

Welcome to



Matrices & Determinants

$$\begin{aligned}2x + 5y + 3z &= -3 \\4x + 0y + 8z &= 0 \\1x + 3y + 0z &= 2\end{aligned}$$

$$\begin{matrix} & \overbrace{\begin{bmatrix} 2 & 5 & 3 \\ 4 & 0 & 8 \\ 1 & 3 & 0 \end{bmatrix}}^A & \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{\vec{x}} & = & \underbrace{\begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix}}_{\vec{v}} \end{matrix}$$

Table of contents



Session 01 03

<u>Introduction</u>	04
<u>Order of Matrix</u>	08
<u>Types of Matrices</u>	09
<u>Principal Diagonal of Matrix</u>	15
<u>Trace of Matrix</u>	16
<u>Types of Matrices</u>	18

Session 02 27

<u>Algebra of Matrices</u>	28
<u>Properties of Addition/ Subtraction of Matrices</u>	32
<u>Matrix Multiplication</u>	34
<u>Properties of Matrix Multiplication</u>	37
<u>Power of a Square Matrix</u>	41

Session 03 48

<u>Polynomial Equation in Matrix</u>	45
<u>Transpose of a Matrix</u>	47
<u>Symmetric and Skew Symmetric Matrices</u>	51
<u>Properties of Trace of a Matrix</u>	60
<u>Determinants</u>	62
<u>Minor of an element</u>	63

Session 04 64

<u>Co-factor of an Element</u>	65
<u>Value of 3 x 3 Matrix Determinant</u>	67
<u>Value of Determinant in terms of Minor and Cofactor</u>	69
<u>Properties of Determinant</u>	74

Session 05 81

<u>Properties of Determinant</u>	82
<u>Properties of Determinant</u>	87
<u>Some important Formulae</u>	97

Session 06 98

<u>Some important Determinants</u>	99
<u>Product of Two Determinants</u>	103
<u>Application of Determinants</u>	107
<u>Differentiation of Determinant</u>	112
<u>Integration/ Summation of Determinant</u>	114

Session 07 119

<u>Singular/Non-Singular Matrix</u>	120
<u>Cofactor Matrix & Adjoint Matrix</u>	121
<u>Properties of Adjoint Matrix</u>	124
<u>Inverse of a Matrix</u>	134
<u>Matrix Properties</u>	138

Session 08 141

<u>Properties of Inverse of Matrix</u>	143
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Session 09 163

<u>Inverse of a Matrix by elementary transformations</u>	164
<u>System of Linear Equations</u>	175
<u>Cramer's Rule</u>	179

Session 10 190

<u>Cramer's Rule</u>	195
<u>System of Linear Equations(Matrix Inversion)</u>	203
<u>Homogeneous System of Linear Equations(Matrix Inversion)</u>	206

Session 11 208

<u>Characteristic Polynomial and Characteristic Equation</u>	209
<u>Cayley-Hamilton Theorem</u>	210
<u>Special Types of Matrices</u>	216





Session 01

Introduction to Matrices

- A rectangular arrangement of $m \cdot n$ numbers (real or complex) or expressions (real or complex valued), having m rows and n columns is called a matrix. ($m, n \in N$)

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

Diagram illustrating the structure of a matrix A with m rows and n columns. The elements are arranged in a grid. The first row is labeled $a_{11}, a_{12}, a_{13}, \dots, a_{1n}$. The second row is labeled $a_{21}, a_{22}, a_{23}, \dots, a_{2n}$. The third row is labeled $\vdots, \vdots, \vdots, \vdots$. The last row is labeled $a_{m1}, a_{m2}, a_{m3}, \dots, a_{mn}$. Arrows point from the labels "Rows" and "Columns" to the corresponding dimensions of the matrix.

- An element of a matrix is denoted by a_{ij} : Element of i^{th} row & j^{th} column.

- A rectangular arrangement of $m \cdot n$ numbers (real or complex) or expressions (real or complex valued), having m rows and n columns is called a matrix. ($m, n \in N$)

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \cdots a_{1n} \\ a_{21} & a_{22} & a_{23} \cdots a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} \cdots a_{mn} \end{bmatrix}$$

Diagram illustrating the structure of a matrix A with m rows and n columns. The elements are arranged in a grid. The first row is labeled $a_{11}, a_{12}, a_{13}, \dots, a_{1n}$. The second row is labeled $a_{21}, a_{22}, a_{23}, \dots, a_{2n}$. The third row is labeled $\vdots, \vdots, \vdots, \vdots$. The last row is labeled $a_{m1}, a_{m2}, a_{m3}, \dots, a_{mn}$. The columns are labeled $a_{11}, a_{21}, \vdots, a_{m1}$ for the first column, $a_{12}, a_{22}, \vdots, a_{m2}$ for the second column, $a_{13}, a_{23}, \vdots, a_{m3}$ for the third column, and $a_{1n}, a_{2n}, \vdots, a_{mn}$ for the n -th column. A bracket on the right side of the matrix is labeled "Rows", and a bracket on the bottom side is labeled "Columns".

- Number of elements in a matrix
= Number of rows \times Number of columns
= $m \times n$