Tracking with Multisensor Out-of-Sequence Measurements with Residual Biases*

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Abstract – *In multisensor target tracking systems measurements from different sensors on the same target exhibit, typically, biases. These biases can be accounted for as fixed random variables by the Schmidt-Kalman filter. Furthermore, measurements from the same target can arrive out of sequence. This "out-of-sequence" measurement (OOSM) problem was recently solved and a procedure for updating the state with a multistep-lag measurement using the simpler "1-step-lag" algorithm was developed for the situation without measurement biases. The present work presents the solution to the combined problem of handling biases from multiple sensors when their measurements arrive out of sequence.*

Keywords: Out-of-sequence measurement, biased measurement, Schmidt-Kalman filter, target tracking.

1 Introduction

In multisensor target tracking systems measurements from different sensors on the same target exhibit, typically, biases. These biases can be accounted for as fixed random variables by the Schmidt-Kalman filter (SKF) [6]. Furthermore, measurements from the same target can arrive out of sequence. Such "out-of-sequence" measurement (OOSM) arrivals occur due to communication delays. This OOSM problem was recently solved and a procedure for updating the state with a multistep lag measurement using the simpler "1-step-lag" algorithm was developed. This work presents the solution to the combined problem of handling biases from multiple sensors when their measurements arrive out of sequence.

The OOSM problem has been discussed in the literature starting with the initial work of [4], who presented an approximate solution to the problem of updating the current state of a target with an OOSM, called "algorithm B". The optimal solution, called "algorithm A" was derived in [2]. It was also shown in [2] that algorithm B is nearly optimal. In all these works it was assumed that the OOSM lag is less than a sampling interval. This has been designated as the "one-step-lag OOSM problem", and thus the corresponding algorithms can be called A1 and B1. The approach presented in [3] obtains the update with an l-step-lag OOSM in a *single step* (a "giant leap"), i.e., it generalized the previous algorithms to an arbitrary l. Furthermore, the resulting algorithms, Al1 and Bl1, have practically the same requirements as those of A1 and B1, respectively, for all $l > 1$. These algorithms have also been shown to perform nearly optimally in [3]. A general optimal solution to the OOSM problem was presented in [8], but it is substantially more complicated than [3].

Section 2 presents the formulation of the OOSM problem for biased multiple sensors. Section 3 presents the multisensor Schmidt-Kalman filter (SKF) in the presence of residual biases. Section 4 derives the modified Joseph form for OOSM with biases—a new result. The combined problem of OOSM with biases from multiple sensors is solved in Section 5 using he SKF/OOSM algorithm. Simulation results are given in Section 6. Section 7 presents a discussion of the results.

2 Formulation of the Problem

The state of the system, x, of dimension n_x, is assumed to evolve from time t_{k-1} to time t_k according to

$$x(k) = F(k, k-1)x(k-1) + v(k, k-1) \qquad (1)$$

where, using only the index of the time arguments, $F(k, k-1)$ is the state transition matrix to time t_k from time t_{k-1} and $v(k, k-1)$ is the (cumulative effect of the) process noise for this interval. The order of the arguments in both F and v follows here the convention for the transition matrices. Typically, the process noise has a single argument, but here two arguments will be needed for clarity.

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The measurement equation is

$$z^{i(k)}(k) = H_x^{i(k)}(k)x(k) + w^{i(k)}(k) + H_b^{i(k)}(k)b^{i(k)} \quad (2)$$

where $i \in \{1, \ldots, N_S\}$, $i(k)$ is the index of the sensor which provided the measurement[1] from time t_k (the "time stamp"), $w^{i(k)}(k)$ is the corresponding measurement noise, modelled to be zero-mean, and $b^{i(k)}$ is the residual bias for this sensor. The dimension of the above measurement is n_{z^i} and the dimension of the bias in this measurement is denoted as n_i. The matrix H_b multiplying the bias has been discussed in [5] for various nonlinear measurements.

It is assumed that bias correction has been done separately (externally to the OOSM problem) following a sensor registration procedure. Consequently, the residual bias, assumed to be a *time-invariant random variable*, is zero-mean

$$E[b^i] = 0 \qquad i \in \{1, \ldots, N_S\} \quad (3)$$

and

$$\mathrm{cov}[b^i, b^j] = E[b^i(b^j)'] = P_{b^i b^j}\delta_{ij} \quad (4)$$

where $i, j \in \{1, \ldots, N_S\}$ and the shorter superscripts are used.

The noises are assumed zero-mean, white with covariances

$$\mathrm{cov}[v(k,j)] = Q(k,j) \quad \mathrm{cov}[w^{i(k)}] = R^{i(k)}(k) \quad (5)$$

and, together with initial state error and the residual biases, mutually uncorrelated.

The time τ, at which the OOSM was made, is assumed to be such that

$$t_{k-l} < \tau < t_{k-l+1} \quad (6)$$

This will require the evaluation of the effect of the process noise over an arbitrary noninteger number of sampling intervals. Note that $l = 1$ corresponds to the case where the lag is a fraction of a sampling interval; for simplicity this is called the "1-step-lag" problem, even though the lag is really a fraction of a time step.

The relationship between the current state $x(k)$ and the state observed by the OOSM is as follows. Similarly to (1), one has

$$x(k) = F(k,\kappa)x(\kappa) + v(k,\kappa) \quad (7)$$

where κ is the discrete time notation for τ. The above can be rewritten backward as

$$x(\kappa) = F(\kappa,k)[x(k) - v(k,\kappa)] \quad (8)$$

where $F(\kappa,k) = F(k,\kappa)^{-1}$ is the backward transition matrix .

[1] The superscript $i(k)$ will be shortened to i wherever this does not cause confusion.

The problem is as follows: At time $t = t_k$ one has

$$\hat{x}(k|k) \triangleq E[x(k)|Z^k] \qquad P(k|k) \triangleq \mathrm{cov}[x(k)|Z^k] \quad (9)$$

based on the (multisensor) cumulative set of measurements at t_k

$$Z^k \triangleq \{z^{i(\ell)}(\ell)\}_{\ell=1}^k \quad (10)$$

Subsequently, the earlier measurement from time τ, denoted from now on with discrete time notation as κ,

$$z^{i(\kappa)}(\kappa) \triangleq H_x^{i(\kappa)}(\kappa)x(\kappa) + w^{i(\kappa)}(\kappa) + H_b^{i(\kappa)}(\kappa)b^{i(\kappa)} \quad (11)$$

arrives after the state estimate (9) has been calculated. We want to update this estimate with the earlier measurement (11), namely, to calculate

$$\hat{x}(k|\kappa) = E[x(k)|Z^\kappa] \qquad P(k|\kappa) = \mathrm{cov}[x(k)|Z^\kappa] \quad (12)$$

where

$$Z^\kappa \triangleq \{Z^k, z(\kappa)\} \quad (13)$$

This update should be done *without* reordering and reprocessing the measurements according to their time stamps.

3 The Multisensor Schmidt-Kalman Filter

This section presents the multisensor Schmidt-Kalman Filter (SKF) for the case of state estimation in the presence of residual biases but without OOSMs. The SKF procedure [6] consists of augmenting the target state vector with the measurement bias vector, calculating the KF gain for this augmented state but then updating only the target state. While the bias is not updated, its covariance stays constant, but the cross-covariance between the bias and the state does change when the state is updated.

In the multisensor case there are, however, as many bias vectors as the number of sensors from which measurements are obtained. Consequently, the straightforward approach would be to augment the target state with all the biases and, while only the target state is updated, the entire updated covariance matrix of such an augmented state has to be calculated, yielding all the updated state-bias crosscovariances. This approach can be, however, very costly because of the possibly high dimension of the augmented state — typically 6 for the target state and with a minimum of 3 bias components from possibly as many as 10 sensors (not an unlikely scenario), one has at least a 36×36-dimensional covariance matrix to be updated. The major problem with this high dimensional matrix occurs in the update with the OOSM, which requires the inversion of the augmented state covariance matrix (which is a full matrix), and this can be computationally expensive for real time implementation.

In the development below it is shown that one can augment the target state only with the bias of the sensor which provided the measurement to be used for the update and a "generic" other sensor. This allows to obtain the updated crosscovariances of the state with all the biases, block by block, rather than having to update the covariance matrix of the state augmented with all the biases. A similar procedure will be used in the update with the OOSM to avoid the need to invert a very large matrix. Furthermore, in the OOSM case, the inversion will have to be done only for the $(n_x \times n_x)$ state covariance matrix , without any augmentation.

Let the augmented state, of dimension $n_x + n_i + n_j$, be

$$\mathbf{x} \triangleq \begin{bmatrix} x \\ b^i \\ b^j \end{bmatrix} \tag{14}$$

where i is the index of the sensor which provided the measurement to be used for the update at time k (the time argument of this index is now dropped for simplicity) and j is the index of a "generic" other sensor. The state equation for this augmented state is

$$\mathbf{x}(k) = \mathbf{F}(k, k-1)\mathbf{x}(k-1) + \mathbf{v}(k, k-1) \tag{15}$$

where

$$\mathbf{F}(k, k-1) \triangleq \begin{bmatrix} F(k, k-1) & 0 & 0 \\ 0 & I_{n_i} & 0 \\ 0 & 0 & I_{n_j} \end{bmatrix} \tag{16}$$

where I_{n_i} denotes the $n_i \times n_i$ identity matrix and

$$\mathbf{v}(k, k-1) \triangleq \begin{bmatrix} v(k, k-1) \\ 0 \\ 0 \end{bmatrix} \tag{17}$$

i.e., the biases are assumed constant between their (external) updates. The measurement at time k is

$$z^i(k) = \mathbf{H}^i(k)\mathbf{x}(k) + w^i(k) \tag{18}$$

where

$$\mathbf{H}^i(k) \triangleq \begin{bmatrix} H_x^i(k) & H_b^i(k) & 0 \end{bmatrix} \tag{19}$$

Let the prediction covariance of $\mathbf{x}(k)$ be

$$\mathbf{P}(k|k-1) \triangleq \begin{bmatrix} P_{xx} & P_{xb^i} & P_{xb^j} \\ P'_{xb^i} & P_{b^ib^i} & 0 \\ P'_{xb^j} & 0 & P_{b^jb^j} \end{bmatrix} \tag{20}$$

where the time arguments have been dropped for simplicity. Note that the bias covariances $P_{b^ib^i}$, $P_{b^jb^j}$ are constant. Then the optimal filter gain for updating $\mathbf{x}(k)$ is

$$\begin{aligned} \mathbf{W}^i(k)^{\mathrm{OPT}} &= \mathbf{P}(k|k-1)\mathbf{H}^i(k)'S^i(k)^{-1} \\ &= \begin{bmatrix} W_x^i(k) \\ W_{b^i}^i(k) \\ W_{b^j}^i(k) \end{bmatrix} \end{aligned} \tag{21}$$

which consists of three blocks.

The idea of the SKF is to use only the top block from the above, i.e., the actual gain will be

$$\mathbf{W}^i(k) = \begin{bmatrix} W_x^i(k) \\ 0 \\ 0 \end{bmatrix} \tag{22}$$

The expression of this block is

$$\begin{aligned} W_x^i(k) &= [P_{xx}(k|k-1)H_x^i(k)' \\ &\quad + P_{xb^i}(k|k-1)H_b^i(k)']S^i(k)^{-1} \end{aligned} \tag{23}$$

where the innovation covariance is

$$\begin{aligned} S^i(k) &= \mathbf{H}^i(k)\mathbf{P}(k|k-1)\mathbf{H}^i(k)' + R^i(k) \\ &= H_x^i(k)P_{xx}(k|k-1)H_x^i(k)' + R^i(k) \\ &\quad + H_x^i(k)P_{xb^i}(k|k-1)H_b^i(k)' \\ &\quad + H_b^i(k)P_{b^ix}(k|k-1)H_x^i(k)' \\ &\quad + H_b^i(k)P_{b^ib^i}H_b^i(k)' \end{aligned} \tag{24}$$

Since (22) is a suboptimal gain, the state covariance[2] update equation to be used in this case is the Joseph form (see, e.g., [1], Eq. (5.2.3-18)), which is the only one valid for an arbitrary gain. Thus

$$\begin{aligned} \mathbf{P}(k|k) &= [I_{n_x+n_i+n_j} - \mathbf{W}^i(k)\mathbf{H}^i(k)]\mathbf{P}(k|k-1) \\ &\quad \cdot [I_{n_x+n_i+n_j} - \mathbf{W}^i(k)\mathbf{H}^i(k)]' \\ &\quad + \mathbf{W}^i(k)R^i(k)\mathbf{W}^i(k)' \end{aligned} \tag{25}$$

Using (19), (20) and (22), the blocks of (25) are obtained as

$$\begin{aligned} P_{xx}(k|k) &= [I_{n_x} - W_x^i(k)H_x^i(k)]P_{xx}(k|k-1) \\ &\quad \cdot [I_{n_x} - W_x^i(k)H_x^i(k)]' - W_x^i(k)H_b^i(k) \\ &\quad \cdot P_{xb^i}(k|k-1)'[I_{n_x} - W_x^i(k)H_x^i(k)]' \\ &\quad - [I_{n_x} - W_x^i(k)H_x^i(k)]P_{xb^i}(k|k-1) \\ &\quad \cdot H_b^i(k)'W_x^i(k)' + W_x^i(k)H_b^i(k)P_{b^ib^i} \\ &\quad \cdot H_b^i(k)'W_x^i(k)' \\ &\quad + W_x^i(k)R^i(k)W_x^i(k)' \end{aligned} \tag{26}$$

$$\begin{aligned} P_{xb^i}(k|k) &= [I_{n_x} - W_x^i(k)H_x^i(k)]P_{xb^i}(k|k-1) \\ &\quad - W_x^i(k)H_b^i(k)P_{b^ib^i} \end{aligned} \tag{27}$$

$$\begin{aligned} P_{xb^j}(k|k) &= [I_{n_x} - W_x^i(k)H_x^i(k)] \\ &\quad \cdot P_{xb^j}(k|k-1) \qquad \forall j \neq i(k) \end{aligned} \tag{28}$$

$$P_{b^ib^i}(k) = P_{b^ib^i}(k-1) = P_{b^ib^i} \tag{29}$$

$$P_{b^jb^j}(k) = P_{b^jb^j}(k-1) = P_{b^jb^j} \tag{30}$$

$$P_{b^ib^j}(k) = 0 \tag{31}$$

The state update is done, in view of (22), according to

$$\hat{x}(k|k) = \hat{x}(k|k-1) + W_x^i(k)\nu^i(k) \tag{32}$$

[2]Actually this is not "state covariance" but "state-error covariance", since the state estimate is not the conditional mean any more due to the use of the suboptimal gain. However, for simplicity we still use the term "state covariance".

where the innovation corresponding to $z^i(k)$ is

$$\nu^i(k) = z^i(k) - H_x^i(k)\hat{x}(k|k-1) \tag{33}$$

The prediction equations, based on the model (15) are the standard ones, namely,

$$\hat{x}(k|k-1) = F(k,k-1)\hat{x}(k-1|k-1) \tag{34}$$

and for the covariance

$$\begin{aligned}\mathbf{P}(k|k-1) &= \mathbf{F}(k,k-1)\mathbf{P}(k-1|k-1)\mathbf{F}(k,k-1)' \\ &\quad +\mathbf{Q}(k,k-1)\end{aligned} \tag{35}$$

where

$$\mathbf{Q}(k,k-1) \triangleq \mathrm{diag}[Q(k,k-1), 0_{n_i}, 0_{n_j}] \tag{36}$$

The blocks of the prediction covariance (35) are calculated as

$$\begin{aligned}P_{xx}(k|k-1) &= F(k,k-1)P_{xx}(k-1|k-1) \\ &\quad \cdot F(k,k-1)' + Q(k,k-1) \quad (37)\\ P_{xb^i}(k|k-1) &= F(k,k-1) \\ &\quad \cdot P_{xb^i}(k-1|k-1) \quad (38)\\ P_{xb^j}(k|k-1) &= F(k,k-1) \\ &\quad \cdot P_{xb^j}(k-1|k-1) \quad (39)\end{aligned}$$

Eqs. (27) and (28) yield the updated crosscovariances of the state with the bias in the measurement used in the update and with each bias in the other measurements, respectively. This procedure avoids having to handle the update of a potentially very large covariance matrix. The crosscovariance of the state with the bias in another sensor's measurement will be needed when that sensor's measurement becomes available for updating the state. Similarly, the predicted crosscovariances are obtained using (38) and (39) and they are the same for all the biases.

Thus, the above equations show how one can obtain the state estimate of the target accounting for all the biases in a multisensor situation, without resorting to state augmentation as far as the computations are concerned. The augmentation was used only to obtain the covariance matrix block updates.

4 Modified Joseph form for OOSM

As discussed above, since the filter gain in the SKF is not optimal, the Joseph form should be used for covariance update. For out-of-sequence measurement (OOSM), the time of the OOSM is not at the current time, so the Joseph should be modified accordingly.

4.1 Modified Joseph form for OOSM without residual biases

Consider an OOSM $z(\kappa)$ ($\kappa < k$). The most recent state estimate after receiving $z(\kappa)$ is given by [3]

$$\hat{x}(k|\kappa) = \hat{x}(k|k) + W(k,\kappa)\nu(\kappa) \tag{40}$$

where $\nu(\kappa)$ is the innovation at time κ of the OOSM, that is

$$\nu(\kappa) = z(\kappa) - H(\kappa)\hat{x}(\kappa|k) \tag{41}$$

Using the suboptimal technique B [2] (performed within 1% of the optimum), the state retrodiction $\hat{x}(\kappa|k)$ is given by

$$\hat{x}(\kappa|k) = F(\kappa,k)\hat{x}(k|k) \tag{42}$$

and $\nu(\kappa)$ is obtained as

$$\begin{aligned}\nu(\kappa) &= z(\kappa) - H(\kappa)F(\kappa,k)\hat{x}(k|k) \\ &= H(\kappa)F(\kappa,k)\left[x(k) - \hat{x}(k|k)\right] \\ &\quad -H(\kappa)F(\kappa,k)v(k,\kappa) + w(\kappa)\end{aligned} \tag{43}$$

which has made use of (8). Substituting (43) into (40), we have

$$\begin{aligned}\hat{x}(k|\kappa) &= \hat{x}(k|k) + W(k,\kappa)H(\kappa)F(\kappa,k) \\ &\quad \cdot [x(k) - \hat{x}(k|k)] - W(k,\kappa)H(\kappa) \\ &\quad \cdot F(\kappa,k)v(k,\kappa) + W(k,\kappa)w(\kappa)\end{aligned} \tag{44}$$

Thus, the estimation error is

$$\begin{aligned}\tilde{x}(k|\kappa) &= x(k) - \hat{x}(k|\kappa) \\ &= \left[I - W(k,\kappa)H(\kappa)F(\kappa,k)\right]\tilde{x}(k|k) \\ &\quad +W(k,\kappa)H(\kappa)F(\kappa,k)v(k,\kappa) \\ &\quad -W(k,\kappa)w(\kappa)\end{aligned} \tag{45}$$

Using (45), the error covariance is given by

$$\begin{aligned}&P(k|\kappa) \\ &= \left[I - W(k,\kappa)H(\kappa)F(\kappa,k)\right]P(k|k) \\ &\quad \cdot \left[I - W(k,\kappa)H(\kappa)F(\kappa,k)\right]' + W(k,\kappa)H(\kappa) \\ &\quad \cdot F(\kappa,k)Q(k,\kappa)\left[W(k,\kappa)H(\kappa)F(\kappa,k)\right]' \\ &\quad +W(k,\kappa)R(\kappa)W(k,\kappa)' + [I - W(k,\kappa)H(\kappa) \\ &\quad \cdot F(\kappa,k)]\,P_{xv}(k,\kappa|k)\left[W(k,\kappa)H(\kappa)F(\kappa,k)\right]' \\ &\quad +W(k,\kappa)H(\kappa)F(\kappa,k)P_{xv}(k,\kappa|k)' \\ &\quad \cdot \left[I - W(k,\kappa)H(\kappa)F(\kappa,k)\right]'\end{aligned} \tag{46}$$

due to the fact that the measurement noise $w(\kappa)$ of the OOSM is independent of the estimation error $\tilde{x}(k|k)$ and the process noise $v(k,\kappa)$. Note that, we have (as in [2] Eq. (22))

$$\begin{aligned}P_{xv}(k,\kappa|k) &= \mathrm{cov}\{\tilde{x}(k|k), v(k,\kappa)\} \\ &= \mathrm{cov}\{x(k), v(k,\kappa)|Z^k\}\end{aligned} \tag{47}$$

since the covariance is independent of the conditioning Z^k. Therefore, when OOSM is considered and the state estimation is given by the technique B, the Joseph form should be modified as in (46).

4.2 Modified Joseph form for OOSM with residual biases

Next, we derive the Joseph form by considering both OOSM and residual biases of the sensors, i.e., (46) for the state augmented with the biases. Using (14), the state equation for this augmented state evolving from κ (the time of the OOSM) to the current time k is given by

$$\mathbf{x}(k) = \mathbf{F}(k,\kappa)\mathbf{x}(\kappa) + \mathbf{v}(k,\kappa) \tag{48}$$

where

$$\mathbf{F}(k,\kappa) = \begin{bmatrix} F(k,\kappa) & 0 & 0 \\ 0 & I_{n_i} & 0 \\ 0 & 0 & I_{n_j} \end{bmatrix} \tag{49}$$

and

$$\mathbf{v}(k,\kappa) = \begin{bmatrix} v(k,\kappa) \\ 0 \\ 0 \end{bmatrix} \tag{50}$$

The corresponding covariance of $\mathbf{v}(k,\kappa)$ is

$$\mathbf{Q}(k,\kappa) = \begin{bmatrix} Q(k,\kappa) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{51}$$

The OOSM at time κ obtained from sensor i is

$$z^i(\kappa) = \mathbf{H}^i(\kappa)\mathbf{x}(\kappa) + w^i(\kappa) \tag{52}$$

where

$$\mathbf{H}^i(\kappa) = \begin{bmatrix} H_x^i(\kappa) & H_b^i(\kappa) & 0 \end{bmatrix} \tag{53}$$

Let the updated covariance of $\mathbf{x}(k)$ be

$$\mathbf{P}(k|k) = \begin{bmatrix} P_{xx}(k|k) & P_{xb^i}(k|k) & P_{xb^j}(k|k) \\ P_{xb^i}(k|k)' & P_{b^ib^i} & 0 \\ P_{xb^j}(k|k)' & 0 & P_{b^jb^j} \end{bmatrix} \tag{54}$$

and the crosscovariance between $\mathbf{x}(k)$ and $\mathbf{v}(k,\kappa)$ be

$$\mathbf{P}_{xv}(k,\kappa|k) = \begin{bmatrix} P_{xv}(k,\kappa|k) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{55}$$

due to the independence between the sensor biases and process noise. The SKF gain using the OOSM $z(\kappa)$ at time k is

$$\mathbf{W}^i(k,\kappa) = \begin{bmatrix} W_x^i(k,\kappa) \\ 0 \\ 0 \end{bmatrix} \tag{56}$$

Then, using the modified Joseph form given in (46), the covariance for the state augmented with residual biases can be written as

$$\begin{aligned}
&\mathbf{P}(k|\kappa) \\
&\quad = \left[I_{n_x+n_i+n_j} - \mathbf{W}^i(k,\kappa)\mathbf{H}^i(\kappa)\mathbf{F}(\kappa,k)\right]\mathbf{P}(k|k) \\
&\qquad \cdot \left[I_{n_x+n_i+n_j} - \mathbf{W}^i(k,\kappa)\mathbf{H}^i(\kappa)\mathbf{F}(\kappa,k)\right]' \\
&\qquad +\mathbf{W}^i(k,\kappa)\mathbf{H}^i(\kappa)\mathbf{F}(\kappa,k)\mathbf{Q}(k,\kappa)\left[\mathbf{W}^i(k,\kappa)\right. \\
&\qquad \cdot \left.\mathbf{H}^i(\kappa)\mathbf{F}(\kappa,k)\right]' + \mathbf{W}^i(k,\kappa)R^i(\kappa)\mathbf{W}^i(k,\kappa)' \\
&\qquad + \left[I_{n_x+n_i+n_j} - \mathbf{W}^i(k,\kappa)\mathbf{H}^i(\kappa)\mathbf{F}(\kappa,k)\right] \\
&\qquad \cdot\mathbf{P}_{xv}(k,\kappa|k)\left[\mathbf{W}^i(k,\kappa)\mathbf{H}^i(\kappa)\mathbf{F}(\kappa,k)\right]' \\
&\qquad +\mathbf{W}^i(k,\kappa)\mathbf{H}^i(\kappa)\mathbf{F}(\kappa,k)\mathbf{P}_{xv}(k,\kappa|k)' \\
&\qquad \cdot \left[I_{n_x+n_i+n_j} - \mathbf{W}^i(k,\kappa)\mathbf{H}^i(\kappa)\mathbf{F}(\kappa,k)\right]'
\end{aligned} \tag{57}$$

Using (48)–(56), the blocks of (57) are obtained as

$$\begin{aligned}
&P_{xx}(k|\kappa) \\
&\quad = \left[I_{n_x} - W_x^i(k,\kappa)H_x^i(\kappa)F(\kappa,k)\right]P_{xx}(k|k) \\
&\qquad \cdot \left[I_{n_x} - W_x^i(k,\kappa)H_x^i(\kappa)F(\kappa,k)\right]' - W_x^i(k,\kappa) \\
&\qquad \cdot H_b^i(\kappa)P_{xb^i}(k|k)'\left[I_{n_x} - W_x^i(k,\kappa)H_x^i(\kappa)F(\kappa,k)\right]' \\
&\qquad - \left[I_{n_x} - W_x^i(k,\kappa)H_x^i(\kappa)F(\kappa,k)\right]P_{xb^i}(k|k)H_b^i(\kappa)' \\
&\qquad \cdot W_x^i(k,\kappa)' + W_x^i(k,\kappa)H_b^i(\kappa)P_{b^ib^i}H_b^i(\kappa)'W_x^i(k,\kappa)' \\
&\qquad +W_x^i(k,\kappa)R^i(\kappa)W_x^i(k,\kappa)' + W_x^i(k,\kappa)H_x^i(\kappa) \\
&\qquad \cdot F(\kappa,k)Q(k,\kappa)\left[W_x^i(k,\kappa)H_x^i(\kappa)F(\kappa,k)\right]' \\
&\qquad + \left[I_{n_x} - W_x^i(k,\kappa)H_x^i(\kappa)F(\kappa,k)\right]P_{xv}(k,\kappa|k) \\
&\qquad \cdot \left[W_x^i(k,\kappa)H_x^i(\kappa)F(\kappa,k)\right]' + W_x^i(k,\kappa)H_x^i(\kappa)F(\kappa,k) \\
&\qquad \cdot P_{xv}(k,\kappa|k)'\left[I_{n_x} - W_x^i(k,\kappa)H_x^i(\kappa)F(\kappa,k)\right]'
\end{aligned} \tag{58}$$

The crosscovariance of the state at k with the bias of the OOSM evolves as

$$\begin{aligned}
P_{xb^i}(k|\kappa) &= \left[I_{n_x} - W_x^i(k,\kappa)H_x^i(\kappa)F(\kappa,k)\right]P_{xb^i}(k|k) \\
&\quad -W_x^i(k,\kappa)H_b^i(\kappa)P_{b^ib^i}
\end{aligned} \tag{59}$$

The crosscovariance of the state at k with the other biases evolve as

$$\begin{aligned}
P_{xb^j}(k|\kappa) &= \left[I_{n_x} - W_x^i(k,\kappa)H_x^i(\kappa)F(\kappa,k)\right] \\
&\quad \cdot P_{xb^j}(k|k) \qquad \forall j \neq i(k)
\end{aligned} \tag{60}$$

and the bias covariances stay unchanged, as in (29)–(31).

5 One-step Algorithm for Multistep-Lag OOSM for Multiple Sensors with Biases—the SKF/OOSM

Using the approach of [3], one can perform in one step the update with an l-step-lag OOSM. Suitable modifications will be made to account for the fact that the measurements are biased. Two procedures, designated as Al1 and Bl1, were presented in [3] for the situation without biases. Both procedures retrodict the current state to the time of the OOSM, calculate the covariance of the retrodicted state, the retrodicted measurement and its the covariance, the crosscovariance between the current state and the retrodicted measurement and, with these, one can perform the direct update of the current state with the OOSM.

As shown in [3], both algorithms, while suboptimal, performed within 1% of the optimum obtained by reordering and reprocessing the measurements, which would not be practical in real systems. In view of their performance and the fact that, in the presence of biases, the statistical relationship between the "equivalent measurement" and the biases is difficult to quantify, the proposed approach is to modify Bl1 to account for the biases.

The suboptimal technique B [2] assumes the retrodicted noise to be zero. The retrodiction of the state to κ from k is[3]

$$\hat{x}(\kappa|k) = F(\kappa,k)\,\hat{x}(k|k) \tag{61}$$

i.e., a linear function of $\hat{x}(k|k)$, rather than an affine function. The covariance of this state retrodiction is

$$\begin{aligned} P_{xx}(\kappa|k) &= F(\kappa,k)[P_{xx}(k|k) + P_{vv}(k,\kappa|k) \\ &\quad -P_{xv}(k,\kappa|k) - P_{xv}(k,\kappa|k)'] \\ &\quad \cdot F(\kappa,k)' \end{aligned} \tag{62}$$

where

$$P_{vv}(k,\kappa|k) = Q(k,\kappa) \tag{63}$$

$$P_{xv}(k,\kappa|k) = P_{xx}(k|k)P_{xx}(k|k-l)^{-1}Q(k,\kappa) \tag{64}$$

are the covariances of the process noise for the retrodiction interval and its crosscovariance with the current state, respectively. Eq. (64) above follows by substituting in Eq. (37) of [3] its preceding Eqs. (24) and (18) and simplifying the result.

The covariance of the retrodicted measurement, as given in Eq. (39) of [3] for the situation *without biases*, is, assuming the OOSM is from sensor i, given by

$$S^i(\kappa) = H_x^i(\kappa)P(\kappa|k)H_x^i(\kappa)' + R^i(\kappa) \tag{65}$$

For the situation of the state augmented with biases, (65) is replaced by

$$\begin{aligned} S^i(\kappa) &= \mathbf{H}^i(\kappa)\mathbf{P}(\kappa|k)\mathbf{H}^i(\kappa)' + R^i(\kappa) \\ &= H_x^i(\kappa)P_{xx}(\kappa|k)H_x^i(\kappa)' + H_x^i(\kappa)P_{xb^i}(\kappa|k) \\ &\quad \cdot H_b^i(\kappa)' + H_b^i(\kappa)P_{b^ix}(\kappa|k)H_x^i(\kappa)' \\ &\quad +H_b^i(\kappa)P_{b^ib^i}H_b^i(\kappa)' + R^i(\kappa) \end{aligned} \tag{66}$$

[3]The superscript ^{B} used in [3] to distinguish between the variables in algorithm versions A and B is dropped, since here we use only algorithm B.

where, using (8), (61), one has

$$\begin{aligned} P_{xb^i}(\kappa|k) &= E\left\{(x(\kappa) - \hat{x}(\kappa|k))b^{i'}\right\} \\ &= E\left\{[F(\kappa,k)(x(k) - v(k,\kappa)) \right. \\ &\quad \left. -F(\kappa,k)\,\hat{x}(k|k)]\,b^{i'}\right\} \\ &= F(\kappa,k)P_{xb^i}(k|k) \end{aligned} \tag{67}$$

because the residual bias and the process noise are independent.

The crosscovariance between the state at k and the OOSM is, for the case without biases, given by Eq. (40) of [3] as

$$\begin{aligned} P_{xz^i}(k,\kappa|k) &= [P_{xx}(k|k) - P_{xv}(k,\kappa|k)] \\ &\quad \cdot F(\kappa,k)'H_x^i(\kappa)' \end{aligned} \tag{68}$$

In the case with biases one has

$$\begin{aligned} P_{xz^i}(k,\kappa|k) &= E\left\{(x(k) - \hat{x}(k|k))(z^i(\kappa) - \hat{z}^i(\kappa|k))'\right\} \\ &= E\{\tilde{x}(k|k)\left[H_x^i(\kappa)F(\kappa,k)\tilde{x}(k|k)\right. \\ &\quad -H_x^i(\kappa)F(\kappa,k)v(k,\kappa) \\ &\quad \left.+w^i(\kappa) + H_b^i(\kappa)b^i\right]'\} \\ &= [P_{xx}(k|k) - P_{xv}(k,\kappa|k)]F(\kappa,k)' \\ &\quad \cdot H_x^i(\kappa)' + P_{xb^i}(k|k)H_b^i(\kappa)' \end{aligned} \tag{69}$$

Therefore, the gain for the update of the current state estimate with the OOSM $z^i(\kappa)$ in the presence of biases is (the first block of $\mathbf{W}^i(k,\kappa)^{\mathrm{OPT}} = P_{\mathbf{x}z^i}(k,\kappa|k)S^i(\kappa)^{-1}$)

$$W_x^i(k,\kappa) = P_{xz^i}(k,\kappa|k)S^i(\kappa)^{-1} \tag{70}$$

with $P_{xz^i}(k,\kappa|k)$ given in (69) and $S^i(\kappa)$ given in (66).

The update with the OOSM $z^i(\kappa)$ of the most recent state estimate $\hat{x}(k|k)$ is thus

$$\hat{x}(k|\kappa) = \hat{x}(k|k) + W_x^i(k,\kappa)\nu^i(\kappa) \tag{71}$$

where the innovation corresponding to the OOSM $z^i(\kappa)$ is

$$\nu^i(\kappa) = z^i(\kappa) - \hat{z}^i(\kappa|k) \tag{72}$$

and the retrodicted OOSM is

$$\hat{z}^i(\kappa|k) = H_x^i(\kappa)\hat{x}(\kappa|k) \tag{73}$$

which uses the retrodicted state $\hat{x}(\kappa|k)$ given in (61). Using the filter gain given in (70) and the (approximated) crosscovariance in (64), the covariance for the state estimate and the crosscovariances of the state with the biases can be obtained from (58)-(60).

As it can be seen from (64), the need to invert the state covariance and the augmentation of the state with all the sensor biases would make the algorithm prohibitive for real-time implementation. The procedure presented above avoids the need to invert the augmented covariance matrix since it does not use any state augmentation.

6 Simulation Results

This example considers a target that moves in a 2–dimensional space with a nearly constant velocity. The target state consists of position and velocity along each coordinate (x and y). The initial target state is [100 m, 9 m/s, 200 m, 5 m/s]. The Power Spectral Density (PSD) of the process noise is $q = 0.5\,\mathrm{m}^2/\mathrm{s}^3$ for both x and y coordinate. The motion model considered is the Discretized Continuous White Noise Accelelration [1] (DCWNA) model. Two GMTI radars are located with nearly perpendicular LOS to the target. One is at (–48, 13) km with a slant range around 50 km to the target and the other is at (–26,–96) km with a slant range around 100 km to the target. The measurements are range (r), azimuth (θ) and range rate ($\dot{r}$) with s.d. $\sigma_r = 10\,\mathrm{m}$, $\sigma_\theta = 1\,\mathrm{mrad}$ and $\sigma_{\dot{r}} = 1\,\mathrm{m/s}$, respectively, for both sensors. The measurement model is

$$z^i = (I_3 + \Lambda^i)h(\mathbf{x} - \mathbf{x}_p^i) + \Delta^i + w^i \qquad i = 1, 2 \tag{74}$$

where

$$\mathbf{x} = \left[\begin{array}{cccc} x & \dot{x} & y & \dot{y} \end{array}\right]' \tag{75}$$

denotes here the (unaugmented) target state and and $\mathbf{x}_p^i$ denotes the ith sensor state, the function $h : \mathcal{R}^4 \to \mathcal{R}^3$ is given by

$$h(\mathbf{x}) = \begin{bmatrix} r \\ \theta \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \sqrt{x^2 + y^2} \\ \tan^{-1} \frac{y}{x} \\ \dot{x}\cos\theta + \dot{y}\sin\theta \end{bmatrix} \tag{76}$$

I_3 denotes the 3×3 identity matrix, Λ represents the scale bias, having the form

$$\Lambda = \begin{bmatrix} \alpha_r & 0 & 0 \\ 0 & \alpha_\theta & 0 \\ 0 & 0 & \alpha_{\dot{r}} \end{bmatrix} \tag{77}$$

with the bias terms on its main diagonal and Δ denotes the offset bias, that is,

$$\Delta = \left[\begin{array}{ccc} \Delta_r & \Delta_\theta & \Delta_{\dot{r}} \end{array}\right]' \tag{78}$$

The order of the measurements arriving at the fusion center is shown in Table 1. Two bias levels are considered, with the bias s.d. given in Table 2.

Table 1: Sensor Indices and Corresponding Time Stamps

Sensor	1	1	2	1	2	1	2
Time (s)	0	5	2.5	10	7.5	15	12.5
Sensor	1	2	1	2	1	2	1
Time (s)	20	17.5	25	22.5	30	27.5	35

The SKF/OOSM algorithm was compared with the (heuristic) Kalman filter with covariance inflation (KFwINF) [7] and the plain Kalman filter without compensation (KFwoINF), in the presence of residual biases. The results are based on 500 Monte Carlo simulations. The two-sided 99% probability region of the NEES [1] (Normalized Estimation Error Square) is [3.68, 4.33] based on the χ^2_{2000} distribution.

Table 2: Bias Standard Deviations for GMTI Measurements

Offset Bias	Δ_r	Δ_θ	$\Delta_{\dot{r}}$
Moderate	10 m	1 mrad	1 m/s
Large	20 m	2 mrad	2 m/s
Scale Bias	α_r	α_θ	$\alpha_{\dot{r}}$
Moderate	1×10^{-4}	1×10^{-4}	1×10^{-4}
Large	2×10^{-4}	2×10^{-4}	2×10^{-4}

From the RMSE in Fig. 1 we can see that SKF improves estimation accuracy compared to KFwoINF and KFwINF. KFwINF is even worse than KFwoINF in this case. With the bias level increasing, the improvement in RMSE using SKF (shown in Fig. 3) becomes more significant. For consistency, from the NEES shown in Fig. 2 we can see that only SKF is consistent. At the beginning, SKF takes some time to become consistent since the initial crosscovariance between the estimation error and the bias is set to be zero[4] and the SKF needs several updates to obtain the correct crosscovariance.

When the bias level increases, SKF needs more updates for the covariance to become consistent (as shown in Fig. 4). KFwoINF is the most inconsistent, with NEES around 40 in Fig. 4 for the case of large bias. KFwINF improves the consistency but is still not consistent, with NEES around 10 in Fig. 4 for the case of large bias.

7 Summary and Conclusions

The single sensor algorithm Bl1, which updates the current state of a target with an OOSM from a single sensor without bias has been extended to the multisensor situation where each sensor exhibits a residual bias. This has been accomplished using the SKF/OOSM algorithm, without having to rely on the augmented state consisting of the target state and the sensor biases, which can become prohibitive for real-time implementation. The simulation results show that, compared to the (heuristic) covariance inflation approach, the major benefit of the SKF/OOSM algorithm is the significant improvement in filter consistency.

[4] The initial crosscovariance between the estimation error and the bias is not available exactly since one needs the true state x to evaluate this crosscovariance due to the scale bias. Using the initial state estimate in the crosscovariance yields the same minor initial inconsistency as when the initial crosscovariance is set to be zero.

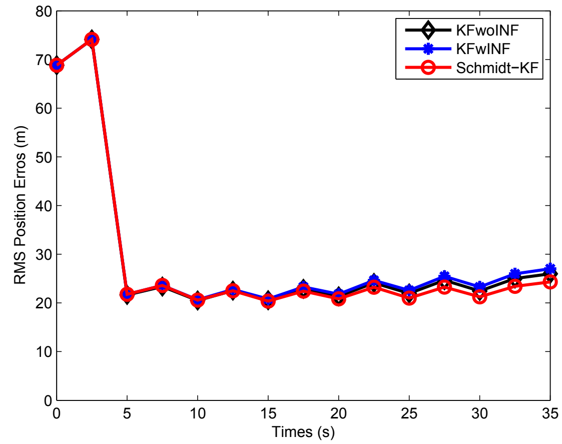

Figure 1: Position RMSE for target with GMTI measurement (moderate bias, OOSM processing).

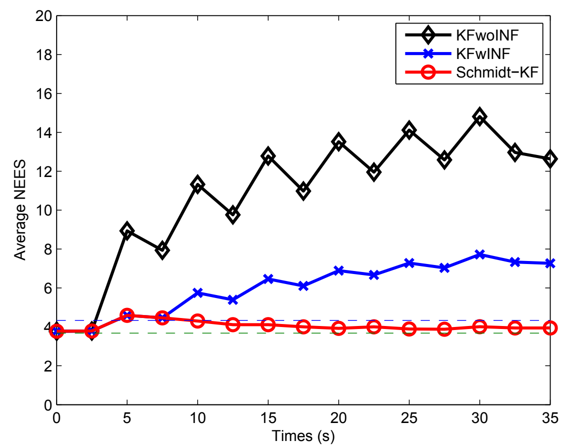

Figure 2: NEES for target with GMTI measurement (moderate bias, OOSM processing).

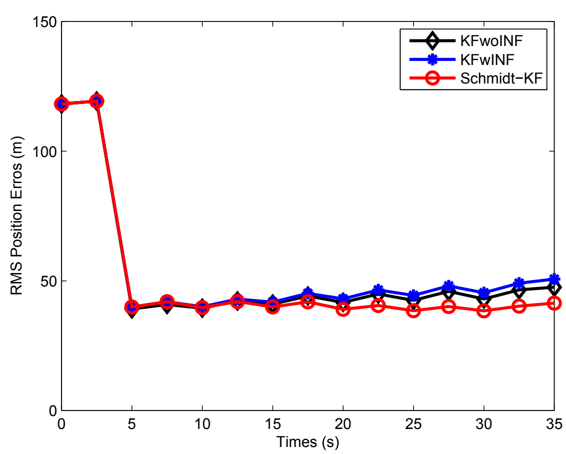

Figure 3: Position RMSE for target with GMTI measurement (large bias, OOSM processing).

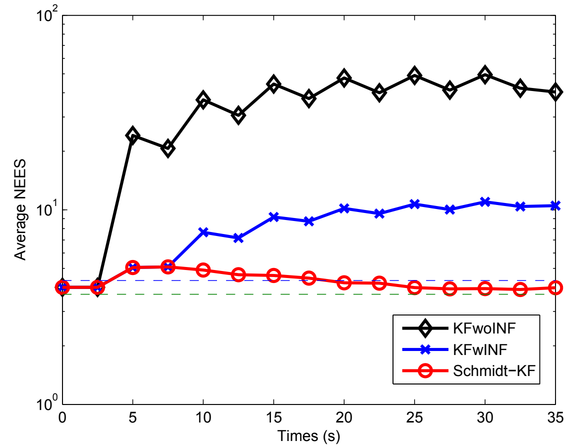

Figure 4: NEES for target with GMTI measurement (large bias, OOSM processing).

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