

Value of determinant in terms of minor and co-factor

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \quad a_{11}C_{21} + a_{12}C_{22} + a_{13}C_{23} = 0$$

Proof: $a_{11}C_{21} + a_{12}C_{22} + a_{13}C_{23}$

$$= a_{11}(a_{32}a_{13} - a_{12}a_{33}) + a_{12}(a_{11}a_{33} - a_{31}a_{13}) + a_{13}(a_{12}a_{31} - a_{11}a_{32})$$

$$= 0$$