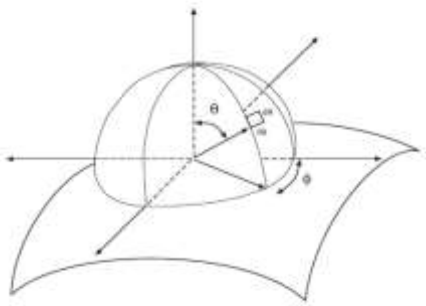
Unit I

Chapter 1: Radiometry – Measuring Light

Light in Space

- The measurement of light is a field in itself, known as radiometry.
- We need a series of units that describe how energy is transferred from light sources to surface patches, and what happens to the energy when it arrives at a surface.
- The first matter to study is the behaviour of light in space**.**
- At each point on a piece of surface is a **hemisphere of directions**, along which light can arrive or leave

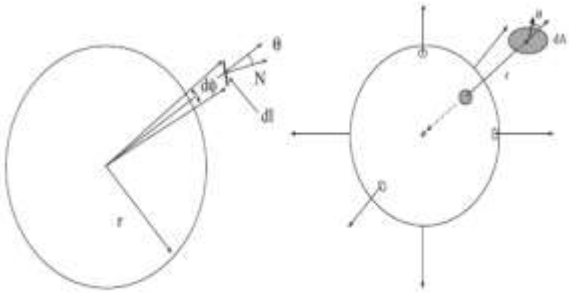


- **Fig: A point on a surface sees the world along a hemisphere of directions centred at the point; the surface normal is used to orient the hemisphere, to obtain the θ, ϕ coordinate system that we use consistently from now on to describe angular coordinates on this hemisphere. Usually in radiation problems we compute the brightness of the surface by summing effects due to all incoming directions, so that the fact we have given no clear way to determine the direction in which ϕ = 0 is not a problem.**
- Two sources that generate the same pattern on this input hemisphere must have the same effect on the surface at this point .This applies to sources, too;
- Two surfaces that generate the same pattern on a source's output hemisphere must receive the same amount of energy from the source.
- This means that the orientation of the surface patch with respect to the direction in which the illumination is travelling is important.
- As a source is tilted with respect to the direction in which the illumination is travelling, it "looks smaller" to a patch of surface.
- Similarly, as a patch is tilted with respect to the direction in which the illumination is travelling, it "looks smaller" to the source.

- The effect is known as foreshortening. Foreshortening is important, because from the point of view of the source a small patch appears the same as a large patch that is heavily foreshortened, and so must receive the same energy.

Solid Angle

- The pattern a source generates on an input hemisphere can be described by the solid angle that the source subtends.
- The angle subtended on the plane by an infinitesimal line segment of length dl at a point p can be obtained by projecting the line segment onto the unit circle whose center is at p; the length of the result is the required angle in radians.
- Because the line segment is infinitesimally short, it subtends an infinitesimally small angle which depends on the distance to the center of the circle and on the orientation of the line:
- $$d\phi = \frac{dl \cos\theta}{r}$$
- The angle subtended by a curve can be obtained by breaking it into infinitesimal segments and summing (integration!).
- Similarly, the solid angle subtended by a patch of surface at a point x is obtained by projecting the patch onto the unit sphere whose center is at x; the area of the result is the required solid angle, whose unit is now steradians. Solid angle is usually denoted by the symbol ω. Notice that solid angle captures the intuition in foreshortening — patches that "look the same" on the input hemisphere subtend the same solid angle.



- The angle subtended by a curve segment at a particular point is obtained by projecting the curve onto the unit circle whose center is at that point, and then measuring the length of the projection. For a small segment, the angle is $(1/r)dl \cos\theta$.
- A sphere, illustrating the concept of solid angle. The small circles surrounding the coordinate axes are to help you see the drawing as a 3D surface. An infinitesimal patch of surface is projected onto the unit sphere centred at the relevant point; the resulting area is the solid angle of the patch. In this case, the patch is small, so that the angle is $(1/r^2)dA \cos\theta$.
- If the area of the patch dA is small (as suggested by the infinitesimal form), then the infinitesimal solid angle it subtends is easily computed in terms of the area of the patch and the distance to it as
- $$d\omega = \frac{dA \cos\theta}{r^2}$$

- Solid angle can be written in terms of the usual angular coordinates on a sphere we have the infinitesimal steps ($d\theta$, $d\phi$) in the angles θ and ϕ cut out a region of solid angle on a sphere given by:

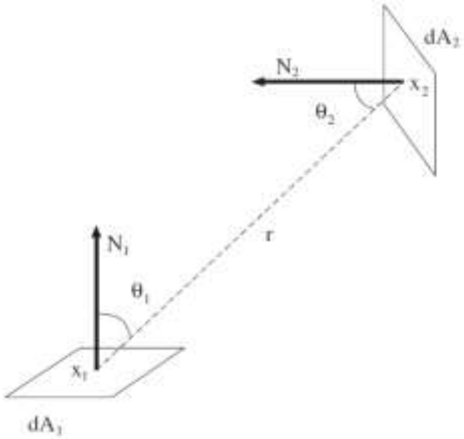
$$d\omega = \sin\theta d\theta d\phi$$

Radiance

- The appropriate unit for measuring the distribution of light in space is radiance, which is defined as the amount of energy travelling at some point in a specified direction, per unit time, per unit area perpendicular to the direction of travel, per unit solid angle.
- The units of radiance are watts per square meter per steradian $(Wm^{-2}sr^{-1})$.
- It is important to remember that the square meters in these units are foreshortened, i.e. perpendicular to the direction of travel.
- This means that a small patch viewing a source frontally collects more energy than the same patch viewing source radiance along a nearly tangent direction.
- The amount of energy a patch collects from a source depends both on how large the source looks from the patch and on how large the patch looks from the source.
- Radiance is a function of position and direction (the torch with a narrow beam is a good model to keep in mind — you can move the torch around, and point the beam in different directions).
- The radiance at a point in space is usually denoted L(x, direction), where x is a coordinate for position, which can be a point in free space or a point on a surface and we use some mechanism for specifying direction.
- One way to specify direction is to use (θ, ϕ) coordinates established using some surface normal. Another is to write x1 → x2, meaning the direction from point x1 to x2. We shall use both, depending on which is convenient for the problem at hand.

Radiance is Constant Along a Straight Line

- For the vast majority of important vision problems, it is safe to assume that light does not interact with the medium through which it travels — i.e. that we are in a vacuum.
- Radiance has the highly desirable property that, for two points p1 and p2 (which have a line of sight between them), the radiance leaving p1 in the direction of p2 is the same as the radiance arriving at p2 from the direction of p1.
- The following figure shows a patch of surface radiating in a particular direction.



- From the definition, if the radiance at the patch is L(x_1, θ, ϕ), then the energy transmitted by the patch into an infinitesimal region of solid angle dω around the direction θ, ϕ in time dt is L(x_1, θ, ϕ)(cos $\theta_1 dA_1$)(dω)(dt),
- (i.e. radiance times the foreshortened area of the patch times the solid angle into which the power is radiated times the time for which the power is radiating).
- Now consider two patches, one at x1 with area $dA_1$and the other at x2 with area dA_2
- To avoid confusion with angular coordinate systems, write the angular direction from x_1 to x_2 as $x_1 \rightarrow x_2$. The angles θ_1 and θ_2 are as defined in figure.
- The radiance leaving X_1 in the direction of x_1 is L(x_1, $x_1 \rightarrow x_2$) and the radiance arriving at x_2 from the direction of x_1 is L(x_2, $x_1 \rightarrow x_2$).

This means that, in time dt, the energy leaving $\boldsymbol{x}_1$ towards $\boldsymbol{x}_2$ is

$$d^3E_{1\to2} = L(\boldsymbol{x}_1, \boldsymbol{x}_1 \to \boldsymbol{x}_2)\cos\theta_1 d\omega_{2(1)} dA_1 dt$$

where $d\omega_{2(1)}$ is the solid angle subtended by patch 2 at patch 1 (energy emitted into this solid angle arrives at 2; all the rest disappears into the void). The notation $d^3E_{1\to2}$ implies that there are three infinitesimal terms involved.

From the expression for solid angle above,

$$d\omega_{2(1)} = \frac{\cos\theta_2 dA_2}{r^2}$$

Now the energy leaving 1 for 2 is:

$$\begin{aligned} d^3E_{1\to2} &= L(\boldsymbol{x}_1, \boldsymbol{x}_1 \to \boldsymbol{x}_2)\cos\theta_1 d\omega_{2(1)} dA_1 dt \\ &= L(\boldsymbol{x}_1, \boldsymbol{x}_1 \to \boldsymbol{x}_2)\frac{\cos\theta_1 \cos\theta_2 dA_2 dA_1 dt}{r^2} \end{aligned}$$

Because the medium is a vacuum, it does not absorb energy, so that the energy arriving at 2 from 1 is the same as the energy leaving 1 in the direction of 2. The energy arriving at 2 from 1 is:

$$\begin{aligned} d^3E_{1\to2} &= L(\boldsymbol{x}_2, \boldsymbol{x}_1 \to \boldsymbol{x}_2)\cos\theta_2 d\omega_{1(2)} dA_2 dt \\ &= L(\boldsymbol{x}_2, \boldsymbol{x}_1 \to \boldsymbol{x}_2)\frac{\cos\theta_2 \cos\theta_1 dA_1 dA_2 dt}{r^2} \end{aligned}$$

which means that $L(\boldsymbol{x}_2, \boldsymbol{x}_1 \to \boldsymbol{x}_2) = L(\boldsymbol{x}_1, \theta, \phi)$, so that *radiance is constant along (unoccluded) straight lines.*

Light at Surfaces

- When light strikes a surface, it may be absorbed, transmitted, or scattered; usually, a combination of these effects occurs.
- For example, light arriving at skin can be scattered at various depths into tissue and reflected from blood or from melanin in there; can be absorbed; or can be scattered tangential to the skin within a film of oil and then escape at some distant point.
- The picture is complicated further by the willingness of some surfaces to absorb light at one wavelength, and then radiate light at a different wavelength as a result.
- This effect, known as fluorescence, is fairly common: scorpions fluoresce visible light under x-ray illumination; human teeth fluoresce faint blue under ultraviolet light and laundry can be made to look bright by washing powders that fluoresce under ultraviolet light. Furthermore, a surface that is warm enough emits light in the visible range.

Simplifying Assumptions

- It is common to assume that all effects are local, and can be explained with a macroscopic model with no fluorescence or emission. This is a reasonable model for the kind of surfaces and decisions that are common in vision.
- In this model: the radiance leaving a point on a surface is due only to radiance arriving at this point (although radiance may change directions at a point on a surface, we assume that it does not skip from point to point);
- We assume that all light leaving a surface at a given wavelength is due to light arriving at that wavelength;
- We assume that the surfaces do not generate light internally, and treat sources separately.

Irradiance

- The appropriate unit for representing incoming power which is irradiance, defined as: incident power per unit area not foreshortened.
- A surface illuminated by radiance Li(x, θ_i, ϕ_i) coming in from a differential region of solid angle $d\omega$ at angles (θ_i, ϕi) receives irradiance L_i(x, θi, ϕi) cos $\theta_i d\omega$
- Where we have multiplied the radiance by the foreshortening factor and by the solid angle to get irradiance. The main feature of this unit is that we could compute all the power incident on a surface at a point by summing the irradiance over the whole input hemisphere — which makes it the natural unit for incoming power.

The BRDF

- The most general model of local reflection is the bidirectional reflectance distribution function, usually abbreviated BRDF.
- The BRDF is defined as the ratio of the radiance in the outgoing direction to the incident irradiance so that, if the surface of the preceding paragraph was to emit radiance Lo(x, θo, ϕo), its BRDF would be:

$$\rho_{bd}(\theta_o, \phi_o, \theta_i, \phi_i) = \frac{L_o(\boldsymbol{x}, \theta_o, \phi_o)}{L_i(\boldsymbol{x}, \theta_i, \phi_i)\cos\theta_i d\omega}$$

- The BRDF has units of inverse steradians (sr−1), and could vary from 0 (no light reflected in that direction) to infinity (unit radiance in an exit direction resulting from arbitrary small radiance in the incoming direction).
- The BRDF is symmetric in the incoming and outgoing direction, a fact known as the Helmholtz reciprocity principle.

Properties of the BRDF

- The radiance leaving a surface due to irradiance in a particular direction is easily obtained from the definition of the BRDF:
- $$L_o(\boldsymbol{x}, \theta_o, \phi_o) = \rho_{bd}(\theta_o, \phi_o, \theta_i, \phi_i) L_i(\boldsymbol{x}, \theta_i, \phi_i)\cos\theta_i d\omega$$
- More interesting is the radiance leaving a surface due to its irradiance (whatever the direction of irradiance). We obtain this by summing over contributions from all incoming directions:
- $$L_o(\boldsymbol{x}, \theta_o, \phi_o) = \int_\Omega \rho_{bd}(\theta_o, \phi_o, \theta_i, \phi_i) L_i(\boldsymbol{x}, \theta_i, \phi_i)\cos\theta_i d\omega$$
- where Ω is the incoming hemisphere. From this we obtain the fact that the BRDF is not an arbitrary symmetric function in four variables.
- To see this, assume that a surface is subjected to a radiance of 1/ cos θi Wm−2 sr−1.
- This means that the total energy arriving at the surface is:
- $$\begin{aligned}\int_\Omega \frac{1}{\cos\theta}\cos\theta d\omega &= \int_0^{2\pi}\int_0^{\frac{\pi}{2}} \sin\theta d\theta d\phi \\ &= 2\pi\end{aligned}$$
- We have assumed that any energy leaving at the surface leaves from the same point at which it arrived, and that no energy is generated within the surface. This means that the total energy leaving the surface must be less than or equal to the amount arriving.

Radiosity

- If the radiance leaving a surface is independent of exit angle, there is no point in describing it using a unit that explicitly depends on direction.
- The appropriate unit is radiosity, defined as the total power leaving a point on a surface per unit area on the surface
- Radiosity, which is usually written as B(x) has units watts per square meter (Wm^{-2}).

- To obtain the radiosity of a surface at a point, we can sum the radiance leaving the surface at that point over the whole exit hemisphere.
- Thus, if x is a point on a surface emitting radiance L(x, θ, ϕ), the radiosity at that point will be:

$$B(\boldsymbol{x}) = \int_{\Omega} L(\boldsymbol{x}, \theta, \phi) \cos\theta d\omega$$

Important Special Cases

- Radiance is a fairly subtle quantity, because it depends on angle.
- This generality is sometimes essential — for example, for describing the distribution of light in space in the torch beam example above.
- As another example, fix a compact disc and illuminate its underside with a torch beam. The intensity and colour of light reflected from the surface depends very strongly on the angle from which the surface is viewed and on the angle from which it is illuminated.
- The CD example is worth trying, because it illustrates how strange the behaviour of reflecting surfaces can be; it also illustrates how accustomed we are to dealing with surfaces that do not behave in this way.
- For many surfaces — cotton cloth is one good example — the dependency of reflected light on angle is weak or non-existent, so that a system of units that are independent of angle is useful.

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The Radiosity of a Surface with Constant Radiance

- One result to remember is the relationship between the radiosity and the radiance of a surface patch where the radiance is independent of angle. In this case Lo(x, θo, ϕo) = Lo(x).
- Now the radiosity can be obtained by summing the radiance leaving the surface over all the directions in which it leaves:

$$B(\boldsymbol{x}) = \int_{\Omega} L_o(\boldsymbol{x}) \cos\theta d\omega$$

$$= L_o(x) \int_0^{\frac{\pi}{2}} \int_0^{2\pi} \cos\theta \sin\theta d\phi d\theta$$
$$= \pi L_o(x)$$

Directional Hemispheric Reflectance

- The BRDF measurements are typically difficult, expensive and not particularly repeatable. This is because surface dirt and aging processes;
- For example, touching a surface will transfer oil to it, typically in little ridges (from the fingertips) which can act as lenses and make significant changes in the directional behaviour of the surface.
- The light leaving many surfaces is largely independent of the exit angle.
- A natural measure of a surface's reflective properties in this case is the **Directional Hemispheric Reflectance**, usually termed ρd_h, defined as: the fraction of the incident irradiance in a given direction that is reflected by the surface, whatever the direction of reflection
- The directional hemispheric reflectance of a surface is obtained by summing the radiance leaving the surface over all directions, and dividing by the irradiance in the direction of illumination, which gives:

$$\begin{aligned}\rho_{dh}(\theta_i, \phi_i) &= \frac{\int_\Omega L_o(x, \theta_o, \phi_o) \cos\theta_o d\omega_o}{L_i(x, \theta_i, \phi_i) \cos\theta_i d\omega_i} \\ &= \int_\Omega \left\{ \frac{L_o(x, \theta_o, \phi_o) \cos\theta_o}{L_i(x, \theta_i, \phi_i) \cos\theta_i d\omega_i} \right\} d\omega_o \\ &= \int_\Omega \rho_{bd}(\theta_o, \phi_o, \theta_i, \phi_i) \cos\theta_o d\omega_o\end{aligned}$$

- This property is dimensionless, and its value will lie between 0 and 1. Directional hemispheric reflectance can be computed for any surface.
- For some surfaces, it will vary sharply with the direction of illumination. A good example is a surface with fine, symmetric triangular grooves which are black on one face and white on the other.
- If these grooves are sufficiently fine, it is reasonable to use a macroscopic description of the surface as flat, and with a directional hemispheric reflectance that is large along a direction pointing towards the white faces and small along that pointing towards the black.

Lambertian Surfaces and Albedo

- For some surfaces the directional hemispheric reflectance does not depend on illumination direction.
- Examples of such surfaces include cotton cloth, many carpets, matte paper and matte paints. A formal model is given by a surface whose BRDF is independent of outgoing direction
- This means the radiance leaving the surface is independent of angle. Such surfaces are known as ideal diffuse surfaces or Lambertian surfaces (after George Lambert, who first formalised the idea).
- It is natural to use radiosity as a unit to describe the energy leaving a Lambertian surface.
- For Lambertian surfaces, the directional hemispheric reflectance is independent of direction.
- In this case the directional hemispheric reflectance is often called their diffuse reflectance or albedo and written ρd. For a Lambertian surface with BRDF $\rho bd(\theta_o, \phi_o, \theta_i, \phi_i) = \rho$, we have:

$$
\begin{aligned}
\rho_d &= \int_\Omega \rho_{bd}(\theta_o, \phi_o, \theta_i, \phi_i) \cos\theta_o d\omega_o \\
&= \int_\Omega \rho \cos\theta_o d\omega_o \\
&= \rho \int_0^{\frac{\pi}{2}} \int_0^{2\pi} \cos\theta_o \sin\theta_o d\theta_o d\phi_o \\
&= \pi\rho
\end{aligned}
$$

This fact is more often used in the form

$$\rho_{brdf} = \frac{\rho_d}{\pi}$$

Specular Surfaces

- A second important class of surfaces are the glossy or mirror-like surfaces, often known as specular surfaces (after the Latin word speculum, a mirror). An ideal specular reflector behaves like an ideal mirror.
- Radiation arriving along a particular direction can leave only along the specular direction, obtained by reflecting the direction of incoming radiation about the surface normal. Usually some fraction of incoming radiation is absorbed; on an ideal specular surface, the same fraction of incoming radiation is absorbed for every direction, the rest leaving along the specular direction.
- The BRDF for an ideal specular surface has a curious form, because radiation arriving in a particular direction can leave in only one direction.

Specular Lobes

- Relatively few surfaces can be approximated as ideal specular reflectors.
- A fair test of whether a flat surface can be approximated as an ideal specular reflector is whether one could safely use it as a mirror.
- Good mirrors are suprisingly hard to make; up until recently, mirrors were made of polished metal.

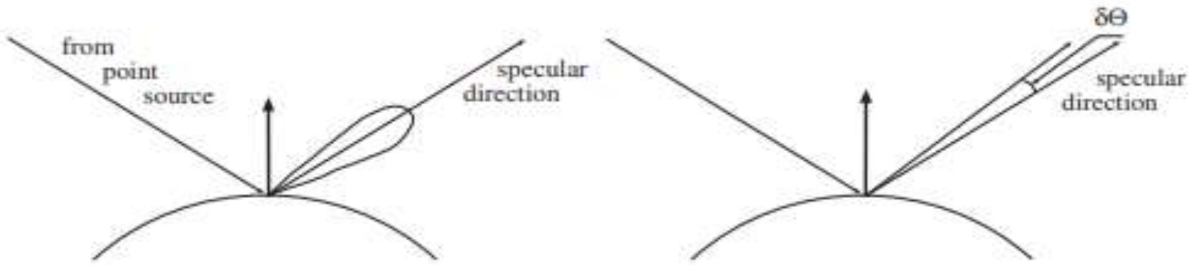

Figure 1.4. Specular surfaces commonly reflect light into a lobe of directions around the specular direction, where the intensity of the reflection depends on the direction, as shown on the left. Phong's model is used to describe the shape of this lobe, in terms of the offset angle from the specular direction.

- Typically, unless the metal is extremely highly polished and carefully maintained, radiation arriving in one direction leaves in a small lobe of directions around the specular direction.
- This results in a typical blurring effect. A good example is the bottom of a flat metal pie dish.
- If the dish is reasonably new, one can see a distorted image of one's face in the surface but it would be difficult to use as a mirror;.
- Larger specular lobes mean that the specular image is more heavily distorted and is darker (because the incoming radiance must be shared over a larger range of outgoing directions). Quite commonly it is possible to see only a specular reflection of relatively bright objects, like sources.
- Thus, in shiny paint or plastic surfaces, one sees a bright blob — often called a specularity — along the specular direction from light sources, but few other specular effects.
- It is not often necessary to model the shape of the specular lobe. When the shape of the lobe is modelled, the most common model is the Phong model, which assumes that only point light sources are specularly reflected.
- In this model, the radiance leaving a specular surface is proportional to $\cos^n(\delta\theta) = \cos^n(\theta_o - \theta_s)$,where θ_o is the exit angle, θ_s is the specular direction and n is a parameter.
- Large values of n lead to a narrow lobe and small, sharp specularities and small values lead to a broad lobe and large specularities with rather fuzzy boundaries

The Lambertian + Specular Model

- Relatively few surfaces are either ideal diffuse or perfectly specular.
- Very many surfaces can be approximated has having a surface BRDF which is a combination of a Lambertian component and a specular component, which usually has some form of narrow lobe.
- Usually, the specular component is weighted by a specular albedo. Again, because specularities tend not to be examined in detail, the shape of this lobe is left unspecified.
- In this case, the surface radiance (because it must now depend on direction) in a given direction is typically approximated as:

  $$L(\boldsymbol{x}, \theta_o, \phi_o) = \rho_d(\boldsymbol{x}) \int_{\Omega} L(\boldsymbol{x}, \theta_i, \phi_i) \cos\theta_i d\omega + \rho_s(\boldsymbol{x}) L(\boldsymbol{x}, \theta_s, \phi_s) \cos^n(\theta_s - \theta_o)$$

  where θ_s, ϕ_s give the specular direction and ρs is the specular albedo.
- It is common not to reason about the exact magnitude of the specular radiance term.
- Using this model implicitly excludes “too narrow” specular lobes, because most algorithms expect to encounter occasional small, compact specularities from light sources.
- Surfaces with too narrow specular lobes (mirrors) produce overwhelming quantities of detail in specularities.
- Similarly, “too broad” lobes are excluded because the specularities would be hard to identify.