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A SCREW DUAL QUATERNION OPERATOR FOR SERIAL ROBOT KINEMATICS

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Abstract. *In this paper we present a new operator to perform robot kinematics based on the dual quaternions algebra. This operator is equivalent to the homogeneous matrix in the Denavit-Hartenberg method (DH). Its definition follows from the same original idea of DH method, then it uses the same DH parameter table to solve robot kinematics. Denavit-Hartenberg is a very understood method in robotic community, so it is clear the applicability of the presented operator. As an illustration, the operator was applied to solve kinematics of the RRC and the anthropomorphic robots.*

Keywords: *Kinematics, Clifford Algebras, Denavit-Hartenberg, Dual Quaternions, Screw Dual Quaternion*

1. INTRODUCTION

Robot kinematics studies positions and orientations of robots. Traditionally, the kinematics of robot manipulators can be obtained by Denavit-Hartenberg method. In this way, the operators which gave the positions and orientations follows from some conventions and are represented by homogeneous matrices.

Robot kinematics is commonly divided into two problems: direct kinematics and inverse kinematics. In the direct kinematics, we have the joint variables and study the position and orientation of the robot end-effector. In the inverse kinematics, the position and orientation of the end-effector is given and the problem is to calculate the joint variables.

Dual quaternions algebra constitutes an alternative tool for kinematics of robots. The definitions, operations and properties of dual quaternions arise from a more general algebra: The *Clifford* algebra [2]. There are many ways to get the dual quaternion algebra from the Clifford algebra, and one initial problem is what representation we establish to work with them.

Dual quaternions are elements composed by scalar, vector and dual numbers. The first work applying dual numbers into kinematics is due to Yang and Freudenstein [11]. After, several authors have shown advantages using dual quaternions algebra in robot kinematics. Some of them have more interest on computational analysis [4, 5], others on singularities treatment [8]. Many other fields of applications includes quaternions and dual quaternions, like Vinde [10], who applies dual quaternions algebra on computer graphic; Dooley and McCarthy [1] apply to track planning.

In this paper we present an operator to perform robot kinematics from coordinate transformation point of view by combining screw motion, Denavit-Hartenberg method and dual quaternions algebra. Therefore, it will be easy to equate robot kinematics in dual quaternions coordinates, and the non desired problems which occurs in traditional methods can be investigated.

We structure the paper as follows: In the section 2.we present, briefly, the well-known Denavit-Hartenberg method. In the section 3.we present the quaternions and dual quaternions algebras and its representation for robot kinematics. In this section we also present the *screw dual quaternion* operator. Section 4presents the application of this operator to solve robot kinematics. In section 5.we present some conclusions and our further work ideas on dual quaternion kinematics.

2. DENAVIT-HARTENBERG METHOD

Denavit-Hartenberg method (DH method) is established from a serie of definitions and conventions. For a robot manipulator with n rotative or prismatic joints, the DH method starts with many orthogonal coordinate systems attached on the robot links and study the relations between each coordinate system, specially between the global coordinate system and the end-effector coordinate system. Let us call a orthogonal coordinate system by *frame*. The torsion angles and the displacements between frames are called DH parameters. Let $\mathcal{B}_i = \{x_i, y_i, z_i\}$ be the frame i. To the end-effector we define $\mathcal{B} = \{x_e, y_e, z_e\}$, where $x_e = x_n$, $y_e = y_n$ and $z_e = z_n$ as shown in figure 1. Traditionally, the global frame and base frame are coincident, $\mathcal{G} = \mathcal{B}_0$.

The main conventions of the DH method can be seen in [6]. After the conventions for the axis of each frame and the origins O_i and O_i', the DH parameters are defined as follow – see figure 2:

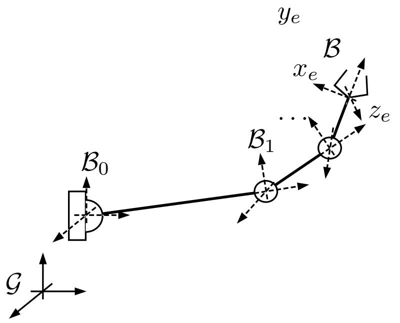

Figure 1. Frames on the robot.

- a_i: distance from O_i and O'_i measured along common normal;
- d_i: distance from O_{i-1} and O'_i measured along z_i;
- α_i: angle between axes z_{i-1} and z_i about axis x_i to be taken positive when rotation is made counter-clockwise; and
- θ_i: angle between axes x_{i-1} and x_i about axis z_{i-1} to be taken positive when rotation is made counter-clockwise.

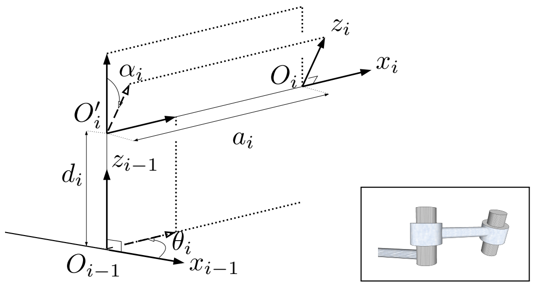

Figure 2. DH parameters.

Using the matrix algebra, the kinematic operator – the coordinate transformation operator – from frame $\mathcal{B}_i$ to frame $\mathcal{B}_{i-1}$ is

$$ {}^{i-1}H_i = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{1} $$

This matrix operator represents the posture of the frame $\mathcal{B}_i$ with respect to the frame $\mathcal{B}_{i-1}$. If ${}^{i-1}p$ represents a point in coordinate frame $\mathcal{B}_{i-1}$, then the transformation from ${}^{i}p$ to ${}^{i-1}p$ is given by ${}^{i-1}p = {}^{i-1}H_i \; {}^{i}p$. Therefore, the robot kinematics is composed by n transformations, that is,

$$ {}^{\mathcal{G}}p_e = {}^{\mathcal{G}}H_e \; {}^{e}p_e, \tag{2} $$

where ${}^{\mathcal{G}}H_e = {}^{0}H_1 \; {}^{1}H_2 \; \cdots \; {}^{n-1}H_e$ is the global operator. The coordinate systems are incorporated locally, so the initial end-effector position is always ${}^{e}p_e = [0 \;\; 0 \;\; 0 \;\; 1]^T$.

3. Screw Dual Quaternion Method

This section presents an operator to describe robot kinematics. Instead of homogeneous matrices, we use Dual Quaternions algebra (DQ) to define robot kinematics.

3.1 Quaternions and Dual Quaternions Algebras

Let $\{1, i, j, k\}$ the standard base of $\mathbb{R}^4$. Quaternions are elements of the form $q = w + x\,i + y\,j + z\,k$, where $(w, x, y, z \in \mathbb{R})$ and $i^2 = j^2 = k^2 = ijk = -1$. The quaternions space is denoted by $\mathbb{H}$. If $w = 0$, the quaternion q correspond to $3D$ vectors and it is called *pure quaternion*. Therefore, it is natural to think quaternions as the sum of scalar and vector $q = w + \mathbf{v} = Sc(q) + Ve(q)$. Let $q = w + x\,i + y\,j + z\,k = w + \mathbf{v}$ and $q_2 = w_2 + x_2\,i + y_2\,j + z_2\,k = w_2 + \mathbf{v}_2$ two quaternions. We have the following basic quaternions operations:

$$ q^* := w - \mathbf{v}, \qquad Sc(q) := \tfrac{q+q^*}{2}, \quad Ve(q) := \tfrac{q-q^*}{2}, \quad \|q\|^2 := qq^* = q^*q, $$
$$ q \pm q_2 := (w \pm w_2) + (\mathbf{v} \pm \mathbf{v}_2) \qquad q\,q_2 := (ww_2 - \mathbf{v}\cdot\mathbf{v}_2) + (w\mathbf{v}_2 + w_2\mathbf{v} + \mathbf{v}\times\mathbf{v}_2). $$

If $\xi \in \mathbb{H}$, then a rotation of ξ is given by $\xi' = q\,\xi\,q^*$, where $q = \cos\frac{\theta}{2} + \mathbf{s}\left(\sin\frac{\theta}{2}\right)$ is the rotation quaternion operator which encodes a rotation by θ around the axis represented by unit direction vector $\mathbf{s}$ [2, 7].

Dual quaternions $\mathbb{H}_2$ are elements which combine quaternions $\mathbb{H}$ and dual numbers $\mathbb{D}$. A dual number $d \in \mathbb{D}$ is an element of the form $d = a + \varepsilon\, b$, where $a, b \in \mathbb{R}$ and $\varepsilon^2 = 0$. The main representation of a dual quaternion is

$$h = w + x\,i + y\,j + z\,k + w_2\varepsilon + x_2\,i\varepsilon + y_2\,j\varepsilon + z_2\,k\varepsilon, \quad (w, x, y, z, w_2, x_2, y_2, z_2 \in \mathbb{R}).$$

but we have the generalized dual number and the generalized quaternion representations, respectively,

$$h = (w + x\,i + y\,j + z\,k) + \varepsilon(w_2 + x_2\,i + y_2\,j + z_2\,k) = q + \varepsilon\,p, \quad (q, p \in \mathbb{H})$$
$$h = (w + w_2) + (x + x_2\varepsilon)i + (y + y_2\varepsilon)j + (z + z_2\varepsilon)k = d + d_1\,i + d_2\,j + d_3\,k, \quad (d, d_1, d_2, d_3 \in \mathbb{D}).$$

Let $h = q + \varepsilon\,p$ and $h_2 = q_2 + \varepsilon\,p_2$ two dual quaternions. We have the basic dual quaternions operations:

$$h^* := q^* + \varepsilon\,p^*, \qquad \overline{h} := q - \varepsilon\,p, \qquad \overline{h^*} := q^* - \varepsilon\,p^*,$$
$$h \pm h_2 := (q \pm q_2) + \varepsilon(p \pm p_2), hh_2 := (qq_2) + \varepsilon(qp_2 + pq_2).$$

A rotation by θ with axis represented by the unit vector $\mathbf{s}$ and a translation by vector $\mathbf{t}$ are performed by the dual quaternions $h_R := \cos\frac{\theta}{2} + \mathbf{s}\left(\sin\frac{\theta}{2}\right) + \varepsilon\,0 + \varepsilon\,\mathbf{0} = q + \varepsilon\,0 + \varepsilon\,\mathbf{0}$ and $h_T := 1 + \mathbf{0} + \varepsilon\,0 + \varepsilon\frac{\mathbf{t}}{2}$. The general displacement operator (rotation follow by translation) is $h := h_T\ h_R = q + \varepsilon\,\frac{tq}{2}$, where $q = \cos\frac{\theta}{2} + \mathbf{s}\left(\sin\frac{\theta}{2}\right)$ and $t = 0 + \mathbf{t}$ are the rotation and translation vector into the quaternions q and t, respectively [2, 7].

A convenient way to represent quaternions q and dual quaternions h is to think about $q \in \mathbb{R}^4$ and $h \in \mathbb{R}^8$. So, $q = [w\ \ x\ \ y\ \ z]^T$ and $h = [w\ \ x\ \ y\ \ z\ \ w_2\ \ x_2\ \ y_2\ \ z_2]^T$ are like matrix representations of quaternions and dual quaternions, and their operations can be easily implemented.

3.2 Screw Dual Quaternion Operator and Kinematics

Let us consider a general displacement operator h. Our idea begins with the screw motion in which the translation is $\mathbf{t} = d\mathbf{s}$, where d is the distance of translation along parallel axis of rotation – screw axis. By this way the general displacement operator becomes $h = q + \varepsilon\,\frac{d}{2}\mathbf{s}q := h_{\mathbf{s}}(d, \theta)$. This is what we call the *screw dual quaternion* operator – SDQ operator.

The second step is composed by SDQ operator and DH parameters. If (d_i, θ_i) and (a_i, α_i) are the DH parameters of a robot – see figure 2, then ${}^{i-1}h_i = h_{\mathbf{z}}(d_i, \theta_i)\ h_{\mathbf{x}}(a_i, \alpha_i)$ is the composition which defines the *screw dual quaternion from DH point of view* operator – SDQ_{DH} operator.

A compact form of the SDQ_{DH} operator is given by ${}^{i-1}h_i = h_{\mathbf{z}}(\hat{\theta}_i)\ h_{\mathbf{x}}(\hat{\alpha}_i)$ where $\hat{\theta}_i = \theta_i + \varepsilon\,d_i$ and $\hat{\alpha}_i = \alpha_i + \varepsilon\,a_i$ are the dual representation of the DH parameters.

Using the Dual Quaternion product, the SDQ_{DH} operator is given by

$$ {}^{i-1}h_i = \begin{bmatrix} \cos\frac{\alpha_i}{2}\cos\frac{\theta_i}{2} \\ \sin\frac{\alpha_i}{2}\cos\frac{\theta_i}{2} \\ \sin\frac{\alpha_i}{2}\sin\frac{\theta_i}{2} \\ \cos\frac{\alpha_i}{2}\sin\frac{\theta_i}{2} \\ -\frac{a_i}{2}\ \sin\frac{\alpha_i}{2}\cos\frac{\theta_i}{2} - \frac{d_i}{2}\ \cos\frac{\alpha_i}{2}\sin\frac{\theta_i}{2} \\ \frac{a_i}{2}\ \cos\frac{\alpha_i}{2}\cos\frac{\theta_i}{2} - \frac{d_i}{2}\ \sin\frac{\alpha_i}{2}\sin\frac{\theta_i}{2} \\ \frac{a_i}{2}\ \cos\frac{\alpha_i}{2}\sin\frac{\theta_i}{2} + \frac{d_i}{2}\ \sin\frac{\alpha_i}{2}\cos\frac{\theta_i}{2} \\ -\frac{a_i}{2}\ \sin\frac{\alpha_i}{2}\sin\frac{\theta_i}{2} + \frac{d_i}{2}\ \cos\frac{\alpha_i}{2}\cos\frac{\theta_i}{2} \end{bmatrix}. \tag{3}$$

Equation (3) corresponds to equation (1) but now by means of dual quaternions algebra. The same DH parameters table of DH method can be used to establish the kinematic operators in dual quaternions coordinates. The action of the SDQ_{DH} operator is shown in figure 3:

Finally, the robot kinematics is given by

$$ {}^{\mathcal{G}}\xi_e = {}^{\mathcal{G}}h_e\ {}^{e}\xi_e\ {}^{\mathcal{G}}\widetilde{h}_e, \tag{4}$$

where ${}^{\mathcal{G}}h_e = {}^{\mathcal{G}}h_1{}^1h_2 \cdots {}^{n-1}h_e$ is the global operator, represented in global frame. ${}^{\mathcal{G}}\xi_e$ and ${}^{e}\xi_e$ are the end-effector information which is under control in global and end-effector frame. For example, ξ_e may be a vector of an orientation frame or a position vector in dual quaternion coordinates.

If ξ_e is the position of the end-effector, then ${}^{\mathcal{G}}\xi_e = [1\ \ 0\ \ 0\ \ 0\ \ 0\ \ x\ \ y\ \ z]^T$ and always ${}^{e}\xi_e = [1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0]^T$. In this case, ${}^{\mathcal{G}}\xi_e(6:8)$ is the $3D$ position vector of end-effector, given from the known join parameters.

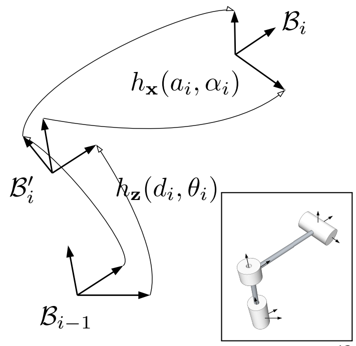

Figure 3. Coordinate transform from frame $\mathcal{B}_i$ to $\mathcal{B}_{i-1}$.

The general conjugate $\widetilde{h}$ depends on the geometrical element encoded into ξ_e [2]. We define ξ_e as a point in dual quaternion coordinates, so $\widetilde{h} = \overline{h^*}$. A full representation of geometrical elements in dual quaternions coordinates and a complete explain about their definitions and the $\widetilde{h}$ definition can be seen in Radavelli [2] and [3].

4. Applications

In this section the methods studied in Section 2and 3are applied on two robots and their results compared. We follow the traditional notations for the joints: R for rotative joints and C for cylindrical ones. Φ represents the kinematic operator free from the algebra under use, being (1) in the DH case and (3) in the DQ case.

In all cases we have the base frame $\mathcal{B}_0 = \mathcal{G}$ and the last frame (end-effector frame) $\mathcal{B}_n$=$\mathcal{B}_e$, which are related by consecutive transformations $\mathcal{G} \to \mathcal{B}_1 \to \mathcal{B}_2 \to \cdots \to \mathcal{B}$. The Φ operator is free from an algebra language. $\Phi = H$ for matrix and $\Phi = h$ for dual quaternions issues.

We perform only the position task of the robots, but if we us the axis orientation information into the dual quaternion ξ, then we also get the orientation of end-effector [2, 3].

4.1 RRC Robot

Consider the RRC robot as shown in Fig. 4, There are two rotative and one cylindrical joints – see figure 4. The DH parameters are presented in table 1.

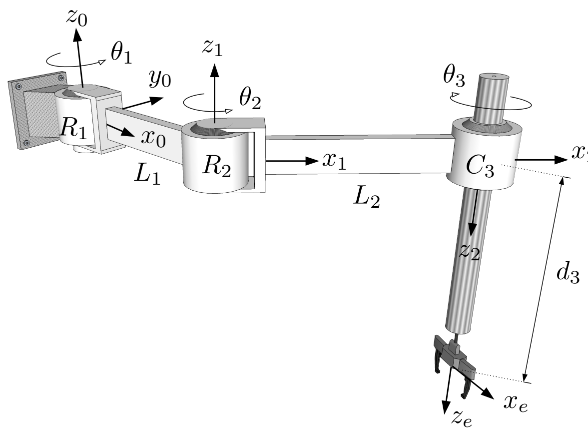

Table 1. DH parameters.

i	d_i	θ_i	a_i	α_i	Operator
1	0	θ_1	L_1	0	$\Phi(0, \theta_1, L_1, 0)$
2	0	θ_2	L_2	π	$\Phi(0, \theta_2, L_2, \pi)$
3	d_3	θ_3	0	0	$\Phi(d_3, \theta_3, 0, 0)$

Figure 4. RRC robot.

Applying DH parameters into (1), we get the homogeneous matrix operators $\Phi = {}^{i-1}H_i$:

$$
{}^{\mathcal{G}}H_1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & L_1\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & 0 & L_1\sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad {}^{1}H_2 = \begin{bmatrix} \cos\theta_2 & \sin\theta_2 & 0 & L_2\cos\theta_2 \\ \sin\theta_2 & -\cos\theta_2 & 0 & L_2\sin\theta_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

$$
{}^{2}H_e = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & 0 \\ \sin\theta_3 & \cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}.
$$

The global transformation operator, which relates the end-effector posture with global frame is

$$
{}^{\mathcal{G}}H_e = \begin{bmatrix} \cos(\theta_1+\theta_2-\theta_3) & \sin(\theta_1+\theta_2-\theta_3) & 0 & L_1\cos\theta_1 + L_2\cos(\theta_1+\theta_2) \\ -\sin(\theta_1+\theta_2-\theta_3) & \cos(\theta_1+\theta_2-\theta_3) & 0 & L_1\sin\theta_1 + L_2\sin(\theta_1+\theta_2) \\ 0 & 0 & -1 & -d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{5}
$$

From the same DH parameters shown in Tab. 1, we get the dual quaternions operators $\Phi = {}^{i-1}h_i$:

$$
{}^{\mathcal{G}}h_1 = \begin{bmatrix} \cos\frac{\theta_1}{2} \\ 0 \\ 0 \\ \sin\frac{\theta_1}{2} \\ 0 \\ \frac{L1}{2}\cos\frac{\theta_1}{2} \\ \frac{L1}{2}\sin\frac{\theta_1}{2} \\ 0 \end{bmatrix}, \quad {}^{1}h_2 = \begin{bmatrix} 0 \\ \cos\frac{\theta_2}{2} \\ \sin\frac{\theta_2}{2} \\ 0 \\ -\frac{L_2}{2}\cos\frac{\theta_2}{2} \\ 0 \\ 0 \\ -\frac{L_2}{2}\sin\frac{\theta_2}{2} \end{bmatrix}, \quad {}^{2}h_e = \begin{bmatrix} \cos\frac{\theta_3}{2} \\ 0 \\ 0 \\ \sin\frac{\theta_3}{2} \\ -\frac{d_3}{2}\sin\frac{\theta_3}{2} \\ 0 \\ 0 \\ \frac{d_3}{2}\cos\frac{\theta_3}{2} \end{bmatrix}. \tag{6}
$$

The transformation in (4) gives the end-effector position ${}^{\mathcal{G}}\xi_e$, with

$$
\begin{aligned}
{}^{\mathcal{G}}\xi_e(6) &= L_1\cos\theta_1 + L_2\cos(\theta_1+\theta_2), \\
{}^{\mathcal{G}}\xi_e(7) &= L_1\sin\theta_1 + L_2\sin(\theta_1+\theta_2), \\
{}^{\mathcal{G}}\xi_e(8) &= -d_3.
\end{aligned}
$$

These results agree with DH ones – see the position vector in last column of (5).

4.2 Anthropomorphic Arm with Spherical Wrist

The Anthropomorphic Arm with Spherical Wrist is an Anthropomorphic Arm coupled with a Spherical Wrist and has six rotative joints – see figure 5 – and the DH parameters are presented in table 2 [6].

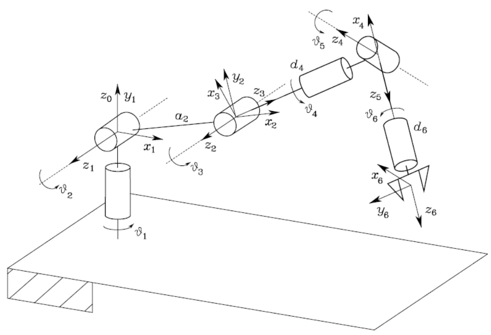

Figure 5. Anthropomorphic Arm with Spherical Wrist robot manipulator.

Table 2. DH parameters.

i	d_i	θ_i	a_i	α_i	Operator
1	0	θ_1	0	$\pi/2$	$\Phi(0,\theta_1,0,\pi/2)$
2	0	θ_2	L_2	0	$\Phi(0,\theta_2,L_2,0)$
3	0	θ_3	0	$\pi/2$	$\Phi(0,\theta_3,0,\pi/2)$
4	d_4	θ_3	0	$-\pi/2$	$\Phi(d_4,\theta_3,0,-\pi/2)$
5	0	θ_3	0	$\pi/2$	$\Phi(0,\theta_3,0,\pi/2)$
6	d_6	θ_3	0	0	$\Phi(d_6,\theta_3,0,0)$

From the DH parameters in table 2, the equations (1) and (3) give the homogeneous matrix and the dual quaternions operators, respectively. So, the anthropomorphic arm kinematics in equations (2) and (4), respectively.

Numerical results are performed. The DH numeric parameters are given as follows:

Example 1 $a_2 = 50$, $d_4 = 30$, $d_6 = 20$, $\theta_1 = 45°$, $\theta_2 = 0°$, $\theta_3 = 0°$, $\theta_4 = 0°$, $\theta_5 = 0°$, $\theta_6 = 0°$;

Example 2 $a_2 = 50$, $d_4 = 30$, $d_6 = 20$, $\theta_1 = 30°$, $\theta_2 = 30°$, $\theta_3 = 30°$, $\theta_4 = 22.8°$, $\theta_5 = 3.6°$, $\theta_6 = 0.5°$;

Example 3 $a_2 = 50$, $d_4 = 30$, $d_6 = 20$, $\theta_1 = 12°$, $\theta_2 = 77.12°$, $\theta_3 = 0°$, $\theta_4 = 89.9°$, $\theta_5 = 1.3°$, $\theta_6 = 163°$;

Example 4 $a_2 = 50$, $d_4 = 30$, $d_6 = 20$, $\theta_1 = 125°$, $\theta_2 = 160°$, $\theta_3 = 90°$, $\theta_4 = 8.1°$, $\theta_5 = 78.5°$, $\theta_6 = 23°$.

The numerical results are presented in tables 3 (DH method) and 4 (DQ method). The algebraic results we got are the same as presented by Sciavicco and Siciliano [6, chap. 2], page 78.

Table 3. DH version to the anthropomorphic arm kinematics.

	x	y	z
Example 1	$35,3553$	$35,3553$	$-50,0000$
Example 2	$75,7150$	$43,1522$	$1,0223$
Example 3	$58,6684$	$12,0065$	$37,5984$
Example 4	$51,3364$	$-68,5015$	$10,4925$

Table 4. DQ version to the anthropomorphic arm kinematics.

	x	y	z
Example 1	$35,3553$	$35,3553$	$-50,0000$
Example 2	$75,7150$	$43,1522$	$1,0223$
Example 3	$58,6684$	$12,0065$	$37,5984$
Example 4	$51,3364$	$-68,5015$	$10,4925$

5. Conclusion

In bibliography review we saw several applications of quaternions and dual quaternions. Quaternions and dual quaternions appears to be flexible to models robot kinematics, and the basic differences in each work is the strategy used to equate the kinematic operator. Unfortunately, in bibliography review some misses were observed, like the transformation of the different kind of geometrical elements in dual quaternions point of view, so as lines, screws and planes.

Dual quaternions are an alternatively tool to perform robot kinematics. A little change in the traditional theory allows us to define a new operator and get different algebraic equations than homogeneous matrices methods. From the presented operator we solve robot kinematics easily. Three examples confirm the powerful applicability of the presented operator. The anthropomorphic arm robot was solved, once we have the answer in an important bibliography, giving our method credibility. From this new kinematic equations, the inverse kinematics and the singularity problems can also be investigated.

From our main idea it is possible to develop other kinematic method, now based on successive screw method [9] and dual quaternions. As a further work, we will explain a successive screw based operator – the SDQ_{SS} operator – and the robot kinematics.

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