

### Exercise 1

A team of three differential-drive mobile robots has to reach the three goal positions  $g^{(1)} = (4, -0.5)$ ,  $g^{(2)} = (7, 0.5)$ ,  $g^{(3)} = (9, 1.5)$ , starting from  $x^{(1)} = (2, 2.5)$ ,  $x^{(2)} = (3, 2)$ ,  $x^{(3)} = (4, 2.5)$ . The scenario is shown in Figure 1. Assign a goal position to each robot considering that:

- Each robot can reach only one goal position.
- We want to minimize the total traveled distance  $d_{trav} = \sum_{i=1}^3 \|x^{(i)} - g^{(i)}\|$ , where  $g^{(i)}$  is the goal position assigned to the  $i^{\text{th}}$  robot.

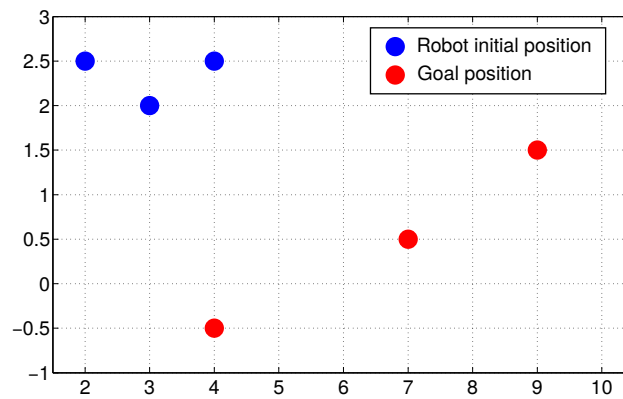


Figure 1: Robot and goal positions.

### Exercise 2

Consider 5 companies that export products ( $x_1$ ) to different countries ( $x_2$ ). In this problem, for each company  $i$ , a 2D feature vector is defined as  $\mathbf{x}^{(i)} = [x_1^{(i)} \ x_2^{(i)}]^T$ . Some observations of  $\mathbf{x}$  are summarized in the following table:

company $i$	product $x_1^{(i)}$	country $x_2^{(i)}$
1	1	1
2	2	3
3	5	7
4	6	5
5	6	7

Cluster those feature vectors using *agglomerative clustering*.

- Calculate the matrix of the initial distances between the clusters (distance of each feature vector from the others) and merge the closest clusters. Use squared Euclidean distance  $d(\mathbf{x}^{(i)}, \mathbf{x}^{(j)}) = \sum_k (x_k^{(i)} - x_k^{(j)})^2$  as metric.
- Calculate the matrix of the distances between the clusters after the first merging using the *single-linkage* algorithm.
- Draw the *Dendrogram* indicating the order in which the merging operations occur. Use the *single-linkage* algorithm.