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| **Lehrstuhl für STEUERUNGS- UND REGELUNGSTECHNIK**<br>Technische Universität München<br>Prof. Dongheui Lee | **MACHINE LEARNING IN ROBOTICS**<br>Exercises 5: PCA and LDA |

Exercise 1

We have a mobile robot which has collected images of objects in its surrounding. The robot will use a clustering algorithm for performing groupng of similar images. Since the image data is high dimensional, a useful preprocessing step is to first project it into a lower dimension space before clustering. Now we have $d \times n$ dimensional data $X$ which has $n$ samples of $d \times 1$ dimensional vectors. These vectors correspond to image data and the dimension $d \gg n$. Now we are interested in calculating the principal components of this data. The principal components correpond to eigenvector of covariance matrix of $C$

$$C = \frac{1}{n-1} * X_c * {X_c}^\top \tag{1}$$

where $X_c$ is obtained by subtracting the mean vector from $X$. Since for large values of $d$ (for grayscale image of size $640 \times 480$, $d = 307200$), this computation can easily hang the onboard system.

A useful trick is to calculate the eigenvector of $C1$ which is $n \times n$

$$C1 = \frac{1}{n-1} * {X_c}^\top * X_c \tag{2}$$

and now if $v$ is an eigenvector of $C1$ with corresponding eigenvalue $\lambda$ then $v1 = X_c * v$ is an eigenvector of $C$ with corresponding eigenvalue $\lambda$. Show that this claim is true.

Exercise 2

Use proof by induction to show that the linear projection onto an M-dimensional subspace that maximizes the variance of the projected data is defined by the M eigenvectors of the data covariance matrix S, corresponding to the M largest eigenvalues.

Exercise 3

Calculate the LDA projection for the following dataset:
Class1 $= \{(4,1),(2,4),(2,3),(3,6),(4,4)\}$ and Class2 $= \{(9,10),(6,8),(9,5),(8,7),(10,8)\}$ and using *1-nerest neighbour classifier* on the projected data, classify the new point $(7,4)$.