DHANAMANJURI UNIVERSITY

Examination- 2024 (June)
Four-year course B.A/B.Sc. 2nd Semester

Name of Programme : B.A/B.SC Mathematics

Paper Type : Core-V (Theory)

Paper Code : CMA-105

Paper Title : Differential equations

Full Marks: 40

Pass Marks: 16 Duration: 2 Hours

The figures in the margin indicate full marks for the questions

Answers the following questions:

- 1. Choose and rewrite the correct answer for the following questions: $1 \times 3 = 3$
 - a) The sum of the order and degree of the differential equation

$$y = x \frac{dy}{dx} + a \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \text{ is}$$
i) 2 ii) 3

iii) 4

iv) 5

b) The complete solution of the differential equation $p^2 + p = 6$ (where $p = \frac{dy}{dx}$) is

i)
$$(y-3x-c)(y-2x-c)=0$$

ii)
$$(y + 3x - c)(y + 2x - c) = 0$$

iii)
$$(y + 3x - c)(y - 2x - c) = 0$$

iv)
$$(y-3x-c)(y+2x-c)=0$$

c) The particular integral of
$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{5x}$$
 is

i)
$$\frac{1}{9}e^{5x}$$

ii)
$$\frac{1}{10}e^{5x}$$

iii)
$$\frac{1}{11}e^{5x}$$

iv)
$$\frac{1}{12}e^{5x}$$

2. Write very short answers on any five of the following: $1 \times 5 = 5$

- a) Write the solution of the differential equation ydx xdy = 0.
- b) When is the equation Pdx + Qdy + Rdz = 0 said to be exact?
- c) Define Clairaut's differential equation.
- d) Write the word equation for drug assimilation into the blood.
- e) Find the complementary function of $(D^2 + D 6)y = x$.
- Define the Wronskian.

3. Answer any two of the following questions:

 $3 \times 2 = 6$

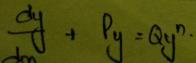
(a) Solve:
$$\frac{dy}{dx} + y \tan x = \sec x$$
.

- b) Solve: $\frac{dx}{mz-ny} = \frac{dy}{nx-lz} = \frac{dz}{ly-mx}$ Interprete your result geometrically.
- c) Solve: $y = 2px xp^2$
- d) Solve the differential equation of exponential decay and radioactivity $\frac{dN}{dt} = -kN$, (where k is a positive of proportionality) with the initial condition $N(t_o) = n_o$.
- (e) If $y_1(x) = \sin 3x$ and $y_2(x) = \cos 3x$ are the two solutions of y'' + 9y = 0, show that $y_1(x)$ and $y_2(x)$ are linearly independent solutions.

4. Answer any two of the following questions:

 $4 \times 2 = 8$

Define Bernoulli's equation. Show that such an equation can be reduced to the form of linear differential equation.



- b) Show that the differential equation $y \sin 2x \, dx (y^2 + \cos^2 x) dy = 0$ and hence solve it and given that y = 1 where x = 0.
- c) Solve the differential equation $p^2 + 2py \cot x = y^2$.
- d) Solve the differential equation $\sin^2 x \frac{d^2y}{dx^2} = 2y$ given that $y = \cot x$ is a solution.
- e) Solve: $\frac{d^2y}{dx^2} \frac{2}{x}\frac{dy}{dx} + \left(1 + \frac{2}{x^2}\right)y = xe^x$.

5. Answer any one questions from the following: $6 \times 1 = 6$

- a) Define exact differential equation. Examine whether the differential equation $x dx + y dy = \frac{a^2(xdy-ydx)}{x^2+y^2}$ is exact or not. If it be exact, then find its solution. Also find the particular solution by giving y = 1 where x = 0.
- Find the orthogonal trajectories of $3a^{3}y^{2} 3a^{3}a^{2}y^{2} + 6y^{2}a^{2} \frac{x^{2}}{a^{2}} + \frac{y^{2}}{a^{2} + \lambda} = 1$, where λ is arbitrary
 - c) Define total differential equation. Show that the total differential equation Pdx + Qdy + Rdz = 0 will be integrable if $P\left(\frac{\partial Q}{\partial z} \frac{\partial R}{\partial y}\right) + Q\left(\frac{\partial R}{\partial x} \frac{\partial p}{\partial z}\right) + R\left(\frac{\partial P}{\partial y} \frac{\partial Q}{\partial x}\right) = 0$ State the names of method that can be used in solving the total differential equations.

6. Answer any one questions from the following: $6 \times 1 = 6$

- Define singular solution. Find the complete primitive and singular solution of $y = px + \sqrt{b^2 + a^2p^2}$ Interprete your result geometrically.
 - b) The acceleration of a moving particle being proportional to the cube of the velocity and negative, find the distance passed over in time t, the initial velocity being v_0 and the distance being measured from the position of the particle at the time t=0.

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- c) A radio active substance has a half-life of h days. Find a formula its mass m in terms of the time t, if the initial mass is m_o . What is its initial decay rate?
- 7. Answer any one questions from the following: $6 \times 1 = 6$
 - a) Solve: $(3x + 2)^2 \frac{d^2y}{dx^2} + 3(3x + 2) \frac{dy}{dx} 36y = 3x^2 + 4x + 1$.
 - Using the method of variation of parameters, solve $\frac{d^2y}{dx^2} + y = x$
 - c) Solve by the method of undetermined co-efficients, the differential equation $\frac{d^2y}{dx^2} + 4y = \sin 2x$.

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