Lehrstuhl für STEUERUNGS- UND REGELUNGSTECHNIK
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MACHINE LEARNING IN ROBOTICS
Exercises 4: Unsupervised Clustering

Exercise 1

A team of three differential-drive mobile robots has to reach the three goal positions $g^{(1)} = (4, -0.5),\ g^{(2)} = (7, 0.5),\ g^{(3)} = (9, 1.5)$, starting from $x^{(1)} = (2, 2.5),\ x^{(2)} = (3, 2),\ x^{(3)} = (4, 2.5)$. The scenario is shown in Figure 1. Assign a goal position to each robot considering that:

i) Each robot can reach only one goal position.

ii) We want to minimize the total traveled distance $d_{trav} = \sum_{i=1}^{3} \|x^{(i)} - g^{(i)}\|$, where $g^{(i)}$ is the goal position assigned to the i$^{\text{th}}$ robot.

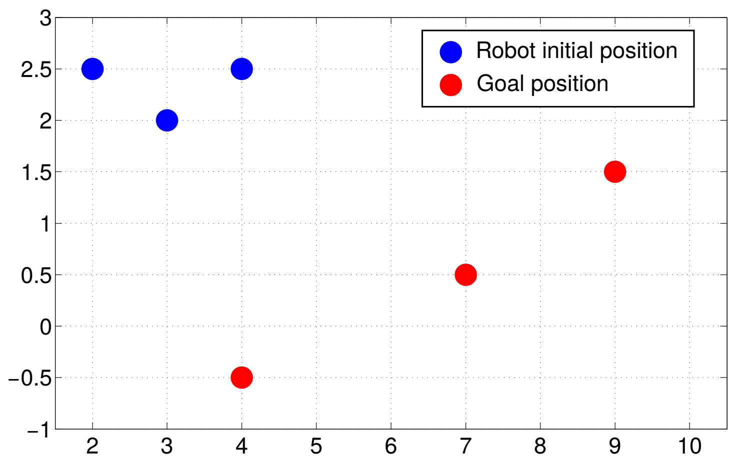

Figure 1: Robot and goal positions.

Solution Exercise 1

We first calculate the closest goal position for each robot.

$x^{(1)}$:

$$d(x^{(1)}, g^{(1)}) = \sqrt{(2-4)^2 + (2.5-(-0.5))^2} = \sqrt{13} \ \Longleftarrow$$
$$d(x^{(1)}, g^{(2)}) = \sqrt{(2-7)^2 + (2.5-0.5)^2} = \sqrt{29}$$
$$d(x^{(1)}, g^{(3)}) = \sqrt{(2-9)^2 + (2.5-1.5)^2} = \sqrt{50}$$

$x^{(2)}$:

$$d(x^{(2)}, g^{(1)}) = \sqrt{(3-4)^2 + (2+0.5)^2} = \sqrt{7.25} \ \Longleftarrow$$
$$d(x^{(2)}, g^{(2)}) = \sqrt{(3-7)^2 + (2-0.5)^2} = \sqrt{18.25}$$
$$d(x^{(2)}, g^{(3)}) = \sqrt{(3-9)^2 + (2-1.5)^2} = \sqrt{36.25}$$

$x^{(3)}$:

$$d(x^{(3)}, g^{(1)}) = \sqrt{(4-4)^2 + (2.5+0.5)^2} = \sqrt{9} \ \Longleftarrow$$
$$d(x^{(3)}, g^{(2)}) = \sqrt{4-7)^2 + (2.5-0.5)^2} = \sqrt{13}$$
$$d(x^{(3)}, g^{(3)}) = \sqrt{(4-9)^2 + (2.5-1.5)^2} = \sqrt{26}$$

As a result, we have that $g^{(1)}$ is the closest goal position to all the robot. To proceed with the assignment of the goal, we can compute the assignment goals-robots that minimizes the traveled distance.

$$(x^{(1)}, g^{(1)}) - (x^{(2)}, g^{(2)}) - (x^{(3)}, g^{(3)}) \Longrightarrow d_{trav} = \sqrt{13} + \sqrt{18.25} + \sqrt{26} = 12.98 \Longleftarrow$$
$$(x^{(1)}, g^{(1)}) - (x^{(2)}, g^{(3)}) - (x^{(3)}, g^{(2)}) \Longrightarrow d_{trav} = \sqrt{13} + \sqrt{36.25} + \sqrt{13} = 13.23$$

$$(x^{(1)}, g^{(2)}) - (x^{(2)}, g^{(1)}) - (x^{(3)}, g^{(3)}) \Longrightarrow d_{trav} = \sqrt{29} + \sqrt{7.25} + \sqrt{26} = 13.18$$
$$(x^{(1)}, g^{(2)}) - (x^{(2)}, g^{(3)}) - (x^{(3)}, g^{(1)}) \Longrightarrow d_{trav} = \sqrt{29} + \sqrt{36.25} + \sqrt{9} = 14.4$$

$$(x^{(1)}, g^{(3)}) - (x^{(2)}, g^{(1)}) - (x^{(3)}, g^{(2)}) \Longrightarrow d_{trav} = \sqrt{50} + \sqrt{7.25} + \sqrt{13} = 13.37$$
$$(x^{(1)}, g^{(3)}) - (x^{(2)}, g^{(2)}) - (x^{(3)}, g^{(1)}) \Longrightarrow d_{trav} = \sqrt{50} + \sqrt{18.25} + \sqrt{9} = 14.34$$

The best assignment is $(x^{(1)}, g^{(1)}) - (x^{(2)}, g^{(2)}) - (x^{(3)}, g^{(3)})$.

NOTE: Hierarchical clustering does not guarantee that d_{trav} is minimal.

Exercise 2

Consider 5 companies that export products (x_1) to different countries (x_2). In this problem, for each company i, a 2D feature vector is defined as $\boldsymbol{x}^{(i)} = [x_1^{(i)}\ x_2^{(i)}]^T$. Some observations of $\boldsymbol{x}$ are summarized in the following table:

company i	product $x_1^{(i)}$	country $x_2^{(i)}$
1	1	1
2	2	3
3	5	7
4	6	5
5	6	7

Cluster those feature vectors using *agglomerative clustering*.

a) Calculate the matrix of the initial distances between the clusters (distance of each feature vector from the others) and merge the closest clusters. Use squared Euclidean distance $d(\boldsymbol{x}^{(i)}, \boldsymbol{x}^{(j)}) = \sum_k (x_k^{(i)} - x_k^{(j)})^2$ as metric.

b) Calculate the matrix of the distances between the clusters after the first merging using the *single-linkage* algorithm.

c) Draw the *Dendrogram* indicating the order in which the merging operations occur. Use the *single-linkage* algorithm.

Solution Exercise 2

a) Matrix of the initial distances:

cluster	1	2	3	4	5
1	0				
2	5	0			
3	52	25	0		
4	41	20	5	0	
5	61	32	1	4	0

The clusters to merge are 3 and 5, since they are the closest ones ($d = 1$).

b) Matrix of the distances using *single-linkage*:

$$d[(3,5),1] = min\left(\|[5\ 7] - [1\ 1]\|^2,\ \|[6\ 7] - [1\ 1]\|^2\right) = 52$$
$$d[(3,5),2] = min\left(\|[5\ 7] - [2\ 3]\|^2,\ \|[6\ 7] - [2\ 3]\|^2\right) = 25$$
$$d[(3,5),4] = min\left(\|[5\ 7] - [6\ 5]\|^2,\ \|[6\ 7] - [6\ 5]\|^2\right) = 4$$

cluster	1	2	3,5	4
1	0			
2	5	0		
3,5	52	25	0	
4	41	20	4	0

Matrix of the distances using *average-linkage*:

$$d[(3,5),1] = \frac{1}{2*1}\left(\|[5\ 7] - [1\ 1]\|^2 + \|[6\ 7] - [1\ 1]\|^2\right) = 56.5$$
$$d[(3,5),2] = \frac{1}{2*1}\left(\|[5\ 7] - [2\ 3]\|^2 + \|[6\ 7] - [2\ 3]\|^2\right) = 28.5$$
$$d[(3,5),4] = \frac{1}{2*1}\left(\|[5\ 7] - [6\ 5]\|^2 + \|[6\ 7] - [6\ 5]\|^2\right) = 4.5$$

cluster	1	2	3,5	4
1	0			
2	5	0		
3,5	56.5	28.5	0	
4	41	20	4.5	0

In both cases the clusters to merge are $3, 5$ and 4, since they are the closest ones ($d = 4$ and $d = 4.5$).

c) Continuing from point b). Matrix of the distances using *single-linkage*:

$$d[(3,4,5),1] = min\left(\|[5\ 7] - [1\ 1]\|^2,\ \|[6\ 5] - [1\ 1]\|^2,\ \|[6\ 7] - [1\ 1]\|^2\right) = 41$$
$$d[(3,4,5),2] = min\left(\|[5\ 7] - [2\ 3]\|^2,\ \|[6\ 5] - [2\ 3]\|^2,\ \|[6\ 7] - [2\ 3]\|^2\right) = 20$$

cluster	1	2	3,4,5
1	0		
2	5	0	
3,4,5	41	20	0

The clusters to merge are 1 and 2, since they are the closest ones ($d = 5$).

Matrix of the distances using *single-linkage*:

$$d[(3,4,5),1] = min\,(\|[5\ 7]-[1\ 1]\|^2,\ \|[6\ 5]-[1\ 1]\|^2,\ \|[6\ 7]-[1\ 1]\|^2) = 41$$
$$d[(3,4,5),2] = min\,(\|[5\ 7]-[2\ 3]\|^2,\ \|[6\ 5]-[2\ 3]\|^2,\ \|[6\ 7]-[2\ 3]\|^2) = 20$$

cluster	1,2	3,4,5
1,2	0	
3,4,5	20	0

The clusters to merge are $1,2$ and $3,4,5$ ($d = 20$).

Dendogram is shown in Figure 2.

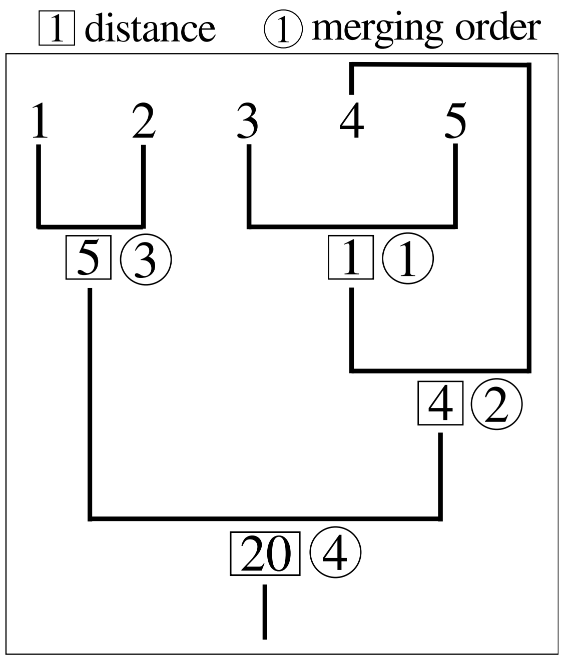

Figure 2: Dendogram.