

Welcome to



Matrices & Determinants

$$2x + 5y + 3z = -3$$

$$4x + 0y + 8z = 0$$

$$1x + 3y + 0z = 2$$

$$A^{-1}A = I$$

$$\begin{matrix} A & \vec{x} & = & \vec{v} \\ \begin{bmatrix} 2 & 5 & 3 \\ 4 & 0 & 8 \\ 1 & 3 & 0 \end{bmatrix} & \begin{bmatrix} x \\ y \\ z \end{bmatrix} & = & \begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix} \end{matrix}$$

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Session 01

Introduction to Matrices

Key Takeaways

- A rectangular arrangement of $m \cdot n$ numbers (real or complex) or expressions (real or complex valued), having m rows and n columns is called a matrix. ($m, n \in N$)

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

Diagram illustrating the structure of a matrix A with m rows and n columns. The elements are arranged in a grid. The first row is labeled $a_{11}, a_{12}, a_{13}, \dots, a_{1n}$. The second row is labeled $a_{21}, a_{22}, a_{23}, \dots, a_{2n}$. The third row is labeled $\vdots, \vdots, \vdots, \vdots$. The last row is labeled $a_{m1}, a_{m2}, a_{m3}, \dots, a_{mn}$. The columns are labeled $a_{11}, a_{21}, \vdots, a_{m1}$ for the first column, $a_{12}, a_{22}, \vdots, a_{m2}$ for the second column, $a_{13}, a_{23}, \vdots, a_{m3}$ for the third column, and $a_{1n}, a_{2n}, \vdots, a_{mn}$ for the n -th column. The label "Rows" is placed to the right of the matrix, and the label "Columns" is placed below the matrix.

- An element of a matrix is denoted by a_{ij} : Element of i^{th} row & j^{th} column.

- A rectangular arrangement of $m \cdot n$ numbers (real or complex) or expressions (real or complex valued), having m rows and n columns is called a matrix. ($m, n \in N$)

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \cdots a_{1n} \\ a_{21} & a_{22} & a_{23} \cdots a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} \cdots a_{mn} \end{bmatrix}$$

Diagram illustrating the structure of a matrix A with m rows and n columns. The elements are arranged in a grid. The first row is $a_{11}, a_{12}, a_{13}, \dots, a_{1n}$. The second row is $a_{21}, a_{22}, a_{23}, \dots, a_{2n}$. The third row is $\vdots, \vdots, \vdots, \vdots$. The last row is $a_{m1}, a_{m2}, a_{m3}, \dots, a_{mn}$. Brackets on the right side of the matrix indicate the rows, and brackets on the bottom side indicate the columns.

- Number of elements in a matrix
= Number of rows \times Number of columns
= $m \times n$



Write $a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}$ for the following matrix:

$$A = \begin{bmatrix} 1 & 0 & 5 \\ -2 & 3 & -8 \end{bmatrix}$$

Solution :

$$a_{11} = 1$$

$$a_{12} = 0$$

$$a_{13} = 5$$

$$a_{21} = -2$$

$$a_{22} = 3$$

$$a_{23} = -8$$



Find the value a_{23} in the following matrix

$$A = \begin{pmatrix} 3 & -4 & 0 \\ -2 & 7 & 10 \\ 5 & -6 & 9 \end{pmatrix}$$

A

-6

B

0

C

10

D

5

Order of a matrix

Order or dimension of a matrix denotes the arrangement of elements as number of rows and number of columns.

- Order = Number of rows \times Number of columns = $m \times n$

Name of a matrix

Order of a matrix

$$A_{m \times n} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

Rows

Columns

- Thus, a matrix can also be represented as $A = [a_{ij}]_{m \times n}$ or $(a_{ij})_{m \times n}$



Types of Matrix:

- Row Matrix (row vector) : A matrix having a single row is called a row matrix.

$$A = [a_{ij}]_{1 \times n} = [a_{11} \quad a_{12} \quad a_{13} \cdots a_{1n}]_{1 \times n}$$

Example: $B = [a \quad b \quad c]_{1 \times 3}$

- Column Matrix (column vector) : A matrix having a single column is called a column matrix.

Example: $B = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}_{4 \times 1}$

$$A = [a_{ij}]_{m \times 1} = \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}_{m \times 1}$$

- Matrices consisting of one row or one column are called vectors.



Types of Matrix:



- Zero Matrix (null matrix) : If all the elements of a matrix are zero, then it is called zero or null matrix

$A = [a_{ij}]_{m \times n}$ is called a zero matrix, if $a_{ij} = 0, \forall i \text{ \& } j$.

Examples:

$$A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$





Vertical Matrix

A matrix of order $m \times n$ is known as vertical matrix if $m > n$, where m is equal to the number of rows and n is equal to the number of columns.

Example:
$$\begin{bmatrix} 2 & 5 \\ 1 & 1 \\ 3 & 6 \\ 2 & 4 \end{bmatrix}$$

- In the matrix example given the number of rows (m) = 4, whereas the number of columns (n) = 2.

Therefore, this makes the matrix a vertical matrix.



Horizontal Matrix

A matrix of order $m \times n$ is known as vertical matrix if $n > m$, where m is equal to the number of rows and n is equal to the number of columns.

Example:
$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 5 & 1 & 1 \end{bmatrix}$$

- In the matrix example given the number of rows (m) = 2, whereas the number of columns (n) = 4.

Therefore, this makes the matrix a horizontal matrix.



If a matrix has 12 elements, then what are the possible orders it can have?

Solution :

Number of elements = Number of rows \times Number of columns

$$12 = m \times n \quad (m, n \in N)$$

Possible Order = $1 \times 12, 2 \times 6, 3 \times 4, 4 \times 3, 6 \times 2, 12 \times 1$



Construct a 2×3 matrix, whose elements are given by $a_{ij} = \frac{(i+2j)}{3}$.

Solution :

$$a_{ij} = \frac{(i+2j)}{3}$$

$$a_{11} = 1$$

$$a_{12} = \frac{5}{3}$$

$$a_{13} = \frac{7}{3}$$

$$a_{21} = \frac{4}{3}$$

$$a_{22} = 2$$

$$a_{23} = \frac{8}{3}$$

$$A = \begin{pmatrix} 1 & \frac{5}{3} & \frac{7}{3} \\ \frac{4}{3} & 2 & \frac{8}{3} \end{pmatrix}$$

- Principal Diagonal of a Matrix: Diagonal containing the elements a_{ij} , where $i = j$ is called principal diagonal of a matrix

Examples:

$$A = \begin{bmatrix} 2 & -6 & 10 \\ 5 & 0 & 7 \\ 19 & -3 & -8 \end{bmatrix}_{3 \times 3}$$

$$B = \begin{pmatrix} 2 & 3 & 4 & -5 \\ 1 & 4 & 0 & 6 \\ -3 & 7 & 8 & 9 \end{pmatrix}_{3 \times 4}$$

Types of Matrix:

- Square Matrix: A matrix where number of rows = number of columns is called square matrix.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \cdots a_{1n} \\ a_{21} & a_{22} & a_{23} \cdots a_{2n} \\ \vdots & \vdots & \vdots \quad \vdots \\ a_{n1} & a_{n2} & a_{n3} \cdots a_{nn} \end{bmatrix}_{n \times n}$$

Example:

$$A = \begin{bmatrix} -4 & 5 & 0 \\ 8 & -1 & 3 \\ 9 & 7 & 2 \end{bmatrix}_{3 \times 3}$$