Executive Summary

The following report will decide which Bonus-Malus scheme to implement for Fantastic Insurance with consideration to profitability and Loimaranta efficiency trade-offs. We first ran 2,000 one-year simulations for the entire 10,000 policy portfolio based off each policyholder's initial levels from the start of 2025.

- **Expected Total Premium 2026**: \$6,271,599
- **Expected Cost Incurred**: \$4,750,000
- **Expected Profit**: \$1,521,599

The simulation also projected the following distribution of policyholders in each of the discount levels at the beginning of year 2026 (rounded up to the nearest integer):

Level	-1	0	1	2	3
Count	5662	1904	1284	869	282

In regards to Loimaranta efficiency, all three different schemes display similar "positively skewed bell-shaped" profiles: $\eta(\lambda)$ (efficiency) all peak in the mid range (where $\lambda = 0.45$-0.55 claims/year) and tapers off in the extreme positive end. The complex scheme for young policy-holders exhibits the highest peak efficiency, at about 0.89 meaning it comes closest to charging high-risk young drivers in direct proportion to their risk while still rewarding the best risks with deep discounts.

A caveat to this would be that a taller Loimaranta peak suggests fairness and not necessarily greater revenue. The complex scheme also offers generous discounts to the majority of well-behaved drivers in the lower levels of the schemes. For the three options, the long-run profit was calculated as:

Scheme	**Long Run Premium**	**Long Run Cost**	**Long Run Profit**
Current 5-Level	\$5,621,715	\$4,750,000	\$871,715.5
Simplified 3-Level	\$6,848,793	\$4,750,000	\$2,098,793.4
Complex Y (7-Level)/ Current O (5-Level)	\$4,931,419	\$4,750,000	\$181,418.7

The recommendation would be to adopt the **simplified 3-level scheme** for 2026.

Pros

- Higher expected profits, roughly \$1.2m higher than the current 5-level scheme
- Operational simplicity
- Clear and easy to understand for policyholders

Cons

- Larger cross subsidy due to lower efficiency, careful policyholders overpay while high risk drivers underpay
- Competitor risk with companies that implement a discount system

1 Task 1

1.1 Probability Transition Matrices

We can find the probability transition matrix for both young and old policyholders under the current scheme by using the provided assumption that the number of claims reported by a policyholder follows a Poisson process ($\lambda_{\text{young}} = 0.22$(per year), $\lambda_{\text{old}} = 0.16$(per year)). Where the probability of 0, 1 and 2 or more claims are

$$\Pr(N_t = 0) = e^{-\lambda},\ \Pr(N_t = 1) = \lambda e^{-\lambda},\ \Pr(N_t \geq 2) = 1 - (1 + \lambda)e^{-\lambda}$$

respectively, where N_t denotes the number of claims reported by one policy-holder.

A function `Mat_og` for the current scheme was then created via R code primarily through the use of `ifelse` statements. There were specific cases that were more manual, such as when a policyholder reaches level 2 or 3 in the current scheme, then they could both reach the maximum penalty level 4 via either one claim or two claims, hence the two probabilities were added together and set as a separate case in `Mat_og`. We then obtain the following transition matrix for the current scheme (see Figure 1 and Figure 2 below).

	-1	0	1	2	3
-1	0.8521438	0.1363430	0.0115132	0.0000000	0.0000000
0	0.8521438	0.0000000	0.1363430	0.0115132	0.0000000
1	0.0000000	0.8521438	0.0000000	0.1363430	0.0115132
2	0.0000000	0.0000000	0.8521438	0.0000000	0.1478562
3	0.0000000	0.0000000	0.0000000	0.8521438	0.1478562

Figure 1: Current Scheme: Old ($\lambda = 0.16$)

	-1	0	1	2	3
-1	0.8025188	0.1765541	0.02092707	0.00000000	0.00000000
0	0.8025188	0.0000000	0.17655414	0.02092707	0.00000000
1	0.0000000	0.8025188	0.00000000	0.17655414	0.02092707
2	0.0000000	0.0000000	0.80251880	0.00000000	0.19748120
3	0.0000000	0.0000000	0.00000000	0.80251880	0.19748120

Figure 2: Current Scheme: Young ($\lambda = 0.22$)

Using similar methods, we attain the probability transition matrices for the simplified and the complex (young only) scheme. For the simplified scheme we attain the following matrices

	0	1	2
0	0.8521438	0.1363430	0.0115132
1	0.8521438	0.0000000	0.1478562
2	0.0000000	0.8521438	0.1478562

Figure 3: Simple Scheme: Old ($\lambda = 0.16$)

	0	1	2
0	0.8025188	0.1765541	0.02092707
1	0.8025188	0.0000000	0.19748120
2	0.0000000	0.8025188	0.19748120

Figure 4: Simple Scheme: Young ($\lambda = 0.22$)

and for the complex scheme applied to young policyholders only (old policyholders retain the current scheme) we get

	-2	-1	0	1	2	3	4
-2	0.8025188	0.1765541	0.02092707	0.00000000	0.00000000	0.00000000	0.00000000
-1	0.8025188	0.0000000	0.17655414	0.02092707	0.00000000	0.00000000	0.00000000
0	0.0000000	0.8025188	0.00000000	0.17655414	0.02092707	0.00000000	0.00000000
1	0.0000000	0.0000000	0.80251880	0.00000000	0.17655414	0.02092707	0.00000000
2	0.0000000	0.0000000	0.00000000	0.80251880	0.00000000	0.17655414	0.02092707
3	0.0000000	0.0000000	0.00000000	0.00000000	0.80251880	0.00000000	0.19748120
4	0.0000000	0.0000000	0.00000000	0.00000000	0.00000000	0.80251880	0.19748120

Figure 5: Complex Scheme (Young only).

1.2 Distribution, Simulation & Expected Profits (2026)

2,000 simulations were run to determine the number of policyholders in each of the levels at the beginning of year 2026 and the expected profit for the current scheme. A loop function was utilised to calculate the expected profit of the current scheme, within the loop there is a counter that tracks which level each policyholder ends up in, which accumulates in a five column vector.

After the loop is completed we get the following distribution of policyholders in each level (rounded up to an integer, which is why it may exceed the total quantity of 10,000 policyholders in the portfolio):

```
       -1         0         1         2         3
5661.8900 1903.7055 1283.8775  868.9125  281.6145
```

Figure 6: Expected Distribution of Policyholders (Current)

Level	-1	0	1	2	3
Count	5662	1904	1284	869	282

Figure 7: Rounded up Expected Distribution of Policyholders (Current)

The expected premium after 2,000 simulations summed to $6,271,599, with the mean cost incurred for the entire portfolio being $4,750,000, we arrive at an expected profit of $1,521,599. The distribution of the premiums from our 2,000 simulation can be viewed in Figure 8.

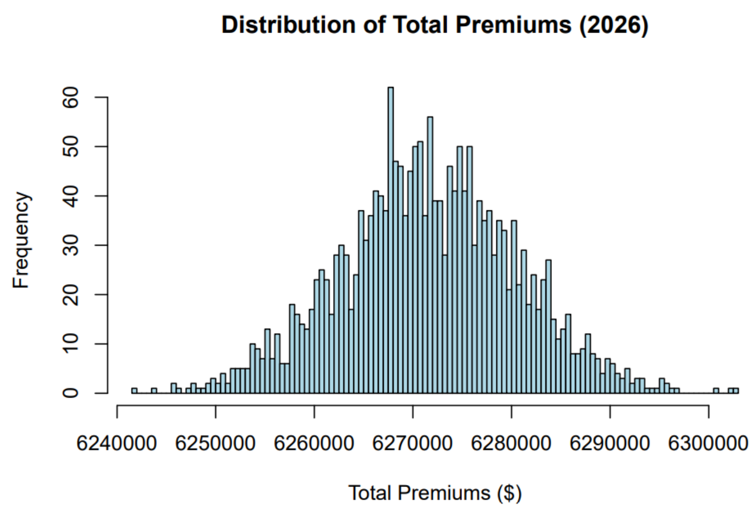

Figure 8: Simulated Premium Distribution

The distribution displayed minimal positive skew of 0.003 and a standard deviation 8910.31. It was noted that throughout multiple 2,000 simulations the mean total premium was consistently within the vicinity of $6,271,000, with a 95% confidence interval suggesting that the true premium mean is within [6271208, 6271989]. We can infer from this that profits would stay positive under almost all simulated outcomes.

2 Task 2

2.1 Loimaranta Efficiency

We can plot the Loimaranta Efficiency for each scheme by first finding the left eigenvectors corresponding to the eigen-value of 1 as

$$Av = \lambda v,$$

but what we require is

$$\pi P = \pi,$$

which is the equivalent of

$$P^T \pi^T = \pi^T,$$

where λ in this context is the eigenvalue 1. We then normalise each eigenvector by dividing each via the sum of all eigenvector elements to attain the limiting probabilities (as we satisfy the conditions of all states communicating in every scheme's transition matrices and each Markov chain is positive recurrent). After defining the discount and penalty factors we plotted each scheme's Loimaranta Efficiency and constructed a table on our specific $\eta(\lambda)$ for each scheme.

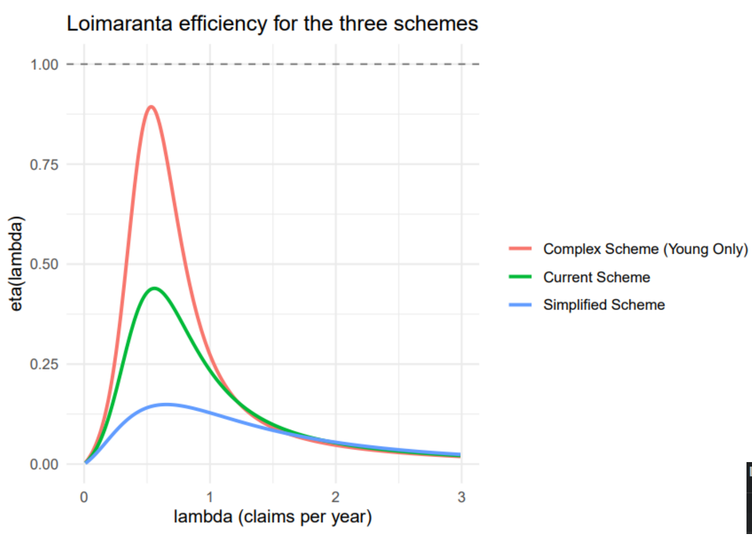

Figure 9: Loimanranta Efficiency Plot

Lambda <dbl>	Current <dbl>	Option_1 <dbl>	Option_2 <dbl>
0.16	0.082	0.049	0.112
0.22	0.141	0.072	0.203

Figure 10: $\eta(\lambda)$ Options

From Figure 9, it is noted that all schemes display similar positively skewed bell shape trends with the scheme exhibiting the highest efficiency response to claims being the complex scheme for young policyholders. From Figure 10 the scheme at the provided λ values achieves a $\eta(\lambda)$ of 0.112 and 0.203 for old and young policyholders respectively. This suggests that the most 'actuarially fair' and risk responsive scheme in general would be the Complex 7-level scheme, but it should be worth noting that the highest efficiency in premium response rates as λ rises does not imply the highest revenue. The current scheme sits at the mid-range, providing moderate deterrence but leaving some subsidy from risky policyholders. The simplified scheme collects only 5% extra premium for a 10% rise in claim frequency, so safer policyholders overpay to make up the shortfall on their riskier policyholders.

2.2 Long-Run Annual Profit & Recommendation

Utilising the limiting probabilities found earlier to find $P(\lambda)$ then scaling it up to the portfolio size and using the expected cost of \$4,750,000, we attain the following long-run profits in Figure 11.

Scheme <chr>	Premium <dbl>	Costs <dbl>	Profit <dbl>
Option 0 (Current Scheme)	5621715	4750000	871715.5
Option 1 (simplified 3-level)	6848793	4750000	2098793.4
Option 2 (complex Y / current O)	4931419	4750000	181418.7

Figure 11: Long Run Profits

The complex scheme yields the lowest profit because its transition probabilities keep most drivers/quickly returns them to the high-discount levels, shrinking premium income. Hence, the recommendation for the 2026 Bonus-Malus scheme that should be implemented is **Option 1**, the simplified 3-level scheme.

Pros: Premiums never drop below base, so long-run revenue climbs to \$6.85m, about \$1.2 m more than the current scheme without changing claim costs. With only three levels the design is simpler to administer and model, lowering call-centre queries and miscoding risk as opposed to a seven-level model like in the complex scheme for young drivers.

Cons: With a peak Loimaranta of just 0.05–0.07 at the provied lambdas, premiums lag behind risk, so low-risk policyholders subsidise riskier ones which over time may discourage good behaviour and shrink retention of profitable customers. This raises further consideration on Fantastic Insurance's emphasis on the trade-off between actuarial fairness and long term profitability. If competitors introduce more attractive discount ladders into the market, those same low-risk policyholders could defect for cheaper quotes, eroding long run expected profit gains.

2.3 Assumptions

- Poisson rate parameters are assumed to be constant, which ignores driver heterogeneity which implies same λ rates after a claim
- We assume constant mean claim cost of \$2,500. A lot of profit modelling revolved around this cost metric, ignores outliers and skew that can influence this value
- Assumption of constant portfolio size which assists in simplifying calculations but is unrealistic for long-run projects as portfolio size would fluctuate due to factors such as non-reporting
- We only model Fantastic Insurance profits as a standalone which is unrealistic and does not consider market sentiment/competitor responses and its impacts on profit forecasting
- We treat each driver's claim frequency λ as exogenous (given from outside the premium system and every loss is reported), this is unrealistic in practice as steep surcharges can change policyholder behaviour in the long run and lead to results such as competitive risk. If resolving a minor incidents privately costs less than the expected future premium increase, a policyholder may choose not to lodge the claim.