Lehrstuhl für STEUERUNGS- UND REGELUNGSTECHNIK Technische Universität München Prof. Dongheui Lee	MACHINE LEARNING IN ROBOTICS Exercises 11: Gaussian Processes

Exercise 1

Linear regression can be transformed into a Gaussian process by assuming a Gaussian prior over the weights $p(\boldsymbol{w}) = \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma})$. Assuming that $f(\boldsymbol{x})$ is linear, i.e. $f(\boldsymbol{x}) = \boldsymbol{x}^T\boldsymbol{w}$, derive the posterior distribution over $\boldsymbol{w}$.

Exercise 2

An autonomous flying robot, while performing a transportation task, encounters a storm. The odometry module of the robot computes and transmits position values (y) every $0.2s$. Due to the electromagnetic interference of the storm, odometry data are influenced by a Gaussian noise.

Being unable to complete the task, the robot activates an emergency landing procedure. To work properly, the landing procedure requires the prediction of the robot position one step $(0.2s)$ in the future. The limited amount of memory of the robots allows to store only the last 3 positions.

To make predictions, the landing strategy uses Gaussian processes, with the radial basis kernel function:

$$k(x, x') = exp\left(-\frac{\|x - x'\|^2}{2l^2}\right) + \sigma_n^2 \delta_{xx'}$$

where x are the position sampling time instants, l defines the lengthscale of the kernel, σ_n^2 the variance of the noise component, and $\delta_{xx'} = 1$ if $x = x'$, 0 otherwise. Given the last 3 positions $\boldsymbol{y} = [1.5,\ 1,\ 0.8]$ with associated times $\boldsymbol{x} = [1,\ 1.2,\ 1.4]$, compute the position at $x^{(3)} = 1.4s$ and $x_t = 1.6s$ assuming:

a) Variance $\sigma_n^2 = 0.1$ and lengthscale $l = 0.1$.

b) Variance $\sigma_n^2 = 0.1$ and lengthscale $l = 0.001$ (close to zero).

c) Variance $\sigma_n^2 = 0.1$ and lengthscale $l = 100$ (very high).

d) State your conclusions regarding the relationship between lenghtscale and obtained results.