Precise Point Positioning with GPS and Galileo

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Final Exam

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- The use of the following material is permitted: 8 handwritten DIN A4 sheets, a formula and a non-programmable pocket calculator. All other items, including in particular books others than formula, cell phones, etc. . . are prohibited.
- Please indicate your name and registration number on all pages. The problems are described on 4 pages. Please also return those pages.
- The examination is subdivided into 6 areas. Each area typically includes several smaller problems, some of which are dependent. Problems that can be solved without requiring the solution of the previous problem are marked with an (*).
- Do as much as you can, the maximum score does not necessarily require the solution of all problems.

I hereby confirm that I have been informed prior to begin of the examination that I have to notify the examination supervisors immediately if sudden illness occurs during the examination. This will be noted in the examination protocol. An application for exam withdrawal has to be filed immediately at the board of examiners being in charge. A medical certificate from one of the physicians acknowledged by the Technische Universität München issued on the same day as the examination must be forwarded without delay. In case the examination is regularly completed despite of illness, a subsequent withdrawal due to illness cannot be accepted. In case the examination is ended due to illness it will not be graded.

Name: ______________________

Matriculation number: ______________________

Course of studies: ______________________

München, ____________ ______________________
(Date) (Signature)

Problem 1 *Area 1: Measurement models*

Precise Point Positioning using precise satellite clock and orbit corrections is becoming very attractive as it enables a precise absolute positioning without the need of raw measurements from a reference station.

a) 4 points (*). Write down a precise model for the carrier phase measurement $\lambda_m \varphi_{r,m}^k$ of receiver r observing satellite k on frequency m. Assign each term to one of the following categories:

- unknown parameters determined by PPP user
- parameters provided by satellite navigation message
- parameters provided by a model
- corrections determined by a network of geodetic reference stations.

b) 2 points (*). Double differences are widely used in geodesy for relative positioning as double differences eliminate the receiver and satellite clock offsets and biases. Why can double difference measurements not be used for absolute position determination?

Problem 2 *Joint PPP and Attitude determination*

Now, a joint PPP and attitude determination with two dual-frequency GNSS receivers shall be performed. Both GNSS receivers are mounted on a rigid body (e.g. ship or car), which is moving in a horizontal plane (i.e. the pitch angle can be neglected). Single difference measurements between a common reference satellite and each satellite are used.

a) 1 point (*). Express the single difference ambiguities $N_2 = (N_2^{12}, \ldots, N_2^{1K})^{\mathrm{T}}$ of receiver 2 in terms of the single difference ambiguities $N_1 = (N_1^{12}, \ldots, N_1^{1K})^{\mathrm{T}}$ of receiver 1 and of the double difference integer ambiguities $N_{12} = (N_{12}^{12}, \ldots, N_{12}^{1K})^{\mathrm{T}}$.

b) 2 points (*). Express the absolute position $\vec{x}_2$ of receiver 2 in terms of the absolute position $\vec{x}_1$ of receiver 1, the baseline length l between both receivers and the heading ψ.

The absolute receiver position $\vec{x}_1$ and velocity $\vec{v}_1$ of receiver 1, the heading ψ, the rate of heading $\dot{\psi}$, the satellite-satellite single difference (SD) slant ionospheric delays I^{1k}, $k \in \{2, \ldots K\}$, the tropospheric zenith delay T_{z}, the single difference ambiguities $N_1 = (N_1^{12}, \ldots, N_1^{1K})^{\mathrm{T}}$, the double difference integer ambiguities $N_{12} = (N_{12}^{12}, \ldots, N_{12}^{1K})^{\mathrm{T}}$, and the code multipath errors $\Delta\rho_{\mathrm{MP},r}^{1k}$, $r \in \{1, 2\}$, $k \in \{2, \ldots, K\}$, shall now be determined in an *extended Kalman filter*.

c) 10 points (*). In this task, a (non-linear) *measurement equation* shall be determined for three types of measurements. Each type of measurements shall be expressed in terms of $\vec{x}_1$, $\vec{v}_1$, ψ, $\dot{\psi}$, I, T_{z}, N_1, N_{12}, $\Delta\rho_{\mathrm{MP},1}^{1k}$ and $\Delta\rho_{\mathrm{MP},2}^{1k}$.

- Determine the measurement equation for the satellite-satellite single difference *pseudorange* measurements of K satellites observed by two dual-frequency GNSS receivers, and re-arrange your measurement equation to bring it into the form

$$z_n = h_n(x_n) + v_n. \tag{2-1}$$

- Determine the measurement equation for the satellite-satellite single difference *carrier phase* measurements of K satellites observed by two dual-frequency GNSS receivers, and rearrange your equation in the same manner as done for the pseudorange measurements.
- Determine the measurement equation for the satellite-satellite single difference *Doppler* measurements of K satellites observed by two dual-frequency GNSS receivers, rearrange your equation again in the same manner as done for the pseudorange measurements.

d) 6 points (*). Write down a *state space model* for determining $\vec{x}_1$, $\vec{v}_1$, ψ, $\dot{\psi}$, I, T_{z}, N_1, N_{12}, $\Delta\rho_{\mathrm{MP},1}^{1k}$ and $\Delta\rho_{\mathrm{MP},2}^{1k}$.

e) 3 points (*). Determine the *predicted* state estimate $\hat{x}_n^-$ at epoch n and also provide its covariance matrix and the residuals of the state prediction.

f) 4 points (*). Determine the *a posteriori* state estimate $\hat{x}_n^+$ at epoch n and also provide its covariance matrix and the residuals of the state update.

Hint: Perform a linearization of $h_n(x_n)$ for determining the covariance matrix.

Problem 3 — *Parameter subspace*

The least-squares estimation of all parameters is not always feasible due to a rank-deficient measurement model. In this case, a mapping from the full parameter space to a subspace is required.

a) 3 points (*). Explain to which parameters the satellite phase biases are implicitly mapped if they are not estimated as state parameters. Which property is lost by this mapping?

b) 3 points (*). The double difference ambiguity states are in general more stable than the single difference ambiguity states. Explain how the higher stability of double difference ambiguities can be exploited in a Kalman filter.

Problem 4 — *Integer ambiguity resolution*

Now, the integer property of the ambiguities shall be considered by mapping the real-valued float solution to an integer solution.

a) 3 points (*). What is the advantage of fixing the ambiguities to integers?

b) 7 points (*). Determine the orthogonal decomposition of the sum of squared errors, i.e. derive

$$\begin{aligned}\|z_n - \tilde{h}_n(\tilde{x}_n) - AN\|^2_{\Sigma^{-1}_{z_n}} &\approx \|\tilde{z}_n - H_n\tilde{x}_n - AN\|^2_{\Sigma^{-1}_{z_n}} \\ &= \|\hat{N} - N\|^2_{\Sigma^{-1}_{\hat{N}}} + \|\check{\tilde{x}}_n(N) - \tilde{x}_n\|^2_{\Sigma^{-1}_{\check{\tilde{x}}_n(N)}} + \|P_A^\perp P_H^\perp \tilde{z}_n\|^2_{\Sigma^{-1}_{z_n}}. \qquad (4\text{-}1)\end{aligned}$$

c) 2 points (*). Explain why the second and third term of Eq. (4-1) can be disregarded for ambiguity resolution.

The integer least-squares estimation of the ambiguities requires a tree search as shown in the following figure.

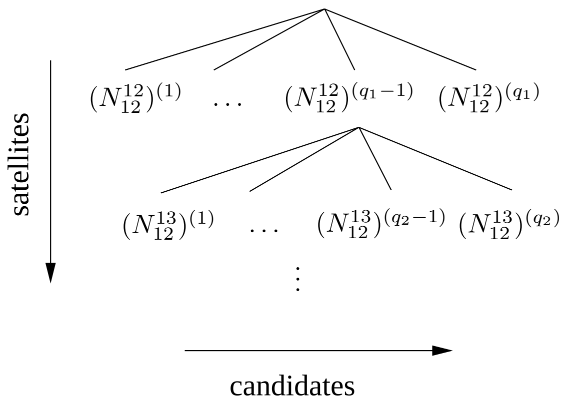

Figure 4-1: Integer tree search for ambiguity resolution

d) 7 points (*). Derive a lower and an upper bound for the search interval of the k-th ambiguity, i.e. prove

$$\begin{aligned} N^k &\leq \hat{N}^{k|1,\ldots,k-1} + \sigma_{\hat{N}^{k|1,\ldots,k-1}}\kappa \\ N^k &\geq \hat{N}^{k|1,\ldots,k-1} + \sigma_{\hat{N}^{k|1,\ldots,k-1}}\kappa, \end{aligned}$$

with

$$\kappa = \sqrt{\chi^2 - \sum_{l=1,l\neq k}^{K} \frac{(N^l - \hat{N}^{l|1,\ldots,l-1})^2}{\sigma^2_{\hat{N}^{l|1,\ldots,l-1}}}}.$$

Hint: Use the triangular decomposition of the sum of squared ambiguity residuals

$$\|\hat{N} - N\|^2_{\Sigma^{-1}_{\hat{N}}} = \sum_{k=1}^{K} \frac{(N^k - \hat{N}^{k|1,\ldots,k-1})^2}{\sigma^2_{\hat{N}^{k|1,\ldots,k-1}}}.$$

The Best Integer Equivariant (BIE) estimator is also an attractive alternative to the conventional integer least-squares estimation. The BIE estimate of the ambiguities is given by

$$\check{\boldsymbol{N}}_{\mathrm{BIE}} = \sum_{\boldsymbol{z}\in\mathcal{Z}^n} \boldsymbol{z} w_{\boldsymbol{z}}(\hat{\boldsymbol{N}})$$

with

$$w_{\boldsymbol{z}}(\hat{\boldsymbol{N}}) = \frac{\exp\left(-\frac{1}{2}\|\hat{\boldsymbol{N}} - \boldsymbol{z}\|^2_{Q^{-1}_{\hat{N}}}\right)}{\sum_{\boldsymbol{z}'\in\mathcal{Z}^n} \exp\left(-\frac{1}{2}\|\hat{\boldsymbol{N}} - \boldsymbol{z}\|^2_{Q^{-1}_{\hat{N}}}\right)}.$$

e) 2 points (*). Explain the advantages and disadvantages of the BIE estimator over the classical integer least-squares estimator.

Problem 5 — *Linear Combinations*

a) 2 points (*). Explain the basic idea of a linear combination: How is it formed, which input is used, what is the aim?

b) 2 points (*). What are important properties of the Melbourne-Wübbena linear combination?

Widelane combinations are used to increase the wavelength-to-noise ratio and, thereby, to simplify ambiguity resolution.

c) 8 points (*). Derive the coefficients of a dual frequency geometry-preserving, ionosphere-free code carrier widelane combination, that combines the ambiguities to $N_{\mathrm{WL}} = N_1 - N_2$ and minimizes the noise variance.

Problem 6 — *Sensor fusion*

The reliability of GNSS-based kinematic PPP can be improved by an accelerometer and a gyroscope, which do not require open sky conditions and whose signals are not affected by multipath. Moreover, these inertial sensors measure with a higher rate of up to 1000 Hz. However, sensor biases and misalignment errors for each sensor in each axis have to be determined with the help of GNSS.

a) 3 points (*). Provide a precise model for the three-dimensional measurement of an accelerometer.

b) 2 points (*). Provide a precise model for the three-dimensional measurement of a gyroscope.

c) 2 points (*). Provide a precise model for the three-dimensional magnetic flux measurement.

d) 4 points (*). Now, sat.-sat. single difference carrier phase, pseudorange and Doppler measurements shall be combined with acceleration, angular rotation rate and magnetic field measurements in a tight coupling. Define a state vector for joint PPP and attitude determination.