1. Let X and Y be topological spaces. We consider in $C(X, Y)$ (with the compact-open topology) the subspace C of constant functions.

a) Prove that if X is locally compact and Hausdorff, then C is a retract of $C(X, Y)$.

b) Under the same hypotheses as the previous item, prove that if X is contractible, then C is a strong deformation retract of $C(X, Y)$.

2. Let $p : E \to B$ be a covering map, where B is path-connected and E is simply connected. Let $b_0 \in B$, $e_0 \in p^{-1}(b_0)$, and $f : B \to B$ be a continuous function with $f(b_0) = b_0$. Recall the pullback

$$E' = E \times_B B = \{(e, b) \in E \times B : p(e) = f(b)\},$$

given by the diagram:

$$\begin{tikzcd} E' \arrow[r, "\overline{f}"] \arrow[d, "p'"] & E \arrow[d, "p"] \\ B \arrow[r, "f"] & B. \end{tikzcd}$$

Prove that $p' : E' \to B$ is a covering map, and $p'_*(\pi_1(E', (e_0, b_0)))$ is the kernel of $f_* : \pi_1(B, b_0) \to \pi_1(B, b_0)$.