

DHANAMANJURI UNIVERSITY

Four-Year Course BA/B.Sc. 2nd Semester

Examination - 2024 (June)

Name of Programme	: B.A/B.Sc Mathematics
Paper Type	: Core-5(Theory)
Paper Code	: CMA-105
Paper Title	: Differential equation
Full mark	: 40
Pass Mark	: 16
Duration	: 2 Hours

The figures in the margin indicate full marks for the questions.

Answer the following questions:

1. Choose and rewrite the correct answer for the following question:

1×3=3

a) The sum of all the order and degree of the differential equation

$$y = x \frac{dy}{dx} + a \sqrt{1 + \left(\frac{dy}{dx} \right)^2} \text{ is}$$

- i) 2
- ii) 3
- iii) 4
- iv) 5

b) The complete solution of the differential equation $p^2 + p = 6$ (where $p = \frac{dy}{dx}$) is

- i) $(y - 3x - c)(y - 2x - c) = 0$
- ii) $(y + 3x - c)(y + 2x - c) = 0$
- iii) $(y + 3x - c)(y - 2x - c) = 0$
- iv) $(y - 3x - c)(y + 2x - c) = 0$

c) The particular integral of $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{5x}$ is

- i) $\frac{1}{9}e^{5x}$
- ii) $\frac{1}{10}e^{5x}$
- iii) $\frac{1}{11}e^{5x}$
- iv) $\frac{1}{12}e^{5x}$

2. Write very short answers on any five of the following:

1×5=5

- a) Write the solution of the differential equation $ydx - xdy = 0$.
- b) When is the equation $Pdx + Qdy + Rdz = 0$ said to be exact?
- c) Define Clairaut's differential equation.
- d) Write the word equation for drug assimilation into the blood.
- e) Find the complementary function of $(D^2 + D - 6)y = x$.
- f) Define the Wronskian.

3. Answer any two of the following questions:

3×2=6

- a) Solve: $\frac{dy}{dx} + y \tan x = \sec x$.
- b) Solve: $\frac{dx}{mz - ny} = \frac{dy}{nx - lz} = \frac{dz}{ly - mx}$ Interpret your result geometrically
- c) Solve: $y = 2px - xp^2$
- d) Solve the differential equation of exponential decay and radioactivity $\frac{dN}{dt} = -kN$, (where k is a positive of proportionality) with the initial condition $N(t_0) = n_0$
- e) if $y_1(x) = \sin 3x$ and $y_2(x) = \cos 3x$ are the two solution of $y'' + 9y = 0$, show that $y_1(x)$ and $y_2(x)$ are linearly independent solutions.

4. Answer any two of the following questions: **$4 \times 2 = 8$**

- a) Define Bernoulli's equation. Show that such an equation can be reduced to the form of linear differential equation.
- b) Show that the differential equation $y \sin 2x dx - (y^2 + \cos^2 x) dy = 0$ and hence solve it and given that $y = 1$ where $x = 0$.
- c) Solve the differential equation $p^2 + 2py \cot x = y^2$.
- d) Solve the differential equation $\sin^2 x \frac{d^2 y}{dx^2} = 2y$ given that $y = \cot x$ is a solution.
- e) Solve: $\frac{d^2 y}{dx^2} - \frac{2}{x} \frac{dy}{dx} + \left(1 + \frac{2}{x^2}\right) y = xe^x$.

5. Answer any one questions from the following: **$6 \times 1 = 6$**

- a) Define exact differential equation. Examine whether the differential equation $x dx + y dy = \frac{a^2 (x dy - y dx)}{x^2 + y^2}$ is exact or not. If it be exact, then find its solution. Also find the particular solution by giving $y = 1$ and $x = 0$.
- b) Find the orthogonal trajectories of $\frac{x^2}{a^2} + \frac{y^2}{a^2 + \lambda} = 1$ where λ is arbitrary
- c) Define total differential equation. Show that the total differential equation $Pdx + Qdy + Rdz = 0$ will be integrable if $P \left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y} \right) + Q \left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z} \right) + R \left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x} \right) = 0$
State the names of method that can be used in solving the total differential equations.

6. Answer any one questions from the following: **$6 \times 1 = 6$**

- [a] Define singular solution. Find the complete primitive and singular solution of $y = px + \sqrt{b^2 + a^2 p^2}$ Interpret your result geometrically.
- b) The acceleration of a moving particle being proportional to the cube of the velocity and negative, find the distance passed over in time t , the initial velocity being v_0 and the distance being measured from the position of the particle at the time $t = 0$.
- c) A radio active substance has a half-life of h days. Find a formula its mass m in terms of the time t , if the initial mass is m_0 . What is its initial decay rate?

7. Answer any one questions from the following:

6×1=6

a) Solve: $(3x + dy2)^2 \frac{d^2y}{dx^2} + 3(3x + 2) \frac{dy}{dx} - 36y = 3x^2 + 4x + 1.$

b) Using the method of variation of parameters, solve $\frac{d^2y}{dx^2} + y = x.$

c) Solve by the method of undetermined co-efficients, the differential equation $\frac{d^2y}{dx^2} + 4y = \sin 2x.$
