If
$$\Delta(r) = \begin{vmatrix} 2^{r-1} & 2 \cdot 3^{r-1} & 4 \cdot 5^{r-1} \\ \alpha & \beta & \gamma \\ 2^n - 1 & 3^n - 1 & 5^n - 1 \end{vmatrix}$$
, then the value of $\sum_{r=1}^n \Delta(r)$



Solution:

$$\sum_{r=1}^{n} \Delta(r) = \begin{vmatrix} \sum_{r=1}^{n} 2^{r-1} & \sum_{r=1}^{n} 2 \cdot 3^{r-1} & \sum_{r=1}^{n} 4 \cdot 5^{r-1} \\ \alpha & \beta & \gamma \\ 2^{n} - 1 & 3^{n} - 1 & 5^{n} - 1 \end{vmatrix} \qquad S_{n} = \frac{a(1 - r^{n})}{1 - r}$$

$$= \begin{vmatrix} 2^{n} - 1 & 3^{n} - 1 & 5^{n} - 1 \\ \alpha & \beta & \gamma \\ 2^{n} - 1 & 3^{n} - 1 & 5^{n} - 1 \end{vmatrix}$$

$$= 0$$



$$\frac{a(1-r^n)}{1-r}$$



$$\alpha + \beta + \gamma$$



$$\alpha 2^n + \beta 3^n + \gamma 4^n$$