

Welcome to



Matrices & Determinants

$$\begin{aligned}2x + 5y + 3z &= -3 \\4x + 0y + 8z &= 0 \\1x + 3y + 0z &= 2\end{aligned}$$

$$\begin{matrix} & \overbrace{\hspace{1cm}}^A & & \overbrace{\hspace{1cm}}^{\vec{x}} & & \overbrace{\hspace{1cm}}^{\vec{v}} \\ \begin{bmatrix} 2 & 5 & 3 \\ 4 & 0 & 8 \\ 1 & 3 & 0 \end{bmatrix} & \begin{bmatrix} x \\ y \\ z \end{bmatrix} & = & \begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix} \end{matrix}$$

$$A^{-1}A = I$$

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Session 01

Introduction to Matrices

Key Takeaways

- A rectangular arrangement of $m \cdot n$ numbers (real or complex) or expressions (real or complex valued), having m rows and n columns is called a matrix. ($m, n \in N$)

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

Diagram illustrating the structure of a matrix A with m rows and n columns. The elements are arranged in a grid. The first row is labeled $a_{11}, a_{12}, a_{13}, \dots, a_{1n}$. The second row is labeled $a_{21}, a_{22}, a_{23}, \dots, a_{2n}$. The third row is labeled $\vdots, \vdots, \vdots, \vdots$. The last row is labeled $a_{m1}, a_{m2}, a_{m3}, \dots, a_{mn}$. The columns are labeled $a_{11}, a_{21}, \vdots, a_{m1}$ for the first column, $a_{12}, a_{22}, \vdots, a_{m2}$ for the second column, $a_{13}, a_{23}, \vdots, a_{m3}$ for the third column, and $a_{1n}, a_{2n}, \vdots, a_{mn}$ for the n -th column. The label "Rows" is placed to the right of the matrix, and the label "Columns" is placed below the matrix.

- An element of a matrix is denoted by a_{ij} : Element of i^{th} row & j^{th} column.

- A rectangular arrangement of $m \cdot n$ numbers (real or complex) or expressions (real or complex valued), having m rows and n columns is called a matrix. ($m, n \in N$)

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

Columns

Rows

The diagram illustrates a matrix A with m rows and n columns. The elements are arranged in a grid, with the first row containing $a_{11}, a_{12}, a_{13}, \dots, a_{1n}$ and the last row containing $a_{m1}, a_{m2}, a_{m3}, \dots, a_{mn}$. The columns are labeled $a_{m1}, a_{m2}, a_{m3}, \dots, a_{mn}$ at the bottom, and the rows are labeled $a_{11}, a_{21}, \vdots, a_{m1}$ on the left. A bracket on the right side of the matrix is labeled 'Rows', and a bracket on the bottom side is labeled 'Columns'.

- Number of elements in a matrix
= Number of rows \times Number of columns
= $m \times n$