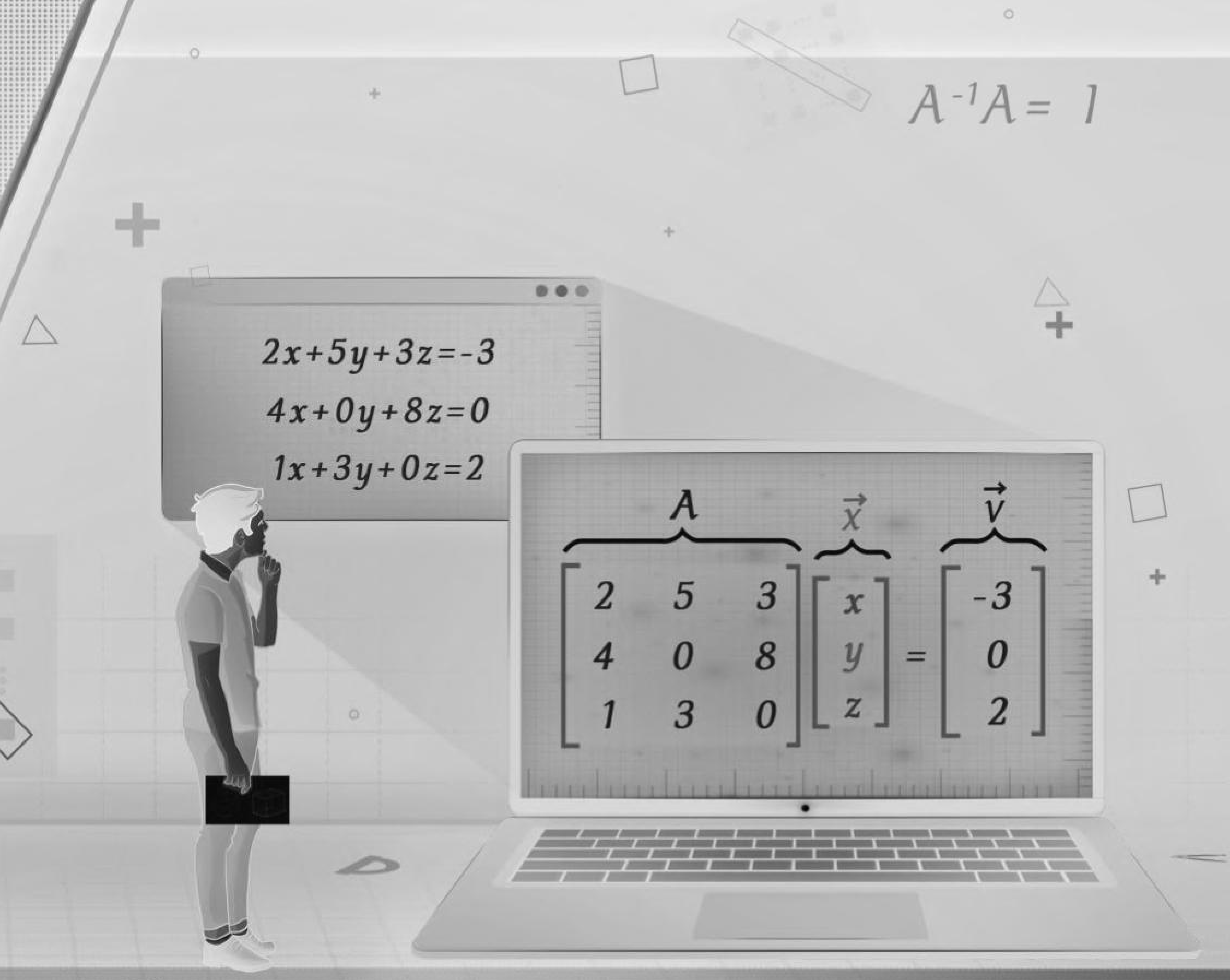
Welcome to

Aakash + BYJU'S LIVE

Matrices & Determinants



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Session 01

Introduction to Matrices

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Key Takeaways

- A rectangular arrangement of $m \cdot n$ numbers (real or complex) or expressions (real or complex valued), having m rows and n columns is called a matrix. $(m, n \in N)$

$$A = \underbrace{\begin{bmatrix} a_{11} & a_{12} & a_{13} \cdots a_{1n} \\ a_{21} & a_{22} & a_{23} \cdots a_{2n} \\ \vdots & \vdots & \vdots \quad\quad \vdots \\ a_{m1} & a_{m2} & a_{m3} \cdots a_{mn} \end{bmatrix}}_{\text{Columns}} \left.\begin{matrix} \leftarrow \\ \leftarrow \\ \\ \leftarrow \end{matrix}\right\} \text{Rows}$$

- An element of a matrix is denoted by

  a_{ij}: Element of i^{th} row & j^{th} column.

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Key Takeaways

- A rectangular arrangement of $m \cdot n$ numbers (real or complex) or expressions (real or complex valued), having m rows and n columns is called a matrix. $(m, n \in N)$

$$A = \begin{array}{cl} \begin{bmatrix} a_{11} & a_{12} & a_{13} \cdots a_{1n} \\ a_{21} & a_{22} & a_{23} \cdots a_{2n} \\ \vdots & \vdots & \vdots \quad\;\; \vdots \\ a_{m1} & a_{m2} & a_{m3} \cdots a_{mn} \end{bmatrix} & \text{Rows} \\ \text{Columns} & \end{array}$$

- Number of elements in a matrix

$$= \text{Number of rows} \times \text{Number of columns}$$
$$= m \times n$$

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Write $a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}$ for the following matrix:

$$A = \begin{bmatrix} 1 & 0 & 5 \\ -2 & 3 & -8 \end{bmatrix}$$

Solution :

$$a_{11} = 1$$

$$a_{12} = 0$$

$$a_{13} = 5$$

$$a_{21} = -2$$

$$a_{22} = 3$$

$$a_{23} = -8$$

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Find the value a_{23} in the following matrix

$$A = \begin{pmatrix} 3 & -4 & 0 \\ -2 & 7 & 10 \\ 5 & -6 & 9 \end{pmatrix}$$

A -6

B 0

C 10

D 5

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Key Takeaways

Order of a matrix

Order or dimension of a matrix denotes the arrangement of elements as number of rows and number of columns.

- Order = Number of rows × Number of columns $= m \times n$

Name of a matrix

$$A_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

Rows

Order of a matrix

Columns

- Thus, a matrix can also be represented as $A = [a_{ij}]_{m\times n}$ or $(a_{ij})_{m\times n}$

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Types of Matrix:

- Row Matrix (row vector) : A matrix having a single row is called a row matrix.

$$A = [a_{ij}]_{1\times n} = [a_{11} \quad a_{12} \quad a_{13} \cdots a_{1n}]_{1\times n}$$

Example: $B = [a \quad b \quad c]_{1\times 3}$

- Column Matrix (column vector) : A matrix having a single column is called a column matrix.

Example: $B = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}_{4\times 1}$

$$A = [a_{ij}]_{m\times 1} = \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}_{m\times 1}$$

- Matrices consisting of one row or one column are called vectors.

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Types of Matrix:

- Zero Matrix (null matrix) : If all the elements of a matrix are zero, then it is called zero or null matrix

$A = [a_{ij}]_{m \times n}$ is called a zero matrix, if $a_{ij} = 0, \forall\, i \,\&\, j$.

Examples:

$$A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

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Vertical Matrix

A matrix of order $m \times n$ is known as vertical matrix if $m > n$, where m is equal to the number of rows and n is equal to the number of columns.

Example: $\begin{bmatrix} 2 & 5 \\ 1 & 1 \\ 3 & 6 \\ 2 & 4 \end{bmatrix}$

- In the matrix example given the number of rows $(m) = 4$, whereas the number of columns $(n) = 2$.

  Therefore, this makes the matrix a vertical matrix.

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Horizontal Matrix

A matrix of order $m \times n$ is known as vertical matrix if $n > m$, where m is equal to the number of rows and n is equal to the number of columns.

Example: $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 5 & 1 & 1 \end{bmatrix}$

- In the matrix example given the number of rows $(m) = 2$, whereas the number of columns $(n) = 4$.

  Therefore, this makes the matrix a horizontal matrix.

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If a matrix has 12 elements, then what are the possible orders it can have?

Solution :

Number of elements = Number of rows × Number of columns

$12 = m \times n \ \ (m, n \in N)$

Possible Order $= 1 \times 12\ , 2 \times 6\ ,3 \times 4\ ,4 \times 3, 6 \times 2,,12 \times 1$

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Construct a 2×3 matrix, whose elements are given by $a_{ij} = \frac{(i+2j)}{3}$.

Solution :

$$a_{ij} = \frac{(i+2j)}{3}$$

$$a_{11} = 1 \qquad a_{12} = \frac{5}{3} \qquad a_{13} = \frac{7}{3}$$

$$a_{21} = \frac{4}{3} \qquad a_{22} = 2 \qquad a_{23} = \frac{8}{3}$$

$$A = \begin{pmatrix} 1 & \frac{5}{3} & \frac{7}{3} \\ \frac{4}{3} & 2 & \frac{8}{3} \end{pmatrix}$$

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Key Takeaways

- Principal Diagonal of a Matrix : Diagonal containing the elements a_{ij}, where $i = j$ is called principal diagonal of a matrix

Examples:

$$A = \begin{bmatrix} 2 & -6 & 10 \\ 5 & 0 & 7 \\ 19 & -3 & -8 \end{bmatrix}_{3\times 3} \qquad B = \begin{pmatrix} 2 & 3 & 4 & -5 \\ 1 & 4 & 0 & 6 \\ -3 & 7 & 8 & 9 \end{pmatrix}_{3\times 4}$$

Types of Matrix:

- Square Matrix : A matrix where number of rows = number of columns is called square matrix.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \cdots a_{1n} \\ a_{21} & a_{22} & a_{23} \cdots a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & a_{n3} \cdots a_{nn} \end{bmatrix}_{n\times n}$$

Example:

$$A = \begin{bmatrix} -4 & 5 & 0 \\ 8 & -1 & 3 \\ 9 & 7 & 2 \end{bmatrix}_{3\times 3}$$

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