Let $f(x) = \begin{vmatrix} \sec x & \cos x & \sec^2 x + \cot x \csc x \\ \cos^2 x & \cos^2 x & \csc^2 x \\ 1 & \cos^2 x & \cos^2 x \end{vmatrix}$, Prove that : $\int_0^{\pi/2} f(x)\, dx = -\left(\dfrac{\pi}{4} + \dfrac{8}{15}\right)$

Solution:

$$f(x) = \left(1 + \frac{\cos^3 x}{\sin^2 x} - \cos^3 x\right)(\cos^2 x - 1)$$

$$= -\sin^2 x \,\frac{\sin^2 x + \cos^3 x - \cos^3 x \sin^2 x}{\sin^2 x}$$

$$f(x) = -\left(\sin^2 x + \cos^5 x\right)$$

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$$\int_0^{\pi/2} f(x)\, dx = \int_0^{\pi/2} \left(\sin^2 x + \cos^5 x\right) dx$$

$$= -\left(\frac{1}{2} \cdot \frac{\pi}{2} + \frac{4 \cdot 2}{5 \cdot 3}\right) = -\left(\frac{\pi}{4} + \frac{8}{15}\right)$$