



Properties of determinant

If any of two rows (or columns) of a determinant are interchanged, then its value gets multiplied by (-1).

$$\Delta = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$\Delta = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \qquad \Delta' = \begin{vmatrix} g & h & i \\ d & e & f \\ a & b & c \end{vmatrix} \Rightarrow \Delta' = -\Delta$$

$$R_1 \leftrightarrow R_2$$

Proof:

$$\Delta_1 = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\Delta_2 = \begin{vmatrix} a_{21} & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\Rightarrow \Delta_1 = a_{11}M_{11} - a_{12}M_{12} + a_{13}M_{13}$$

With respect to second row
$$\Rightarrow \Delta_2 = -a_{11}M_{11} + a_{12}M_{12} - a_{13}M_{13}$$

$$\Rightarrow \Delta_2 = -\Delta_1$$