

Let S be the set of all integer solutions (x, y, z) , of the system of equations $x - 2y + 5z = 0$; $-2x + 4y + z = 0$; $-7x + 14y + 9z = 0$, such that $15 \leq x^2 + y^2 + z^2 \leq 150$. Then, the number of elements in the set S is ____

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Solution: $x - 2y + 5z = 0 \dots (i)$

$$-2x + 4y + z = 0 \dots (ii)$$

$$15 \leq x^2 + y^2 + z^2 \leq 150$$

$$-7x + 14y + 9z = 0 \dots (iii)$$

$$\Delta = \begin{vmatrix} 1 & -2 & 5 \\ -2 & 4 & 1 \\ -7 & 14 & 9 \end{vmatrix} = 0$$

Let $x = k$, in (i) & (ii)

$$k - 2y + 5z = 0 \Rightarrow 2y - 5z = k$$

$$\Rightarrow -2k + 4y + z = 0 \Rightarrow 4y + z = 2k$$

$$\Rightarrow z = 0, y = \frac{k}{2} \text{ Since } x, y, z \text{ are integers, } k = \text{even integer}$$