UNIVERSITY OF NEW SOUTH WALES

ACTL2111 - FINANCIAL MATHEMATICS FOR ACTUARIES

# 2111 Assignment 2025 T1

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# Context

Sharawata and Hamunyari have a 20% down payment for their first residential home with a value of $1,260,000. They will need to borrow $1,008,000 from their bank over a 30 year loan tenure. Initial rate is 5.97% convertible monthly for an effective annual rate of 6.14%, with an interest rate collar around floored at 2.5% per annum and capped at 7.5% per annum.

# Task 1

We are provided the discretised equation of the stochastic differential equation for mortgage rates and its respective parameters for the nominal rate $r_n$ (see Appendix A, B and C).

Using Excel, we can then simulate the nominal rates convertible monthly via the formula seen in Appendix A. IF statements were applied to verify if the nominal rates fell below the floor of 2.5% or exceeded the rate ceiling of 7.5%, and were replaced with the floor/ceiling itself instead if rates exceeded these provided bounds. Then the effective rates are simply these collared rates divided by 12 to find the effective monthly interest rate. Note that an overarching assumption made for this task and for the remainder of the report is that there are no leap years involved in any calculation process.

In doing so, we get the following graph:

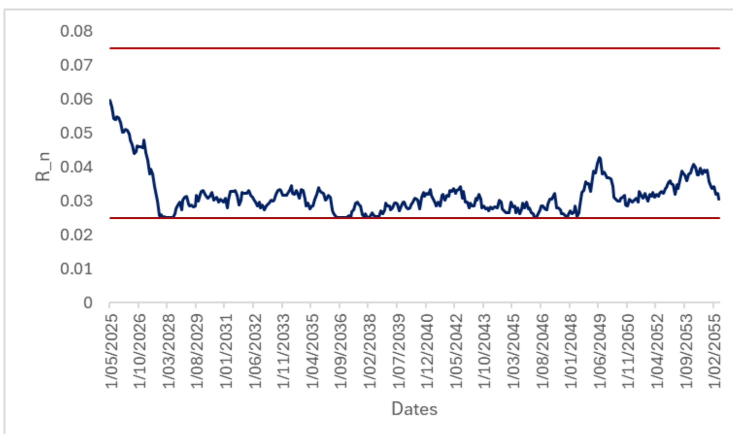



Figure 1: Rates Simulation

The CIR simulation (see Figure 1) displays an overall downwards trend for the first 3 years then tapers out to prolonged low interest rates. The rates frequently reach the floor in the simulations, but it is inferred from the CIR simulations that the rates were intended to mean-revert back to the low long-term rate of $\theta_r = 2.95\%$, with the average nominal rate of this specific simulation being $3.18\%$.

The rate ceiling was never reached, suggesting that the stochastic interest rate volatility parameter $\sigma_r$ was not extreme enough to induce such changes. The interest rates however, was floored on multiple observations (in this simulation there were 15 counts), suggesting that the collar significantly reduces volatility in interest payments and further supports the notion of stability in low long-term interest rates as stated earlier, which has beneficial implications for a retiring couple in this context.

# Task 2 & 3

A single mortgage calculator was developed to account for both Task 2 and 3 scenarios, where changes in payment frequencies, interest rates, tenures of loans and loan amounts were all dynamically accounted for.

The mortgage calculator relies on a similar table to that crafted in Task 1 as a reference point, where only nominal un-collared and collared rates are depicted. The effective periodic interest rates are shown in the actual mortgage calculator itself, dividing by the compounding frequency selected in the TOGGLE drop-down.

Effective periodic rates remained low throughout the loan repayment schedule. Payments remained consistent, but as the low interest rates cause the outstanding loan balance to accrue at a slower rate than the payments made, an increasingly larger proportion of payments go towards principal rather than interest as time progresses. The borrower also benefits from this collar as the rates do not aggressively vary and each interest accrual period is controlled by the effective rate of the collars, benefiting the couple in the form of 'cheap' debt.

Utilising a case of a 30 year loan, payable fortnightly, the payments stay consistent at around a range of $\$2,000 - \$2,100$. With the effective collar rate applied, repayments are stable even if rates are stochastic as the collars reduces its impact on the loan balance, which is favourable to Sharawata and Hamunyari due to its predictable repayment schedule.

Similar trends of consistency can be seen in daily and monthly frequencies of payment (see Appendix D and Appendix E). We can see that nominal payments experience a steep drop that then stabilises for the remainder of the graph, frequently paying at the lowest rate acceptable by the rate collars, again reaffirming the fact that the collars protect borrowers from interest rate volatility and that the set of rates $r_t$ eventually averages out to the a low long run mean; $\theta_r$ at speed $\kappa_r$.

# Task 4 & 5

We are again, provided the discretised version of the equation for instantaneous volatility $v_n$ and $S_n$, the house price at time $n$ (see Appendix F, G and H)

Using Excel and a macro that simulates the formula in Appendix F, we can conduct 1000 simulations and take the median price for the couple since age 30 up until the maximum attainable age of 95 to analyse the evolution of home equity.

Assumptions that were present in this section was the fact that we assume the couple to reach the maximum attainable age of 95 for calculation simplicity. Another implicit assumption made in this process is that the homes are perfectly liquid and can be sold immediately, with reference to our calculated median house prices.

We can then compare the trend of median house prices to the trend of the fortnightly withdrawals from retirement until death at age 95 to get:

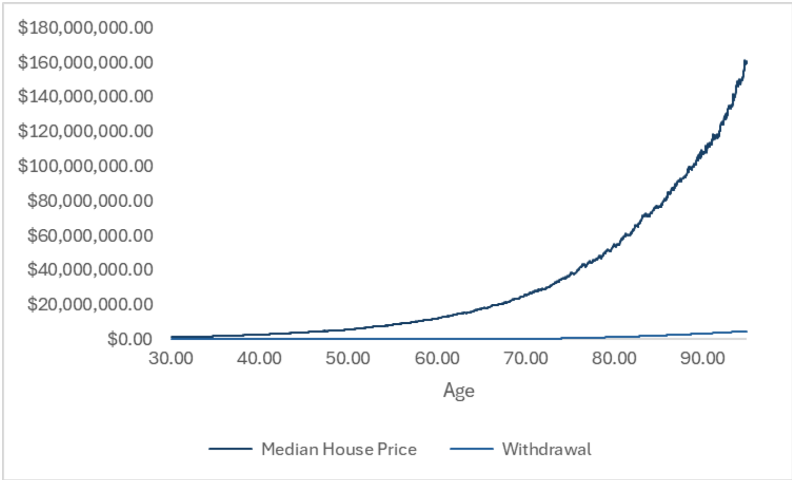



Figure 2: Evolution of Home Equity and Reverse Mortgage Process

The trend observed in Figure 2 is recorded again when comparing the mortgage reversal process with the 30% Equity Cap throughout the course of the couple's lifespan.

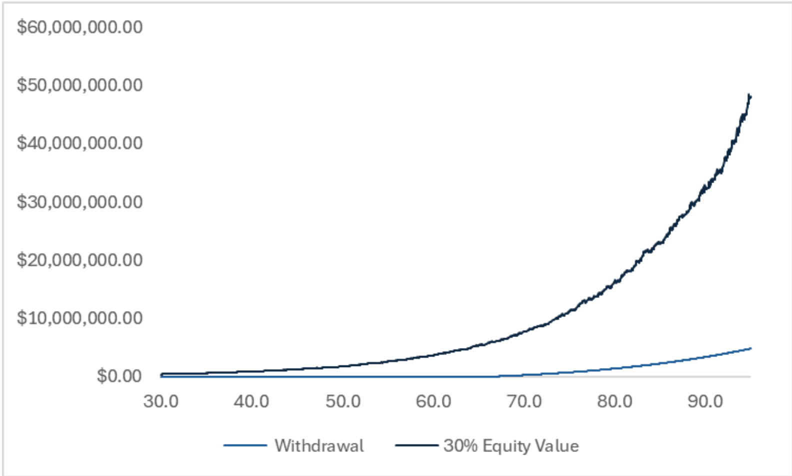



Figure 3: Equity Cap and Reverse Mortgage Trend

A trivial deduction that can be made from this dataset is that the 30% withdrawal equity cap grows proportionately with property values compared to a flat nominal rate of 3.95% per annum for interest. Figure 3 further suggests that the growth of withdrawals will never reach this exponential growth of property prices, hence for as long as the couple are alive, they are eligible to continue withdrawing equity from their property, in this simulation they have been modelled to have only taken out a total of 3% total equity by age 95 and are highly unlikely to have depleted 30% of total home equity in any simulation (see Appendix I for contrast).

The steady increase to a cumulative withdrawal amount of $4.7m is explained by the flat increase of withdrawing an extra $100 every time the couple gets older by a year. This approach suggests a conservative mortgage reversal strategy, which could be due to factors such as pacing withdrawals with a risk adverse mindset post-retirement.

Another key factor for this conservative mortgage reversal strategy from the couple would be their equity preservation motivations for their beneficiaries. Following 1000 Monte Carlo simulations, the expected home value excluding outliers at the time of the couple's death is $159,666,226. The cumulative withdrawal balance is $4,789,966.33 and through the provided assumptions, we deduce that after the bank receives the outstanding payments, that the equity remaining payable to beneficiaries is $154,876,259.81 (see Appendix J), reinforcing the view that the 30% equity cap will not be reached in this mortgage reversal strategy.

# Appendix

Appendix A:

$$r_{n+1} = r_n + \kappa_r(\theta_r - r_n)\Delta_n + \sigma_r\sqrt{r_n}\Delta W_1(n), \quad \text{for} \quad n = 0, 1, 2, \cdots, N-1$$

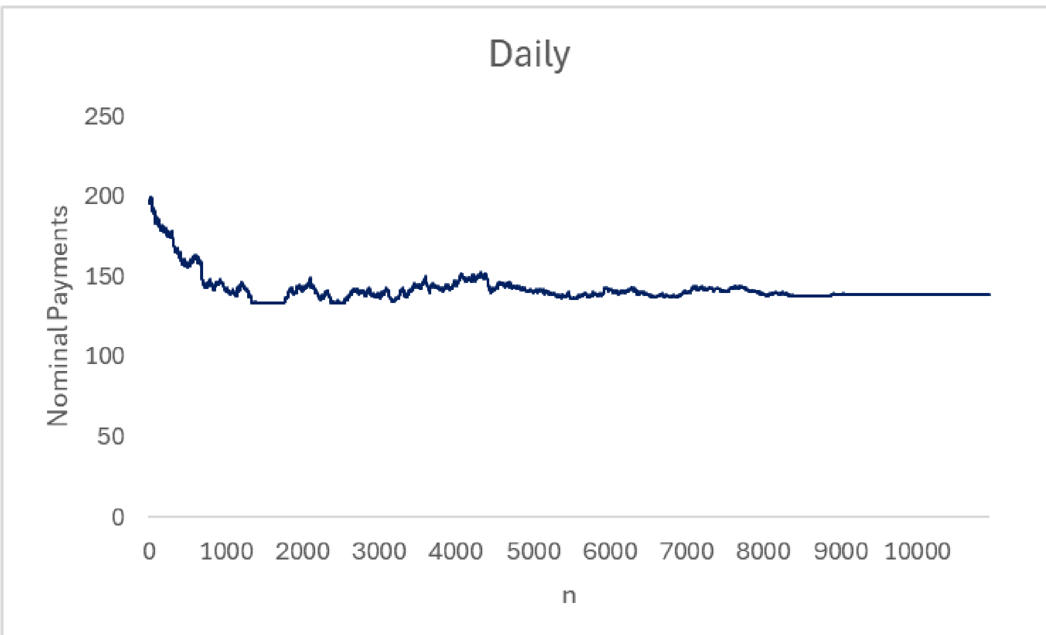
Appendix B:

$$\Delta W_1(n) = \sqrt{\Delta_n} \times \text{NORMINV}(\text{RAND}(), 0, 1)$$

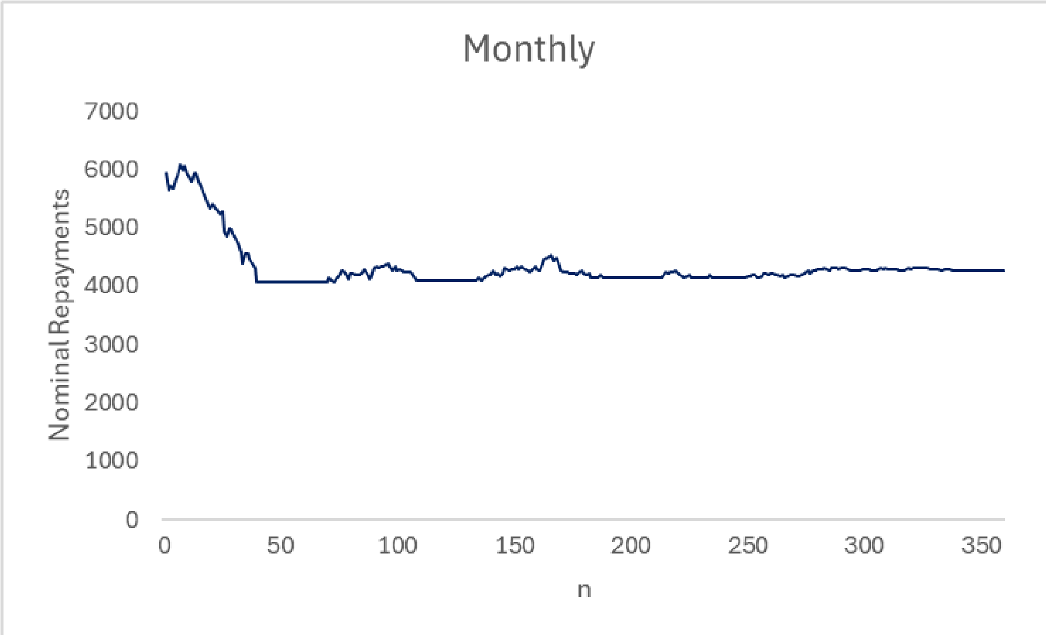
Appendix C:

| | $\Delta_n$ | $\theta_r$ | $r_0$ | $\kappa_r$ | $\sigma_r$ |
|---|---|---|---|---|---|
| Daily | 1/365 | 0.2956 | 0.059594 | 0.40569 | 0.02956 |
| Weekly | 1/52 | 0.029567 | 0.059623 | 0.40705 | 0.029567 |
| Fortnightly | 1/26 | 0.029576 | 0.059657 | 0.408643 | 0.029576 |
| Monthly | 1/12 | 0.029595 | 0.059737 | 0.412392998 | 0.029595 |
| Quarterly | 0.25 | 0.029668 | 0.060035 | 0.426727679 | 0.029668 |
| Semi-Annually | 0.5 | 0.029778 | 0.060485 | 0.449489743 | 0.029778 |
| Annually | 1 | 0.03 | 0.0614 | 0.5 | 0.02 |

Appendix D:



Appendix E:




Appendix F:

$$S_{n+1} = S_n + \mu S_n \Delta_n + \sqrt{v_{n+1}} S_n \Delta W_2(n)$$

Appendix G:

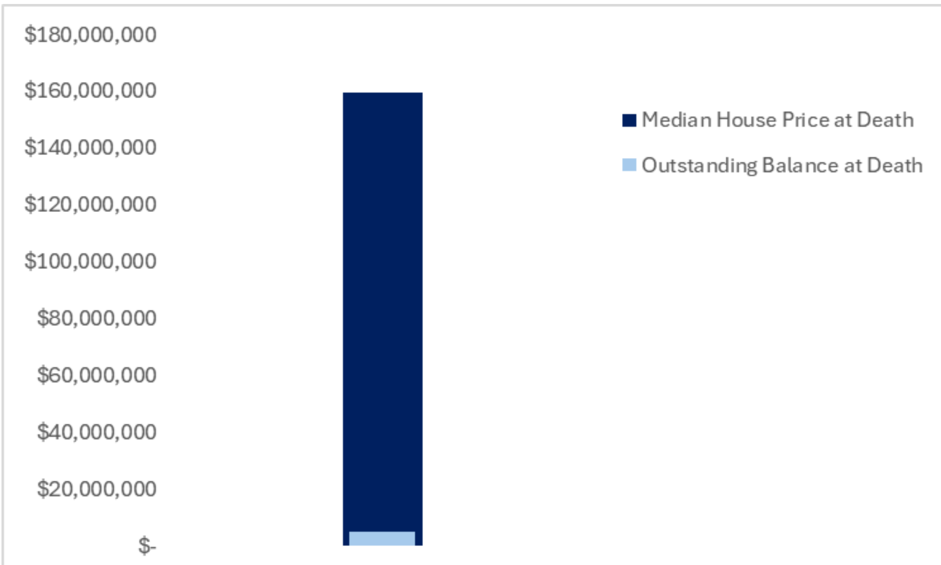
$$v_{n+1} = v_n + \kappa_v(\theta_v - v_n)\Delta_n + \rho\sigma_v\sqrt{v_n}\Delta W_2(n) + \sigma_v\sqrt{1-\rho^2}\sqrt{v_n}\Delta W_3(n)$$

Appendix H:

| | $\mu$ | $\theta_v$ | $v_0$ | $\kappa_v$ | $\sigma_v$ |
|---|---|---|---|---|---|
| Daily | 0.095322625 | 0.041144262 | 0.03922282 | 0.4056904 | 0.019803164 |
| Weekly | 0.09539758 | 0.041158223 | 0.039235508 | 0.407050013 | 0.019806398 |
| Fortnightly | 0.095485086 | 0.041174512 | 0.03925031 | 0.408643183 | 0.01981017 |
| Monthly | 0.095689685 | 0.041212551 | 0.039284877 | 0.412392998 | 0.019818976 |
| Quarterly | 0.096454756 | 0.041354253 | 0.039413626 | 0.426727679 | 0.019851726 |
| Semi-Annually | 0.097617696 | 0.041568025 | 0.039607805 | 0.449489743 | 0.019900988 |
| Annually | 0.10 | 0.042 | 0.04 | 0.5 | 0.02 |

Table 2: Constant parameters for the house price and variance processes. All rates are nominal per annum. For all compounding frequencies, $S_0 = \$1,260,000$ and $\rho = -0.5$

Appendix I:




Appendix J:

| | |
|---|---|
| **Median House Price at Death** | $ 159,666,226.14 |
| **Outstanding Balance at Death** | $ 4,789,966.33 |
| **Q5. Expected Credit to Beneficiary** | $ 154,876,259.81 |