

If $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$ is a root of the polynomial $x^3 - 6x^2 + 7x + k = 0$, then the value of k is:

Solution: $A^3 - 6A^2 + 7A + kI = 0 \dots (i)$ $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$

$$\Rightarrow \lambda^3 - 6\lambda^2 + 7\lambda + 2 = 0 \rightarrow \text{characteristic equation}$$

\therefore By Cayley – Hamilton Theorem ,

$$A^3 - 6A^2 + 7A + 2I = 0 \dots (ii)$$

By (i) & (ii), $k = 2$