

Key Takeaways

Properties of Inverse of a matrix

If A is a non-singular matrix,

$$(A^{-1})^{-1} = A$$

If $A = \text{diag}(a_1, a_2, \dots, a_n)$, then $A^{-1} = \text{diag}(a_1^{-1}, a_2^{-1}, \dots, a_n^{-1})$

Proof: $A = \begin{pmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{pmatrix}$ $|A| = a_1 \cdot a_2 \cdot a_3$, $|A| \neq 0 \Rightarrow A^{-1} = \frac{\text{adj}(A)}{|A|}$

$$\Rightarrow \text{adj}(A) = \begin{pmatrix} a_2 a_3 & 0 & 0 \\ 0 & a_1 a_3 & 0 \\ 0 & 0 & a_1 a_2 \end{pmatrix} \Rightarrow A^{-1} = \frac{1}{a_1 a_2 a_3} \begin{pmatrix} a_2 a_3 & 0 & 0 \\ 0 & a_1 a_3 & 0 \\ 0 & 0 & a_1 a_2 \end{pmatrix} \Rightarrow A^{-1} = \begin{pmatrix} \frac{1}{a_1} & 0 & 0 \\ 0 & \frac{1}{a_2} & 0 \\ 0 & 0 & \frac{1}{a_3} \end{pmatrix}$$