

```
In [1]: from dsc80_utils import *
```

Lecture 12 – Text Features

DSC 80, Fall 2025

Agenda

- Text features.
- Bag of words.
- Cosine similarity.
- TF-IDF.
 - Example: State of the Union addresses .

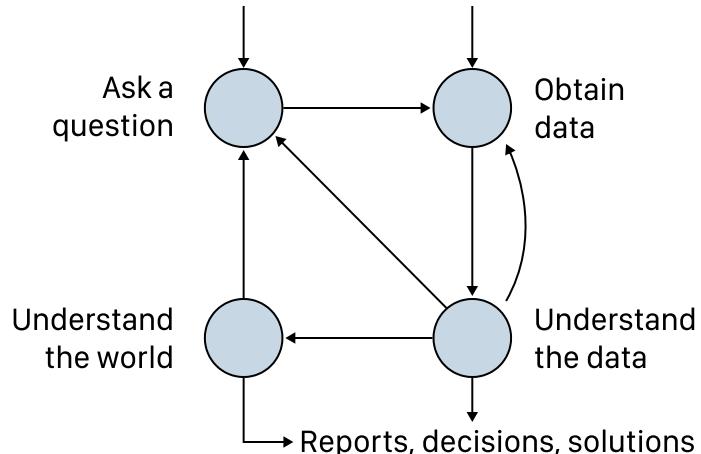
Question

`^\w{2,5}.\d*/[^A-Z5]{1,}`

Select all strings below that contain any match with the regular expression above.

- "billy4/Za"
- "billy4/za"
- "DAI_s2154/pacific"
- "daisy/ZZZZZ"
- "bi/_lly98"
- "!@__!14-atlantic"

Text features



Processing math: 100%

Review: Regression and features

- In DSC 40A, our running example was to use **regression** to predict a data scientist's salary, given their GPA, years of experience, and years of education.
- After minimizing empirical risk to determine optimal parameters, w_0^*, \dots, w_3^* , we made predictions using:

$$\text{predicted salary} = w_0^* + w_1^* \cdot \text{GPA} + w_2^* \cdot \text{experience} + w_3^* \cdot \text{education}$$

- GPA, years of experience, and years of education are **features** – they represent a data scientist as a vector of *numbers*.
 - e.g. Your feature vector may be [3.5, 1, 7].
- **This approach requires features to be numeric.**

Moving forward

Suppose we'd like to predict the **sentiment** of a piece of text from 1 to 10.

- 10: Very positive (happy).
- 1: Very negative (sad, angry).

Example:

- Input: "DSC 80 is a pretty good class."
- Output: 7.
- We can frame this as a regression problem, but we can't directly use what we learned in 40A, because here our inputs are **text**, not **numbers**.

Text features

- **Big question: How do we represent a text document as a feature vector of numbers?**
- If we can do this, we can:
 - use a text document as input in a regression or classification model (in a few lectures).

Processing math: 100%

- **quantify** the similarity of two text documents (today).

Example: San Diego employee salaries

- [Transparent California](#) publishes the salaries of all City of San Diego employees.
- Let's look at the 2022 data.

```
In [2]: salaries = pd.read_csv('https://transcal.s3.amazonaws.com/public/export/san-salaries['Employee Name'] = salaries['Employee Name'].str.split().str[0] + '
```

```
In [3]: salaries.head()
```

Out [3]:

	Employee Name	Job Title	Base Pay	Overtime Pay	...	Year	Notes	Agency	Status
0	Mara XXXX	City Attorney	227441.53	0.00	...	2022	NaN	San Diego	FT
1	Todd XXXX	Mayor	227441.53	0.00	...	2022	NaN	San Diego	FT
2	Terence XXXX	Assistant Police Chief	227224.32	0.00	...	2022	NaN	San Diego	FT
3	Esmeralda XXXX	Police Sergeant	124604.40	162506.54	...	2022	NaN	San Diego	FT
4	Marcelle XXXX	Assistant Retirement Administrator	279868.04	0.00	...	2022	NaN	San Diego	FT

5 rows × 13 columns

Aside on privacy and ethics

- Even though the data we downloaded is publicly available, employee names still correspond to real people.
- Be careful when dealing with PII (personably identifiable information).
 - Only work with the data that is needed for your analysis.
 - Even when data is public, people have a reasonable right to privacy.
- Remember to think about the impacts of your work **outside** of your Jupyter Notebook.

Goal: Quantifying similarity

Processing math: 100%

- Our goal is to describe, numerically, how **similar** two job titles are.
- For instance, our similarity metric should tell us that 'Deputy Fire Chief' and 'Fire Battalion Chief' are more similar than 'Deputy Fire Chief' and 'City Attorney'.
- **Idea:** Two job titles are similar if they contain shared words, regardless of order. So, to measure the similarity between two job titles, we could **count the number of words they share in common**.
- Before we do this, we need to be confident that the job titles are clean and consistent – let's explore.

Exploring job titles

```
In [4]: jobtitles = salaries['Job Title']
jobtitles.head()
```

```
Out[4]: 0                 City Attorney
        1                     Mayor
        2      Assistant Police Chief
        3             Police Sergeant
        4 Assistant Retirement Administrator
Name: Job Title, dtype: object
```

How many employees are in the dataset? How many **unique** job titles are there?

```
In [5]: jobtitles.shape[0], jobtitles.nunique()
```

```
Out[5]: (12831, 611)
```

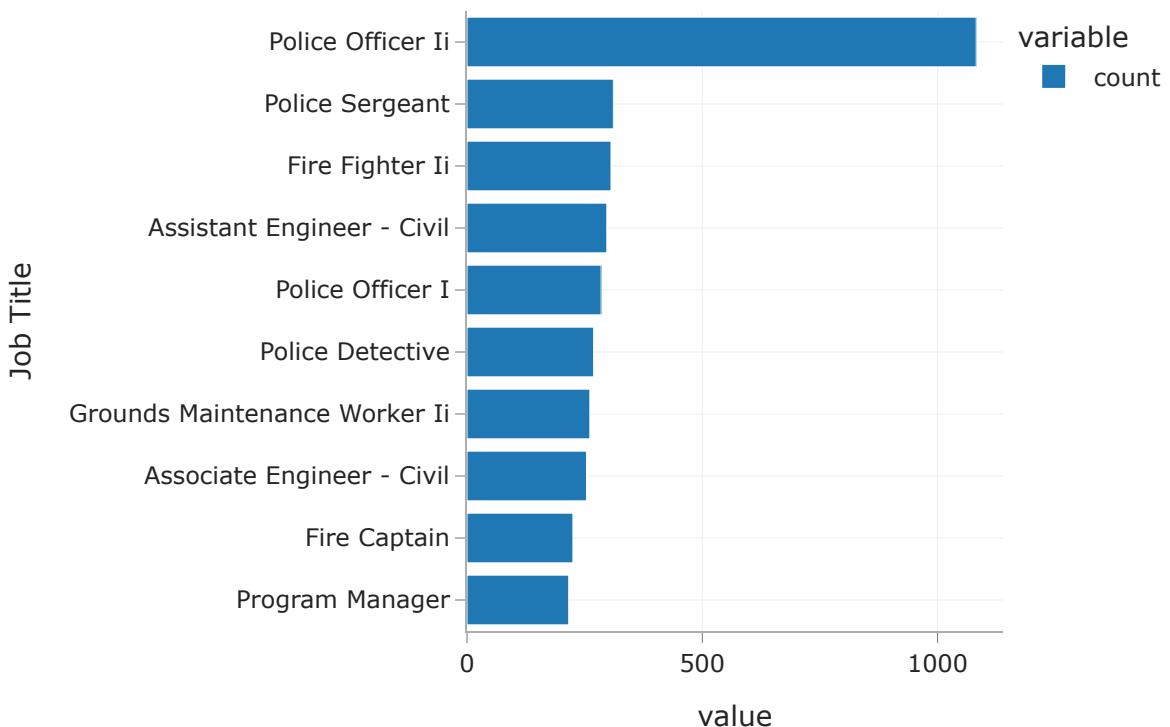
What are the most common job titles?

```
In [6]: jobtitles.value_counts().iloc[:100]
```

```
Out[6]: Job Title
        Police Officer Ii          1082
        Police Sergeant            311
        Fire Fighter Ii           306
        ...
        Public Works Supervisor     29
        Project Assistant           29
        Associate Engineer - Traffic 29
Name: count, Length: 100, dtype: int64
```

```
In [7]: jobtitles.value_counts().iloc[:10].sort_values().plot(kind='barh')
```

Processing math: 100%



Are there any missing job titles?

```
In [8]: jobtitles.isna().sum()
```

```
Out[8]: np.int64(0)
```

Fortunately, no.

Canonicalization

Remember, our goal is ultimately to count the number of shared words between job titles. But before we start counting the number of shared words, we need to consider the following:

- Some job titles may have **punctuation**, like `'-'` and `'&'`, which may count as words when they shouldn't.
 - `'Assistant - Manager'` and `'Assistant & Manager'` should count as the same job title.
- Some job titles may have "**glue" words**", like `'to'` and `'the'`, which (we can argue) also shouldn't count as words.
 - `'Assistant To The Manager'` and `'Assistant Manager'` should count as the same job title.

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- If we just want to focus on the titles themselves, then perhaps **roman numerals** should be removed: that is, 'Police Officer II' and 'Police Officer I' should count as the same job title.

Let's address the above issues. The process of converting job titles so that they are always represented the same way is called **canonicalization**.

Punctuation

Are there job titles with unnecessary punctuation that we can remove?

- To find out, we can write a regular expression that looks for characters other than letters, numbers, and spaces.
- We can use regular expressions with the `.str` methods we learned earlier in the quarter just by using `regex=True`.

```
In [9]: # Uses character class negation.
jobtitles.str.contains(r'^[A-Za-z0-9 ]+', regex=True).sum()
```

```
Out[9]: np.int64(922)
```

```
In [10]: jobtitles[jobtitles.str.contains(r'^[A-Za-z0-9 ]+', regex=True)].head()
```

```
Out[10]: 137      Park & Recreation Director
248      Associate Engineer – Mechanical
734      Associate Engineer – Electrical
882      Associate Engineer – Traffic
1045     Associate Engineer – Civil
Name: Job Title, dtype: object
```

It seems like we should replace these pieces of punctuation with a single space.

"Glue" words

Are there job titles with "glue" words in the middle, such as 'Assistant to the Manager'?

To figure out if any titles contain the word 'to', we **can't** just do the following, because it will evaluate to `True` for job titles that have 'to' anywhere in them, even if not as a standalone word.

```
In [11]: # Why are we converting to lowercase?
jobtitles.str.lower().str.contains('to').sum()
```

```
Out[11]: np.int64(1577)
```

Processing math: 100%

```
In [12]: jobtitles[jobtitles.str.lower().str.contains('to')]
```

```
Out[12]: 0           City Attorney
        4       Assistant Retirement Administrator
        8           Retirement Administrator
        ...
       12778           Test Monitor I
      12812           Custodian I
      12826       Word Processing Operator
Name: Job Title, Length: 1577, dtype: object
```

Instead, we need to look for 'to' separated by word boundaries.

```
In [13]: jobtitles.str.lower().str.contains(r'\bto\b', regex=True).sum()
```

```
Out[13]: np.int64(10)
```

```
In [14]: jobtitles[jobtitles.str.lower().str.contains(r'\bto\b', regex=True)]
```

```
Out[14]: 1638           Assistant To The Chief Operating Officer
      2183           Principal Assistant To City Attorney
      2238           Assistant To The Director
      ...
      6594           Confidential Secretary To Chief Operating Officer
      6832           Confidential Secretary To Mayor
      11028           Confidential Secretary To Mayor
Name: Job Title, Length: 10, dtype: object
```

We can look for other filler words too, like 'the' and 'for' .

```
In [15]: jobtitles[jobtitles.str.lower().str.contains(r'\bthe\b', regex=True)]
```

```
Out[15]: 1638   Assistant To The Chief Operating Officer
      2238   Assistant To The Director
      5609   Assistant To The Director
      6544   Assistant To The Director
Name: Job Title, dtype: object
```

```
In [16]: jobtitles[jobtitles.str.lower().str.contains(r'\bfor\b', regex=True)]
```

```
Out[16]: 3449   Assistant For Community Outreach
      6889   Assistant For Community Outreach
      10810  Assistant For Community Outreach
Name: Job Title, dtype: object
```

We should probably remove these "glue" words.

Roman numerals (e.g. "li")

Lastly, let's try and identify job titles that have roman numerals at the end, like 'i' (1), 'ii' (2), 'iii' (3), or 'iv' (4). As before, we'll convert to lowercase first.

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```
In [17]: jobtitles[jobtitles.str.lower().str.contains(r'\bi+v?\b', regex=True)]
```

```
Out[17]: 5                  Police Officer Ii
          10                 Police Officer Ii
          48                 Fire Prevention Inspector Ii
          ...
          12822                Clerical Assistant Ii
          12828                Police Officer Ii
          12830                Police Officer I
Name: Job Title, Length: 6087, dtype: object
```

Let's get rid of those numbers, too.

Fixing punctuation and removing "glue" words and roman numerals

Let's put the preceding three steps together and canonicalize job titles by:

- converting to lowercase,
- removing each occurrence of 'to', 'the', and 'for',
- replacing each non-letter/digit/space character with a space,
- replacing each sequence of roman numerals – either 'i', 'ii', 'iii', or 'iv' at the end with nothing, and
- replacing each sequence of multiple spaces with a single space.

```
In [18]: jobtitles = (
    jobtitles
    .str.lower()
    .str.replace(r'\bto\b|\bthe\b|\bfor\b', '', regex=True)
    .str.replace(r'\b[A-Za-z0-9 ]\b', ' ', regex=True)
    .str.replace(r'\bi+v?\b', ' ', regex=True)
    .str.replace(r'\s+', ' ', regex=True)                      # '+' matches 1 or n
    .str.strip()                                              # Removes leading/trailing whitespace
)
```

```
In [19]: jobtitles.sample(5)
```

```
Out[19]: 3382      associate engineer civil
          7033      equipment operator
          1884      police officer
          2501      associate engineer civil
          10110     clerical assistant
Name: Job Title, dtype: object
```

```
In [20]: (jobtitles == 'police officer').sum()
```

```
Out[20]: np.int64(1378)
```

Possible issue: inconsistent representations

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Another possible issue is that some job titles may have inconsistent representations of the same word (e.g. 'Asst.' vs 'Assistant').

```
In [21]: jobtitles[jobtitles.str.contains('asst')].value_counts()
```

```
Out[21]: Series([], Name: count, dtype: int64)
```

```
In [22]: jobtitles[jobtitles.str.contains('assistant')].value_counts().head()
```

```
Out[22]: Job Title
library assistant      385
assistant engineer civil    297
clerical assistant      111
assistant center director    50
assistant chemist        45
Name: count, dtype: int64
```

The 2020 salaries dataset had several of these issues, but fortunately they appear to be fixed for us in the 2022 dataset (thanks, Transparent California).

Bag of words

Text similarity

Recall, our idea is to measure the similarity of two job titles by counting the number of shared words between the job titles. How do we actually do that, for all of the job titles we have?

A counts matrix

Let's create a "counts" matrix, such that:

- there is 1 row per job title,
- there is 1 column per **unique** word that is used in job titles, and
- the value in row `title` and column `word` is the number of occurrences of `word` in `title`.

Such a matrix might look like:

	senior	lecturer	teaching	professor	assistant	associate
senior lecturer	1	1	0	0	0	0
assistant teaching professor	0	0	1	1	1	0
associate professor	0	0	0	1	0	1

Processing math: 100%

	senior	lecturer	teaching	professor	assistant	associate
senior assistant to the assistant professor	1	0	0	1	2	0

Then, we can make statements like: "assistant teaching professor" is more similar to "associate professor" than to "senior lecturer".

Creating a counts matrix

First, we need to determine all words that are used across all job titles.

```
In [23]: jobtitles.str.split()
```

```
Out[23]: 0           [city, attorney]
         1           [mayor]
         2           [assistant, police, chief]
         ...
        12828       [police, officer]
        12829       [police, dispatcher]
        12830       [police, officer]
Name: Job Title, Length: 12831, dtype: object
```

```
In [24]: # The .explode method concats the lists together.
all_words = jobtitles.str.split().explode()
all_words
```

```
Out[24]: 0           city
         0           attorney
         1           mayor
         ...
        12829       dispatcher
        12830       police
        12830       officer
Name: Job Title, Length: 30057, dtype: object
```

Next, to determine the columns of our matrix, we need to find a list of all **unique** words used in titles. We can do this with `np.unique`, but `value_counts` shows us the distribution, which is interesting.

```
In [25]: unique_words = all_words.value_counts()
unique_words
```

```
Out[25]: Job Title
police      2299
officer     1608
assistant    1267
...
relations    1
dna          1
gardener     1
Name: count, Length: 330, dtype: int64
```

Note that in `unique_words.index`, job titles are sorted by number of occurrences!

For each of the 330 unique words that are used in job titles, we can count the number of occurrences of the word in each job title.

- `'deputy fire chief'` contains the word `'deputy'` once, the word `'fire'` once, and the word `'chief'` once.
- `'assistant managers assistant'` contains the word `'assistant'` twice and the word `'managers'` once.

```
In [26]: # Created using a dictionary to avoid a "DataFrame is highly fragmented" warning
counts_dict = {}
for word in unique_words.index:
    re_pat = fr'\b{word}\b'
    counts_dict[word] = jobtitles.str.count(re_pat).astype(int).tolist()

counts_df = pd.DataFrame(counts_dict).set_index(jobtitles)
```

```
In [27]: counts_df.head()
```

	police	officer	assistant	fire	...	emergency	relations	dna	gardener
Job Title									
city attorney	0	0	0	0	...	0	0	0	C
mayor	0	0	0	0	...	0	0	0	C
assistant police chief	1	0	1	0	...	0	0	0	C
police sergeant	1	0	0	0	...	0	0	0	C
assistant retirement administrator	0	0	1	0	...	0	0	0	C

5 rows × 330 columns

```
In [28]: counts_df.shape
```

```
Out[28]: (12831, 330)
```

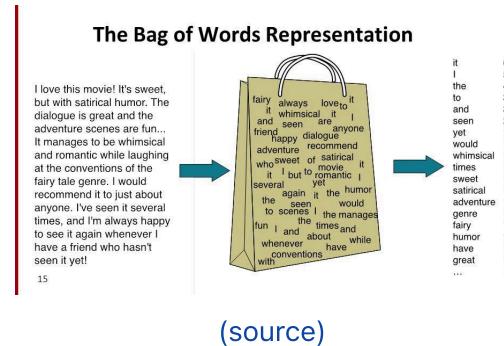
`counts_df` has one row for all 12831 employees, and one column for each unique word that is used in a job title.

Its third row, for example, tells us that the third job title contains `'police'` once and `'assistant'` once.

Dig of words

Processing math: 100%

- The **bag of words** model represents texts (e.g. job titles, sentences, documents) as **vectors of word counts**.
 - The "counts" matrices we have worked with so far were created using the bag of words model.
 - The bag of words model defines a **vector space** in $R^{\text{number of unique words}}$.
 - In the matrix on the previous slide, each **row** was a vector corresponding to a specific job title.
- It is called "bag of words" because it doesn't consider **order**.



Aside: Interactive bag of words demo

Check [this](#) site out – it automatically generates a bag of words matrix for you!

Bag of Words Demo

Edit the sentences and watch the corresponding vocabulary and cell-counts update:

Count Method: Count Binary

	i	love	dogs
Sentence 1: <input type="text" value="I"/>	1	0	0
Sentence 2: <input type="text" value="love dogs"/>	0	1	1
Sentence 3: <input type="text" value="dogs dogs dogs"/>	0	0	3

(source)

Cosine similarity

Question: What job titles are most similar to 'deputy fire chief'?

- Remember, our idea was to count the number of shared words between two job titles.

Processing math: 100% We now have access to `counts_df`, which contains a row vector for each job title.

- How can we use it to count the number of shared words between two job titles?

Counting shared words

To start, let's compare the row vectors for 'deputy fire chief' and 'fire battalion chief'.

```
In [29]: dfc = counts_df.loc['deputy fire chief'].iloc[0]
dfc
```

```
Out[29]: police      0
officer      0
assistant     0
...
relations    0
dna          0
gardener     0
Name: deputy fire chief, Length: 330, dtype: int64
```

```
In [30]: fbc = counts_df.loc['fire battalion chief'].iloc[0]
fbc
```

```
Out[30]: police      0
officer      0
assistant     0
...
relations    0
dna          0
gardener     0
Name: fire battalion chief, Length: 330, dtype: int64
```

We can stack these two vectors horizontally.

```
In [31]: pair_counts = (
    pd.concat([dfc, fbc], axis=1)
    .sort_values(by=['deputy fire chief', 'fire battalion chief'], ascending=False)
    .head(10)
    .T
)

pair_counts
```

```
Out[31]:   fire  chief  deputy  battalion  ...  assistant  engineer  civil  technician
```

	fire	chief	deputy	battalion	...	assistant	engineer	civil	technician
deputy fire chief	1	1	1	0	...	0	0	0	0
fire battalion chief	1	1	0	1	...	0	0	0	0

9 rows × 10 columns

Processing math: 100%

'deputy fire chief' and 'fire battalion chief' have 2 shared words in common. One way to arrive at this result mathematically is by taking their **dot product**:

```
In [32]: np.dot(pair_counts.iloc[0], pair_counts.iloc[1])
```

```
Out[32]: np.int64(2)
```

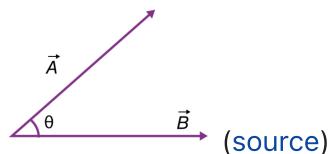
Recall: The dot product

- Recall, if $\vec{a} = [a_1 \ a_2 \ \dots \ a_n]^T$ and $\vec{b} = [b_1 \ b_2 \ \dots \ b_n]^T$ are two vectors, then their **dot product** $\vec{a} \cdot \vec{b}$ is defined as:

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + \dots + a_n b_n$$

- The dot product also has a **geometric** interpretation. If $|\vec{a}|$ and $|\vec{b}|$ are the L_2 norms (lengths) of \vec{a} and \vec{b} , and θ is the angle between \vec{a} and \vec{b} , then:

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos\theta$$



- $\cos\theta$ is equal to its maximum value (1) when $\theta = 0$, i.e. when \vec{a} and \vec{b} point in the same direction.

Cosine similarity and bag of words

To measure the similarity between two word vectors, instead of just counting the number of shared words, we should compute their **normalized** dot product, also known as their **cosine similarity**.

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

- If all elements in \vec{a} and \vec{b} are non-negative, then $\cos\theta$ ranges from 0 to 1.

Processing math: 100%



Key idea: The larger $\cos\theta$ is, the more similar the two vectors are!

- It is important to normalize by the lengths of the vectors, otherwise texts with more words will have artificially high similarities with other texts.

Normalizing

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

- Why can't we just use the dot product – that is, why must we divide by $|\vec{a}| |\vec{b}|$?
- Consider the following example:

	big	data	science
big big big big data	4	1	0
big data science	1	1	1
science big data	1	1	1
Pair	Dot Product		Cosine Similarity
big data science and big big big big data	5		0.7001
big data science and science big data	3		1

- 'big big big big data' has a large dot product with 'big data science' just because it has the word 'big' four times. But intuitively, 'big data science' and 'science big data' should be as similar as possible, since they're permutations of the same sentence.
- So, make sure to compute the cosine similarity – don't just use the dot product!

Note: Sometimes, you will see the **cosine distance** being used. It is the complement of cosine similarity:

$$\text{dist}(\vec{a}, \vec{b}) = 1 - \cos\theta$$

If $\text{dist}(\vec{a}, \vec{b})$ is small, the two word vectors are similar.

Processing math: 100% Recipe for computing similarities

Given a set of documents, to find the **most similar** text to one document d in particular:

- Use the bag of words model to create a counts matrix, in which:
 - there is 1 row per document,
 - there is 1 column per **unique** word that is used across documents, and
 - the value in row `doc` and column `word` is the number of occurrences of `word` in `doc`.
- Compute the cosine similarity between d 's row vector and all other documents' row vectors.
- The other document with the greatest cosine similarity is the most similar, under the bag of words model.

Example: Global warming

Consider the following three documents.

```
In [33]: sentences = pd.Series([
    'I really really want global peace',
    'I must enjoy global warming',
    'We must solve climate change'
])

sentences
```

```
Out[33]: 0    I really really want global peace
1          I must enjoy global warming
2          We must solve climate change
dtype: object
```

Let's represent each document using the bag of words model.

```
In [34]: unique_words = sentences.str.split().explode().value_counts()
```

```
Out[34]: I           2
really       2
global       2
..
solve        1
climate      1
change       1
Name: count, Length: 12, dtype: int64
```

```
In [35]: counts_dict = {}
for word in unique_words.index:
    re_pat = fr'\b{word}\b'
```

```
counts_dict[word] = sentences.str.count(re_pat).astype(int).tolist()
counts_df = pd.DataFrame(counts_dict).set_index(sentences)
```

In [36]: counts_df

Out[36]:

	I	really	global	must	...	We	solve	climate	change
I really really want global peace	1	2	1	0	...	0	0	0	0
I must enjoy global warming	1	0	1	1	...	0	0	0	0
We must solve climate change	0	0	0	1	...	1	1	1	1

3 rows × 12 columns

Let's now find the cosine similarity between each document.

In [37]: counts_df

Out[37]:

	I	really	global	must	...	We	solve	climate	change
I really really want global peace	1	2	1	0	...	0	0	0	0
I must enjoy global warming	1	0	1	1	...	0	0	0	0
We must solve climate change	0	0	0	1	...	1	1	1	1

3 rows × 12 columns

```
In [38]: def sim_pair(s1, s2):
    return np.dot(s1, s2) / (np.linalg.norm(s1) * np.linalg.norm(s2))
```

```
In [39]: # Look at the documentation of the .corr method to see how this works!
counts_df.T.corr(sim_pair)
```

Out[39]:

	I really really want global peace	I must enjoy global warming	We must solve climate change
I really really want global peace	1.00	0.32	0.0
I must enjoy global warming	0.32	1.00	0.2
We must solve climate change	0.00	0.20	1.0

Processing math: 100%

Issue: Bag of words only encodes the **words** that each document uses, not their **meanings**.

- "I really really want global peace" and "We must solve climate change" have similar meanings, but have no shared words, and thus a low cosine similarity.
- "I really really want global peace" and "I must enjoy global warming" have very different meanings, but a relatively high cosine similarity.

Pitfalls of the bag of words model

Remember, the key assumption underlying the bag of words model is that **two documents are similar if they share many words in common**.

- The bag of words model doesn't consider **order**.
 - The job titles '`deputy fire chief`' and '`chief fire deputy`' are treated as the same.
- The bag of words model doesn't consider the **meaning** of words.
 - '`I love data science`' and '`I hate data science`' share 75% of their words, but have very different meanings.
- The bag of words model treats all words as being equally important.
 - '`deputy`' and '`fire`' have the same importance, even though '`fire`' is probably more important in describing someone's job title.
 - **Let's address this point.**

Question 😊

From the Fa23 final:

Consider the following corpus:

Document number	Content
1	yesterday rainy today sunny
2	yesterday sunny today sunny
3	today rainy yesterday today
4	yesterday yesterday today today

1. Using a bag-of-words representation, which two documents have the largest dot product?
2. Using a bag-of-words representation, what is the cosine similarity between documents 2 and 3?

Processing math: 100%

TF-IDF

The importance of words

Issue: The bag of words model doesn't know which words are "important" in a document. Consider the following document:

"my brother has a friend named **billy** who has an uncle named **billy**"

How do we determine which words are important?

- The most common words ("the", "has") often **don't** have much meaning!
- The very rare words are also less important!

Goal: Find a way of quantifying the importance of a word in a document by **balancing the above two factors**, i.e. find the word that **best summarizes** a document.

Term frequency

- The **term frequency** of a word (term) t in a document d , denoted $\text{tf}(t, d)$ is the proportion of words in document d that are equal to t .

$$\text{tf}(t, d) = \frac{\text{\# of occurrences of } t \text{ in } d}{\text{total \# of words in } d}$$

- **Example:** What is the term frequency of "billy" in the following document?

"my brother has a friend named **billy** who has an uncle named **billy**"

- **Answer:** $\frac{2}{13}$.

- Intuition: Words that occur often within a document are important to the document's meaning.
 - If $\text{tf}(t, d)$ is large, then word t occurs often in d .
 - If $\text{tf}(t, d)$ is small, then word t does not occur often in d .

- Issue: "has" also has a TF of $\frac{2}{13}$, but it seems less important than "billy".

Inverse document frequency

Processing math: 100%

- The **inverse document frequency** of a word t in a set of documents d_1, d_2, \dots is

$$\text{idf}(t) = \log \left(\frac{\text{total } \# \text{ of documents}}{\# \text{ of documents in which } t \text{ appears}} \right)$$

- Example:** What is the inverse document frequency of "billy" in the following three documents?
 - "my brother has a friend named **billy** who has an uncle named **billy**"
 - "my favorite artist is named jilly boel"
 - "why does he talk about someone named **billy** so often"

- Answer:** $\log\left(\frac{3}{2}\right) \approx 0.4055$.

- Intuition: If a word appears in every document (like "the" or "has"), it is probably not a good summary of any one document.
 - If $\text{idf}(t)$ is large, then t is rarely found in documents.
 - If $\text{idf}(t)$ is small, then t is commonly found in documents.
 - Think of $\text{idf}(t)$ as the "rarity factor" of t across documents – the larger $\text{idf}(t)$ is, the more rare t is.

Intuition

$$\text{tf}(t, d) = \frac{\# \text{ of occurrences of } t \text{ in } d}{\text{total } \# \text{ of words in } d}$$

$$\text{idf}(t) = \log \left(\frac{\text{total } \# \text{ of documents}}{\# \text{ of documents in which } t \text{ appears}} \right)$$

Goal: Quantify how well word t **summarizes** document d .

- If $\text{tf}(t, d)$ is small, then t doesn't occur very often in d , so t can't be a good summary of d .
- If $\text{idf}(t)$ is small, then t occurs often amongst all documents, and so it is not a good summary of any one document.
- If $\text{tf}(t, d)$ and $\text{idf}(t)$ are both large, then t **occurs often in d but rarely overall**. This makes t **a good summary** of document d .

Processing math: 100%

Term frequency-inverse document frequency

The **term frequency-inverse document frequency (TF-IDF)** of word t in document d is the product:

$$\begin{aligned} \text{tfidf}(t, d) &= \text{tf}(t, d) \cdot \text{idf}(t) \\ &= \frac{\# \text{ of occurrences of } t \text{ in } d}{\text{total } \# \text{ of words in } d} \cdot \log \left(\frac{\text{total } \# \text{ of documents}}{\# \text{ of documents in which } t \text{ appears}} \right) \end{aligned}$$

- If $\text{tfidf}(t, d)$ is large, then t is a good summary of d , because t occurs often in d but rarely across all documents.
- TF-IDF is a **heuristic** – it has no probabilistic justification.
- To know if $\text{tfidf}(t, d)$ is large for one particular word t , we need to compare it to $\text{tfidf}(t_i, d)$, for several different words t_i .

Computing TF-IDF

Question: What is the TF-IDF of "global" in the second sentence?

In [40]: sentences

Out[40]:

0	I really really want global peace
1	I must enjoy global warming
2	We must solve climate change

dtype: object

Answer:

In [41]:

```
tf = sentences.iloc[1].count('global') / len(sentences.iloc[1].split())
tf
```

Out[41]: 0.2

In [48]:

```
idf = np.log(len(sentences)) / sentences.str.contains('global').sum()
idf
```

Out[48]: np.float64(0.4054651081081644)

In [49]:

```
tf * idf
```

Out[49]:

0	0.00
1	0.08
2	0.00

Processing math: 100% type: float64

Question: Is this big or small? Is "global" the **best** summary of the second sentence?

TF-IDF of all words in all documents

On its own, the TF-IDF of a word in a document doesn't really tell us anything; we must compare it to TF-IDFs of other words in that same document.

In [50]: `sentences`

```
Out[50]: 0    I really really want global peace
          1          I must enjoy global warming
          2          We must solve climate change
         dtype: object
```

In [51]: `unique_words = np.unique(sentences.str.split().explode())`
`unique_words`

```
Out[51]: array(['I', 'We', 'change', 'climate', 'enjoy', 'global', 'must', 'peace',
   'really', 'solve', 'want', 'warming'], dtype=object)
```

In [52]: `tfidf_dict = {}`
`for word in unique_words:`
 `re_pat = fr'\b{word}\b'`
 `tf = sentences.str.count(re_pat) / sentences.str.split().str.len()`
 `idf = np.log(len(sentences) / sentences.str.contains(re_pat).sum())`
 `tfidf_dict[word] = tf * idf`
`tfidf = pd.DataFrame(tfidf_dict).set_index(sentences)`

In [53]: `tfidf`

	I	We	change	climate	...	really	solve	want	warming
I really really want global peace	0.07	0.00	0.00	0.00	...	0.37	0.00	0.18	0.00
I must enjoy global warming	0.08	0.00	0.00	0.00	...	0.00	0.00	0.00	0.22
We must solve climate change	0.00	0.22	0.22	0.22	...	0.00	0.22	0.00	0.00

3 rows × 12 columns

Interpreting TF-IDFs

In [54]: `display_df(tfidf, cols=12)`

Processing math: 100%

	I	We	change	climate	enjoy	global	must	peace	really	solve	want
I really really want global peace	0.07	0.00	0.00	0.00	0.00	0.07	0.00	0.18	0.37	0.00	0.18
I must enjoy global warming	0.08	0.00	0.00	0.00	0.22	0.08	0.08	0.00	0.00	0.00	0.00
We must solve climate change	0.00	0.22	0.22	0.22	0.00	0.00	0.08	0.00	0.00	0.22	0.00

The above DataFrame tells us that:

- the TF-IDF of 'really' in the first sentence is ≈ 0.37 ,
- the TF-IDF of 'climate' in the second sentence is 0.

Note that there are two ways that $\text{tfidf}(t, d) = \text{tf}(t, d) \cdot \text{idf}(t)$ can be 0:

- If t appears in every document, because then $\text{idf}(t) = \log(\frac{\# \text{ documents}}{\# \text{ documents}}) = \log(1) = 0$.
- If t does not appear in document d , because then $\text{tf}(t, d) = \frac{0}{\text{len}(d)} = 0$.

The word that **best summarizes** a document is the word with the highest TF-IDF for that document:

In [55]: `display_df(tfidf, cols=12)`

	I	We	change	climate	enjoy	global	must	peace	really	solve	want
I really really want global peace	0.07	0.00	0.00	0.00	0.00	0.07	0.00	0.18	0.37	0.00	0.18
I must enjoy global warming	0.08	0.00	0.00	0.00	0.22	0.08	0.08	0.00	0.00	0.00	0.00
We must solve climate change	0.00	0.22	0.22	0.22	0.00	0.00	0.08	0.00	0.00	0.22	0.00

In [56]: `+df.idxmax(axis=1)`
Processing math: 100%

```
Out[56]: I really really want global peace    really
          I must enjoy global warming        enjoy
          We must solve climate change       We
          dtype: object
```

Look closely at the rows of `tfidf` – in documents 2 and 3, the max TF-IDF is not unique!

Example: State of the Union addresses

State of the Union addresses

The 2024 State of the Union address was delivered on March 7th, 2024; last year's was on February 7th, 2023.

```
In [57]: from IPython.display import YouTubeVideo
YouTubeVideo('gzcBTUvVp7M')
```

Out[57]:



The data

```
In [63]: from pathlib import Path
sotu_txt = Path('data') / 'stateoftheunion1790-2023.txt'
sotu = sotu_txt.read_text()
```

```
In [64]: len(sotu)
```

Out[64]: 10577941

The entire **corpus** (another word for "set of documents") is over 10 million characters long... let's not display it in our notebook.

Processing math: 100%

```
In [65]: print(sotu[:1600])
```

The Project Gutenberg EBook of Complete State of the Union Addresses, from 1790 to the Present. Speeches beginning in 2002 are from UCSB The American Presidency Project.

Speeches from 2018–2023 were manually downloaded from whitehouse.gov.

Character set encoding: UTF8

The addresses are separated by three asterisks

CONTENTS

- George Washington, State of the Union Address, January 8, 1790
- George Washington, State of the Union Address, December 8, 1790
- George Washington, State of the Union Address, October 25, 1791
- George Washington, State of the Union Address, November 6, 1792
- George Washington, State of the Union Address, December 3, 1793
- George Washington, State of the Union Address, November 19, 1794
- George Washington, State of the Union Address, December 8, 1795
- George Washington, State of the Union Address, December 7, 1796
- John Adams, State of the Union Address, November 22, 1797
- John Adams, State of the Union Address, December 8, 1798
- John Adams, State of the Union Address, December 3, 1799
- John Adams, State of the Union Address, November 11, 1800
- Thomas Jefferson, State of the Union Address, December 8, 1801
- Thomas Jefferson, State of the Union Address, December 15, 1802
- Thomas Jefferson, State of the Union Address, October 17, 1803
- Thomas Jefferson, State of the Union Address, November 8, 1804
- Thomas Jefferson, State of the Union Address, December 3, 1805
- Thomas Jefferson, State of the Union Address, December 2, 1806
- Thomas Jefferson, State of the Union Address, October 27, 1807
- Thomas Jefferson, State of the Union Address,

Each speech is separated by '`***`'.

```
In [68]: speeches = sotu.split('\n***\n')[1:]
```

```
In [69]: len(speeches)
```

```
Out[69]: 233
```

Note that each "speech" currently contains other information, like the name of the president and the date of the address.

```
In [70]: print(speeches[-1][:1000])
```

State of the Union Address
 Joseph R. Biden Jr.
 February 7, 2023

Mr. Speaker. Madam Vice President. Our First Lady and Second Gentleman. Members of Congress and the Cabinet. Leaders of our military. Mr. Chief Justice, Associate Justices, and retired Justices of the Supreme Court. And you, my fellow Americans.

I start tonight by congratulating the members of the 118th Congress and the new Speaker of the House, Kevin McCarthy. Mr. Speaker, I look forward to working together.

I also want to congratulate the new leader of the House Democrats and the first Black House Minority Leader in history, Hakeem Jeffries.

Congratulations to the longest serving Senate leader in history, Mitch McConnell.

And congratulations to Chuck Schumer for another term as Senate Majority Leader, this time with an even bigger majority.

And I want to give special recognition to someone who I think will be considered the greatest Speaker in the history of this country, Nancy Pelosi.

The story of Amer

Let's extract just the speech text.

```
In [71]: import re
def extract_struct(speech):
    L = speech.strip().split('\n', maxsplit=3)
    L[3] = re.sub(r"^[A-Za-z ]", ' ', L[3]).lower()
    return dict(zip(['speech', 'president', 'date', 'contents'], L))
```

```
In [72]: speeches_df = pd.DataFrame(list(map(extract_struct, speeches)))
speeches_df
```

	speech	president	date	contents
0	State of the Union Address	George Washington	January 8, 1790	fellow citizens of the senate and house of re...
1	State of the Union Address	George Washington	December 8, 1790	fellow citizens of the senate and house of re...
2	State of the Union Address	George Washington	October 25, 1791	fellow citizens of the senate and house of re...
...
230	State of the Union Address	Joseph R. Biden Jr.	April 28, 2021	thank you thank you thank you good to be b...
231	State of the Union Address	Joseph R. Biden Jr.	March 1, 2022	madam speaker madam vice president and our ...
232	State of the Union Address	Joseph R. Biden Jr.	February 7, 2023	mr speaker madam vice president our firs...

233 rows × 4 columns

Processing math: 100%

Finding the most important words in each speech

Here, a "document" is a speech. We have 233 documents.

In [73]: `speeches_df`

	speech	president	date	contents
0	State of the Union Address	George Washington	January 8, 1790	fellow citizens of the senate and house of re...
1	State of the Union Address	George Washington	December 8, 1790	fellow citizens of the senate and house of re...
2	State of the Union Address	George Washington	October 25, 1791	fellow citizens of the senate and house of re...
...
230	State of the Union Address	Joseph R. Biden Jr.	April 28, 2021	thank you thank you thank you good to be b...
231	State of the Union Address	Joseph R. Biden Jr.	March 1, 2022	madam speaker madam vice president and our ...
232	State of the Union Address	Joseph R. Biden Jr.	February 7, 2023	mr speaker madam vice president our firs...

233 rows × 4 columns

A rough sketch of what we'll compute:

```
for each word t:
    for each speech d:
        compute tfidf(t, d)
```

In [74]: `unique_words = speeches_df['contents'].str.split().explode().value_counts()
Take the top 500 most common words for speed
unique_words = unique_words.iloc[:500].index
unique_words`

Out[74]: `Index(['the', 'of', 'to', 'and', 'in', 'a', 'that', 'for', 'be', 'our',
 ...
 'desire', 'call', 'submitted', 'increasing', 'months', 'point', 'true',
 'throughout', 'set', 'object'],
 dtype='object', name='contents', length=500)`

💡 Pro-Tip: Using `tqdm`

This code takes a while to run, so we'll use the `tqdm` package to track its progress.
(Install with `mamba install tqdm` if needed).

Processing math: 100%

```
In [75]: from tqdm.notebook import tqdm

tfidf_dict = {}
tf_denom = speeches_df['contents'].str.split().str.len()

# Wrap the sequence with `tqdm()` to display a progress bar
for word in tqdm(unique_words):
    re_pat = fr' {word} ' # Imperfect pattern for speed.
    tf = speeches_df['contents'].str.count(re_pat) / tf_denom
    idf = np.log(len(speeches_df)) / speeches_df['contents'].str.contains(re_
    tfidf_dict[word] = tf * idf

0% | 0/500 [00:00<?, ?it/s]
```

```
In [76]: tfidf = pd.DataFrame(tfidf_dict)
tfidf.head()
```

```
Out[76]:   the   of   to   and   ...    trust  throughout      set   object
0   0.0   0.0   0.0   0.0   ...  4.29e-04  0.00e+00  0.00e+00  2.04e-03
1   0.0   0.0   0.0   0.0   ...  0.00e+00  0.00e+00  0.00e+00  1.06e-03
2   0.0   0.0   0.0   0.0   ...  4.06e-04  0.00e+00  3.48e-04  6.44e-04
3   0.0   0.0   0.0   0.0   ...  6.70e-04  2.17e-04  0.00e+00  7.09e-04
4   0.0   0.0   0.0   0.0   ...  2.38e-04  4.62e-04  0.00e+00  3.77e-04
```

5 rows × 500 columns

Note that the TF-IDFs of many common words are all 0!

Summarizing speeches

By using `idxmax`, we can find the word with the highest TF-IDF in each speech.

```
In [77]: summaries = tfidf.idxmax(axis=1)
summaries
```

```
Out[77]: 0        object
1        convention
2        provision
...
230       it's
231       tonight
232       it's
Length: 233, dtype: object
```

What if we want to see the 5 words with the highest TF-IDFs, for each speech?

```
In [78]: def five_largest(row):
    return ', '.join(row.index[row.argsort()][-5:])
```

Processing math: 100%

```
In [79]: keywords = tfidf.apply(five_largest, axis=1)
keywords_df = pd.concat([
    speeches_df['president'],
    speeches_df['date'],
    keywords
], axis=1)
```

```
In [80]: keywords_df
```

Out[80]:

	president	date	0
0	George Washington	January 8, 1790	your, proper, regard, ought, object
1	George Washington	December 8, 1790	case, established, object, commerce, convention
2	George Washington	October 25, 1791	community, upon, lands, proper, provision
...
230	Joseph R. Biden Jr.	April 28, 2021	get, americans, percent, jobs, it's
231	Joseph R. Biden Jr.	March 1, 2022	let, jobs, americans, get, tonight
232	Joseph R. Biden Jr.	February 7, 2023	down, percent, jobs, tonight, it's

233 rows × 3 columns

Uncomment the cell below to see every single row of `keywords_df`.

```
In [81]: # display_df(keywords_df, rows=233)
```

Aside: What if we remove the log from $\text{idf}(t)$?

Let's try it and see what happens.

```
In [82]: tfidf_nl_dict = {}
tf_denom = speeches_df['contents'].str.split().str.len()

for word in tqdm(unique_words):
    re_pat = fr' {word} # Imperfect pattern for speed.
    tf = speeches_df['contents'].str.count(re_pat) / tf_denom
    idf_nl = len(speeches_df) / speeches_df['contents'].str.contains(re_pat)
    tfidf_nl_dict[word] = tf * idf_nl

0% | 0/500 [00:00<?, ?it/s]
```

```
In [83]: tfidf_nl = pd.DataFrame(tfidf_nl_dict)
Processing math: 100% df_nl.head()
```

Out[83]:

	the	of	to	and	...	trust	throughout	set	object
0	0.09	0.06	0.05	0.04	...	1.47e-03	0.00e+00	0.00e+00	5.78e-03
1	0.09	0.06	0.03	0.03	...	0.00e+00	0.00e+00	0.00e+00	2.99e-03
2	0.11	0.07	0.04	0.03	...	1.39e-03	0.00e+00	1.30e-03	1.82e-03
3	0.09	0.07	0.04	0.03	...	2.29e-03	7.53e-04	0.00e+00	2.01e-03
4	0.09	0.07	0.04	0.02	...	8.12e-04	1.60e-03	0.00e+00	1.07e-03

5 rows × 500 columns

In [84]:

```
keywords_nl = tfidf_nl.apply(five_largest, axis=1)
keywords_nl_df = pd.concat([
    speeches_df['president'],
    speeches_df['date'],
    keywords_nl
], axis=1)
keywords_nl_df
```

Out[84]:

	president	date	0
0	George Washington	January 8, 1790	a, and, to, of, the
1	George Washington	December 8, 1790	in, and, to, of, the
2	George Washington	October 25, 1791	a, and, to, of, the
...
230	Joseph R. Biden Jr.	April 28, 2021	of, it's, and, to, the
231	Joseph R. Biden Jr.	March 1, 2022	we, of, to, and, the
232	Joseph R. Biden Jr.	February 7, 2023	a, of, and, to, the

233 rows × 3 columns

The role of log in $\text{idf}(t)$

$$\text{tfidf}(t, d) = \text{tf}(t, d) \cdot \text{idf}(t)$$

$$= \frac{\text{\# of occurrences of } t \text{ in } d}{\text{total \# of words in } d} \cdot \log \left(\frac{\text{total \# of documents}}{\text{\# of documents in which } t \text{ appears}} \right)$$

- Remember, for any positive input x , $\log(x)$ is (much) smaller than x .
- In $\text{idf}(t)$, the log "dampens" the impact of the ratio $\frac{\text{\# documents}}{\text{\# documents with } t}$.
- If a word is very common, the ratio will be close to 1. The log of the ratio will be close to 0.

Processing math: 100%

```
In [85]: (1000 / 999)
```

```
Out[85]: 1.001001001001001
```

```
In [86]: np.log(1000 / 999)
```

```
Out[86]: np.float64(0.001000500333583622)
```

- If a word is very common (e.g. 'the'), removing the log multiplies the statistic by a large factor.
- If a word is very rare, the ratio will be very large. However, for instance, a word being seen in **2 out of 50** documents is not very different than being seen in **2 out of 500** documents (it is very rare in both cases), and so $\text{idf}(t)$ should be similar in both cases.

```
In [87]: (50 / 2)
```

```
Out[87]: 25.0
```

```
In [88]: (500 / 2)
```

```
Out[88]: 250.0
```

```
In [89]: np.log(50 / 2)
```

```
Out[89]: np.float64(3.2188758248682006)
```

```
In [90]: np.log(500 / 2)
```

```
Out[90]: np.float64(5.521460917862246)
```

Question 🤔

From the Fa23 final:

Consider the following corpus:

Document number	Content
1	yesterday rainy today sunny
2	yesterday sunny today sunny
3	today rainy yesterday today
4	yesterday yesterday today today

Which words have a TF-IDF score of 0 for all four documents?

Summary

- One way to turn documents, like `'deputy fire chief'`, into feature vectors, is to count the number of occurrences of each word in the document, ignoring order. This is done using the **bag of words** model.
- To measure the similarity of two documents under the bag of words model, compute the cosine similarity of their two word vectors.
- Term frequency-inverse document frequency (TF-IDF) is a statistic that tries to quantify how **important** a word (term) is to a document. It balances:
 - **how often a word appears in a particular document**, $\text{tf}(t, d)$, with
 - **how often a word appears across documents**, $\text{idf}(t)$.
- For a given document, the word with the highest TF-IDF is thought to "best summarize" that document.

Next time

Modeling and feature engineering.