



Some Important Determinants

$$\begin{aligned} & \bullet \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c) \\ & \bullet \underbrace{\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}}_{\text{Degree} = 4} = \underbrace{(a-b)(b-c)(c-a)}_{\text{Degree} = 3} \underbrace{(a+b+c)}_{\text{Linear term}} \end{aligned}$$

Proof:

$$\Delta = 1(b^1c^3 - b^3c)$$

Put $a = b \Rightarrow \Delta = 0 \Rightarrow (a - b)$ is a factor of Δ

$b = c \Rightarrow \Delta = 0 \Rightarrow (b - c)$ is a factor of Δ

$c = a \Rightarrow \Delta = 0 \Rightarrow (c - a)$ is a factor of Δ

