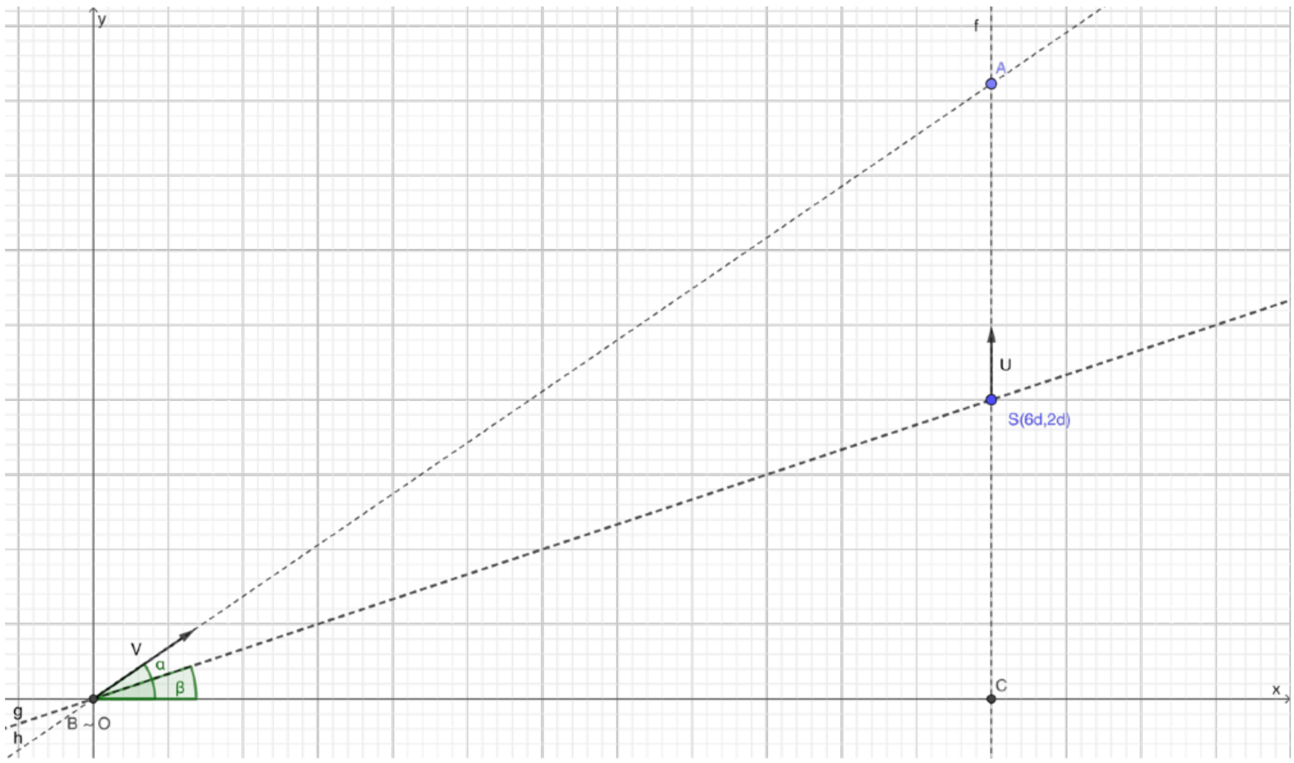
A motor boat sights a ship travelling due north with constant velocity $U\ km/h$. The coordinates of the ship at time of sighting are $(6d, 2d)$, with respect to cartesian axes Ox, Oy in the east and north directions respectively, where the origin O is taken in the boat and distances are measured in kilo-meters. The boat immediately begins to move with constant velocity $V km/h$, in a direction making an acute angle α north of east, so as to intercept the ship. Given that $\alpha = \tan^{-1}\left(\frac{3}{4}\right)$, sketch the path of the boat relative to the ship. Hence, find the value of V, in terms of U, and show that the time taken for the interception is $\frac{5d}{2U}$ hours.

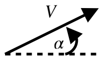
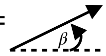


COMMENTARY: AT POINT A THE SHIP AND THE BOAT INTERCEPT, OS IS GIVEN AS THE DISPLACEMENT BY THE BOAT RELATIVE TO THE SHIP WHILST OA IS GIVEN AS THE DISPLACEMENT (OF THE BOAT) RELATIVE TO EARTH.

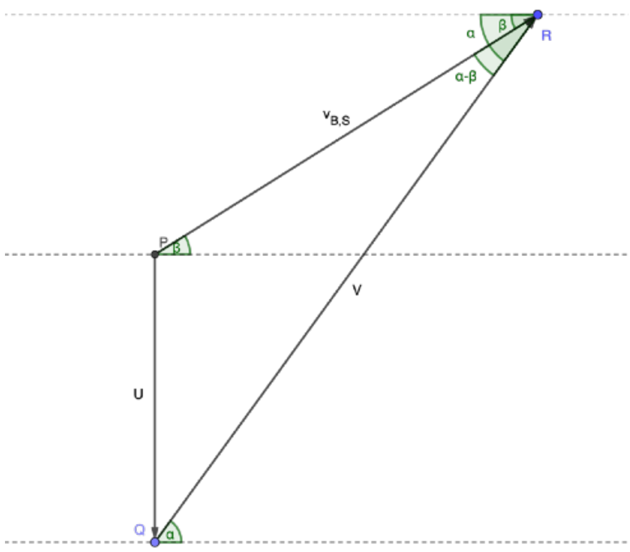
AT REST THE BOAT IS INCLINED AT AN ANGLE β TO THE HORIZONTAL. WHERE $\beta = \tan^{-1}\left(\frac{1}{3}\right)$.

AS THE BOAT STARTS TO COURSE THROUGH, IT MUST ACCOMPANY AN ANGLE α TO NORTH OF EAST WHICH IS GIVEN ABOVE IN THE QUESTION (I.E. AS SOON AS WHEN THE BOAT SEE'S THE SHIP WITHIN ITS VICINITY) OR OTHERWISE THE BOAT WOULD NOT MEET THE SHIP.

Continued….

$v_{B,E} =$  , $v_{B,S} =$  , $v_{S,E} = U$ 

$\therefore v_{B,S} = v_{B,E} + v_{S,E}$



By sine rule;

$$\frac{U}{\sin(\alpha-\beta)} = \frac{V}{\cos\beta} \to U = \frac{V\sin(\alpha-\beta)}{\cos\beta}$$

$$\therefore U = V(\sin\alpha - \tan\beta\cos\alpha)$$

By $\tan\alpha, \tan\beta$;

$$\therefore U = V\left(\frac{9-4}{15}\right) \to V = 3U$$

For time;

$$t = \frac{|S_{B,S}|}{|v_{B,S}|} = \frac{OS}{|v_{B,S}|}$$

$$t = \frac{2d\sqrt{10}}{\sqrt{U^2+V^2-2UV\cos\left(\frac{\pi}{2}-\alpha\right)}}$$

$$t = \frac{10d\sqrt{2}}{U\sqrt{10-6\cdot\frac{3}{5}}} = \frac{10d\sqrt{2}}{U\sqrt{32}} = \frac{5d}{2U}$$