4 illustrates traffic flow in the distance headway mode on a single-lane road at a certain time t. The upper part of the figure shows desired positions of the vehicles, the lower part shows the actual positions and the distance delays are shown in the middle of the figure.

It follows from (6) that for all $k \ge 1$ the distance delays $\delta_k^s(t) = d_k^s(t) - a_k^s(t)$ can be calculated as

$$\delta_k^s(t) = 0,$$

if $d_k^s(t) < a_{k-1}^s(t) - f_{k-1}^s(t)$, and

$$\delta_k^s(t) = d_k^s(t) + f_{k-1}^s(t) - a_{k-1}^s(t),$$

if $d_k^s(t) \ge a_{k-1}^s(t) - f_{k-1}^s(t)$. Therefore the time delays satisfy the following recursion

$$\delta_0^s(t) = 0,$$

$$\delta_k^s(t) = (\delta_{k-1}^s(t) + f_{k-1}^s(t) - \tau_k^s(t))^+,\ k \ge 1. \qquad (9)$$

Here $\tau_k^s(t) = d_{k-1}^s(t) - d_k^s(t)$ is the distance between desired positions of *k*-th vehicle and (*k*-1)-th vehicle at time *t*.

Consider a single server queueing system $Q^s(t)$ defined by inter-arrival times $T_k^s(t) = \tau_k^s(t),\ k \ge 1$, and service times $S_k^s(t) = f_k^s(t),\ \ k \ge 0$. Then the waiting time of the customers, $W_k^s(t)$, satisfy the Lindley recursion [6]:

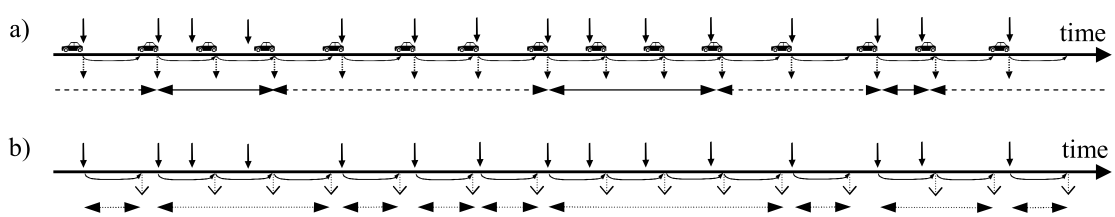

Figure 5. a) Desired positions (↓), safe following distances ( ), actual positions (↓), congestion periods (◄—►), and free flow periods (◄- - -►) for a traffic flow in the distance headway mode; b) Arrivals (↓), service times ( ), departures (↓) and busy periods (◄···►) in the associated single server queueing system $Q^t(s)$.

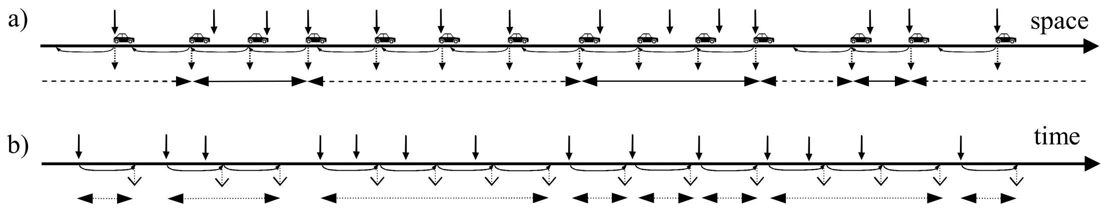

Figure 6. a) Desired positions (↓), safe following distances (⌣), actual positions (⇣), congestion periods (◄—►), and free flow periods (◄- - -►) for a traffic flow in the distance headway mode; b) Arrivals (↓), service time (⌣), departures (↓) and busy periods (◄····►) in the associated single server queueing system $Q^s(t)$.

$$W_0^s(t) = 0,$$

$$W_k^s(t) = (W_{k-1}^s(t) + S_{k-1}^s(t) - T_k^s(t))^+, \ k \geq 1, \qquad (10)$$

which is exactly the same as recursion (9) for the distance delays.

III. VEHICLE PLATOON ANALYSIS

Recursions (4) and (9) for time and distance delays in the traffic flow are the same as recursions (5) and (10) for the waiting times in single server queues. In this section we look at this similarity in details and use it for the vehicle platoon analysis.

We define congestion period of a traffic flow in the time headway mode at a position *s* as follows. Congestion period starts when a vehicle, which is in the free driving state and for which the follower vehicle is in the following state, arrives to the position s. Congestion period ends when the first vehicle, for which the follower vehicle is in the free driving state, arrives to the position *s*. Intervals between two subsequent congestion periods will be called as free flow periods.

Figure 5-a shows congestion and free flow periods for a traffic flow in the time headway mode and Figure 5-b shows busy periods in the associated single server queueing system $Q^t(s)$.

Figure 6-a shows congestion and free flow periods for a traffic flow in the distance headway mode and Figure 6-b shows busy periods in the associated single server queueing system $Q^s(t)$. Notice the inverse order of customer arrivals in $Q^s(t)$ compared with vehicle positions on the space axis.

Analysis of traffic flow in the time headway and the distance headway modes are very similar. Accordingly, here we derive formulas for vehicle platoons for a traffic flow in the time headway mode. We introduce the following notations:

$N_b^t(s)$ - the mean number of customers served during a busy period in $Q^t(s)$;

$\tilde{N}_b^t(s)$ - the conditional mean number of customers served during a busy period in $Q^t(s)$, given this number is greater than one;

$N_c^t(s)$ - the mean number of vehicles in a congestion period;

$N_f^t(s)$ - the mean number of vehicles in a free flow period;

$T_b^t(s)$ - the mean length of a busy period in $Q^t(s)$;

$\tilde{T}_b^t(s)$ - the conditional mean length of a busy period in $Q^t(s)$, given the number of customers served during this busy period is greater than one;

$T_c^t(s)$ - the mean length of a congestion period;

$T_f^t(s)$ - the mean length of a free flow period.

Assume that the safe following times $f_k^t(s)$, $k \geq 1$, of vehicles are independent and identically distributed random variables with probability distribution function $B^t(s,x)$, Laplace-Stieltjes transform

$$\beta^t(s,\sigma) = \int_0^\infty e^{-x\sigma} B^t(s,dx),$$

and the mean safe following time

$$b^t(s) = -\frac{d}{d\sigma}\beta^t(s,\sigma)\bigg|_{\sigma=0}. \qquad (11)$$

We also assume that intervals between desired arrival times, $\tau_k^t(s)$, $k \geq 1$, are independent random variables exponentially distributed with parameter $\lambda^t(s)$. This parameter gives the mean number of vehicles passing the position *s* per unit of time and it is called “the flow” in the traffic flow literature [1].

Consider a traffic flow in the equilibrium condition, when $\rho^t(s) = \lambda^t(s) b^t(s) < 1$. Queueing system $Q^t(s)$ associated with the time headway mode is of M/G/1-type. It has the following characteristics of the busy periods [6]. The mean number of customers served during a busy period is given by

$$N_b^t(s) = \frac{1}{1-\rho^t(s)}, \qquad (12)$$

and the mean length of a busy period by

$$T_b^t(s) = \frac{b^t(s)}{1-\rho^t(s)}. \qquad (13)$$

It follows from (11) that the probability $\pi^t(s)$ of no arrivals during the service of the customer starting a busy period in $Q^t(s)$ is given by

$$\pi^t(s) = \beta^t(s, \lambda^t(s)) .$$

The number of vehicles in a free flow period has geometric distribution with the probability of success $1-\pi^t(s)$. Therefore the mean number of vehicles in a free flow period can be calculated as

$$N_f^t(s) = \frac{1}{1-\pi^t(s)} . \tag{14}$$

and the mean length of a free flow period as

$$T_f^t(s) = m_f^t(s) N_f^t(s) = \frac{m_f^t(s)}{1-\pi^t(s)} . \tag{15}$$

Here $m_f^t(s)$ is the mean distance between vehicles in a free flow period. It can be calculated as the conditional mean distance between vehicles, given this distance is greater than the safe following distance i.e.,

$$m_f^t(s) = \frac{1}{\pi^t(s)} \int_0^\infty x\lambda^t(s) e^{-x\lambda^t(s)} B^t(s,x)dx . \tag{16}$$

Notice that each congestion period in a traffic flow corresponds to the starting at the same time busy period of the associated queueing system shortened by cutting off the last service time. The conditional mean number of customers served during a busy period in $Q^t(s)$, given this number is greater than one, can be found from the following equation:

$$N_b^t(s) = \pi^t(s) \cdot 1 + (1-\pi^t(s)) \widetilde{N}_b^t(s) \tag{17}$$

Since $\widetilde{N}_b^t(s) = N_c^t(s) + 1$, it follows from (12), (15) and (17) that the mean number of vehicles in a congestion period is given by:

$$N_c^t(s) = \frac{\rho^t(s)}{(1-\pi^t(s))(1-\rho^t(s))} . \tag{18}$$

Equation for the conditional mean length of a busy period in $Q^t(s)$ is similar to equation (16):

$$\begin{aligned} T_b^t(s) &= \int_0^\infty x e^{-x\lambda^t(s)} B^t(s,dx) + (1-\pi^t(s)) \cdot \widetilde{T}_b^t(s) \\ &= \gamma_f^t(s) + (1-\pi^t(s)) \cdot \widetilde{T}_b^t(s) , \end{aligned} \tag{19}$$

where $\gamma_f^t(s) = -\frac{d}{d\sigma} \beta^t(s,\sigma) \Big|_{\sigma=\lambda^t(s)}$. The first term in (20) corresponds to a busy period consisting of one service and the second term corresponds to the case when more customers are served. The mean service time of the last customer in a busy period in $Q^t(s)$ is the conditional mean service time, given no arrivals during the service. Using memoryless property of exponential distribution we calculate mean service time of the last customer in a busy period as:

$$m_b^t(s) = \frac{1}{\pi^t(s)} \int_0^\infty x e^{-x\lambda^t(s)} B^t(s,dx) = \frac{\gamma_f^t(s)}{\pi^t(s)} . \tag{20}$$

The length of a congestion period in a traffic flow can be calculated by subtraction of the last service time from the length of corresponding busy period of the system $Q^t(s)$. From (20) it follows that

$$T_c^t(s) = \widetilde{T}_b^t(s) - m_b^t(s) = \frac{T_b^t(s) - m_b^t(s)}{1-\pi^t(s)} . \tag{21}$$

Similar results for a traffic flow in the distance headway mode one can get by switching *s* and *t* in all formulas of this section.

IV. Conclusion

Influence of the leading vehicle to the follower and stochastic variations of headways lead to the formation of vehicle platoons. In this paper we use traffic flow model proposed in [6] for the analysis of the vehicle platoons in a single lane traffic flows. We derive formulas for the mean number of vehicles in a platoon, mean number of subsequent vehicles in a free flow state, and the mean length of a platoon and inter-platoon distance. In future research we plan selection of probability distribution of safe following time and/or probability distribution of safe following distance that conform traffic flow data and calibrate proposed model.

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