DHANAMANJURI UNIVERSITY
DECEMBER-2022

Name of Programme	:	**B.A/B.Sc. Mathematics (Honours)**
Semester	:	**3rd**
Paper Type	:	**Core – V (Theory)**
Paper Code	:	**CMA-205**
Paper Title	:	**Theory of Real Functions**

Full Marks : 100

Pass Marks : 40

Duration : 3 Hours

The figures in the margin indicate full marks for the questions

Answer all the questions:

1. Give the definitions of the limits: 4

   a) $\lim_{x \to c} f(x) = l$ and

   b) $\lim_{x \to \infty} f(x) = l$

2. Using $\varepsilon - \delta$ definition, show that $\lim_{x \to 0} x \sin \frac{1}{x} = 0.$ 4

3. Show that $\lim_{x \to 4} \frac{1}{(x-4)^2} = \infty$ 4

4. Prove that the function $f(x) = \sin x$ is uniformly continuous on $[0, \infty[$ 4

5. Prove that a function which is uniformly continuous on an interval is continuous on that interval. 4

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6. Using Taylor's Theorem with $n = 2$, approximate $\sqrt[3]{1+x}$, $x > -1$. 4

Or

Expand the function $\sin x$ in powers of x in a finite series with Lagrange form of remainder. 4

7. Show that the function f defined on $\mathbb{R}$ by

$$f(x) = \begin{cases} 1, & \text{when } x \text{ is irrational} \\ -1, & \text{when } x \text{ is rational} \end{cases}$$ is discontinuous at every point. 6

8. Let $f : I \to \mathbb{R}$ be differentiable on the interval I. Then, prove that f is increasing on I if and only if $f'(x) \geq 0$ for all $x \in I$. 6

9. State Cauchy's necessary and sufficient condition for the existence of a limit. Hence, show that $\lim_{x \to 0} \sin \frac{1}{x}$ does not exist. 8

10. If a function f is continuous on a closed interval $[a, b]$ and $f(a)$ and $f(b)$ are of opposite signs, then prove that there exists at least one point $\alpha \in]a, b[$ such that $f(\alpha) = 0$. 8

Or

Prove that if a function is continuous in a closed interval, then it is bounded therein. 8

11. Prove that a function f defined on an interval I is continuous at a point $c \in I$ iff forevery sequence $\{c_n\}$ in I converging to c, $\lim_{n \to \infty} f(c_n) = f(c)$. 8

12. State and prove Caratheodory's Theorem. 8

13. State and prove Rolle's Theorem. Also, write the geometrical interpretation of the theorem. 8

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14. Let $I \subseteq \mathbb{R}$ be an interval, let $f: I \to \mathbb{R}$, let $c \in I$, and assume that f has a derivative at c. Then, prove that: 8

a) If $f'(c) > 0$, then there is a number $\delta > 0$ such that $f(x) > f(c)$ for $x \in I$ such that $c < x < c + \delta$.

b) If $f'(c) < 0$, then there is a number $\delta > 0$ such that $f(x) > f(c)$ for $x \in I$ such that $c - \delta < x < c$.

Or

State and prove Darboux's Theorem.

15. State and prove Taylor's Theorem in finite form with Lagrange form of remainder. 8

16. Let I be an open interval and let $f: I \to \mathbb{R}$, have a second derivative on I. Then, prove that f is a convex function on I if and only if $f''(x) \geq 0$ for all $x \in I$. 8

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$|f(x) - f(c)|$

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