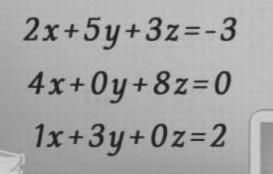
Welcome to



Matrices & Determinants





 $A^{-1}A = 1$

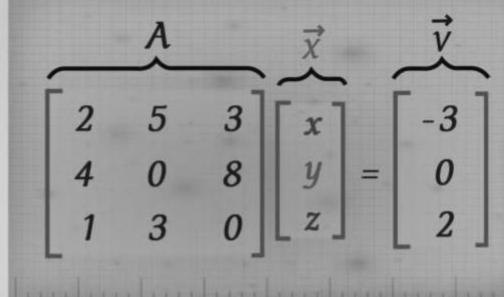


Table of contents

Session 01	03	Session 04	64	Session 08	141
Introduction	04	Co-factor of an Element	65	Properties of Inverse of Matrix	143
Order of Matrix	08	Value of 3 x 3 Matrix Determinant	67		
<u>Types of Matrices</u>	09	Value of Determinant in terms of Minor and Cofactor	69	Cossion 00	163
Principal Diagonal of Matrix	15	Properties of Determinant	74	Session 09	103
Trace of Matrix	16	Session 05	81	Inverse of a Matrix by elementary transformations	164
<u>Types of Matrices</u>	18	Properties of Determinant	82	System of Linear Equations	175
Session 02	27	Properties of Determinant	87	Cramer's Rule	179
Algebra of Matrices	28	Some important Formulae	97		2 00 4 52
Properties of Addition/ Subtraction of Matrices	32	Session 06	98	Session 10	190
Matrix Multiplication	34	Some important Determinants	99	<u>Cramer's Rule</u>	195
Properties of Matrix	37	Product of Two Determinants	103	System of Linear Equations(203
Multiplication Device of a Servera Matrix	41	Application of Determinants	107	Matrix Inversion)	
<u>Power of a Square Matrix</u>	41	<u>Differentiation of Determinant</u>	112	<u>Homogeneous System of</u> Linear Equations(Matrix	206
Session 03	48	Integration/ Summation of Determinant	114	Inversion)	
Polynomial Equation in Matrix	45	Coopies OT	119		
<u>Transpose of a Matrix</u>	47	Session 07	119	Session 11	208
Symmetric and Skew	51	Singular/Non-Singular Matrix	120		200
Symmetric Matrices		Cofactor Matrix & Adjoint Matrix	121	Characteristic Polynomial and Characteristic Equation	209
Properties of Trace of a Matrix	60	Properties of Adjoint Matrix	124		210
<u>Determinants</u>	62	<u>Inverse of a Matrix</u>	134	<u>Cayley-Hamilton Theorem</u>	
Minor of an element	63	Matrix Properties	138	Special Types of Matrices	216







• A rectangular arrangement of $m \cdot n$ numbers (real or complex) or expressions (real or complex valued), having m rows and n columns is called a matrix. $(m, n \in N)$

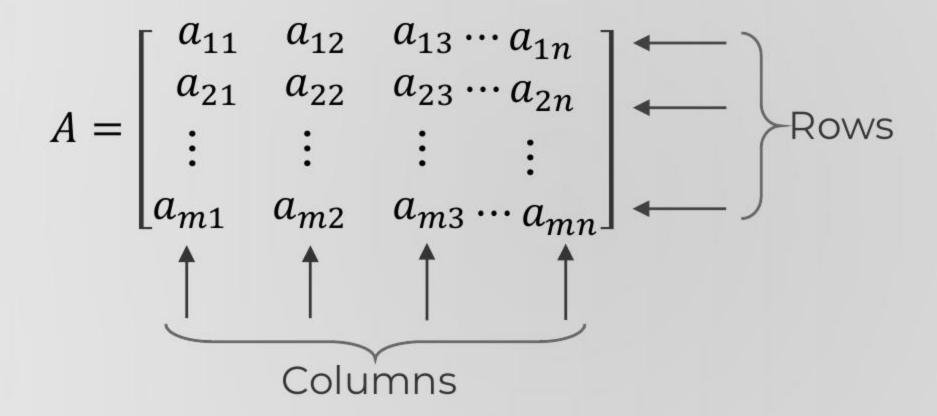
$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \cdots a_{1n} \\ a_{21} & a_{22} & a_{23} \cdots a_{2n} \\ \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} \cdots a_{mn} \end{bmatrix}$$
Rows

Columns

• An element of a matrix is denoted by a_{ij} : Element of i^{th} row & j^{th} column.



• A rectangular arrangement of $m \cdot n$ numbers (real or complex) or expressions (real or complex valued), having m rows and n columns is called a matrix. $(m, n \in N)$



- Number of elements in a matrix
 - = Number of rows x Number of columns
 - $= m \times n$





Write a_{11} , a_{12} , a_{13} , a_{21} , a_{22} , a_{23} for the following matrix:

+

$$A = \begin{bmatrix} 1 & 0 & 5 \\ -2 & 3 & -8 \end{bmatrix}$$

Solution:

$$a_{11} = 1$$

$$a_{12} = 0$$

$$a_{13} = 5$$

$$a_{21} = -2$$

$$a_{22} = 3$$

$$a_{23} = -8$$



Find the value a_{23} in the following matrix



$$A = \begin{pmatrix} 3 & -4 & 0 \\ -2 & 7 & 10 \\ 5 & -6 & 9 \end{pmatrix}$$

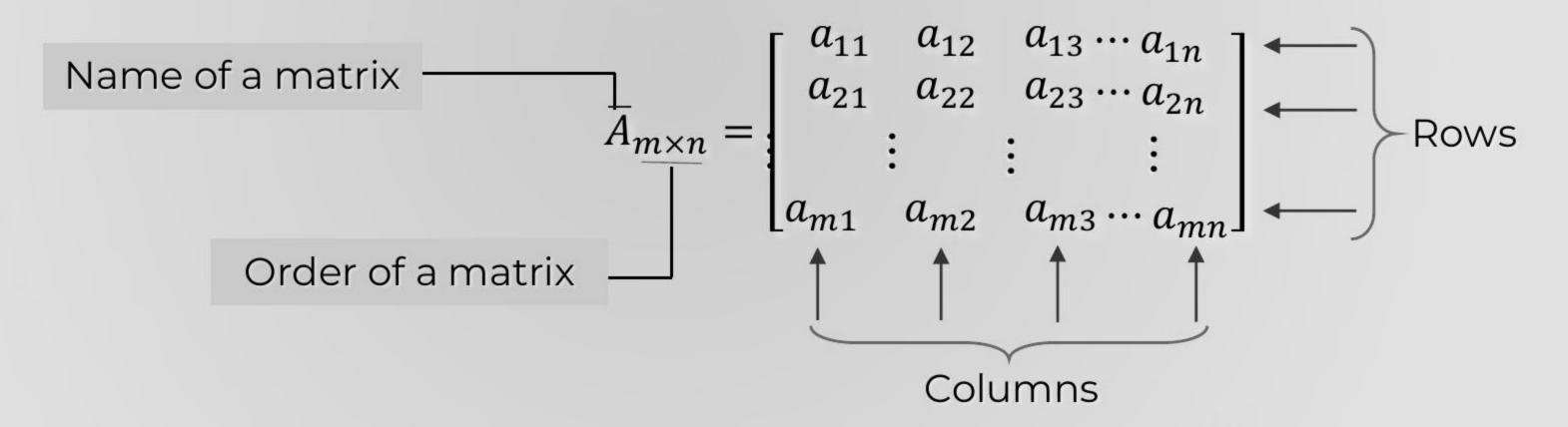
- A –6
- B 0
- C 10
- D 5



Order of a matrix

Order or dimension of a matrix denotes the arrangement of elements as number of rows and number of columns.

• Order = Number of rows \times Number of columns = $m \times n$



• Thus, a matrix can also be represented as $A = \left[a_{ij}\right]_{m \times n}$ or $(a_{ij})_{m \times n}$



Types of Matrix:



Row Matrix (row vector): A matrix having a single row is called a row matrix.

$$A = [a_{ij}]_{1 \times n} = [a_{11} \quad a_{12} \quad a_{13} \cdots a_{1n}]_{1 \times n}$$

Example:
$$B = [a \ b \ c]_{1\times 3}$$

 Column Matrix (column vector): A matrix having a single column is called a column matrix.

Example:
$$B = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}_{4 \times 1}$$

$$A = \begin{bmatrix} a_{ij} \end{bmatrix}_{m \times 1} = \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}_{m \times 1}$$

Matrices consisting of one row or one column are called vectors.



Types of Matrix:



 Zero Matrix (null matrix): If all the elements of a matrix are zero, then it is called zero or null matrix

$$A = \left[a_{ij}\right]_{m \times n}$$

is called a zero matrix, if $a_{ij} = 0$, $\forall i \& j$.

Examples:

$$A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



Vertical Matrix



A matrix of order $m \times n$ is known as vertical matrix if m > n, where m is equal to the number of rows and n is equal to the number of columns.

• In the matrix example given the number of rows (m) = 4, whereas the number of columns (n) = 2.

Therefore, this makes the matrix a vertical matrix.



Horizontal Matrix



A matrix of order $m \times n$ is known as vertical matrix if n > m, where m is equal to the number of rows and n is equal to the number of columns.

Example:
$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 5 & 1 & 1 \end{bmatrix}$$

• In the matrix example given the number of rows (m) = 2, whereas the number of columns (n) = 4.

Therefore, this makes the matrix a horizontal matrix.





If a matrix has 12 elements, then what are the possible orders it can have?

Solution:

Number of elements = Number of rows x Number of columns

$$12 = m \times n \ (m, n \in N)$$

Possible Order = 1×12 , 2×6 , 3×4 , 4×3 , 6×2 , 12×1





Construct a 2 × 3 matrix, whose elements are given by $a_{ij} = \frac{(i+2j)}{3}$.

Solution:

$$a_{ij} = \frac{(i+2j)}{3}$$

$$a_{11} = 1$$

$$a_{12} = \frac{5}{3}$$

$$a_{11} = 1 \qquad \qquad a_{12} = \frac{5}{3} \qquad \qquad a_{13} = \frac{7}{3}$$

$$a_{21} = \frac{4}{3}$$

$$a_{22} = 2$$

$$a_{21} = \frac{4}{3}$$
 $a_{22} = 2$ $a_{23} = \frac{8}{3}$

$$A = \begin{pmatrix} 1 & \frac{5}{3} & \frac{7}{3} \\ \frac{4}{3} & 2 & \frac{8}{3} \end{pmatrix}$$





Principal Diagonal of a Matrix: Diagonal containing the elements a_{ij} , where i = j is called principal diagonal of a matrix Examples:

$$A = \begin{bmatrix} 2 & -6 & 10 \\ 5 & 0 & 7 \\ 19 & -3 & -8 \end{bmatrix}_{3 \times 3} \qquad B = \begin{pmatrix} 2 & 3 & 4 & -5 \\ 1 & 4 & 0 & 6 \\ -3 & 7 & 8 & 9 \end{pmatrix}_{3 \times 4}$$

Types of Matrix:

 Square Matrix: A matrix where number of rows = number of columns is called square matrix.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \cdots a_{1n} \\ a_{21} & a_{22} & a_{23} \cdots a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & a_{n3} \cdots a_{nn} \end{bmatrix}_{n \times n}$$
 Example:
$$A = \begin{bmatrix} -4 & 5 & 0 \\ 8 & -1 & 3 \\ 9 & 7 & 2 \end{bmatrix}_{3 \times 3}$$

$$A = \begin{bmatrix} -4 & 5 & 0 \\ 8 & -1 & 3 \\ 9 & 7 & 2 \end{bmatrix}_{3 \times 3}$$