DHANAMANJURI UNIVERSITY

JUNE-2022

Name of Programme	:	**B.A/B.Sc. Mathematics (Honours)**
Semester	:	**II**
Paper Type	:	**Core III**
Paper Code	:	**CMA-103**
Paper Title	:	**Real Analysis**
Full Marks	:	**100**

Duration : 3 Hours

The figures in the margin indicate full marks for the questions

Answer all the questions:

1. State the completeness property of $\mathbb{R}$. Prove that if A and B are nonempty subsets of $\mathbb{R}$ that satisfy the property: $a \leq b$ for all $a \in A$ and all $b \in B$, then sub $A \leq \inf B$. 4

2. Show that the series $\frac{1.2}{3^2.4^2} + \frac{3.4}{5^2.6^2} + \frac{5.6}{7^2.8^2} + \cdots$ is convergent. 4

3. Prove that a sequence cannot converge to more than one limit. 5

4. What is an absolutely convergent series? Prove that every absolutely convergent series is convergent. 5

5. Prove that an upper bound u of a non empty set S in $\mathbb{R}$ is the supremum of S if an only if for every $\varepsilon > 0$ there exists $s_\varepsilon \in S$ such that $u - \varepsilon < s_\varepsilon$. 6

6. Prove the following: 6

   i) If $a \in \mathbb{R}$ and $a \neq 0$, then $a^2 > 0$.

   ii) $1 > 0$

   iii) If $n \in \mathbb{N}$, then $n > 0$.

7. Prove that the following statement are equivalent. 6
   i) S is a countable set.
   ii) There exists a surjection of $\mathbb{N}$ into S
   iii) There exists a ~~surjection~~ injection of S into $\mathbb{N}$
8. State and prove the Archimedean Property. 6
9. State and prove Sandwich Theorem for limits. 6
10. Prove that a necessary and sufficient condition for the convergence of a monotonic sequence is that it is bounded. 6
11. State and prove Cantor's Theorem. 7
12. If $I_n = [a_n, b_n], n \in \mathbb{N}$, is a nested sequence of closed bounded intervals, then prove that there exists a number $\xi \in \mathbb{R}$ such that $\xi \in I_n$ for all $n \in \mathbb{N}$. 7
13. If S is a subset of $\mathbb{R}$ that contains at least two points and has the property: If $x, y \in S$ and $x < y$, then $[x, y] \subseteq S$, then prove that S is an interval. 8

   Or

   State and prove the Density Theorem for rational numbers.
14. State and prove Bolzano-Weierstrass Theorem for sequences. 8

   Or

   State and prove Cauchy's General Principal of Convergence.
15. State and prove Leibnitz Test for the convergence of an alternating series. 8
16. State and prove Cauchy's Root Test for the convergence of positive term series. 8

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$a_k \le b_n \le b_k$. upper bound

$a_k \le$ lower