1.

$$\begin{aligned} \min \quad & f(x) = 4x_1 - 3x_2 \\ \text{s.t.} \quad & -(\mathrm{x}_1 - 3)^2 + \mathrm{x}_2 + 1 \geq 0 \\ & 4 - x_1 - x_2 \geq 0 \\ & x_2 + 7 \geq 0 \end{aligned}$$

$$L(x_1, x_2, \mu_1, \mu_2, \mu_3) = 4x_1 - 3x_2 + \mu_1 \left[-(x_1 - 3)^2 + x_2 + 1\right] + \mu_2(4 - x_1 - x_2) + \mu_3(x_2 + 7)$$

$$\begin{aligned} \frac{\partial L}{\partial x_1} &= 4 - 2\mu_1(x_1 - 3) - \mu_2 = 0 \\ \frac{\partial L}{\partial x_2} &= -3 + \mu_1 - \mu_2 + \mu_3 = 0 \\ \mu_1 \left[-(x_1 - 3)^2 + x_2 + 1\right] &= 0 \\ \mu_2(4 - x_1 - x_2) &= 0 \\ \mu_3(x_2 + 7) &= 0 \\ \mu_1, \mu_2, \mu_3 &\geq 0 \end{aligned}$$

$$\mu_1 = \mu_2 = \mu_3 = 0 \rightarrow \text{不满足 KT 条件}$$

$$\mu_1 = \mu_2 = 0, x_2 + 7 = 0 \rightarrow \text{不满足 KT 条件}$$

$$\mu_1 = \mu_3 = 0, 4 - x_1 - x_2 = 0 \rightarrow \text{不满足 KT 条件}$$

$$\mu_1 = 0, 4 - x_1 - x_2 = 0, x_2 + 7 = 0 \rightarrow x_1 = 11, x_2 = -7, \mu_2 = 4, \mu_3 = 1$$

$$\mu_2 = \mu_3 = 0, \left[-(x_1 - 3)^2 + x_2 + 1\right] = 0 \rightarrow x_1 = \frac{7}{3}, x_2 = -\frac{5}{9}, \mu_1 = -3$$

$$\mu_2 = 0, \left[-(x_1 - 3)^2 + x_2 + 1\right] = 0, x_2 + 7 = 0 \rightarrow \text{不满足 KT 条件}$$

$$\mu_3 = 0, \left[-(x_1 - 3)^2 + x_2 + 1\right] = 0, 4 - x_1 - x_2 = 0 \rightarrow x_1 = 1, x_2 = 3, \mu_1 = -\frac{1}{3}, \mu_2 = \frac{8}{3}$$

$$x_1 = 4, x_2 = 0, \mu_1 = \frac{1}{3}, \mu_2 = \frac{10}{3}$$

$$\left[-(x_1 - 3)^2 + x_2 + 1\right] = 0, 4 - x_1 - x_2 = 0, x_2 + 7 = 0 \rightarrow \text{不满足 KT 条件}$$

因此结果是$x_1^* = 1, x_2^* = 3, f(x^*) = -5$

2.

$$\begin{aligned} \max \quad & f(x,y) = xy \\ \text{s.t.} \quad & \mathrm{x} + \mathrm{y}^2 \leq 2 \\ & x, y \geq 0 \end{aligned}$$

$$L(x,y,\mu_1,\mu_2,\mu_3) = xy + \mu_1(x+y^2-2) + \mu_2(-x) + \mu_3(-y)$$

$$\frac{\partial L}{\partial x} = y + \mu_1 - \mu_2 = 0$$

$$\frac{\partial L}{\partial y} = x + 2\mu_1 y - \mu_3 = 0$$

$$\mu_1(x+y^2-2) = 0$$

$$\mu_2 x = 0$$

$$\mu_3 y = 0$$

$$\mu_1, \mu_2, \mu_3 \geq 0$$

$$\mu_1 = \mu_2 = \mu_3 = 0 \rightarrow x = 0, y = 0$$

$$\mu_1 = \mu_2 = 0, y = 0 \rightarrow x = \mu_3, y = 0$$

$$\mu_1 = \mu_3 = 0, x = 0 \rightarrow x = 0, y = \mu_2$$

$$\mu_1 = 0, x = 0, y = 0 \rightarrow x = 0, y = 0, \mu_2 = 0, \mu_3 = 0$$

$$\mu_2 = \mu_3 = 0, (x+y^2-2) = 0 \rightarrow \text{不满足 KT 条件}$$

$$\mu_2 = 0, (x+y^2-2) = 0, y = 0 \rightarrow x = 2, y = 0, \mu_1 = 0, \mu_3 = 2$$

$$\mu_3 = 0, (x+y^2-2) = 0, x = 0 \rightarrow x = 0, y = \sqrt{2}, \mu_1 = 0, \mu_2 = \sqrt{2}$$

$$(x+y^2-2) = 0, x = 0, y = 0 \rightarrow \text{不满足 KT 条件}$$

综上满足 KT 条件的点有$(0,0), (0,\sqrt{2}), (2,0)$