Let $a, b, c \in R$ be all non – zero and satisfy $a^3 + b^3 + c^3 = 2$. If the matrix $A = \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$ satisfies $A^T A = I$, then a value of abc can be :

Solution: $A^T A = I$

$\Rightarrow |A^T A| = |I|$

$\Rightarrow |A^T||A| = 1$

$\Rightarrow |A|^2 = 1$

$\Rightarrow |A| = \pm 1$

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 3abc - a^3 - b^3 - c^3$$

$\Rightarrow 3abc - a^3 - b^3 - c^3 = \pm 1$

$\Rightarrow 3abc = 1, 3 \quad \Rightarrow abc = \frac{1}{3}, 1$

A $\frac{2}{3}$

B $-\frac{1}{3}$

C 3

D $\frac{1}{3}$

Return To Top