Let S be the set of all integer solutions (x, y, z), of the system of equations $x - 2y + 5z = 0$; $-2x + 4y + z = 0$; $-7x + 14y + 9z = 0$, such that $15 \leq x^2 + y^2 + z^2 \leq 150$. Then, the number of elements in the set S is ___

JEE MAIN Apr 2019

Solution:

$$\Delta = \begin{vmatrix} 1 & -2 & 5 \\ -2 & 4 & 1 \\ -7 & 14 & 9 \end{vmatrix} \qquad \Delta = 0 \qquad x = k,$$

$$z = 0, y = \frac{k}{2}$$

Since x, y, z are integers , k = even integer

$$15 \leq \frac{5k^2}{4} \leq 150$$

$$\Rightarrow 12 \leq k^2 \leq 120 \quad \Rightarrow k^2 \in [12, 120]$$

$$k \in \{\pm 4, \pm 6, \pm 8, \pm 10\}$$

N number of elements in the set S is $= 8$.

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