If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}$ & $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $A^{-1} = \frac{1}{6}(A^2 + cA + dI)$, then the ordered pair (c, d) is:

A $(-6, -11)$

B $(6, -11)$

C $(-6, 11)$

D $(6, 11)$

Solution:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix} \quad A^{-1} = \frac{1}{6}(A^2 + cA + dI) \quad (c, d) = ?$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 1 \\ 0 & -2 & 4-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)\big((1-\lambda)(4-\lambda) + 2\big) = 0$$

$$\Rightarrow (1-\lambda)(\lambda^2 - 5\lambda + 6) = 0$$

$$\Rightarrow \lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$$ → characteristic equation

By Cayley – Hamilton theorem,

$$A^3 - 6A^2 + 11A - 6I = 0$$

Return To Top