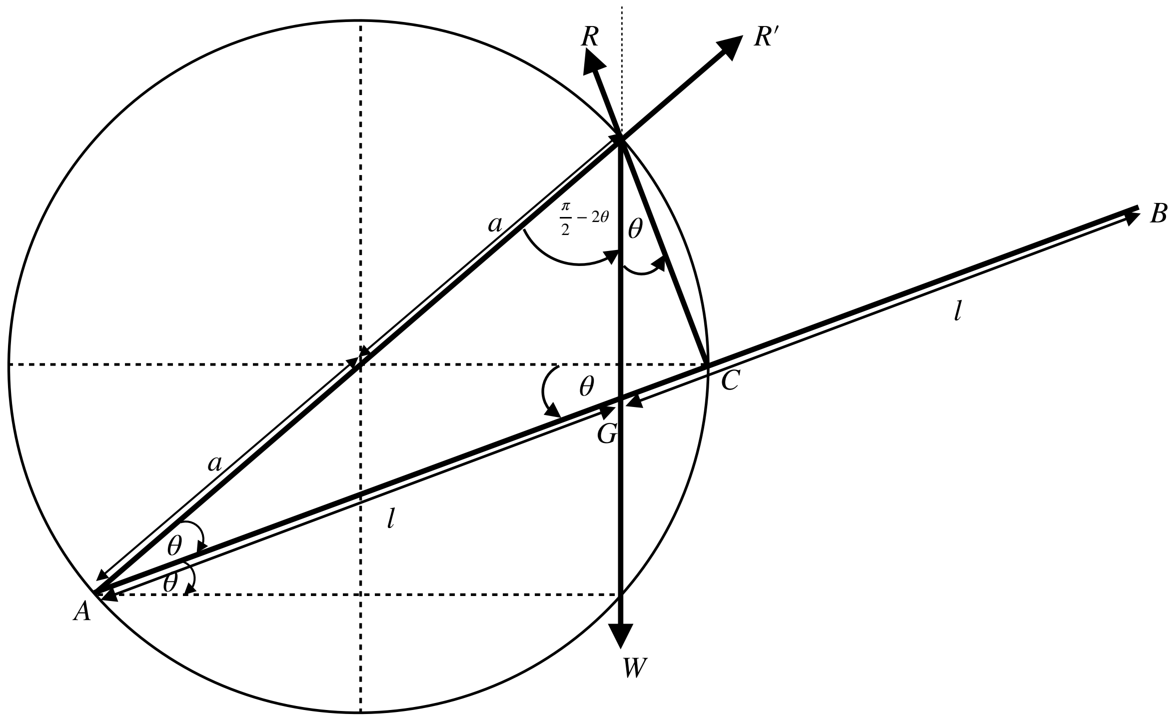


2003 A/L

A thin smooth hemispherical bowl of radius a is fixed with its rim uppermost and horizontal. A uniform rod of AB of weight W and length $2l(>2a)$ rests with the end A on the surface of the bowl and a point C of the rod in contact with the rim. Indicate the forces acting on the rod.

By taking moments about A, show that the reaction R at C is of magnitude $\dfrac{Wl}{2a}$,

For equilibrium;

$$\overset{\curvearrowleft}{A} = R \cdot 2a\cos\theta - W \cdot l\cos\theta = 0$$

$$\therefore R = \frac{Wl}{2a}$$

Also obtain another relation between R & W,

COMMENTARY: TO FIND THE ANOTHER RELATION USE EITHER LAMI'S OR COMPONENTS

By Lami's theorem; $\dfrac{R}{\sin\left(\pi - \left(\frac{\pi}{2} - 2\theta\right)\right)} = \dfrac{W}{\sin\left(\frac{\pi}{2} - \theta\right)} \rightarrow R = \dfrac{W\cos 2\theta}{\cos\theta} = \dfrac{Wl}{2a}$

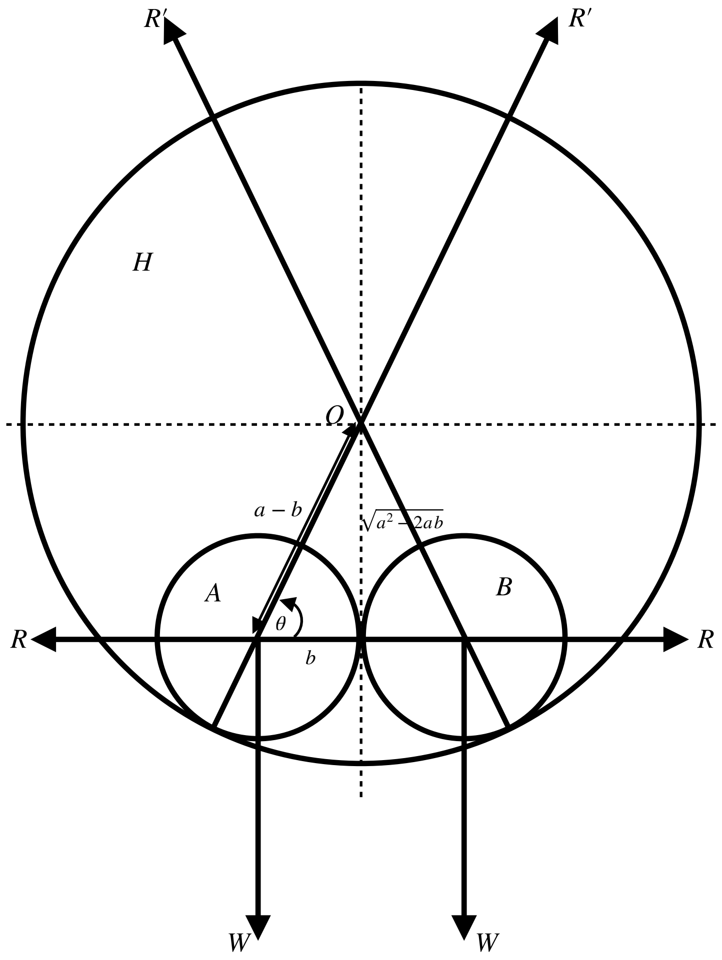
Hence, Show that the length CB is $\dfrac{1}{4}\left(7l - \sqrt{l^2 + 32a^2}\right)$,

COMMENTARY: FIND $\cos\theta$ USING THE RELATIONS FOUND ABOVE

TRIGONOMETRY: Acute angle case; only positive quantity is considered

By $R = \dfrac{W\cos 2\theta}{\cos\theta} = \dfrac{Wl}{2a}$; $2a\cos 2\theta = l\cos\theta \rightarrow 4a\cos^2\theta - l\cos\theta - 2a = 0 \rightarrow \cos\theta = \dfrac{l + \sqrt{l^2 + 32a^2}}{8a}\left(\because \theta < \dfrac{\pi}{2}\right)$

$$\therefore CB = AB - AC = 2\left(l - a\left(\frac{l + \sqrt{l^2 + 32a^2}}{8a}\right)\right) = \frac{1}{4}\left(7l - \sqrt{l^2 + 32a^2}\right)$$



2008 A/L

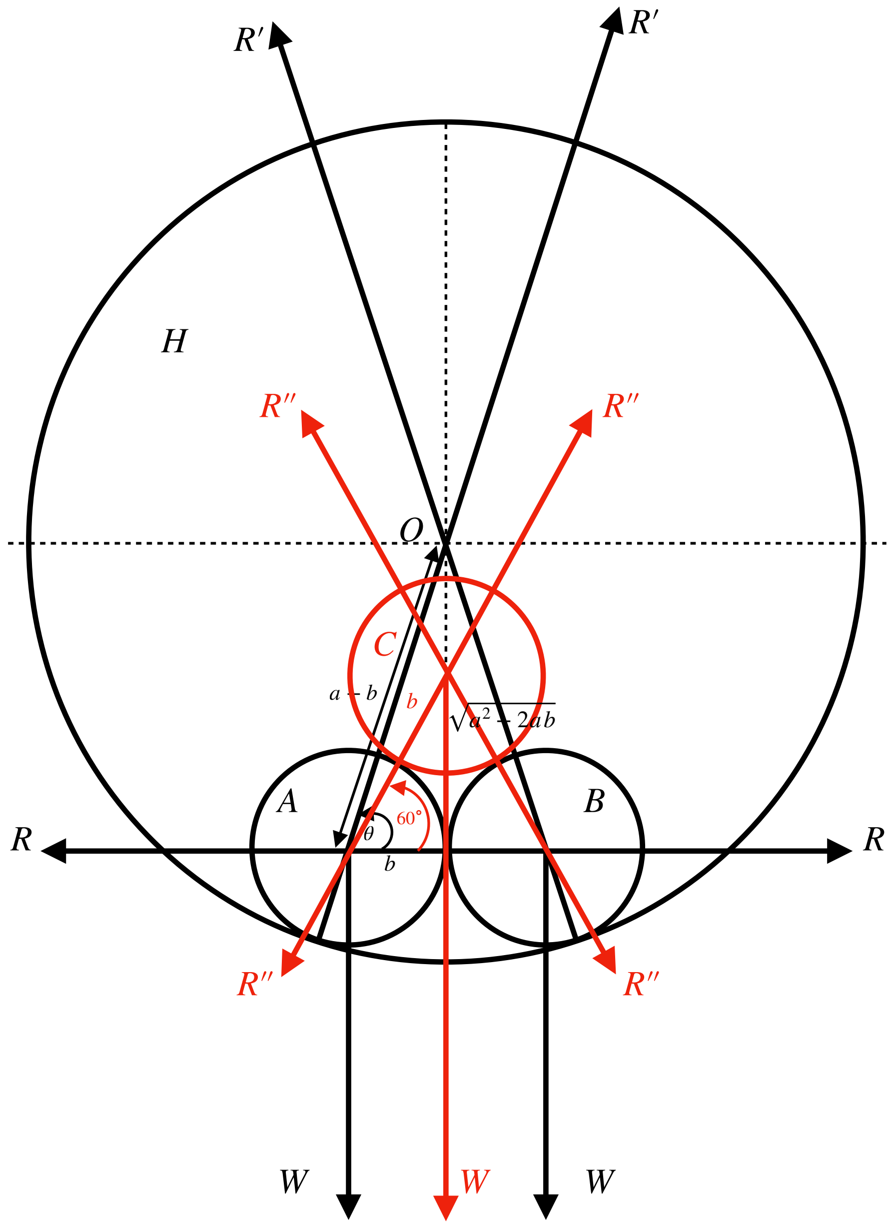
A Smooth hollow right circular cylinder H of radius a is fixed with its axis horizontal. Two equal smooth uniform right circular cylinders A and B, each of radius $b \left(< \dfrac{a}{2} \right)$ and weight W are placed symmetrically inside H so that they are in equilibrium with their axes parallel to that of H.

Show that the reaction between A and B is $\dfrac{bW}{\sqrt{a(a-2b)}}$,

Using Lami's Theorem on A;

$$\frac{R}{\cos\theta} = \frac{W}{\sin\theta} \to R = W\cot\theta = \frac{Wb}{\sqrt{a(a-2b)}}$$

Continue in page 3….



Another cylinder C equal to each of A and B is gently placed symmetrically on them (Denoted in Red), with its axis parallel to that of H. Show that equilibrium is possible with A and B in contact, only if $a < b(1 + 2\sqrt{7})$,

COMMENTARY: FOR THE SYSTEM TO BE IN EQUILIBRIUM IN THIS CONFIGURATION THE THEN THERE MUST BE A REACTION BETWEEN A AND B. I.E. $R > 0$

For equilibrium C; $\overrightarrow{Y} = 0;\ 2R'' \cos 30° - W = 0 \rightarrow R'' = \dfrac{W}{\sqrt{3}}$

For ABC;

COMMENTARY: NEGLECT R'' SINCE R'' IS INDEPENDENT OF THE WHOLE SYSTEM.

$\overrightarrow{Y} = 0;\ 2R' \sin\theta - 3W = 0 \rightarrow R' = \dfrac{3W}{\sin\theta}$

By A;

$\overrightarrow{X} = 0; -R'' \cos 60° - R + R' \cos\theta = 0 \rightarrow R = \dfrac{3W}{2}\cot\theta - \dfrac{W}{2\sqrt{3}}$

By $R > 0$;

$$\frac{3b}{\sqrt{a(a-2b)}} > \frac{1}{\sqrt{3}} \rightarrow 27b^2 > a^2 - 2ab \rightarrow (a-b)^2 < 28b^2$$

$\therefore a - b < 2\sqrt{7}b \rightarrow a < b(1 + 2\sqrt{7})$