Let $f : \mathbb{N} \to \mathbb{N}$ be an injective function. The following relation R is defined on $\mathbb{N}$:

$$mRn \Leftrightarrow f(m) \mid f(n).$$

(a) Prove that R is a partial order relation.

(b) For the function $f : \mathbb{N} \to \mathbb{N}$ defined as $f(n) = 12n + 20$, characterize all $n \in \mathbb{N}$ such that $1Rn$.

Determine all the values of $n \in \mathbb{N}$ for which

$$n^{1021} \equiv 22 \pmod{55}$$

and

$$5 \mid n \cdot 2^n - 3n^5.$$