If the system of linear equations $x + y + z = 5$; $x + 2y + 2z = 6$ & $x + 3y + \lambda z = \mu$, ($\lambda, \mu \in \mathbb{R}$) has infinitely many solutions, then the value of $\lambda + \mu$ is:

Solution:

$$\left.\begin{array}{l} x + 3y + \lambda z = \mu \\ x + y + z = 5 \\ x + 2y + 2z = 6 \end{array}\right\} \text{infinitely many solutions}$$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 3 & \lambda \end{vmatrix} = 0 \quad \Rightarrow \lambda = 3 \qquad \Delta = \Delta_x = \Delta_y = \Delta_z = 0$$

$$\Delta_z = \begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & 6 \\ 1 & 3 & \mu \end{vmatrix} = 0$$

$$\Rightarrow 2\mu - 18 - (\mu - 6) + 5(3 - 2) = 7$$

$$\Rightarrow \mu - 7 = 0 \quad \Rightarrow \mu = 7$$

Putting $\lambda = 3$ and $\mu = 7$