



If  $a, b, c$  are all different and  $\begin{vmatrix} a & a^3 & a^4 - 1 \\ b & b^3 & b^4 - 1 \\ c & c^3 & c^4 - 1 \end{vmatrix} = 0$ , then the value of  $abc(ab + bc + ca)$  is equal to:

Solution:

$$\Rightarrow abc \begin{vmatrix} b+a & b^2+a^2+ab \\ c+a & c^2+a^2+ac \end{vmatrix} = \begin{vmatrix} 1 & b^2+a^2+ab \\ 1 & c^2+a^2+ac \end{vmatrix}$$

Applying  $R_2 \rightarrow R_2 - R_1$  and then cancelling  $c - b$  on both sides, we get

$$\Rightarrow abc \begin{vmatrix} b+a & b^2+a^2+ab \\ 1 & a+b+c \end{vmatrix} = \begin{vmatrix} 1 & b^2+a^2+ab \\ 0 & a+b+c \end{vmatrix}$$

$$\therefore abc(ab + b^2 + bc + a^2 + ab + ac - b^2 - c^2 - ab) = a + b + c$$

$$\Rightarrow abc(ab + bc + ca) = a + b + c$$

A

$$a - b - c$$

B

$$a - b + c$$

C

$$a + b + c$$

D

$$0$$