

Properties of adjoint matrix

- Let $A = [a_{ij}]_n$ be a square matrix.

$$\boxed{\text{adj}(\text{adj}(A)) = |A|^{n-2} A}$$

Proof:

$$A \text{adj}(A) = |A|I$$

$$\Rightarrow \text{adj}(A) \text{adj}(\text{adj}(A)) = |\text{adj}(A)|I$$

$$\Rightarrow A \text{adj}(A) \text{adj}(\text{adj}(A))$$

$$\Rightarrow |A| \text{adj}(\text{adj}(A)) = A|A|^{n-1}$$

$$\Rightarrow \text{adj}(\text{adj}(A)) = |A|^{n-2} A$$

$$A \rightarrow \text{adj}(A)$$

$$|\text{adj}(A)| = |A|^{n-1}$$

$$A \text{adj}(A) = |A|I_n = \text{adj}(A) A$$