If a, b, c are all different and $\begin{vmatrix} a & a^3 & a^4-1 \\ b & b^3 & b^4-1 \\ c & c^3 & c^4-1 \end{vmatrix} = 0$, then the value of $abc(ab+bc+ca)$ is equal to:

A. $a-b-c$

B. $a-b+c$

C. $a+b+c$

D. 0

Solution:

$$\Rightarrow \begin{vmatrix} a & a^3 & a^4 \\ b & b^3 & b^4 \\ c & c^3 & c^4 \end{vmatrix} + \begin{vmatrix} a & a^3 & -1 \\ b & b^3 & -1 \\ c & c^3 & -1 \end{vmatrix} = 0$$

Taking a, b, c from the first determinant and apply $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$ in both determinants

$$\Rightarrow abc \begin{vmatrix} 1 & a^2 & a^3 \\ 0 & b^2-a^2 & b^3-a^3 \\ 0 & c^2-a^2 & c^3-a^3 \end{vmatrix} - \begin{vmatrix} a & a^3 & 1 \\ b-a & b^3-a^3 & 0 \\ c-a & c^3-a^3 & 0 \end{vmatrix} = 0$$

As a, b, c are all distinct and cancelling out $b-a$ and $c-a$

$$\Rightarrow abc \begin{vmatrix} b+a & b^2+a^2+ab \\ c+a & c^2+a^2+ac \end{vmatrix} = \begin{vmatrix} 1 & b^2+a^2+ab \\ 1 & c^2+a^2+ac \end{vmatrix}$$

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