DHANAMANJURI UNIVERSITY

Examination- 2024 (June)

Four-year course BA/B.SC 2nd Semester

Name of Programme : BA/B.Sc. Mathematics
Paper Type : Core-IV (Theory)
Paper Code : CMA-104
Paper Title : Real Analysis
Full Marks : 40
Pass Marks : 16 **Duration: 2 Hours**

The figures in the margin indicate full marks for the questions
Answers all the following questions:

1. Choose and rewrite the correct answer for each of the following questions: $1 \times 3 = 3$

a) Which of the following is an open set
   i) $(0,2) \cup [-2,0)$
   ii) $(0,2] \cup [-2,0)$
   iii) $(0,2] \cup (-2,0)$
   iv) $[0,2) \cup (-2,0]$

b) For the sequence $\{u_n\}$ where
$$u_n = \begin{cases} 2 & , \ if \ \ n = 1 \\ least\ prime\ factor\ of\ n, & if \ \ n \geq 2 \end{cases}$$
then the number of limit points is
   i) 0
   ii) 1
   iii) 2
   iv) infinite

c) The geometric infinite series $1 + x + x^2 + x^3 + \cdots$ is convergent when
   i) $x = 1$
   ii) $|x| < 1$
   iii) $x > 1$
   iv) $x < -1$

2. Write any five short answer from the following questions: $1 \times 5 = 5$

a) Give an example of a non-empty set which has no supremum.
b) Define deleted neighbourhood of a point.
c) Define an open cover of a set.
d) Give an example of an oscillatory sequence.
e) Give an example of an unbounded sequence which has limit point.
f) Give an example of a conditionally convergent series.

3. Write any two answer from the following questions: $3 \times 2 = 6$

a) Define a countable set. Give an example of a countable set which has infinite number of elements. Is the set of irrational numbers countable?
b) Show that the derived set of every set is a closed set.
c) Show that the sequence $\{u_n\}$ where
$$\{u_n\} = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+n}$$
is convergent.
d) Show that a necessary condition for the convergence of a series $\sum u_n$ is $\lim_{n\to\infty} u_n = 0$. By giving an example, show that $\lim_{n\to\infty} u_n = 0$ is not a sufficient condition for the convergence of $\sum u_n$.
e) Show that every absolutely convergent series is convergent.

4. Write any two answers from the following questions: $4 \times 2 = 8$

a) State and prove Archimedean property of $\mathbb{R}$.
b) Every convergent sequence is a Cauchy sequence but not the converse. Prove it.

c) Define subsequence of a given sequence. Show that every subsequence of a convergent sequence converge to the same limit.

d) State and prove Cauchy root test.

e) Test the convergent of the series.

$$x+\frac{3}{5}x^2+\frac{8}{10}x^3+\frac{15}{17}x^4+\cdots+\frac{n^2-1}{n+1}x^n+\cdots$$

where $x>0$.

5. **Answer any one question:** $6\times 1=6$

a) Define an open set. Show that finite intersection of open sets is an open set. Further show that arbitrary intersection of open set need not be open. Also give an example, where intersection of infinite collection of open sets is an open set.

b) Define limit point of a set. Show that every infinite bounded set has a limit point.

c) State and prove Heine Borel Theorem.

6. **Answer any one question:** $6\times 1=6$

a) State and prove Bolzano-Weierstrass theorem for sequence.

b) Define monotonic increasing sequence. Show that a necessary and sufficient condition for a monotonic sequence to be convergent is that it is bounded.

c) State nested interval theorem and prove the same.

7. **Answer any one question:** $6\times 1=6$

a) Discuss the convergence and divergence of the $p-series$ $i.e.,\ \frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\cdots+\frac{1}{n^p}+\cdots$

b) State and prove D'Alembert's ratio test.

c) State Leibnitz theorem for alternating series and prove the same.

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