Dynamic Control of A Space Robot System with No Thrust Jets Controlled Base

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ABSTRACT

In this paper we discuss dynamic control of a free-flying space robot system where the base attitude is not controlled by thrust jets. Without external forces and moments, the system is governed by linear and angular momentum conservation laws. We first derive the system dynamic formulations in joint space and in inertia space, based on Lagrangian dynamics. Then we discuss the fact that dynamics of a space robot system can not be linearly parameterized, as opposed to the case of a fixed-based robot. Revealing this property is significant since the linearity of parameterization has been used as a prerequisite for various adaptive and nonlinear control schemes currently used in the robot control. Based on the dynamic model of the space robot system, a simple linear control scheme is presented for the normal regulation problem for tasks in space, such as holding lights for illuminating objects or handing an astronaut tools in extra-vehicular activity. A globally stable dynamic control scheme is proposed for trajectory tracking applications, such as catching moving objects or structure inspection for the space station. The dynamic control algorithm exhibits a fast and accurate motion response even when the mass/inertia ratio of the base with respect to the robot is low. The effectiveness of the proposed algorithms are demonstrated by simulation studies.

1 Introduction

Robotic technology offers two potential benefits for future space exploration. One benefit is minimizing the risk that astronauts face, since the environment for humans in space is inhospitable, such as high vacuum, extreme glare and temperature, possibly high level of radiation. The other benefit is increasing the productivity of the mission, because the extra-vehicular activity (EVA) consumes considerable time and can require the degree of dexterity exceeding human capability. The use of robots, however, is a challenge problem for control of both robot and space vehicle that the robot is attached to as a base. Considerable research efforts have been directed to man-machine interfacing of telerobotics [3, 21], control of light-weight flexible space robots [5], and development of space robot concept [15, 4, 21].

Only recently the research on dynamics and control of a space robot system by considering the interaction between the space robot and the base (spacecraft, space station, or satellite) has begun. Due to the dynamic interaction, the motion of space robots can alter the base trajectory. On the other hand, the robot end-effector may miss the desired target due to the motion of the base. This mutual dependence severely affects the performance of both robot and base, especially when the mass and the moment of inertia of the robot and payload are not negligible in comparison to the base. Any inefficiency in the planning and control can considerably risk the success of the space mission.

Lindberg, Longman, and Zedd [10] addressed several issues related to dynamics and kinematics of a space robot system when the base is controlled in orientation but free in translation, and when the base is free in both orientation and translation. In the paper [9], Longman discussed the kinematic relationship in joint space and inertia space, called *forward and inverse kinetics problems*, and the workspace of a space robot. Vafa and Dubowsky [19, 20], and Papadopoulos and Dubowsky [13] introduced the concept of *Virtual Manipulator* to represent the dynamics of a space robot system. The virtual manipulator concept makes it possible to reproduce the kinematic behavior of a space robot by the kinematics of a modified fixed-base robot. They applied this concept to plan robot motions that minimize disturbances to the spacecraft, and recently to study the singularity problem of space robots. Due to the conservation of angular momentum, non-integrable velocity constraints result in the *nonholonomic nature* of a space robot, and were discussed by Nakamura and Mukherjee [12]. Umetani and Yoshida [18] proposed a resolved rate and acceleration control based on the *Generalized Jacobian Matrix* of a space robot. Masutani, Miyazaki, and Arimoto [11] addressed feedback control problems of a space robot system. Alexander and Cannon [2], and Ullman and Cannon [16] provided an experimental study in autonomous navigation and control of a free-flying space robot.

In this paper, we address the problem of controlling a space robot when the base is not controlled by thrust jets. The base attitude can be normally controlled by reaction wheels or thrust jets. Reaction wheels can be installed in three perpendicular directions and their speeds can be altered to control the orientation of the body. In this case, the linear and angular momenta are conserved because no external force is applied. When thrust jets are used to control the attitude of the base, however, linear and/or angular momenta are not conserved, due to external thrust forces. We consider the case when the robot works on a base with no thrust jets in this paper. Therefore, the robot is completely free-flying in zero-gravity environment. This unique feature results in two properties of the space robot with respect to the fixed-base robot. First, the kinematic mapping from joint space to Cartesian (inertial) space, or vice versa, is no longer unique and is in relation to not noly the current positions, but also to the past path that the robot follows. Second, the kinematic relationship is dependent on dynamic parameters, such as the mass and the moment of inertia.

In this paper, first we systematically formulate kinematics and dynamics equations of a free-flying space robot system, based on linear and angular momentum conservation law and Lagrangian dynamics. Then the dynamic properties of the system are studied. It is found that the free-flying space robot dynamics cannot be linearly expressed in terms of dynamic parameters, such as the mass and inertia of the robot. In other words, it is impossible to properly choose a set of combinations of dynamic parameters, such that the dynamics can be represented by product of two functions, one with dynamic parameters, and the other with no dynamic parameters. This results in infeasibility of many control schemes, such as most existing adaptive control schemes, which use this property as a prerequisite.

Based on the derived dynamic model in inertia space, two control schemes are proposed for normal regulation and trajectory tracking problems. Applications of regulation control can be found in various tasks in space, such as lights holding for illuminating objects, or handing tools to astronauts in extra-vehicle activity. For inspection of the structure or surface of the space station, or parts transporting, trajectory tracking is essential. The PD-based regulator is simple to implement and works well for the large mass/inertia ratio (bass/robot). The tracking controller makes full use of dynamic model, therefore allows more accurate and fast motion even for the small mass/inertia ratio (bass/robot). The effectiveness of the control schemes proposed is demonstrated by simulation results.

2 Kinematics of the Space Robot System

In this section, we will discuss kinematics of the free-flying space robot system. As pointed out previously, since the kinematics in this case is actually mass and inertia related, it is completely different from the conventional fixed-based robots. Our discussion in this section is focused on the motion relationship between the joint space and inertia space, without explicitly involving acceleration. Some papers, such as [9, 17, 11], have derived kinematics equations, but for the purpose of developing dynamic control scheme and easy understanding the discussion in the following sections, we provide a uniform formulation for kinematics of a space robot system.

A space robot attached to spacecraft on the orbit is considered as a free-flying system in the non-gravitational environment. The system is modeled as a set of $n+1$ rigid bodies connected by n joints, which are numbered from 1 to n. A joint variable vector $\mathbf{q} = (q_1, q_2, \cdots, q_n)^T$ is used to represent those joint displacements. Each body is numbered from 0 to n, and the base can be named as B in particular. The mass and inertia of ith body are denoted by m_i and $\mathbf{I}_i$, respectively.

We define two coordinate frames, the inertia coordinate $\sum_I$ on the orbit, and the base coordinate $\sum_B$ attached on the base body with its origin at the centroid of the base. As shown in Figure 1, let $\mathbf{R}_i$ and $\mathbf{r}_i$ be the position vectors pointing the centroid of ith body with reference to $\sum_I$ and $\sum_B$ respectively, then

$$\mathbf{R}_i = \mathbf{r}_i + \mathbf{R}_B \tag{1}$$

where $\mathbf{R}_B$ is the position vector pointing the centroid of the base with reference to $\sum_I$.

In general, the desired motion of a manipulator is specified in terms of hand trajectory in inertia coordinate, while the servo control system requires that the reference inputs be specified in joint coordinate. The relationship between the inertia coordinates, such as lift p_x, sweep p_y, reach p_z, yaw α, pitch β, and roll γ, and the joint angular motion in fixed-base case is inherently nonlinear

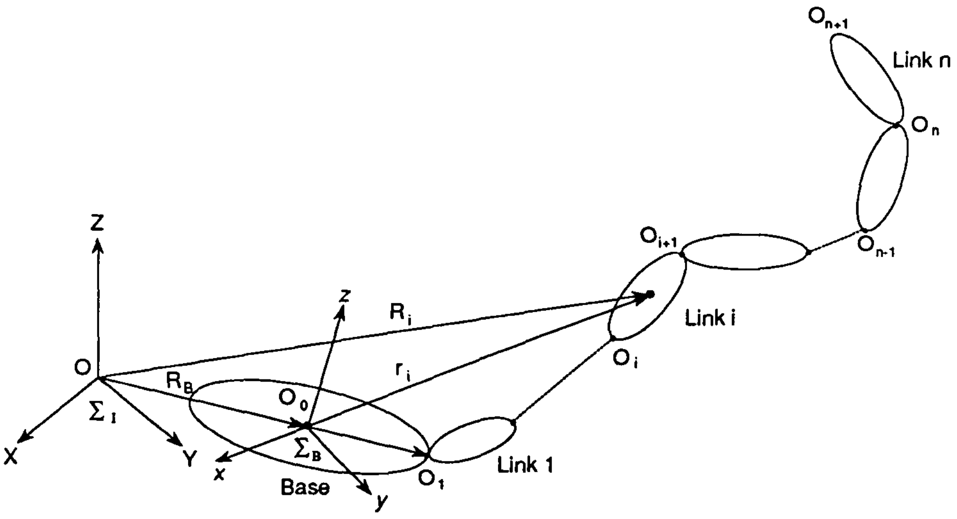

Figure 1: A free-flying space robot system model.

and can be generally expressed by a nonlinear vector function

$$\mathbf{X}(t) = f(\mathbf{q}(t)) \tag{2}$$

where f is a vector function, and $\mathbf{X}(t)$ and $\mathbf{q}(t)$ are vectors representting state variables in the inertia coordinates $\mathbf{X}(t) = (p_x,\, p_y,\, p_z,\, \alpha,\, \beta,\, \gamma)^T$, and in joint coordinates $\mathbf{q}(t) = (q_1, q_2, \cdots, q_n)^T$.

In free-flying case, however, it must be noted that even the forward kinematics is difficult to solve. The position and orientation of the hand may not have a closed-form solution because it depends on the inertia property which changes with configurations. Thus the solution is function of the history of the configurations [9]. The inverse kinematics problem is even more difficult to tackle.

One possible solution to this dilemma is to discuss the kinematics problem indirectly by motion rates rather than by displacements. It has been found [17] that the motion rate of the hand and that of joint variables can be linearized excluding from their history. If the trajectory at the initial state and the joint motion rate at each step are determined, the trajectory of position and orientation of the hand in inertia space can be obtained by numerical integration.

Let $\mathbf{V}_i$ and $\boldsymbol{\Omega}_i$ be linear and angular velocities of ith body with respect to $\sum_I$, $\mathbf{v}_i$ and ω_i with respect to $\sum_B$. Then we have

$$\begin{aligned} \mathbf{V}_i &= \mathbf{v}_i + \mathbf{V}_B + \Omega_B \times \mathbf{r}_i \\ \boldsymbol{\Omega}_i &= \omega_i + \Omega_B \end{aligned} \tag{3}$$

where $\mathbf{V}_B$ and $\boldsymbol{\Omega}_B$ are linear and angular velocities of the centroid of the base with respect to $\sum_I$, and operator '×' represents outer product of R^3 vector. The velocities in base coordinates $\mathbf{v}_i$ and ω_i can be represented by

$$\begin{bmatrix} \mathbf{v}_i \\ \omega_i \end{bmatrix} = \mathbf{J}_i(\mathbf{q})\dot{\mathbf{q}} \tag{4}$$

where $\mathbf{J}_i(\mathbf{q})$ is the Jacobian of the ith body,

$$\mathbf{J}_i = \begin{bmatrix} \mathbf{J}_{Li}(\mathbf{q}) \\ \mathbf{J}_{Ai}(\mathbf{q}) \end{bmatrix} \tag{5}$$

$$\mathbf{v}_i = \mathbf{J}_{Li}(\mathbf{q})\dot{\mathbf{q}} \tag{6}$$

$$\omega_i = \mathbf{J}_{Ai}(\mathbf{q})\dot{\mathbf{q}} \tag{7}$$

In what follows, we derive the relationship between motion rate in inertia coordinates and that in joint coordinates. The linear and angular momenta **P** and **L** are defined as

$$\mathbf{P} = \sum_{i=0}^{n} m_i \mathbf{V}_i \tag{8}$$

$$\mathbf{L} = \sum_{i=0}^{n} \mathbf{I}_i^B \boldsymbol{\Omega}_i + m_i \mathbf{R}_i \times \mathbf{V}_i \tag{9}$$

where $\mathbf{I}_i^B$ is the inertia tensor in Σ_B. Now we define the following for the centroid of whole system,

$$m_c = \sum_{i=0}^{n} m_i \tag{10}$$

$$\mathbf{r}_c = \sum_{i=1}^{n} m_i \mathbf{r}_i / m_c \tag{11}$$

$$\mathbf{J}_c = \sum_{i=1}^{n} m_i \mathbf{J}_{Li} / m_c \tag{12}$$

Substituting Equations (10,11,12) to (8,9) yields

$$\begin{bmatrix} \mathbf{P} \\ \mathbf{L} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_V & \mathbf{H}_{V\Omega} \\ \mathbf{H}_{V\Omega}^T & \mathbf{H}_{\Omega} \end{bmatrix} \begin{bmatrix} \mathbf{V}_B \\ \boldsymbol{\Omega}_B \end{bmatrix} + \begin{bmatrix} \mathbf{H}_{Vq} \\ \mathbf{H}_{\Omega q} \end{bmatrix} \dot{\mathbf{q}} + \begin{bmatrix} \mathbf{O}_3 \\ \mathbf{R}_B \times \mathbf{P} \end{bmatrix} \tag{13}$$

Each block submatrix is determined by

$$\mathbf{H}_V = m_c \mathbf{U}_3 \qquad \in \Re^{3\times 3} \tag{14}$$

$$\mathbf{H}_{V\Omega} = -m_c [\mathbf{r}_c \times] \qquad \in \Re^{3\times 3} \tag{15}$$

$$\mathbf{H}_{Vq} = m_c \mathbf{J}_c \qquad \in \Re^{3\times n} \tag{16}$$

$$\mathbf{H}_{\Omega} = \sum_{i=1}^{n} [\mathbf{I}_i^B + m_i \mathbf{D}(\mathbf{r}_i)] + \mathbf{I}_B \qquad \in \Re^{3\times 3} \tag{17}$$

$$\mathbf{H}_{\Omega q} = \sum_{i=1}^{n} [\mathbf{I}_i^B \mathbf{J}_{Ai} + m_i [\mathbf{r}_i \times] \mathbf{J}_{Li}] \qquad \in \Re^{3\times n} \tag{18}$$

where $\mathbf{U}_3$ is a 3×3 unity matrix, $\mathbf{O}_3$ is a 3×3 zero matrix. The matrix functions $[\mathbf{r}\times]$ and $\mathbf{D}(\mathbf{r})$ for a vector $\mathbf{r} = [r_x, r_y, r_z]^T$ are defined as

$$[\mathbf{r}\times] = \begin{bmatrix} 0 & -r_z & r_y \\ r_z & 0 & -r_x \\ -r_y & r_x & 0 \end{bmatrix} \tag{19}$$

$$\mathbf{D}(\mathbf{r}) = [\mathbf{r}\times]^T[\mathbf{r}\times] = \begin{bmatrix} r_y^2 + r_z^2 & -r_x r_y & -r_x r_z \\ -r_x r_y & r_x^2 + r_z^2 & -r_y r_z \\ -r_x r_z & -r_y r_z & r_x^2 + r_y^2 \end{bmatrix} \tag{20}$$

Because in free-flying case we assume that there are no gravitational force and no external forces acting on the system, therefore, the linear and angular momenta of the system are conserved. Without loss of generality we assume that the system is stationary in the initial state. This implies that total linear and angular momenta are zero. Substituting $\mathbf{P} = \mathbf{O}_3$ and $\mathbf{L} = \mathbf{O}_3$ into (13) we obtain

$$\begin{aligned} \mathbf{V}_B &= \mathbf{H}_L(\mathbf{q})\dot{\mathbf{q}} && (21) \\ \omega_B &= \mathbf{H}_A(\mathbf{q})\dot{\mathbf{q}} && (22) \end{aligned}$$

where

$$\begin{aligned} \begin{bmatrix} \mathbf{H}_L \\ \mathbf{H}_A \end{bmatrix} &= -\begin{bmatrix} \mathbf{H}_V & \mathbf{H}_{V\Omega} \\ \mathbf{H}_{V\Omega}^T & \mathbf{H}_\Omega \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{H}_{Vq} \\ \mathbf{H}_{\Omega q} \end{bmatrix} && (23) \\ &= \begin{bmatrix} -\mathbf{J}_c - [\mathbf{r}_c\times]\mathbf{H}_B^{-1}\mathbf{H}_M \\ -\mathbf{H}_B^{-1}\mathbf{H}_M \end{bmatrix} && (24) \\ \mathbf{H}_B &= \mathbf{H}_\Omega - m_c\mathbf{D}(\mathbf{r}_c) && (25) \\ \mathbf{H}_M &= \mathbf{H}_{\Omega q} - m_c[\mathbf{r}_c\times]\mathbf{J}_c && (26) \end{aligned}$$

Therefore,

$$\begin{aligned} \mathbf{V}_E &= \mathbf{v}_E + \mathbf{V}_B + \boldsymbol{\Omega}_B \times \mathbf{r}_E \\ &= \mathbf{N}_L\dot{\mathbf{q}} && (27) \\ \boldsymbol{\Omega}_E &= \omega_E + \boldsymbol{\Omega}_B \\ &= \mathbf{N}_A\dot{\mathbf{q}} && (28) \\ \begin{bmatrix} \mathbf{V}_E \\ \boldsymbol{\Omega}_E \end{bmatrix} &= \mathbf{N}(\mathbf{q})\dot{\mathbf{q}} && (29) \end{aligned}$$

where

$$\begin{aligned} \mathbf{N}(\mathbf{q}) &= \begin{bmatrix} \mathbf{N}_L(\mathbf{q}) \\ \mathbf{N}_A(\mathbf{q}) \end{bmatrix} && (30) \\ \mathbf{N}_L &= \mathbf{J}_L + \mathbf{H}_L - [\mathbf{r}_E\times]\mathbf{H}_A \\ &= \mathbf{J}_L - \mathbf{J}_c - [(\mathbf{r}_E - \mathbf{r}_c)\times]\mathbf{H}_B^{-1}\mathbf{H}_M && (31) \\ \mathbf{N}_A &= \mathbf{J}_A + \mathbf{H}_A \end{aligned}$$

$$\begin{aligned} &= \mathbf{J}_A - \mathbf{H}_B^{-1}\mathbf{H}_M \qquad (32) \\ \mathbf{J}(\mathbf{q}) &= \begin{bmatrix} \mathbf{J}_L(\mathbf{q}) \\ \mathbf{J}_A(\mathbf{q}) \end{bmatrix} \end{aligned}$$

and $\mathbf{r}_E$ is the hand position with respect to the base, **J** is the conventional fixed-based Jacobian of the robot with respect to the base, **N** is so-called the generalized Jacobian [17] which is not only a function of robot motion but also function of the base motion.

Let $\tau = [\tau_1, \cdots, \tau_n]^T$ be a vector of joint torques corresponding to the joint coordinates $\mathbf{q}$, and $\mathbf{F} = [F_x, F_y, F_z, T_x, T_y, T_z]^T$ be a vector of the generalized force in the inertia coordinates. Assume that there exist virtual displacements $\delta\mathbf{x}$ and $\delta\mathbf{q}$ in inertia space and in joint space, respectively. The relationship between $\delta\mathbf{x}$ and $\delta\mathbf{q}$ is

$$\delta\mathbf{x} = \mathbf{N}\delta\mathbf{q} \qquad (33)$$

where N is the generalized Jacobian. By the virtual work principle,

$$\delta_{work} = \tau^T\delta\mathbf{q} - \mathbf{F}^T\delta\mathbf{x} = (\tau - \mathbf{N}^T\mathbf{F})^T\delta\mathbf{q} = 0$$

for all $\delta\mathbf{q}$, therefore,

$$\tau = \mathbf{N}^T\mathbf{F} \qquad (34)$$

This relationship will be used in dynamic control in the following sections.

3 Dynamics of the Space Robot System

Based on Lagrangian dynamics, we formulate the dynamics equation in joint space as follows. It is known that the total kinetic energy of the space robot system is given in terms of variables in joint space by

$$\mathbf{K} = \frac{1}{2}\dot{\mathbf{q}}^T\mathbf{M}(\mathbf{q})\dot{\mathbf{q}} \qquad (35)$$

where $\mathbf{M}(\mathbf{q})$ is a 6×6 symmetric kinetic energy matrix.

Since the space robot works in non-gravitational environment and we assume that no elastic body is involved, the potential energy can be regarded as zero. Thus, the Lagrangian formulation,

$$\frac{d}{dt}(\frac{\partial\mathbf{K}}{\partial\dot{\mathbf{q}}}) - \frac{\partial\mathbf{K}}{\partial\mathbf{q}} = \tau \qquad (36)$$

yields the motion equation

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{B}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} = \tau \qquad (37)$$

where

$$\mathbf{B}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} = \dot{\mathbf{M}}\dot{\mathbf{q}} - \frac{\partial}{\partial\mathbf{q}}(\frac{1}{2}\dot{\mathbf{q}}^T\mathbf{M}\dot{\mathbf{q}}) \qquad (38)$$

and

$$\mathbf{M} = \mathbf{H}_q - m_c\mathbf{J}_c^T\mathbf{J}_c - \mathbf{H}_M^T\mathbf{H}_B^{-1}\mathbf{H}_M \qquad (39)$$

is symmetric and positive definite, and

$$\mathbf{H}_q = \sum_{i=1}^{n}(\mathbf{J}_{Ai}^T \mathbf{I}_i^B \mathbf{J}_{Ai} + m_i \mathbf{J}_{Li}^T \mathbf{J}_{Li}) \tag{40}$$

is the inertia matrix of the robot when the base is fixed.

From (37) to (40) we can observe the following properties of the dynamics model for a free-flying space robot.

- As in the fixed-based robot system, M *and* B *are not independent, and* $\dot{\mathrm{M}} - 2\mathrm{B}$ *are skew-symmetry.* This can be easily seen by energy conservation principle, i.e.,

  $$\frac{1}{2}\frac{d}{dt}(\dot{\mathbf{q}}^T \mathbf{M}\dot{\mathbf{q}}) = \dot{\mathbf{q}}^T \tau$$

  which implies

  $$\dot{\mathbf{q}}^T(\dot{\mathbf{M}} - 2\mathbf{B})\dot{\mathbf{q}} = \mathbf{0}$$

- The inertia matrix M is composed of two parts, manipulator inertia matrix M_1 which is completely independent of the base dynamics and only a function of the robot motion with respect to the base coordinates, and coupled inertia matrix M_2 which takes the dynamic interaction into consideration. That is,

  $$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 \tag{41}$$

  $$\mathbf{M}_1 = \mathbf{H}_q \tag{42}$$

  $$\mathbf{M}_2 = -m_c \mathbf{J}_c^T \mathbf{J}_c - \mathbf{H}_M^T \mathbf{H}_B^{-1} \mathbf{H}_M \tag{43}$$

  Since

  $$\mathbf{B}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} = \dot{\mathbf{M}}\dot{\mathbf{q}} - \frac{\partial}{\partial \mathbf{q}}(\frac{1}{2}\dot{\mathbf{q}}^T \mathbf{M}\dot{\mathbf{q}}) \tag{44}$$

  if M can be decomposed into two parts, M_1 and M_2, then B can always be decomposed into two parts B_1 and B_2. In this way, dynamics of the system can also be viewed as a combination of two subsystems corresponding to the manipulator part and interaction part.

  $$\begin{aligned} \tau &= \tau_1 + \tau_2 \\ \tau_1 &= \mathbf{M}_1\ddot{\mathbf{q}} + \mathbf{B}_1(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} \\ \tau_2 &= \mathbf{M}_2\ddot{\mathbf{q}} + \mathbf{B}_2(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} \end{aligned}$$

  The first part τ_1 is linear in terms of robot dynamic parameters such as mass and inertia tensor, and nonlinear in terms of kinematic parameters. The second part τ_2 is completely nonlinear in terms of both kinematic and dynamic parameters, as we will discuss later in detail. By kinematic parameters we mean joint variables and geometrical parameters of links.

- When the base mass/inertia is sufficiently large compared to that of the robot and the load being manipulated, *the effect of the dynamics of the second part is reduced*, as M_2 is inversely proportional to m_c and I_c which are total mass and moment of inertia of the system respectively. The first part dynamics τ_1 does not change since it is independent of the base parameters. The first part dynamics τ_1 therefore dominates the whole system.

- The most important property of the free-flying space robot system that will be addressed in detail is *nonlinearity of parameterization* with respect to the dynamic parameters which makes the control of the space robot system much different from that of the fixed based robot system. We will discuss this issue in the following section, nonlinearity of parameterization.

In the similar way, we can obtain the dynamic equation of a space robot system in inertia space. The detail derivation can be found in Appendix 1.

$$\mathbf{H}\ddot{\mathbf{x}} + \mathbf{C}(\mathbf{x},\dot{\mathbf{x}})\dot{\mathbf{x}} = \mathbf{F} \tag{45}$$

where

$$\mathbf{C}(\mathbf{x},\dot{\mathbf{x}})\dot{\mathbf{x}} = \dot{\mathbf{H}}\dot{\mathbf{x}} - \frac{\partial}{\partial \mathbf{x}}(\frac{1}{2}\dot{\mathbf{x}}^T\mathbf{H}\dot{\mathbf{x}}) \tag{46}$$

and $\mathbf{H}$ is symmetric and positive definite, $\dot{\mathbf{H}} - 2\mathbf{C}$ is skew-symmetry. For simplicity, we include all derivation process in Appendix 1. The relationship between joint space dynamics and inertia space dynamics can be summarized as follows.

$$\dot{\mathbf{x}} = \mathbf{N}(\mathbf{q})\dot{\mathbf{q}} \tag{47}$$

$$\ddot{\mathbf{x}} = \mathbf{N}(\mathbf{q})\ddot{\mathbf{q}} + \dot{\mathbf{N}}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} \tag{48}$$

$$\mathbf{F} = \mathbf{N}^{-T}(\mathbf{q})\tau \tag{49}$$

$$\mathbf{H}(\mathbf{x}) = \mathbf{N}^{-T}(\mathbf{q})\mathbf{M}(\mathbf{q})\mathbf{N}^{-1}(\mathbf{q}) \tag{50}$$

$$\mathbf{C}(\mathbf{x},\dot{\mathbf{x}})\dot{\mathbf{x}} = \mathbf{N}^{-T}(\mathbf{q})\mathbf{B}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} - \mathbf{H}(\mathbf{x})\dot{\mathbf{N}}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} \tag{51}$$

Obviously, the inertia matrix $\mathbf{H}$ is even more complicated than its counterpart in joint space M because it involves inversion of the generalized Jacobian matrix.

4 Nonlinearity of Parameterization

In this section we discuss one of fundamental differences in dynamics between the fixed-based robot and the free-flying space robot, ***nonlinearity of parameterization.*** We first discuss the linearity of parameterization in dynamics of the fixed-base robot and its importance in control. Then we demonstrate the nonlinearity of parameterization in dynamics of the free-flying space robot.

The property of linear parameterization of the fixed-based manipulator dynamics is important for both controller design and dynamics analysis [8]. It has been used as one of prerequisite conditions in design of most of nonlinear dynamic control and adaptive control for conventional robot manipulators [7, 14]. Let's take adaptive control as an example to illustrate the importance of linear parameterization of manipulator dynamics.

Present robot adaptive control schemes can be generally classified into three categories, *direct, indirect*, and *composite adaptive control.* Nearly all of these algorithms are based on the property of linear parameterization in the robot dynamics, i.e., possibility of selecting a proper set of equivalent parameters such that the robot dynamics depends linearly on these parameters. These controllers can take full consideration of the nonlinear time-varying and coupled dynamics, without unrealistic assumptions on linearization of robot dynamics, or on decoupled joint motions, or on slow variation of the inertia matrix used by early-day robot adaptive controllers.

According to Equation (35), the system dynamics can be linearly parameterized if both $\mathbf{M}$ and $\mathbf{B}$ can be linearly parameterized. From Equation (36), linear parameterization of $\mathbf{B}$ depends on that of $\mathbf{M}$. Therefore, the problem of linear parameterization of space robot dynamics, can be reduced to the problem of linear parameterization of inertia matrix of the free-flying robot system since the space robot works in zero-gravity environment and no gravity force is involved in dynamics equation. If the property of linear parameterization were still valid, most of existing algorithms for controlling the fixed-based robot would be applicable to the free-flying space robot systems. Unfortunately, this property is no longer valid here. We will demonstrate that it is impossible to linearly parameterize dynamics of the free-flying space robot system, in theory and by case study.

As we have derived, the inertia matrix $\mathbf{M}$ can be represented by

$$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 \tag{52}$$

$$\mathbf{M}_1 = \mathbf{H}_q \tag{53}$$

$$\mathbf{M}_2 = -m_c \mathbf{J}_c^T \mathbf{J}_c - \mathbf{H}_M^T \mathbf{H}_B^{-1} \mathbf{H}_M \tag{54}$$

Therefore, the problem of linear parameterization of inertia matrix becomes the possibility of linear parameterization of $\mathbf{M}_2$, since $\mathbf{M}_1$ is the inertia matrix of the fixed-base robot which can be linearly parameterized [8]. Now we examine whether $\mathbf{M}_2$ can be linearly parameterized in terms of dynamic parameters.

Refer to Equation (12),

$$\mathbf{J}_c = \sum_{i=1}^{n} m_i \mathbf{J}_{Li} / m_c \tag{55}$$

thus,

$$m_c \mathbf{J}_c^T \mathbf{J}_c = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{1}{m_c} m_i m_j \mathbf{J}_{Li}^T \mathbf{J}_{Lj} = \sum_{i=1}^{n} \sum_{j=1}^{n} R_{ij} \frac{m_i m_j}{m_c} \tag{56}$$

where R_{ij} is a function of geometric parameters and joint variables, and independent of dynamic parameters. Therefore, the first term in $\mathbf{M}_2$ can be linearly determined by choosing a set of parameters $m_i m_j / m_c$ $(i, j = 1, \ldots, n)$.

Then we examine the second term, $\mathbf{H}_M^T \mathbf{H}_B^{-1} \mathbf{H}_M$, which is much more complicated. From previous derivation, we have

$$\mathbf{H}_B = \sum_{i=1}^{n} (\mathbf{I}_i + m_i \mathbf{D}(\mathbf{r}_i)) + \mathbf{I}_B - m_c \mathbf{D}(\mathbf{r}_c) \tag{57}$$

$$\mathbf{H}_B^{-1} = \frac{\mathbf{H}_B^*}{\det(\mathbf{H}_B)} \tag{58}$$

where $\mathbf{H}_B^*$ is the adjoint of the matrix $\mathbf{H}_B$.

It can be observed that the linear parameterization of the second term is possible only if $\det(\mathbf{H}_B)$ can be reduced from all elements of $\mathbf{H}_M^T \mathbf{H}_B^* \mathbf{H}_M$, or if it can be expressed as a product of two scalar functions with only one containing dynamic parameters. Because $\mathbf{H}_B$ is time varying and coupled, $\det(\mathbf{H}_B)$ can not be eliminated from $\mathbf{H}_M^T \mathbf{H}_B^* \mathbf{H}_M$. Furthermore, $\det(\mathbf{H}_B)$ can not be expressed as a linear scalar function with respect to any set of combinations of dynamic parameters. Therefore, the second term of the inertia matrix, $\mathbf{H}_M^T \mathbf{H}_B^{-1} \mathbf{H}_M$ can not be linearly parameterized in terms of dynamic parameters.

From the 2-D example given in the case study section, we can take a closer look at property of the term $\mathbf{H}_M^T \mathbf{H}_B^{-1} \mathbf{H}_M$ where $\mathbf{H}_B$ is reduced to a scaler,

$$H_B = I_0 + I_1 + I_2 + m_1(x_1^2 + y_1^2) + m_2(x_2^2 + y_2^2) + m_c(r_{cx}^2 + r_{cy}^2) \tag{59}$$

$$\mathbf{H}_M = [I_1 + I_2, I_2] + m_1[-r_{1cy}, r_{1cx}]\mathbf{J}_1 + m_2[-r_{2cy}, r_{2cx}]\mathbf{J}_2 \tag{60}$$

It can be easily seen that $\mathbf{H}_M^T \mathbf{H}_M$ cannot be reduced by H_B. On the other hand, H_B can not be expressed as a product of two scalar functions with one including combinations of dynamic parameters $(I_0, I_1, I_2, m_c, m_1, m_2)$, and the other excluding these parameters. Thus, it is impossible to linearly parameterize space robot dynamics even for the simple 2-D case.

It is noted that the previous analysis is for the inertia matrix in *joint* space. In inertia space, the inertia matrix $\mathbf{H} = \mathbf{N}^{-T}\mathbf{M}\mathbf{N}^{-1}$ is even more complicated, due to inversion of the generalized Jacobian matrix $\mathbf{N}$. In the same manner, it can be shown that the dynamics equation in inertia space can also not be linearly parameterized for the space robot system.

5 Dynamic Control Algorithms

Based on previous dynamic formulations, in this section we derive control schemes for the free-flying space robot system.

It must be noted that the desired position or trajectory of the space robot system usually is specified in inertia space, while it can not be uniquely mapped into the joint space where control is executed. Using the pseudo-inverse method, we can find at least one solution of the joint angles based on the given position in inertia space [1]. This, however, requires frequent calibration and computational complexity. Therefore, we propose our control schemes in inertia space, other than in joint space.

We consider two basic cases in controlling the space robot system, *regulation* and *tracking*. In *regulation*, we position the robot hand to the desired location, or target location, in inertia space. This type of tasks can be useful for holding lights illuminating the objects or handing an astronaut some tools in extra-vehicular activity. In *tracking*, we control the robot hand motion to follow the given trajectory. This type of task is essential in inspection of space station structure and surface, part transport, or light assembly. In what follows, we first develop a simple PD control scheme for regulation control, and then propose a globally stable dynamic control scheme for tracking applications.

5.1 Regulation

Let $\mathbf{x}_t$ and $\mathbf{x}$ be the generalized displacement vectors of the *target* which the robot is to reach, and of the robot *hand* in inertia space, respectively. The difference between them is denoted by

$$\mathbf{e}_p = \mathbf{x} - \mathbf{x}_t \tag{61}$$

The objective of regulation is to control robot such that $\mathbf{x}$ converges to the given $\mathbf{x}_t$, assuming $\mathbf{x}_t$ is within the nonsingular workspace of the robot.

Theorem 1 *If the following regulator in inertia space is applied to the dynamic system (45) in inertia space,*

$$\mathbf{F} = -(\mathbf{K}_p \mathbf{e}_p + \mathbf{K}_d \dot{\mathbf{e}}_p) \tag{62}$$

where K_p *and* K_d *are constant, diagonal gain matrices with positive* k_{pj} *and* k_{dj} *on the diagonals, or a corresponding regulator in joint space is given by*

$$\tau = \mathrm{N}^T\mathbf{F} = -\mathrm{N}^T(\mathbf{K}_p\mathbf{e}_p + \mathbf{K}_d\dot{\mathbf{e}}_p) \tag{63}$$

then the equilibrium state $\mathrm{x}_t = \mathrm{x}$ *is asymptotically stable, if* N *is of full rank.*

Proof

A Lyapunov candidate is taken as

$$V_1(t) = \frac{1}{2}(\dot{\mathbf{x}}^T\mathbf{H}\dot{\mathbf{x}} + \mathbf{e}_p^T\mathbf{K}_p\mathbf{e}_p) \tag{64}$$

Using the skew symmetry of the matrix $(\dot{\mathbf{H}} - 2\mathbf{C})$, the time derivative of $V_1(t)$ becomes

$$\dot{V}_1(t) = \dot{\mathbf{x}}^T(\mathbf{F} + \mathbf{K}_p\mathbf{e}_p) \tag{65}$$

Substituting (62) into (65) yields

$$\dot{V}_1(t) = -\dot{\mathbf{x}}^T\mathbf{K}_d\dot{\mathbf{x}} \leq 0 \tag{66}$$

Thus, $\dot{V}_1 = 0$ iff $\dot{\mathbf{x}} = 0$. This implies that $\ddot{\mathbf{x}} = -\mathbf{H}^{-1}\mathbf{K}_p\mathbf{e}_p$. Therefore, one has $\dot{V}_1 \equiv 0$ only if $\mathbf{e}_p = 0$.

□

Note that the position convergence of the simple PD controller (62) does not require a priori knowledge of system dynamic parameters. However, the controller (63) in joint space does require a priori knowledge of dynamic parameters, because the generalized Jacobian N is invloved. For a given set of control gains, its transient response also depends on system dynamic parameters, as well as the modeling error.

5.2 Tracking

The objective of tracking is to constrain the robot hand motion to follow the given trajectory in inertia space which is determined by task specifications. For example, in the task of catching a moving object in space, the trajectory and velocity of the robot before and after catching are determined by motion requirement of catching and trajectory of the object. Measurement for the robot hand position in inertia space may not be an easy problem, since a wide range sensor may not be found in space applications. Laser range finder may be a good candidate as proposed in [6]. In the case that the hand position can not be directly measured, it still can be determined by using joint measurements, base location, and the generalized Jacobian matrix. This requires, however, that the base location in inertia space is measurable, and at the same time introduces observing errors. From now on we assume the measurement of the robot hand in inertia space is available. Moreover, we assume that the desired $\mathbf{x}_d$, $\dot{\mathbf{x}}_d$ and $\ddot{\mathbf{x}}_d$ are bounded.

Theorem 2 *For the dynamic system (45), if we apply a dynamic control law in inertia space*

$$\mathbf{F} = \mathbf{H}(\ddot{\mathbf{x}}_d + \mathbf{K}_d\dot{\mathbf{e}}_p + \mathbf{K}_p\mathbf{e}_p) + \mathbf{C}(\mathbf{x},\dot{\mathbf{x}})\dot{\mathbf{x}} \tag{67}$$

and the corresponding control torques in joint space is given by

$$\tau = \mathrm{N}^T\mathbf{F} = \mathrm{N}^T[\mathbf{H}(\ddot{\mathbf{x}}_d + \mathbf{K}_d\dot{\mathbf{e}}_p + \mathbf{K}_p\mathbf{e}_p) + \mathbf{C}(\mathbf{x},\dot{\mathbf{x}})\dot{\mathbf{x}}] \tag{68}$$

then the equilibrium state is asymptotically stable, if N *is of full rank.*

Proof

If we substitute (67) into (45), then we get

$$\ddot{\mathbf{e}}_p + \mathbf{K}_d \dot{\mathbf{e}}_p + \mathbf{K}_p \mathbf{e}_p = \mathbf{0} \tag{69}$$

where $\mathbf{K}_p$ and $\mathbf{K}_d$ can be so chosen that the system has the desired performance by pole assignment.

We select a Lyapunov candidate as

$$V_2(t) = \frac{1}{2}(\mathbf{e}_p^T \mathbf{e}_p + \mathbf{e}_p^T \mathbf{K}_p \mathbf{e}_p) \tag{70}$$

Differentiating it with respect to time,

$$\dot{V}_2(t) = \dot{\mathbf{e}}_p^T (\ddot{\mathbf{e}}_p + \mathbf{K}_p \mathbf{e}_p) \tag{71}$$

i.e.

$$\dot{V}_2(t) = -\dot{\mathbf{x}}^T \mathbf{K}_d \dot{\mathbf{x}} \leq 0 \tag{72}$$

$\dot{V}_2 = 0$ implies that $\dot{\mathbf{x}} = \dot{\mathbf{x}}$ and $\mathbf{x} = \mathbf{x}_t$. This proves that the equilibrium state is asymptotically stable.

□

In the above control scheme, we have incorporated complete dynamics of the system. With the knowledge of complete system dynamics, the robot can follow a given trajectory and enhance the desired motion performance. Most of existing approaches for the free-flying space robot system are kinematics-based control scheme, though they may involve the generalized Jacobian matrix which is dynamics dependent [11, 18]. The effectiveness of dynamic control can be demonstrated in simulation study of the following section.

6 Simulation Study

To better understand the space robot dynamics and demonstrate the validity of the proposed control laws, we discuss a case study in this section. The model of a two-dimensional free-flying space robot system model is shown in Figure 2. For simplicity, we locate the robot base at the centroid of the base, and links of the robot are represented by lumped mass.

The position vectors of the base, link 1 and link 2 in inertia space are denoted by $\mathbf{R}_0$, $\mathbf{R}_1$ and $\mathbf{R}_2$, respectively. The position vectors of link 1 and link 2 in the base frame are $\mathbf{r}_1$ and $\mathbf{r}_2$,

$$\mathbf{R}_0 = \begin{bmatrix} X_0 \\ Y_0 \end{bmatrix} \qquad \mathbf{R}_1 = \begin{bmatrix} X_1 \\ Y_1 \end{bmatrix} \qquad \mathbf{R}_2 = \begin{bmatrix} X_2 \\ Y_2 \end{bmatrix} \tag{73}$$

$$\mathbf{r}_1 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} r + l_1 c_1 \\ l_1 s_1 \end{bmatrix} \tag{74}$$

$$\mathbf{r}_2 = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} r + l_1 c_1 + l_2 c_{12} \\ l_1 s_1 + l_2 s_{12} \end{bmatrix} \tag{75}$$

where $c_1 = cos(q_1)$, $s_{12} = sin(q_1 + q_2)$, etc.

We further simplify the model by assuming $l_1 = l_2 = l$ and $m_1 = m_2 = m$ thereafter.

$$\mathbf{v}_1 = \dot{\mathbf{r}}_1 = \mathbf{J}_1 \dot{\mathbf{q}} \tag{76}$$

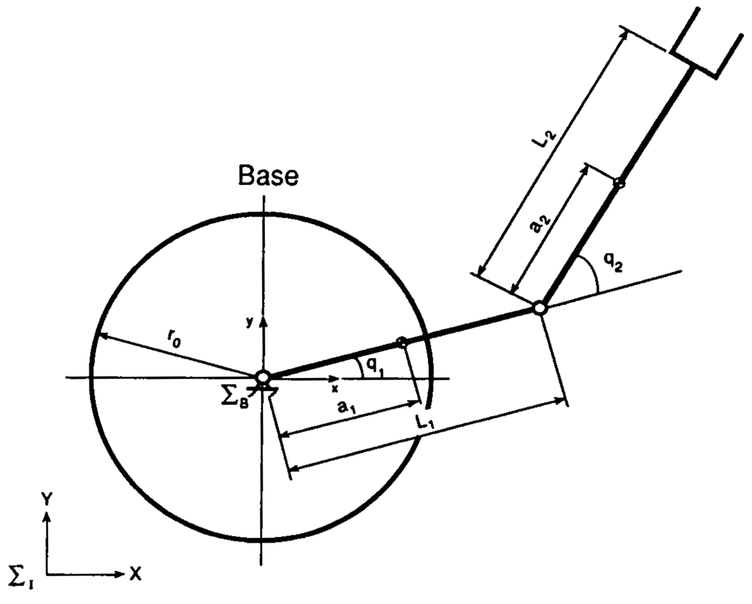

Figure 2: A free-flying planar space robot system.

$$\mathbf{v}_2 = \dot{\mathbf{r}}_2 = \mathbf{J}_2\dot{\mathbf{q}} \tag{77}$$

$$\omega_1 = \mathbf{J}_{A1}\dot{\mathbf{q}} \tag{78}$$

$$\omega_2 = \mathbf{J}_{A2}\dot{\mathbf{q}} \tag{79}$$

where

$$\dot{\mathbf{q}} = [\dot{q}_1, \dot{q}_2]^T \tag{80}$$

$$\mathbf{J}_1 = \begin{bmatrix} -ls_1 & 0 \\ lc_1 & 0 \end{bmatrix} \qquad \mathbf{J}_2 = \begin{bmatrix} -ls_1 - ls_{12} & -ls_{12} \\ lc_1 + lc_{12} & lc_{12} \end{bmatrix} \tag{81}$$

$$\mathbf{J}_{A1} = [1,0] \qquad \mathbf{J}_{A2} = [1,1] \tag{82}$$

The centroid of the whole system can be determined as follows

$$\begin{aligned} m_c &= m_0 + m + m \\ \mathbf{r}_c &= \frac{m}{m_c}\mathbf{r}_1 + \frac{m}{m_c}\mathbf{r}_2 \\ \mathbf{J}_c &= \frac{m}{m_c}\mathbf{J}_1 + \frac{m}{m_c}\mathbf{J}_2 \end{aligned}$$

The velocities of two links are

$$\begin{aligned} \mathbf{V}_1 &= \mathbf{v}_1 + \mathbf{V}_0 + \Omega_0 \times \mathbf{r}_1 \\ &= \mathbf{V}_0 + \mathbf{J}_1\dot{\mathbf{q}} - \mathbf{r}_1 \times \Omega_0 \\ \mathbf{V}_2 &= \mathbf{V}_0 + \mathbf{J}_2\dot{\mathbf{q}} - \mathbf{r}_2 \times \Omega_0 \end{aligned}$$

$$
\begin{aligned}
\Omega_1 &= \Omega_0 + \omega_1 \\
&= \Omega_0 + \mathbf{J}_{A1}\dot{\mathbf{q}} \\
\Omega_2 &= \Omega_0 + \mathbf{J}_{A2}\dot{\mathbf{q}}
\end{aligned}
$$

where $\Omega_0 = \dot{q}_0$.

Following derivations in Section 3 we have

$$
\begin{aligned}
\mathbf{H}_V &= m_c U_2 \\
\mathbf{H}_{V\Omega} &= \begin{bmatrix} -my_1 - my_2 \\ mx_1 - mx_2 \end{bmatrix} \\
H_\Omega &= I_0 + I_1 + I_2 + m(x_1^2 + y_1^2) + m(x_2^2 + y_2^2) \\
\mathbf{H}_{Vq} &= m\mathbf{J}_1 + m\mathbf{J}_2 \\
\mathbf{H}_{\Omega q} &= (I_1 + I_2, I_2) + m(-y_1, x_1)\mathbf{J}_1 + m(-y_2, x_2)\mathbf{J}_2 \\
H_B &= H_\Omega - m_c D(r_c) \\
D(r_c) &= r_{cx}^2 + r_{cy}^2 \\
\mathbf{H}_M &= [I_1 + I_2, I_2] + m[-(r_1 - r_c)_y, (r_1 - r_c)_x]\mathbf{J}_1 + m[-(r_2 - r_c)_y, (r_2 - r_c)_x]\mathbf{J}_2
\end{aligned}
$$

The generalized Jacobian can then be determined by

$$
\begin{aligned}
\mathrm{N} &= [\mathrm{N}_L, \mathrm{N}_A]^T \\
\mathrm{N}_L &= \mathbf{J} - \mathbf{J}_c - [(r_2 - r_c)_y, -(r_2 - r_c)_x]^T H_B^{-1}\mathbf{H}_M \\
\mathrm{N}_A &= \mathbf{J}_A - H_B^{-1}\mathbf{H}_M
\end{aligned}
$$

The joint space system dynamics can then be computed by using Equations (35) to (38) and the above equations, and the corresponding dynamics equation in inertia space can be obtained by (45) to (51).

Based on the above planar model, we conduct a set of simulation studies to verify our control algorithms. In the simulation we assume that $m = 50(kg)$, $l = 2.5(m)$, $a_1 = a_2 = 1.25(m)$, $I_1 = I_2 = 26(kgm^2)$ for robot parameters. Initially the coordinate $\sum_I$ coincides with $\sum_B$ and the robot hand is at (2.5,0) in inertia space. In the following discussion the mass/inertia ratio is referred to as the mass/inertia ratio of the base with respect to the robot.

The base is completely free-flying, therefore it moves and rotates in response of the robot motion. Figure 3 depicts the space robot system motion trajectory in 60 seconds when the dynamic control algorithm is applied to the system, with the mass/inertia ratio $m_0/m = I_0/I = 2$. The base exhibits not only translation but also rotation when the robot hand tracks along a specified horizontal straight-line trajectory.

Figure 4 shows position errors of the robot hand when the PD control with gains $\mathrm{K_p} = diag[400, 400]$ and $\mathrm{K_d} = diag[40, 40]$ is selected. The target trajectory for robot hand in inertia space is $X_t = 2.5 + 0.02t$, $Y_t = 0$. The mass/inertia ratio of the base with respect to the robot is 10 in this case. It is found that the simple PD control algorithm works well and the position error converges to zero. When the mass/inertia ratio is decreased, however, e.g., $m_0/m = I_0/I = 2$, the simple PD algorithm may cause system instability, which can be confirmed by Figure 5 where excessive torques is needed to correct divergent joint error due to instability. Under the exactly same condition, Figure 6 shows the required joint torques using the dynamic control algorithm, and it can be found that the torques converges as the error converges to zero. Various simulations have shown that the PD control works well when the mass/inertia ratio is not too small, and the

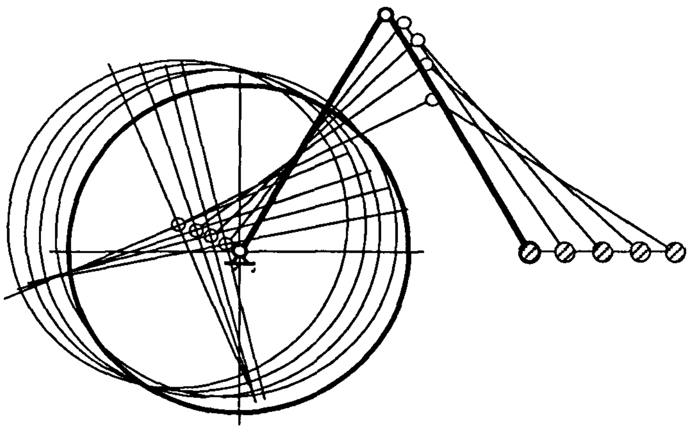

Figure 3: Motion trajectory of a planar free-flying robot system.

dynamic control algorithm present an excellent tracking performance even when the mass/inertia ratio is small.

For the case that mass/inertia ratio is large, e.g., $m_0/m = I_0/I = 100$, the required input torques are plotted in Figures 7, and 8. We observe that, though the PD algorithm works well, it requires larger joint torques and longer transient response time than dynamic control scheme.

Step input system responses are also studied. Figure 9 shows that the motion trajectories in X direction under different control algorithms. The commanded step input is from $2.5(m)$ to $3(m)$. It is found that system response (dotted line) is unstable using the PD algorithm. However, desired critical damping transient response performance can be achieved by using the dynamic control algorithm.

Physically how much mass/inertia ratio of the base to the robot is desirable and feasible, is still to be determined. When the base is a space station, the ratio has been suggested to be as low as 3. Intuitively, the lower ratio causes more significant effect of dynamic interaction on the system performance. Figure 10 shows the base rotation angles when the mass/inertia ratio is 2, 10, and 100, respectively. It is found, no surprisingly, that the less the ratio, the larger the base rotation angle as a response of the robot motion.

7 Conclusions and Future Work

The dynamic control of a space robot system with on thrust jets controlled base has been studied in this paper. We presented systematic derivations of kinematics and dynamics of the system. We found that the system dynamics can not be linearly parameterized, which makes fundamental differences on the control of a fixed-base robot and a free-flying space robot. Based on the dynamic model derived in inertia space, we proposed a PD-based control algorithm for regulation and a dynamic control algorithm for trajectory tracking applications in space. The regulation controller is simple and easy to implement. The dynamic controller provides a stable and fast system response even when the mass/inertia ratio of the base with respect to the robot is small. The property of nonlinear parameterization has been demonstrated in theory and by a case study. The validity of the proposed control algorithms are verified by simulations using a planar space robot system.

Some future research issues can be addressed as follows. First, in space applications, velocity measurements may not be available or inevitably contaminated by noise. It is therefore worthwhile to accurately estimate the velocity from direct available measurements such as position in inertia space. Some nonlinear observers, such as pseudo-linearization technique, sliding observers, and smooth nonlinear observers can be adopted to this purpose. Second, the dynamic control algorithm is computationally expensive. On-line computation of inertia matrix and inversion of generalized Jacobian is required. It can be improved by developing efficient algorithms similar to recursive N-E formulation in fixed-base. Third, it may be impossible to cancel nonlinear and time-varying effect by using the proposed control law, because an accurate estimation of **H** and **C** may not be available. In this case, adaptive control is desirable. Adaptive control algorithms have been proposed for space robot system with an attitude controlled base in [22]. For completely free-flying space robot system, however, adaptive control scheme is still demanded.

8 Appendix 1: Dynamic Equations

The dynamic equation in inertia space is developed by using Lagrangian formulation as follows.

The kinetic energy is

$$\mathbf{T} = 1/2 \sum_{i=1}^{6} \sum_{j=1}^{6} \mathbf{H}_{ij} \dot{\mathbf{x}}_i \dot{\mathbf{x}}_j \tag{83}$$

Before using the Lagrangian formulation we must compute

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\right) = \frac{d}{dt}\left(\sum_{j=1}^{6} \mathbf{H}_{ij} \dot{\mathbf{x}}_j\right) = \sum_{j=1}^{6} \mathbf{H}_{ij} \ddot{\mathbf{x}}_j + \sum_{j=1}^{6} \dot{\mathbf{H}}_{ij} \dot{\mathbf{x}}_j \tag{84}$$

where $\mathbf{H}_{ij}$ is a function of $x_1, \cdots, x_6$ (i.e., $p_x, p_y, p_z, \alpha, \beta, \gamma$), and

$$\frac{d\mathbf{H}_{ij}}{dt} = \sum_{k=1}^{6} \frac{\partial \mathbf{H}_{ij}}{\partial x_k} \frac{dx_k}{dt} = \sum_{k=1}^{6} \frac{\partial \mathbf{H}_{ij}}{\partial x_k} \dot{x}_k \tag{85}$$

and

$$\frac{\partial T}{\partial x_i} = \frac{\partial}{\partial x_i}\left(1/2 \sum_{j=1}^{6} \sum_{k=1}^{6} \mathbf{H}_{jk} \dot{\mathbf{x}}_j \dot{\mathbf{x}}_k\right) = 1/2 \sum_{j=1}^{6} \sum_{k=1}^{6} \frac{\partial \mathbf{H}_{jk}}{\partial x_i} \dot{\mathbf{x}}_j \dot{\mathbf{x}}_k \tag{86}$$

Therefore, the dynamic equations are represented by a set of equations

$$\sum_{j=1}^{6} \mathbf{H}_{ij} \ddot{x}_j + \sum_{j=1}^{6} \mathbf{C}_{ij} \dot{x}_j = \mathbf{F}_i, \qquad i = 1, \cdots, 6. \tag{87}$$

or

$$\sum_{j=1}^{6} \mathbf{H}_{ij} \ddot{x}_j + \sum_{j=1}^{6} \sum_{k=1}^{6} h_{ijk} \dot{x}_j \dot{x}_k = \mathbf{F}_i, \qquad i = 1, \cdots, 6. \tag{88}$$

where

$$h_{ijk} = \frac{\partial \mathbf{H}_{ij}}{\partial x_k} - 1/2 \frac{\partial \mathbf{H}_{jk}}{\partial x_i} \tag{89}$$

are the Christoffel coefficients. And

$$\begin{aligned}
\sum_{j=1}^{6} \mathbf{C}_{ij} &= 1/2 \sum_{j=1}^{6} \sum_{k=1}^{6} \frac{\partial \mathbf{H}_{ij}}{\partial x_k} \dot{\mathbf{x}}_j \dot{\mathbf{x}}_k + 1/2 \sum_{k=1}^{6} \sum_{j=1}^{6} \left(\frac{\partial \mathbf{H}_{ik}}{\partial x_j} - \frac{\partial \mathbf{H}_{jk}}{\partial x_i} \right) \dot{\mathbf{x}}_k \dot{\mathbf{x}}_j \\
\mathbf{C}_{ij} &= 1/2 \sum_{k=1}^{6} \frac{\partial \mathbf{H}_{ij}}{\dot{\mathbf{x}}_k} + 1/2 \sum_{k=1}^{6} \left(\frac{\partial \mathbf{H}_{ik}}{\partial x_j} - \frac{\partial \mathbf{H}_{jk}}{\partial x_i} \right) \dot{\mathbf{x}}_k \\
\dot{\mathbf{H}}_{ij} - 2\mathbf{C}_{ij} &= \sum_{k=1}^{6} \left(\frac{\partial \mathbf{H}_{ik}}{\partial x_j} - \frac{\partial \mathbf{H}_{jk}}{\partial x_i} \right) \dot{\mathbf{x}}_k \\
\dot{\mathbf{H}}_{ij} - 2\mathbf{C}_{ij} &= -(\dot{\mathbf{H}}_{ji} - 2\mathbf{C}_{ji}) \qquad (90)
\end{aligned}$$

Therefore, $\dot{\mathbf{H}} - 2\mathbf{C}$ is skew-symmetry.

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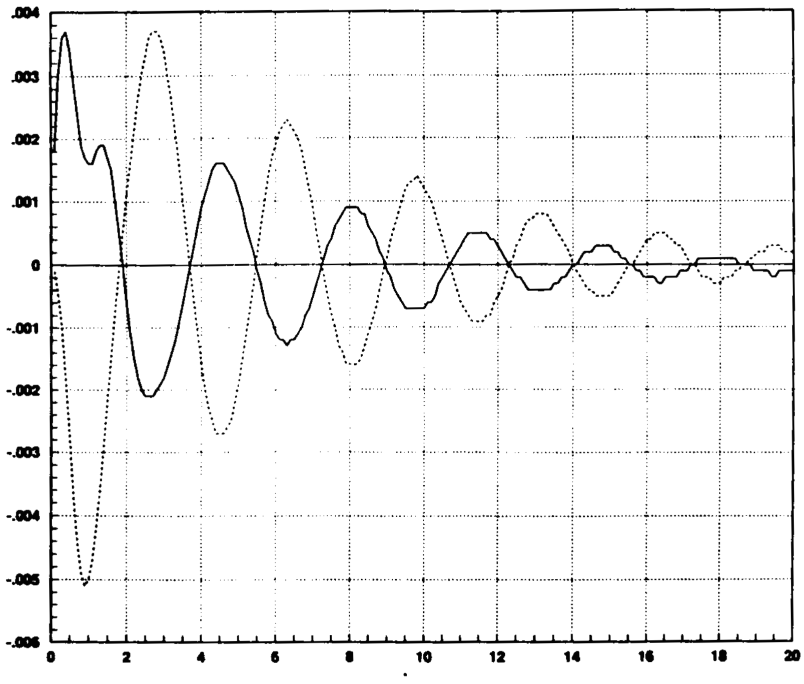

Figure 4 Position errors of the robot hand using the PD regulation control

(m_0/m_1 = 10, I_0/I_1 = 10)

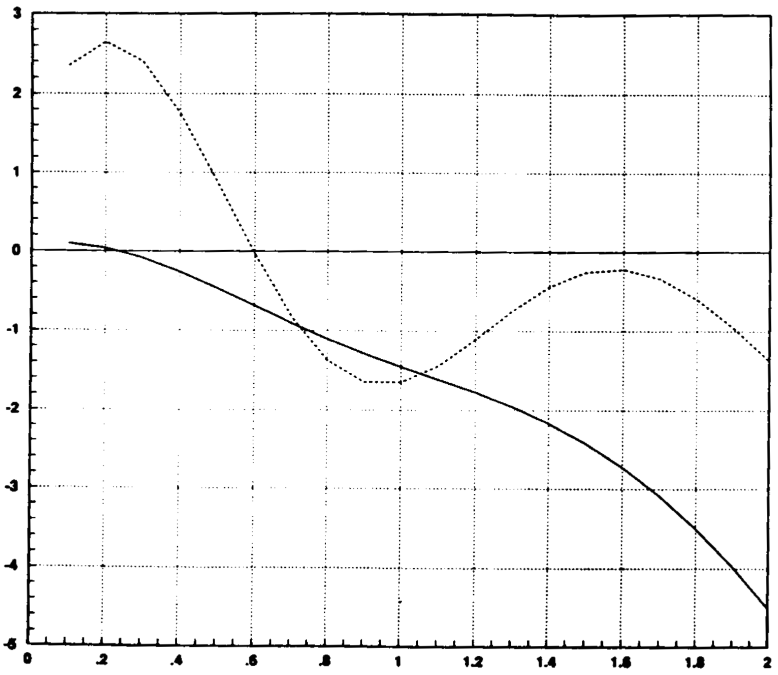

Figure 5 Input torques of robot joints using the PD regulation control

(m_0/m_1 = 2, I_0/I_1 = 2)

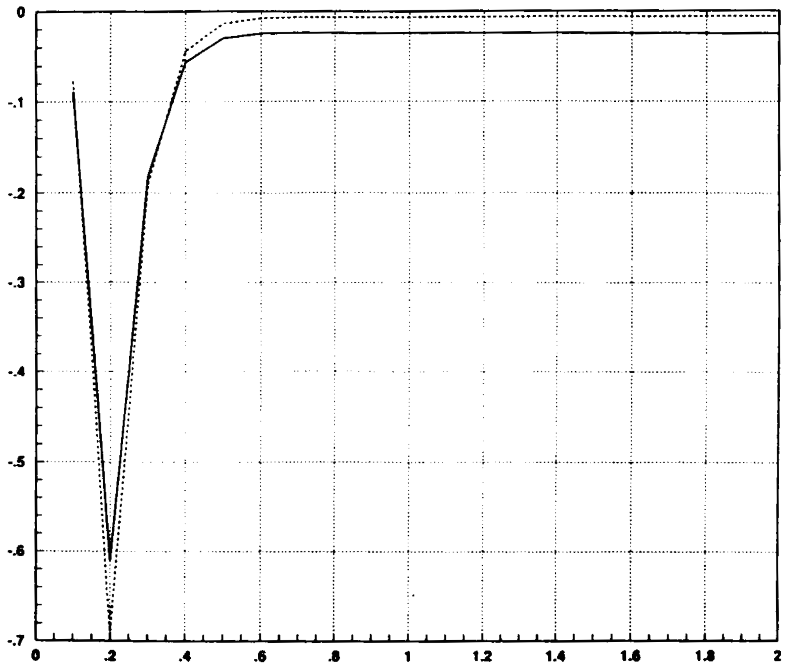

Figure 6 Input torques of robot joints using the dynamic control scheme

$(m_0/m_1 = 2, I_0/I_1 = 2)$

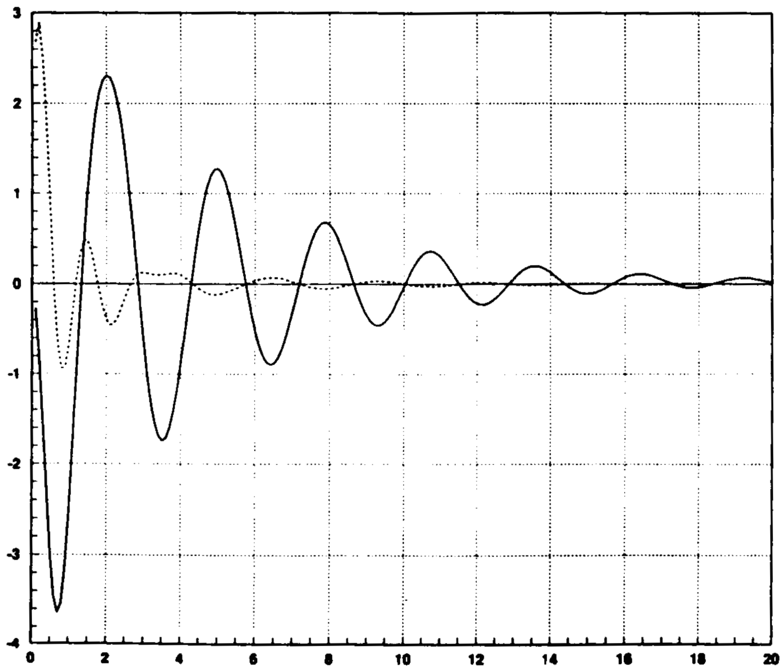

Figure 7 Input torques of robot joints using the PD regulation control

$(m_0/m_1 = 100, I_0/I_1 = 100)$

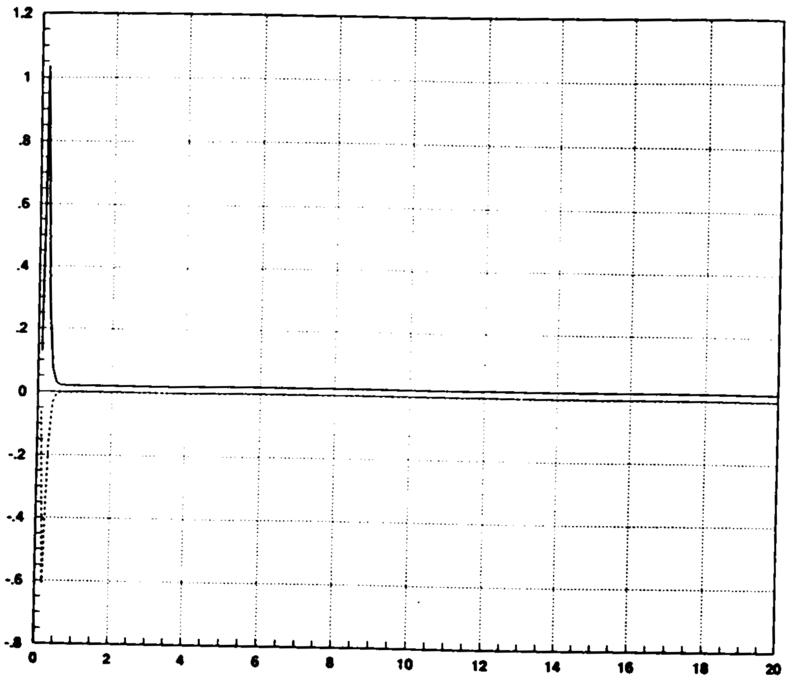

Figure 8 Input torques of robot joints using the dynamic control scheme

$(m_0/m_1 = 100, I_0/I_1 = 100)$

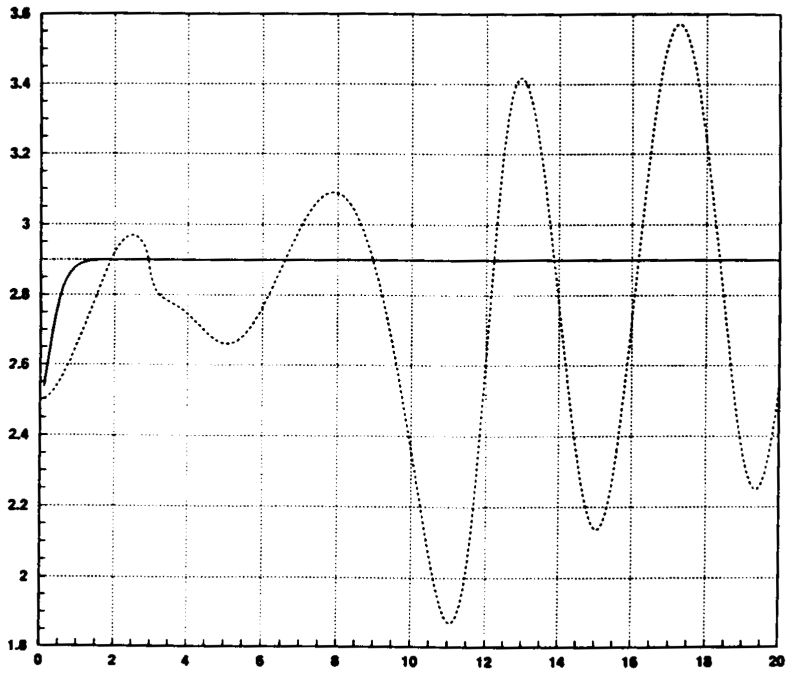

Figure 9 Step input response in position X using different control schemes

$(m_0/m_1 = 5, I_0/I_1 = 5)$

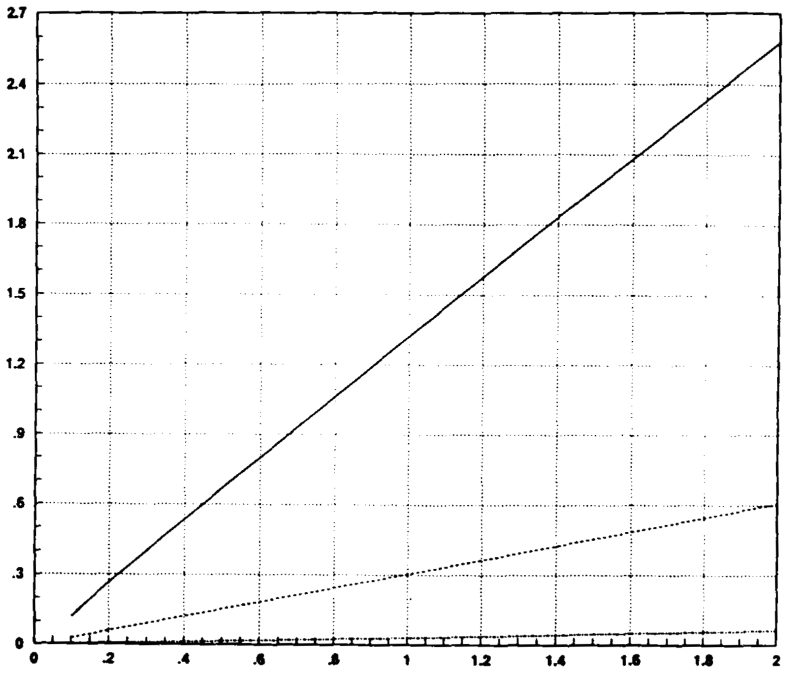

Figure 10 Base rotation angle under different mass/inertia ratios