Noise Models in Feature-based Stereo Visual Odometry

Pablo F. Alcantarilla[†] and Oliver J. Woodford[‡]

Abstract -- Feature-based visual structure and motion reconstruction pipelines, common in visual odometry and large-scale reconstruction from photos, use the location of corresponding features in different images to determine the 3D structure of the scene, as well as the camera parameters associated with each image. The noise model, which defines the likelihood of the location of each feature in each image, is a key factor in the accuracy of such pipelines, alongside optimization strategy. Many different noise models have been proposed in the literature; in this paper we investigate the performance of several. We evaluate these models specifically w.r.t. stereo visual odometry, as this task is both simple (camera intrinsics are constant and known; geometry can be initialized reliably) and has datasets with ground truth readily available (KITTI Odometry and New Tsukuba Stereo Dataset). Our evaluation shows that noise models which are more adaptable to the varying nature of noise generally perform better.

I. Introduction

Inverse problems—given a set of observed measurements, infer the physical system that generated them—are common in computer vision and robotics. One of the keys to solving such problems is knowing how accurate each measurement is. This is captured in a noise model: a distribution over expected measured values, given a true value.

In visual reconstruction tasks the measurements are images and the physical system is scene geometry, lighting and camera position. *Direct* methods [1], [2] phrase the problem in exactly this way, seeking the scene/camera model which best recreates all the pixel values of the images. The source of noise in this case is well understood: mostly electronic noise (for electronic sensors). It is also straightforward to measure. However, most direct methods simply approximate the noise with a Gaussian or Laplacian, and this seems to be sufficient.

In contrast, feature-based methods [3], [4], [5] pre-process the images, extracting a set of interest points, or features, from each and tracking or matching these across images. In this case, the inverse problem to be solved is: Given feature image locations, compute their 3D locations and the camera poses. This gives rise to the following notion of a feature noise model, illustrated in Fig. 1: Features are assumed to be points in space, the true 3D locations of which are unknown variables. These features project onto each image plane to give a true image location. The "measurements" are the computed image locations of these features, and are a noisy version of the true values. The noise model is therefore a probability distribution over the expected image location of a measurement, given the true image location.

This work was done while both authors were employees at Toshiba Research Europe Ltd., Cambridge, United Kingdom. Contact information: pablofdezalc@gmail.com † , o.j.woodford.98@cantab.net ‡ .

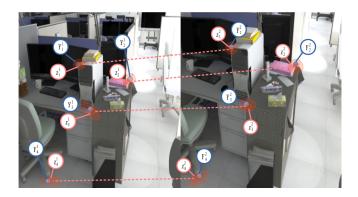


Fig. 1. Given a set of 2D feature correspondences z_i (red circles) between two images, the inverse problem to be solved is to estimate the camera poses and the 3D locations of the features Y_i . The true 3D locations project onto each image plane to give a true image location (blue circles). The noise model represents a probability distribution over the error \mathbf{e} between the true and measured feature locations. The size of the ellipses depends on the uncertainty in the feature locations.

In this case, the nature of the noise distribution is not so obvious. It is certainly not well approximated by a Gaussian, since errors in tracking or matching can lead to gross errors, or outliers.

Several noise models, along with associated optimization strategies, have been suggested in the literature, which we discuss in detail in $\S II$. However, there is currently lacking an up-to-date evaluation and comparison of these noise models. Previous approaches such as [6], evaluate the performance of RANSAC-based [7] noise models for line fitting and planar homography estimation. Recently in [8], a match selection and refinement strategy for two-view Structure from Motion (SfM) problems was presented. That work analyzes the trade-offs of quality versus quantity of feature matches applied to different noise models.

In this work, we present a thorough explanation of feature noise models and unify the explanation to describe how each one specifies a given distribution on the measurement errors. We evaluate published feature noise models along with optimization strategies applied to the problem of feature-based stereo visual odometry (VO) [9], [10]. This is a simple visual reconstruction task; camera intrinsics are known and constant, stereo gives a good initial structure estimate, and the baseline between frames is small. It also has good test datasets and performance benchmarks available, in the form of the KITTI Odometry benchmark [11] and the New Tsukuba Stereo dataset (synthetic) [12].

The next section provides an overview of noise models used in feature-based visual reconstruction, focusing on those we evaluate and applied to the problem of stereo VO. In $\S III$ we describe our evaluation framework. In $\S IV$ we present the results of our evaluation, before concluding in the final section.

II. FEATURE NOISE MODELS

In this section we review feature noise models in the literature. Many visual reconstruction methods do not in fact mention probability distributions. Some talk about cost functions and optimizations; others simply present algorithms. However, all these methods do imply assumptions about the nature of the noise. In order to present these assumptions on an even field, we cast them all as explicit probability distributions.

To simplify things, we refer directly to the difference in image coordinates between the true and measured feature locations, which we denote as e (see Fig. 1). We therefore assume that our reader is familiar with the projection of 3D points in world coordinates onto image planes [13]. Since many noise models are defined by summed costs, *i.e.* the negative log likelihood of probability, we represent such cost functions by $\rho(e)$, and define a helper function,

$$\operatorname{prob}(\rho(\mathbf{e})) = \frac{\exp(-\rho(\mathbf{e}))}{\int \exp(-\rho(\mathbf{x})) \, d\mathbf{x}}, \tag{1}$$

to convert a cost to a probability.

The set of variables, Θ , used to compute the measurement errors, $\{e_i\}_{i=1}^N$, consists of camera pose variables (extrinsics) \mathcal{X} , 3D feature locations Y, and optionally camera intrinsics and noise model parameters. The cost, which is the negative log of the probability of the measurements, for a given Θ is defined as

$$E(\Theta) = \underbrace{\sum_{i=1}^{N} \rho(\mathbf{e}_i)}_{\text{Data costs}} - \underbrace{N \cdot \log\left(\int \exp\left(-\rho(\mathbf{x})\right) d\mathbf{x}\right)}_{\text{Noise model cost (usually constant)}}.$$
 (2)

Two aspects of feature-based visual reconstruction make the above cost multi-modal, irrespective of the noise models used. Firstly, the projection of features onto image planes and the camera rotations are both non-linear operations; convex noise models do not therefore lead to a convex cost w.r.t. the variables. Secondly, the presence of outliers encourage spurious local minima.

Most visual reconstruction pipelines therefore have two distinct stages (following the feature extraction stage): An *initialization* stage, where approximate 3D feature locations and camera poses are computed. The main tasks here are to efficiently find a starting point in the convergence basin of the global minimum of the cost function, and to reject outlier measurements in the process. Then a *refinement* stage, where the remaining measurements are used to iteratively improve output variables. The main task here is to maximize the accuracy of the final solution.

Since the noise models of the first stage need to be robust to outliers, whilst those of the second stage need to maximize accuracy of inliers, the models used in each stage are often different. We therefore split our discussion of models into those two stages.

A. Initialization

The majority of initialization methods in visual reconstruction pipelines use a *hypothesize-and-test* framework to select an initial set of variables Θ : many sets of values for the variables are computed, and the set with the lowest cost (computed using Eq. (2)) is selected. This helps to ensure the initial values lie within the convergence basin of the global minimum. The noise models below are all used within this framework.

1) RANSAC: RAndom SAmple Consensus [7] is the original hypothesize-and-test framework. Its score is the number of errors below a given threshold, T. This corresponds to the following cost function:

$$\rho(\mathbf{e}) = \begin{cases} 0 & \text{if } ||\mathbf{e}||^2 < T^2, \\ 1 & \text{otherwise.} \end{cases}$$
 (3)

The threshold T quantifies the maximum deviation attributable to the effects of noise and is used to classify each correspondence between inlier or outlier. Both are assumed to be distributed uniformly within their domains.

The choice of this threshold T has a tremendous impact on the accuracy of the estimated variables. If T is too small, little data is used to estimate the variables, which can lead to inaccuracies; on the other hand, if T is too large, the estimated variables may be corrupted by outliers.

2) MSAC: Torr et al. [14] recognised that a uniform distribution is a poor approximation for inliers. They instead proposed the MSAC cost function, which is a slight variation on RANSAC that assumes that inliers are distributed normally, thus:

$$\rho(\mathbf{e}) = \begin{cases} \|\mathbf{e}\|^2 & \text{if } \|\mathbf{e}\|^2 < T^2, \\ T^2 & \text{otherwise.} \end{cases}$$
 (4)

3) **MLESAC**: In the same paper, Torr *et al.* [14] take a more probabilistic approach with their maximum likelihood estimator, MLESAC. Similarly to MSAC, it assumes that inliers are distributed normally, though with a full covariance matrix, and that outliers are distributed uniformly. However, rather than assuming (or implying) a prior probability for a measurement being an inlier, they introduce a new variable, γ , to represent this. This leads to the following noise model:

$$\rho(\mathbf{e}) = -\log \left[\gamma \operatorname{prob} \left(\frac{1}{2} \mathbf{e}^{\mathsf{T}} \Sigma^{-1} \mathbf{e} \right) + (1 - \gamma) \frac{1}{\nu} \right]$$
 (5)

where ν is the space within which the outliers are believed to fall uniformly and Σ is the covariance matrix of the inlier noise. For each hypothesis, the value of γ that maximizes likelihood is computed using Expectation-Maximization (EM) from some initial value, e.g. 0.5. The benefit of this approach is that it can adapt to varying levels of outlier contamination.

¹The method for computing plausible sets of values is application specific. We describe our approach for stereo VO in §*III*.

- 4) AMLESAC: MLESAC assumes prior knowledge of the inlier noise distribution, through the parameter Σ . In certain scenarios this distribution may not be known, or may even vary according to the conditions. To overcome this, Adaptive MLESAC [15] has the same noise model as Eq. (5), but additionally optimizes Σ .
- 5) AC-RANSAC: AC-RANSAC [16], [17] uses the *a contrario* methodology. For each hypothesis, this approach computes an inlier/outlier adaptive threshold based on the *Helmholtz principle*—whenever some large deviation from randomness occurs, a structure is perceived. AC-RANSAC seeks to minimize the number of false alarms (*NFA*).

$$\hat{\Theta} = \underset{\Theta}{\operatorname{arg\,min}} NFA(\mathbf{e}, q) \le \epsilon$$
 (6)

where ϵ is the minimum number of false alarms in order to consider a set of variables Θ as valid and it is usually set to 1. A false alarm is defined as a set of variables that is actually due to chance. This requires the definition of a rigidity measure e and a background noise model \mathcal{H}_0 , that models random correspondences assuming that points are uniformly distributed in their respective images. The *NFA* is defined as:

$$NFA(\mathbf{e},q) = (N-N_s)\binom{N}{q}\binom{q}{N_s}\left(\mathbf{e}_q^d\alpha_0\right)^{q-N_s} \tag{7}$$

where:

- N is the total number of correspondences between two images.
- N_s is the cardinal of a RANSAC sample.
- q is the total number of hypothesized inlier correspondences
- e_q is the q-th lowest error among all N correspondences and depends on the set of variables Θ .
- d is the error dimension.
- α₀ is the probability of a random correspondence of having error of one pixel. In the case of stereo VO, this probability is computed as the ratio between the volume

In the particular case of stereo VO, d=3 for rectified stereo pairs and α_0 can be computed as the ratio between the volume of a sphere of unit radius divided by the volume given by the image dimensions and the disparity range, *i.e.* $4\pi/(3 \cdot \text{Width} \cdot \text{Height} \cdot \text{Disparity})$.

6) **ERODE**: In contrast to the above noise models, the *Efficient and Robust Outlier Detector* [18] is not used in a hypothesize-and-test framework. Instead, it uses a robust yet convex cost function, the Pseudo-Huber kernel, to reduce the number of local minima, and performs an optimization on the variables from a single starting point. The cost function is given by:

$$\rho(\mathbf{e}) = 2b^2 \left(\sqrt{1 + \frac{\|\mathbf{e}\|^2}{b^2}} - 1 \right)$$
 (8)

where b is a user-defined parameter that tunes the shape of the function. ERODE is faster compared to RANSAC and other consensus approaches and achieves similar accuracy when the initial camera pose is close to the real one.

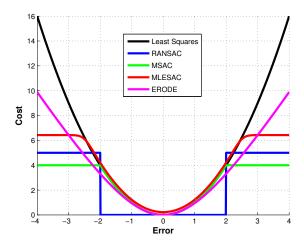


Fig. 2. Cost functions $\rho(\cdot)$ for several noise models: RANSAC, MSAC, MLESAC and ERODE with b=2. We also show the least squares cost function for illustrative purposes.

Other robust costs with adaptive thresholds have been proposed in the literature such as [19] and more recently [20]. These methods overcome the limitations of globally-fixed thresholds, yielding better precision. RECON [19] is built on the observation that those sets of variables generated from uncontaminated minimal subsets of correspondences are consistent in terms of the behavior of their residuals, while contaminated subsets exhibit uncorrelated behavior. Cohen and Zach [20] propose an improvement of RANSAC which also optimizes over a discrete number of outlier thresholds in a very efficient manner, using the likelihood-ratio test.

Table I shows a summary of the robust initialization methods considered in this paper, showing their cost functions and main configuration parameters. Fig. 2 depicts the cost functions for some of the noise models considered in this paper.

	Cost Function:	Parameters
RANSAC	Inliers Count	T
MSAC	Truncated Quadratic	T
MLESAC	Neg. Log Gaussian+Uniform	Σ , ν
ERODE	Pseudo-Huber	b, T
AC-RANSAC	Number of False Alarms	α_0,ϵ

TABLE I

ROBUST INITIALIZATION: COST FUNCTIONS AND MAIN CONFIGURATION

PARAMETERS.

B. Refinement

In the refinement stage, the goal is to maximize the accuracy of the final solution using a good initial set of variables. It is typically a minimization of the cost in equation (2) using standard nonlinear least-squares optimizers such as Gauss-Newton or Levenberg-Marquardt.

1) Handling outliers: Three of the noise models from $\S II - A$ robustly handle outliers, as well as defining a sensible noise model for inliers: MSAC, MLESAC and ERODE.

These can be used directly in a refinement optimization. RANSAC's uniform inlier model does not lend itself to refinement, and AC-RANSAC does not define an inlier noise model. These approaches therefore need a different noise model for the refinement stage. Both use the same strategy, which is to remove from the refinement optimization those measurements deteremined to be outliers during the initialization stage, *i.e.* those measurements for which $\|e\|$ was above the outlier threshold for the selected Θ . The remaining measurements are then given a simple, uniform gaussian cost:

$$\rho(\mathbf{e}) = \|\mathbf{e}\|^2. \tag{9}$$

- 2) Variables optimized over: The refinement optimization may not necessarily refine all the variables. As a minimum, the optimization refines camera pose variables, \mathcal{X} , between pairs of frames. 3D structure, Y, can be added, creating a two-view Bundle Adjustment (BA) optimization [13], [21]. It is also possible to include the parameters of the noise distribution (if there are any) as variables. Finally, more frames can be introduced into the refinement [22], [23]. Every additional variable comes at the expense of higher computational demand. In our experiments we will investigate three levels of variable refinement:
 - 1) **Motion only**: The 3D structure Y and noise parameters are fixed, and only camera poses \mathcal{X} are optimized.
 - 2) **Motion and Structure (BA)**: The noise parameters are fixed, while camera poses \mathcal{X} and 3D structure Y are optimized.
 - 3) **Motion, Structure and Inlier Noise Distribution**: We change the inlier noise distribution from a uniform gaussian to a Cauchy distribution with full covariance:

$$\rho(\mathbf{e}) = -\log\left(1 + \mathbf{e}^{\mathsf{T}} \Sigma^{-1} \mathbf{e}\right). \tag{10}$$

Furthermore, the parameters of the inverse covariance, Σ^{-1} , are optimized in addition to the camera poses $\mathcal X$ and the 3D structure Y. Since the noise distribution is changing in the optimization, the right hand term of equation (2) is not constant, and therefore should be included in the optimization; for the Cauchy distribution this term is $-\log(|\Sigma|) \cdot N/(d+1)$.

Estimating the inlier noise distribution can help when this distribution is unknown or variable and non-uniform, *e.g.* with occasional motion blur. We use the robust Cauchy distribution to add further protection against the presence of outliers.

III. STEREO VISUAL ODOMETRY

The goal of stereo VO is to estimate the camera motion $\mathcal{X}_k = \{R, \mathbf{t}\}$ between two consecutive frames (k-1, k) in the Euclidean group SE(3) of 3D poses, by minimizing the reprojection error given a set of N correspondences between 3D points $Y_i \in \mathbb{R}^3$ and stereo image measurements $z_i \in \mathbb{R}^6$. Stereo calibration parameters are obtained in a prior calibration process. The calibration parameters comprise of

camera focal length f, principal point (u_0, v_0) , stereo baseline B and distortion parameters. Given the stereo calibration parameters, *stereo rectification* is typically performed to simplify the stereo correspondence problem.

Stereo VO has the benefit over monocular VO that an initial estimate of scene geometry, *i.e.* 3D points, can be computed from the 2D correspondences from the stereo image pair at each timeframe, independently of any camera motion between frames. Current feature-based stereo VO approaches [3], [4], [5] generally follow these three steps:

- Putative correspondences: A tracking [24], [25] or matching [26], [27] method establishes an initial set of putative 2D point correspondences between consecutive stereo frames. A single putative correspondence, comprises of a set of stereo image measurements z_i = [u_l^{k-1}, u_r^{k-1}, v^{k-1}, u_l^k, u_r^k, v^k] and triangulated 3D points Y_i^{k-1} with respect to stereo frame k 1. The subscripts l and r denote measurements from the left and right images respectively.
- Robust initialization: An inlier subset of the correspondences and an approximate initial camera motion and 3D scene geometry are computed.
- 3) Model Refinement: The camera motion, and optionally 3D scene geometry and inliers noise distribution, are refined by minimizing the 2D reprojection error of the inlier point correspondences in the image domain.

In stereo VO, the error e between the predicted and measured image feature locations for a particular correspondence is defined as:

$$\mathbf{e}_{i} = z_{i}^{k} - \pi \left(\mathcal{X}_{k} Y_{i}^{k-1} \right) \tag{11}$$

where the function $\pi(\cdot)$ is the stereo projection function that projects a 3D point from the stereo frame k-1 into the left and right images at frame k given the stereo calibration parameters. In the case of rectified stereo, this error has dimension d=3. When refining also for the structure parameters, the error e takes into account the measurements in the two views z_i^{k-1} and z_i^k , therefore having a dimension d=6. In this case, the camera pose for the frame k-1 is set to a canonical pose with identity rotation matrix and zero translation vector.

IV. EXPERIMENTAL RESULTS

In this section we show the results of our noise model evaluation w.r.t. feature-based stereo VO, alongside optimization strategies. First, we describe some details of our evaluation in $\S{IV}-A$. Then we show experimental results for the robust initialization and refinement steps in $\S{IV}-C$ and $\S{IV}-D$ respectively.

A. Preliminaries

In our evaluation we consider two stereo datasets: the KITTI odometry benchmark [11] and the New Tsukuba Stereo dataset [12]. The KITTI Odometry benchmark comprises 20 sequences captured from a front facing car mounted camera; ground truth pose information is available for the

 $^{^2}$ We use a triangular parameterization of the square root of the inverse covariance matrix Σ^{-1} .

first 11 sequences. The New Tsukuba Stereo dataset is a synthetic dataset for the purpose of stereo matching and camera tracking evaluation. The dataset comprises 4 different illumination settings, from which we show average results on the *fluorescent* and *daylight* settings.

- 1) LIBVISO2: We compare our stereo visual odometry approaches with the LIBVISO2 library [4]. LIBVISO2 is a publicly available library that performs feature-based stereo VO, detecting corners and blobs from the input images and using horizontal and vertical Sobel filter responses as descriptors. An initial camera motion is estimated by means of a RANSAC approach using a globally-fixed threshold T equal to 2 pixels. The camera motion is then refined using only the inliers.
- 2) Feature Detection, Description and Matching: In our experiments, we use A-KAZE [26] as our feature detector and descriptor method due to its good detector repeatability and highly discriminative binary descriptors. We first match the binary descriptors between the left and right images of the stereo pair using a brute-force approach and consistency check between the two views to establish a set of stereo matches. Then, for obtaining a set of temporal putative correspondences, we perform the same matching strategy considering the left image as the reference one and again using a brute-force approach and consistency check between the two consecutive stereo frames.
- 3) Initial Camera Motion: For each hypothesis in consesus-based methods, an initial motion is computed from a minimal set of N_s correspondences. We obtain an initial guess for the camera motion by using a Perspective-n-Point (PnP) algorithm [28] with $N_s=4$ points as minimal sample and considering only the measurements from the left camera. ERODE considers an initial camera pose with identity rotation matrix and zero translation vector, assuming that the camera motion between consecutive frames is small.
- 4) Inlier/Outlier Classification: We consider two different values of the threshold parameter T to highlight the sensitivity of the estimated camera motion with respect to the threshold parameter in RANSAC and MSAC. The first threshold is set to T=2 pixels. The second value comes from the cumulative chi-squared distribution assuming that the measurement error is Gaussian with zero mean and standard deviation $\sigma=1$ pixels. Under this assumption, the value of the threshold parameter is 2.79 pixels [13].
- 5) Evaluation Criteria: We use the same evaluation method recommended by the KITTI dataset: we compute translational and rotational errors for all possible subsequences of length $(100, 150, 200, \ldots, 800)$ meters and take the average. Since the travelled distance in the New Tsukuba Stereo dataset is smaller, we consider all possible subsequences of length $(5, 10, 50, 100, 150, \ldots, 400)$ centimeters.

B. Bias in the KITTI Odometry Benchmark

Although not mentioned in [4], the LIBVISO2 library includes a heuristic feature weighting scheme with the aim of adding some robustness against calibration errors. Each

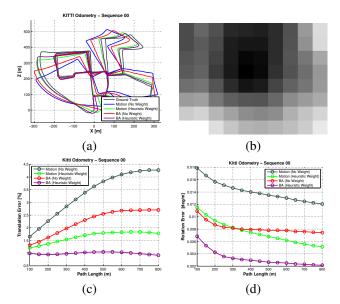


Fig. 3. Effect of feature weighting in the KITTI Odometry benchmark for the sequence 00. (a) Trajectories (b) Reprojection error histogram for the left camera (c) Average translation errors (d) Average rotation errors.

stereo measurement has an associated scalar weight $\boldsymbol{w_i}$ defined as:

$$w_i = (|u_L - u_0|/u_0 + 0.05)^{-1}$$
(12)

where u_0 is the camera horizontal principal point. Those measurements that are closer to the principal point have a higher weight in the optimization, whereas measurements closer to image boundaries have lower weights since they are prone to be more erroneous due to calibration errors. Fig. 3 shows the effect of this weighting scheme in the camera motion accuracy, considering two types of refinement: motion only and BA.

As can be observed in Fig. 3(c-d), there is a very significant improvement in accuracy when using the heuristic weighting scheme. An explanation of this improvement in accuracy is because the reprojection error distribution varies across the stereo image pair in the KITTI dataset [29], possibly due to poor camera calibration. Our experiments corroborate this, revealing that there is a reprojection error bias when using the ground truth camera motion, which tends to be stronger for observations closer to the image borders, as shown in Fig. 3(b). On the other hand, reprojection errors are uniformly distributed in the New Tsukuba Stereo dataset, as expected since this is a synthetic dataset. In the rest of our experiments on the KITTI dataset, we will use the heuristic feature weighting scheme.

C. Initialization

We performed 1000 iterations for RANSAC, MSAC, MLESAC and AC-RANSAC using the same set of minimal correspondences at each iteration. Fig. 4 depicts average translation and rotation errors for the KITTI training sequences and for the New Tsukuba Stereo dataset.

According to the results in Fig. 4, MSAC and AC-RANSAC are the two best performing methods. In the KITTI

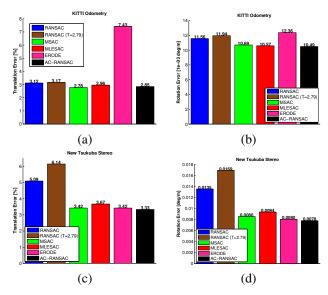


Fig. 4. Robust initialization results. Top: Average errors in translation (a) and rotation (b) for the KITTI Odometry training sequences (00-10) Bottom: Average errors in translation (c) and rotation (d) for the and New Tsukuba Stereo dataset, *daylight* and *fluorescent* illumination settings.

dataset, MSAC is slightly better in translation (2.78% versus 2.85%) but AC-RANSAC is better in rotation (10.49E-03 deg/m versus 10.69E-03 deg/m). In the New Tsukuba Stereo dataset, AC-RANSAC outperforms MSAC and all the other methods both in translation and rotation. ERODE obtains bad performance in the KITTI dataset. The reason is because ERODE fails to compute a good camera motion when the motion between consecutive frames is large, as the initialization is far from the real camera motion and the cost function is multi-modal. In these scenarios, consensus based methods such as RANSAC are necessary in order to generate a good camera motion hypothesis. On the other hand, ERODE performs well in the New Tsukuba Stereo dataset, where the motion between frames is smaller than in the KITTI dataset.

Fig. 5 depicts the adaptive thresholds per frame returned by AC-RANSAC for the first sequence from the KITTI odometry benchmark and the *fluorescent* sequence from the New Tsukuba Stereo dataset. As has been shown in Fig. 4, the choice of a proper threshold value T is critical in order to get good accuracy. This value may vary between different datasets and between different image pairs within the same sequence. AC-RANSAC solves this problem by finding an adaptive threshold for each pair of frames.

D. Refinement

Fig. 6 depicts average translation and rotation errors for the KITTI odometry training sequences, considering the different model refinement strategies (motion, BA, BA plus inlier noise distribution). We do not show results for ERODE in the refinement evaluation for the KITTI dataset since this method fails to obtain good initial results. Best results are again obtained with MSAC and AC-RANSAC with BA plus inlier noise distribution refinement, giving errors of 0.84%

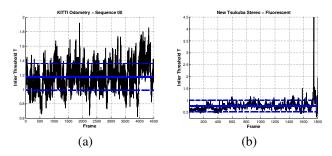


Fig. 5. Adaptive threshold *T* returned by AC-RANSAC per frame: (a) KITTI Odometry - Sequence 00 (b) New Tsukuba Stereo - Fluorescent. The solid blue line denotes the mean threshold for the sequence, while the discontinuous blue lines show the mean plus/minus the standard deviation of the threshold.

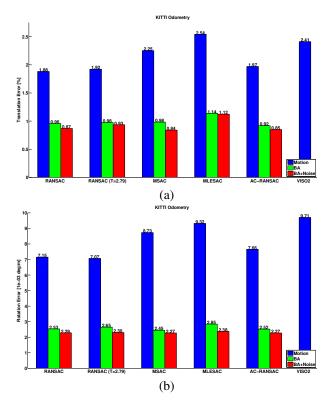


Fig. 6. Average errors in translation (a) and rotation (b) for the KITTI odometry training sequences (00-10), considering three different refinement strategies: motion only, BA and BA plus inlier noise distribution.

in translation and 2.27E-03 deg/m in rotation.

Table II shows the improvements in camera motion accuracy by considering the BA and the BA plus inlier noise distribution refinement strategies w.r.t. the motion only refinement. The improvement in % is computed as $100 \cdot ((A_{motion}/A_{ref^*}) - 1)$, where A_{motion} and A_{ref^*} denote the accuracy of the motion only refinement and BA or BA plus inlier noise distribution strategies respectively. BA, *i.e.* including 3D structure in the refinement optimization, considerably improves the camera motion accuracy for all the cases. In addition, BA plus inlier noise distribution also helps to further improve the camera motion accuracy.

Fig. 7 depicts average translation and rotation errors for the New Tsukuba Stereo *daylight* and *fluorescent* sequences.

		BA	BA+Inlier Noise
RANSAC	Translation	95.83 %	116.09 %
	Rotation	182.61 %	213.59 %
RANSAC	Translation	95.91 %	106.45 %
T = 2.79	Rotation	166.79 %	207.39 %
MSAC	Translation	129.59 %	167.85 %
	Rotation	256.32 %	284.58 %
MLESAC	Translation	122.81 %	126.78 %
	Rotation	227.01 %	291.59 %
AC-RANSAC	Translation	114.13 %	131.76 %
	Rotation	203.96 %	237.45 %

TABLE II

MODEL REFINEMENT IMPROVEMENTS IN THE KITTI ODOMETRY TRAINING SEQUENCES (00-10) W.R.T. MOTION ONLY REFINEMENT.

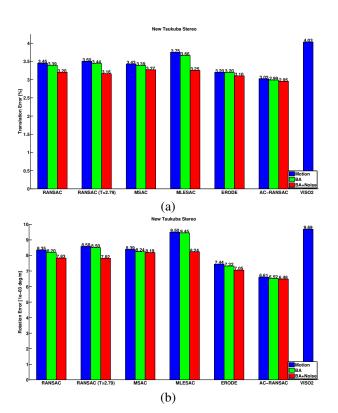


Fig. 7. Average errors for the New Tsukuba Stereo dataset, *daylight* and *fluorescent* sequences. (a) Translation error (b) Rotation error.

In this case, AC-RANSAC alongside BA plus inlier noise distribution refinement is the best performing method with errors of 2.95% in translation and 6.48E-03 deg/m.

Table III shows the improvements in camera motion accuracy by considering the BA and the BA plus inlier noise distribution refinement strategies w.r.t. the motion only refinement in the New Tsukuba Stereo dataset. While significant, the improvements due to the refinement of 3D structure and inlier noise distribution are much larger in the KITTI dataset than in the New Tsukuba Stereo. The main explanation is that KITTI is a real dataset and therefore calibration is more prone to errors than in the New Tsukuba dataset, which is a synthetic dataset. Therefore, image observations and 3D points are noisier in KITTI than in the New Tsukuba Stereo

dataset.

		BA	BA+Inlier Noise
RANSAC	Translation	1.77 %	7.81 %
	Rotation	1.83 %	6.64 %
RANSAC	Translation	1.74 %	10.76 %
T = 2.79	Rotation	0.94 %	9.72 %
MSAC	Translation	1.18 %	4.89 %
	Rotation	1.82 %	2.44 %
MLESAC	Translation	3.01 %	16.00 %
	Rotation	0.53 %	15.29 %
ERODE	Translation	0.00 %	3.22 %
	Rotation	1.63 %	5.53 %
AC-RANSAC	Translation	1.00 %	2.37 %
	Rotation	1.38 %	2.01 %

TABLE III

MODEL REFINEMENT IMPROVEMENTS IN THE NEW TSUKUBA STEREO DATASET W.R.T. MOTION ONLY REFINEMENT.

We performed the same evaluation using the LIBVISO2 library. LIBVISO2 obtained a mean average error in translation of 2.41% and 9.71E-03 deg/m in rotation for the KITTI Odometry training dataset. Regarding the experiments in the New Tsukuba Stereo dataset, LIBVISO2 obtained a mean average error in translation 4.03% of and 9.69E-03 deg/m in rotation. Fig. 8 shows an example of the estimated trajectories using AC-RANSAC and the different refinement strategies w.r.t. to LIBVISO2 for the KITTI Odometry Sequence 05 and for the New Tsukuba Stereo daylight setting. According to the results, our different refinement techniques in our stereo VO evaluation greatly outperform the results of LIBVISO2 in the two analyzed datasets.

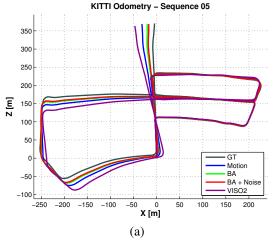
E. Timing Evaluation

Fig. 9(a) depicts a timing evaluation for the robust initialization and refinement strategies. ERODE is the fastest method. MLESAC is the slowest method since the inlier probability, γ , needs to be estimated per camera motion hypothesis using the EM algorithm. AC-RANSAC is also expensive since it needs to sort the residuals per hypothesis. RANSAC and MSAC obtain similar timing results. Note that it is possible to use other acceleration schemes to speed-up consensus methods such as [30]. In the case of AC-RANSAC, the acceleration scheme ORSA [16] speeds-up the robust initialization since once it finds a good hypothesis, the procedure is restarted for a reduced number of iterations, drawing new samples from the best inlier set found so far.

Fig. 9(b) depicts timing results for the refinement strategies, where the refinement of motion, structure and inlier noise distribution strategy is the most computationally expensive strategy.

V. CONCLUSIONS

In this paper we have performed an evaluation of image feature localisation noise models, alongside optimization strategies, in the context of feature-based stereo VO. Our evaluation shows that noise models that are more adaptable to the varying nature of the noise generally perform better.



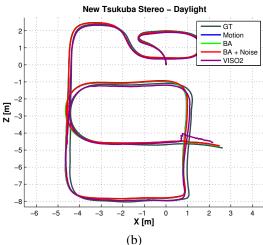
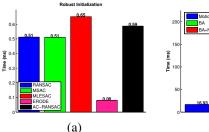


Fig. 8. Reconstructed trajectories considering AC-RANSAC alongside refinement strategies and VISO2: (a) KITTI Odometry - Sequence 05 (b) New Tsukuba Stereo - Daylight.

Despite the bias in the KITTI Odometry evaluation, our results are confirmed using the New Tsukuba Stereo dataset. In addition, we believe this paper presents the first use of AC-RANSAC, and also the first joint optimization of BA and inlier noise distribution parameters, in stereo VO.

REFERENCES

- [1] A. I. Comport, E. Malis, and P. Rives, "Real-time quadrifocal visual odometry," *Intl. J. of Robotics Research*, vol. 29, pp. 245–266, 2010.
- [2] S. Lovegrove, A. J. Davison, and J. I.-Guzmán, "Accurate visual odometry from a rear parking camera," in *IEEE Intelligent Vehicles* Symposium (IV), 2011, pp. 788–793.
- [3] C. Beall, B. J. Lawrence, V. Ila, and F. Dellaert, "3D reconstruction of underwater structures," in *IEEE/RSJ Intl. Conf. on Intelligent Robots* and Systems (IROS), 2010.
- [4] A. Geiger, J. Ziegler, and C. Stiller, "StereoScan: Dense 3d reconstruction in real-time," in *IEEE Intelligent Vehicles Symposium (IV)*, 2011, pp. 963–968.
- [5] H. Badino, A. Yamamoto, and T. Kanade, "Visual odometry by multi-frame feature integration," in *ICCV International Workshop on Computer Vision for Autonomous Driving*, Sydney, Australia, 2013.
- [6] S. Choi, T. Kim, and W. Yu, "Performance evaluation of RANSAC family," in *British Machine Vision Conf. (BMVC)*, 2009.
- [7] R. Bolles and M. Fischler, "A RANSAC-based approach to model fitting and its application to finding cylinders in range data," in *Intl. Joint Conf. on AI (IJCAI)*, Vancouver, Canada, 1981, pp. 637–643.



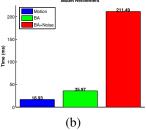


Fig. 9. (a) Robust initialization timing results: average computation time in ms per RANSAC iteration. (b) Model refinement timing results: average computation time in ms for each optimization strategy. In both cases, 1000 putative correspondences were considered. Timing results were obtained using a single threaded implementation on a 3.00GHz desktop computer.

- [8] Z. Liu, P. Monasse, and R. Marlet, "Match selection and refinement for highly accurate two-view structure from motion," in Eur. Conf. on Computer Vision (ECCV), 2014, pp. 818–833.
- [9] D. Nistér, O. Naroditsky, and J. Bergen, "Visual odometry," in *IEEE Conf. on Computer Vision and Pattern Recognition (CVPR)*, 2004, pp. 652–659.
- [10] D. Scaramuzza and F. Fraundorfer, "Visual odometry: Part i the first 30 years and fundamentals," *IEEE Robotics and Automation Magazine*, vol. 18, no. 4, pp. 80–92, 2011.
- [11] A. Geiger, P. Lenz, C. Stiller, and R. Urtasun, "Vision meets robotics: The KITTI dataset," *Intl. J. of Robotics Research*, 2013.
- [12] M. Peris, A. Maki, S. Martull, Y. Ohkawa, and K. Fukui, "Towards a simulation driven stereo vision system," in *Intl. Conf. on Pattern Recognition (ICPR)*, Tsukuba, Japan, 2012.
- [13] R. I. Hartley and A. Zisserman, Multiple View Geometry in Computer Vision, 2nd ed. Cambridge University Press, 2004.
- [14] P. H. S. Torr and A. Zisserman, "MLESAC: A new robust estimator with application to estimating image geometry," *Computer Vision and Image Understanding*, vol. 78, pp. 138–156, 2000.
- [15] A. Konouchine, V. Gaganov, and V. Veznevets, "AMLESAC: A new maximum likelihood robust estimator," in *In Proceedings of the Inter*national Conference on Computer Graphics and Vision (GraphiCon), 2005.
- [16] L. Moisan and B. Stival, "A probabilistic criterion to detect rigid point matches between two images and estimate the fundamental matrix," *Intl. J. of Computer Vision*, vol. 57, no. 3, pp. 201–218, 2004.
- [17] P. Moulon, P. Monasse, and R. Marlet, "Adaptive structure from motion with a contrario model estimation," in *Asian Conf. on Computer Vision (ACCV)*, 2012, pp. 257–270.
- [18] F.-A. Moreno, J. Blanco, and J. González-Jiménez, "ERODE: An efficient and robust outlier detector and its application to stereovisual odometry," in *IEEE Intl. Conf. on Robotics and Automation (ICRA)*, 2013, pp. 4676–4682.
- [19] R. Raguram and J. M. Frahm, "RECON: Scale-adaptive robust estimation via residual consensus," in *Intl. Conf. on Computer Vision (ICCV)*, 2011.
- [20] A. Cohen and C. Zach, "The likelihood-ratio test and efficient robust estimation," in *Intl. Conf. on Computer Vision (ICCV)*, 2015, pp. 2282– 2290.
- [21] B. Triggs, P. McLauchlan, R. Hartley, and A. Fitzgibbon, "Bundle adjustment – a modern synthesis," in *Vision Algorithms: Theory and Practice*, ser. LNCS, W. Triggs, A. Zisserman, and R. Szeliski, Eds. Springer Verlag, 2000, vol. 1883, pp. 298–372.
- [22] E. Mouragnon, M. Lhuillier, M. Dhome, F. Dekeyser, and P. Sayd, "Generic and real-time structure from motion using local bundle adjustment," vol. 27, no. 8, pp. 1178–1193, 2009.
- [23] V. Indelman, R. Roberts, C. Beall, and F. Dellaert, "Incremental Light Bundle Adjustment," in *British Machine Vision Conf. (BMVC)*, 2012.
- [24] A. E. Johnson, S. B. Goldberg, C. Yang, and L. H. Matthies, "Robust and efficient stereo feature tracking for visual odometry," in *IEEE Intl. Conf. on Robotics and Automation (ICRA)*, 2008, pp. 39–46.
- [25] J. Shi and C. Tomasi, "Good features to track," in *Intl. Conf. on Pattern Recognition (ICPR)*, 1994, pp. 539–600.
- [26] P. F. Alcantarilla, J. Nuevo, and A. Bartoli, "Fast explicit diffusion for accelerated features in nonlinear scale spaces," in *British Machine Vision Conf. (BMVC)*, Bristol, UK, 2013.

- [27] D. G. Lowe, "Distinctive image features from scale-invariant key-
- [28] X. S. Gao, X.-R. Hou, J. Tang, and H.-F. Chang, "Complete solution classification for the perspective-three-point problem," *IEEE Trans. Pattern Anal. Machine Intell.*, vol. 25, no. 8, pp. 930–943, 2003.
 [29] I. Krešo and S. Šegvić, "Improving the egomotion estimation by correcting the calibration bias," in *International Conference on Computer Vision Theory and Applications (MSARD)*, 2015.
- Vision Theory and Applications (VISAPP), 2015.

 [30] O. Chum and J. Matas, "Matching with PROSAC-progressive sample
- consensus," in IEEE Conf. on Computer Vision and Pattern Recognition (CVPR), 2005, pp. 220–226.