Communication

Elimination of AC Interference in Electrocardiogram Using IIR Notch Filter with Transient Suppression

Soo-Chang Pei and Chien-Cheng Tseng

Abstract—**In this paper, a technique for suppressing the transient states of IIR notch filter is investigated. This technique uses the vector projection to find better initial values for notch filters. When a notch or comb filter is used to eliminate power line (AC) interference in the recording of electrocardiograms (ECG), the performance of the notch filter with transient suppression is better than that of the conventional notch filter with arbitrary initial condition. The improvements with this technique are at the cost of additional computation load at the beginning of filtering.**

I. Introduction

A major problem in the recording of electrocardiograms (ECG's) is that the measurement signal is degraded by additive 50- or 60-Hz power line (AC) interference [1], [2]. A well-known method capable of reducing AC interference is the use of a notch filter characterized by a unit gain at all frequencies except at notch frequency where gain is zero. So far, many good digital IIR and FIR notch filters have been proposed for this application [3]–[5].

The problem tackled in this paper is motivated by the work in which we use notch filters to remove 60-Hz interference in ECG signal. When a corrupted signal passes through an IIR notch filter for the removing of sinusoidal interference, the transient response distorts the filter output on the start-up. Typically, a notch filter with the narrower 3-dB rejection bandwidth has the longer transient response at the filter output. However, we usually prefer a notch filter with narrower 3-dB rejection bandwidth to faithfully separate sinusoidal component and broadband component. Hence, a trade-off between the duration of transient and the distortion of separateness must be considered. In this paper, a technique is proposed to suppress transient response.

The technique proposed in this paper is divided into the following two steps. First, we use vector projection to decompose the first M samples of input signal into the sinusoidal interference component and the ECG signal component. Next, we use the ECG signal component as initial values of the notch filter to perform the filtering operation. Consequently, transient response is reduced dramatically. Although additional computation cost is required on the start-up of filtering, it is small and not a problem for real time processing.

II. Transient State in Notch Filter

In this section, the transient response of notch filter will be studied. The transfer function of second-order IIR notch filter is given by [4]

$$Y(z) = \frac{1}{2}\frac{(1+a_2) - 2a_1 z^{-1} + (1+a_2)z^{-2}}{1 - a_1 z^{-1} + a_2 z^{-2}} X(z) \qquad (1)$$

where $X(z)$ and $Y(z)$ are z transforms of notch filter input $x(n)$ and output $y(n)$, respectively. The notch frequency ω_0 and 3-dB rejection bandwidth Ω relate to the filter coefficients a_1 and a_2 by the following:

$$a_1 = \frac{2\cos(w_0)}{1+\tan\left(\frac{\Omega}{2}\right)}$$

$$a_2 = \frac{1-\tan\left(\frac{\Omega}{2}\right)}{1+\tan\left(\frac{\Omega}{2}\right)}. \qquad (2)$$

In this paper, we assume that the input signal $x(n)$ of notch filter is a sinusoidal interference $d(n)$ with frequency ω_0 embedded in an ECG signal $s(n)$, i.e.,

$$x(n) = s(n) + d(n) \qquad (3)$$

where $d(n)$ is given by

$$d(n) = A_0 \sin(\omega_0 n + \phi). \qquad (4)$$

When 3-dB rejection bandwidth Ω is very small, the notch filter output $y(n)$ is almost the same as $s(n)$. Now, we summarize the notch filtering operation with arbitrary initial states as follows:

1) Filtering Process 1: Given the measurement signal $x(n)$, the number of samples is N, the notch frequency is ω_0, and the bandwidth is Ω. In practice, if the sampling frequency, sinusoidal frequency, and notch bandwidth are f_s, f_d, and BW Hz, then $\omega_0 = 2\pi(f_d/f_s)$ and $\Omega = 2\,\pi\,(BW/f_s)$.

1) Use (2) to calculate filter coefficient a_1 and a_2.
2) Choose arbitrary initial $y(-1)$ and $y(-2)$.
3) For $n = 0$ to N, the output $y(n)$ is given by

$$\begin{aligned} y(n) =& \tfrac{1}{2}\left[(1+a_2)x(n) - 2a_1 x(n-1) + (1+a_2)x(n-2)\right] \\ &+ a_1 y(n-1) - a_2 y(n-2). \end{aligned} \qquad (5)$$

As an example, we use Filtering process 1 to remove the 60-Hz power line interference in the recording of ECG signal, i.e., $f_d = 60$. The samples used in this example have a quantization on eight bits and the sampling rate f_s is 800 Hz. Fig. 1(a) shows an ECG waveform with an excessive amount of 60-Hz interference. Fig. 1(b)–(e) shows the outputs of Filtering process 1 with various 3-dB notch rejection bandwidth BW and initial $y(-1)$, $y(-2)$. From these results, it is clear that the severe transient states appear at the beginning of notch filter output. Also, the narrower bandwidth BW the notch filter has, the longer duration of transient states the notch output has. Because we usually want the bandwidth Ω to be very narrow for reducing the distortion of ECG signal, a technique for transient states suppression must be developed. In the next section, we will propose a suppression technique to achieve this purpose.

III. Notch Filtering with Transient State Suppression

In this section, we first develop a suppression technique to reduce the transient states in notch filter output. Then, various ECG signals are used to evaluate the performance of proposed notch filtering. Finally, we extend the suppression technique to the high order notch filters which are used to remove harmonic interference.

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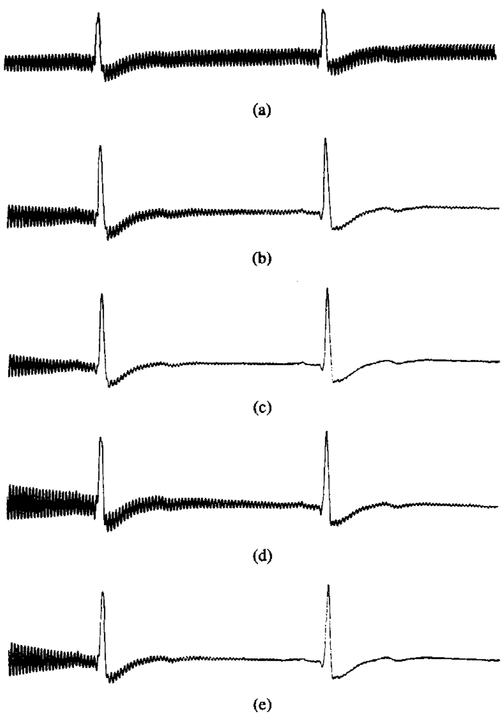

Fig. 1. Transient states in the notch filter output. (a) ECG waveform with 60-Hz interference. (b) Notch filter output with $y(-1)= 0$, $y(-2)= 0$, and $BW = 0.4$ Hz. (c) Notch filter output with $y(-1)= 0$, $y(-2)= 0$, and $BW = 0.8$ Hz. (d) Notch filter output with $y(-1)= 10$, $y(-2)= 20$, and $BW = 0.4$ Hz. (e) Notch filter output with $y(-1)= 10$, $y(-2)= 20$, and $BW = 0.8$ Hz.

A. Suppression Technique

The basic procedure of suppression technique is now described as follows. At the beginning, we arrange first M input signal samples into the following $M \times 1$ data vector:

$$X = [x(0) \quad x(1) \quad x(2) \quad \cdots \quad x(M-1)]^t. \tag{6}$$

From (3), this vector can be rewritten as

$$X = S + D \tag{7}$$

where vectors S and D are

$$S = [s(0) \quad s(1) \quad s(2) \quad \cdots \quad s(M-1)]^t \tag{8}$$

$$D = [d(0) \quad d(1) \quad d(2) \quad \cdots \quad d(M-1)]^t. \tag{9}$$

If we define matrix A as

$$A = \begin{bmatrix} 1 & \cos(\omega_0) & \cos(2\omega_0) & \cdots & \cos[(M-1)\omega_0] \\ 0 & \sin(\omega_0) & \sin(2\omega_0) & \cdots & \sin[(M-1)\omega_0] \end{bmatrix}^t$$

then vector D is in the column space of matrix A for any phase ϕ of $d(n)$. This is because the following expression is valid:

$$\begin{aligned} d(n) &= A_0 \sin(\omega_0 n + \phi) \\ &= A_0 \cos(\phi) \sin(\omega_0 n) + A_0 \sin(\phi) \cos(\omega_0 n) \end{aligned} \tag{10}$$

i.e., $d(n)$ is a linear combination of $\cos(\omega_0 n)$ and $\sin(\omega_0 n)$. The definition of column space of a matrix can be found in [6, p. 66]. Thus,

TABLE I
THE MEAN SQUARE ERROR E FOR VARIOUS CASES ($n_0 = 0$, $B = 800$)

	E (initial value zeroed)	E (transient suppression)
Case 1	36.9135	0.1277
Case 2	36.6771	17.5499
Case 3	40.9621	2.0222
Case 4	36.9486	1.2617
Case 5	40.8813	30.7916

we can use projection operation to decompose X into sinusoidal part D and signal part S. If we define projection operator P as

$$P = A(A^t A)^{-1} A^t \tag{11}$$

then the sinusoidal part can be estimated by

$$\hat{D} = PX. \tag{12}$$

Moreover, the signal part can be obtained by

$$\begin{aligned} \hat{S} &= X - \hat{D} \\ &= (I - P)X \end{aligned} \tag{13}$$

where I is an identity matrix. Instead of using an arbitrary initial in Filtering process 1, we use $\hat{S}$ as an initial to perform the filtering operation. We summarize this new filtering process as follows:

1) Filtering Process 2: Given signal $x(n)$ and ω_0, Ω, N, M

1) Use (2) to calculate filter coefficients a_1 and a_2.
2) Construct input data vector X and projection matrix P.
3) Calculate first M output samples by $[y(0) \quad y(1) \quad \cdots \quad y(M-1)]^t = (I-P)X$.
4) For $n = M+1$ to N, the output $y(n)$ is given by

$$\begin{aligned} y(n) = \tfrac{1}{2}[&(1+a_2)x(n) - 2a_1 x(n-1) + (1+a_2)x(n-2)] \\ &+ a_1 y(n-1) - a_2 y(n-2). \end{aligned} \tag{14}$$

2) Remark 1: In the above, it seems to imply that the projection matrix P needs to be recalculated for each signal $x(n)$. This would involve inverting a 2×2 matrix. Actually, matrix P is only dependent on M and on ω_0, which do not change from ECG to ECG. Hence, matrix $I - P$ could be pre-stored in a data array, and the proposed method really only requires about M^2 floating point multiplications to obtain an initial M data values.

3) Remark 2: In the above, we have not stated how to choose M. In our experience, when M is chosen as an integer from 5–15, the Filtering process 2 will perform very well.

B. Testing Examples

Now, various ECG signals are used to test this new filtering technique. These ECG signals include the following five cases:

1) ECG signal began at flat segment.
2) ECG signal began at the QRS complex.
3) ECG signal with atrial flutter.

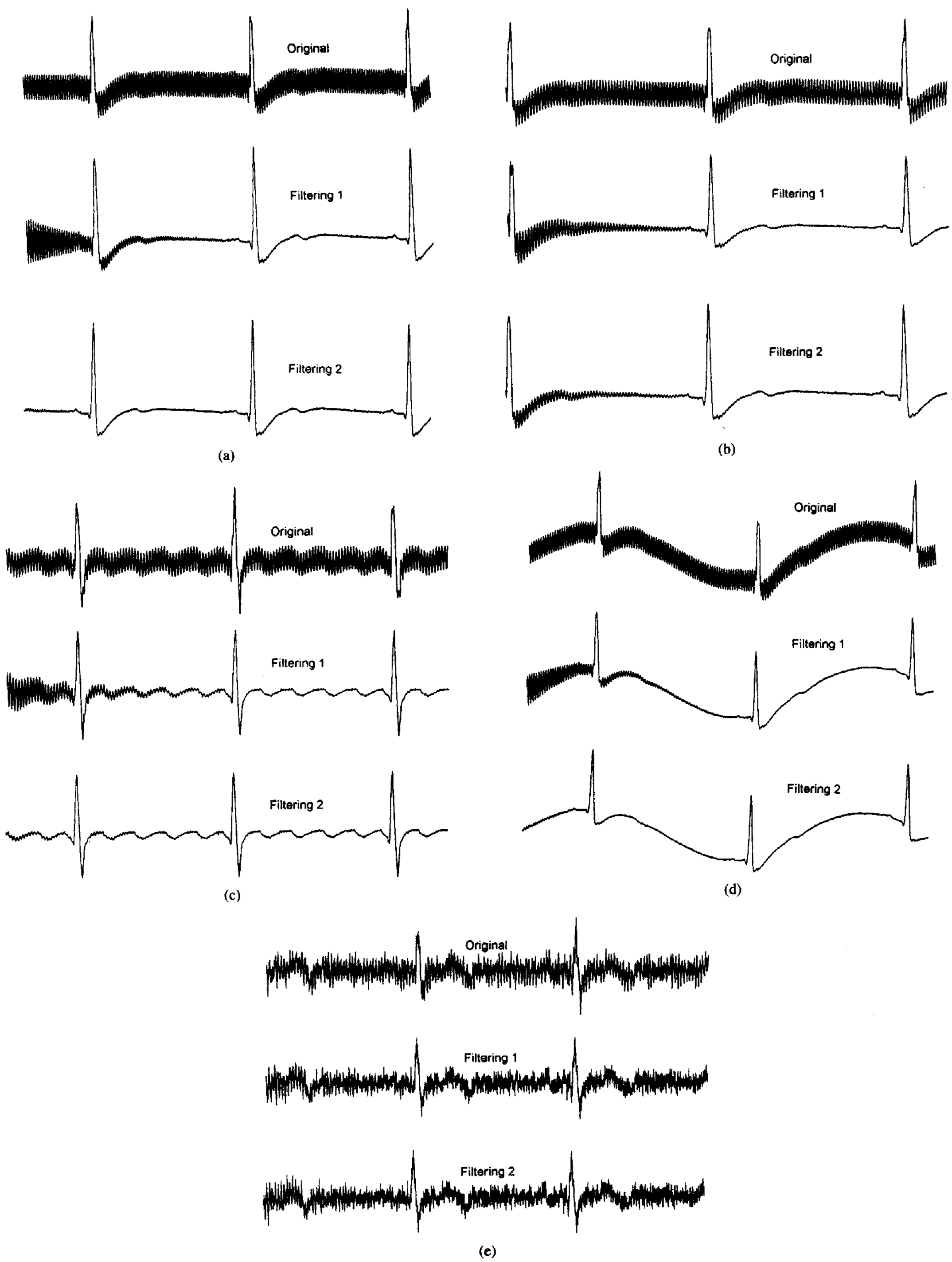

Fig. 2. Comparison between Filtering process 1 and Filtering process 2. "Original" denotes ECG waveform with sinusoidal interference. "Filtering process 1" denotes notch filter output with initial values zeroed. "Filtering process 2" denotes notch filter output with transient suppression. (a) Case 1. (b) Case 2. (c) Case 3. (d) Case 4. (e) Case 5.

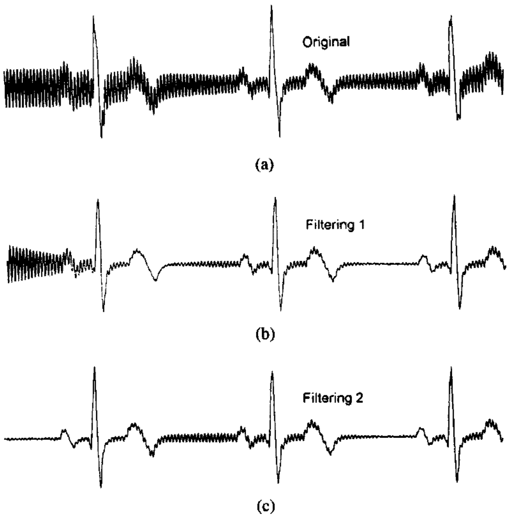

Fig. 3. Actual measurement signal test. (a) ECG waveform with AC interference. (b) Notch filter output with initial values zeroed. (c) Notch filter output with transient suppression.

4) ECG signal with respiratory baseline shifts.
5) ECG signal with muscle tremor.

In order to compare Filtering process 1 and Filtering process 2 quantitatively for all cases, the AC interference $d(n)$ in this illustration is artificially added. Thus, the ECG signal $s(n)$ is known in advance. A good way to evaluate the performance of the proposed approach is to calculate the mean square error of the different test cases. The mean square error used here is defined as

$$E = \frac{1}{B} \sum_{n=n_0+1}^{n_0+B} |y(n) - s(n)|^2 \tag{15}$$

where B is the window size and n_0 is the starting time. Usually, the smaller the E value is, the closer the notch filter output $y(n)$ is to the ECG signal $s(n)$. That is, a small E value implies a small transient state. Fig. 2 shows various notch filter inputs $x(n)$ and outputs $y(n)$ with $M = 10$ and $BW = 0.8$ Hz. It is clear that the transient states have been reduced markedly. Moreover, Table I lists the mean square error E for various cases with $n_0 = 0$, $B = 800$. For all cases, it is obvious that Filtering process 2 always has smaller E than Filtering process 1 with initial values set to zero. Finally, we use the actual measurement signal $x(n)$ to test the proposed technique. In this case, corrupted interference is actual AC noise. Fig. 3 shows the outputs of the notch filter with initial values zeroed and the suppression technique. The rejection bandwidth $BW = 0.8$ Hz and $M = 10$. From this result, it is obvious that the transient states can be suppressed by our proposed technique.

C. Extension to High Order Notch Filter

In general, the second-order notch filter only removes one sinusoidal interference. When ECG signal is corrupted by harmonic interference, then high order notch filter or comb filter must be used [2]. In this case, the proposed suppression technique can be extended directly. Without loss of generality, we investigate a sixth-order notch filter as follows. From the results in [7], a transfer function

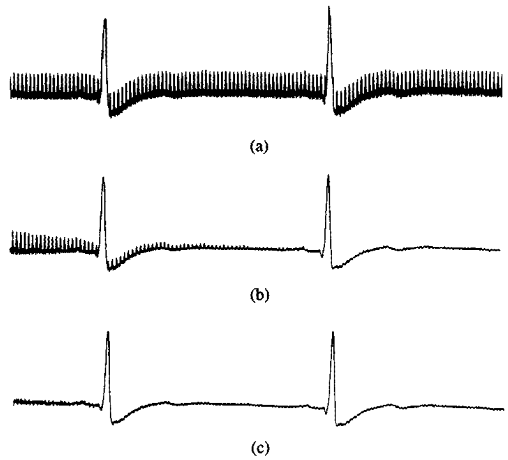

Fig. 4. Transient states in high order notch filter. (a) ECG waveform with harmonic interference. (b) Notch filter output with initial values zeroed. (c) Notch filter output with transient suppression.

of sixth-order notch filter is given by

$$H(z) = \frac{Q(z)}{Q(\rho z)} \tag{16}$$

where ρ is the pole radius and

$$Q(z) = 1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_2 z^{-4} + a_1 z^{-5} + z^{-6}. \tag{17}$$

This sixth-order notch filter is not a generalization of the second-order notch filter in (1) which has been used in [4]. From [7], the notch frequencies ω_1, ω_2, ω_3, and 3-dB rejection bandwidth Ω relate to the filter coefficients a_1, a_2, a_3, and ρ by the following:

$$a_1 = -2(\cos\omega_1 + \cos\omega_2 + \cos\omega_3) \tag{18}$$

$$a_2 = 3 + 4(\cos\omega_1 \cos\omega_2 + \cos\omega_2 \cos\omega_3 + \cos\omega_3 \cos\omega_1) \tag{19}$$

$$a_3 = -4(\cos\omega_1 + \cos\omega_2 + \cos\omega_3) - 8\cos\omega_1 \cos\omega_2 \cos\omega_3 \tag{20}$$

$$\rho = 1 - \frac{\Omega}{\pi}. \tag{21}$$

Moreover, the input signal has the form

$$x(n) = s(n) + \sum_{i=1}^{3} A_i \sin(\omega_i n + \phi_i). \tag{22}$$

As with the second-order notch filter, transient states appear in the notch filter output if initial values are set to zero. Now we summarize a technique to suppress transients.

1) Filtering Process 3: Given signal $x(n)$ and Ω, M, N, ω_i ($i = 1, 2, 3$)

1) Use (18)–(21) to calculate filter parameters ρ, and a_i ($i = 1, 2, 3$).
2) Construct input data vector X and matrix A

$$A = \begin{bmatrix} 1 & \cos(\omega_1) & \cos(2\omega_1) & \cdots & \cos[(M-1)\omega_1] \\ 0 & \sin(\omega_1) & \sin(2\omega_1) & \cdots & \sin[(M-1)\omega_1] \\ 1 & \cos(\omega_2) & \cos(2\omega_2) & \cdots & \cos[(M-1)\omega_2] \\ 0 & \sin(\omega_2) & \sin(2\omega_2) & \cdots & \sin[(M-1)\omega_2] \\ 1 & \cos(\omega_3) & \cos(2\omega_3) & \cdots & \cos[(M-1)\omega_3] \\ 0 & \sin(\omega_3) & \sin(2\omega_3) & \cdots & \sin[(M-1)\omega_3] \end{bmatrix}^t.$$

3) Calculate projection matrix $P = A(A^tA)^{-1}A^t$.
4) Calculate first M output samples by $[y(0) \quad y(1) \quad \cdots \quad y(M-1)]^t = (I-P)X$.
5) For $n = M+1$ to N, the output $y(n)$ is given by

$$\begin{aligned} y(n) =& x(n) + x(n-6) \\ &+ \sum_{i=1}^{2} \cdot a_i[x(n-i) + x(n+i-6) \\ &- \rho^i y(n-i) - \rho^{6-i} y(n+i-6)] \\ &+ a_3[x(n-3) - \rho^3 y(n-3)] - \rho^6 y(n-6). \end{aligned}$$

As in the second-order notch filter case, the matrix P can be computed in advance because it only depends on M, ω_1, ω_2, and ω_3. Hence, the 6×6 matrix inversion does not need to be performed for each data set. Now, we use an example to test this new filtering technique. Fig. 4(a) shows an input signal waveform which is ECG corrupted by a harmonic with frequencies $w_1 = 50$ Hz, $w_2 = 100$ Hz, and $w_3 = 150$ Hz. Figs. 4(b) and (c) show the outputs of the notch filter with initial values zeroed and the suppression technique. The rejection bandwidth is $BW = 0.4$ Hz and $M = 15$. From this result, it is obvious that the transient states are almost completely removed by our proposed technique in the high order notch filter case.

IV. Conclusion

This paper presents a technique to suppress transient states of IIR notch filters. Experimental results show that this technique can improve the performance of IIR notch filters for 60-Hz interference canceling application if the proper computation load increases. In addition, it is interesting to extend this technique to the general class of input signals and interferences in biomedical applications. This topic will be studied in the future.

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Signal-to-Noise Ratio Enhancement of Cardiac Late Potentials Using Ensemble Correlation

Roozbeh Atarius and Leif Sörnmo

***Abstract*—An optimal one-weight "filter" is presented for the purpose of improving the signal-to-noise ratio of averaged ECG recordings in the analysis of late potentials. Based on a simple statistical model, the filter is estimated from the ensemble correlation of available beats. The correlation estimator is found by a maximum likelihood procedure in which the observed signal is assumed to have a Gaussian distribution. The performance of the optimal filter is studied in relation to an ensemble with individual or subaveraged beats.**

I. Introduction

Time-coherent averaging is a standard technique used for improving the signal-to-noise ratio (SNR) of repetitive bioelectric signals such as evoked potentials and cardiac late potentials. Further improvement of the SNR of averaged data has, however, been achieved by exploiting the fact that adjacent samples in the average are correlated. This may be done by employing a so-called "*a posteriori* Wiener filter," a technique which has been widely studied in the literature, e.g., [1] and [2]. The design involves the computation of the spectrum for the averaged data and the average of the spectra from individual realizations. The filter has been implemented as a time-varying filter in which the spectral estimation is performed for successive intervals of the observed data [3], [4]. In doing so, the filter becomes better suited for the processing of nonstationary data. The application of *a posteriori* Wiener filtering to averaged data with low SNR has been found to reduce the variance of the signal estimate, but at the expense of an increased bias [5].

A key issue in the analysis of late potentials in signal-averaged ECG recordings is finding that transition in time where late potentials terminate and after which only noise is present. This problem has previously been treated by us within the framework of maximum likelihood (ML) estimation [6]. In that case the object was to design a function which accurately reflects the amplitude of late potentials and which is suitable for endpoint determination. This paper describes a new method for improving the SNR of an averaged ECG which is based on a simple, statistical model similar to the one described in [6]. However, the idea here is to estimate the signal itself (i.e., late potentials), rather than to design a certain function for endpoint determination. Based on the model, the resulting minimum mean square error estimator is a one-weight filter which weights each sample of the beat average in relation to the correlation across the ensemble of beats. The present type of filtering could be interpreted as a weighting of the beat average by a function which is related to the local SNR of the beat (cf., [7]). Accordingly, intervals which are noisy are assigned weights close to zero while the intervals containing late potentials have weights which are close to unity. The ensemble correlation is estimated by an ML procedure from the ensemble of beats and could be based on either individual or subaveraged beats. The relative merits of using subaveraging are investigated in terms of mean and variance of the resulting filter estimate.

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