

If $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$ is a root of the polynomial $x^3 - 6x^2 + 7x + k = 0$, then the value of k is:

A

2

B

4

C

-2

D

1

Solution:

$$x^3 - 6x^2 + 7x + k = 0 \quad A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$$

$$A^3 - 6A^2 + 7A + kI = 0 \quad \dots (i)$$

In order to get characteristics equation $|A - \lambda I| = 0$

$$|A - \lambda| = \begin{vmatrix} 1 - \lambda & 0 & 2 \\ 0 & 2 - \lambda & 1 \\ 2 & 0 & 3 - \lambda \end{vmatrix} = 0$$

$$\Rightarrow (1 - \lambda)((2 - \lambda)(3 - \lambda) - 0) + 2(0 - 2(2 - \lambda)) = 0$$

$$\Rightarrow (2 - \lambda)((1 - \lambda)(3 - \lambda) - 4) = 0 \Rightarrow (2 - \lambda)(\lambda^2 - 4\lambda - 1) = 0$$

$$\Rightarrow \lambda^3 - 6\lambda^2 + 7\lambda + 2 = 0 \rightarrow \text{characteristic equation}$$