A General Formulation for Under-Actuated Manipulators

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Abstract

Under-actuated and macro-mini manipulators represent common characteristics in dynamics. This paper introduces a general formulation of kinematics and dynamics for manipulator systems which are composed of two types generalized coordinate, active and passive, or primary and secondary. The Generalized Jacobian Matrix and the Genralized Inertia Tensor, originally proposed for free-floating space manipulators, are shown to be redefined for general under-actuated manipulator systems. The Reaction Null-Space and reactionless motion which are recently studied focusing to flexible-base manipulators, the compensability which is originally discussed for flexible-arm manipulators, and the measure of dynamic coupling, are proposed as examples of common properties for general under-actuated systems.

1 Introduction

The interest toward complex robot systems is expanding for new application areas. A class of such robot systems are so-called under-actuated systems, characterized by the number of control actuators being less than the number of degree of freedom. Typical examples are serial manipulators with passive joints [1][2] and free-flying space manipulators. It is obvious for the manipulators with passive joints that the number of controllable joints are less than the number of degrees of freedom (DOF) of the system. For the free-flying space manipulators (**Figure 1**), the number of controllable joints are n in general but the number of system DOF is $n+6$ including the position and orientation of the base body in inertial space.

Another example of an under-actuated system is a dextrous manipulator arm mounted on a passive flexible base (**Figure 2**). In literature, such a system is known under the name *long-reach manipulator* [3]-[5], or *flexible structure mounted manipulator system (FSMS)* [9].

Recently, research on long-reach manipulators is gaining momentum due to a number of prospective applications, such as nuclear waste cleanup [3]-[8], space robotics [10], [11], power line maintenance and forestry operation [12].

A major issue concerning long-reach manipulators is dynamic interaction between the motion of the dexterous manipulator, located at the end of a flexible long-reach base, and the vibrations of the flexible base due to induced reaction. One can find that the problem here is similar to the problem in free-floating manipulators at the point that the dynamic reaction of the manipulator arm induces the interactive motion in the supporting base, which is usually undesirable for dexterous operations. The difference between these two systems is that the manipulator base of a free-flying manipulator is a floating inertia, but that of a long-reach manipulator is an inertia-spring-damper system.

One more example of an under-actuated system is a flexible-arm manipulator, the system DOF of which is composed of the controllable joints plus the elastic displacements of each flexible link. The joint motion and elastic displacements are dynamically interacting in a fashion similar to other under-actuated systems. Compared with long-reach manipulators which have a concentrated compliance at the base, flexible-arm manipulators have distributed compliance along the kinematic chain.

The above systems have been studied independently although, they are considered members of the same family, named as *under-actuated system*, and eventually they show a lot of similarity in the dynamic formulation of the system. The purpose of this paper is to find and compare common dynamic characteristics over these systems, highlighting the fact that a concept originally developed for a particular system, a free-flying system for example, is applicable to the other systems.

Figure 3 shows a whole family of fully/under-actuated manipulator systems. The top four represent under-actuated systems, where the top two are systems with *free* joints and the next two with *compliant* or visco-elastic joints. On the other hand, the bottom two represent fully-actuated system. From the top to bottom, the left column is a group of *separable* systems, which means that free or compliant (passive),

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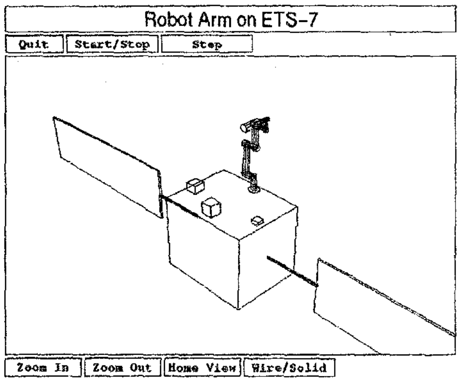

Figure 1 An example of Free-Floating Space Manipulator

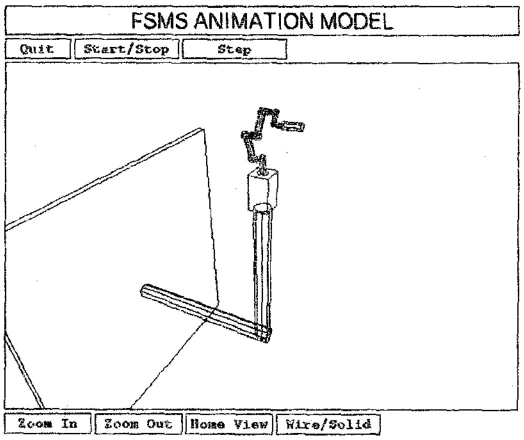

Figure 2 An example of Flexible-Base Manipulator

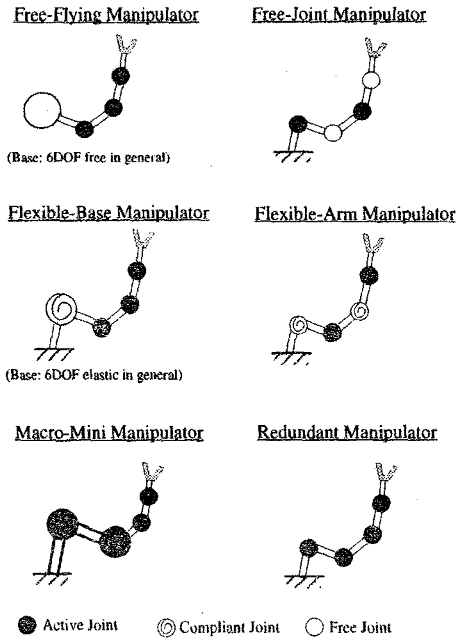

Figure 3 A family of under-actuated manipulator systems including macro-mini and redundant manipulators

or macro-part joints are separately located from active or micro-part joints. The right column is a group of *distributed* systems, where passive or macro joints are distributed among other active joints.

For example, a free-flying manipulator is a system composed by a free base (which can be modeled as 6 DOF free joints in general) and active manipulator(s). But once the free joints are distributed, the system can be regarded as a serial manipulator with free joints, and both two systems basically have common dynamics. A long-reach manipulator is a system composed by a compliant base (which can be modeled as a 6 DOF visco-elastic beam in general) and active manipulator(s). But when the compliant joints are distributed, the system can be regarded as a manipulator with compliant joints or a flexible-arm manipulator.

If not containing any passive element it is difficult to distinguish between separable and distributed systems, but a manipulator system which is composed by a base part for coarse relocation and a terminal part for fine manipulation, is usually called macro-mini (or macro-micro) system. A typical composition of a macro-mini manipulator is a 6 DOF macro arm plus a 6 DOF mini arm. But when it makes no difference between macro and mini joints, the system is considered as a 12 DOF redundant manipulator arm, which is depicted at the right-bottom of Figure 3.

This paper develops a general formulation for under-actuated manipulator systems, paying attention to the fact that all the systems in this class have common characteristics in dynamic formulation. The author highlights that the Generalized Jacobian Matrix [13][14] and the Generalized Inertia Tensor [15]–[18], which are originally developed for a free-flying space manipulator, are applicable to other systems. As some examples of common properties for general under-actuated systems, the Reaction Null-Space [21] [23], kinematic compensability [25][27], and the measure of dynamic coupling are introduced.

2 Formulation

2.1 General Case

As a general discussion, let us consider a system whose motion is described by n degrees of freedom of the generalized coordinate $\mathbf{q} \in R^n$ for *active* joints and m degrees of freedom of the generalized coordinate $\mathbf{p} \in R^m$ for *passive* joints. If the system is a macro-mini system with active joints all, $\mathbf{q} \in R^n$ should be macro part or *primary* joints and $\mathbf{p} \in R^m$ should be mini part or *secondary* joints. Now, define $\mathbf{F}_q$ as active (or primary) force/torque (twist) generated on coordinate $\mathbf{q}$, and $\mathbf{F}_p$ as a passive (or secondary) force/torque exerted on coordinate $\mathbf{p}$. Also, define $\mathbf{x}$ as a coordinate of a point of interest (the operational coordinate) composed by $\mathbf{p}$ and $\mathbf{q}$, and let an external force/torque $\mathbf{F}_{ex}$ be applied on $\mathbf{x}$. Hence, the applied external force/torque is decomposed as $\mathbf{J}_q^T\mathbf{F}_{ex}$ and $\mathbf{J}_p^T\mathbf{F}_{ex}$ onto each gereralized coordinate using corresponding Jacobian matrices.

The equation of motion of such system is generally expressed as:

$$\begin{bmatrix} \mathbf{H}_p & \mathbf{H}_{pq} \\ \mathbf{H}_{pq}^T & \mathbf{H}_q \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{p}} \\ \ddot{\mathbf{q}} \end{bmatrix} + \begin{bmatrix} \mathbf{c}_p \\ \mathbf{c}_q \end{bmatrix} = \begin{bmatrix} \mathbf{F}_p \\ \mathbf{F}_q \end{bmatrix} + \begin{bmatrix} \mathbf{J}_p^T \\ \mathbf{J}_q^T \end{bmatrix} \mathbf{F}_{ex} \tag{1}$$

where $\mathbf{H}_p$, $\mathbf{H}_q$, $\mathbf{H}_{pq}$ are inertia matrices. $\mathbf{c}_p$, $\mathbf{c}_q$ are non-linear Coriolis and centrifugel forces and they can include gravity forces if necessary.

Kinematic relationship among $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{x}$ is expressed using Jacobians as:

$$\dot{\mathbf{x}} = \mathbf{J}_p\dot{\mathbf{p}} + \mathbf{J}_q\dot{\mathbf{q}} \tag{2}$$

$$\ddot{\mathbf{x}} = \mathbf{J}_p\ddot{\mathbf{p}} + \dot{\mathbf{J}}_p\dot{\mathbf{p}} + \mathbf{J}_q\ddot{\mathbf{q}} + \dot{\mathbf{J}}_q\dot{\mathbf{q}} \tag{3}$$

The above set of equations are commonly applicable for any type of manipulator systems described in Figure 3.

In the following subsections, we will look at specific examples of free-floating manipulators, flexible-base manipulators and flexible-arm manipulators.

2.2 Free-Floating Manipulator

Now, let us consider a free-floating system composed by a single robot base which is floating in the inertial space without any external force or torque, and a serial manipulator arm at whose endpoint an external force/torque may apply. For such a space manipulator, use the following symbols replaced from Equation (1).

$\mathbf{p}$	$\rightarrow$	$\mathbf{x}_b$	:position/orientation of the base
$\mathbf{q}$	$\rightarrow$	$\boldsymbol{\phi}$	:joint angle of the arm
$\mathbf{x}$	$\rightarrow$	$\mathbf{x}_h$	:position/orientation of the endpoint
$\mathbf{F}_p$	$\rightarrow$	$\mathbf{0}$	:external force/torque on the base
$\mathbf{F}_q$	$\rightarrow$	$\boldsymbol{\tau}$	:joint torque of the arm
$\mathbf{F}_{ex}$	$\rightarrow$	$\mathbf{F}_h$	:external force/torque on the endpoint

Then we obtain the following equations:

$$\begin{bmatrix} \mathbf{H}_b & \mathbf{H}_{bm} \\ \mathbf{H}_{bm}^T & \mathbf{H}_m \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{x}}_b \\ \ddot{\boldsymbol{\phi}} \end{bmatrix} + \begin{bmatrix} \mathbf{c}_b \\ \mathbf{c}_m \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\tau} \end{bmatrix} + \begin{bmatrix} \mathbf{J}_b^T \\ \mathbf{J}_m^T \end{bmatrix} \mathbf{F}_h \tag{4}$$

$$\dot{\mathbf{x}}_h = \mathbf{J}_m\dot{\boldsymbol{\phi}} + \mathbf{J}_b\dot{\mathbf{x}}_b \tag{5}$$

$$\ddot{\mathbf{x}}_h = \mathbf{J}_m\ddot{\boldsymbol{\phi}} + \dot{\mathbf{J}}_m\dot{\boldsymbol{\phi}} + \mathbf{J}_b\ddot{\mathbf{x}}_b + \dot{\mathbf{J}}_b\dot{\mathbf{x}}_b \tag{6}$$

The above is a familiar expression for space manipulators with external force/torque inputs [16].

For a space free-floating manipulator, there is no gravity exerting on the system, then the non-linear term becomes $\mathbf{c}_b = \dot{\mathbf{H}}_b\dot{x}_b + \dot{\mathbf{H}}_{bm}\dot{\boldsymbol{\phi}}_b$. Integrating the upper set of equation in (4) with respect to time, we obtain the total momentum of the system as:

$$\mathcal{L} = \int \mathbf{J}_b^T\mathbf{F}_h dt = \mathbf{H}_b\dot{\mathbf{x}}_b + \mathbf{H}_{bm}\dot{\boldsymbol{\phi}} \tag{7}$$

From equations (5) and (7), we can eliminate the coordinate of the manipulator base $\dot{\mathbf{x}}_b$, which is a passive and unactuated coordinate, as:

$$\dot{\mathbf{x}}_h = \hat{\mathbf{J}}\dot{\boldsymbol{\phi}} + \dot{\mathbf{x}}_{h0} \tag{8}$$

where

$$\hat{\mathbf{J}} = \mathbf{J}_m - \mathbf{J}_b\mathbf{H}_b^{-1}\mathbf{H}_{bm} \tag{9}$$

and

$$\dot{\mathbf{x}}_{h0} = \mathbf{J}_b\mathbf{H}_b^{-1}\mathcal{L} \tag{10}$$

Since $\mathbf{H}_b$ is an inertia tensor of a single rigid body (the manipulator base), it is always positive definte then its inverse exists.

The matrix $\hat{\mathbf{J}}$ was first introduced in [13][14] and termed the *Generalized Jacobian Matrix*. In its original definition, no external force/torque acting on the system was assumed. In fact, if $\mathbf{F}_h = \mathbf{0}$, the term $\dot{\mathbf{x}}_{h0}$ is constant, and particularly if the system has zero initial momentum, $\dot{\mathbf{x}}_{h0} = \mathbf{0}$, then Equation (8) becomes very simple. However, note that in the definition of (9), zero or constant momentum is not a condition. It is interesting to know that the Generalized Jacobian Matrix can be defined from the other approach without momutum equation.

From the upper and lower sets of equations in (4), we can eliminate $\ddot{\mathbf{x}}_b$ and obtain the following expression including the matrices $\hat{\mathbf{G}}$ and $\hat{\mathbf{H}}$:

$$\hat{\mathbf{H}}\ddot{\boldsymbol{\phi}} + \hat{\mathbf{c}} = \boldsymbol{\tau} + \hat{\mathbf{J}}\mathbf{F}_h \tag{11}$$

where

$$\hat{\mathbf{H}} = \mathbf{H}_m - \mathbf{H}_{bm}^T\mathbf{H}_b^{-1}\mathbf{H}_{bm} \tag{12}$$

The matrix $\hat{\mathbf{H}}$ is known as the Generalized Inertia Tensor for space manipulators [15]. This derivation using the dynamic equation (4), instead of the momentum equation (7), provides an alternative definition of the Generalized Jacobian Matrix.

Note that the Generalized Jacobian Matrix has been considered to require momentum conservation, and there is a discussion that it is questionable to use the Generalized Jacobian for impact analysis [17][18] because the momentum is modified by impact, however this new definition clearly suggests that momentum conservation is not needed for the Generalized Jacobian Matrix.

Note also that, as suggested in Equation (11), the transpose of the Generalized Jacobian maps the external force/torque at endpoint $\mathbf{F}_h$ onto the joint space involving the reaction effect of the base.

Here a new definition is made for general under-actuated manipulator systems:

Definition: For under-actuated (or macro-mini) systems whose dynamics and kinematics are described by Equations (1)-(3),

$$\hat{\mathbf{H}} = \mathbf{H}_q - \mathbf{H}_{pq}^T \mathbf{H}_p^{-1} \mathbf{H}_{pq} \tag{13}$$

is the **Generalized Inertia Tensor** which directly relates the applied force/torque on the active (primary) coordinate $\mathbf{F}_q$ to the acceleration of the corresponding coordinate $\ddot{\mathbf{q}}$, involving the reaction dynamic effect regarding the passive (secondary) coordinate $\mathbf{p}$.
And

$$\hat{\mathbf{J}} = \mathbf{J}_q - \mathbf{J}_p \mathbf{H}_p^{-1} \mathbf{H}_{pq} \tag{14}$$

is the **Generalized Jacobian Matrix** which directly relates the endpoint velocity $\dot{\mathbf{x}}$ to the velocity of the active (primary) coordinate $\dot{\mathbf{q}}$, or using its transposed form, the external force/torque at endpoint $\mathbf{F}_{ex}$ to the static force/torque on the active (primary) coordinate $\mathbf{F}_q$, involving the reaction dynamic effect regarding the passive (secondary) coordinate $\mathbf{p}$.

The above definition is generally valid for any manipulator system shown in Figure 3.

Similar expressions to (4)-(6) can be obtained for a serial manipulator with some free joints [1] [2].

2.3 Flexible-Base Manipulator

Next, let us consider a flexible-base manipulator composed by a single manipulator base which is constrained by a flexible-beam or a spring and damper (visco-elastic) system, and a serial manipulator arm at whose endpoint an external force/torque may apply. For such a flexible-base manipulator, use the following symbols replaced from Equation (1).

$\mathbf{p}$	$\rightarrow$	$\mathbf{x}_b$	:position/orientation of the base
$\mathbf{q}$	$\rightarrow$	ϕ	:joint angle of the arm
$\mathbf{x}$	$\rightarrow$	$\mathbf{x}_h$	:position/orientation of the endpoint
$\mathbf{F}_p$	$\rightarrow$	$\mathbf{F}_b$	:force/torque to deflect the flexible base
$\mathbf{F}_q$	$\rightarrow$	τ	:joint torque of the arm
$\mathbf{F}_{ex'}$	$\rightarrow$	$\mathbf{F}_h$	:external force/torque on the endpoint

Then we obtain the following equations:

$$\begin{bmatrix} \mathbf{H}_b & \mathbf{H}_{bm} \\ \mathbf{H}_{bm}^T & \mathbf{H}_m \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{x}}_b \\ \ddot{\boldsymbol{\phi}} \end{bmatrix} + \begin{bmatrix} \mathbf{c}_b \\ \mathbf{c}_m \end{bmatrix} = \begin{bmatrix} \mathbf{F}_b \\ \boldsymbol{\tau} \end{bmatrix} + \begin{bmatrix} \mathbf{J}_b^T \\ \mathbf{J}_m^T \end{bmatrix} \mathbf{F}_h \tag{15}$$

$$\dot{\mathbf{x}}_h = \mathbf{J}_m \dot{\boldsymbol{\phi}} + \mathbf{J}_b \dot{\mathbf{x}}_b \tag{16}$$

$$\ddot{\mathbf{x}}_h = \mathbf{J}_m \ddot{\boldsymbol{\phi}} + \dot{\mathbf{J}}_m \dot{\boldsymbol{\phi}} + \mathbf{J}_b \ddot{\mathbf{x}}_b + \dot{\mathbf{J}}_b \dot{\mathbf{x}}_b \tag{17}$$

Here with a gravity force $\mathbf{g}$ in Cartesian space, the term $\mathbf{c}_b$ is generally expressed as:

$$\mathbf{c}_b = \mathbf{f}(\mathbf{x}_b, \boldsymbol{\phi}, \dot{\mathbf{x}}_b, \dot{\boldsymbol{\phi}}) + \mathbf{g}(\mathbf{x}_b, \boldsymbol{\phi}) \tag{18}$$

The difference from the space manipulators is the base constraint force $\mathbf{F}_b$. Let $\mathbf{D}_b, \mathbf{K}_b$ be damping and spring factors of the flexible-base, the constraint force $\mathbf{F}_b$ is expressed as:

$$\mathbf{F}_b = -\mathbf{D}_b \dot{\mathbf{x}}_b - \mathbf{K}_b \Delta \mathbf{x}_b \tag{19}$$

where $\Delta \mathbf{x}_b$ denotes an elastic base displacement from its neutral position.

Since the base is constrained, the total momentum is not conserved and it is meaningless to check the system momentum $\mathcal{L}$. However, it is important to pay attention to a partial momentum $\mathcal{L}_m$ for the part of the manpulator arm:

$$\mathcal{L}_m = \mathbf{H}_{bm} \dot{\boldsymbol{\phi}} \tag{20}$$

which is termed the *coupling momentum* [23].

Its time derivative describes the dynamic reaction from the manipulator arm onto the base: $\mathbf{F}_m$.

$$\mathbf{F}_m = \mathbf{H}_{bm} \ddot{\boldsymbol{\phi}} + \dot{\mathbf{H}}_{bm} \dot{\boldsymbol{\phi}} \tag{21}$$

Using $\mathbf{F}_b$ and $\mathbf{F}_m$, the upper set of equation in (15) is rearranged as:

$$\mathbf{H}_b \ddot{\mathbf{x}}_b + \mathbf{D}_b \dot{\mathbf{x}}_b + \mathbf{K}_b \Delta \mathbf{x}_b = -\mathbf{g} - \mathbf{F}_m + \mathbf{J}_b^T \mathbf{F}_h \tag{22}$$

Equations (15) or (22) are familiar expressions for the flexible-base manipulators [6] [7].

From the top and bottom equations of (15), we can eliminate the unactuated coordinate $\mathbf{x}_b$, to obtain:

$$\hat{\mathbf{H}} \ddot{\boldsymbol{\phi}} + \hat{\mathbf{c}} = \boldsymbol{\tau} + \hat{\mathbf{J}}^T \mathbf{F}_h + \mathbf{R} \mathbf{F}_b \tag{23}$$

where we meet the Generalized Inertia Tensor and Generalized Jacobian Matrix again, and

$$\mathbf{R} = \mathbf{H}_{bm}^T \mathbf{H}_b^{-1} \tag{24}$$

2.4 Flexible-Arm Manipulator

Now, let us consider a manipulator system whose arms are composed by a flexible link. The flexible link has infinite number of vibrational degrees of freedom (mode) actually, but it can be approximately modeled as a successive chain of finite number of virtual elastic joints. Using this model, useful models for the dynamics of flexible-arm manipulators have been developed by Yoshikawa and Hosoda [26] and Uchiyama and Konno [28]. Here we find these formulations have exactly the same structure as other under-actuated systems.

Let us replace the symbols from Equation (1) as:

$\mathbf{p}$	$\to$	$\mathbf{e}$	:elastic deflection of the flexible links
$\mathbf{q}$	$\to$	ϕ	:active joint angle
$\mathbf{x}$	$\to$	$\mathbf{x}_h$	:position/orientation of the endpoint
$\mathbf{F}_p$	$\to$	$\mathbf{F}_e$	:forces to deflect the flexible links
$\mathbf{F}_q$	$\to$	τ	:joint torque
$\mathbf{F}_{ex}$	$\to$	$\mathbf{F}_h$	:external force/torque on the endpoint

Then we obtain the following equations:

$$\begin{bmatrix} \mathbf{H}_e & \mathbf{H}_{em} \\ \mathbf{H}_{em}^T & \mathbf{H}_m \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{e}} \\ \ddot{\boldsymbol{\phi}} \end{bmatrix} + \begin{bmatrix} \mathbf{c}_e \\ \mathbf{c}_m \end{bmatrix} = \begin{bmatrix} \mathbf{F}_e \\ \boldsymbol{\tau} \end{bmatrix} + \begin{bmatrix} \mathbf{J}_e^T \\ \mathbf{J}_m^T \end{bmatrix} \mathbf{F}_h \tag{25}$$

$$\dot{\mathbf{x}}_h = \mathbf{J}_m \dot{\boldsymbol{\phi}} + \mathbf{J}_e \dot{\mathbf{e}} \tag{26}$$

$$\ddot{\mathbf{x}}_h = \mathbf{J}_m \ddot{\boldsymbol{\phi}} + \dot{\mathbf{J}}_m \dot{\boldsymbol{\phi}} + \mathbf{J}_e \ddot{\mathbf{e}} + \dot{\mathbf{J}}_e \dot{\mathbf{e}} \tag{27}$$

The forces to deflect the flexible links are generally expressed with stiffness and damping matrices as:

$$\mathbf{F}_e = -\mathbf{D}_e \dot{\mathbf{e}} - \mathbf{K}_e \Delta \mathbf{e} \tag{28}$$

where $\Delta\mathbf{e}$ denotes elastic displacements from a neutral position.

3 Common Properties

As we discussed in the above section, free-floating manipulators, flexible-base manipulators, and flexible-arm manipulators have a common structure in the equations of motion. In this section, we emphasize the case that an idea originally developed for a specific system among them can be useful for others, then considered as a common property for a general under-actuated system.

3.1 Reaction Null-Space

The "Reaction Null-Space" is a useful idea to discuss the coupling and decoupling of dynamic interaction between a manipulator and its base. The reaction null-space concept has its roots in the earlier work on free-floating space manipulator by Nenchev et al, where the Fixed-Attitude-Restricted (FAR) Jacobian has been proposed as means to plan [19] and control [20] manipulator motion that does not disturb the attitude of the free-floating base. Application of the reaction null space with relation to impact dynamics can be found in [21]. In a recent study [23] the authors emphasized the fact that the reaction null space concept is general, and can be applied to a broad class of moving base manipulators.

Now let us recall the definition of the reaction null-space.

In the system described by Equations (16)-(18), let us consider the case where the external force/torque at endpoint $\mathbf{F}_h = \mathbf{0}$, say free space motion of the manipulator without any contact with environment.

In Equation (20), if the coupling momentum $\mathcal{L}_m$ is constant, then $\mathbf{F}_m = \mathbf{0}$ in (21) and no reaction force or torque is induced on the base. In case the number of degrees of freedom of the active manipulator joints n is grater than that of the base coordinate m, the solution for the manipulator operation to satisfy $\bar{\mathcal{L}}_m$ =const is given by:

$$\dot{\boldsymbol{\phi}}_c = \mathbf{H}_{bm}^+ \bar{\mathcal{L}}_m + (\mathbf{E} - \mathbf{H}_{bm}^+ \mathbf{H}_{bm}) \boldsymbol{\xi} \tag{29}$$

where $(\cdot)^+$ indicates pseudo-inverse, $\boldsymbol{\xi} \in R^n$ is an arbitrary vector.

The component $(\mathbf{E} - \mathbf{H}_{bm}^+ \mathbf{H}_{bm})$ suggests the mapping onto the null space of the inertia matrix $\mathbf{H}_{bm}$ and this inertial null space is termed "Reaction Null-Space."

In the special case when $\bar{\mathcal{L}}_m = \mathbf{0}$, Equation (29) becomes much simpler as:

$$\dot{\boldsymbol{\phi}}_{ns} = (\mathbf{E} - \mathbf{H}_{bm}^+ \mathbf{H}_{bm}) \boldsymbol{\xi}. \tag{30}$$

As long as we operate the manipulator joints using the joint velocities given by (30), no reaction force or torque is generated on the base, therefore no reactive motion or vibration is observed in the base. If it is integrable, the integration of (30) yields *Reactionless Paths* which are joint space trajectories not exciting the base motion. The paths are succesfully demonstrated by hardware experiments with 2 DOF manipulator arm against 1 DOF flexible base [22][24].

On the other hand, the first term of Equation (29) suggests maximum interaction with the base, in contrast with the second term for the reaction null-space. This maximum interaction characteristics can be used to an effective damping of the base vibration for elastically constraint systems. For example, using the measurement of the base displacement $\Delta\mathbf{x}_b$ as a feedback signal and G as a gain matrix, we have a simple, but effective vibration surpression law:

$$\dot{\boldsymbol{\theta}}_v = G \mathbf{H}_{bm}^+ \Delta \mathbf{x}_b \tag{31}$$

The above control space is perpendicular to the reaction null-space. Therefore these two operations (30) and (31) can be easily superimposed without interfering each other, just by simple addition.

$$\dot{\boldsymbol{\theta}}_c = \dot{\boldsymbol{\theta}}_v + \dot{\boldsymbol{\theta}}_{ns}$$

$$\dot{\boldsymbol{\theta}}_c = g \mathbf{H}_{bm}^+ \Delta \mathbf{x}_b + (\mathbf{E} - \mathbf{H}_{bm}^+ \mathbf{H}_{bm}) \boldsymbol{\xi} \tag{32}$$

The idea of reaction null-space is useful for the *seperable* class of under-actuated systems where active joints and a passive base are physically separated. For a space robot which has a 6 DOF manipulator arm on a floating base satellite such as Figure 2, $n = 6$ and $m = 6$, therefore the reaction null-space does not exist. However, if we pay attention to the base attitude only, allowing the base translation during the manipulation, then $m = 3$ and we have the reaction null-space and can find reactionless paths of the manipulator arm that do not disturb the attitude of the base satellite.

3.2 Compensability

Compensability is proposed for a flexible-arm manipulator [25] and for a system composed by flexible-macro and rigid-micro manipulators [27].

The definition of compensability comes from Equation (26) with small deformation from their nominal positions:

$$\delta \mathbf{x}_h = \mathbf{J}_m \delta \boldsymbol{\phi} + \mathbf{J}_e \delta \mathbf{e} \tag{33}$$

When the flexible deflection $\delta \mathbf{e}$ is compensated by the joint action $\delta \boldsymbol{\phi}$, we can get $\delta \mathbf{x}_h = 0$. For this case, the amount of compensation is

$$\delta \boldsymbol{\phi} = -\mathbf{J}_m^+ \mathbf{J}_e \delta \mathbf{e} \tag{34}$$

Such compensation is possible only when

$$\mathrm{range}(\mathbf{J}_m) \supseteq \mathrm{range}(\mathbf{J}_e) \tag{35}$$

The required effort for the compensation is evaluated by the following index:

$$I_c = \frac{1}{|\mathbf{J}_m^+ \mathbf{J}_e \mathbf{J}_e^T \mathbf{J}_m^{+T}|^{1/2}} \tag{36}$$

The same evaluation is possible for flexible-base manipulators by replacing the variables as:

$$\mathbf{e} \rightarrow \mathbf{x}_b$$

$$\mathbf{J}_e \rightarrow \mathbf{J}_b$$

3.3 Measure of Dynamic Coupling

The dynamic coupling between the active coordinate and the passive coordinate is evaluated in some papers [7][22][29].

If we follow Torres [7] here, the passive coordinate displacement induced by the reaction of active joints is quasi-statically expressed as:

$$\delta \mathbf{x}_b = -\mathbf{H}_b^{-1} \mathbf{H}_{bm} \delta \boldsymbol{\phi}, \tag{37}$$

which leads a *Coupling Map* for flexible-base manipulators.

The matrix combination $\mathbf{H}_b^{-1}\mathbf{H}_{bm}$ and equivalent expressions are found in equations (9)(12)(13)(14)(24), therefore this combination is significant to represent the degree of dynamic coupling. For example, in (9) and (12) if $\mathbf{H}_b^{-1}\mathbf{H}_{bm}$ is a null matrix, the generalized Jacobian matrix and the generalized inertia tensor coincide those of ground based manipulators without any dynamic coupling with the base. As the eigenvalues of $\mathbf{H}_b^{-1}\mathbf{H}_{bm}$ get larger, the dynamic coupling effects become significant. We therefore propose an index measure for dynamic coupling in a similar fashion to the compensability measure.

$$I_d = \frac{1}{|\mathbf{H}_b^{-1} \mathbf{H}_{bm} \mathbf{H}_{bm}^T \mathbf{H}_b^{-T}|^{1/2}} \tag{38}$$

This measure will be particularly useful for free-floating and free-joint manipulators, because dynamic reaction effects are dominant in such systems with *free* passive elements.

4 Conclusion

This paper discusses a general formulation for general under-actuated manipulator systems, paying attention to the fact that all the systems in this class have common characteristics in dynamic formulation. The author highlights that the Generalized Jacobian Matrix and the Generalized Inertia Tensor, which are originally developed for a free-flying space manipulator, are redefined from the general form of dynamics equation (1). In addition, the author points interesting properties which are originally developed for a specific system but can be commonly used for any other under-actuated systems. Such examples are:

1. The Reaction Null-Space and reactionless motion have recently studied focusing to flexible-base manipulators, but the ideas are useful for free-floating manipulators to find manipulator paths which do not disturb the attitude of the base satellite.

2. The compensability has proposed originally for flexible-arm manipulators, but it is directly applied for flexible-base manipulators and may also be useful for free-floating and free-joint manipulators.

3. The measure of dynamic coupling is proposed to evaluate the degree of dynamic coupling effects. This idea will be particularly useful for free-floating and free-joint manipulators.

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