| Lehrstuhl für STEUERUNGS- UND REGELUNGSTECHNIK<br>Technische Universität München<br>Prof. Dongheui Lee | MACHINE LEARNING IN ROBOTICS<br>Exercises: HMM |
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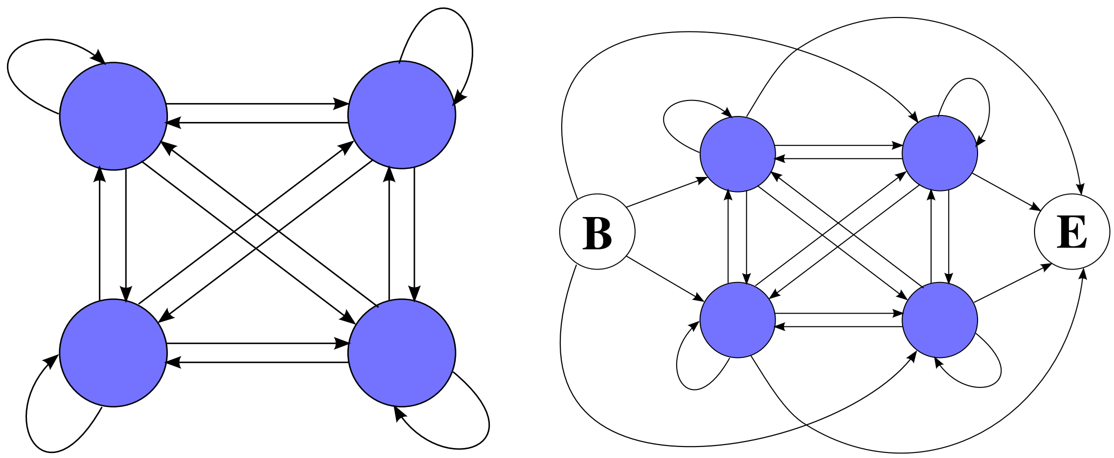
Exercise 1

Two HMMs with different structure are shown in Fig. 1.




(a) (b)

Figure 1: HMMs with different structures.

a) Consider the HMM in Fig. 1(a). Show that the sum of the probability of all possible state sequences $\mathcal{Q} = [q_1, \ldots, q_L]$ of length $L$ is equal to $1$.

b) Assume that the HMM has a begin (B) and an end (E) state, as in Fig. 1(b). The end state has probability $\varepsilon$. Show that the sum of the probability over all state sequences $\mathcal{Q} = [q_1, \ldots, q_L]$ of length $L$ (and properly terminating by making a transition to the end state) is $p = \varepsilon(1-\varepsilon)^{L-1}$. Use this result to show that the sum of the probability over all possible state sequences of any length is $1$.

   Hint: Use the result $\sum_{i=0}^{\infty} x^i = 1/(1-x)$ for $0 < x < 1$.

Exercise 2

A Hidden Markov Model with 2 states $\{s_1 = H, s_2 = C\}$ and 3 possible observations based on the number of observed sizes $\{small, medium, large\}$ is given with transition probability matrix:

$$A = \begin{pmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{pmatrix}$$

and observation matrix:

$$B = \begin{pmatrix} 0.1 & 0.4 & 0.5 \\ 0.7 & 0.2 & 0.1 \end{pmatrix}$$

The prior probabilities of states are $\pi_1 = 0.6, \pi_2 = 0.4$.

a) Compute the probability of the state sequence $\{q_1 = H \ q_2 = H \ q_3 = C \ q_4 = C\}$.

b) Compute the probability $P(o_1 = S \ o_2 = M \ o_3 = S \ o_4 = L \mid q_1 = H \ q_2 = H \ q_3 = C \ q_4 = C)$.