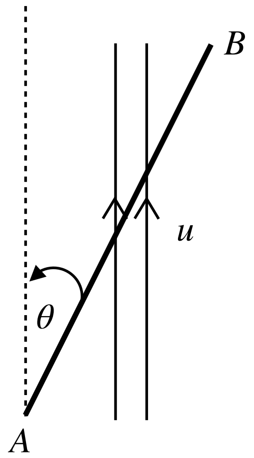
An aeroplane whose speed in still air is v is flying from a city A to a city B which is θ from north to east; when there's a constant wind due north with speed u. Determine whether the aeroplane can course through for the following conditions.

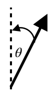
I. $u < v$
II. $u \sin\theta = v$
III. $u \sin\theta > v$
IV. $u \sin\theta < v < u$
V. $u = v$

If the distance between cities A and B is d then find the time taken to course from city A to B.



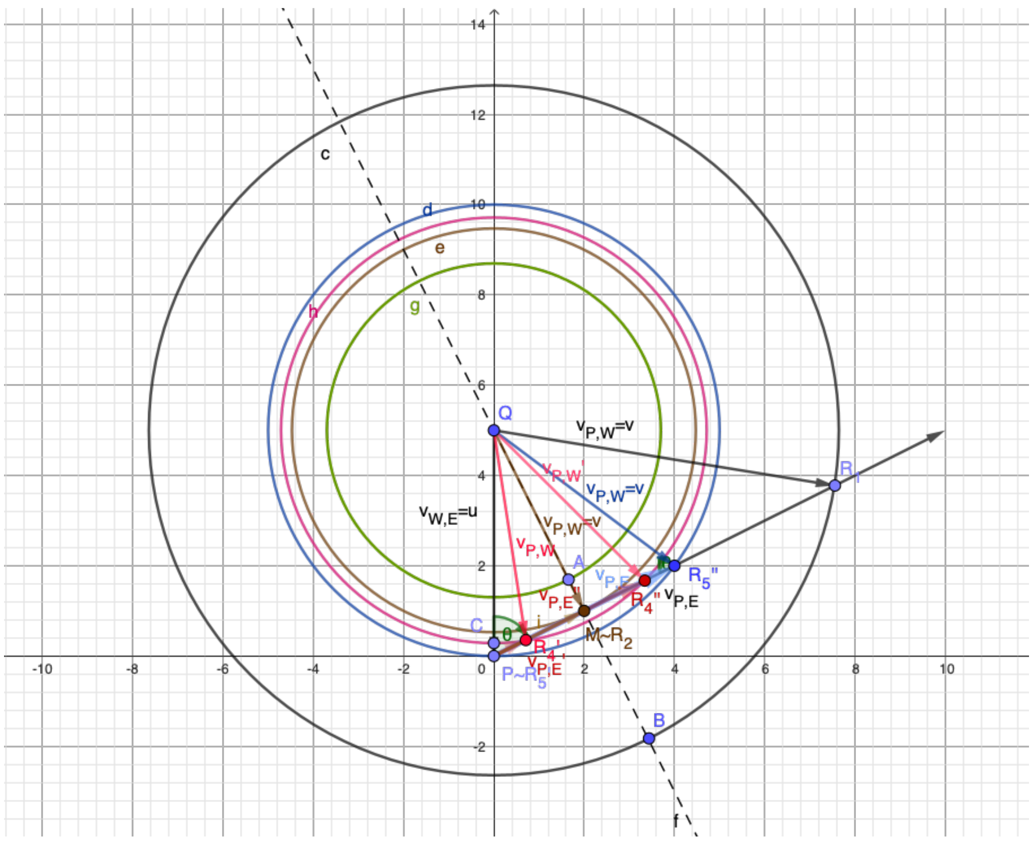
$v_{W,E} = \uparrow u$

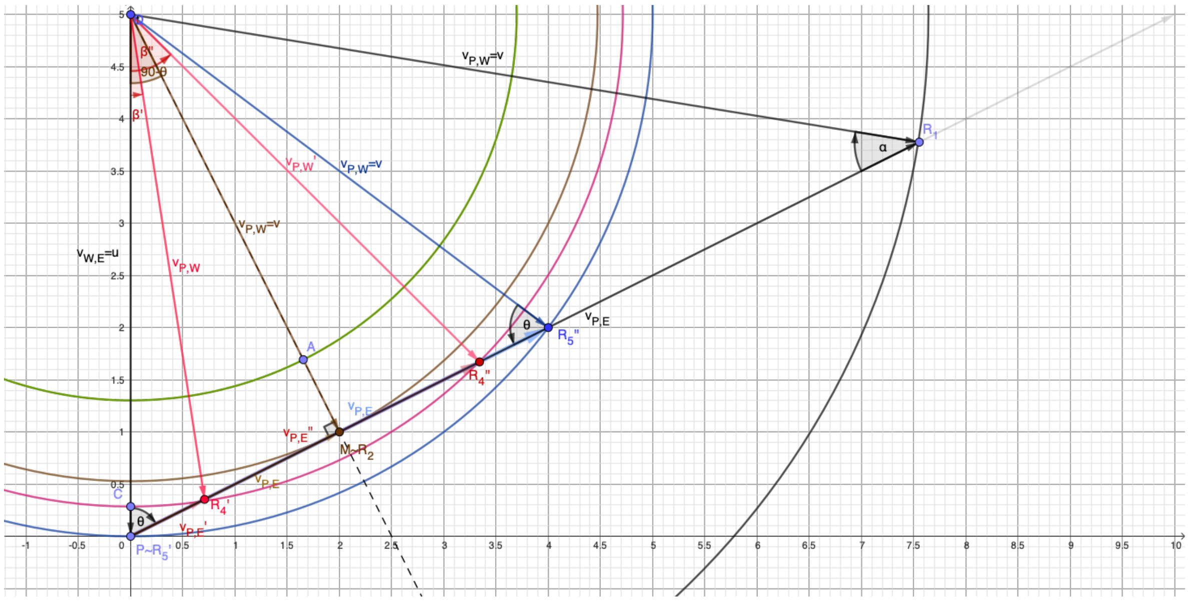
$|v_{P,W}| = v$

$v_{P,E} =$ 

By relative velocity principle;

$v_{P,E} = v_{P,W} + v_{W,E}$





For I. ;

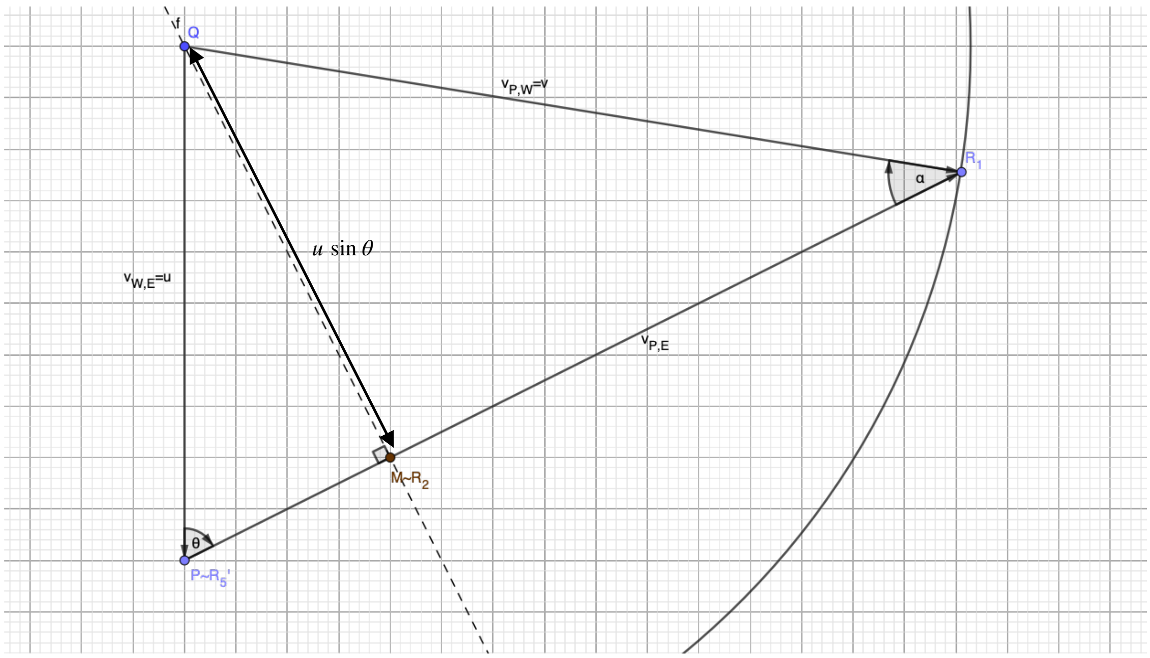
There's a common point R_1 of the side PR_1 on the circumference of the circle whose center is at Q. Therefore a $\triangle PQR_1$ exists such that PR_1 is real. i.e. aeroplane can course from A to B.

$$\therefore u\sin\theta = v\sin\alpha \rightarrow \sin\alpha = \frac{u}{v}\sin\theta$$

For time;

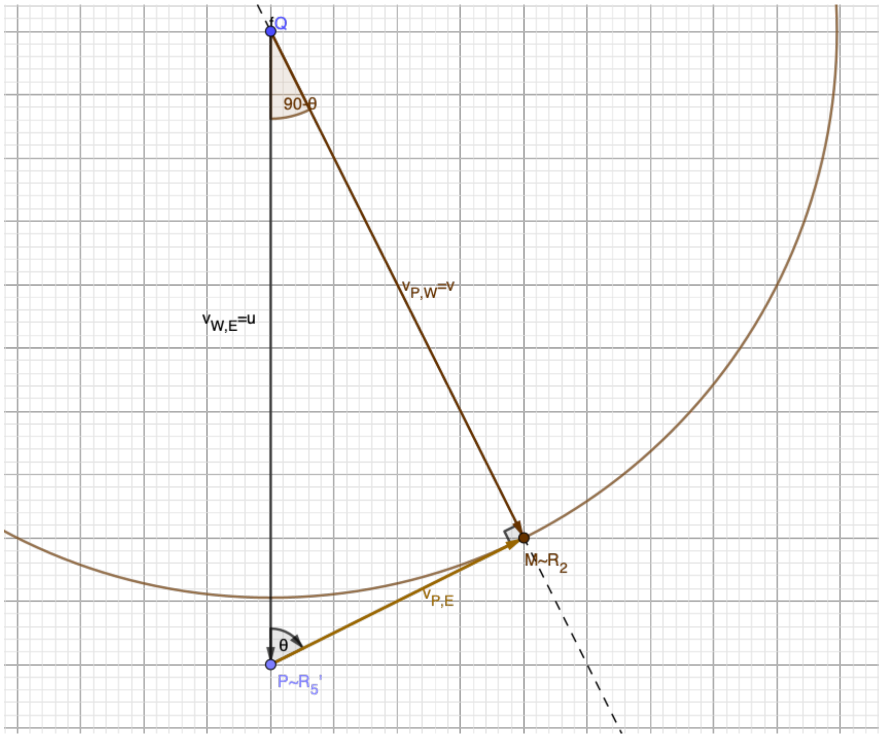
$$t_{AB} = \frac{d}{|v_{P.E}|} = \frac{d}{PR_1} = \frac{d}{u\cos\theta + v\cos\alpha} = \frac{d}{u\cos\theta + \sqrt{v^2 - u^2\sin^2\theta}}$$

$$\therefore t_{AB} = \frac{d(u\cos\theta - \sqrt{v^2 - u^2\sin^2\theta})}{u^2 - v^2}$$



For II.;

If $v = u \sin \theta$ the side PR_2 is the tangent to the circle such that PQR_2 exists. Also $PR_2 \perp QR_2$.
i.e. the aeroplane can fly from A-B if the direction of the plane with respect to wind is $90° - \theta$ from south to east.

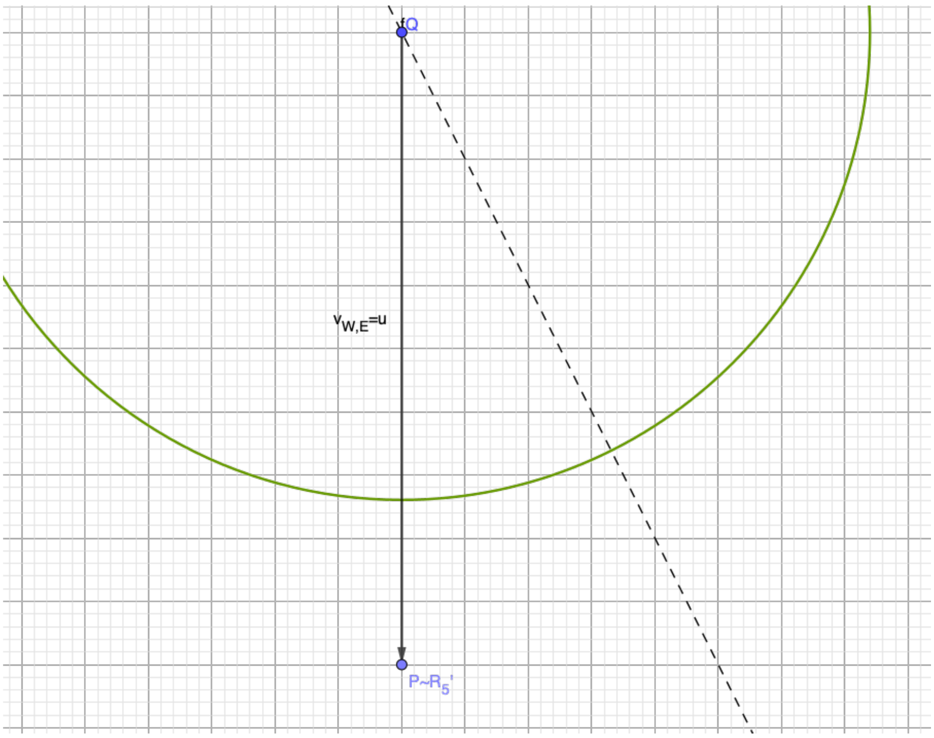


$$\therefore |v_{P,E}| = PR_2 = \sqrt{u^2 - v^2}$$

$$i.e.\ t_{AB} = \frac{d}{\sqrt{u^2 - v^2}}$$

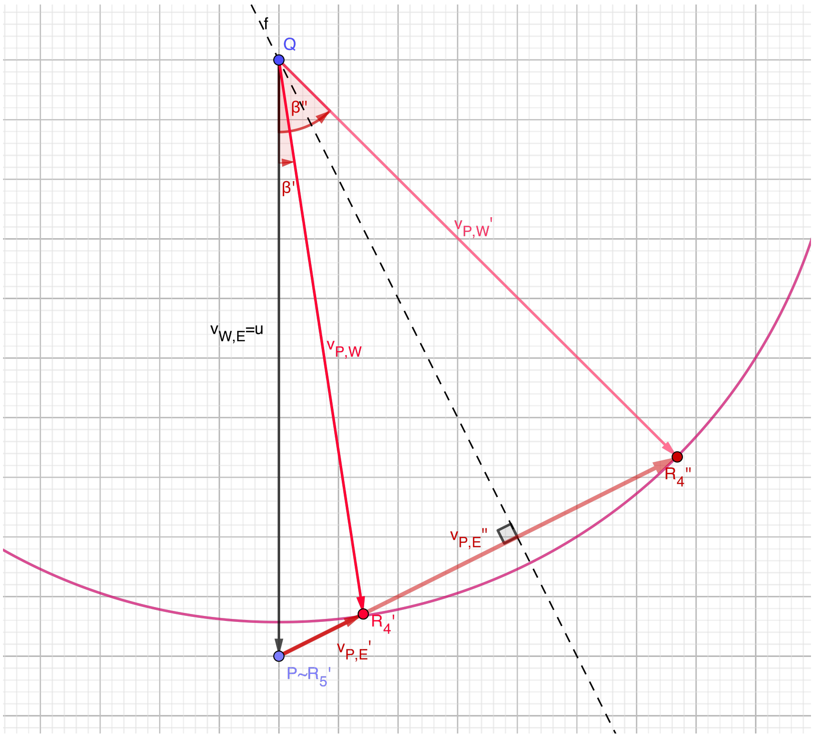
For III.;

If $v < u \sin\theta$ a $\triangle PQR_3$ does not exist I.e. the aeroplane cannot course from A - B



For IV.;

For the above case there are two possible triangles such that the aeroplane can course in between A and B.



Case 1:

If the direction of the plane with respect to wind is β' from south to east then the plane courses with a speed $PR_4' = |v_{P,E}'|$;

$$\therefore PR_4' = |v_{P,E}'| = u\cos\theta - \sqrt{v^2 - u^2\sin^2\theta}$$

Then; $t_{AB} = \dfrac{d(u\cos\theta + \sqrt{v^2 - u^2\sin^2\theta})}{u^2 - v^2}$

Case 2:

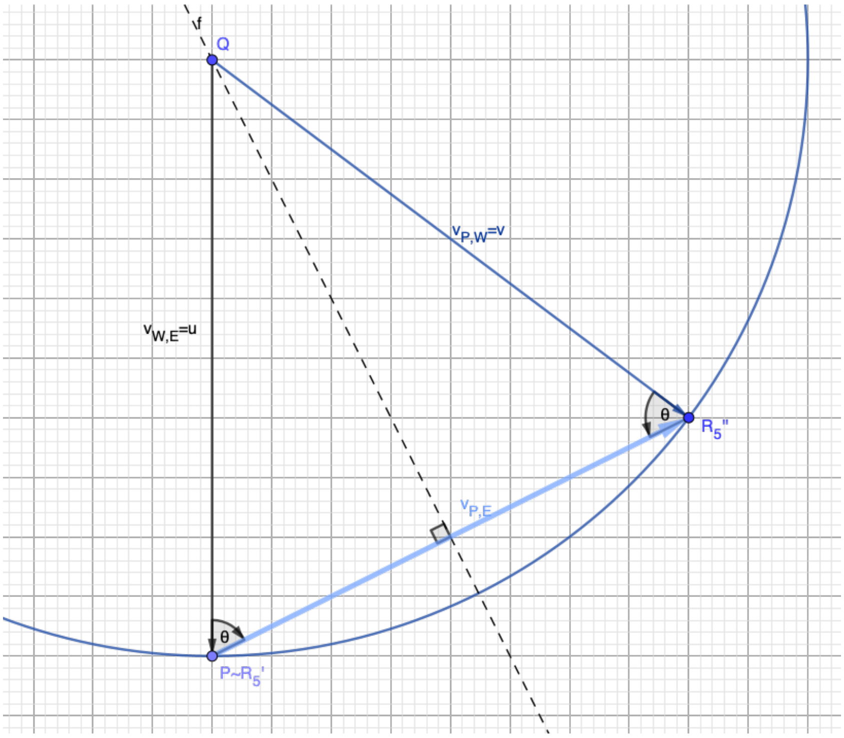
If the direction of the plane with respect to wind is β'' from south to east then the plane courses with the speed $PR_4'' = |v_{P,E}''|$;

$$\therefore PR_4'' = |v_{P,E}''| = u\cos\theta + \sqrt{v^2 - u^2\sin^2\theta}$$

Then; $t_{AB}' = \dfrac{d(u\cos\theta - \sqrt{v^2 - u^2\sin^2\theta})}{u^2 - v^2}$

Since $|v_{P,E}''| > |v_{P,E}'|$; $t_{AB}' < t_{AB}$

For V.;



There are two points on the circumference R_5', R_5''. Of these points $\triangle PQR_5'$ does not exist. Therefore the plane would not course in the direction due south (i.e. plane with respect to wind's direction). So only possibility that plane would course is on the $\triangle PQR_5''$.

$$\therefore |v_{P,E}| = 2u\cos\theta$$

$$\therefore t_{AB} = \frac{d}{2u\cos\theta}$$