If $|A| = \begin{vmatrix} e^{-t} & e^{-t}\cos t & e^{-t}\sin t \\ e^{-t} & -e^{-t}\cos t - e^{-t}\sin t & e^{-t}\cos t - e^{-t}\sin t \\ e^{-t} & 2e^{-t}\sin t & -2e^{-t}\cos t \end{vmatrix}$, then A is

JEE MAIN JAN 2019

$$|A| = \begin{vmatrix} e^{-t} & e^{-t}\cos t & e^{-t}\sin t \\ e^{-t} & -e^{-t}\cos t - e^{-t}\sin t & e^{-t}\cos t - e^{-t}\sin t \\ e^{-t} & 2e^{-t}\sin t & -2e^{-t}\cos t \end{vmatrix}$$

$$\Rightarrow |A| = e^{-3t}\begin{vmatrix} 1 & \cos t & \sin t \\ 1 & -\cos t - \sin t & \cos t - \sin t \\ 1 & 2\sin t & -2\cos t \end{vmatrix}$$

$$R_1 = R_1 + R_2 + \frac{1}{2}R_3$$

$$\Rightarrow |A| = e^{-3t}\begin{vmatrix} \frac{5}{2} & 0 & 0 \\ 1 & -\cos t - \sin t & \cos t - \sin t \\ 1 & 2\sin t & -2\cos t \end{vmatrix}$$

$$\Rightarrow |A| = e^{-3t} \cdot \frac{5}{2}(2\cos^2 t + 2\sin t\cos t - 2\sin t\cos t + 2\sin^2 t)$$

$$\Rightarrow |A| = e^{-3t}(5) \neq 0 \quad \therefore A \text{ is invertible for all } t \in \mathbb{R}$$

A. Non-invertible for any $t \in \mathbb{R}$

B. Invertible only if $t = \frac{\pi}{2}$

C. Invertible only if $t = \pi$

D. Invertible for all $t \in \mathbb{R}$

Return To Top