Solve the system of equations :

$x + y + z = 6 \; ; \; x - y + z = 2 ; \; 2x + y - z = 1$, using matrix inverse.

Solution:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix}$$

$$C_3 \rightarrow C_2 + C_3 \quad |A| = \begin{vmatrix} 1 & 1 & 2 \\ 1 & -1 & 0 \\ 2 & 1 & 0 \end{vmatrix} \quad |A| = 6$$

$$|A| = 2(1 + 2) = 6 \neq 0 \quad \text{(Unique solution)}$$

$$\therefore X = A^{-1}B$$

$$\text{Adj } A = \begin{bmatrix} 0 & 2 & 2 \\ 3 & -3 & 0 \\ 3 & 1 & -2 \end{bmatrix} \quad A^{-1} = \frac{1}{6} \begin{bmatrix} 0 & 2 & 2 \\ 3 & -3 & 0 \\ 3 & 1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 0 & 2 & 2 \\ 3 & -3 & 0 \\ 3 & 1 & -2 \end{bmatrix} \begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix} \Rightarrow X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 6 \\ 12 \\ 18 \end{bmatrix} \Rightarrow x = 1, \; y = 2, \; z = 3$$