Let $\lfloor x \rfloor$ denote the largest integer not greater than $x$. Prove that for all positive integers $a_1, a_2, \ldots, a_n$, the following inequality holds:

$$\left\lfloor \frac{a_1^2}{a_2} \right\rfloor + \left\lfloor \frac{a_2^2}{a_3} \right\rfloor + \cdots + \left\lfloor \frac{a_n^2}{a_1} \right\rfloor \geq a_1 + a_2 + \cdots + a_n$$

Find all functions $f$ from the rational numbers to the rational numbers such that $f(x+y) + f(x-y) = 2f(x) + 2f(y)$ for all rational numbers $x$, $y$.