

Four-year course B.A/B.Sc. 2nd Semester

Name of Programme	:	B.A/B.SC Mathematics
Paper Type	:	Core-VI (Theory)
Paper Code	:	CMA-106
Paper Title	:	Vector Analysis & Solid Geometry
Full Marks	:	40
Pass Marks	:	16
		Duration: 2 Hours

The figures in the margin indicate full marks for the questions
Answers the following questions:

- 1. Choose and rewrite the correct answer:** **$1 \times 3 = 3$**

a) If $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$, then $\text{div } \vec{a}$ is:

- [illegible]

b) The equation of the cylinder generated by the lines parallel to the z-axis and passing through the curve of intersection of the plane $lx + my + nz = p$ and the surface $ax^2 + by^2 + cz^2 = 1$ is:

- $$\begin{aligned} \text{i)} \quad & ax^2 + by^2 + c \left(\frac{p-lx-my}{n} \right)^2 = 1, \\ \text{ii)} \quad & cz^2 + a \left(\frac{p-nz}{l} \right)^2 = 1, \\ \text{iii)} \quad & cz^2 + by^2 + a \left(\frac{p-nz-my}{l} \right)^2 = 1, \\ \text{iv)} \quad & cz^2 + b \left(\frac{p-nz}{l} \right)^2 = 1. \end{aligned}$$

c) The equation of hyperbolic paraboloid is:

$$\text{i) } \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1,$$

$$\text{ii) } \frac{x^2}{a^2} = \frac{2z}{c} - \frac{y^2}{b^2},$$

$$\text{iii) } \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{2z}{c},$$

$$\text{iv) } \frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{2z}{c}.$$

2. Write very short answers for each of the following: $1 \times 5 = 5$

- Define reciprocal vectors.
- Give an example of a vector point function.
- Write the formula to find the radius of a sphere whose equation is given by $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$.
- Define a right circular cone.
- What do you mean by hyperboloid of one sheet?

3. Write short answers (any two) of the following: $3 \times 2 = 6$

- The acceleration of a particle at time t is given by

$$\vec{a} = 18\cos t \cdot \hat{i} - 8\sin 2t \cdot \hat{j} + 6t \cdot \hat{k}$$

 If the velocity \vec{v} be zero at time $t = 0$, find \vec{v} at any time t .
- Find the equation of the sphere on which the circle given by
 $x + y + z + 3 = 0$ and $x^2 + y^2 + z^2 = 9$ is a great circle.
- The axis of a right circular cylinder is $\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-3}{2}$ and its radius is 5. Find its equation.

4. Answer any two of the following: $4 \times 2 = 8$

- Using Green's theorem, evaluate $\int_C (x^2 y dx + x^2 dy)$, where C is boundary described counterclockwise of the triangle with vertices $(0,0)$, $(1,0)$ and $(1,1)$.
- Find the equation of the curve in which the plane $z = h$ cuts the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ and find the area enclosed by the curve.

- c) Prove that the point $(1,2,3)$ and $(1,2,1)$ are conjugate with respect to the conicoid $2x^2 + 3y^2 - 4z^2 = 2$.

5. Answer any one of the following:

$6 \times 1 = 6$

- a) Evaluate: $\int_C \vec{F} d\vec{r}$, where $F(x, y, z) = -y^2\hat{i} + x\hat{j} + z^2\hat{k}$ and C is the curve of intersection of the plane $y + z = 2$ and the cylinder $x^2 + y^2 = 1$.
- b) Apply Stock's theorem to find the value of $\int_C (ydx + zdy + xdz)$, where C is the curve of $x^2 + y^2 + z^2 = a^2$ and $x + z = a$.

6. Answer any one of the following:

$6 \times 1 = 6$

- a) Find the equation of the tangent plane to the sphere $x^2 + y^2 + z^2 = 14$ at the point $(1, -2, 3)$.
- b) Prove that the plane $ax + by + cz = 0$ cuts the cone $yz + zx + xy = 0$ in perpendicular generators, if $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$.

7. Answer any one of the following:

$6 \times 1 = 6$

- a) Find the equation of the tangent plane to the paraboloid

$$\frac{x^2}{5} - \frac{y^2}{3} = 2z \text{ parallel to the plane } 2x - 3y + z = 0.$$

- b) Prove that the plane $x + y + z = 1$ touches the conicoid

$$\frac{x^2}{3} + \frac{y^2}{2} - \frac{z^2}{4} = 1.$$
