HW7

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Find a function $\phi(x, y)$ that is harmonic, i.e., a solution of Laplace's equation, in the unit disk $|z| < 1$ and takes on the boundary values

$$\begin{aligned} \phi(x, y) &= 0, \text{if } z = e^{i\theta}, 0 < \theta < \pi \\ \phi(x, y) &= 1, \text{if } z = e^{i\theta}, \pi < \theta < 2\pi \end{aligned} \tag{0.1}$$

Use the mapping

$$w = f(x) = i\frac{1 - z}{1 + z}$$

to map the interior of the disk to the upper half plane $\Im(w) > 0$