

If A is idempotent and $(I + A)^{10} = I + kA$, then k is:

A

1023

B

2047

C

1024

D

2048

Solution: $(I + A)^{10} = {}^{10}C_0 I + {}^{10}C_1 I \cdot A + {}^{10}C_2 I \cdot A^2 + \dots + {}^{10}C_{10} A^{10} \quad A^n = A$

$$= I + {}^{10}C_1 A + {}^{10}C_2 A + \dots + {}^{10}C_{10} A \quad A^n = A$$

$$= I + ({}^{10}C_1 + {}^{10}C_2 + \dots + {}^{10}C_{10})A$$

$$= I + (2^{10} - 1)A$$

$$= I + (1024 - 1)A$$

$$\therefore k = 1023$$