Lehrstuhl für STEUERUNGS- UND REGELUNGSTECHNIK Technische Universität München Prof. Dongheui Lee	**MACHINE LEARNING IN ROBOTICS** Exercises 1: Linear Regression

Exercise 1

Given the dataset shown in table 1 and illustrated in figure 1, we want to predict the output value for $x = 1$. We assume a linear regression model.

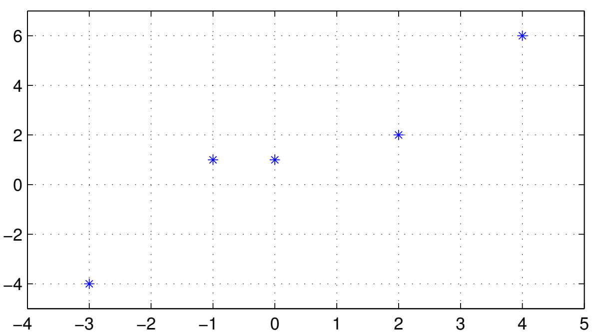

Figure 1: Training dataset

input x	-3	-1	0	2	4
output y	-4	1	1	2	6

Table 1: Data

a) Let's assume $f(x) = wx$ as a regression model with unknown parameter w. Find w which fits the data best in the sense of the Euclidean norm.

b) Let's assume $f(x) = w_0 + w_1 x$ as a regression model with unknown parameter vector $\boldsymbol{w} = [w_0 \quad w_1]^T$. By the use of the normal equation, find the best $\boldsymbol{w}$.

c) Predict the output value of the system for $x = 1$ using both regression models (a) and (b).

d) Let's assume the regression model as in (a). Now, compute the unknown parameter w by the gradient descent algorithm. Start with an initial value of $w = 0$ and use the learning rate $\alpha = 0.1$. Compute the first 2 iterations.

Exercise 2

The kinematic of a differential-drive mobile robot like that in figure 2 is described in the discrete-time by the set of equations

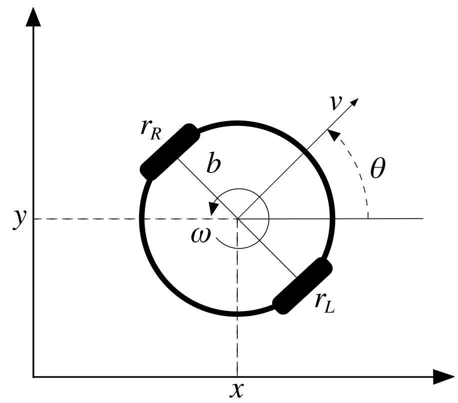

Figure 2: Top-view sketch of a differential-drive mobile robot with relevant variables.

$$\begin{cases} x^{(t+1)} = x^{(t)} + v^{(t)} cos(\theta^{(t)} + \omega^{(t)} \frac{\Delta T}{2}) \Delta T \\ y^{(t+1)} = y^{(t)} + v^{(t)} sin(\theta^{(t)} + \omega^{(t)} \frac{\Delta T}{2}) \Delta T \\ \theta^{(t+1)} = \theta^{(t)} + \omega^{(t)} \Delta T \end{cases}$$

where ΔT is the sample time. The relation between the linear v and angular ω velocities of the robot and the velocity of the wheels (ω_R and ω_L) is

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} \begin{bmatrix} \omega_R \\ \omega_L \end{bmatrix} = \boldsymbol{W} \begin{bmatrix} \omega_R \\ \omega_L \end{bmatrix}$$

Given m motion trajectories $T_r = \left[\left\{ x_1^{(t)}, y_1^{(t)}, \theta_1^{(t)}, \omega_{R,1}^{(t)}, \omega_{L,1}^{(t)} \right\}_{t=0}^{n}, \ldots, \left\{ x_m^{(t)}, y_m^{(t)}, \theta_m^{(t)}, \omega_{R,1}^{(t)}, \omega_{L,1}^{(t)} \right\}_{t=0}^{n} \right]$, estimate the unknown parameters $\boldsymbol{W}$ using least square regression. (Hint: $[w_{11}, w_{12}]$ and $[w_{21}, w_{22}]$ can be separately estimated.)

Exercise 3

Given the dataset in Exercise 1, we want to predict the output value for $x = 1$ using a quadratic regression model.

a) Let's assume $f(x) = w_1 x + w_2 x^2$ as a regression model with unknown parameter vector $\boldsymbol{w} = [w_1 \quad w_2]^T$. Find $\boldsymbol{w}$ which fits the data best in the sense of the Euclidean norm.

c) Predict the output value of the system for $x = 1$ using the regression model (a).