1. 已知今日有雨明日也有雨的概率为 0.7。今日无雨明日有雨的概率为 0.5。
可得转移矩阵

	明日无雨	明日有雨
今日无雨	0.5	0.5
今日有雨	0.3	0.7

要求星期一有雨，星期三也有雨的概率，即求$P_{1,1}^2$, 计算可得$P_{1,1}^2 = 0.64$

2. 由题意可知转移矩阵为状态为 $0 \sim n$ 的 $n \times n$ 矩阵。且有 $p_{0,0} = 1, p_{n,n-1} = 1$。转移矩阵描述如下:

$$
\begin{array}{cccccccc}
1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
p & 0 & 1-p & 0 & \cdots & 0 & 0 & 0 \\
0 & p & 0 & 1-p & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & p & 0 & 1-p \\
0 & 0 & 0 & 0 & \cdots & 0 & 1 & 0
\end{array}
$$

令 A_0 表示最终落入状态 0 的事件，记 $q_{i,0} = \Pr(A_0|X_1 = i)$, 当 $i = 0$ 时，$q_{0,0} = 1$

$$q_{i,0} = \Pr(A_0|x_1 = i) = \sum_{j=0}^{n} \Pr(A_0|x_1 = i, x_2 = j) \cdot \Pr(x_2 = j|x_1 = i) = \sum_{j=0}^{n} \Pr(A_0|x_1 = j) \cdot p_{i,j}$$

该方程组表示为如下的矩阵

$$
\begin{array}{ccccccccc}
1 & p-1 & 0 & 0 & \cdots & 0 & 0 & 0 & p \\
-p & 1 & p-1 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & -p & 1 & p-1 & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & -p & 1 & p-1 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & -1 & 1 & 0
\end{array}
$$

解得 $q_{n,0} = q_{n-1,0} = q_{n-2,0} = \cdots = q_{1,0}$
代入 $q_{1,0} + (p-1)q_{2,0} = p$, 解得 $q_{n,0} = q_{n-1,0} = q_{n-2,0} = \cdots = q_{1,0} = 1$
$\Pr(A_0) = \sum_{i=1}^{n} \Pr(A_0|x_1 = i) p(x_1 = i) = \frac{1}{n} \sum_{i=1}^{n} \Pr(A_0|x_1 = i) = 1$
所以蚂蚁被吃掉的概率为 1

3. 已知蜘蛛起始位置在 0，蜘蛛的转移矩阵为 $\begin{smallmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{smallmatrix}$

$$p_{0,0}^{(n)} = \sum_i p_{0,i}^{(1)} p_{i,0}^{(n-1)} = \sum_i p_{0,i} (\sum_j p_{i,j}^{(1)} p_{j,0}^{(n-2)}) = \sum_i \sum_j p_{0,i} p_{i,j} p_{j,0}^{(n-2)} = \cdots = \sum \cdots \sum p_{0,i} p_{i,j} \cdots p_{k,0}$$

$$= \sum_{k=0}^{\frac{n}{2}} C_n^{2k} (0.3)^{2k} \times (0.7)^{n-2k}$$

$$p_{0,1}^{(n)} = 1 - p_{0,0}^{(n)} = \sum_{k=1}^{\frac{n}{2}} C_n^{2k-1} (0.3)^{2k-1} \times (0.7)^{n-2k+1}$$

蚂蚁的起始位置在 1，蚂蚁的转移矩阵为 $\begin{smallmatrix} 0.3 & 0.7 \\ 0.7 & 0.3 \end{smallmatrix}$

$$p_{1,0} = \sum_{k=1}^{\frac{n}{2}} C_n^{2k-1} (0.7)^{2k-1} \times (0.3)^{n-2k+1}$$

$$p_{1,1} = \sum_{k=0}^{\frac{n}{2}} C_n^{2k} (0.7)^{2k} \times (0.3)^{n-2k}$$