



Key Takeaways



Some Important Determinants

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 3abc - a^3 - b^3 - c^3 = -(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

Proof:

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \quad C_1 \rightarrow C_1 + C_2 + C_3$$

$$= \begin{vmatrix} a+b+c & b & c \\ a+b+c & c & a \\ a+b+c & a & b \end{vmatrix} = (a+b+c) \begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix}$$

$$= (a+b+c)(ab + bc + ca - a^2 - b^2 - c^2)$$

$$= -(a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$$