DHANAMANJURI UNIVERSITY
DECEMBER-2022

Name of Programme	:	B.A/B.Sc. Mathematics (Honours)
Semester	:	3^{rd}
Paper Type	:	Core – VII (Theory)
Paper Code	:	CMA-207
Paper Title	:	Partial Differential Equations and Laplace Transform

Full Marks : 100

Pass Marks : 40

Duration: 3 Hours

The figures in the margin indicate full marks for the questions

Answer all the questions:

1. **Choose and rewrite the correct answer for each of the following:** **1 x 4 = 4**

a) The partial differential equation formed by eliminating the arbitrary constants a and c from the equation $x^2+y^2+(z-c)^2=a^2$ is

i) $xp-yq=0$ ii) $xp+yq=0$

iii) $yp-xq=0$ iv) $yp+xq=0$

$q=a$

$p=\frac{1}{a}$

$ax+\frac{y}{a}$

b) The complete integral of $pq=1$ is

i) $z=ax+\frac{by}{a}+c$ ii) $z=\frac{a}{x}+by+c$

iii) $z=ax+\frac{y}{a}+b$ iv) $z=ax+\frac{y}{b}+c$

c) If $F(D,D')$ be homogeneous function of D and D' of degree n with constant coefficients, where $D\equiv\frac{\partial}{\partial x}$, $D'\equiv\frac{\partial}{\partial y}$ and $F(a,b)=0$, then the particular integral of the equation $F(D,D')z=\phi(ax+by)$ is

i) $\frac{x^n}{a^n\cdot n!}\phi(ax+by)$ ii) $\frac{x^n}{b^n\cdot n!}\phi(ax+by)$

iii) $\frac{y^n}{a^n\cdot n!}\phi(ax+by)$ iv) $\frac{y^n}{b^n\cdot n!}\phi(ax+by)$

d) The Laplace Transform of te^{at} is

i) $\frac{1}{s-a}$ ii) $\frac{1}{s+a}$

iii) $\frac{1}{(s-a)^2}$ iv) $\frac{1}{(s+a)^2}$

2. **Write very short answer for each of the following:** **1 x 12 = 12**

a) What is a linear partial differential equation?

b) What is the geometrical interpretation of Lagrange's first order linear partial differential equation?

c) If Jacobian of two functions $\xi = \xi(x,y)$ and $\eta = \eta(x,y)$ with respect to two variables x and y is zero, what cay you say about the two functions ξ and η.

d) What is the geometrical interpretation of singular integral of a non-linear partial differential equation of first order?

e) What are compatible systems of first order partial differential equations?

f) Write the complete integral of the equation of the form $f(p,q)=0$?

g) Write the general form of a second order linear partial differential equation in two independent variables.

h) Determine the region in which the equation $(x^2-1)u_{xx}+2yu_{xy}-u_{yy}=0$ is hyperbolic.

i) Write λ-quadratic of the Monge's equation $Rr+Ss+Tt+U(rt-s^2)=V$, where the symbols have their usual meanings.

j) Define Laplace Transform of a function $f(t)$

k) Define function of class A.

l) Find the inverse Laplace Transform of $\frac{s}{s^2-a^2}$

3. **Choose any twelve and rewrite short answers:** **3 x 12 = 36**

a) Find the partial differential equation by eliminating the arbitrary constants a and b from the equation $(x-a)^2+(y-b)^2+z^2=r^2$.

b) Form a partial differential equation by eliminating the arbitrary function F from the equation $F(u,v)=0$, where u and v are known functions of x, y and z.

c) Find the general solution of the equation:
$x(y^2+z)p-y(x^2+z)q=z(x^2-y^2)$

d) How do you obtain general integral of a first order non-linear partial differential equation? What is the geometrical interpretation of a general integral?

e) Find the complete integral of the equation: $z^2(p^2z^2+q^2)=1$

f) Find the complete integral of the equation:
$\frac{p^2}{x}-\frac{q^2}{y}=\frac{1}{z}\left(\frac{1}{x}+\frac{1}{y}\right)$

g) Find the complete integral of the Clairaut Equation by using Charpit's Method. What does the complete integral of a Clairaut Equation represent geometrically?

h) Show that $\begin{bmatrix} A^* & B^*/2 \\ B^*/2 & C^* \end{bmatrix}=\begin{bmatrix} \xi_x & \xi_y \\ \eta_x & \eta_y \end{bmatrix}\begin{bmatrix} A & B/2 \\ B/2 & C \end{bmatrix}\begin{bmatrix} \xi_x & \xi_y \\ \eta_x & \eta_y \end{bmatrix}^T$ where
$A^*=A\xi_x^2+B\xi_x\xi_y+C\xi_y^2$ $B^*=2A\xi_x\eta_x+B(\xi_x\eta_y+\xi_y\eta_x)+2C\xi_y\eta_y$
and $C^*=A\eta_x^2+B\eta_x\eta_y+C\eta_y^2$

i) If $F(D,D')=D-mD'$, find the particular integral of the equation $F(D,D')z=f(x,y)$ using Lagrange's Method, where
$D\equiv\frac{\partial}{\partial x},\ D'\equiv\frac{\partial}{\partial y}$

j) Find the general solution of the equation: $r+5s+6t=(y-2x)^{-1}$

k) Find the general solution of the equation:
$(2DD'+D'^2-3D')z=5\cos(3x-2y)$

l) Show that $f(x)=x^n$ is of exponential order as $x\to\infty$, n being any positive integer.

m) If $L\{F(t)\} = f(s)$ and $G(t) = \begin{cases} F(t-a), & t > a \\ 0 & , t < a \end{cases}$, prove that

$L\{G(t)\} = e^{-as} f(s)$.

n) Show that $\int_0^\infty te^{-3t} \sin t \, dt = \frac{3}{50}$.

o) Evaluate : $L^{-1}\left\{\frac{1}{s^2(s+1)^2}\right\}$

4. Answer any two of the following: **6 x 2 = 12**

a) Solve: $y^2(x-y)p + x^2(y-x)q = z(x^2+y^2)$

b) Find the general integral of the partial differential equation $(2xy-1)p + (z-2x^2)q = 2(x-yz)$ and also the particular integral which passes through the line $x=1,\ y=0$.

c) Reduce the equation $u_x - u_y = u$ to canonical form and obtain the general solution.

d) Solve the equation $y^2u_x^2 + x^2u_y^2 = (xyu)^2$, $u(x,0) = 3e^{\frac{x^2}{4}}$ by the method of separation of variables of the form $u(x,y) = f(x)\cdot g(y)$

5. Answer any two of the following questions: **6 x 2 = 12**

a) Prove that the necessary and sufficient condition for the first order PDEs: $f(x,y,z,p,q)=0$ and $g(x,y,z,p,q)=0$ to be compatible is that the Jacobi bracket

$$[f,g] \equiv \frac{\partial(f,g)}{\partial(x,p)} + p\frac{\partial(f,g)}{\partial(z,p)} + \frac{\partial(f,g)}{\partial(y,q)} + q\frac{\partial(f,g)}{\partial(z,q)} = 0$$

b) Describe Charpit's method for solving the first order non-linear PDE.

c) Apply Charpit's method to solve the equation:

$2z + p^2 + qy + 2y^2 = 0$

d) Apply Jacobi's method to find the complete integral of:

$p_1p_2p_3 = z^3x_1x_2x_3$

$\frac{2z}{z^2} + \frac{p^2}{z^2} + \frac{qy}{z} + 2y^2 = 0$.

$L\{t\,F(t)\}$

6. **Answer any two of the following questions:** **$6 \times 2 = 12$**

a) Reduce to canonical form and find the general solution of the equation $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = 0$.

b) Solve: $r + s - 6t = y\cos x$

c) Solve: $x^2\dfrac{\partial^2 z}{\partial x^2} - 4xy\dfrac{\partial^2 z}{\partial x\partial y} + 4y^2\dfrac{\partial^2 z}{\partial y^2} + 6y\dfrac{\partial z}{\partial y} = x^3y^4$

d) Apply Monge's method to find the general integral of $y^2r - 2ys + t = p + 6y$

7. **Answer any two of the following questions:** **$6 \times 2 = 12$**

a) Prove that if a function $F(t)$ is piece-wise continuous in every finite interval in the range $t \geq 0$ and is of exponential order a as $t \to \infty$, then Laplace transform of $F(t)$ exists for all $s > a$.

b) If $L\{F(t)\} = f(s)$, prove that $L\{t^nF(t)\} = (-1)^n \dfrac{d^n}{ds^n} f(s)$, n=1, 2, 3......

c) Apply convolution theorem to evaluate

$$L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$$

d) Apply Laplace transform to solve $\dfrac{d^2y}{dt^2} + y = 6\cos 2t$

given that $y = 3$, $\dfrac{dy}{dt} = 1$ when $t = 0$.

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