### GRewriter: Practical Query Rewriting with Automatic Ruleset Expansion in GaussDB (Appendix)

### **ABSTRACT**

This document contains the appendix of VLDB'25 industrial track submission #2123. Section 1 presents the proof of the theorem about rule verification in the paper. Section 2 describes the G-DSL in detail. Section 3 lists the rewrite rules discovered by our work.

### 1 PROOF THE VERIFICATION ALGORITHM

This section provides the detailed proof of the following theorem in the paper, which enables us to prove rules by proving concrete SQL query pairs via existing verifiers, such as SQLSolver [1].

THEOREM 1.1. For each enumerated rewrite rule, if each representative pair of queries is equivalent, the rule is correct.

Before proving the above theorem, we will first prove several lemmas, which are required to prove the theorem. Then, we will present the proof of the theorem.

Lemma 1.2. For each non-representative pair of queries  $p_1$ , it always belong to one of the following cases: 1)  $p_1$  is the same as some representative pair of queries  $p_2$  after renaming its attributes and tables. 2)  $p_1$  is almost the same as some representative pair of queries  $p_2$  after renaming its attributes and tables except for expressions and schemas. Note that renaming there means that there is a one-to-one name mapping between names used in  $p_1$  and another representative query pair.

PROOF. The lemma can be proved based on the construction method of representative query pairs and non-representative query pairs. Both types of query pairs are generated by assigning symbols in the rule to concrete attributes, tables, and expressions. Thus, these generated query pairs all satisfy the same syntax structures specified by plan templates and constraints. They can only differ in 1) concrete names of attributes and tables; 2) concrete expressions; 3) concrete schemas. Therefore, after renaming, the non-representative query pair  $p_1$  can always be transformed into a representative query pair  $p_2$  except for the concrete expressions and schemas. In conclusion, the lemma is proved.

LEMMA 1.3. If a query pair  $p_1$  is the same as another query pair  $p_2$  after renaming the attributes and tables, then the equivalence of  $p_2$  can imply the equivalence of  $p_1$ .

PROOF. Whether two queries are equivalent depends on the syntax structures of queries and constraints rather than the concrete names. If two queries are equivalent, renaming of attributes and tables via a one-to-one names mapping will not corrupt the equivalence. Therefore, since queries in  $p_1$  are the same as queries in  $p_2$  after renaming, the equivalence of  $p_2$  can imply the equivalence of  $p_1$ . In conclusion, the lemma is proved.

LEMMA 1.4. If a query pair  $p_1$  is the same as another query pair  $p_2$  after renaming the attributes and tables except for expressions, then the equivalence of  $p_2$  can imply the equivalence of  $p_1$ .

PROOF. According to the construction method of representative query pairs, the expressions in  $p_2$  are uninterpreted user-defined functions. When verifying  $p_2$ , the verifier will treat these functions as random functions. Thus, if  $p_2$  is equivalent, replacing these functions with any concrete expressions will not change the equivalence. The equivalence of  $p_2$  can imply the equivalence of  $p_1$  after renaming. Furthermore, according to Lemma 1.3, the equivalence of  $p_1$  after renaming can imply the equivalence of  $p_1$ . In conclusion, the lemma is proved.

LEMMA 1.5. If a query pair  $p_1$  is the same as another query pair  $p_2$  after renaming the attributes and tables except for schemas, then the equivalence of  $p_2$  can imply the equivalence of  $p_1$ .

PROOF. According to the construction method of representative query pairs, the schemas of  $p_1$  and  $p_2$  differ in two aspects. First, the difference comes from some attributes not used in queries. Since these attributes are not used in queries, the equivalence of  $p_2$  can imply the equivalence of  $p_1$  after renaming. Then, according to Lemma 1.3, the equivalence of  $p_1$  after renaming can further imply the equivalence of  $p_1$ . Second, the difference comes from the size of attribute lists in some schemas. These attribute list correspond to the symbolic attribute lists in the rule. Since existing verifiers for concrete queries treat such attribute lists as a whole, the sizes of such concrete attribute lists will not affect the equivalence of queries. Therefore, the equivalence of  $p_2$  can imply the equivalence of  $p_1$  after renaming. Then, according to Lemma 1.3, the equivalence of  $p_1$  after renaming can further imply the equivalence of  $p_1$  after renaming can further imply the equivalence of  $p_1$  after renaming can further imply the equivalence of  $p_1$ . In conclusion, the lemma is proved.

LEMMA 1.6. For each non-representative query pair  $p_1$ , there exists a representative query pair  $p_2$  such that the equivalence of  $p_2$  can imply the equivalence of  $p_1$ .

PROOF. According to Lemma 1.2, each non-representative query pair  $p_1$  belongs to one of the following cases: 1)  $p_1$  is the same as some representative pair of queries  $p_2$  after renaming its attributes and tables. 2)  $p_1$  is almost the same as some representative pair of queries  $p_2$  after renaming its attributes and tables except for expressions and schemas. For case 1, the lemma is proved by Lemma 1.3. For case 2, the lemma is proved by Lemma 1.4 and Lemma 1.5. Therefore, the lemma is proved.

Based on the above lemmas, we can finally prove the following theorem in the paper.

THEOREM 1.7. For each candidate rewrite rule, if each representative pair of queries is equivalent, the rule is correct. PROOF. According to the definition of rule correctness, we prove the theorem by proving that each pair of queries before and after the rewrite are equivalent. All query pairs can be classfied into two types: representative query pairs and non-representative query pairs. The premise indicates that all representative query pairs are equivalent. For each non-representative query pair, Lemma 1.6 indicates that there exists a representative query pair such that the equivalence of the representative query pair can imply the equivalence of the non-representative query pair. Therefore, all non-representative query pairs are equivalent. In conclusion, all query pairs are equivalent, which means that the rule is correct.  $\qed$ 

**Example.** Take the rule below as an example, which simply removes redundant projections.

```
Proj<_ a0 r2>(Proj<_ a0 r1>(Input<r0>))|
Proj<_ a0 r1>(Input<r0>)|
AttrsSub(a0, r0);
```

We can verify it by the following steps. First, we enumerate all possible schemas. We know that the columns represent by a0 is a subset of table r0. Thus we have to enumerate if r0 has other columns except columns in a0.

```
CREATE TABLE R0(C0 INT);
CREATE TABLE R0(C0 INT, C1 INT);
```

Second, according to each enumerated schemas, we generate the SQL pair by simply assign concrete columns to the symbols.

```
SELECT C0 FROM (SELECT C0 FROM R0);

SELECT C0 FROM R0;

SELECT C1 FROM (SELECT C1 FROM R0);

SELECT C1 FROM R0;
```

Finally, we use existing verifier SQLSolver [1] to verify the equivalence of each pair of queries. Since each query pair is equivalent, the rule is correct.

### 2 G-DSL

### 2.1 Syntax in YAML

```
Rule : SourceTemplate '|' DestinationTemplate
    '|' G-Constraints

SourceTemplate: Node
TargetTemplate: Node

Node : NodeType '<' Symbols '>' '(' SubNodes ')'

NodeType : 'Filter' | 'Proj' | 'Input' | 'Join_left'
    | 'Join_inner' | 'Union' | 'Union_all' | 'Limit'
    | 'Agg_max' | 'Agg_min' | 'Agg_count' | 'Agg_avg'
    | 'Agg_sum' | 'Agg' | 'Sort_asc' | 'Sort_desc'
    | 'Insub' | 'Exists' | 'Except' | 'Intersect'
    | 'Join_cross' | 'Join_right'

Symbols : Symbol | Symbol ' ' Symbols |

Symbol : [a-z][0-9]+
```

```
SubNodes : Node | Node ', 'SubNodes |
G-Constraints : G-Constraint
    | G-Constraint ';' G-Constraints |
G-Constraint : Destructure | Definition | Constraint
Destructure : Expression ':=' Symbol
Definition : Symbol ':=' Expression
Constraint : ConstraintType '(' Symbols ')'
    | '!' ConstraintType '(' Symbols ')'
Expression: Symbol
    | ExpressionType '<' Infos '>' '(' Expressions ')'
Expressions: Expression
    | Expression, Expressions |
ExpressionType: 'Sublink' | 'Eq' | 'And' | 'Const'
    | 'CTE' | 'Or' | 'Plus' | 'Minus' | 'Div'
   | 'Mul' | 'Target' | 'List' | 'IsNull' | 'IsNotNull'
    | 'LT' | 'GT' | 'LE' | 'GE' | 'Like' | 'Star'
    | 'FuncCall'
ConstraintType: 'AttrsSub' | 'AttrsEq' | 'ExpressionEq'
    | 'Unique' | 'NotNull' | 'Indexed' | 'TableEq'
    | 'ExpressionSub' | 'Reference'
```

### 2.2 Node Types

**Filter.** The Filter operator is used to apply a condition (typically a WHERE clause) to a dataset, filtering out rows that do not satisfy the condition. It is responsible for selecting only those rows that meet specific criteria.

**Proj (Projection).** The Proj operator refers to the projection operation in relational algebra, which is used to select specific columns from a dataset. It reduces the number of columns in the output, keeping only the ones required by the query.

**Input.** The Input operator represents the input or source data in a query plan. It can be thought of as the starting point or base relation in a query, from which further operations (like filtering, joining, etc.) are performed.

Join\_left (Left Join). The Join\_left operator performs a left outer join between two tables. It returns all rows from the left table and the matching rows from the right table. If no match is found, NULL values are returned for the columns from the right table.

**Join\_inner** (**Inner Join**). The Join\_inner operator performs an inner join between two tables, returning only the rows that have matching values in both tables. If no match exists, those rows are excluded from the result.

**Union.** The Union operator combines the result sets of two queries, eliminating duplicate rows from the final result. It returns the distinct rows from both input datasets, assuming the datasets have the same number of columns with compatible data types.

**Union\_all.** The Union\_all operator is similar to Union, but it does not remove duplicates. It returns all rows from both input datasets, including duplicate rows.

**Limit.** The Limit operator is used to restrict the number of rows returned in a query result. It is often used to paginate large datasets or to retrieve a fixed number of records from the query result.

**Agg\_max** (**Aggregate Maximum**). The Agg\_max operator computes the maximum value of a specified column for a given group of rows. It is commonly used in aggregate functions like MAX() in SOL.

**Agg\_min** (**Aggregate Minimum**). The Agg\_min operator computes the minimum value of a specified column for a given group of rows. It is used in aggregate functions like MIN() to find the smallest value in a set of values.

**Agg\_count (Aggregate Count).** The Agg\_count operator counts the number of rows in a dataset or the number of non-null values in a specified column. It is used in aggregate functions like COUNT() to determine the total number of records or specific values.

**Agg\_avg** (**Aggregate Average**). The Agg\_avg operator computes the average value of a specified column for a group of rows. It is used in the AVG() aggregate function to calculate the mean value of a set of values.

**Agg\_sum (Aggregate Sum).** The Agg\_sum operator computes the sum of the values in a specified column for a group of rows. It is used in aggregate functions like SUM() to calculate the total of numeric values.

**Agg (General Aggregation).** The Agg operator represents general aggregation operations that compute values (like sum, count, average, etc.) based on specific columns or groups of rows. It groups rows based on certain criteria and applies aggregate functions.

**Sort\_asc (Sort Ascending).** The Sort\_asc operator sorts the rows in ascending order based on a specified column. The sorting places the smallest values first and the largest values last.

**Sort\_desc (Sort Descending).** The Sort\_desc operator sorts the rows in descending order based on a specified column. It places the largest values first and the smallest values last.

**Insub (Subquery with IN).** The Insub operator represents the use of a subquery in an IN condition, checking if a value is present in a set of values returned by a subquery.

**Exists.** The Exists operator is used to check if a subquery returns any results. It returns true if the subquery produces one or more rows, and false if the subquery returns no rows.

**Except.** The Except operator returns the rows from the first query that are not present in the second query. It eliminates any rows that appear in both queries, effectively subtracting one result set from another.

**Intersect.** The Intersect operator returns the common rows from both queries. It finds the intersection of two result sets, returning only the rows that are present in both datasets.

**Join\_cross (Cross Join).** The Join\_cross operator performs a cross join between two tables, producing a Cartesian product of the rows from both tables. Each row from the first table is paired with every row from the second table, resulting in all possible combinations of rows.

Join\_right (Right Join). The Join\_right operator performs a right outer join, returning all rows from the right table and the matching rows from the left table. If no match is found in the left table, NULL values are returned for the left table's columns

### 3 RULES

```
rule 1:
Exists(Agg_average<a4 a5 r6 e2 a6 r7>(Exists(Union (Input<r3>,Input<r4>),Input<r5>)),Input<r8>)|
Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input <r0>))|
AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a6);
AttrsEq(a3,a4);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r8);
TableEq(r3,r4);TableEq(r3,r5);TableEq(r3,r8);
TableEq(r4,r5);TableEq(r4,r8);TableEq(r5,r8);
Unique(r3,a4);Unique(r3,a5)|
```

```
rule 2:
Filter<e3 _>(Filter<e4 _>(Agg<_ a3 _ e0 a4 r4 e1
a5 r5>(Input<r3>)));e0:=FuncCall<count>(a4);e3:=
Sublink<EXISTS Union_all<>(Input<r7>,Input<r8>)>;
e4:=Sublink<EXISTS Input<r6>>|
Agg<_ a5 _ e2 a3 r1 e1 a3 r2>(Input<r3>);e2:=
FuncCall<count>(a3)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r3,r8);TableEq(r6,r7);
TableEq(r6,r8);TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 3:
Filter<e3 _>(Agg<_ a3 _ e0 a4 r4 e1 a5 r5>(Input< r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS
Filter<e4 _>(Input<r6>)>;e4:=Sublink<EXISTS
Union_all<>(Input<r7>,Input<r8>)>|

Agg<_ a5 _ e2 a3 r1 e1 a3 r2>(Input<r3>);e2:=
FuncCall<count>(a3)|

AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r3,r8);TableEq(r6,r7);
TableEq(r6,r8);TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 4:
rule 4:
Filter<e3 _>(Agg<_ a4 _ e0 a5 r4 e2 a6 r5>(Input<
r3>));e0:=FuncCall<count>(a5);e3:=Sublink<EXISTS
Input<r6>>|
Filter<e2 a4>(Agg<_ a4 _ e1 a6 r1 e2 a4 r2>(Input<
r3>));e1:=FuncCall<count>(a6)|
AttrsEq(a4,a5);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
TableEq(r3,r6);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 5:
Filter<e3 _>(Agg<_ a4 _ e0 a5 r6 e2 a6 r7>(
Union_all<>(Filter<e4 _>(Input<r3>),Input<r5>)));
e0:=FuncCall<avg>(a5);e3:=Sublink<EXISTS Input<r8
>>;e4:=Sublink<EXISTS Input<r4>>|
Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a4 r2>(Input<r3>));e1:=FuncCall<avg>(a5)|
```

```
AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4);
NotNull(r3,a4); NotNull(r3,a5); TableEq(r3,r4);
TableEq(r3,r5); TableEq(r3,r8); TableEq(r4,r5);
TableEq(r4,r8); TableEq(r5,r8); Unique(r3,a4); Unique
(r3,a5)|
rule 6:
Exists(Agg_count < a3 a4 r4 e1 a5 r5 > (Input < r3 >),
Union(Input<r6>,Exists(Input<r7>,Input<r8>)))|
                                                           rule 11:
Agg\_count < a0 a1 r1 e0 a2 r2 > (Input < r0 >) |
AttrsEq(a0,a5); AttrsEq(a1,a5); AttrsEq(a2,a3);
AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);
NotNull(r3,a3); NotNull(r3,a4); PredicateEq(e0,e1);
TableEq(r0,r3); TableEq(r0,r6); TableEq(r0,r7);
TableEq(r0,r8); TableEq(r3,r6); TableEq(r3,r7);
TableEq(r3,r8); TableEq(r6,r7); TableEq(r6,r8);
TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
                                                           (r3,a5)|
rule 7:
Agg < a4 = e0 a5 r6 e2 a6 r7>(Filter<e3 _>(Input < e0
                                                           rule 12:
r3>));e0:=FuncCall<count>(a5);e3:=Sublink<EXISTS
Union_all <>(Input <r4>, Input <r5>)>|
Filter<e2 a4>(Agg<_ a6 _ e1 a4 r1 e2 a6 r2>(Input<
r3>));e1:=FuncCall<count>(a4)|
AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4);
NotNull(r3,a4); NotNull(r3,a5); TableEq(r3,r4);
TableEq(r3,r5); TableEq(r4,r5); Unique(r3,a4); Unique
(r3,a5)|
                                                           (r3,a5)|
rule 8:
Filter<e3 _>(Agg<_ a4 _ e0 a5 r5 e2 a6 r6>(
                                                           rule 13:
Union_all <> (Input <r3>, Input <r4>))); e0:=FuncCall <
avg>(a5);e3:=Sublink<EXISTS Filter<e4 _>(Input<r7</pre>
>)>;e4:=Sublink<EXISTS Input<r8>>|
```

# rule 8: Filter<e3 \_>(Agg<\_ a4 \_ e0 a5 r5 e2 a6 r6>( Union\_all<>(Input<r3>,Input<r4>)));e0:=FuncCall< avg>(a5);e3:=Sublink<EXISTS Filter<e4 \_>(Input<r7 >));e4:=Sublink<EXISTS Input<r8>>| Filter<e2 a4>(Agg<\_ a4 \_ e1 a5 r1 e2 a6 r2>(Input< r3>));e1:=FuncCall<avg>(a5)| AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4); NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4); TableEq(r3,r7);TableEq(r3,r8);TableEq(r4,r7); TableEq(r4,r8);TableEq(r7,r8);Unique(r3,a4);Unique(r3,a5)|

```
rule 9:
Agg_sum<a4 a5 r7 e2 a6 r8>(Exists(Input<r3>,Exists
(Input<r4>,Union(Input<r5>,Input<r6>))))|
Filter<e1 a3>(Agg_sum<a0 a1 r1 e0 a2 r2>(Input<r0
>))|
AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a4);
AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r6);
TableEq(r3,r4);TableEq(r4,r6);TableEq(r5,r6);
Unique(r3,a4);Unique(r3,a5)|
```

```
rule 10:
Exists(Agg_count<a3 a4 r4 e1 a5 r5>(Input<r3>),
Union(Exists(Input<r6>,Input<r7>),Input<r8>))|
Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|
```

```
AttrsEq(a0,a5); AttrsEq(a1,a5); AttrsEq(a2,a3); AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3); NotNull(r3,a3); NotNull(r3,a4); PredicateEq(e0,e1); TableEq(r0,r3); TableEq(r0,r6); TableEq(r0,r7); TableEq(r0,r8); TableEq(r3,r6); TableEq(r3,r7); TableEq(r3,r8); TableEq(r6,r7); TableEq(r6,r8); TableEq(r7,r8); Unique(r3,a3); Unique(r3,a4)|
```

```
rule 11:
Agg<_ a4 _ e0 a5 r6 e2 a6 r7>(Union_all<>(Input<r3
>,Filter<e3 _>(Input<r4>)));e0:=FuncCall<avg>(a5);
e3:=Sublink<EXISTS Input<r5>>|
Filter<e2 a4>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input<
r3>));e1:=FuncCall<avg>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r4,r5);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 12:
Filter<e3 _>(Agg<_ a4 _ e0 a5 r5 e2 a6 r6>(
Union_all<>(Input<r3>, Input<r4>)));e0:=FuncCall<
avg>(a5);e3:=Sublink<EXISTS Input<r7>>|
Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input<
r3>));e1:=FuncCall<avg>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r7);TableEq(r4,r7);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 13:
Filter<e3 _>(Agg<_ a3 _ e0 a4 r4 e1 a5 r5>(Input<
r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS
Union_all<>(Union_all<>(Input<r6>,Input<r7>),Input
<r8>)>|
Agg<_ a5 _ e2 a5 r1 e1 a3 r2>(Input<r3>);e2:=
FuncCall<count>(a5)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r3,r8);TableEq(r6,r7);
TableEq(r6,r8);TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 14:

Agg_count<a3 a4 r7 e1 a5 r8>(Exists(Input<r3>,
Exists(Input<r4>,Union(Input<r5>,Input<r6>))))|

Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|

AttrsEq(a0,a5);AttrsEq(a1,a3);AttrsEq(a2,a5);

AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);

NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);

TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);

TableEq(r0,r6);TableEq(r3,r4);TableEq(r3,r5);

TableEq(r3,r6);TableEq(r4,r5);TableEq(r4,r6);

TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 15:
2    Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Union_all<>(
    Union_all<>(Filter<e3 _>(Input<r3>),Input<r5>),
    Input<r6>));e0:=FuncCall<avg>(a5);e3:=Sublink<
EXISTS Input<r4>>|
```

```
Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input<
r3>));e1:=FuncCall<avg>(a5)|
AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4);
NotNull(r3,a4); NotNull(r3,a5); TableEq(r3,r4);
TableEq(r3,r5); TableEq(r3,r6); TableEq(r4,r5);
TableEq(r4,r6); TableEq(r5,r6); Unique(r3,a4); Unique
(r3,a5)|
rule 16:
Agg_count < a3 a4 r7 e1 a5 r8 > (Exists (Input < r3 > ,
Exists(Union(Input<r4>, Input<r5>), Input<r6>)))|
Agg\_count < a0 a1 r1 e0 a2 r2 > (Input < r0 >) |
AttrsEq(a0,a3); AttrsEq(a1,a3); AttrsEq(a2,a3);
AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);
NotNull(r3,a3); NotNull(r3,a4); PredicateEq(e0,e1);
TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r5);
TableEq(r0,r6); TableEq(r3,r4); TableEq(r3,r5);
TableEq(r3,r6); TableEq(r4,r5); TableEq(r4,r6);
TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
rule 17:
```

```
rule 17:
Agg<_ a3 _ e0 a4 r7 e1 a5 r8>(Filter<e3 _>(Input<
r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS
Filter<e4 _>(Union_all<>(Input<r4>,Input<r5>))>;e4
:=Sublink<EXISTS Input<r6>>|
Agg<_ a5 _ e2 a3 r1 e1 a3 r2>(Input<r3>);e2:=
FuncCall<count>(a3)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 18:

Exists(Agg_count<a4 a5 r4 e2 a6 r5>(Input<r3>),
Union(Input<r6>,Input<r7>))|

Filter<e1 a3>(Agg_count<a0 a1 r1 e0 a2 r2>(Input<
r0>))|

AttrsEq(a0,a4);AttrsEq(a1,a4);AttrsEq(a2,a6);
AttrsEq(a3,a4);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r6);TableEq(r0,r7);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r6,r7);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 19:
Filter<e3 _>(Agg<_ a4 _ e0 a5 r4 e2 a6 r5>(Input< r3>));e0:=FuncCall<sum>(a5);e3:=Sublink<EXISTS
Union_all<>(Input<r6>,Union_all<>(Input<r7>,Input< r8>))>|
Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a4 r2>(Input< r3>));e1:=FuncCall<sum>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r3,r8);TableEq(r6,r7);
TableEq(r6,r8);TableEq(r7,r8);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 20:
```

```
Filter<e3 _>(Agg<_ a3 _ e0 a4 r6 e1 a5 r7>(Filter<
e4 _>(Input<r3>)));e0:=FuncCall<count>(a4);e3:=
Sublink<EXISTS Input<r8>>;e4:=Sublink<EXISTS
Union_all<>(Input<r4>,Input<r5>)>|
agg<_ a3 _ e2 a5 r1 e1 a5 r2>(Input<r3>);e2:=
FuncCall<count>(a5)|
attrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r8);TableEq(r4,r5);
TableEq(r4,r8);TableEq(r5,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 21:
Agg<_ a3 _ e0 a4 r6 e1 a5 r7>(Filter<e3 _>(Input<
r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS
Union_all<>(Input<r4>,Input<r5>)>|
Agg<_ a5 _ e2 a5 r1 e1 a3 r2>(Input<r3>);e2:=
FuncCall<count>(a5)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r4,r5);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 22:

Filter<e3 _>(Agg<_ a4 _ e0 a5 r6 e2 a6 r7>(Filter<
e4 _>(Union_all<>(Input<r3>,Input<r4>))));e0:=
FuncCall<avg>(a5);e3:=Sublink<EXISTS Input<r8>>;e4
:=Sublink<EXISTS Input<r5>>|
Filter<e2 a4>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input<
r3>));e1:=FuncCall<avg>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r8);TableEq(r4,r5);
TableEq(r4,r8);TableEq(r5,r8);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 23:
Agg<_ a3 _ e0 a4 r7 e1 a5 r8>(Filter<e3 _>(Filter<e4 _>(Input<r3>)));e0:=FuncCall<count>(a4);e3:=
Sublink<EXISTS Input<r6>>;e4:=Sublink<EXISTS
Union_all<>(Input<r4>,Input<r5>)>|
Agg<_ a3 _ e2 a5 r1 e1 a5 r2>(Input<r3>);e2:=
FuncCall<count>(a5)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 24:

Filter<e3 _>(Agg<_ a3 _ e0 a4 r5 e1 a5 r6>(

Union_all<>(Input<r3>,Input<r4>)));e0:=FuncCall<
max>(a4);e3:=Sublink<EXISTS Input<r7>>|
Agg<_ a5 _ e2 a4 r1 e1 a3 r2>(Input<r3>);e2:=
FuncCall<max>(a4)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r4);
TableEq(r3,r7);TableEq(r4,r7);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 25:
Filter<e3 _>(Agg<_ a3 _ e0 a4 r4 e1 a5 r5>(Input< r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS
Union_all<>(Input<r6>,Filter<e4 _>(Input<r7>))>;e4
:=Sublink<EXISTS Input<r8>|
Agg<_ a5 _ e2 a5 r1 e1 a3 r2>(Input<r3>);e2:=
FuncCall<count>(a5)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r3,r8);TableEq(r6,r7);
TableEq(r6,r8);TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 26:

Filter<e3 _>(Agg<_ a4 _ e0 a5 r5 e2 a6 r6>(Filter< e4 _>(Input<r3>)));e0:=FuncCall<sum>(a5);e3:=
Sublink<EXISTS Union_all<>(Input<r7>,Input<r8>)>;
e4:=Sublink<EXISTS Input<r4>|
Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input< r3>));e1:=FuncCall<sum>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r7);TableEq(r3,r8);TableEq(r4,r7);
TableEq(r4,r8);TableEq(r7,r8);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 27:

Agg<_ a4 _ e0 a5 r5 e2 a6 r6>(Union_all<>(Input<r3 >,Input<r4>));e0:=FuncCall<avg>(a5)|

Filter<e2 a4>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input< r3>));e1:=FuncCall<avg>(a5)|

AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);

NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);

Unique(r3,a4);Unique(r3,a5)|
```

```
rule 28:
Agg<_ a3 _ e0 a4 r7 e1 a5 r8>(Filter<e3 _>(Input<
r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS
Filter<e4 _>(Union_all<>(Input<r4>,Input<r5>))>;e4
:=Sublink<EXISTS Input<r6>>|
Agg<_ a5 _ e2 a3 r1 e1 a5 r2>(Input<r3>);e2:=
FuncCall<count>(a3)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 29:
Exists(Agg_max<a3 a4 r5 e1 a5 r6>(Union(Input<r3>,
Input<r4>)),Input<r7>)|
Agg_max<a0 a1 r1 e0 a2 r2>(Input<r0>)|
AttrsEq(a0,a5);AttrsEq(a1,a4);AttrsEq(a2,a3);
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);
TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r7);
TableEq(r3,r4);TableEq(r3,r7);TableEq(r4,r7);
Unique(r3,a3);Unique(r3,a4)|
```

```
rule 30:
Exists(Agg_count<a3 a4 r5 e1 a5 r6>(Exists(Input< r3>,Input<r4>)),Union(Input<r7>,Input<r8>))|
Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|
AttrsEq(a0,a5);AttrsEq(a1,a3);AttrsSub(a5,a3);
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);
TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r7);
TableEq(r0,r8);TableEq(r3,r4);TableEq(r3,r7);
TableEq(r3,r8);TableEq(r4,r7);TableEq(r4,r8);
TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 31:

Exists(Agg_count<a4 a5 r4 e2 a6 r5>(Input<r3>),

Union(Input<r6>,Input<r7>))|

Filter<e1 a3>(Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>))|

AttrsEq(a0,a4); AttrsEq(a1,a4); AttrsEq(a2,a6);

AttrsEq(a3,a6); AttrsSub(a4,r3); AttrsSub(a5,r3);

AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);

PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,r3); TableEq(r0,r7); TableEq(r3,r6);

TableEq(r3,r7); TableEq(r6,r7); Unique(r3,a4); Unique(r3,a5)|
```

```
rule 32:
2    Agg<_ a4 _ e0 a5 r6 e2 a6 r7>(Filter<e3 _>(Input<
r3>));e0:=FuncCall<count>(a5);e3:=Sublink<EXISTS
    Union_all<>(Input<r4>,Input<r5>)>|
3    Filter<e2 a4>(Agg<_ a4 _ e1 a4 r1 e2 a4 r2>(Input<
r3>));e1:=FuncCall<count>(a4)|
4    AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r4,r5);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 33:
Agg_average < a4 a5 r5 e2 a6 r6 > (Union(Input < r3 > ,
    Input < r4 > )) |
Filter < e1 a3 > (Agg_average < a0 a1 r1 e0 a2 r2 > (Input < r0 > )) |
AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a6);
AttrsEq(a3,a6); AttrsSub(a4,r3); AttrsSub(a5,r3);
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,r3); TableEq(r0,r4); TableEq(r3,r4); Unique(r3,a4);
Unique(r3,a5) |
```

```
rule 35:
Filter<e3 _>(Agg<_ a4 _ e0 a5 r5 e1 a6 r6>(Proj<
ea3 a3 r4>(Input<r3>)));e0:=FuncCall<count>(a5);e3
:=Sublink <EXISTS Union_all <> (Input <r7>, Input <r8>)
>|
Agg<_ a6 _ e2 a4 r1 e1 a4 r2>(Input<r3>);e2:=
FuncCall < count > (a4) |
AttrsSub(a3,r3); AttrsSub(a4,r4); AttrsSub(a5,r4);
AttrsSub(a6,a4); NotNull(r3,a3); TableEq(r3,r7);
TableEq(r3,r8); TableEq(r7,r8); Unique(r3,a3)|
rule 36:
Agg<_ a4 _ e0 a5 r5 e2 a6 r6>(Filter<e3 _>(Input<
r3>)); e0:=FuncCall < count > (a5); e3:=Sublink < EXISTS
Filter<e2 a4>(Agg<_ a4 _ e1 a6 r1 e2 a4 r2>(Input<
r3>));e1:=FuncCall<count>(a6)|
AttrsEq(a4,a5); AttrsSub(a4,r3); AttrsSub(a5,r3);
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
TableEq(r3,r4);Unique(r3,a4);Unique(r3,a5)|
rule 37:
Exists(Agg_count<a3 a4 r4 e1 a5 r5>(Input<r3>),
Exists(Union(Input<r6>, Input<r7>), Input<r8>))|
Agg\_count < a0 a1 r1 e0 a2 r2 > (Input < r0 >) |
AttrsEq(a0,a5); AttrsEq(a1,a5); AttrsEq(a2,a3);
AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);
NotNull(r3,a3); NotNull(r3,a4); PredicateEq(e0,e1);
Table Eq(r0,r3); Table Eq(r0,r6); Table Eq(r0,r7);\\
TableEq(r0,r8); TableEq(r3,r6); TableEq(r3,r7);
TableEq(r3,r8); TableEq(r6,r7); TableEq(r6,r8);
TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
rule 38:
```

```
rule 38:
Agg_average<a4 a5 r7 e2 a6 r8>(Union(Union(Input<
r3>,Exists(Input<r4>,Input<r5>)),Input<r6>))|
Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input
<r0>))|
AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a4);
AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,
r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r6);
TableEq(r3,r4);TableEq(r3,r5);TableEq(r3,r6);
TableEq(r4,r5);TableEq(r4,r6);TableEq(r5,r6);
Unique(r3,a4);Unique(r3,a5)|
```

```
rule 39:

Agg<_ a3 _ e0 a4 r7 e1 a5 r8>(Filter<e3 _>(Input< r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS
Union_all<>(Union_all<>(Input<r4>,Input<r5>),Input
<r6>)>|

Agg<_ a3 _ e2 a5 r1 e1 a5 r2>(Input<r3>);e2:=
FuncCall<count>(a5)|

AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 40:
Agg_average<a4 a5 r7 e2 a6 r8>(Union(Union(Exists(Input<r3>,Input<r4>),Input<r5>),Input<r6>))|
Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input<r0>))|
AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a4);
AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r6);
TableEq(r3,r4);TableEq(r3,r5);TableEq(r3,r6);
TableEq(r4,r5);TableEq(r4,r6);TableEq(r5,r6);
Unique(r3,a4);Unique(r3,a5)|
```

```
rule 41:
Agg_average<a4 a5 r7 e2 a6 r8>(Union(Input<r3>,
Exists(Union(Input<r4>,Input<r5>),Input<r6>)))|
Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input<r0>))|
AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a4);
AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r6);
TableEq(r3,r4);TableEq(r3,r5);TableEq(r3,r6);
TableEq(r4,r5);TableEq(r4,r6);TableEq(r5,r6);
Unique(r3,a4);Unique(r3,a5)|
```

```
rule 42:
rile 42:
filter<e3 _>(Agg<_ a3 _ e0 a4 r4 e1 a5 r5>(Input<
r3>)); e0:=FuncCall<count>(a4); e3:=Sublink<EXISTS
Union_all<>(Input<r6>,Union_all<>(Input<r7>,Input<
r8>))>|
Agg<_ a5 _ e2 a3 r1 e1 a5 r2>(Input<r3>); e2:=
FuncCall<count>(a3)|
AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);
NotNull(r3,a3); NotNull(r3,a4); TableEq(r3,r6);
TableEq(r3,r7); TableEq(r3,r8); TableEq(r6,r7);
TableEq(r6,r8); TableEq(r7,r8); Unique(r3,a3); Unique(r3,a4)|
```

```
rule 44:
Agg_average<a4 a5 r7 e2 a6 r8>(Exists(Union(Input<
r3>,Input<r4>),Union(Input<r5>,Input<r6>)))|
Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input
<r0>))|
```

```
AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a4);
AttrsEq(a3,a6); AttrsSub(a4,r3); AttrsSub(a5,r3);
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,
r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r0,r6);
TableEq(r3,r4); TableEq(r3,r5); TableEq(r3,r6);
TableEq(r4,r5); TableEq(r4,r6); TableEq(r5,r6);
Unique(r3.a4):Unique(r3.a5)|
rule 45:
Exists(Agg_average < a4 a5 r6 e2 a6 r7 > (Union(Union(
Input <r3>, Input <r4>), Input <r5>)), Input <r8>)|
Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input
<r0>))|
AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a4);
AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
\label{eq:predicateEq} PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,e2); TableEq(r0,e2
r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r0,r8);
TableEq(r3,r4); TableEq(r3,r5); TableEq(r3,r8);
TableEq(r4,r5); TableEq(r4,r8); TableEq(r5,r8);
Unique(r3,a4);Unique(r3,a5)|
```

```
rule 46:

Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Union_all<>(
Union_all<>(Input<r3>,Input<r4>),Filter<e3 _>(
Input<r5>)));e0:=FuncCall<avg>(a5);e3:=Sublink<
EXISTS Input<r6>|

Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input<
r3>));e1:=FuncCall<avg>(a5)|

AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 47:

Agg_max<a3 a4 r7 e1 a5 r8>(Union(Exists(Input<r3>,
Union(Input<r4>,Input<r5>)),Input<r6>))|

Agg_max<a0 a1 r1 e0 a2 r2>(Input<r0>)|

AttrsEq(a0,a3);AttrsEq(a1,a4);AttrsEq(a2,a5);
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);

NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);

TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);

TableEq(r3,r6);TableEq(r3,r4);TableEq(r3,r5);

TableEq(r3,r6);TableEq(r4,r5);TableEq(r4,r6);

TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
```

```
Filter<e3 _>(Agg<_ a3 _ e0 a4 r5 e1 a5 r6>(Filter<
e4 _>(Input<r3>)));e0:=FuncCall<count>(a4);e3:=
Sublink<EXISTS Union_all<>(Input<r7>,Input<r8>)>;
e4:=Sublink<EXISTS Input<r4>>|
Agg<_ a3 _ e2 a3 r1 e1 a5 r2>(Input<r3>);e2:=
FuncCall<count>(a3)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r4);
TableEq(r3,r7);TableEq(r3,r8);TableEq(r4,r7);
TableEq(r4,r8);TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

rule 48:

```
rule 49:
    Exists(Agg_count<a4 a5 r4 e2 a6 r5>(Input<r3>),
    Union(Input<r6>,Input<r7>))|
    Filter<e1 a3>(Agg_count<a0 a1 r1 e0 a2 r2>(Input<
r0>))|
    AttrsEq(a0,a4);AttrsEq(a1,a6);AttrsEq(a2,a6);
    AttrsSub(a3,a4);AttrsSub(a4,r3);AttrsSub(a5,r3);
    AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
    PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r7);TableEq(r3,r6);
    TableEq(r3,r7);TableEq(r6,r7);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 50:
2 Agg<_ a4 _ e0 a5 r6 e2 a6 r7>(Filter<e3 _>(Input< r3>)); e0:=FuncCall<sum>(a5); e3:=Sublink<EXISTS
Union_all<>(Input<r4>,Input<r5>)>|
5 Filter<e2 a4>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input< r3>)); e1:=FuncCall<sum>(a5)|
6 AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4);
NotNull(r3,a4); NotNull(r3,a5); TableEq(r3,r4);
TableEq(r3,r5); TableEq(r4,r5); Unique(r3,a4); Unique(r3,a5)|
```

```
rule 51:
Agg<_ a3 _ e0 a4 r7 e1 a5 r8>(Filter<e3 _>(Input<
r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS
Union_all<>(Filter<e4 _>(Input<r4>),Input<r6>)>;e4
:=Sublink<EXISTS Input<r5>>|
Agg<_ a5 _ e2 a3 r1 e1 a5 r2>(Input<r3>);e2:=
FuncCall<count>(a3)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 52:

2 Agg_average<a4 a5 r7 e2 a6 r8>(Union(Input<r3>,
Exists(Input<r4>,Exists(Input<r5>,Input<r6>))))|
5 Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input
<r0>))|
6 AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a6);
AttrsEq(a3,a6); AttrsSub(a4,r3); AttrsSub(a5,r3);
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r0,r6);
TableEq(r3,r4); TableEq(r4,r6); TableEq(r5,r6);
Unique(r3,a4); Unique(r3,a5)|
```

```
rule 53:
Exists(Agg_count<a3 a4 r4 e1 a5 r5>(Input<r3>),
Union(Exists(Input<r6>,Input<r7>),Input<r8>))|
Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|
```

```
(Appendix)
AttrsEq(a0, a5); AttrsEq(a1, a3); AttrsEq(a2, a5);
AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);
NotNull(r3,a3); NotNull(r3,a4); PredicateEq(e0,e1);
TableEq(r0,r3); TableEq(r0,r6); TableEq(r0,r7);
TableEq(r0,r8); TableEq(r3,r6); TableEq(r3,r7);
TableEq(r3,r8); TableEq(r6,r7); TableEq(r6,r8);
TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
rule 54:
Agg_count < a4 a5 r6 e2 a6 r7 > (Exists (Input < r3 > ,
Union(Input<r4>, Input<r5>)))|
Filter<e1 a3>(Agg_count<a0 a1 r1 e0 a2 r2>(Input<
r0>))|
AttrsEq(a0,a4); AttrsEq(a1,a4); AttrsEq(a2,a4);
AttrsEq(a3,a4);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
\label{eq:predicateEq} PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,e2); TableEq(r0,e2
r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r3,r4);
TableEq(r3,r5); TableEq(r4,r5); Unique(r3,a4); Unique
```

```
rule 55:

Exists(Agg_count<a4 a5 r4 e2 a6 r5>(Input<r3>),
Union(Input<r6>,Input<r7>))|

Filter<e1 a3>(Agg_count<a0 a1 r1 e0 a2 r2>(Input<
r0>))|

AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a6);
AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r7);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r6,r7);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 56:
2 Agg<_ a3 _ e0 a4 r7 e1 a5 r8>(Union_all<>(Filter<
e3 _>(Input<r3>),Input<r6>));e0:=FuncCall<max>(a4)
;e3:=Sublink<EXISTS Union_all<>(Input<r4>,Input<r5
>)>|
3 Agg<_ a3 _ e2 a4 r1 e1 a5 r2>(Input<r3>);e2:=
FuncCall<max>(a4)|
4 AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 57:
Agg<_ a4 _ e0 a5 r6 e2 a6 r7>(Union_all<>(Filter< e3 _>(Input<r3>),Input<r5>));e0:=FuncCall<avg>(a5);e3:=Sublink<EXISTS Input<r4>>|
Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input< r3>));e1:=FuncCall<avg>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r4,r5);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 58:

Exists(Agg_average<a4 a5 r5 e2 a6 r6>(Union(Input<
r3>,Input<r4>)),Union(Input<r7>,Input<r8>))|
```

```
Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input <r0>))|
AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a6);
AttrsEq(a3,a4); AttrsSub(a4,r3); AttrsSub(a5,r3);
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r7); TableEq(r0,r8);
TableEq(r3,r4); TableEq(r3,r7); TableEq(r3,r8);
TableEq(r4,r7); TableEq(r4,r8); TableEq(r7,r8);
Unique(r3,a4); Unique(r3,a5)|
```

```
rule 59:
rule 59:
Filter<e3 _>(Filter<e4 _>(Agg<_ a4 _ e0 a5 r4 e2 a6 r5>(Input<r3>)));e0:=FuncCall<sum>(a5);e3:=
Sublink<EXISTS Union_all<>(Input<r7>, Input<r8>)>;
e4:=Sublink<EXISTS Input<r6>|
Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a4 r2>(Input<r3>));e1:=FuncCall<sum>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r3,r8);TableEq(r6,r7);
TableEq(r6,r8);TableEq(r7,r8);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 60:
Exists(Agg_sum<a4 a5 r6 e2 a6 r7>(Exists(Input<r3 >,Union(Input<r4>,Input<r5>))),Input<r8>)|
Filter<e1 a3>(Agg_sum<a0 a1 r1 e0 a2 r2>(Input<r0 >))|
AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a6);
AttrsEq(a3,a4);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r8);
TableEq(r3,r4);TableEq(r3,r5);TableEq(r3,r8);
TableEq(r4,r5);TableEq(r4,r8);TableEq(r5,r8);
Unique(r3,a4);Unique(r3,a5)|
```

```
rule 61:

Exists(Agg_count<a3 a4 r4 e1 a5 r5>(Input<r3>),

Union(Input<r6>,Union(Input<r7>,Input<r8>)))|

Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|

AttrsEq(a0,a3);AttrsEq(a1,a3);AttrsEq(a2,a5);

AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);

NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);

TableEq(r0,r3);TableEq(r0,r6);TableEq(r0,r7);

TableEq(r0,r8);TableEq(r3,r6);TableEq(r3,r7);

TableEq(r3,r8);TableEq(r6,r7);TableEq(r6,r8);

TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 62:
2 Agg_average<a4 a5 r7 e2 a6 r8>(Union(Exists(Exists (Input<r3>,Input<r4>),Input<r5>),Input<r6>))|
3 Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input<r0>))|
```

```
AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a6);
AttrsEq(a3,a4); AttrsSub(a4,r3); AttrsSub(a5,r3);
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,
r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r0,r6);
TableEq(r3,r4); TableEq(r3,r5); TableEq(r3,r6);
TableEq(r4,r5); TableEq(r4,r6); TableEq(r5,r6);
Unique(r3,a4);Unique(r3,a5)|
rule 63:
Agg_count < a5 a6 r7 e2 a7 r8 > (Exists(Proj_simple <_
a4 r4>(Input<r3>),Union(Input<r5>,Input<r6>)))|
Filter<e1 a3>(Agg_count<a0 a1 r1 e0 a2 r2>(Input<
r0>))|
AttrsEq(a0, a5); AttrsEq(a1, a5); AttrsEq(a2, a5);
AttrsEq(a3,a5); AttrsSub(a4,r3); AttrsSub(a5,r4);
AttrsSub(a6,r4); AttrsSub(a7,a5); NotNull(r3,a4);
PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,
r3); TableEq(r0,r5); TableEq(r0,r6); TableEq(r3,r5);
TableEq(r3,r6); TableEq(r5,r6); Unique(r3,a4)|
```

rule 64:
Filter<e3 \_>(Agg<\_ a4 \_ e0 a5 r4 e2 a6 r5>(Input< r3>)); e0:=FuncCall<count>(a5); e3:=Sublink<EXISTS
Union\_all<>(Input<r6>,Input<r7>)>|
Filter<e2 a4>(Agg<\_ a6 \_ e1 a5 r1 e2 a4 r2>(Input< r3>)); e1:=FuncCall<count>(a5)|
AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4);
NotNull(r3,a4); NotNull(r3,a5); TableEq(r3,r6);
TableEq(r3,r7); TableEq(r6,r7); Unique(r3,a4); Unique(r3,a5)|

rule 65:

Agg<\_ a4 \_ e0 a5 r6 e2 a6 r7>(Filter<e3 \_>(Input< r3>));e0:=FuncCall<count>(a5);e3:=Sublink<EXISTS
Union\_all<>(Input<r4>,Input<r5>)>|

Filter<e2 a6>(Agg<\_ a6 \_ e1 a6 r1 e2 a6 r2>(Input< r3>));e1:=FuncCall<count>(a6)|

AttrsEq(a4,a5);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);

TableEq(r3,r4);TableEq(r3,r5);TableEq(r4,r5);
Unique(r3,a4);Unique(r3,a5)|

rule 66:
 Agg\_average<a4 a5 r6 e2 a6 r7>(Union(Input<r3>,
 Union(Input<r4>,Input<r5>)))|
 Filter<e1 a3>(Agg\_average<a0 a1 r1 e0 a2 r2>(Input
 <r0>))|
 AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a6);
 AttrsEq(a3,a6); AttrsSub(a4,r3); AttrsSub(a5,r3);
 AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
 PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r3,r4);
 TableEq(r3,r5); TableEq(r4,r5); Unique(r3,a4); Unique(r3,a5)|

```
rule 67:

Agg_count<a4 a5 r6 e2 a6 r7>(Exists(Input<r3>,
Union(Input<r4>,Input<r5>)))|

Filter<e1 a3>(Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>))|
```

```
AttrsEq(a0,a4); AttrsEq(a1,a4); AttrsEq(a2,a6); AttrsEq(a3,a6); AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5); PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r3,r4); TableEq(r3,r5); TableEq(r4,r5); Unique(r3,a4); Unique(r3,a5)|
```

rule 68:

Agg\_average<a4 a5 r6 e2 a6 r7>(Exists(Union(Input< r3>,Input<r4>),Input<r5>))|

Filter<e1 a3>(Agg\_average<a0 a1 r1 e0 a2 r2>(Input < r0>))|

AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a4);
AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r4,r5);Unique(r3,a4);Unique(r3,a5)|

```
rule 69:
Agg_average<a4 a5 r7 e2 a6 r8>(Union(Input<r3>,
Exists(Union(Input<r4>,Input<r5>),Input<r6>)))|
Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input</ri>
<r0>))|
AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a6);
AttrsEq(a3,a4);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r6);
TableEq(r3,r4);TableEq(r3,r5);TableEq(r5,r6);
Unique(r3,a4);Unique(r3,a5)|
```

rule 70:
Exists(Agg\_count<a3 a4 r4 e1 a5 r5>(Input<r3>),
Union(Exists(Input<r6>,Input<r7>),Input<r8>))|
Agg\_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|
AttrsEq(a0,a3);AttrsEq(a1,a3);AttrsSub(a5,a3);
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);
TableEq(r0,r3);TableEq(r0,r6);TableEq(r0,r7);
TableEq(r0,r8);TableEq(r3,r6);TableEq(r3,r7);
TableEq(r3,r8);TableEq(r6,r7);TableEq(r6,r8);
TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|

```
rule 71:

rule 71:

rilter<e3 _>(Agg<_ a3 _ e0 a4 r4 e1 a5 r5>(Input< r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS
Union_all<>(Input<r6>,Input<r7>)>|

Agg<_ a3 _ e2 a3 r1 e1 a3 r2>(Input<r3>);e2:=
FuncCall<count>(a3)|

AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r6,r7);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 72:
Agg_count<a3 a4 r7 e1 a5 r8>(Exists(Input<r3>,
Union(Input<r4>,Union(Input<r5>,Input<r6>))))|
```

```
Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|
AttrsEq(a0,a5);AttrsEq(a1,a3);AttrsEq(a2,a3);
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);
TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);
TableEq(r0,r6);TableEq(r3,r4);TableEq(r3,r5);
TableEq(r3,r6);TableEq(r4,r5);TableEq(r4,r6);
TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|

rule 73:
Filter<e3 _>(Agg<_ a3 _ e0 a4 r4 e1 a5 r5>(Input<r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS
Union_all<>(Filter<e4 _>(Input<r6>),Input<r8>)>;e4
:=Sublink<EXISTS Input<r7>>|
Agg<_ a5 _ e2 a3 r1 e1 a5 r2>(Input<r3>);e2:=
```

AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);

TableEq(r6,r8); TableEq(r7,r8); Unique(r3,a3); Unique

NotNull(r3,a3); NotNull(r3,a4); TableEq(r3,r6);

TableEq(r3,r7); TableEq(r3,r8); TableEq(r6,r7);

FuncCall < count > (a3) |

(r3,a4)|

```
rule 75:
Agg<_ a4 _ e0 a5 r6 e2 a6 r7>(Filter<e3 _>(Input<
r3>));e0:=FuncCall<count>(a5);e3:=Sublink<EXISTS
Union_all<>(Input<r4>,Input<r5>)>|
Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a4 r2>(Input<
r3>));e1:=FuncCall<count>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r4,r5);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 76:

Exists(Agg_sum<a4 a5 r4 e2 a6 r5>(Input<r3>), Union (Exists(Input<r6>, Input<r7>), Input<r8>))|

Filter<e1 a3>(Agg_sum<a0 a1 r1 e0 a2 r2>(Input<r0
>))|

AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a4);

AttrsEq(a3,a6); AttrsSub(a4,r3); AttrsSub(a5,r3);

AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);

PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,r3); TableEq(r0,r6); TableEq(r0,r7); TableEq(r0,r8);

TableEq(r3,r6); TableEq(r6,r8); TableEq(r7,r8);

Unique(r3,a4); Unique(r3,a5)|
```

```
rule 77:
```

```
2 Agg<_ a4 _ e0 a5 r7 e1 a6 r8>(Filter<e3 _>(Proj<
ea3 a3 r4>(Input<r3>)));e0:=FuncCall<count>(a5);e3
:=Sublink<EXISTS Union_all<>(Input<r5>,Input<r6>)
>|
3 Agg<_ a4 _ e2 a4 r1 e1 a6 r2>(Input<r3>);e2:=
FuncCall<count>(a4)|
4 AttrsSub(a3,r3);AttrsSub(a4,r4);AttrsSub(a5,r4);
AttrsSub(a6,a4);NotNull(r3,a3);TableEq(r3,r5);
TableEq(r3,r6);TableEq(r5,r6);Unique(r3,a3)|
```

```
rule 78:

Filter<e3 _>(Agg<_ a3 _ e0 a4 r4 e1 a5 r5>(Input< r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS
Union_all<>(Input<r6>,Filter<e4 _>(Input<r7>))>;e4
:=Sublink<EXISTS Input<r8>>|

Agg<_ a3 _ e2 a3 r1 e1 a5 r2>(Input<r3>);e2:=
FuncCall<count>(a3)|

AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r3,r8);TableEq(r6,r7);
TableEq(r6,r8);TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 79:
Filter<e3 _>(Filter<e4 _>(Agg<_ a3 _ e0 a4 r4 e1
a5 r5>(Input<r3>)));e0:=FuncCall<count>(a4);e3:=
Sublink<EXISTS Input<r8>>;e4:=Sublink<EXISTS
Union_all<>(Input<r6>,Input<r7>)>|
Agg<_ a3 _ e2 a3 r1 e1 a5 r2>(Input<r3>);e2:=
FuncCall<count>(a3)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r3,r8);TableEq(r6,r7);
TableEq(r6,r8);TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 80:

Exists(Exists(Agg_count<a3 a4 r4 e1 a5 r5>(Input<ra>
r3>),Union(Input<r6>,Input<r7>)),Input<r8>)|

Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|

AttrsEq(a0,a3);AttrsEq(a1,a5);AttrsEq(a2,a5);

AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);

NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);

TableEq(r0,r3);TableEq(r0,r6);TableEq(r0,r7);

TableEq(r0,r8);TableEq(r3,r6);TableEq(r3,r7);

TableEq(r3,r8);TableEq(r6,r7);TableEq(r6,r8);

TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 81:
Exists(Agg_sum<a4 a5 r4 e2 a6 r5>(Input<r3>),Union
(Input<r6>,Union(Input<r7>,Input<r8>)))|
Filter<e1 a3>(Agg_sum<a0 a1 r1 e0 a2 r2>(Input<r0
>))|
AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a4);
AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r6);TableEq(r0,r7);TableEq(r0,r8);
TableEq(r3,r6);TableEq(r3,r7);TableEq(r7,r8);
Unique(r3,a4);Unique(r3,a5)|
```

```
rule 82:
Filter<e3 _>(Agg<_ a4 _ e0 a5 r4 e2 a6 r5>(Input<
r3>));e0:=FuncCall<sum>(a5);e3:=Sublink<EXISTS
Filter<e4 _>(Union_all<>(Input<r6>,Input<r7>))>;e4
:=Sublink<EXISTS Input<r8>>|
Filter<e2 a4>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input<
r3>));e1:=FuncCall<sum>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r3,r8);TableEq(r6,r7);
TableEq(r6,r8);TableEq(r7,r8);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 83:

Agg_count<a3 a4 r7 e1 a5 r8>(Exists(Input<r3>,
Union(Union(Input<r4>,Input<r5>),Input<r6>)))|

Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|

AttrsEq(a0,a5);AttrsEq(a1,a3);AttrsSub(a5,a3);

AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);

NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);

TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);

TableEq(r0,r6);TableEq(r3,r4);TableEq(r3,r5);

TableEq(r3,r6);TableEq(r4,r5);TableEq(r4,r6);

TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 84:

Exists(Agg_count<a4 a5 r4 e2 a6 r5>(Input<r3>),
    Union(Input<r6>, Input<r7>))|

Filter<e1 a3>(Agg_count<a0 a1 r1 e0 a2 r2>(Input<
r0>))|

AttrsEq(a0,a4);AttrsEq(a1,a6);AttrsEq(a2,a6);
    AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);
    AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
    PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r6);TableEq(r0,r7);TableEq(r3,r6);
    TableEq(r3,r7);TableEq(r6,r7);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 85:

Agg_count<a4 a5 r7 e1 a6 r8>(Exists(Proj_simple<_a3 r4>(Input<r3>),Union(Input<r5>,Input<r6>)))|

Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|

AttrsEq(a0,a6);AttrsEq(a1,a6);AttrsEq(a2,a4);

AttrsSub(a3,r3);AttrsSub(a4,r4);AttrsSub(a5,r4);

AttrsSub(a6,a4);NotNull(r3,a3);PredicateEq(e0,e1);

TableEq(r0,r3);TableEq(r0,r5);TableEq(r0,r6);

TableEq(r3,r5);TableEq(r3,r6);TableEq(r5,r6);

Unique(r3,a3)|
```

```
rule 86:

Exists(Agg_sum<a4 a5 r4 e2 a6 r5>(Input<r3>),Union (Input<r6>,Union(Input<r7>,Input<r8>)))|

Filter<e1 a3>(Agg_sum<a0 a1 r1 e0 a2 r2>(Input<r0
>))|
```

```
AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a6); AttrsEq(a3,a4); AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5); PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,r3); TableEq(r0,r6); TableEq(r0,r7); TableEq(r0,r8); TableEq(r3,r6); TableEq(r3,r7); TableEq(r3,r8); TableEq(r6,r7); TableEq(r6,r8); TableEq(r7,r8); Unique(r3,a4); Unique(r3,a5)|
```

```
rule 87:

Exists(Agg_count<a4 a5 r4 e2 a6 r5>(Input<r3>),
Union(Input<r6>,Input<r7>))|

Filter<e1 a3>(Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>))|

AttrsEq(a0,a6);AttrsEq(a1,a4);AttrsEq(a2,a6);
AttrsEq(a3,a4);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r6);TableEq(r0,r7);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r6,r7);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 88:
Exists(Agg_average < a4 a5 r4 e2 a6 r5>(Input < r3>),
    Union(Union(Input < r6>, Input < r7>), Input < r8>))|
Filter < e1 a3>(Agg_average < a0 a1 r1 e0 a2 r2>(Input < r0>))|
AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a4);
AttrsEq(a3,a6); AttrsSub(a4,r3); AttrsSub(a5,r3);
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,r3); TableEq(r0,r6); TableEq(r0,r7); TableEq(r0,r8);
TableEq(r3,r6); TableEq(r3,r7); TableEq(r3,r8);
TableEq(r6,r7); TableEq(r6,r8); TableEq(r7,r8);
Unique(r3,a4); Unique(r3,a5)|
```

```
rule 89:

2 Agg_count<a3 a4 r6 e1 a5 r7>(Exists(Input<r3>,
    Union(Input<r4>,Input<r5>)))|

3 Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|

4 AttrsEq(a0,a3); AttrsEq(a1,a5); AttrsEq(a2,a5);
    AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);
    NotNull(r3,a3); NotNull(r3,a4); PredicateEq(e0,e1);
    TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r5);
    TableEq(r3,r4); TableEq(r3,r5); TableEq(r4,r5);
    Unique(r3,a3); Unique(r3,a4)|
```

```
rule 90:
2 Agg<_ a4 _ e0 a5 r6 e2 a6 r7>(Union_all<>(Input<r3
>,Union_all<>(Input<r4>,Input<r5>)));e0:=FuncCall<
avg>(a5)|
3 Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input<
r3>));e1:=FuncCall<avg>(a5)|
4 AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r4,r5);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 91:
```

```
(Appendix)
Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Filter<e3 _>(
Union_all <>(Union_all <>(Input <r3>, Input <r4>), Input
<r5>)));e0:=FuncCall<avg>(a5);e3:=Sublink<EXISTS
Input <r6>>1
Filter < e2 a6 > (Agg < _ a4 _ e1 a5 r1 e2 a4 r2 > (Input <
r3>));e1:=FuncCall<avg>(a5)|
AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4);
NotNull(r3,a4); NotNull(r3,a5); TableEq(r3,r4);
TableEq(r3,r5); TableEq(r3,r6); TableEq(r4,r5);
TableEq(r4,r6); TableEq(r5,r6); Unique(r3,a4); Unique
(r3, a5)
rule 92:
Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Union_all<>(Filter<
e3 _>(Filter<e4 _>(Input<r3>)),Input<r6>));e0:=
FuncCall < avg > (a5); e3:=Sublink < EXISTS Input < r5 >>; e4
:=Sublink<EXISTS Input<r4>>|
Filter < e2 a4 > (Agg < _ a4 _ e1 a5 r1 e2 a6 r2 > (Input <
r3>));e1:=FuncCall<avg>(a5)|
AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4);
NotNull(r3,a4); NotNull(r3,a5); TableEq(r3,r4);
TableEq(r3,r5); TableEq(r3,r6); TableEq(r4,r5);
TableEq(r4,r6); TableEq(r5,r6); Unique(r3,a4); Unique
```

```
rule 93:

Agg<_ a4 _ e0 a5 r6 e2 a6 r7>(Filter<e3 _>(Input< r3>)); e0:=FuncCall<sum>(a5); e3:=Sublink<EXISTS
Union_all<>(Input<r4>, Input<r5>)>|

Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a4 r2>(Input< r3>)); e1:=FuncCall<sum>(a5)|

AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4);
NotNull(r3,a4); NotNull(r3,a5); TableEq(r3,r4);
TableEq(r3,r5); TableEq(r4,r5); Unique(r3,a4); Unique(r3,a5)|
```

(r3,a5)|

```
rule 94:
Agg<_ a4 _ e0 a5 r6 e2 a6 r7>(Union_all<>(
Union_all<>(Input<r3>,Input<r4>),Input<r5>));e0:=
FuncCall<avg>(a5)|
Filter<e2 a4>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input<
r3>));e1:=FuncCall<avg>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r4,r5);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 95:

Agg_average<a4 a5 r7 e2 a6 r8>(Union(Exists(Input<r3>,Union(Input<r4>,Input<r5>)),Input<r6>))|

Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input<r0>))|

AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a6);

AttrsSub(a6,a4);AttrsSub(a4,r3);AttrsSub(a5,r3);

AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);

PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r6);

TableEq(r3,r4);TableEq(r3,r5);TableEq(r3,r6);

TableEq(r4,r5);TableEq(r4,r6);TableEq(r5,r6);

Unique(r3,a4);Unique(r3,a5)|
```

```
rule 96:
Agg_count<a3 a4 r7 e1 a5 r8>(Exists(Input<r3>,
Exists(Union(Input<r4>,Input<r5>),Input<r6>)))|
Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|
AttrsEq(a0,a3);AttrsEq(a1,a5);AttrsEq(a2,a5);
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);
TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);
TableEq(r0,r6);TableEq(r3,r4);TableEq(r3,r5);
TableEq(r3,r6);TableEq(r4,r5);TableEq(r4,r6);
TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 97:

Agg_average<a4 a5 r7 e2 a6 r8>(Union(Exists(Union(Input<r3>,Input<r4>),Input<r5>),Input<r6>))|

Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input<r0>)|

AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a4);
AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);

PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r6);

TableEq(r3,r4);TableEq(r3,r5);TableEq(r3,r6);

TableEq(r4,r5);TableEq(r4,r6);TableEq(r5,r6);

Unique(r3,a4);Unique(r3,a5)|
```

```
rule 98:
Agg<_ a3 _ e0 a4 r7 e1 a5 r8>(Filter<e3 _>(Input<
r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS
Union_all<>(Union_all<>(Input<r4>,Input<r5>),Input
<r6>)>|
Agg<_ a5 _ e2 a5 r1 e1 a3 r2>(Input<r3>);e2:=
FuncCall<count>(a5)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 99:
Agg_count<a3 a4 r7 e1 a5 r8>(Exists(Exists(Input < r3>,Union(Input < r4>,Input < r5>)),Input < r6>))|
Agg_count < a0 a1 r1 e0 a2 r2>(Input < r0>)|
AttrsEq(a0,a5);AttrsEq(a1,a3);AttrsSub(a2,a5);
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);
TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);
TableEq(r0,r6);TableEq(r3,r4);TableEq(r3,r5);
TableEq(r3,r6);TableEq(r4,r5);TableEq(r4,r6);
TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 100:
Filter<e3 _>(Filter<e4 _>(Agg<_ a4 _ e0 a5 r5 e2 a6 r6>(Union_all<>(Input<r3>,Input<r4>))));e0:=
FuncCall<avg>(a5);e3:=Sublink<EXISTS Input<r8>>;e4
:=Sublink<EXISTS Input<r7>>|
Filter<e2 a4>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input<r3>)));e1:=FuncCall<avg>(a5)|
```

```
rule 101:
Agg_count<a3 a4 r7 e1 a5 r8>(Exists(Exists(Input<r3>,Union(Input<r4>,Input<r5>)),Input<r6>))|
Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|
AttrsEq(a0,a3);AttrsEq(a1,a3);AttrsEq(a2,a3);
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);
TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);
TableEq(r0,r6);TableEq(r3,r4);TableEq(r3,r5);
TableEq(r3,r6);TableEq(r4,r5);TableEq(r4,r6);
```

AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4);

NotNull(r3,a4); NotNull(r3,a5); TableEq(r3,r4);

TableEq(r3,r7); TableEq(r3,r8); TableEq(r4,r7);
TableEq(r4,r8); TableEq(r7,r8); Unique(r3,a4); Unique

```
rule 102:
Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Filter<e3 _>(Filter<e4 _>(Input<r3>)));e0:=FuncCall<sum>(a5);e3:=
Sublink<EXISTS Input<r6>>;e4:=Sublink<EXISTS
Union_all<>(Input<r4>,Input<r5>)>|
Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input<r3>));e1:=FuncCall<sum>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a4);Unique(r3,a5)|
```

TableEq(r5,r6); Unique(r3,a3); Unique(r3,a4)|

```
rule 103:
    Exists(Exists(Agg_count<a4 a5 r4 e2 a6 r5>(Input< r3>),Input<r6>),Input<r7>)|
    Filter<e1 a3>(Agg_count<a0 a1 r1 e0 a2 r2>(Input< r0>))|
    AttrsEq(a0,a4);AttrsEq(a0,a5);AttrsEq(a1,a6);
    AttrsEq(a2,a4);AttrsEq(a2,a5);AttrsEq(a3,a4);
    AttrsSub(a5,r3);AttrsSub(a4,r3);
    AttrsSub(a5,r3);AttrsSub(a6,a4);NotNull(r3,a4);
    NotNull(r3,a5);PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r7);
    TableEq(r3,r6);TableEq(r3,r7);TableEq(r6,r7);
    Unique(r3,a4);Unique(r3,a5)|
```

```
rule 104:
Agg_count<a4 a5 r6 e2 a6 r7>(Exists(Input<r3>,
    Union(Input<r4>,Input<r5>)))|
Filter<e1 a3>(Agg_count<a0 a1 r1 e0 a2 r2>(Input<
r0>))|
AttrsEq(a0,a6);AttrsEq(a1,a6);AttrsEq(a2,a4);
AttrsEq(a3,a4);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r4,r5);Unique(r3,a4);Unique(r3,a5)|
```

```
Filter<e3 _>(Agg<_ a4 _ e0 a5 r4 e2 a6 r5>(Input<
r3>));e0:=FuncCall<sum>(a5);e3:=Sublink<EXISTS
Union_all<>(Filter<e4 _>(Input<r6>),Input<r8>)>;e4
:=Sublink<EXISTS Input<r7>>|
Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input<
r3>));e1:=FuncCall<sum>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r6);
TableEq(r6,r8);TableEq(r7,r8);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 106:
Agg_count<a4 a5 r6 e2 a6 r7>(Exists(Input<r3>,
Union(Input<r4>,Input<r5>)))|
Filter<e1 a3>(Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>))|
AttrsEq(a0,a6); AttrsEq(a1,a4); AttrsEq(a2,a6);
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r3,r4);
TableEq(r3,r5); TableEq(r4,r5); Unique(r3,a4); Unique(r3,a5)|
```

```
rule 107:
Filter<e3 _>(Agg<_ a4 _ e0 a5 r4 e2 a6 r5>(Input< r3>));e0:=FuncCall<count>(a5);e3:=Sublink<EXISTS
Union_all<>(Input<r6>,Input<r7>)>|
Filter<e2 a4>(Agg<_ a6 _ e1 a6 r1 e2 a6 r2>(Input< r3>));e1:=FuncCall<count>(a6)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r6,r7);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 108:

Exists(Exists(Agg_count < a3 a4 r4 e1 a5 r5>(Input < r3>), Union(Input < r6>, Input < r7>)), Input < r8>)|

Agg_count < a0 a1 r1 e0 a2 r2>(Input < r0>)|

AttrsEq(a0,a5); AttrsEq(a1,a5); AttrsEq(a2,a3);

AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);

NotNull(r3,a3); NotNull(r3,a4); PredicateEq(e0,e1);

TableEq(r0,r3); TableEq(r0,r6); TableEq(r0,r7);

TableEq(r0,r8); TableEq(r3,r6); TableEq(r3,r7);

TableEq(r7,r8); Unique(r3,a3); Unique(r3,a4)|
```

```
rule 109:
Filter<e3 _>(Agg<_ a4 _ e0 a5 r4 e2 a6 r5>(Input<
r3>));e0:=FuncCall<sum>(a5);e3:=Sublink<EXISTS
Union_all<>(Input<r6>,Input<r7>)>|
Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a4 r2>(Input<
r3>));e1:=FuncCall<sum>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r6,r7);Unique(r3,a4);Unique(r3,a5)|
```

```
1 rule 110:
```

```
2 Agg_sum<a4 a5 r7 e2 a6 r8>(Exists(Input<r3>,Union(Input<r4>,Exists(Input<r5>,Input<r6>))))|
3 Filter<e1 a3>(Agg_sum<a0 a1 r1 e0 a2 r2>(Input<r0>)))|
4 AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a6);
AttrsEq(a3,a4);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r6);
TableEq(r3,r4);TableEq(r3,r5);TableEq(r3,r6);
TableEq(r4,r5);TableEq(r4,r6);TableEq(r5,r6);
Unique(r3,a4);Unique(r3,a5)|
```

```
rule 111:

Agg_sum<a4 a5 r7 e2 a6 r8>(Exists(Input<r3>,Union(Input<r4>,Union(Input<r5>,Input<r6>))))|

Filter<e1 a3>(Agg_sum<a0 a1 r1 e0 a2 r2>(Input<r0 >))|

AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a6);

AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);

AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);

PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r6);

TableEq(r3,r4);TableEq(r3,r5);TableEq(r3,r6);

TableEq(r4,r5);TableEq(r4,r6);TableEq(r5,r6);

Unique(r3,a4);Unique(r3,a5)|
```

```
rule 112:
Exists(Exists(Agg_count < a3 a4 r4 e1 a5 r5>(Input < r3>), Union(Input < r6>, Input < r7>)), Input < r8>)|
Agg_count < a0 a1 r1 e0 a2 r2>(Input < r0>)|
AttrsEq(a0,a3); AttrsEq(a1,a3); AttrsEq(a2,a5);
AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);
NotNull(r3,a3); NotNull(r3,a4); PredicateEq(e0,e1);
TableEq(r0,r3); TableEq(r0,r6); TableEq(r0,r7);
TableEq(r0,r8); TableEq(r3,r6); TableEq(r3,r7);
TableEq(r7,r8); Unique(r3,a3); Unique(r3,a4)|
```

```
rule 113:
rule 113:
rilter<e3 _>(Agg<_ a4 _ e0 a5 r4 e2 a6 r5>(Input<
r3>));e0:=FuncCall<count>(a5);e3:=Sublink<EXISTS
Union_all<>(Input<r6>,Input<r7>)>|
Filter<e2 a4>(Agg<_ a4 _ e1 a6 r1 e2 a6 r2>(Input<
r3>));e1:=FuncCall<count>(a6)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r6,r7);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 114:
2 Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Filter<e3 _>(
Union_all<>(Input<r3>,Filter<e4 _>(Input<r4>))));
e0:=FuncCall<avg>(a5);e3:=Sublink<EXISTS Input<r6
>>;e4:=Sublink<EXISTS Input<r5>>|
Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a4 r2>(Input<
r3>));e1:=FuncCall<avg>(a5)|
```

```
AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4);
NotNull(r3,a4); NotNull(r3,a5); TableEq(r3,r4);
TableEq(r3,r5); TableEq(r3,r6); TableEq(r4,r5);
TableEq(r4,r6); TableEq(r5,r6); Unique(r3,a4); Unique(r3,a5)|
```

```
rule 115:

2 Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Filter<e3 _>(Input< r3>));e0:=FuncCall<sum>(a5);e3:=Sublink<EXISTS
Filter<e4 _>(Input<r4>)>;e4:=Sublink<EXISTS
Union_all<>(Input<r5>,Input<r6>)>|
Filter<e2 a4>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input< r3>));e1:=FuncCall<sum>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 116:

Exists(Agg_sum<a4 a5 r4 e2 a6 r5>(Input<r3>), Union (Exists(Input<r6>,Input<r7>),Input<r8>))|

Filter<e1 a3>(Agg_sum<a0 a1 r1 e0 a2 r2>(Input<r0
>))|

AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a6);

AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);

PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r6);TableEq(r0,r7);TableEq(r0,r8);

TableEq(r3,r6);TableEq(r6,r8);TableEq(r7,r8);

Unique(r3,a4);Unique(r3,a5)|
```

```
rule 117:
Agg_average<a4 a5 r7 e2 a6 r8>(Union(Exists(Union(Input<r3>,Input<r4>),Input<r5>),Input<r6>))|
Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input<r0>))|
AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a6);
AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r6);
TableEq(r3,r4);TableEq(r4,r6);TableEq(r5,r6);
Unique(r3,a4);Unique(r3,a5)|
```

```
rule 118:

Agg_average<a4 a5 r7 e2 a6 r8>(Exists(Union(Input<r3>,Union(Input<r4>,Input<r5>)),Input<r6>))|

Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input<r0>))|

AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a6);

AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);

AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);

PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r6);

TableEq(r3,r4);TableEq(r3,r5);TableEq(r3,r6);

TableEq(r4,r5);TableEq(r4,r6);TableEq(r5,r6);

Unique(r3,a4);Unique(r3,a5)|
```

```
rule 119:
Filter<e3 _>(Agg<_ a3 _ e0 a4 r4 e1 a5 r5>(Input< r3>)); e0:=FuncCall<count>(a4); e3:=Sublink<EXISTS
Filter<e4 _>(Union_all<>(Input<r6>,Input<r7>))); e4
:=Sublink<EXISTS Input<r8>>|
Agg<_ a3 _ e2 a3 r1 e1 a5 r2>(Input<r3>); e2:=
FuncCall<count>(a3)|
AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);
NotNull(r3,a3); NotNull(r3,a4); TableEq(r3,r6);
TableEq(r3,r7); TableEq(r3,r8); TableEq(r6,r7);
TableEq(r6,r8); TableEq(r7,r8); Unique(r3,a3); Unique(r3,a4)|
```

```
rule 120:
Agg_average < a4 a5 r7 e2 a6 r8 > (Union(Input < r3 > ,
Exists(Input < r4 > ,Union(Input < r5 > ,Input < r6 > ))))|
Filter < e1 a3 > (Agg_average < a0 a1 r1 e0 a2 r2 > (Input < r0 > ))|
AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a6);
AttrsEq(a3,a4); AttrsSub(a4,r3); AttrsSub(a5,r3);
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r0,r6);
TableEq(r3,r4); TableEq(r4,r6); TableEq(r5,r6);
Unique(r3,a4); Unique(r3,a5)|
```

```
rule 121:
Agg<_ a3 _ e0 a4 r6 e1 a5 r7>(Filter<e3 _>(Input<
r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS
Union_all<>(Input<r4>,Input<r5>)>|
Agg<_ a3 _ e2 a3 r1 e1 a5 r2>(Input<r3>);e2:=
FuncCall<count>(a3)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r4,r5);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 122:

Exists(Agg_count<a3 a4 r4 e1 a5 r5>(Input<r3>),

Exists(Input<r6>,Union(Input<r7>,Input<r8>)))|

Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|

AttrsEq(a0,a3);AttrsEq(a1,a3);AttrsEq(a2,a5);

AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);

NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);

TableEq(r0,r3);TableEq(r0,r6);TableEq(r0,r7);

TableEq(r0,r8);TableEq(r3,r6);TableEq(r3,r7);

TableEq(r3,r8);TableEq(r6,r7);TableEq(r6,r8);

TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 123:
Agg<_ a3 _ e0 a4 r7 e1 a5 r8>(Filter<e3 _>(Input<
r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS
Filter<e4 _>(Union_all<>(Input<r4>,Input<r5>))>;e4
:=Sublink<EXISTS Input<r6>>|
Agg<_ a3 _ e2 a5 r1 e1 a5 r2>(Input<r3>);e2:=
FuncCall<count>(a5)|
```

```
AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3); NotNull(r3,a3); NotNull(r3,a4); TableEq(r3,r4); TableEq(r3,r5); TableEq(r3,r6); TableEq(r4,r5); TableEq(r4,r6); TableEq(r5,r6); Unique(r3,a3); Unique(r3,a4)|
```

```
rule 124:

Agg_average<a4 a5 r7 e2 a6 r8>(Union(Exists(Input< r3>,Input<r4>),Union(Input<r5>,Input<r6>)))|

Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input < r0>))|

AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a4);
AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r6);
TableEq(r3,r4);TableEq(r3,r5);TableEq(r3,r6);
TableEq(r4,r5);TableEq(r4,r6);TableEq(r5,r6);
Unique(r3,a4);Unique(r3,a5)|
```

```
rule 125:

2 Agg<_ a4 _ e0 a5 r7 e1 a6 r8>(Proj<ea3 a3 r6>(
Filter<e3 _>(Input<r3>)));e0:=FuncCall<count>(a5);
e3:=Sublink<EXISTS Union_all<>(Input<r4>,Input<r5
>))|

3 Agg<_ a6 _ e2 a4 r1 e1 a6 r2>(Input<r3>);e2:=
FuncCall<count>(a4)|

4 AttrsSub(a3,r3);AttrsSub(a4,r6);AttrsSub(a5,r6);
AttrsSub(a6,a4);NotNull(r3,a3);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r4,r5);Unique(r3,a3)|
```

```
rule 126:
Filter<e3 _>(Filter<e4 _>(Agg<_ a3 _ e0 a4 r4 e1
a5 r5>(Input<r3>)));e0:=FuncCall<count>(a4);e3:=
Sublink<EXISTS Input<r8>>;e4:=Sublink<EXISTS
Union_all<>(Input<r6>,Input<r7>)>|
Agg<_ a5 _ e2 a5 r1 e1 a3 r2>(Input<r3>);e2:=
FuncCall<count>(a5)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r3,r8);TableEq(r6,r7);
TableEq(r6,r8);TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 127:
Filter<e3 _>(Agg<_ a4 _ e0 a5 r4 e2 a6 r5>(Input< r3>));e0:=FuncCall<sum>(a5);e3:=Sublink<EXISTS
Union_all<>(Union_all<>(Input<r6>,Input<r7>),Input
<r8>)>|
Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a4 r2>(Input< r3>));e1:=FuncCall<sum>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r7,r8);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 128:
Exists(Agg_average<a4 a5 r5 e2 a6 r6>(Union(Input<
r3>,Input<r4>)),Input<r7>)|
```

```
Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input
<r0>))|
AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a6);
AttrsEq(a3,a4); AttrsSub(a4,r3); AttrsSub(a5,r3);
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,
r3); TableEq(r0,r4); TableEq(r0,r7); TableEq(r3,r4);
TableEq(r3,r7); TableEq(r4,r7); Unique(r3,a4); Unique
(r3,a5)|
rule 129:
Agg<_ a3 _ e0 a4 r7 e1 a5 r8>(Filter<e3 _>(Input<
r3>)); e0:=FuncCall <count>(a4); e3:=Sublink <EXISTS
Union_all <> (Input <r4>, Filter <e4 _> (Input <r5>))>; e4
:=Sublink<EXISTS Input<r6>>|
Agg<_ a3 _ e2 a3 r1 e1 a5 r2>(Input<r3>);e2:=
FuncCall < count > (a3) |
AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);
NotNull(r3,a3); NotNull(r3,a4); TableEq(r3,r4);
TableEq(r3,r5); TableEq(r3,r6); TableEq(r4,r5);
TableEq(r4,r6); TableEq(r5,r6); Unique(r3,a3); Unique
(r3,a4)|
rule 130:
Filter<e3 _>(Filter<e4 _>(Agg<_ a4 _ e0 a5 r4 e2
a6 r5>(Input<r3>)));e0:=FuncCall<avg>(a5);e3:=
Sublink<EXISTS Union_all<>(Input<r7>, Input<r8>)>;
e4:=Sublink<EXISTS Input<r6>>|
Filter < e2 a6 > (Agg < _ a4 _ e1 a5 r1 e2 a4 r2 > (Input <
r3>));e1:=FuncCall<avg>(a5)|
AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4);
NotNull(r3,a4); NotNull(r3,a5); TableEq(r3,r6);
TableEq(r3,r7); TableEq(r3,r8); TableEq(r6,r7);
TableEq(r6,r8); TableEq(r7,r8); Unique(r3,a4); Unique
(r3,a5)|
rule 131:
Agg_average < a4 a5 r7 e2 a6 r8 > (Union(Union(Input <
r3>, Input <r4>), Union(Input <r5>, Input <r6>)))|
Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input
<r0>))|
AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a6);
AttrsEq(a3,a4); AttrsSub(a4,r3); AttrsSub(a5,r3);
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,
r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r0,r6);
TableEq(r3,r4); TableEq(r3,r5); TableEq(r3,r6);
TableEq(r4,r5); TableEq(r4,r6); TableEq(r5,r6);
Unique(r3,a4);Unique(r3,a5)|
rule 132:
Exists(Exists(Agg_count < a3 a4 r4 e1 a5 r5>(Input <
r3>), Input <r6>), Union(Input <r7>, Input <r8>))|
Agg\_count < a0 a1 r1 e0 a2 r2 > (Input < r0 >) |
AttrsEq(a0,a5); AttrsEq(a1,a3); AttrsEq(a2,a3);
AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);
NotNull(r3,a3); NotNull(r3,a4); PredicateEq(e0,e1);
TableEq(r0,r3); TableEq(r0,r6); TableEq(r0,r7);
TableEq(r0,r8); TableEq(r3,r6); TableEq(r3,r7);
TableEq(r3,r8); TableEq(r6,r7); TableEq(r6,r8);
TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 133:
Agg_count < a3 a4 r7 e1 a5 r8 > (Exists (Input < r3 > ,
Union(Union(Input<r4>, Input<r5>), Input<r6>)))|
Agg_count < a0 a1 r1 e0 a2 r2 > (Input < r0 >) |
AttrsEq(a0,a3); AttrsEq(a1,a5); AttrsEq(a2,a5);
AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);
NotNull(r3,a3); NotNull(r3,a4); PredicateEq(e0,e1);
TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r5);
TableEq(r0,r6); TableEq(r3,r4); TableEq(r3,r5);
TableEq(r3,r6); TableEq(r4,r5); TableEq(r4,r6);
TableEq(r5,r6); Unique(r3,a3); Unique(r3,a4)|
rule 134:
Agg_average < a4 a5 r6 e2 a6 r7 > (Union(Input < r3 > ,
Exists(Input<r4>, Input<r5>)))|
Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input
<r0>))|
AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a6);
AttrsEq(a3,a4); AttrsSub(a4,r3); AttrsSub(a5,r3);
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,
r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r3,r4);
TableEq(r3,r5); TableEq(r4,r5); Unique(r3,a4); Unique
(r3,a5)|
rule 135:
Agg_max < a3 a4 r5 e1 a5 r6 > (Union(Input < r3 > , Input <
r4>))|
Agg_max<a0 a1 r1 e0 a2 r2>(Input<r0>)|
AttrsEq(a0,a3); AttrsEq(a1,a4); AttrsEq(a2,a5);
AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);
NotNull(r3,a3); NotNull(r3,a4); PredicateEg(e0,e1);
TableEq(r0,r3); TableEq(r0,r4); TableEq(r3,r4);
Unique(r3,a3);Unique(r3,a4)|
rule 136:
Filter<e3 _>(Agg<_ a4 _ e0 a5 r5 e2 a6 r6>(
Union_all <>(Input <r3>, Input <r4>))); e0:=FuncCall 
avg>(a5);e3:=Sublink<EXISTS Union_all<>(Input<r7>,
Input < r8 > ) > |
Filter<e2 a4>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input<
r3>));e1:=FuncCall<avg>(a5)|
AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4);
NotNull(r3,a4); NotNull(r3,a5); TableEq(r3,r4);
TableEq(r3,r7); TableEq(r3,r8); TableEq(r4,r7);
TableEq(r4,r8); TableEq(r7,r8); Unique(r3,a4); Unique
(r3,a5)|
rule 137:
Agg_sum<a4 a5 r6 e2 a6 r7>(Exists(Input<r3>,Union(
Input < r4 > , Input < r5 > ) ) |
Filter<e1 a3>(Agg_sum<a0 a1 r1 e0 a2 r2>(Input<r0
AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a6);
AttrsEq(a3,a4); AttrsSub(a4,r3); AttrsSub(a5,r3);
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,
r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r3,r4);
TableEq(r3,r5); TableEq(r4,r5); Unique(r3,a4); Unique
(r3,a5)|
```

```
rule 138:
Filter<e3 _>(Agg<_ a4 _ e0 a5 r4 e2 a6 r5>(Input<
r3>)); e0:=FuncCall <count>(a5); e3:=Sublink <EXISTS
Union_all <>(Input < r6>, Input < r7>)>|
Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input<
r3>));e1:=FuncCall<count>(a5)|
AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4);
NotNull(r3,a4); NotNull(r3,a5); TableEq(r3,r6);
TableEq(r3,r7); TableEq(r6,r7); Unique(r3,a4); Unique
(r3,a5)|
rule 139:
Agg<_a4 _e0 a5 r7 e1 a6 r8>(Proj<ea3 a3 r6>(
Filter<e3 _>(Input<r3>)));e0:=FuncCall<count>(a5);
e3:=Sublink<EXISTS Union_all<>(Input<r4>,Input<r5
>)>|
Agg<_ a6 _ e2 a6 r1 e1 a4 r2>(Input<r3>);e2:=
FuncCall < count > (a6)|
AttrsSub(a3,r3); AttrsSub(a4,r6); AttrsSub(a5,r6);
AttrsSub(a6,a4); NotNull(r3,a3); TableEq(r3,r4);
TableEq(r3,r5); TableEq(r4,r5); Unique(r3,a3)|
```

```
rule 140:
Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Union_all<>(
Union_all<>(Filter<e3 _>(Input<r3>),Input<r5>),
Input<r6>));e0:=FuncCall<avg>(a5);e3:=Sublink<
EXISTS Input<r4>|
Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a4 r2>(Input<
r3>));e1:=FuncCall<avg>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 141:
Agg_count<a3 a4 r7 e1 a5 r8>(Exists(Input<r3>,
Exists(Union(Input<r4>,Input<r5>),Input<r6>)))|
Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|
AttrsEq(a0,a5);AttrsEq(a1,a3);AttrsEq(a2,a3);
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);
TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);
TableEq(r0,r6);TableEq(r3,r4);TableEq(r3,r5);
TableEq(r3,r6);TableEq(r4,r5);TableEq(r4,r6);
TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 142:
Agg_count<a3 a4 r7 e1 a5 r8>(Exists(Input<r3>,
Union(Input<r4>,Exists(Input<r5>,Input<r6>))))|
Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0)|
AttrsEq(a0,a5);AttrsEq(a1,a5);AttrsEq(a2,a3);
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);
TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);
TableEq(r0,r6);TableEq(r3,r4);TableEq(r3,r5);
TableEq(r3,r6);TableEq(r4,r5);TableEq(r4,r6);
TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|</pre>
```

```
rule 143:
Agg_average<a4 a5 r7 e2 a6 r8>(Union(Input<r3>,
Exists(Input<r4>,Union(Input<r5>,Input<r6>))))|

Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input
<r0>))|

AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a6);
AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r6);
TableEq(r3,r4);TableEq(r3,r5);TableEq(r3,r6);
TableEq(r4,r5);TableEq(r4,r6);TableEq(r5,r6);
Unique(r3,a4);Unique(r3,a5)|
```

```
rule 144:

Agg_max<a3 a4 r7 e1 a5 r8>(Union(Exists(Exists(Input<r3>,Input<r4>),Input<r5>),Input<r6>))|

Agg_max<a0 a1 r1 e0 a2 r2>(Input<r0>)|

AttrsEq(a0,a3);AttrsEq(a1,a4);AttrsEq(a2,a5);

AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);

NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);

TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);

TableEq(r0,r6);TableEq(r3,r4);TableEq(r4,r6);

TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 146:

Exists(Agg_count<a3 a4 r4 e1 a5 r5>(Input<r3>),

Union(Union(Input<r6>,Input<r7>),Input<r8>))|

Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|

AttrsEq(a0,a5);AttrsEq(a1,a3);AttrsEq(a2,a5);

AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);

NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);

TableEq(r0,r3);TableEq(r0,r6);TableEq(r0,r7);

TableEq(r0,r8);TableEq(r3,r6);TableEq(r3,r7);

TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 147:

Exists(Agg_count<a3 a4 r6 e1 a5 r7>(Exists(Input<r3>,Union(Input<r4>,Input<r5>))),Input<r8>)|

Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|
```

```
AttrsEq(a0,a5); AttrsEq(a1,a3); AttrsEq(a2,a5);
AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);
NotNull(r3,a3); NotNull(r3,a4); PredicateEq(e0,e1);
TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r5);
TableEq(r0,r8); TableEq(r3,r4); TableEq(r3,r5);
TableEq(r3,r8); TableEq(r4,r5); TableEq(r4,r8);
TableEq(r5,r8);Unique(r3,a3);Unique(r3,a4)|
rule 148:
Agg<_ a3 _ e0 a4 r7 e1 a5 r8>(Union_all<>(Filter<
e3 _>(Filter<e4 _>(Input<r3>)),Input<r6>));e0:=
FuncCall <max > (a4); e3:=Sublink < EXISTS Input < r5 >>; e4
:=Sublink<EXISTS Input<r4>>|
Agg<_ a3 _ e2 a4 r1 e1 a5 r2>(Input<r3>);e2:=
FuncCall < max > (a4) |
AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);
NotNull(r3,a3); NotNull(r3,a4); TableEq(r3,r4);
TableEq(r3,r5); TableEq(r3,r6); TableEq(r4,r5);
TableEq(r4,r6); TableEq(r5,r6); Unique(r3,a3); Unique
(r3,a4)|
```

```
rule 149:

Agg<_ a4 _ e0 a5 r6 e2 a6 r7>(Filter<e3 _>(Input< r3>)); e0:=FuncCall<count>(a5); e3:=Sublink<EXISTS
Union_all<>(Input<r4>,Input<r5>)>|

Filter<e2 a4>(Agg<_ a6 _ e1 a5 r1 e2 a6 r2>(Input< r3>)); e1:=FuncCall<count>(a5)|

AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4);
NotNull(r3,a4); NotNull(r3,a5); TableEq(r3,r4);
TableEq(r3,r5); TableEq(r4,r5); Unique(r3,a4); Unique(r3,a5)|
```

```
rule 150:

Exists(Agg_count<a3 a4 r4 e1 a5 r5>(Input<r3>),
Exists(Union(Input<r6>,Input<r7>),Input<r8>))|

Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|

AttrsEq(a0,a5);AttrsEq(a1,a3);AttrsSub(a2,a5);
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);

NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);
TableEq(r0,r3);TableEq(r0,r6);TableEq(r0,r7);
TableEq(r0,r8);TableEq(r3,r6);TableEq(r3,r7);
TableEq(r3,r8);TableEq(r6,r7);TableEq(r6,r8);
TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 151:
Agg<_ a3 _ e0 a4 r7 e1 a5 r8>(Filter<e3 _>(Input<
r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS
Union_all<>(Input<r4>,Filter<e4 _>(Input<r5>))>;e4
:=Sublink<EXISTS Input<r6>>|
Agg<_ a3 _ e2 a5 r1 e1 a5 r2>(Input<r3>);e2:=
FuncCall<count>(a5)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 152:
2 Agg_average<a4 a5 r7 e2 a6 r8>(Union(Union(Exists(Input<r3>,Input<r4>),Input<r5>),Input<r6>))|
```

```
Filter < e1 a3 > (Agg_average < a0 a1 r1 e0 a2 r2 > (Input < r0 > )) |

AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a6);

AttrsEq(a3,a4); AttrsSub(a4,r3); AttrsSub(a5,r3);

AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);

PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0, r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r0,r6);

TableEq(r3,r4); TableEq(r3,r5); TableEq(r3,r6);

Unique(r3,a4); Unique(r3,a5)|
```

```
rule 153:

Exists(Agg_count<a3 a4 r5 e1 a5 r6>(Exists(Input<r3>, Input<r4>)), Union(Input<r7>, Input<r8>))|

Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|

AttrsEq(a0,a3); AttrsEq(a1,a3); AttrsEq(a2,a3);

AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);

NotNull(r3,a3); NotNull(r3,a4); PredicateEq(e0,e1);

TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r7);

TableEq(r0,r8); TableEq(r3,r4); TableEq(r3,r7);

TableEq(r7,r8); Unique(r3,a3); Unique(r3,a4)|
```

```
rule 154:

Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Filter<e3 _>(Input< r3>));e0:=FuncCall<sum>(a5);e3:=Sublink<EXISTS

Union_all<>(Union_all<>(Input<r4>,Input<r5>),Input
<r6>)>|

Filter<e2 a4>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input< r3>));e1:=FuncCall<sum>(a5)|

AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);

NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);

TableEq(r3,r5);TableEq(r5,r6);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 155:

Exists(Agg_max<a3 a4 r5 e1 a5 r6>(Union(Input<r3>,
Input<r4>)),Input<r7>)|

Agg_max<a0 a1 r1 e0 a2 r2>(Input<r0>)|

AttrsEq(a0,a3);AttrsEq(a1,a4);AttrsEq(a2,a5);

AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);

NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);

TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r7);

TableEq(r3,r4);TableEq(r3,r7);TableEq(r4,r7);
Unique(r3,a3);Unique(r3,a4)|
```

```
rule 156:

Agg_count<a3 a4 r7 e1 a5 r8>(Exists(Exists(Input<ra>r3>,Union(Input<r4>,Input<r5>)),Input<r6>))|

Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|

AttrsEq(a0,a3);AttrsEq(a1,a3);AttrsEq(a2,a5);

AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);

NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);

TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);

TableEq(r0,r6);TableEq(r3,r4);TableEq(r3,r5);

TableEq(r3,r6);TableEq(r4,r5);TableEq(r4,r6);

TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 157:
```

```
Agg<_ a4 _ e0 a5 r6 e2 a6 r7>(Union_all<>(
Union_all <> (Input < r3>, Input < r4>), Input < r5>)); e0:=
FuncCall <avg>(a5)|
Filter < e2 a6 > (Agg < a4 e1 a5 r1 e2 a6 r2 > (Input <
r3>));e1:=FuncCall<avg>(a5)|
AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4);
NotNull(r3,a4); NotNull(r3,a5); TableEq(r3,r4);
TableEq(r3,r5); TableEq(r4,r5); Unique(r3,a4); Unique
(r3,a5)|
rule 158:
Agg<_ a4 _ e0 a5 r7 e1 a6 r8>(Proj<ea3 a3 r6>(
Filter<e3 _>(Input<r3>)));e0:=FuncCall<count>(a5);
e3:=Sublink<EXISTS Union_all<>(Input<r4>,Input<r5
>)>|
Agg<_ a4 _ e2 a4 r1 e1 a4 r2>(Input<r3>);e2:=
FuncCall < count > (a4) |
AttrsSub(a3,r3); AttrsSub(a4,r6); AttrsSub(a5,r6);
AttrsSub(a6,a4); NotNull(r3,a3); TableEq(r3,r4);
TableEq(r3,r5); TableEq(r4,r5); Unique(r3,a3)|
rule 159:
Agg\_count < a4 \ a5 \ r7 \ e1 \ a6 \ r8 > (Proj\_simple < \_ \ a3 \ r6 > (
```

### | rule 159: | Agg\_count<a4 a5 r7 e1 a6 r8>(Proj\_simple<\_ a3 r6>(Exists(Input<r3>,Union(Input<r4>,Input<r5>)))| | Agg\_count<a0 a1 r1 e0 a2 r2>(Input<r0>)| | AttrsEq(a0,a4);AttrsEq(a1,a4);AttrsEq(a2,a6); | AttrsSub(a3,r3);AttrsSub(a4,r6);AttrsSub(a5,r6); | AttrsSub(a6,a4);NotNull(r3,a3);PredicateEq(e0,e1); | TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5); | TableEq(r3,r4);TableEq(r3,r5);TableEq(r4,r5); | Unique(r3,a3)|

```
rule 160:
Agg_average<a4 a5 r7 e2 a6 r8>(Union(Input<r3>,
Exists(Union(Input<r4>,Input<r5>),Input<r6>)))|
Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input
<r0>))|
AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a6);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r6);
TableEq(r1,r5);TableEq(r1,r6);TableEq(r5,r6);
Unique(r3,a4);Unique(r3,a5)|
```

```
rule 161:
Exists(Agg_sum<a4 a5 r4 e2 a6 r5>(Input<r3>),Union
(Input<r6>,Input<r7>))|
Filter<e1 a3>(Agg_sum<a0 a1 r1 e0 a2 r2>(Input<r0
>))|
AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a4);
AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r6);TableEq(r0,r7);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r6,r7);Unique(r3,a4);Unique(r3,a5)|
```

rule 162:

```
Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Union_all<>(Input<r3
>,Filter<e3 _>(Input<r4>)));e0:=FuncCall<avg>(a5);
e3:=Sublink<EXISTS Union_all<>(Input<r5>,Input<r6
>)>1
Filter < e2 a6 > (Agg < _ a4 _ e1 a5 r1 e2 a6 r2 > (Input <
r3>));e1:=FuncCall<avg>(a5)|
AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4);
NotNull(r3,a4); NotNull(r3,a5); TableEq(r3,r4);
TableEq(r3,r5); TableEq(r3,r6); TableEq(r4,r5);
TableEq(r4,r6); TableEq(r5,r6); Unique(r3,a4); Unique
(r3, a5)|
rule 163:
Filter<e3 _>(Agg<_ a3 _ e0 a4 r4 e1 a5 r5>(Input<
r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS
Union_all <> (Input <r6>, Union_all <> (Input <r7>, Input <
r8>))>|
Agg<_ a3 _ e2 a5 r1 e1 a5 r2>(Input<r3>);e2:=
FuncCall < count > (a5) |
AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);
NotNull(r3,a3); NotNull(r3,a4); TableEq(r3,r6);
TableEq(r3,r7); TableEq(r3,r8); TableEq(r6,r7);
```

```
rule 164:

Agg<_ a4 _ e0 a5 r6 e2 a6 r7>(Filter<e3 _>(Input< r3>));e0:=FuncCall<count>(a5);e3:=Sublink<EXISTS
Union_all<>(Input<r4>,Input<r5>)>|

Filter<e2 a4>(Agg<_ a6 _ e1 a6 r1 e2 a4 r2>(Input< r3>));e1:=FuncCall<count>(a6)|

AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r4,r5);Unique(r3,a4);Unique(r3,a5)|
```

TableEq(r6,r8); TableEq(r7,r8); Unique(r3,a3); Unique

(r3.a4)|

```
rule 165:

Agg<_ a3 _ e0 a4 r6 e1 a5 r7>(Union_all<>(Input<r3 >, Union_all<>(Input<r4>, Input<r5>))); e0:=FuncCall<
max>(a4)|

Agg<_ a3 _ e2 a4 r1 e1 a5 r2>(Input<r3>); e2:=
FuncCall<max>(a4)|

AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);
NotNull(r3,a3); NotNull(r3,a4); TableEq(r3,r4);
TableEq(r3,r5); TableEq(r4,r5); Unique(r3,a3); Unique(r3,a4)|
```

```
rule 166:

rule 166:

rilter<e3 _>(Agg<_ a4 _ e0 a5 r4 e2 a6 r5>(Input< r3>)); e0:=FuncCall<sum>(a5); e3:=Sublink<EXISTS

Filter<e4 _>(Union_all<>(Input<r6>, Input<r7>))>; e4
:=Sublink<EXISTS Input<r8>>|

Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input< r3>)); e1:=FuncCall<sum>(a5)|

AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4);

NotNull(r3,a4); NotNull(r3,a5); TableEq(r3,r6);

TableEq(r3,r7); TableEq(r3,r8); TableEq(r6,r7);

TableEq(r6,r8); TableEq(r7,r8); Unique(r3,a4); Unique(r3,a5)|
```

```
rule 167:

Agg_count<a3 a4 r7 e1 a5 r8>(Exists(Input<r3>,
Union(Exists(Input<r4>, Input<r5>), Input<r6>)))|

Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|

AttrsEq(a0,a3); AttrsEq(a1,a3); AttrsEq(a2,a5);

AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);

NotNull(r3,a3); NotNull(r3,a4); PredicateEq(e0,e1);

TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r5);

TableEq(r0,r6); TableEq(r3,r4); TableEq(r3,r5);

TableEq(r3,r6); TableEq(r4,r5); TableEq(r4,r6);

TableEq(r5,r6); Unique(r3,a3); Unique(r3,a4)|
```

```
rule 168:

Exists(Exists(Agg_sum<a4 a5 r4 e2 a6 r5>(Input<r3 >),Input<r6>),Union(Input<r7>,Input<r8>))|

Filter<e1 a3>(Agg_sum<a0 a1 r1 e0 a2 r2>(Input<r0 >))|

AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a6);

AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);

AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);

PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r6);TableEq(r0,r7);TableEq(r0,r8);

TableEq(r3,r6);TableEq(r3,r7);TableEq(r3,r8);

TableEq(r6,r7);TableEq(r6,r8);TableEq(r7,r8);

Unique(r3,a4);Unique(r3,a5)|
```

```
rule 169:

Exists(Agg_average<a4 a5 r6 e2 a6 r7>(Union(Exists (Input<r3>,Input<r4>),Input<r5>)),Input<r8>)|

Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input <r0>))|

AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a6);

AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);

PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0, r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r8);

TableEq(r4,r5);TableEq(r4,r8);TableEq(r5,r8);

Unique(r3,a4);Unique(r3,a5)|
```

```
rule 170:

Exists(Agg_count<a5 a6 r5 e2 a7 r6>(Proj_simple<_a4 r4>(Input<r3>)), Union(Input<r7>, Input<r8>))|

Filter<e1 a3>(Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>))|

AttrsEq(a0,a5); AttrsEq(a1,a5); AttrsEq(a2,a5);

AttrsEq(a3,a5); AttrsSub(a4,r3); AttrsSub(a5,r4);

AttrsSub(a6,r4); AttrsSub(a7,a5); NotNull(r3,a4);

PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,r3); TableEq(r0,r7); TableEq(r7,r8); Unique(r3,a4)|
```

```
rule 171:

2 Agg<_ a4 _ e0 a5 r7 e1 a6 r8>(Filter<e3 _>(Proj<
case = ea3 a3 r4>(Input<r3>)));e0:=FuncCall<count>(a5);e3
:=Sublink<EXISTS Union_all<>(Input<r5>,Input<r6>)
>|

3 Agg<_ a4 _ e2 a6 r1 e1 a6 r2>(Input<r3>);e2:=
FuncCall<count>(a6)|
```

```
AttrsSub(a6,a4);NotNull(r3,a3);TableEq(r3,r5);
TableEq(r3,r6);TableEq(r5,r6);Unique(r3,a3)|

rule 172:
Exists(Agg_sum<a4 a5 r4 e2 a6 r5>(Input<r3>),Union (Exists(Input<r6>,Input<r7>),Input<r8>))|
Filter<e1 a3>(Agg_sum<a0 a1 r1 e0 a2 r2>(Input<r0
>))|
```

AttrsSub(a3,r3); AttrsSub(a4,r4); AttrsSub(a5,r4);

Filter<e1 a3>(Agg\_sum<a0 a1 r1 e0 a2 r2>(Input<r0 >))|
AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a6);
AttrsEq(a3,a4); AttrsSub(a4,r3); AttrsSub(a5,r3);
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,r3); TableEq(r0,r6); TableEq(r0,r7); TableEq(r0,r8);
TableEq(r3,r6); TableEq(r3,r7); TableEq(r3,r8);
TableEq(r6,r7); TableEq(r6,r8); TableEq(r7,r8);
Unique(r3,a4); Unique(r3,a5)|

```
rule 173:
Agg<_ a3 _ e0 a4 r7 e1 a5 r8>(Filter<e3 _>(Input<
r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS
Union_all<>(Union_all<>(Input<r4>,Input<r5>),Input
<r6>)>|
Agg<_ a3 _ e2 a3 r1 e1 a5 r2>(Input<r3>);e2:=
FuncCall<count>(a3)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 174:
Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Union_all<>(
    Union_all<>(Input<r3>,Input<r4>),Union_all<>(Input
<r5>,Input<r6>)));e0:=FuncCall<avg>(a5)|

Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a4 r2>(Input<
r3>));e1:=FuncCall<avg>(a5)|

AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 175:

Exists(Agg_count<a4 a5 r4 e2 a6 r5>(Input<r3>),
Union(Input<r6>,Input<r7>))|

Filter<e1 a3>(Agg_count<a0 a1 r1 e0 a2 r2>(Input<
r0>))|

AttrsEq(a0,a6);AttrsEq(a1,a4);AttrsEq(a2,a4);

AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);

PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r6);TableEq(r0,r7);TableEq(r3,r6);

TableEq(r3,r7);TableEq(r6,r7);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 176:
2 Agg<_ a4 _ e0 a5 r6 e2 a6 r7>(Union_all<>(Input<r3 >,Union_all<>(Input<r4>,Input<r5>)));e0:=FuncCall<avg>(a5)|
```

```
Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a4 r2>(Input<
                                                                                    Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Filter<e3 _>(Input<
r3>));e1:=FuncCall<avg>(a5)|
                                                                                     r3>));e0:=FuncCall<sum>(a5);e3:=Sublink<EXISTS
                                                                                     Union_all<>(Input<r4>,Union_all<>(Input<r5>,Input<
AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4);
NotNull(r3,a4); NotNull(r3,a5); TableEq(r3,r4);
                                                                                     r6>))>l
TableEq(r3,r5); TableEq(r4,r5); Unique(r3,a4); Unique
                                                                                     Filter < e2 a6 > (Agg < _ a4 _ e1 a5 r1 e2 a4 r2 > (Input <
(r3,a5)|
                                                                                     r3>));e1:=FuncCall<sum>(a5)|
                                                                                     AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4);
                                                                                     NotNull(r3,a4); NotNull(r3,a5); TableEq(r3,r4);
rule 177:
                                                                                     TableEq(r3,r5); TableEq(r3,r6); TableEq(r4,r5);
Agg<_ a4 _ e0 a5 r6 e2 a6 r7>(Filter<e3 _>(Input<
                                                                                     TableEq(r4,r6); TableEq(r5,r6); Unique(r3,a4); Unique
r3>));e0:=FuncCall<count>(a5);e3:=Sublink<EXISTS
                                                                                     (r3, a5)|
Union_all<>(Input<r4>,Input<r5>)>|
Filter<e2 a4>(Agg<_ a6 _ e1 a5 r1 e2 a4 r2>(Input<
                                                                                    rule 182:
r3>));e1:=FuncCall<count>(a5)|
                                                                                     Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Filter<e3 _>(Input<
AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4);
                                                                                     r3>));e0:=FuncCall<sum>(a5);e3:=Sublink<EXISTS
NotNull(r3,a4); NotNull(r3,a5); TableEq(r3,r4);
TableEq(r3,r5); TableEq(r4,r5); Unique(r3,a4); Unique
                                                                                     Union_all <> (Input <r4>, Filter <e4 _> (Input <r5>))>; e4
(r3,a5)|
                                                                                     :=Sublink<EXISTS Input<r6>>|
                                                                                     Filter<e2 a4>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input<
                                                                                     r3>));e1:=FuncCall<sum>(a5)|
rule 178:
                                                                                     AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4);
Agg_average < a4 a5 r7 e2 a6 r8 > (Union(Union(Union(
                                                                                     NotNull(r3,a4); NotNull(r3,a5); TableEq(r3,r4);
Input <r3>, Input <r4>), Input <r5>), Input <r6>))|
                                                                                     TableEq(r3,r5); TableEq(r3,r6); TableEq(r4,r5);
Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input
                                                                                     TableEq(r4,r6); TableEq(r5,r6); Unique(r3,a4); Unique
<r0>))|
                                                                                     (r3,a5)|
AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a6);
AttrsEq(a3,a4); AttrsSub(a4,r3); AttrsSub(a5,r3);
                                                                                     rule 183:
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
                                                                                    Agg<_a3 _e0 a4 r7 e1 a5 r8>(Filter<e3 _>(Input<
\label{eq:predicateEq} PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,e2); TableEq(r0,e2
                                                                                     r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS
r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r0,r6);
                                                                                     Union_all <> (Input <r4>, Filter <e4 _> (Input <r5>))>; e4
TableEq(r3,r4); TableEq(r3,r5); TableEq(r3,r6);
                                                                                     :=Sublink<EXISTS Input<r6>>|
TableEq(r4,r5); TableEq(r4,r6); TableEq(r5,r6);
                                                                                     Agg<_ a5 _ e2 a3 r1 e1 a3 r2>(Input<r3>);e2:=
Unique(r3,a4);Unique(r3,a5)|
                                                                                     FuncCall < count > (a3) |
                                                                                     AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);
rule 179:
                                                                                     NotNull(r3,a3); NotNull(r3,a4); TableEq(r3,r4);
Agg_average < a4 a5 r7 e2 a6 r8 > (Union(Exists(Exists
                                                                                     TableEq(r3,r5); TableEq(r3,r6); TableEq(r4,r5);
(Input <r3>, Input <r4>), Input <r5>), Input <r6>))|
                                                                                     TableEq(r4,r6); TableEq(r5,r6); Unique(r3,a3); Unique
Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input
                                                                                     (r3,a4)|
<r0>))|
AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a6);
                                                                                     rule 184:
AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);
                                                                                     Agg<_ a5 _ e0 a6 r7 e2 a7 r8>(Filter<e3 _>(Proj<
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
                                                                                     ea4 a4 r4>(Input<r3>)));e0:=FuncCall<sum>(a6);e3:=
PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,
                                                                                     Sublink < EXISTS Union_all <> (Input < r5>, Input < r6>) > |
r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r0,r6);
                                                                                     Filter < e2 a7 > (Agg < a5 e1 a6 r1 e2 a5 r2 > (Input <
TableEq(r3,r4); TableEq(r3,r5); TableEq(r3,r6);
                                                                                     r3>));e1:=FuncCall<sum>(a6)|
TableEq(r4,r5); TableEq(r4,r6); TableEq(r5,r6);
                                                                                     AttrsSub(a4,r3); AttrsSub(a5,r4); AttrsSub(a6,r4);
Unique(r3,a4);Unique(r3,a5)|
                                                                                     AttrsSub(a7,a5); NotNull(r3,a4); TableEq(r3,r5);
                                                                                     TableEq(r3,r6); TableEq(r5,r6); Unique(r3,a4) |
rule 180:
Exists(Agg_count < a3 a4 r6 e1 a5 r7 > (Exists(Input <
                                                                                     rule 185:
r3>,Union(Input<r4>,Input<r5>))),Input<r8>)|
                                                                                     Exists(Agg_max<a3 a4 r6 e1 a5 r7>(Union(Input<r3>,
Agg\_count < a0 a1 r1 e0 a2 r2 > (Input < r0 >) |
                                                                                     Union(Input<r4>, Input<r5>))), Input<r8>)|
AttrsEq(a0, a5); AttrsEq(a1, a5); AttrsEq(a2, a3);
                                                                                     Agg_max < a0 a1 r1 e0 a2 r2 > (Input < r0 >) |
AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);
                                                                                     AttrsEq(a0,a3); AttrsEq(a1,a4); AttrsEq(a2,a5);
NotNull(r3,a3); NotNull(r3,a4); PredicateEq(e0,e1);
                                                                                     AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);
TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r5);
                                                                                     NotNull(r3,a3); NotNull(r3,a4); PredicateEq(e0,e1);
TableEq(r0,r8); TableEq(r3,r4); TableEq(r3,r5);
                                                                                     TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r5);
TableEq(r3,r8); TableEq(r4,r5); TableEq(r4,r8);
                                                                                     TableEq(r0,r8); TableEq(r3,r4); TableEq(r3,r5);
TableEq(r5,r8);Unique(r3,a3);Unique(r3,a4)|
                                                                                     TableEq(r3,r8); TableEq(r4,r5); TableEq(r4,r8);
```

1 rule 181:

TableEq(r5,r8);Unique(r3,a3);Unique(r3,a4)|

```
rule 186:
Agg_count<a3 a4 r7 e1 a5 r8>(Exists(Input<r3>,
Union(Input<r4>,Exists(Input<r5>,Input<r6>))))|
Agg\_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|
AttrsEq(a0,a5); AttrsEq(a1,a3); AttrsEq(a2,a3);
AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);
NotNull(r3,a3); NotNull(r3,a4); PredicateEq(e0,e1);
TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r5);
TableEq(r0,r6); TableEq(r3,r4); TableEq(r3,r5);
TableEq(r3,r6); TableEq(r4,r5); TableEq(r4,r6);
TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
rule 187:
Agg<_ a3 _ e0 a4 r7 e1 a5 r8>(Filter<e3 _>(Input<
r3>)); e0:=FuncCall <count>(a4); e3:=Sublink <EXISTS
Union_all <> (Input <r4>, Union_all <> (Input <r5>, Input <
r6>))>|
Agg<_ a3 _ e2 a5 r1 e1 a5 r2>(Input<r3>);e2:=
FuncCall < count > (a5) |
AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);
NotNull(r3,a3); NotNull(r3,a4); TableEq(r3,r4);
TableEq(r3,r5); TableEq(r3,r6); TableEq(r4,r5);
TableEq(r4,r6); TableEq(r5,r6); Unique(r3,a3); Unique
(r3,a4)|
```

```
rule 188:

Agg_sum<a4 a5 r7 e2 a6 r8>(Exists(Input<r3>,Exists(Union(Input<r4>,Input<r5>),Input<r6>)))|

Filter<e1 a3>(Agg_sum<a0 a1 r1 e0 a2 r2>(Input<r0 >))|

AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a6);

AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);

AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);

PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r6);

TableEq(r3,r4);TableEq(r4,r6);TableEq(r5,r6);

Unique(r3,a4);Unique(r3,a5)|
```

```
rule 189:

Agg_average<a4 a5 r7 e2 a6 r8>(Union(Union(Input<r3>,Exists(Input<r4>,Input<r5>)),Input<r6>))|

Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input<r0>))|

AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a6);

AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);

AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);

PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r6);

TableEq(r3,r4);TableEq(r3,r5);TableEq(r3,r6);

TableEq(r4,r5);TableEq(r4,r6);TableEq(r5,r6);

Unique(r3,a4);Unique(r3,a5)|
```

```
rule 190:
Filter<e3 _>(Agg<_ a4 _ e0 a5 r4 e2 a6 r5>(Input< r3>)); e0:=FuncCall<sum>(a5); e3:=Sublink<EXISTS
Union_all<>(Input<r6>,Filter<e4 _>(Input<r7>))>; e4
:=Sublink<EXISTS Input<r8>|
Filter<e2 a4>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input< r3>)); e1:=FuncCall<sum>(a5)|
```

```
AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5); TableEq(r3,r6); TableEq(r3,r7); TableEq(r3,r8); TableEq(r6,r7); TableEq(r6,r8); TableEq(r7,r8); Unique(r3,a4); Unique(r3,a5)|
```

```
rule 191:
Agg<_ a4 _ e0 a5 r7 e1 a6 r8>(Filter<e3 _>(Proj<
ea3 a3 r4>(Input<r3>)));e0:=FuncCall<count>(a5);e3
:=Sublink<EXISTS Union_all<>(Input<r5>,Input<r6>)
>|
Agg<_ a6 _ e2 a4 r1 e1 a4 r2>(Input<r3>);e2:=
FuncCall<count>(a4)|
AttrsSub(a3,r3);AttrsSub(a4,r4);AttrsSub(a5,r4);
AttrsSub(a6,a4);NotNull(r3,a3);TableEq(r3,r5);
TableEq(r3,r6);TableEq(r5,r6);Unique(r3,a3)|
```

```
rule 192:
Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Union_all<>(Input<r3
>,Filter<e3 _>(Input<r4>)));e0:=FuncCall<avg>(a5);
e3:=Sublink<EXISTS Filter<e4 _>(Input<r5>)>;e4:=
Sublink<EXISTS Input<r6>>|
Filter<e2 a4>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input<
r3>));e1:=FuncCall<avg>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 193:
Exists(Agg_sum<a4 a5 r4 e2 a6 r5>(Input<r3>),Union
(Input<r6>,Union(Input<r7>,Input<r8>)))|
Filter<e1 a3>(Agg_sum<a0 a1 r1 e0 a2 r2>(Input<r0
>))|
AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a6);
AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r6);TableEq(r0,r7);TableEq(r0,r8);
TableEq(r3,r6);TableEq(r3,r7);TableEq(r3,r8);
TableEq(r6,r7);TableEq(r6,r8);TableEq(r7,r8);
Unique(r3,a4);Unique(r3,a5)|
```

```
rule 194:
Agg<_ a4 _ e0 a5 r6 e2 a6 r7>(Filter<e3 _>(Input<
r3>));e0:=FuncCall<count>(a5);e3:=Sublink<EXISTS
Union_all<>(Input<r4>,Input<r5>)>|
Filter<e2 a4>(Agg<_ a4 _ e1 a4 r1 e2 a6 r2>(Input<
r3>));e1:=FuncCall<count>(a4)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r4,r5);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 195:

Exists(Agg_count<a4 a5 r5 e1 a6 r6>(Proj_simple<_

a3 r4>(Input<r3>)),Union(Input<r7>,Input<r8>))|

Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|
```

```
Agg_count<a3 a4 r7 e1 a5 r8>(Exists(Input<r3>,
Union(Input<r4>, Union(Input<r5>, Input<r6>))))|
Agg\_count < a0 a1 r1 e0 a2 r2 > (Input < r0 >) |
AttrsEq(a0,a3); AttrsEq(a1,a5); AttrsEq(a2,a5);
AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);
NotNull(r3,a3); NotNull(r3,a4); PredicateEq(e0,e1);
TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r5);
TableEq(r0,r6); TableEq(r3,r4); TableEq(r3,r5);
{\tt TableEq(r3,r6);TableEq(r4,r5);TableEq(r4,r6);}\\
TableEq(r5,r6); Unique(r3,a3); Unique(r3,a4)|
rule 197:
Agg_average < a4 a5 r6 e2 a6 r7 > (Union(Input < r3 > ,
Exists(Input<r4>, Input<r5>)))|
Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input
AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a4);
AttrsEq(a3,a6); AttrsSub(a4,r3); AttrsSub(a5,r3);
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,
r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r3,r4);
TableEq(r3,r5); TableEq(r4,r5); Unique(r3,a4); Unique
(r3,a5)|
rule 198:
Filter<e3 _>(Agg<_ a4 _ e0 a5 r4 e2 a6 r5>(Input<
r3>));e0:=FuncCall<sum>(a5);e3:=Sublink<EXISTS
Filter<e4 _>(Input<r6>)>; e4:=Sublink<EXISTS
Union_all <>(Input <r7>, Input <r8>)>|
Filter < e2 a4 > (Agg < a4 e1 a5 r1 e2 a6 r2 > (Input <
r3>));e1:=FuncCall<sum>(a5)|
AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4);
NotNull(r3,a4); NotNull(r3,a5); TableEq(r3,r6);
TableEq(r3,r7); TableEq(r3,r8); TableEq(r6,r7);
TableEq(r6,r8); TableEq(r7,r8); Unique(r3,a4); Unique
(r3,a5)|
rule 199:
Agg<_ a3 _ e0 a4 r5 e1 a5 r6>(Union_all<>(Input<r3
>, Input <r4>)); e0:=FuncCall <max >(a4)|
Agg<_ a3 _ e2 a4 r1 e1 a5 r2>(Input<r3>);e2:=
FuncCall < max > (a4) |
AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);
NotNull(r3,a3); NotNull(r3,a4); TableEq(r3,r4);
Unique(r3,a3);Unique(r3,a4)|
rule 200:
Filter<e3 _>(Agg<_ a3 _ e0 a4 r4 e1 a5 r5>(Input<
r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS
Union_all <> (Filter <e4 _> (Input <r6>), Input <r8>)>; e4
:=Sublink<EXISTS Input<r7>>|
Agg<_ a5 _ e2 a5 r1 e1 a3 r2>(Input<r3>);e2:=
FuncCall < count > (a5) |
```

AttrsEq(a0,a6);AttrsEq(a1,a6);AttrsEq(a2,a4);

TableEq(r3,r7); TableEq(r3,r8); TableEq(r7,r8);

Unique(r3,a3)|

rule 196:

AttrsSub(a3,r3); AttrsSub(a4,r4); AttrsSub(a5,r4);

AttrsSub(a6,a4); NotNull(r3,a3); PredicateEq(e0,e1);
TableEq(r0,r3); TableEq(r0,r7); TableEq(r0,r8);

```
rule 201:

Agg_average<a4 a5 r7 e2 a6 r8>(Exists(Union(Input<r3>,Input<r4>),Union(Input<r5>,Input<r6>)))|

Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input<r0>))|

AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a6);
AttrsEq(a3,a4);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r6);
TableEq(r3,r4);TableEq(r3,r5);TableEq(r3,r6);
TableEq(r4,r5);TableEq(r4,r6);TableEq(r5,r6);
Unique(r3,a4);Unique(r3,a5)|
```

AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);

TableEq(r6, r8); TableEq(r7, r8); Unique(r3, a3); Unique

NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r6); TableEq(r3,r7);TableEq(r3,r8);TableEq(r6,r7);

rule 202:
Filter<e3 \_>(Agg<\_ a4 \_ e0 a5 r6 e2 a6 r7>(Filter<
e4 \_>(Union\_all<>(Input<r3>,Input<r4>))));e0:=
FuncCall<avg>(a5);e3:=Sublink<EXISTS Input<r8>>;e4
:=Sublink<EXISTS Input<r5>>|
Filter<e2 a6>(Agg<\_ a4 \_ e1 a5 r1 e2 a6 r2>(Input<
r3>));e1:=FuncCall<avg>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r8);TableEq(r4,r5);
TableEq(r4,r8);TableEq(r5,r8);Unique(r3,a4);Unique(r3,a5)|

```
rule 203:

rule 203:

Filter<e3 _>(Agg<_ a3 _ e0 a4 r6 e1 a5 r7>(Filter< e4 _>(Input<r3>)));e0:=FuncCall<count>(a4);e3:=

Sublink<EXISTS Input<r8>>;e4:=Sublink<EXISTS

Union_all<>(Input<r4>,Input<r5>)>|

Agg<_ a5 _ e2 a3 r1 e1 a3 r2>(Input<r3>);e2:=

FuncCall<count>(a3)|

AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);

NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r4);

TableEq(r3,r5);TableEq(r3,r8);TableEq(r4,r5);

TableEq(r4,r8);TableEq(r5,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
2 Agg<_ a3 _ e0 a4 r7 e1 a5 r8>(Union_all<>(Filter<
e3 _>(Union_all<>(Input<r3>,Input<r4>)),Input<r6>)
);e0:=FuncCall<max>(a4);e3:=Sublink<EXISTS Input<
r5>|
3 Agg<_ a5 _ e2 a4 r1 e1 a3 r2>(Input<r3>);e2:=
FuncCall<max>(a4)|
4 AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
```

rule 204:

```
rule 205:
```

```
2 Agg<_ a3 _ e0 a4 r7 e1 a5 r8>(Filter<e3 _>(Input<
r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS
Union_all<>(Union_all<>(Input<r4>, Input<r5>), Input
<r6>)>|
3 Agg<_ a5 _ e2 a3 r1 e1 a5 r2>(Input<r3>);e2:=
FuncCall<count>(a3)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 206:
Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Filter<e3 _>(Input<
r3>)); e0:=FuncCall<sum>(a5); e3:=Sublink<EXISTS
Union_all<>(Union_all<>(Input<r4>, Input<r5>), Input
<r6>)>|
Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input<
r3>)); e1:=FuncCall<sum>(a5)|
AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4);
NotNull(r3,a4); NotNull(r3,a5); TableEq(r3,r4);
TableEq(r3,r5); TableEq(r3,r6); TableEq(r4,r5);
TableEq(r4,r6); TableEq(r5,r6); Unique(r3,a4); Unique(r3,a5)|
```

```
rule 207:

Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Union_all<>(Input<r3 >,Filter<e3 _>(Filter<e4 _>(Input<r4>))));e0:=
FuncCall<avg>(a5);e3:=Sublink<EXISTS Input<r6>);e4 :=Sublink<EXISTS Input<r5>|
Filter<e2 a4>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input< r3>));e1:=FuncCall<avg>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 208:
Filter<e3 _>(Filter<e4 _>(Agg<_ a4 _ e0 a5 r4 e2 a6 r5>(Input<r3>)));e0:=FuncCall<count>(a5);e3:= Sublink<EXISTS Input<r7>>;e4:=Sublink<EXISTS Input</ri>

**r6>|

Filter<e2 a4>(Agg<_ a4 _ e1 a6 r1 e2 a4 r2>(Input< r3>));e1:=FuncCall<count>(a6)|

**AttrsEq(a4,a5);AttrsSub(a4,r3);AttrsSub(a5,r3); AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5); TableEq(r3,r6);TableEq(r3,r7);TableEq(r6,r7); Unique(r3,a4);Unique(r3,a5)|
```

```
rule 209:

Filter<e3 _>(Agg<_ a3 _ e0 a4 r4 e1 a5 r5>(Input< r3>)); e0:=FuncCall<count>(a4); e3:=Sublink<EXISTS
Union_all<>(Union_all<>(Input<r6>, Input<r7>), Input
<r8>)>|

Agg<_ a3 _ e2 a3 r1 e1 a5 r2>(Input<r3>); e2:=
```

FuncCall < count > (a3) |

```
AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3); NotNull(r3,a3); NotNull(r3,a4); TableEq(r3,r6); TableEq(r3,r7); TableEq(r3,r8); TableEq(r6,r7); TableEq(r6,r8); TableEq(r7,r8); Unique(r3,a3); Unique(r3,a4)|
```

```
rule 210:

2 Agg<_ a4 _ e0 a5 r6 e2 a6 r7>(Filter<e3 _>(Input< r3>));e0:=FuncCall<count>(a5);e3:=Sublink<EXISTS
Union_all<>(Input<r4>,Input<r5>)>|

5 Filter<e2 a6>(Agg<_ a4 _ e1 a4 r1 e2 a6 r2>(Input< r3>));e1:=FuncCall<count>(a4)|

4 AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r4,r5);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 211:
    Exists(Agg_count<a4 a5 r4 e2 a6 r5>(Input<r3>),
    Input<r6>)|
    Filter<e1 a3>(Agg_count<a0 a1 r1 e0 a2 r2>(Input<
    r0>))|
    AttrsEq(a0,a6);AttrsEq(a1,a6);AttrsEq(a2,a6);
    AttrsEq(a3,a6);AttrsEq(a4,a5);AttrsSub(a4,r3);
    AttrsSub(a5,r3);AttrsSub(a6,a4);NotNull(r3,a4);
    NotNull(r3,a5);PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r6);TableEq(r3,r6);
    Unique(r3,a4);Unique(r3,a5)|
```

```
rule 213:

Agg_sum<a4 a5 r7 e2 a6 r8>(Exists(Input<r3>,Union(Input<r4>,Union(Input<r5>,Input<r6>))))|

Filter<e1 a3>(Agg_sum<a0 a1 r1 e0 a2 r2>(Input<r0 >))|

AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a6);

AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);

PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r6);

TableEq(r3,r4);TableEq(r3,r5);TableEq(r5,r6);

Unique(r3,a4);Unique(r3,a5)|
```

```
rule 214:

Exists(Agg_count<a3 a4 r6 e1 a5 r7>(Exists(Input< r3>,Union(Input<r4>,Input<r5>))),Input<r8>)|

Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|
```

```
AttrsEq(a0,a3); AttrsEq(a1,a5); AttrsEq(a2,a5);
AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);
NotNull(r3,a3); NotNull(r3,a4); PredicateEq(e0,e1);
TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r5);
TableEq(r0,r8); TableEq(r3,r4); TableEq(r3,r5);
TableEq(r3,r8); TableEq(r4,r5); TableEq(r4,r8);
TableEq(r5,r8);Unique(r3,a3);Unique(r3,a4)|
rule 215:
Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Filter<e3 _>(Input<
r3>)); e0:=FuncCall<sum>(a5); e3:=Sublink<EXISTS
Filter < e4 _>(Input < r4>)>; e4:=Sublink < EXISTS
Union_all <> (Input <r5>, Input <r6>)>|
Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a4 r2>(Input<
r3>));e1:=FuncCall<sum>(a5)|
AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4);
NotNull(r3,a4); NotNull(r3,a5); TableEq(r3,r4);
TableEq(r3,r5); TableEq(r3,r6); TableEq(r4,r5);
TableEq(r4,r6); TableEq(r5,r6); Unique(r3,a4); Unique
(r3,a5)|
rule 216:
Agg<_ a3 _ e0 a4 r7 e1 a5 r8>(Filter<e3 _>(Filter<
e4 _>(Input<r3>)));e0:=FuncCall<count>(a4);e3:=
Sublink < EXISTS Input < r6 >>; e4: = Sublink < EXISTS
Union_all <>(Input <r4>, Input <r5>)>|
Agg<_ a3 _ e2 a3 r1 e1 a3 r2>(Input<r3>);e2:=
FuncCall < count > (a3) |
AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);
NotNull(r3,a3); NotNull(r3,a4); TableEq(r3,r4);
TableEq(r3,r5); TableEq(r3,r6); TableEq(r4,r5);
TableEq(r4,r6); TableEq(r5,r6); Unique(r3,a3); Unique
(r3,a4)|
```

```
rule 217:

Exists(Agg_count<a3 a4 r4 e1 a5 r5>(Input<r3>),
Union(Input<r6>,Union(Input<r7>,Input<r8>)))|

Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|

AttrsEq(a0,a5);AttrsEq(a1,a3);AttrsEq(a2,a5);
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);
TableEq(r0,r3);TableEq(r0,r6);TableEq(r0,r7);
TableEq(r0,r8);TableEq(r3,r6);TableEq(r3,r7);
TableEq(r3,r8);TableEq(r6,r7);TableEq(r6,r8);
TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 218:

Exists(Exists(Agg_count<a3 a4 r4 e1 a5 r5>(Input<r3>), Union(Input<r6>, Input<r7>)), Input<r8>)|

Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|

AttrsEq(a0,a3); AttrsEq(a1,a3); AttrsEq(a2,a3);

AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);

NotNull(r3,a3); NotNull(r3,a4); PredicateEq(e0,e1);

TableEq(r0,r3); TableEq(r0,r6); TableEq(r0,r7);

TableEq(r0,r8); TableEq(r3,r6); TableEq(r3,r7);

TableEq(r7,r8); Unique(r3,a3); Unique(r3,a4)|
```

```
2 Agg<_ a4 _ e0 a5 r6 e2 a6 r7>(Filter<e3 _>(Input<
r3>));e0:=FuncCall<count>(a5);e3:=Sublink<EXISTS
Union_all<>(Input<r4>,Input<r5>)>|
Filter<e2 a4>(Agg<_ a4 _ e1 a6 r1 e2 a6 r2>(Input<
r3>));e1:=FuncCall<count>(a6)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r4,r5);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 220:

Exists(Agg_count<a4 a5 r5 e1 a6 r6>(Proj_simple<_ a3 r4>(Input<r3>)),Union(Input<r7>,Input<r8>))|

Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|

AttrsEq(a0,a4);AttrsEq(a1,a4);AttrsEq(a2,a4);

AttrsSub(a3,r3);AttrsSub(a4,r4);AttrsSub(a5,r4);

AttrsSub(a6,a4);NotNull(r3,a3);PredicateEq(e0,e1);

TableEq(r0,r3);TableEq(r0,r7);TableEq(r0,r8);

TableEq(r3,r7);TableEq(r3,r8);TableEq(r7,r8);

Unique(r3,a3)|
```

```
rule 221:
Filter<e3 _>(Agg<_ a3 _ e0 a4 r4 e1 a5 r5>(Input<
r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS
Union_all<>(Input<r6>,Input<r7>)>|
Agg<_ a3 _ e2 a3 r1 e1 a5 r2>(Input<r3>);e2:=
FuncCall<count>(a3)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r6,r7);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 222:

Agg_max < a3 a4 r7 e1 a5 r8 > (Union(Input < r3 > ,Exists(Union(Input < r4 > ,Input < r5 > ),Input < r6 > )))|

Agg_max < a0 a1 r1 e0 a2 r2 > (Input < r0 > )|

AttrsEq(a0,a3); AttrsEq(a1,a4); AttrsEq(a2,a5);

AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);

NotNull(r3,a3); NotNull(r3,a4); PredicateEq(e0,e1);

TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r5);

TableEq(r0,r6); TableEq(r3,r4); TableEq(r4,r6);

TableEq(r5,r6); Unique(r3,a3); Unique(r3,a4)|
```

```
rule 223:
Agg_average<a4 a5 r7 e2 a6 r8>(Union(Exists(Input< r3>,Exists(Input<r4>,Input<r5>)),Input<r6>))|
Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input < r0>))|
AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a6);
AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r6);
TableEq(r3,r4);TableEq(r3,r5);TableEq(r5,r6);
Unique(r3,a4);Unique(r3,a5)|
```

1 rule 224:

```
(Appendix)
Agg<_ a4 _ e0 a5 r6 e2 a6 r7>(Filter<e3 _>(Input<
r3>));e0:=FuncCall<count>(a5);e3:=Sublink<EXISTS
Union_all <>(Input <r4>, Input <r5>)>|
Filter < e2 a4 > (Agg < a4 e1 a5 r1 e2 a6 r2 > (Input <
r3>));e1:=FuncCall<count>(a5)|
AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5); TableEq(r4,r5); Unique(r3,a4); Unique
(r3,a5)|
rule 225:
Exists(Agg_count < a3 a4 r6 e1 a5 r7 > (Exists(Input <
r3>,Union(Input<r4>,Input<r5>))),Input<r8>)|
Agg\_count < a0 a1 r1 e0 a2 r2 > (Input < r0 >) |
AttrsEq(a0,a3); AttrsEq(a1,a3); AttrsEq(a2,a3);
AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);
NotNull(r3,a3); NotNull(r3,a4); PredicateEq(e0,e1);
TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r5);
TableEq(r0,r8); TableEq(r3,r4); TableEq(r3,r5);
TableEq(r3,r8); TableEq(r4,r5); TableEq(r4,r8);
TableEq(r5,r8);Unique(r3,a3);Unique(r3,a4)|
rule 226:
Agg<_ a3 _ e0 a4 r7 e1 a5 r8>(Union_all<>(Filter<
e3 _>(Union_all <>(Input <r3>, Input <r4>)), Input <r6>)
```

);e0:=FuncCall<max>(a4);e3:=Sublink<EXISTS Input< r5>>| Agg<\_ a3 \_ e2 a4 r1 e1 a5 r2>(Input<r3>);e2:= FuncCall < max > (a4) | AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3); NotNull(r3,a3); NotNull(r3,a4); TableEq(r3,r4);TableEq(r3,r5); TableEq(r3,r6); TableEq(r4,r5); TableEq(r4,r6); TableEq(r5,r6); Unique(r3,a3); Unique (r3,a4)|

```
rule 227:
Exists(Agg_count<a4 a5 r4 e2 a6 r5>(Input<r3>),
Input < r6>)|
Filter < e1 a3 > (Agg_count < a0 a1 r1 e0 a2 r2 > (Input <
r0>))|
AttrsEq(a0,a4); AttrsEq(a0,a5); AttrsEq(a1,a6);
AttrsEq(a2,a4); AttrsEq(a2,a5); AttrsEq(a3,a4);
AttrsEq(a3,a5); AttrsEq(a4,a5); AttrsSub(a4,r3);
AttrsSub(a5,r3); AttrsSub(a6,a4); NotNull(r3,a4);
NotNull(r3, a5); PredicateEq(e0, e2); PredicateEq(e1,
e2); TableEq(r0,r3); TableEq(r0,r6); TableEq(r3,r6);
Unique(r3,a4);Unique(r3,a5)|
```

```
rule 228:
Agg_average < a4 a5 r7 e2 a6 r8 > (Exists (Union (Input <
r3>, Input <r4>), Union(Input <r5>, Input <r6>)))|
Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input
AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a6);
AttrsEq(a3,a6); AttrsSub(a4,r3); AttrsSub(a5,r3);
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,
r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r0,r6);
TableEq(r3,r4); TableEq(r3,r5); TableEq(r3,r6);
TableEq(r4,r5); TableEq(r4,r6); TableEq(r5,r6);
Unique(r3,a4);Unique(r3,a5)|
```

```
rule 229:
Filter<e3 _>(Filter<e4 _>(Agg<_ a4 _ e0 a5 r4 e2
a6 r5>(Input<r3>)));e0:=FuncCall<avg>(a5);e3:=
Sublink<EXISTS Union_all<>(Input<r7>, Input<r8>)>;
e4:=Sublink<EXISTS Input<r6>>|
Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input<
r3>));e1:=FuncCall<avg>(a5)|
AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4);
NotNull(r3,a4); NotNull(r3,a5); TableEq(r3,r6);
TableEq(r3,r7); TableEq(r3,r8); TableEq(r6,r7);
TableEq(r6,r8); TableEq(r7,r8); Unique(r3,a4); Unique
(r3,a5)|
```

```
rule 230:
Filter<e3 _>(Agg<_ a4 _ e0 a5 r5 e2 a6 r6>(
Union_all <> (Input <r3>, Input <r4>))); e0:=FuncCall <
avg>(a5);e3:=Sublink<EXISTS Input<r7>>|
Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a4 r2>(Input<
r3>));e1:=FuncCall<avg>(a5)|
AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4);
NotNull(r3,a4); NotNull(r3,a5); TableEq(r3,r4);
TableEq(r3,r7); TableEq(r4,r7); Unique(r3,a4); Unique
(r3,a5)|
```

```
rule 231:
Exists(Exists(Agg_average < a4 a5 r4 e2 a6 r5 > (Input
<r3>), Input <r6>), Union(Input <r7>, Input <r8>))|
Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input
<r0>))|
AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a4);
AttrsEq(a3,a6); AttrsSub(a4,r3); AttrsSub(a5,r3);
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,
r3); TableEq(r0,r6); TableEq(r0,r7); TableEq(r0,r8);
TableEq(r3,r6); TableEq(r3,r7); TableEq(r3,r8);
TableEq(r6,r7); TableEq(r6,r8); TableEq(r7,r8);
Unique(r3,a4);Unique(r3,a5)|
```

```
rule 232:
Agg_average < a4 a5 r7 e2 a6 r8 > (Union(Exists(Input <
r3>, Exists(Input<r4>, Input<r5>)), Input<r6>))|
Filter < e1 a3 > (Agg_average < a0 a1 r1 e0 a2 r2 > (Input
<r0>))|
AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a4);
AttrsEq(a3,a6); AttrsSub(a4,r3); AttrsSub(a5,r3);
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,
r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r0,r6);
TableEq(r3,r4); TableEq(r3,r5); TableEq(r3,r6);
TableEq(r4,r5); TableEq(r4,r6); TableEq(r5,r6);
Unique(r3,a4);Unique(r3,a5)|
```

```
rule 233:
Exists(Agg_average < a4 a5 r6 e2 a6 r7 > (Union(Exists
(Input < r3>, Input < r4>), Input < r5>)), Input < r8>)|
Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input
<r0>))|
```

```
AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a6);
AttrsEq(a3,a4); AttrsSub(a4,r3); AttrsSub(a5,r3);
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,
r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r0,r8);
TableEq(r3,r4); TableEq(r3,r5); TableEq(r3,r8);
TableEq(r4,r5); TableEq(r4,r8); TableEq(r5,r8);
Unique(r3,a4);Unique(r3,a5)|
rule 234:
Agg_average < a4 a5 r7 e2 a6 r8 > (Union(Union(Input <
\verb|r3>, \verb|Input<|r4>|), \verb|Union(Input<|r5>, \verb|Input<|r6>|))||
Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input
<r0>))|
AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a4);
AttrsEq(a3,a6); AttrsSub(a4,r3); AttrsSub(a5,r3);
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,
r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r0,r6);
TableEq(r3,r4); TableEq(r3,r5); TableEq(r3,r6);
TableEq(r4,r5); TableEq(r4,r6); TableEq(r5,r6);
Unique(r3,a4);Unique(r3,a5)|
```

rule 235:

Filter<e3 \_>(Agg<\_ a4 \_ e0 a5 r5 e2 a6 r6>(
Union\_all<>(Input<r3>,Input<r4>)));e0:=FuncCall<
avg>(a5);e3:=Sublink<EXISTS Input<r7>>|

Filter<e2 a4>(Agg<\_ a4 \_ e1 a5 r1 e2 a6 r2>(Input<
r3>));e1:=FuncCall<avg>(a5)|

AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);

NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);

TableEq(r3,r7);TableEq(r4,r7);Unique(r3,a4);Unique(r3,a5)|

```
rule 236:

rule 236:

Filter<e3 _>(Agg<_ a3 _ e0 a4 r5 e1 a5 r6>(Filter< e4 _>(Input<r3>)));e0:=FuncCall<count>(a4);e3:=

Sublink<EXISTS Union_all<>(Input<r7>, Input<r8>)>;
e4:=Sublink<EXISTS Input<r4>>|

Agg<_ a3 _ e2 a3 r1 e1 a3 r2>(Input<r3>);e2:=

FuncCall<count>(a3)|

AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);

NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r4);

TableEq(r3,r7);TableEq(r3,r8);TableEq(r4,r7);

TableEq(r4,r8);TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 237:
Filter<e3 _>(Agg<_ a4 _ e0 a5 r4 e2 a6 r5>(Input< r3>)); e0:=FuncCall<count>(a5); e3:=Sublink<EXISTS
Union_all<>(Input<r6>, Input<r7>)>|
Filter<e2 a4>(Agg<_ a4 _ e1 a6 r1 e2 a4 r2>(Input< r3>)); e1:=FuncCall<count>(a6)|
AttrsEq(a4,a5); AttrsSub(a4,r3); AttrsSub(a5,r3);
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
TableEq(r3,r6); TableEq(r3,r7); TableEq(r6,r7);
Unique(r3,a4); Unique(r3,a5)|
```

```
rule 238:

Exists(Agg_count<a4 a5 r4 e2 a6 r5>(Input<r3>),

Union(Input<r6>,Input<r7>))|
```

```
Filter<e1 a3>(Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>))|
AttrsEq(a0,a6); AttrsEq(a1,a6); AttrsEq(a2,a4);
AttrsEq(a3,a6); AttrsSub(a4,r3); AttrsSub(a5,r3);
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,r3); TableEq(r0,r6); TableEq(r0,r7); TableEq(r3,r6);
TableEq(r3,r7); TableEq(r6,r7); Unique(r3,a4); Unique(r3,a5)|
```

```
rule 239:

Agg_sum<a4 a5 r7 e2 a6 r8>(Exists(Input<r3>,Union(Union(Input<r4>,Input<r5>),Input<r6>)))|

Filter<e1 a3>(Agg_sum<a0 a1 r1 e0 a2 r2>(Input<r0 >))|

AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a6);

AttrsEq(a3,a4);AttrsSub(a4,r3);AttrsSub(a5,r3);

AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);

PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r6);

TableEq(r3,r4);TableEq(r3,r5);TableEq(r3,r6);

TableEq(r4,r5);TableEq(r4,r6);TableEq(r5,r6);

Unique(r3,a4);Unique(r3,a5)|
```

```
rule 240:

Agg_count <a4 a5 r6 e2 a6 r7>(Exists(Input <r3>,
Union(Input <r4>,Input <r5>)))|

Filter <e1 a3>(Agg_count <a0 a1 r1 e0 a2 r2>(Input < r0>))|

AttrsEq(a0,a4); AttrsEq(a1,a6); AttrsEq(a2,a4);
AttrsEq(a3,a6); AttrsSub(a4,r3); AttrsSub(a5,r3);
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r3,r4);
TableEq(r3,r5); TableEq(r4,r5); Unique(r3,a4); Unique(r3,a5)|
```

```
rule 241:
Agg_average<a4 a5 r7 e2 a6 r8>(Exists(Union(Union(Input<r3>,Input<r4>),Input<r5>),Input<r6>))|
Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input<r0>))|
AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a6);
AttrsSub(a3,a4);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r6);
TableEq(r3,r4);TableEq(r3,r5);TableEq(r5,r6);
Unique(r3,a4);Unique(r3,a5)|
```

```
rule 242:

Agg_average<a4 a5 r7 e2 a6 r8>(Union(Exists(Input<r3>,Exists(Input<r4>,Input<r5>)),Input<r6>))|

Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input<r0>))|
```

```
(Appendix)
AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a6);
AttrsEq(a3,a4); AttrsSub(a4,r3); AttrsSub(a5,r3);
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,
r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r0,r6);
TableEq(r3,r4); TableEq(r3,r5); TableEq(r3,r6);
TableEq(r4,r5); TableEq(r4,r6); TableEq(r5,r6);
Unique(r3,a4);Unique(r3,a5)|
rule 243:
Filter<e3 _>(Agg<_ a4 _ e0 a5 r5 e2 a6 r6>(
\label{linear_section} \\ \mbox{Union\_all} <> (\mbox{Input} < \mbox{r3}>, \mbox{Input} < \mbox{r4}>))); e0:= \mbox{FuncCall} < \\ \mbox{FuncCall} < \mbox{FuncCall} < \mbox{FuncCall} < 
avg>(a5);e3:=Sublink<EXISTS Filter<e4 _>(Input<r7</pre>
>)>;e4:=Sublink<EXISTS Input<r8>>|
Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a4 r2>(Input<
r3>));e1:=FuncCall<avg>(a5)|
AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4);\\
NotNull(r3,a4); NotNull(r3,a5); TableEq(r3,r4);
TableEq(r3,r7); TableEq(r3,r8); TableEq(r4,r7);
TableEq(r4,r8); TableEq(r7,r8); Unique(r3,a4); Unique
(r3, a5)
```

```
rule 244:

Exists(Exists(Agg_count<a3 a4 r4 e1 a5 r5>(Input< r3>),Input<r6>),Union(Input<r7>,Input<r8>))|

Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|

AttrsEq(a0,a3);AttrsEq(a1,a3);AttrsEq(a2,a3);

AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);

NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);

TableEq(r0,r3);TableEq(r0,r6);TableEq(r0,r7);

TableEq(r0,r8);TableEq(r3,r6);TableEq(r3,r7);

TableEq(r3,r8);TableEq(r6,r7);TableEq(r6,r8);

TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 245:
rule 245:
filter<e3 _>(Agg<_ a4 _ e0 a5 r4 e2 a6 r5>(Input<
r3>));e0:=FuncCall<count>(a5);e3:=Sublink<EXISTS
Union_all<>(Input<r6>,Input<r7>)>|
filter<e2 a4>(Agg<_ a4 _ e1 a4 r1 e2 a6 r2>(Input<
r3>));e1:=FuncCall<count>(a4)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r6,r7);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 246:
Filter<e3 _>(Agg<_ a3 _ e0 a4 r6 e1 a5 r7>(Filter< e4 _>(Input<r3>)));e0:=FuncCall<count>(a4);e3:=
Sublink<EXISTS Input<r8>>;e4:=Sublink<EXISTS
Union_all<>(Input<r4>,Input<r5>)>|
Agg<_ a5 _ e2 a3 r1 e1 a5 r2>(Input<r3>);e2:=
FuncCall<count>(a3)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r8);TableEq(r4,r5);
TableEq(r4,r8);TableEq(r5,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 247:
2 Agg_average<a4 a5 r6 e2 a6 r7>(Union(Union(Input<rr>
r3>,Input<r4>),Input<r5>))|
```

```
Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input <r0>))|

AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a4);

AttrsEq(a3,a6); AttrsSub(a4,r3); AttrsSub(a5,r3);

AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);

PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r3,r4);

TableEq(r3,r5); TableEq(r4,r5); Unique(r3,a4); Unique(r3,a5)|
```

```
rule 248:
Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Union_all<>(
   Union_all<>(Input<r3>,Filter<e3 _>(Input<r4>)),
   Input<r6>));e0:=FuncCall<avg>(a5);e3:=Sublink<
   EXISTS Input<r5>>|
Filter<e2 a4>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input<
r3>));e1:=FuncCall<avg>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 249:
Agg<_ a4 _ e0 a5 r6 e2 a6 r7>(Union_all<>(Input<r3
>,Filter<e3 _>(Input<r4>)));e0:=FuncCall<avg>(a5);
e3:=Sublink<EXISTS Input<r5>>|
Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input<
r3>));e1:=FuncCall<avg>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r4,r5);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 250:
2 Agg<_ a4 _ e0 a5 r7 e1 a6 r8>(Filter<e3 _>(Proj<
ea3 a3 r4>(Input<r3>)));e0:=FuncCall<count>(a5);e3
:=Sublink<EXISTS Union_all<>(Input<r5>,Input<r6>)
>|
3 Agg<_ a4 _ e2 a4 r1 e1 a4 r2>(Input<r3>);e2:=
FuncCall<count>(a4)|
4 AttrsSub(a3,r3);AttrsSub(a4,r4);AttrsSub(a5,r4);
AttrsSub(a6,a4);NotNull(r3,a3);TableEq(r3,r5);
TableEq(r3,r6);TableEq(r5,r6);Unique(r3,a3)|
```

```
rule 251:

Exists(Agg_count<a3 a4 r4 e1 a5 r5>(Input<r3>),
Exists(Input<r6>,Union(Input<r7>,Input<r8>)))|

Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|
AttrsEq(a0,a3);AttrsEq(a1,a5);AttrsEq(a2,a5);
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);
TableEq(r0,r3);TableEq(r0,r6);TableEq(r0,r7);
TableEq(r0,r8);TableEq(r3,r6);TableEq(r3,r7);
TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 252:
```

```
2 Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Union_all<>(Input<r3 >,Filter<e3 _>(Input<r4>)));e0:=FuncCall<avg>(a5); e3:=Sublink<EXISTS Filter<e4 _>(Input<r5>)>;e4:= Sublink<EXISTS Input<r6>>|
3 Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a4 r2>(Input< r3>));e1:=FuncCall<avg>(a5)|
4 AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 253:

Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Union_all<>(Filter< e3 _>(Input<r3>),Input<r6>));e0:=FuncCall<avg>(a5);e3:=Sublink<EXISTS Union_all<>(Input<r4>,Input<r5>>>|

Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a4 r2>(Input< r3>));e1:=FuncCall<avg>(a5)|

AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 254:
Agg_average<a4 a5 r7 e2 a6 r8>(Exists(Union(Exists
(Input<r3>,Input<r4>),Input<r5>),Input<r6>))|

Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input
<r0>))|

AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a6);
AttrsEq(a3,a4);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r6);
TableEq(r3,r4);TableEq(r3,r5);TableEq(r3,r6);
TableEq(r4,r5);TableEq(r4,r6);TableEq(r5,r6);
Unique(r3,a4);Unique(r3,a5)|
```

```
rule 255:
Agg_max<a3 a4 r5 e1 a5 r6>(Union(Input<r3>,Input<
r4>))|
Agg_max<a0 a1 r1 e0 a2 r2>(Input<r0>)|
AttrsEq(a0,a5);AttrsEq(a1,a4);AttrsEq(a2,a3);
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);
TableEq(r0,r3);TableEq(r0,r4);TableEq(r3,r4);
Unique(r3,a3);Unique(r3,a4)|
```

```
2 Agg_average<a4 a5 r7 e2 a6 r8>(Exists(Union(Union(
   Input<r3>, Input<r4>), Input<r5>), Input<r6>))|
3 Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input
   <r0>))|
```

rule 256:

```
AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a4); AttrsEq(a3,a6); AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5); PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r0,r6); TableEq(r3,r4); TableEq(r3,r5); TableEq(r3,r6); TableEq(r4,r5); TableEq(r4,r6); TableEq(r3,a4); Unique(r3,a5)|
```

```
rule 257:
Exists(Agg_sum<a4 a5 r4 e2 a6 r5>(Input<r3>),Union
(Input<r6>,Exists(Input<r7>,Input<r8>)))|
Filter<e1 a3>(Agg_sum<a0 a1 r1 e0 a2 r2>(Input<r0
>))|
AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a6);
AttrsEq(a3,a4);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r6);TableEq(r0,r7);TableEq(r0,r8);
TableEq(r3,r6);TableEq(r3,r7);TableEq(r7,r8);
Unique(r3,a4);Unique(r3,a5)|
```

```
rule 258:
Agg<_ a3 _ e0 a4 r7 e1 a5 r8>(Filter<e3 _>(Filter<
e4 _>(Input<r3>)));e0:=FuncCall<count>(a4);e3:=
Sublink<EXISTS Input<r6>>;e4:=Sublink<EXISTS
Union_all<>(Input<r4>,Input<r5>)>|
Agg<_ a3 _ e2 a3 r1 e1 a5 r2>(Input<r3>);e2:=
FuncCall<count>(a3)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 259:
Filter<e3 _>(Agg<_ a3 _ e0 a4 r4 e1 a5 r5>(Input<
r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS
Union_all<>(Input<r6>,Filter<e4 _>(Input<r7>))>;e4
:=Sublink<EXISTS Input<r8>|
Agg<_ a5 _ e2 a3 r1 e1 a3 r2>(Input<r3>);e2:=
FuncCall<count>(a3)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r3,r8);TableEq(r6,r7);
TableEq(r6,r8);TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 260:
2 Agg_average<a4 a5 r7 e2 a6 r8>(Exists(Union(Exists (Input<r3>,Input<r4>),Input<r5>),Input<r6>))|
3 Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input<r0>))|
```

```
AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a4);
AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r6);
TableEq(r3,r4);TableEq(r3,r5);TableEq(r3,r6);
TableEq(r4,r5);TableEq(r4,r6);TableEq(r5,r6);
Unique(r3,a4);Unique(r3,a5)|

rule 261:
Agg_average<a4 a5 r7 e2 a6 r8>(Union(Input<r3>,
Union(Union(Input<r4>,Input<r5>),Input<r6>)))|
```

Union(Union(Input<r4>,Input<r5>),Input<r6>)))|
Filter<e1 a3>(Agg\_average<a0 a1 r1 e0 a2 r2>(Input
<r0>))|
AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a6);
AttrsEq(a3,a4);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,
r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r6);
TableEq(r3,r4);TableEq(r4,r6);TableEq(r5,r6);
Unique(r3,a4);Unique(r3,a5)|

```
rule 262:

Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Union_all<>(
Union_all<>(Input<r3>,Input<r4>),Union_all<>(Input
<r5>,Input<r6>)));e0:=FuncCall<avg>(a5)|

Filter<e2 a4>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input<
r3>));e1:=FuncCall<avg>(a5)|

AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a4);Unique
(r3,a5)|
```

```
rule 263:
Agg_count<a3 a4 r7 e1 a5 r8>(Exists(Exists(Input< r3>,Union(Input<r4>,Input<r5>)),Input<r6>))|
Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|
AttrsEq(a0,a5);AttrsEq(a1,a3);AttrsEq(a2,a3);
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);
TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);
TableEq(r0,r6);TableEq(r3,r4);TableEq(r3,r5);
TableEq(r3,r6);TableEq(r4,r5);TableEq(r4,r6);
TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 264:

Agg_count<a3 a4 r7 e1 a5 r8>(Exists(Input<r3>,
Union(Exists(Input<r4>,Input<r5>),Input<r6>)))|

Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|

AttrsEq(a0,a5);AttrsEq(a1,a5);AttrsEq(a2,a3);
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);
TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);
TableEq(r0,r6);TableEq(r3,r4);TableEq(r3,r5);
TableEq(r3,r6);TableEq(r4,r5);TableEq(r4,r6);
TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 265:
```

```
2 Agg_count<a4 a5 r5 e2 a6 r6>(Exists(Input<r3>, Input<r4>))|
5 Filter<e1 a3>(Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>))|
6 AttrsEq(a0,a6); AttrsEq(a1,a6); AttrsEq(a2,a6);
7 AttrsEq(a3,a6); AttrsEq(a4,a5); AttrsSub(a4,r3);
8 AttrsSub(a5,r3); AttrsSub(a6,a4); NotNull(r3,a4);
8 NotNull(r3,a5); PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,r3); TableEq(r0,r4); TableEq(r3,r4);
9 Unique(r3,a4); Unique(r3,a5)|
```

```
rule 266:
Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Union_all<>(Input<r3
>,Filter<e3 _>(Filter<e4 _>(Input<r4>))));e0:=
FuncCall<avg>(a5);e3:=Sublink<EXISTS Input<r6>>;e4
:=Sublink<EXISTS Input<r5>>|
Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a4 r2>(Input<
r3>));e1:=FuncCall<avg>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 267:
Filter<e3 _>(Agg<_ a4 _ e0 a5 r4 e2 a6 r5>(Input<
r3>));e0:=FuncCall<avg>(a5);e3:=Sublink<EXISTS
Union_all<>(Union_all<>(Input<r6>,Input<r7>),Input
<r8>)>|
Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a4 r2>(Input<
r3>));e1:=FuncCall<avg>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r3,r8);TableEq(r6,r7);
TableEq(r6,r8);TableEq(r7,r8);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 268:
Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Union_all<>(Filter<
e3 _>(Input<r3>),Filter<e4 _>(Input<r5>)));e0:=
FuncCall<avg>(a5);e3:=Sublink<EXISTS Input<r4>>;e4
:=Sublink<EXISTS Input<r6>>|
Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a4 r2>(Input<
r3>));e1:=FuncCall<avg>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 269:
Agg_average<a4 a5 r7 e2 a6 r8>(Union(Exists(Input<
r3>,Input<r4>),Exists(Input<r5>,Input<r6>)))|
Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input
<r0>))|
```

```
AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a4); AttrsEq(a3,a6); AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5); PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r0,r6); TableEq(r3,r4); TableEq(r3,r5); TableEq(r3,r6); TableEq(r4,r5); TableEq(r4,r6); TableEq(r3,a4); Unique(r3,a5)|
```

```
rule 270:

Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Union_all<>(
Union_all<>(Input<r3>,Union_all<>(Input<r4>,Input<
r5>)),Input<r6>));e0:=FuncCall<avg>(a5)|

Filter<e2 a4>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input<
r3>));e1:=FuncCall<avg>(a5)|

AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);

NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);

TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);

TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 271:

Exists(Agg_count<a4 a5 r5 e1 a6 r6>(Proj_simple<_ a3 r4>(Input<r3>)),Union(Input<r7>,Input<r8>))|

Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|

AttrsEq(a0,a4);AttrsEq(a1,a4);AttrsEq(a2,a6);

AttrsSub(a3,r3);AttrsSub(a4,r4);AttrsSub(a5,r4);

AttrsSub(a6,a4);NotNull(r3,a3);PredicateEq(e0,e1);

TableEq(r0,r3);TableEq(r0,r7);TableEq(r0,r8);

TableEq(r3,r7);TableEq(r3,r8);TableEq(r7,r8);

Unique(r3,a3)|
```

```
rule 272:
Agg_count<a4 a5 r7 e1 a6 r8>(Proj_simple<_ a3 r6>(
    Exists(Input<r3>, Union(Input<r4>, Input<r5>))))|
Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|
AttrsEq(a0,a6); AttrsEq(a1,a6); AttrsEq(a2,a4);
AttrsSub(a3,r3); AttrsSub(a4,r6); AttrsSub(a5,r6);
AttrsSub(a6,a4); NotNull(r3,a3); PredicateEq(e0,e1);
TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r5);
TableEq(r3,r4); TableEq(r3,r5); TableEq(r4,r5);
Unique(r3,a3)|
```

```
rule 273:
Agg_count<a3 a4 r7 e1 a5 r8>(Exists(Input<r3>,
Union(Input<r4>,Union(Input<r5>,Input<r6>))))|
Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|
AttrsEq(a0,a3);AttrsEq(a1,a3);AttrsEq(a2,a5);
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);
TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);
TableEq(r0,r6);TableEq(r3,r4);TableEq(r3,r5);
TableEq(r3,r6);TableEq(r4,r5);TableEq(r4,r6);
TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 274:
Filter<e3 _>(Agg<_ a5 _ e0 a6 r5 e2 a7 r6>(Proj< ea4 a4 r4>(Input<r3>)));e0:=FuncCall<count>(a6);e3
:=Sublink<EXISTS Union_all<>(Input<r7>,Input<r8>)
>|
```

```
Filter<e2 a7>(Agg<_ a5 _ e1 a6 r1 e2 a5 r2>(Input< r3>));e1:=FuncCall<count>(a6)|
AttrsSub(a4,r3);AttrsSub(a5,r4);AttrsSub(a6,r4);
AttrsSub(a7,a5);NotNull(r3,a4);TableEq(r3,r7);
TableEq(r3,r8);TableEq(r7,r8);Unique(r3,a4)|
```

```
rule 275:

Exists(Agg_sum<a4 a5 r4 e2 a6 r5>(Input<r3>),Union
(Input<r6>,Exists(Input<r7>,Input<r8>)))|

Filter<e1 a3>(Agg_sum<a0 a1 r1 e0 a2 r2>(Input<r0
>))|

AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a4);
AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r6);TableEq(r0,r7);TableEq(r0,r8);
TableEq(r3,r6);TableEq(r3,r7);TableEq(r3,r8);
TableEq(r6,r7);TableEq(r6,r8);TableEq(r7,r8);
Unique(r3,a4);Unique(r3,a5)|
```

```
rule 276:
Agg<_ a3 _ e0 a4 r7 e1 a5 r8>(Union_all<>(Input<r3
>,Filter<e3 _>(Union_all<>(Input<r4>,Input<r5>))))
;e0:=FuncCall<max>(a4);e3:=Sublink<EXISTS Input<r6
>>|
Agg<_ a3 _ e2 a4 r1 e1 a5 r2>(Input<r3>);e2:=
FuncCall<max>(a4)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 277:
Agg<_ a4 _ e0 a5 r6 e2 a6 r7>(Filter<e3 _>(Input<
r3>));e0:=FuncCall<count>(a5);e3:=Sublink<EXISTS
Union_all<>(Input<r4>,Input<r5>)>|
Filter<e2 a6>(Agg<_ a6 _ e1 a5 r1 e2 a4 r2>(Input<
r3>));e1:=FuncCall<count>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r4,r5);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 278:

Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Union_all<>(Filter<
e3 _>(Input<r3>),Union_all<>(Input<r5>,Input<r6>))
);e0:=FuncCall<avg>(a5);e3:=Sublink<EXISTS Input<
r4>|

Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a4 r2>(Input<
r3>));e1:=FuncCall<avg>(a5)|

AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a4);Unique(r3.a5)|
```

```
rule 279:
Agg_average < a4 a5 r7 e2 a6 r8 > (Union(Input < r3 > ,
Union(Input < r4 > , Union(Input < r5 > , Input < r6 > ))))|
```

```
Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input <r0>))|

4 AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a6);
   AttrsEq(a3,a4); AttrsSub(a4,r3); AttrsSub(a5,r3);
   AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
   PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r0,r6);
   TableEq(r3,r4); TableEq(r3,r5); TableEq(r3,r6);
   TableEq(r4,r5); TableEq(r4,r6); TableEq(r5,r6);
   Unique(r3,a4); Unique(r3,a5)|
```

```
rule 280:

Filter<e3 _>(Agg<_ a4 _ e0 a5 r6 e2 a6 r7>(
Union_all<>(Input<r3>,Union_all<>(Input<r4>,Input<
r5>))));e0:=FuncCall<avg>(a5);e3:=Sublink<EXISTS
Input<r8>|

Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a4 r2>(Input<
r3>));e1:=FuncCall<avg>(a5)|

AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r8);TableEq(r4,r5);
TableEq(r4,r8);TableEq(r5,r8);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 281:
    Exists(Agg_average<a4 a5 r5 e2 a6 r6>(Union(Input< r3>,Input<r4>)),Exists(Input<r7>,Input<r8>))|
    Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input < r0>))|
    AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a6);
    AttrsEq(a3,a4);AttrsSub(a4,r3);AttrsSub(a5,r3);
    AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
    PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r7);TableEq(r0,r8);
    TableEq(r3,r4);TableEq(r4,r8);TableEq(r7,r8);
    Unique(r3,a4);Unique(r3,a5)|
```

```
rule 282:
Agg_count<a4 a5 r6 e2 a6 r7>(Exists(Input<r3>,
Union(Input<r4>,Input<r5>)))|
Filter<e1 a3>(Agg_count<a0 a1 r1 e0 a2 r2>(Input< r0>))|
AttrsEq(a0,a6);AttrsEq(a1,a4);AttrsEq(a2,a4);
AttrsEq(a3,a4);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r4,r5);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 283:

Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Filter<e3 _>(
    Union_all<>(Filter<e4 _>(Input<r3>),Input<r5>)));
    e0:=FuncCall<avg>(a5);e3:=Sublink<EXISTS Input<r6
    >>;e4:=Sublink<EXISTS Input<r4>>|
    Filter<e2 a4>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input<
r3>));e1:=FuncCall<avg>(a5)|
```

```
AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4);
NotNull(r3,a4); NotNull(r3,a5); TableEq(r3,r4);
TableEq(r3,r5); TableEq(r3,r6); TableEq(r4,r5);
TableEq(r4,r6); TableEq(r5,r6); Unique(r3,a4); Unique(r3,a5)|
```

```
rule 284:
Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Union_all<>(Filter<
e3 _>(Union_all<>(Input<r3>,Input<r4>)),Input<r6>)
);e0:=FuncCall<avg>(a5);e3:=Sublink<EXISTS Input<
r5>>|
Filter<e2 a4>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input<
r3>));e1:=FuncCall<avg>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 285:

Agg<_ a3 _ e0 a4 r7 e1 a5 r8>(Filter<e3 _>(Input< r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS

Union_all<>(Filter<e4 _>(Input<r4>),Input<r6>)>;e4
:=Sublink<EXISTS Input<r5>|

Agg<_ a3 _ e2 a3 r1 e1 a3 r2>(Input<r3>);e2:=
FuncCall<count>(a3)|

AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);

NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r4);

TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);

TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
```

```
Filter<e3 _>(Agg<_ a3 _ e0 a4 r5 e1 a5 r6>(
Union_all<>(Input<r3>,Input<r4>)));e0:=FuncCall<
max>(a4);e3:=Sublink<EXISTS Union_all<>(Input<r7>,
Input<r8>)>|

Agg<_ a3 _ e2 a4 r1 e1 a5 r2>(Input<r3>);e2:=
FuncCall<max>(a4)|

AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r4);
TableEq(r3,r7);TableEq(r3,r8);TableEq(r4,r7);
TableEq(r4,r8);TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

rule 286:

```
rule 287:

Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Filter<e3 _>(Input< r3>));e0:=FuncCall<sum>(a5);e3:=Sublink<EXISTS

Union_all<>(Union_all<>(Input<r4>,Input<r5>),Input
<r6>)>|

Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a4 r2>(Input< r3>));e1:=FuncCall<sum>(a5)|

AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);

NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);

TableEq(r3,r5);TableEq(r5,r6);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 288:
```

```
2 Agg<_ a3 _ e0 a4 r7 e1 a5 r8>(Filter<e3 _>(Input<
 r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS
 Filter<e4 _>(Input<r4>)>; e4:=Sublink<EXISTS
 Union_all <>(Input <r5>, Input <r6>)>|
 Agg<_ a5 _ e2 a3 r1 e1 a3 r2>(Input<r3>);e2:=
 FuncCall < count > (a3) |
 AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);
 NotNull(r3,a3); NotNull(r3,a4); TableEq(r3,r4);
 TableEq(r3,r5); TableEq(r3,r6); TableEq(r4,r5);
 TableEq(r4,r6); TableEq(r5,r6); Unique(r3,a3); Unique
 (r3,a4)|
 rule 289:
 Agg_max < a3 a4 r6 e1 a5 r7 > (Union(Union(Input < r3 > ,
 Input < r4 > ) , Input < r5 > ) ) |
 Agg_max<a0 a1 r1 e0 a2 r2>(Input<r0>)|
 AttrsEq(a0,a3); AttrsEq(a1,a4); AttrsEq(a2,a5);
 AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);
 NotNull(r3,a3); NotNull(r3,a4); PredicateEq(e0,e1);
 TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r5);
 TableEq(r3,r4); TableEq(r3,r5); TableEq(r4,r5);
 Unique(r3,a3);Unique(r3,a4)|
 rule 290:
```

### rule 290: Agg\_count<a3 a4 r7 e1 a5 r8>(Exists(Input<r3>, Union(Input<r4>,Exists(Input<r5>,Input<r6>))))| Agg\_count<a0 a1 r1 e0 a2 r2>(Input<r0>)| AttrsEq(a0,a3);AttrsEq(a1,a3);AttrsEq(a2,a5); AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3); NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1); TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5); TableEq(r0,r6);TableEq(r3,r4);TableEq(r3,r5); TableEq(r3,r6);TableEq(r4,r5);TableEq(r4,r6); TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|

```
rule 291:
    Exists(Agg_average<a3 a4 r4 e1 a5 r5>(Input<r3>),
    Union(Input<r6>,Union(Input<r7>,Input<r8>)))|
    Agg_max<a0 a1 r1 e0 a2 r2>(Input<r0>)|
    AttrsEq(a0,a3);AttrsEq(a0,a4);AttrsEq(a1,a3);
    AttrsEq(a1,a4);AttrsSub(a3,r3);AttrsEud(a2,a4);
    AttrsSub(a5,a3);NotNull(r3,a3);NotNull(r3,a4);
    PredicateEq(e0,e1);TableEq(r0,r3);TableEq(r0,r6);
    TableEq(r0,r7);TableEq(r0,r8);TableEq(r6,r7);
    TableEq(r6,r8);TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 292:

Exists(Exists(Agg_count < a3 a4 r4 e1 a5 r5>(Input < r3>), Union(Input < r6>, Input < r7>)), Input < r8>)|

Agg_count < a0 a1 r1 e0 a2 r2>(Input < r0>)|

AttrsEq(a0, a5); AttrsEq(a1, a3); AttrsEq(a2, a5);

AttrsSub(a3, r3); AttrsSub(a4, r3); AttrsSub(a5, a3);

NotNull(r3, a3); NotNull(r3, a4); PredicateEq(e0, e1);

TableEq(r0, r3); TableEq(r0, r6); TableEq(r0, r7);

TableEq(r3, r8); TableEq(r6, r7); TableEq(r6, r8);

TableEq(r7, r8); Unique(r3, a3); Unique(r3, a4)|
```

```
rule 293:

Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Union_all<>(
Union_all<>(Input<r3>,Filter<e3 _>(Input<r4>)),
Input<r6>));e0:=FuncCall<avg>(a5);e3:=Sublink<
EXISTS Input<r5>>|
Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input<
r3>));e1:=FuncCall<avg>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 294:
Agg_count<a4 a5 r6 e2 a6 r7>(Exists(Input<r3>,
Union(Input<r4>,Input<r5>)))|
Filter<e1 a3>(Agg_count<a0 a1 r1 e0 a2 r2>(Input<
r0>))|
AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a6);
AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r4,r5);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 295:
Filter<e3 _>(Filter<e4 _>(Agg<_ a4 _ e0 a5 r5 e2
a6 r6>(Union_all<>(Input<r3>,Input<r4>))));e0:=
FuncCall<avg>(a5);e3:=Sublink<EXISTS Input<r8>>;e4
:=Sublink<EXISTS Input<r7>>|
Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input<
r3>));e1:=FuncCall<avg>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r7);TableEq(r3,r8);TableEq(r4,r7);
TableEq(r4,r8);TableEq(r7,r8);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 296:
    Exists(Agg_average<a4 a5 r6 e2 a6 r7>(Union(Input< r3>,Union(Input<r4>,Input<r5>))),Input<r8>)|
    Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input < r0>))|
    AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a4);
    AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);
    AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
    PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r8);
    TableEq(r3,r4);TableEq(r3,r5);TableEq(r5,r8);
    Unique(r3,a4);Unique(r3,a5)|
```

```
rule 297:
Exists(Agg_count<a4 a5 r5 e1 a6 r6>(Proj_simple<_
a3 r4>(Input<r3>)),Union(Input<r7>,Input<r8>))|
Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|
```

```
AttrsEq(a0,a6); AttrsEq(a1,a4); AttrsEq(a2,a4); AttrsSub(a3,r3); AttrsSub(a4,r4); AttrsSub(a5,r4); AttrsSub(a6,a4); NotNull(r3,a3); PredicateEq(e0,e1); TableEq(r0,r3); TableEq(r0,r7); TableEq(r0,r8); TableEq(r3,r7); TableEq(r7,r8); Unique(r3,a3)|
```

```
rule 298:
Filter<e3 _>(Filter<e4 _>(Agg<_ a3 _ e0 a4 r4 e1
a5 r5>(Input<r3>)));e0:=FuncCall<count>(a4);e3:=
Sublink<EXISTS Union_all<>(Input<r7>,Input<r8>)>;
e4:=Sublink<EXISTS Input<r6>>|
Agg<_ a5 _ e2 a3 r1 e1 a5 r2>(Input<r3>);e2:=
FuncCall<count>(a3)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r3,r8);TableEq(r6,r7);
TableEq(r6,r8);TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 299:

Filter<e3 _>(Agg<_ a3 _ e0 a4 r4 e1 a5 r5>(Input<
r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS
Filter<e4 _>(Union_all<>(Input<r6>,Input<r7>))>;e4
:=Sublink<EXISTS Input<r8>|

Agg<_ a5 _ e2 a5 r1 e1 a3 r2>(Input<r3>);e2:=
FuncCall<count>(a5)|

AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r3,r8);TableEq(r6,r7);
TableEq(r6,r8);TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 300:
Agg<_ a4 _ e0 a5 r7 e1 a6 r8>(Filter<e3 _>(Proj<
ea3 a3 r4>(Input<r3>)));e0:=FuncCall<count>(a5);e3
:=Sublink<EXISTS Union_all<>(Input<r5>,Input<r6>)
>|
Agg<_ a6 _ e2 a4 r1 e1 a6 r2>(Input<r3>);e2:=
FuncCall<count>(a4)|
AttrsSub(a3,r3);AttrsSub(a4,r4);AttrsSub(a5,r4);
AttrsSub(a6,a4);NotNull(r3,a3);TableEq(r3,r5);
TableEq(r3,r6);TableEq(r5,r6);Unique(r3,a3)|
```

```
rule 301:

Agg<_ a3 _ e0 a4 r7 e1 a5 r8>(Filter<e3 _>(Input< r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS
Filter<e4 _>(Union_all<>(Input<r4>,Input<r5>)));e4
:=Sublink<EXISTS Input<r6>>|

Agg<_ a3 _ e2 a3 r1 e1 a3 r2>(Input<r3>);e2:=
FuncCall<count>(a3)|

AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
```

```
Agg<_ a3 _ e0 a4 r7 e1 a5 r8>(Filter<e3 _>(Filter<
e4 _>(Input<r3>)));e0:=FuncCall<count>(a4);e3:=
Sublink < EXISTS Union_all <> (Input < r5>, Input < r6>) >;
e4:=Sublink<EXISTS Input<r4>>|
Agg<_ a3 _ e2 a5 r1 e1 a5 r2>(Input<r3>);e2:=
FuncCall < count > (a5) |
AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);
NotNull(r3,a3); NotNull(r3,a4); TableEq(r3,r4);
TableEq(r3,r5); TableEq(r3,r6); TableEq(r4,r5);
TableEq(r4,r6); TableEq(r5,r6); Unique(r3,a3); Unique
(r3, a4)|
rule 303:
Agg_max < a3 a4 r6 e1 a5 r7 > (Union(Input < r3 > , Union(
Input < r4>, Input < r5>)))|
Agg_max<a0 a1 r1 e0 a2 r2>(Input<r0>)|
AttrsEq(a0,a3); AttrsEq(a1,a4); AttrsEq(a2,a5);
```

AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);

TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r5);
TableEq(r3,r4); TableEq(r3,r5); TableEq(r4,r5);

Unique(r3,a3);Unique(r3,a4)|

NotNull(r3,a3); NotNull(r3,a4); PredicateEq(e0,e1);

```
rule 304:

Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Union_all<>(
    Union_all<>(Union_all<>(Input<r3>,Input<r4>),Input
    <r5>),Input<r6>));e0:=FuncCall<avg>(a5)|

Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input<
r3>));e1:=FuncCall<avg>(a5)|

AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 305:

Exists(Agg_max<a3 a4 r6 e1 a5 r7>(Union(Union(Input<r3>,Input<r4>),Input<r5>)),Input<r8>)|

Agg_max<a0 a1 r1 e0 a2 r2>(Input<r0>)|

AttrsEq(a0,a3);AttrsEq(a1,a4);AttrsEq(a2,a5);

AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);

NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);

TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);

TableEq(r0,r8);TableEq(r3,r4);TableEq(r4,r8);

TableEq(r5,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 307:
```

```
2 Agg<_ a4 _ e0 a5 r6 e2 a6 r7>(Union_all<>(Filter<
 e3 _>(Input<r3>),Input<r5>));e0:=FuncCall<avg>(a5)
 ;e3:=Sublink<EXISTS Input<r4>>|
 Filter < e2 a4 > (Agg < a4 e1 a5 r1 e2 a6 r2 > (Input <
 r3>));e1:=FuncCall<avg>(a5)|
 AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4);
 NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
 TableEq(r3,r5); TableEq(r4,r5); Unique(r3,a4); Unique
 (r3,a5)|
 rule 308:
 Agg_average < a4 a5 r6 e2 a6 r7 > (Union(Union(Input <
 r3>,Input<r4>),Input<r5>))|
 Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input
 <r0>))|
 AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a6);
 AttrsEq(a3,a4); AttrsSub(a4,r3); AttrsSub(a5,r3);
 AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
 PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,
 r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r3,r4);
 TableEq(r3,r5); TableEq(r4,r5); Unique(r3,a4); Unique
 (r3,a5)|
 rule 309:
 Exists(Agg_count<a3 a4 r4 e1 a5 r5>(Input<r3>),
```

## rule 309: Exists(Agg\_count<a3 a4 r4 e1 a5 r5>(Input<r3>), Union(Exists(Input<r6>,Input<r7>),Input<r8>))| Agg\_count<a0 a1 r1 e0 a2 r2>(Input<r0>)| AttrsEq(a0,a5);AttrsEq(a1,a3);AttrsEq(a2,a3); AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3); NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1); TableEq(r0,r3);TableEq(r0,r6);TableEq(r0,r7); TableEq(r0,r8);TableEq(r3,r6);TableEq(r3,r7); TableEq(r3,r8);TableEq(r6,r7);TableEq(r6,r8); TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|

```
rule 310:

Exists(Agg_average<a3 a4 r4 e1 a5 r5>(Input<r3>),
Union(Input<r6>,Union(Input<r7>,Input<r8>)))|

Agg_max<a0 a1 r1 e0 a2 r2>(Input<r0>)|

AttrsEq(a0,a3); AttrsEq(a0,a4); AttrsEq(a1,a5);
AttrsEq(a2,a3); AttrsEq(a2,a4); AttrsEq(a3,a4);

AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);

NotNull(r3,a3); NotNull(r3,a4); PredicateEq(e0,e1);

TableEq(r0,r3); TableEq(r0,r6); TableEq(r0,r7);

TableEq(r0,r8); TableEq(r3,r6); TableEq(r3,r7);

TableEq(r7,r8); Unique(r3,a3); Unique(r3,a4)|
```

```
rule 311:
Agg_count<a4 a5 r6 e2 a6 r7>(Exists(Input<r3>,
Union(Input<r4>,Input<r5>)))|
Filter<e1 a3>(Agg_count<a0 a1 r1 e0 a2 r2>(Input<
r0>))|
AttrsEq(a0,a6);AttrsEq(a1,a5);AttrsEq(a2,a4);
AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r4,r5);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 312:

Exists(Agg_count<a5 a6 r5 e2 a7 r6>(Proj_simple<_a4 r4>(Input<r3>)),Union(Input<r7>,Input<r8>))|

Filter<e1 a3>(Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>))|

AttrsEq(a0,a5);AttrsEq(a1,a6);AttrsEq(a2,a5);

AttrsSub(a6,r4);AttrsSub(a4,r3);AttrsSub(a5,r4);

AttrsSub(a6,r4);AttrsSub(a7,a5);NotNull(r3,a4);

PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r7);

TableEq(r3,r8);TableEq(r7,r8);Unique(r3,a4)|
```

```
rule 313:

Exists(Agg_count<a3 a4 r4 e1 a5 r5>(Input<r3>),
Union(Input<r6>,Input<r7>))|

Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|

AttrsEq(a0,a5); AttrsEq(a1,a3); AttrsEq(a2,a5);

AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);

NotNull(r3,a3); NotNull(r3,a4); PredicateEq(e0,e1);

TableEq(r0,r3); TableEq(r0,r6); TableEq(r0,r7);

TableEq(r3,r6); TableEq(r3,r7); TableEq(r6,r7);
Unique(r3,a3); Unique(r3,a4)|
```

```
rule 314:

Agg_average<a4 a5 r7 e2 a6 r8>(Exists(Union(Input<r3>,Exists(Input<r4>,Input<r5>)),Input<r6>))|

Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input<r0>))|

AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a4);

AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);

AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);

PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r6);

TableEq(r3,r4);TableEq(r3,r5);TableEq(r3,r6);

TableEq(r4,r5);TableEq(r4,r6);TableEq(r5,r6);

Unique(r3,a4);Unique(r3,a5)|
```

```
rule 315:
rule 315:
rilter<e3 _>(Agg<_ a5 _ e0 a6 r5 e2 a7 r6>(Proj< ea4 a4 r4>(Input<r3>)));e0:=FuncCall<count>(a6);e3 :=Sublink<EXISTS Union_all<>(Input<r7>,Input<r8>)
rilter<e2 a5>(Agg<_ a5 _ e1 a5 r1 e2 a7 r2>(Input< r3>));e1:=FuncCall<count>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r4);AttrsSub(a6,r4);
AttrsSub(a7,a5);NotNull(r3,a4);TableEq(r3,r7);
TableEq(r3,r8);TableEq(r7,r8);Unique(r3,a4)|
```

```
rule 316:

rule 316:

Filter<e3 _>(Filter<e4 _>(Agg<_ a4 _ e0 a5 r4 e2 a6 r5>(Input<r3>)));e0:=FuncCall<sum>(a5);e3:=
Sublink<EXISTS Input<r8>>;e4:=Sublink<EXISTS
Union_all<>(Input<r6>,Input<r7>)>|
Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a4 r2>(Input<r3>));e1:=FuncCall<sum>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r6);
TableEq(r6,r8);TableEq(r7,r8);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 317:

Agg_average<a4 a5 r7 e2 a6 r8>(Union(Union(Union(Input<r3>,Input<r4>),Input<r5>),Input<r6>))|

Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input<r0>))|

AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a4);

AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);

AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);

PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r6);

TableEq(r3,r4);TableEq(r3,r5);TableEq(r3,r6);

TableEq(r4,r5);TableEq(r4,r6);TableEq(r5,r6);

Unique(r3,a4);Unique(r3,a5)|
```

```
rule 318:
Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Union_all<>(
    Union_all<>(Input<r3>,Input<r4>),Filter<e3 _>(
    Input<r5>)));e0:=FuncCall<avg>(a5);e3:=Sublink</br>
    EXISTS Input<r6>|
Filter<e2 a4>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input<r3>));e1:=FuncCall<avg>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 319:
Agg_average<a4 a5 r6 e2 a6 r7>(Exists(Union(Input< r3>,Input<r4>),Input<r5>))|
Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input <r0>))|
AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a6);
AttrsEq(a3,a4);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r4,r5);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 320:
Filter<e3 _>(Agg<_ a3 _ e0 a4 r5 e1 a5 r6>(Filter<
    e4 _>(Input<r3>)));e0:=FuncCall<count>(a4);e3:=
    Sublink<EXISTS Union_all<>(Input<r7>,Input<r8>)>;
    e4:=Sublink<EXISTS Input<r4>>|
Agg<_ a3 _ e2 a5 r1 e1 a5 r2>(Input<r3>);e2:=
FuncCall<count>(a5)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r4);
TableEq(r3,r7);TableEq(r3,r8);TableEq(r4,r7);
TableEq(r4,r8);TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 321:
Agg<_ a4 _ e0 a5 r6 e2 a6 r7>(Filter<e3 _>(Input<
r3>));e0:=FuncCall<count>(a5);e3:=Sublink<EXISTS
Union_all<>(Input<r4>,Input<r5>)>|
Filter<e2 a4>(Agg<_ a6 _ e1 a4 r1 e2 a4 r2>(Input<
r3>));e1:=FuncCall<count>(a4)|
```

```
AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5); TableEq(r3,r4); TableEq(r3,r5); TableEq(r4,r5); Unique(r3,a4); Unique(r3,a5)|
```

```
rule 322:

Filter<e3 _>(Agg<_ a4 _ e0 a5 r4 e2 a6 r5>(Input<
r3>));e0:=FuncCall<sum>(a5);e3:=Sublink<EXISTS
Union_all<>(Filter<e4 _>(Input<r6>),Input<r8>)>;e4
:=Sublink<EXISTS Input<r7>>|

Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a4 r2>(Input<
r3>));e1:=FuncCall<sum>(a5)|

4 AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r3,r8);TableEq(r6,r7);
TableEq(r6,r8);TableEq(r7,r8);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 323:
Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Filter<e3 _>(Input<
r3>));e0:=FuncCall<sum>(a5);e3:=Sublink<EXISTS
Filter < e4 _>(Input < r4>)>; e4:=Sublink < EXISTS
Union_all <> (Input <r5>, Input <r6>)>|
Filter < e2 a6 > (Agg < a4 e1 a5 r1 e2 a6 r2 > (Input <
r3>));e1:=FuncCall<sum>(a5)|
AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4);
NotNull(r3,a4); NotNull(r3,a5); TableEq(r3,r4);
TableEq(r3,r5); TableEq(r3,r6); TableEq(r4,r5);
TableEq(r4,r6); TableEq(r5,r6); Unique(r3,a4); Unique
(r3,a5)Filter<e3 _>(Agg<_ a4 _ e0 a5 r4 e2 a6 r5>(
Input <r3>)); e0:=FuncCall <count >(a5); e3:=Sublink <</pre>
EXISTS Union_all <> (Input < r6 > , Input < r7 > ) > |
Filter<e2 a6>(Agg<_ a4 _ e1 a6 r1 e2 a4 r2>(Input<
r3>));e1:=FuncCall<count>(a6)|
AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4);
NotNull(r3,a4); NotNull(r3,a5); TableEq(r3,r6);
TableEq(r3,r7); TableEq(r6,r7); Unique(r3,a4); Unique
(r3,a5)|
```

```
rule 324:
Agg<_ a4 _ e0 a5 r6 e2 a6 r7>(Filter<e3 _>(Input<
r3>));e0:=FuncCall<count>(a5);e3:=Sublink<EXISTS
Union_all<>(Input<r4>,Input<r5>)>|
Filter<e2 a4>(Agg<_ a4 _ e1 a6 r1 e2 a4 r2>(Input<
r3>));e1:=FuncCall<count>(a6)|
AttrsEq(a4,a5);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
TableEq(r3,r4);TableEq(r3,r5);TableEq(r4,r5);
Unique(r3,a4);Unique(r3,a5)|
```

```
rule 325:
Agg_average<a4 a5 r7 e2 a6 r8>(Union(Input<r3>,
Exists(Input<r4>,Exists(Input<r5>,Input<r6>))))|
Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input<r0>)))|
```

```
AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a6);
AttrsEq(a3,a4); AttrsSub(a4,r3); AttrsSub(a5,r3);
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,
r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r0,r6);
TableEq(r3,r4); TableEq(r3,r5); TableEq(r3,r6);
 TableEq(r4,r5); TableEq(r4,r6); TableEq(r5,r6);
Unique(r3,a4);Unique(r3,a5)|
rule 326:
Exists(Exists(Agg_sum < a4 a5 r4 e2 a6 r5 > (Input < r3
>),Union(Input<r6>,Input<r7>)),Input<r8>)|
Filter < e1 a3 > (Agg_sum < a0 a1 r1 e0 a2 r2 > (Input < r0
>))|
AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a6);
AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
\label{eq:predicateEq} PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,e2); TableEq(r0,e2
r3); TableEq(r0,r6); TableEq(r0,r7); TableEq(r0,r8);
TableEq(r3,r6); TableEq(r3,r7); TableEq(r3,r8);
TableEq(r6,r7); TableEq(r6,r8); TableEq(r7,r8);
Unique(r3,a4);Unique(r3,a5)|
```

rule 327:
Filter<e3 \_>(Agg<\_ a3 \_ e0 a4 r4 e1 a5 r5>(Input< r3>)); e0:=FuncCall<count>(a4); e3:=Sublink<EXISTS
Filter<e4 \_>(Union\_all<>(Input<r6>,Input<r7>))>; e4
:=Sublink<EXISTS Input<r8>>|
Agg<\_ a5 \_ e2 a3 r1 e1 a5 r2>(Input<r3>); e2:=
FuncCall<count>(a3)|
AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);
NotNull(r3,a3); NotNull(r3,a4); TableEq(r3,r6);
TableEq(r3,r7); TableEq(r3,r8); TableEq(r6,r7);
TableEq(r6,r8); TableEq(r7,r8); Unique(r3,a3); Unique(r3,a4)|

```
rule 328:

Filter<e3 _>(Agg<_ a3 _ e0 a4 r4 e1 a5 r5>(Input< r3>)); e0:=FuncCall<count>(a4); e3:=Sublink<EXISTS

Filter<e4 _>(Input<r6>)>; e4:=Sublink<EXISTS

Union_all<>(Input<r7>, Input<r8>)>|

Agg<_ a3 _ e2 a5 r1 e1 a5 r2>(Input<r3>); e2:=
FuncCall<count>(a5)|

AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);

NotNull(r3,a3); NotNull(r3,a4); TableEq(r3,r6);

TableEq(r3,r7); TableEq(r3,r8); TableEq(r6,r7);

TableEq(r6,r8); TableEq(r7,r8); Unique(r3,a3); Unique(r3,a4)|
```

```
rule 329:

Exists(Agg_count<a3 a4 r4 e1 a5 r5>(Input<r3>),

Union(Input<r6>,Exists(Input<r7>,Input<r8>)))|

Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|

AttrsEq(a0,a3);AttrsEq(a1,a3);AttrsEq(a2,a5);

AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);

NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);

TableEq(r0,r3);TableEq(r0,r6);TableEq(r0,r7);

TableEq(r0,r8);TableEq(r3,r6);TableEq(r3,r7);

TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 330:
Filter<e3 _>(Filter<e4 _>(Agg<_ a4 _ e0 a5 r4 e2
a6 r5>(Input<r3>)));e0:=FuncCall<avg>(a5);e3:=
Sublink<EXISTS Union_all<>(Input<r7>, Input<r8>)>;
e4:=Sublink<EXISTS Input<r6>>|
Filter<e2 a4>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input<
r3>));e1:=FuncCall<avg>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r3,r8);TableEq(r6,r7);
TableEq(r6,r8);TableEq(r7,r8);Unique(r3,a4);Unique(r3,a5)|
```

rule 331:
Filter<e3 \_>(Agg<\_ a4 \_ e0 a5 r4 e2 a6 r5>(Input< r3>));e0:=FuncCall<count>(a5);e3:=Sublink<EXISTS
Union\_all<>(Input<r6>,Input<r7>)>|
Filter<e2 a4>(Agg<\_ a6 \_ e1 a5 r1 e2 a6 r2>(Input< r3>));e1:=FuncCall<count>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r6,r7);Unique(r3,a4);Unique(r3,a5)|

```
rule 332:

Agg_count<a4 a5 r6 e2 a6 r7>(Exists(Input<r3>,
Union(Input<r4>,Input<r5>)))|

Filter<e1 a3>(Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>))|

AttrsEq(a0,a4); AttrsEq(a1,a4); AttrsEq(a2,a4);
AttrsEq(a3,a6); AttrsSub(a4,r3); AttrsSub(a5,r3);
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r3,r4);
TableEq(r3,r5); TableEq(r4,r5); Unique(r3,a4); Unique(r3,a5)|
```

```
rule 333:
filter<e3 _>(Agg<_ a3 _ e0 a4 r5 e1 a5 r6>(Filter<
e4 _>(Input<r3>)));e0:=FuncCall<count>(a4);e3:=
Sublink<EXISTS Union_all<>(Input<r7>,Input<r8>)>;
e4:=Sublink<EXISTS Input<r4>>|
Agg<_ a5 _ e2 a3 r1 e1 a3 r2>(Input<r3>);e2:=
FuncCall<count>(a3)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r4);
TableEq(r3,r7);TableEq(r3,r8);TableEq(r4,r7);
TableEq(r4,r8);TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 334:

Agg_count < a3 a4 r7 e1 a5 r8 > (Exists (Exists (Input < r3 > ,Input < r4 > ),Union (Input < r5 > ,Input < r6 > ))) |

Agg_count < a0 a1 r1 e0 a2 r2 > (Input < r0 > ) |

AttrsEq(a0,a3); AttrsEq(a1,a3); AttrsEq(a2,a5);

AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);

NotNull(r3,a3); NotNull(r3,a4); PredicateEq(e0,e1);

TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r5);

TableEq(r0,r6); TableEq(r3,r4); TableEq(r3,r5);

TableEq(r3,r6); TableEq(r4,r5); TableEq(r4,r6);

TableEq(r5,r6); Unique(r3,a3); Unique(r3,a4) |
```

```
rule 335:

Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Union_all<>(Filter< e3 _>(Input<r3>),Input<r6>));e0:=FuncCall<avg>(a5);e3:=Sublink<EXISTS Filter<e4 _>(Input<r4>)>;e4:= Sublink<EXISTS Input<r5>|

Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a6 r2>(Input< r3>));e1:=FuncCall<avg>(a5)|

AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 336:
Agg_count<a4 a5 r7 e1 a6 r8>(Proj_simple<_ a3 r6>(
Exists(Input<r3>,Union(Input<r4>,Input<r5>))))|
Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|
AttrsEq(a0,a6);AttrsEq(a1,a4);AttrsEq(a2,a6);
AttrsSub(a3,r3);AttrsSub(a4,r6);AttrsSub(a5,r6);
AttrsSub(a6,a4);NotNull(r3,a3);PredicateEq(e0,e1);
TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);
TableEq(r3,r4);TableEq(r3,r5);TableEq(r4,r5);
Unique(r3,a3)|
```

```
rule 337:

Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Union_all<>(Input<r3 >,Union_all<>(Input<r4>,Filter<e3 _>(Input<r5>))));
;e0:=FuncCall<avg>(a5);e3:=Sublink<EXISTS Input<r6 >>|

Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a4 r2>(Input< r3>));e1:=FuncCall<avg>(a5)|

AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);

NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);

TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);

TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 338:

2 Exists(Agg_sum<a4 a5 r5 e2 a6 r6>(Exists(Input<r3 >,Input<r4>)),Union(Input<r7>,Input<r8>))|

3 Filter<e1 a3>(Agg_sum<a0 a1 r1 e0 a2 r2>(Input<r0 >))|

4 AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a4);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r7);TableEq(r0,r8);
TableEq(r3,r4);TableEq(r4,r8);TableEq(r7,r8);
Unique(r3,a4);Unique(r3,a5)|
```

```
rule 339:

Agg_average<a4 a5 r6 e2 a6 r7>(Union(Input<r3>,
Union(Input<r4>,Input<r5>)))|

Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input</br>
```

```
AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a4); AttrsEq(a3,a6); AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5); PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r3,r4); TableEq(r3,r5); TableEq(r4,r5); Unique(r3,a4); Unique(r3,a5)|
```

```
rule 340:
Agg<_ a4 _ e0 a5 r7 e2 a6 r8>(Union_all<>(
Union_all<>(Input<r3>,Input<r4>),Filter<e3 _>(
Input<r5>)));e0:=FuncCall<avg>(a5);e3:=Sublink<
EXISTS Input<r6>|
Filter<e2 a6>(Agg<_ a4 _ e1 a5 r1 e2 a4 r2>(Input<r3>));e1:=FuncCall<avg>(a5)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 341:
rule 341:
Filter<e3 _>(Agg<_ a3 _ e0 a4 r4 e1 a5 r5>(Input<
r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS
Filter<e4 _>(Input<r6>)>;e4:=Sublink<EXISTS
Union_all<>(Input<r7>,Input<r8>)>|
Agg<_ a5 _ e2 a3 r1 e1 a5 r2>(Input<r3>);e2:=
FuncCall<count>(a3)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r3,r8);TableEq(r6,r7);
TableEq(r6,r8);TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 342:

Agg_count<a3 a4 r7 e1 a5 r8>(Exists(Input<r3>,
Union(Exists(Input<r4>,Input<r5>),Input<r6>)))|

Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|

AttrsEq(a0,a5);AttrsEq(a1,a3);AttrsEq(a2,a3);

AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);

NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);

TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);

TableEq(r0,r6);TableEq(r3,r4);TableEq(r3,r5);

TableEq(r3,r6);TableEq(r4,r5);TableEq(r4,r6);

TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 343:

Exists(Agg_count<a3 a4 r4 e1 a5 r5>(Input<r3>),
Exists(Union(Input<r6>,Input<r7>),Input<r8>))|
Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|
AttrsEq(a0,a3);AttrsEq(a1,a3);AttrsEq(a2,a5);
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);
TableEq(r0,r3);TableEq(r0,r6);TableEq(r0,r7);
TableEq(r0,r8);TableEq(r3,r6);TableEq(r3,r7);
TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 344:
Exists(Agg_sum<a4 a5 r4 e2 a6 r5>(Input<r3>),Union
(Union(Input<r6>,Input<r7>),Input<r8>))|
```

```
Filter<e1 a3>(Agg_sum<a0 a1 r1 e0 a2 r2>(Input<r0
 >))|
 AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a4);
 AttrsEq(a3,a6); AttrsSub(a4,r3); AttrsSub(a5,r3);
 AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
 PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,
 r3); TableEq(r0,r6); TableEq(r0,r7); TableEq(r0,r8);
 TableEq(r3,r6); TableEq(r3,r7); TableEq(r3,r8);
 TableEq(r6,r7); TableEq(r6,r8); TableEq(r7,r8);
 Unique(r3,a4); Unique(r3,a5)|
 rule 345:
 Agg_count < a3 a4 r7 e1 a5 r8 > (Exists (Exists (Input <
 r3>, Union(Input<r4>, Input<r5>)), Input<r6>))|
 Agg\_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|
 AttrsEq(a0,a3); AttrsEq(a1,a5); AttrsEq(a2,a5);
 AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);
 NotNull(r3,a3); NotNull(r3,a4); PredicateEq(e0,e1);
 TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r5);
 TableEq(r0,r6); TableEq(r3,r4); TableEq(r3,r5);
 TableEq(r3,r6); TableEq(r4,r5); TableEq(r4,r6);
 TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
 rule 346:
 Agg_average < a4 a5 r6 e2 a6 r7 > (Union(Input < r3 > ,
```

```
rule 346:
Agg_average<a4 a5 r6 e2 a6 r7>(Union(Input<r3>,
Union(Input<r4>,Input<r5>)))|
Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input
<r0>))|
AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a6);
AttrsEq(a3,a4);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r4,r5);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 347:

Exists(Agg_max < a3 a4 r5 e1 a5 r6 > (Union(Input < r3 > ,
Input < r4 > )), Union(Input < r7 > ,Input < r8 > )) |

Agg_max < a0 a1 r1 e0 a2 r2 > (Input < r0 > ) |

AttrsEq(a0,a3); AttrsEq(a1,a4); AttrsEq(a2,a5);

AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);

NotNull(r3,a3); NotNull(r3,a4); PredicateEq(e0,e1);

TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r7);

TableEq(r0,r8); TableEq(r3,r4); TableEq(r3,r7);

TableEq(r7,r8); Unique(r3,a3); Unique(r3,a4) |
```

```
rule 348:

Agg_average<a4 a5 r7 e2 a6 r8>(Union(Exists(Exists (Input<r3>,Input<r4>),Input<r5>),Input<r6>))|

Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input<r0>))|

AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a4);

AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);

AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);

PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r6);

TableEq(r3,r4);TableEq(r3,r5);TableEq(r3,r6);

TableEq(r4,r5);TableEq(r4,r6);TableEq(r5,r6);

Unique(r3,a4);Unique(r3,a5)|
```

```
rule 349:
Filter<e3 _>(Agg<_ a3 _ e0 a4 r6 e1 a5 r7>(Filter<
e4 _>(Input<r3>)));e0:=FuncCall<count>(a4);e3:=
Sublink<EXISTS Input<r8>>;e4:=Sublink<EXISTS
Union_all<>(Input<r4>,Input<r5>)>|
Agg<_ a3 _ e2 a3 r1 e1 a5 r2>(Input<r3>);e2:=
FuncCall<count>(a3)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r8);TableEq(r4,r5);
TableEq(r4,r8);TableEq(r5,r8);Unique(r3,a4)|
```

```
rule 350:
Agg<_ a3 _ e0 a4 r7 e1 a5 r8>(Filter<e3 _>(Input<
r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS
Union_all<>(Input<r4>,Filter<e4 _>(Input<r5>))>;e4
:=Sublink<EXISTS Input<r6>>|
Agg<_ a5 _ e2 a3 r1 e1 a5 r2>(Input<r3>);e2:=
FuncCall<count>(a3)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 351:
Filter<e3 _>(Agg<_ a3 _ e0 a4 r4 e1 a5 r5>(Input<
r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS
Union_all<>(Filter<e4 _>(Input<r6>),Input<r8>));e4
:=Sublink<EXISTS Input<r7>>|
Agg<_ a5 _ e2 a3 r1 e1 a3 r2>(Input<r3>);e2:=
FuncCall<count>(a3)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r3,r8);TableEq(r6,r7);
TableEq(r6,r8);TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 352:
Agg_count<a5 a6 r7 e2 a7 r8>(Exists(Proj_simple<_
    a4 r4>(Input<r3>),Union(Input<r5>,Input<r6>)))|
Filter<e1 a3>(Agg_count<a0 a1 r1 e0 a2 r2>(Input<
    r0>))|
AttrsEq(a0,a5);AttrsEq(a1,a5);AttrsEq(a2,a7);
AttrsEq(a3,a5);AttrsSub(a4,r3);AttrsSub(a5,r4);
AttrsSub(a6,r4);AttrsSub(a7,a5);NotNull(r3,a4);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r5);TableEq(r0,r6);TableEq(r3,r6);
```

```
rule 353:

Agg_average<a4 a5 r7 e2 a6 r8>(Union(Input<r3>,
Union(Exists(Input<r4>,Input<r5>),Input<r6>)))|

Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input<r0>))|
```

```
(Appendix)
AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a6);
AttrsEq(a3,a4); AttrsSub(a4,r3); AttrsSub(a5,r3);
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,
r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r0,r6);
TableEq(r3,r4); TableEq(r3,r5); TableEq(r3,r6);
TableEq(r4,r5); TableEq(r4,r6); TableEq(r5,r6);
Unique(r3,a4);Unique(r3,a5)|
rule 354:
Exists(Agg_sum < a4 a5 r4 e2 a6 r5>(Input < r3>), Union
(Input < r6 > , Input < r7 > )) |
Filter < e1 a3 > (Agg_sum < a0 a1 r1 e0 a2 r2 > (Input < r0
>))|
AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a6);
AttrsEq(a3,a4);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,
r3); TableEq(r0,r6); TableEq(r0,r7); TableEq(r3,r6);
TableEq(r3,r7); TableEq(r6,r7); Unique(r3,a4); Unique
(r3,a5)|
```

```
rule 355:

Exists(Agg_count<a3 a4 r4 e1 a5 r5>(Input<r3>),
Union(Input<r6>,Exists(Input<r7>, Input<r8>)))|

Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|

AttrsEq(a0,a3);AttrsEq(a1,a5);AttrsEq(a2,a5);
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);

NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);
TableEq(r0,r3);TableEq(r0,r6);TableEq(r0,r7);
TableEq(r0,r8);TableEq(r3,r6);TableEq(r3,r7);
TableEq(r3,r8);TableEq(r6,r7);TableEq(r6,r8);
TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 356:
Agg_average<a4 a5 r7 e2 a6 r8>(Union(Input<r3>,
Union(Union(Input<r4>,Input<r5>),Input<r6>)))|
Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input</r>
<rr>

4 AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a4);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r6);
TableEq(r4,r5);TableEq(r4,r6);TableEq(r5,r6);
Unique(r3,a4);Unique(r3,a5)|
```

```
rule 357:
Filter<e3 _>(Agg<_ a3 _ e0 a4 r4 e1 a5 r5>(Input<
r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS
Union_all<>(Input<r6>,Filter<e4 _>(Input<r7>)));e4
:=Sublink<EXISTS Input<r8>|
Agg<_ a5 _ e2 a3 r1 e1 a5 r2>(Input<r3>);e2:=
FuncCall<count>(a3)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r3,r8);TableEq(r6,r7);
TableEq(r6,r8);TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 358:

Agg_count<a4 a5 r7 e1 a6 r8>(Exists(Proj_simple<_ a3 r4>(Input<r3>),Union(Input<r5>,Input<r6>)))|

Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|

AttrsEq(a0,a4);AttrsEq(a1,a4);AttrsEq(a2,a6);

AttrsSub(a3,r3);AttrsSub(a4,r4);AttrsSub(a5,r4);

AttrsSub(a6,a4);NotNull(r3,a3);PredicateEq(e0,e1);

TableEq(r0,r3);TableEq(r0,r5);TableEq(r0,r6);

TableEq(r3,r5);TableEq(r3,r6);TableEq(r5,r6);

Unique(r3,a3)|
```

```
rule 359:

Agg_average<a4 a5 r7 e2 a6 r8>(Exists(Union(Exists (Input<r3>,Input<r5>),Input<r6>))|

Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input <r0>))|

AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a6);
AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);

PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r6);

TableEq(r3,r4);TableEq(r3,r5);TableEq(r3,r6);

TableEq(r4,r5);TableEq(r4,r6);TableEq(r5,r6);

Unique(r3,a4);Unique(r3,a5)|
```

```
rule 360:
Agg<_ a3 _ e0 a4 r7 e1 a5 r8>(Filter<e3 _>(Input< r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS
Union_all<>(Input<r4>,Filter<e4 _>(Input<r5>))>;e4
:=Sublink<EXISTS Input<r6>>|
Agg<_ a3 _ e2 a3 r1 e1 a3 r2>(Input<r3>);e2:=
FuncCall<count>(a3)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 361:
Agg_sum<a5 a6 r7 e2 a7 r8>(Exists(Proj_simple<_ a4 r4>(Input<r3>),Union(Input<r5>,Input<r6>)))|
Filter<e1 a3>(Agg_sum<a0 a1 r1 e0 a2 r2>(Input<r0 >))|
AttrsEq(a0,a5);AttrsEq(a1,a6);AttrsEq(a2,a5);
AttrsSub(a3,a7);AttrsSub(a4,r3);AttrsSub(a5,r4);
AttrsSub(a6,r4);AttrsSub(a7,a5);NotNull(r3,a4);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r5);TableEq(r0,r6);TableEq(r3,r6);
```

```
rule 362:
Filter<e3 _>(Agg<_ a3 _ e0 a4 r5 e1 a5 r6>(Filter<
e4 _>(Input<r3>)));e0:=FuncCall<count>(a4);e3:=
Sublink<EXISTS Union_all<>(Input<r7>,Input<r8>)>;
e4:=Sublink<EXISTS Input<r4>>|
Agg<_ a5 _ e2 a3 r1 e1 a5 r2>(Input<r3>);e2:=
FuncCall<count>(a3)|
```

```
rule 363:
Exists(Exists(Agg_count<a3 a4 r4 e1 a5 r5>(Input<r3>),Input<r6>),Union(Input<r7>,Input<r8>))|
Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|
AttrsEq(a0,a3);AttrsEq(a1,a3);AttrsEq(a2,a5);
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);
TableEq(r0,r3);TableEq(r0,r6);TableEq(r0,r7);
TableEq(r0,r8);TableEq(r3,r6);TableEq(r6,r8);
```

AttrsSub(a3,r3); AttrsSub(a4,r3); AttrsSub(a5,a3);

TableEq(r4,r8); TableEq(r7,r8); Unique(r3,a3); Unique

NotNull(r3,a3); NotNull(r3,a4); TableEq(r3,r4);

TableEq(r3,r7); TableEq(r3,r8); TableEq(r4,r7);

```
rule 364:
Filter<e3 _>(Agg<_ a4 _ e0 a5 r4 e2 a6 r5>(Input< r3>));e0:=FuncCall<count>(a5);e3:=Sublink<EXISTS
Union_all<>(Input<r6>,Input<r7>)>|
Filter<e2 a6>(Agg<_ a4 _ e1 a4 r1 e2 a4 r2>(Input< r3>));e1:=FuncCall<count>(a4)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r6,r7);Unique(r3,a4);Unique(r3,a5)|
```

TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|

```
rule 365:
Filter<e3 _>(Agg<_ a4 _ e0 a5 r5 e1 a6 r6>(Proj< ea3 a3 r4>(Input<r3>)));e0:=FuncCall<count>(a5);e3 :=Sublink<EXISTS Union_all<>(Input<r7>,Input<r8>)>|
Agg<_ a4 _ e2 a4 r1 e1 a6 r2>(Input<r3>);e2:= FuncCall<count>(a4)|
AttrsSub(a3,r3);AttrsSub(a4,r4);AttrsSub(a5,r4);
AttrsSub(a6,a4);NotNull(r3,a3);TableEq(r3,r7);
TableEq(r3,r8);TableEq(r7,r8);Unique(r3,a3)|
```

```
rule 366:
Agg<_ a4 _ e0 a5 r6 e2 a6 r7>(Filter<e3 _>(Input<
r3>));e0:=FuncCall<count>(a5);e3:=Sublink<EXISTS
Union_all<>(Input<r4>,Input<r5>)>|
Filter<e2 a6>(Agg<_ a6 _ e1 a6 r1 e2 a4 r2>(Input<
r3>));e1:=FuncCall<count>(a6)|
AttrsSub(a4,r3);AttrsSub(a5,r3);AttrsSub(a6,a4);
NotNull(r3,a4);NotNull(r3,a5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r4,r5);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 367:

Agg_count<a4 a5 r6 e2 a6 r7>(Exists(Input<r3>,
Union(Input<r4>,Input<r5>)))|

Filter<e1 a3>(Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>))|
```

```
AttrsEq(a0,a6); AttrsEq(a1,a4); AttrsEq(a2,a4); AttrsEq(a3,a6); AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5); PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r3,r4); TableEq(r3,r5); TableEq(r4,r5); Unique(r3,a4); Unique(r3,a5)|
```

```
rule 368:
Filter<e3 _>(Agg<_ a3 _ e0 a4 r4 e1 a5 r5>(Input<
r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS
Union_all<>(Input<r6>,Filter<e4 _>(Input<r7>))>;e4
:=Sublink<EXISTS Input<r8>|
Agg<_ a3 _ e2 a3 r1 e1 a3 r2>(Input<r3>);e2:=
FuncCall<count>(a3)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r3,r8);TableEq(r6,r7);
TableEq(r6,r8);TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 369:
Agg<_ a3 _ e0 a4 r7 e1 a5 r8>(Filter<e3 _>(Input<
r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS
Union_all<>(Filter<e4 _>(Input<r4>),Input<r6>)>;e4
:=Sublink<EXISTS Input<r5>>|
Agg<_ a3 _ e2 a3 r1 e1 a5 r2>(Input<r3>);e2:=
FuncCall<count>(a3)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r3,r6);TableEq(r4,r5);
TableEq(r4,r6);TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 370:

Agg_count<a3 a4 r7 e1 a5 r8>(Exists(Input<r3>,
Union(Input<r4>,Exists(Input<r5>,Input<r6>))))|

Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|

AttrsEq(a0,a3);AttrsEq(a1,a3);AttrsEq(a2,a3);

AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);

NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);

TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);

TableEq(r0,r6);TableEq(r3,r4);TableEq(r4,r6);

TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 371:

Agg_average<a4 a5 r7 e2 a6 r8>(Union(Exists(Input<r3>,Union(Input<r4>,Input<r5>)),Input<r6>))|

Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input<r0>))|

AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a4);

AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);

AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);

PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r6);

TableEq(r3,r4);TableEq(r3,r5);TableEq(r3,r6);

TableEq(r4,r5);TableEq(r4,r6);TableEq(r5,r6);

Unique(r3,a4);Unique(r3,a5)|
```

```
rule 372:
Agg_count<a4 a5 r6 e2 a6 r7>(Exists(Input<r3>,
Union(Input<r4>,Input<r5>)))|
Filter<e1 a3>(Agg_count<a0 a1 r1 e0 a2 r2>(Input<
r0>))|
AttrsEq(a0,a6);AttrsEq(a1,a5);AttrsEq(a2,a4);
AttrsSub(a6,a4);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r3,r4);
TableEq(r3,r5);TableEq(r4,r5);Unique(r3,a4);Unique(r3,a5)|
```

```
rule 373:

Agg_average<a4 a5 r7 e2 a6 r8>(Union(Input<r3>,
Union(Input<r4>,Exists(Input<r5>,Input<r6>))))|

Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input</ri>
<r0>))|

AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a4);
AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r6);
TableEq(r3,r4);TableEq(r3,r5);TableEq(r3,r6);
TableEq(r4,r5);TableEq(r4,r6);TableEq(r5,r6);
Unique(r3,a4);Unique(r3,a5)|
```

```
rule 374:
Exists(Exists(Agg_sum<a4 a5 r4 e2 a6 r5>(Input<r3 >),Input<r6>),Union(Input<r7>,Input<r8>))|
Filter<e1 a3>(Agg_sum<a0 a1 r1 e0 a2 r2>(Input<r0 >))|
AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a4);
AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r6);TableEq(r0,r7);TableEq(r0,r8);
TableEq(r3,r6);TableEq(r3,r7);TableEq(r3,r8);
TableEq(r6,r7);TableEq(r6,r8);TableEq(r7,r8);
Unique(r3,a4);Unique(r3,a5)|
```

```
rule 375:
Exists(Agg_sum<a4 a5 r6 e2 a6 r7>(Exists(Input<r3 >,Union(Input<r4>,Input<r5>))),Input<r8>)|
Filter<e1 a3>(Agg_sum<a0 a1 r1 e0 a2 r2>(Input<r0 >))|
AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a6);
AttrsEq(a3,a6);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r8);
TableEq(r3,r4);TableEq(r3,r5);TableEq(r3,r8);
Unique(r3,a4);Unique(r3,a5)|
```

```
rule 376:
2 Agg_average<a4 a5 r7 e2 a6 r8>(Union(Input<r3>,
Union(Input<r4>,Exists(Input<r5>,Input<r6>))))|
3 Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input<r0>)))|
```

```
AttrsEq(a0,a4); AttrsEq(a1,a5); AttrsEq(a2,a6); AttrsEq(a3,a6); AttrsSub(a4,r3); AttrsSub(a5,r3); AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5); PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r0,r6); TableEq(r3,r4); TableEq(r3,r5); TableEq(r3,r6); TableEq(r4,r5); TableEq(r4,r6); TableEq(r3,a4); Unique(r3,a5)|
```

```
rule 377:

Agg_count<a4 a5 r6 e2 a6 r7>(Exists(Input<r3>,
Union(Input<r4>,Input<r5>)))|

Filter<e1 a3>(Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>))|

AttrsEq(a0,a6); AttrsEq(a1,a4); AttrsEq(a2,a6);
AttrsEq(a3,a4); AttrsSub(a4,r3); AttrsSub(a5,r3);
AttrsSub(a6,a4); NotNull(r3,a4); NotNull(r3,a5);
PredicateEq(e0,e2); PredicateEq(e1,e2); TableEq(r0,r3); TableEq(r0,r4); TableEq(r0,r5); TableEq(r3,r4);
TableEq(r3,r5); TableEq(r4,r5); Unique(r3,a4); Unique(r3,a5)|
```

```
rule 378:
Filter<e3 _>(Agg<_ a3 _ e0 a4 r4 e1 a5 r5>(Input< r3>));e0:=FuncCall<count>(a4);e3:=Sublink<EXISTS
Filter<e4 _>(Input<r6>)>;e4:=Sublink<EXISTS
Union_all<>(Input<r7>,Input<r8>)>|
Agg<_ a3 _ e2 a3 r1 e1 a3 r2>(Input<r3>);e2:=
FuncCall<count>(a3)|
AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);
NotNull(r3,a3);NotNull(r3,a4);TableEq(r3,r6);
TableEq(r3,r7);TableEq(r3,r8);TableEq(r6,r7);
TableEq(r6,r8);TableEq(r7,r8);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 379:
Agg_average<a4 a5 r7 e2 a6 r8>(Union(Union(Input< r3>,Union(Input<r4>,Input<r5>)),Input<r6>))|
Filter<e1 a3>(Agg_average<a0 a1 r1 e0 a2 r2>(Input< r0>))|
AttrsEq(a0,a4);AttrsEq(a1,a5);AttrsEq(a2,a6);
AttrsEq(a3,a4);AttrsSub(a4,r3);AttrsSub(a5,r3);
AttrsSub(a6,a4);NotNull(r3,a4);NotNull(r3,a5);
PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);TableEq(r0,r6);
TableEq(r3,r4);TableEq(r3,r5);TableEq(r3,r6);
TableEq(r4,r5);TableEq(r4,r6);TableEq(r5,r6);
Unique(r3,a4);Unique(r3,a5)|
```

```
rule 380:

Agg_count<a3 a4 r7 e1 a5 r8>(Exists(Exists(Input< r3>,Input<r4>),Union(Input<r5>,Input<r6>)))|

Agg_count<a0 a1 r1 e0 a2 r2>(Input<r0>)|

AttrsEq(a0,a3);AttrsEq(a1,a3);AttrsEq(a2,a3);

AttrsSub(a3,r3);AttrsSub(a4,r3);AttrsSub(a5,a3);

NotNull(r3,a3);NotNull(r3,a4);PredicateEq(e0,e1);

TableEq(r0,r3);TableEq(r0,r4);TableEq(r0,r5);

TableEq(r0,r6);TableEq(r3,r4);TableEq(r4,r6);

TableEq(r5,r6);Unique(r3,a3);Unique(r3,a4)|
```

```
rule 381:
Exists(Agg_count<a4 a5 r4 e2 a6 r5>(Input<r3>),
Input<r6>)|
Filter<e1 a3>(Agg_count<a0 a1 r1 e0 a2 r2>(Input<
r0>))|
AttrsEq(a0,a6);AttrsEq(a1,a6);AttrsEq(a2,a6);
AttrsEq(a3,a6);AttrsEq(a4,a5);AttrsSub(a4,r3);
AttrsSub(a5,r3);AttrsSub(a6,a4);NotNull(r3,a4);
NotNull(r3,a5);PredicateEq(e0,e2);PredicateEq(e1,e2);TableEq(r0,r3);TableEq(r0,r6);TableEq(r3,r6);
Unique(r3,a4);Unique(r3,a5)
```

### **REFERENCES**

[1] Haoran Ding, Zhaoguo Wang, Yicun Yang, Dexin Zhang, Zhenglin Xu, Haibo Chen, Ruzica Piskac, and Jinyang Li. 2023. Proving query equivalence using linear integer arithmetic. *Proceedings of the ACM on Management of Data* 1, 4 (2023), 1–26