Lehrstuhl für STEUERUNGS- UND REGELUNGSTECHNIK Technische Universität München Prof. Dongheui Lee	MACHINE LEARNING IN ROBOTICS Exercises 8: Non-parametric Density Estimation

Exercise 1

In density estimation with Parzen windows, we distribute hypercubes of fixed size in the dataspace and the number of samples involved in these hypercubes is estimated based on the given data distribution. Assume that an 1D dataset is observed and the number of samples per hypercube is given by $K = \sum_{i=1}^{n} k(\frac{x-x^{(i)}}{h})$ where $k(u) = \begin{cases} 1 & |u| \leq 1/2 \\ 0 & otherwise \end{cases}$

a) Prove that the pdf of the data distribution is given by $p(x) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{h}\cdot k(\frac{x-x^{(i)}}{h})$.

b) Analyze the effect that the window width has on the estimation of the function $p(x)$.

Exercise 2

Consider a histogram-like density model in which the space x is divided into fixed regions for which the density $p(x)$ takes the constant value h_k over the k^{th} region, and that the volume of region k is denoted Δ_k . Suppose we have a set of n observations of x such that n_k of these observations fall in region k. Using a Lagrange multiplier to enforce the normalization constraint on the density, derive an expression for the maximum likelihood estimator for the h_k.

Exercise 3

Show that the K-nearest-neighbour density model defines an improper distribution whose integral over all space is divergent.