Evaluate $\begin{vmatrix} 1 & \cos(B-A) & \cos(C-A) \\ \cos(A-B) & 1 & \cos(C-B) \\ \cos(A-C) & \cos(B-C) & 1 \end{vmatrix}$

A $\cos A \cos B \cos C$

B 1

C 0

D $\cos A+\cos B+\cos C$

$$\begin{vmatrix} 1 & \cos(B-A) & \cos(C-A) \\ \cos(A-B) & 1 & \cos(C-B) \\ \cos(A-C) & \cos(B-C) & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \cos(A-A) & \cos(B-A) & \cos(C-A) \\ \cos(A-B) & \cos(B-B) & \cos(C-B) \\ \cos(A-C) & \cos(B-C) & \cos(C-C) \end{vmatrix}$$

$$= \begin{vmatrix} \cos A\cos A+\sin A\sin A & \cos B\cos A+\sin B\sin A & \cos C\cos A+\sin C\sin A \\ \cos A\cos B+\sin A\sin B & \cos B\cos B+\sin B\sin B & \cos C\cos B+\sin C\sin B \\ \cos A\cos C+\sin A\sin C & \cos B\cos C+\sin B\sin C & \cos C\cos C+\sin C\sin C \end{vmatrix}$$

$$= \begin{vmatrix} \cos A & \sin A & 1 \\ \cos B & \sin B & 1 \\ \cos C & \sin C & 1 \end{vmatrix} \times \begin{vmatrix} \cos A & \cos B & \cos C \\ \sin A & \sin B & \sin C \\ 0 & 0 & 0 \end{vmatrix} = 0$$

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