

Doppler-Aided Positioning

Improving Single-Frequency RTK in the Urban Environment

Mojtaba Bahrami and Marek Ziebart

WHAT DO A GPS RECEIVER, a policeman's speed gun, a weather radar, and some medical diagnostic equipment have in common? Give up? They all make use of the Doppler effect. First proposed in 1842 by the Austrian mathematician and physicist, Christian Doppler, it is the change in the perceived frequency of a wave when the transmitter and the receiver are in relative motion.

Doppler introduced the concept in an attempt to explain the shift in the color of light from certain binary stars. Three years later, the effect was tested for sound



INNOVATION INSIGHTS
with Richard Langley

Doppler measurements can be used to smooth pseudoranges.

waves by the Dutch scientist Christophorus Buys Ballot. We have all heard the Doppler shift of a train whistle or a siren with their descending tones as the train or emergency vehicle passes us. Doctors use Doppler sonography — also known as Doppler ultrasound — to provide information about the flow of blood and the movement of inner areas of the body with the moving reflectors changing the received ultrasound frequencies. Similarly, some speed guns use the Doppler effect to measure the speed of vehicles or baseballs and Doppler weather radar measures the relative velocity of particles in the air.

The beginning of the space age heralded a new application of the Doppler effect. By

measuring the shift in the received frequency of the radio beacon signals transmitted by Sputnik I from a known location, scientists were able to determine the orbit of the satellite. And shortly thereafter, they determined that if the orbit of a satellite was known, then the position of a receiver could be determined from the shift. That realization led to the development of the United States Navy Navigation Satellite System, commonly known as Transit, with the first satellite being launched in 1961. Initially classified, the system was made available to civilians in 1967 and was widely used for navigation and precise positioning until it was shut down in 1996. The Soviet Union developed a similar system called Tsikada and a special military version called Parus. These systems are also assumed to be no longer in use — at least for navigation.

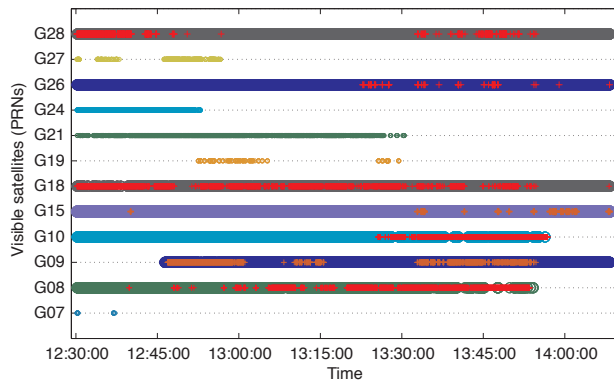
GPS and other global navigation satellite systems use the Doppler shift of the received carrier frequencies to determine the velocity of a moving receiver. Doppler-derived velocity is far more accurate than that obtained by simply differencing two position estimates. But GPS Doppler measurements can be used in other ways, too. In this month's column, we look at how Doppler measurements can be used to smooth noisy code-based pseudoranges to improve the precision of autonomous positioning as well as to improve the availability of single-frequency real-time kinematic positioning, especially in urban environments.

"Innovation" is a regular column that features discussions about recent advances in GPS technology and its applications as well as the fundamentals of GPS positioning. The column is coordinated by Richard Langley of the Department of Geodesy and Geomatics Engineering at the University of New Brunswick, who welcomes your comments and topic ideas. To contact him, see the "Contributing Editors" section on page 6.

Real-time kinematic (RTK) techniques enable centimeter-level, relative positioning. The technology requires expensive, dedicated, dual-frequency, geodetic-quality receivers. However, myriad industrial and engineering applications would benefit from small-size, cost-effective, single-frequency, low-power, and high-accuracy RTK satellite positioning. Can such a sensor be developed and will it deliver? If feasible, such an instrument would find many applications within urban environments — but here the barriers to success are higher. In this article, we show how some of the problems can be overcome.

Single-Frequency RTK

Low-cost single-frequency (L1) GPS receivers have attained mass-market status in the consumer industry. Notwithstanding current levels of maturity in GPS hardware and algorithms, these receivers still suffer from large positioning errors. Any positioning accuracy improvement for mass-market receivers is of great practical importance, especially for many applications demanding small size, cost-effectiveness, low power consumption, and highly accurate GPS positioning and navigation. Examples include mobile mapping technology; machine control; agriculture fertilization and yield monitoring; forestry; utility services; intelligent transportation systems; civil engineering projects; unmanned aerial vehicles; automated continuous monitoring of landslides, avalanches, ground subsidence, and river level; and monitoring deformation of built structures. Moreover, today an ever-increasing number of smartphones and handsets come equipped with a GPS receiver. In those devices, the increasing sophistication of end-user applica-



▲ **FIGURE 1** Satellite tracking and carrier-phase anomaly summary during the observation time-span. These data were collected in a dense urban environment in both static and kinematic mode. The superimposed red-points show epochs where carrier-phase observables are either missing or contaminated with cycle slips, loss of locks, and/or other measurement anomalies.

tions and refinement of map databases are continually tightening the accuracy requirements for GPS positioning.

For single-frequency users, the RTK method does appear to offer the promise of highly precise position estimates for stationary and moving receivers and can even be considered a candidate for integration within mobile handhelds. Moreover, the RTK approach is attractive because the potential of the existing national infrastructures such as Great Britain's Ordnance Survey National GNSS Network-RTK (OSNet) infrastructure, as well as enabling technologies such as the Internet and the cellular networks, can be exploited to deliver RTK corrections and provide high-precision positioning and navigation.

The basic premise of relative (differential) positioning techniques such as RTK is that many of the sources of GNSS measurement errors including the frequency-dependent error (the ionospheric delay) are spatially correlated. By performing relative positioning between receivers, the correlated measurement errors are completely cancelled or greatly reduced, resulting in a significant increase in the positioning accuracy and precision.

Single-Frequency Challenges. Although RTK positioning is a well-established and routine technology, its effective implementation for low-cost, single-frequency L1 receivers poses many serious challenges, especially in difficult and degraded signal environments for GNSS such as urban canyons. The most serious challenge is the use of only the L1 frequency for carrier-phase integer ambiguity resolution and validation. Unfortunately, users with single-frequency capability do not have frequency diversity and many options in forming useful functions and combinations for pseudorange and carrier-phase observables. Moreover, observations from a single-frequency, low-cost receiver are typically "biased" due to the high level of multipath and/or receiver signal-tracking anomalies and also the low-cost patch antenna design that is typically used. In addition, in those receivers, measurements are typically contaminated with high levels of noise due to the

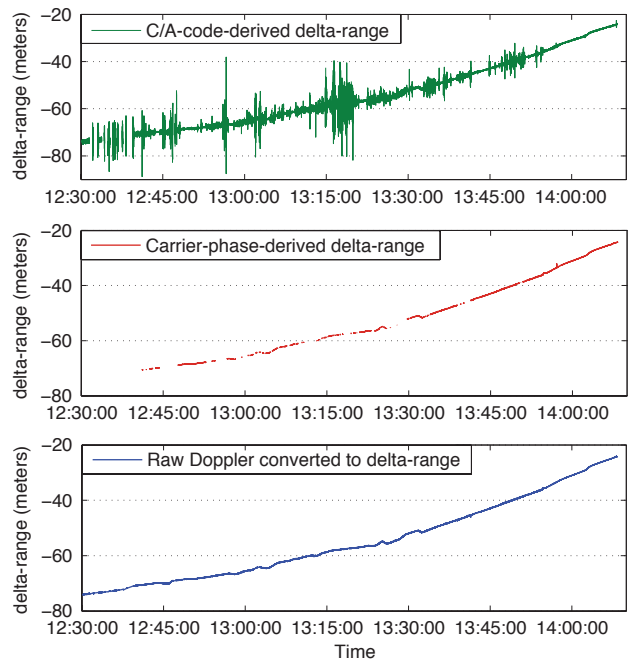
low-cost hardware design compared to the high-end receivers. This makes the reliable fixing of the phase ambiguities to their correct integer values, for single-frequency users, a non-trivial problem. As a consequence, the reliability of single-frequency observations to resolve ambiguities on the fly in an operational environment is relatively low compared to the use of dual-frequency observations from geodetic-quality receivers. Improving performance will be difficult, unless high-level noise and multipath can be dealt with effectively or unless ambiguity resolution techniques can be devised that are more robust and are less sensitive to the presence of biases and/or high levels of noise in the observations.

Traditionally, single-frequency RTK positioning requires long uninterrupted initialization times to obtain reliable results, and hence have a time-to-fix ambiguities constraint. Times of 10 to 25 minutes are common. Observations made at tens of continuous epochs are used to determine reliable estimates of the integer phase ambiguities. In addition, these continuous epochs must be free from cycle slips, loss of lock, and interruptions to the carrier-phase signals for enough satellites in view during the ambiguity fixing procedure. Otherwise, the ambiguity resolution will fail to fix the phase ambiguities to correct integer values. To overcome these drawbacks and be able to determine the integer phase ambiguities and thus the precise relative positions, observations made at only one epoch (single-epoch) can be used in resolving the integer phase ambiguities. This allows instantaneous RTK positioning and navigation for single-frequency users such that the problem of cycle slips, discontinuities, and loss of lock is eliminated. However, for single-frequency users, the fixing of the phase ambiguities to their correct integer values using a single epoch of observations is a non-trivial problem; indeed, it is considered the most challenging scenario for ambiguity resolution at the present time.

Instantaneous RTK positioning relies fundamentally upon the inversion of both carrier-phase measurements and code measurements (pseudoranges) and successful instantaneous ambiguity resolution. However, in this approach, the probability of fixing ambiguities to correct integer values is dominated by the relatively imprecise pseudorange measurements. This is more severe in urban areas and difficult environments where the level of noise and multipath on pseudoranges is high. This problem may be overcome partially by carrier smoothing of pseudoranges in the range/measurement domain using, for example, the Hatch filter. While carrier-phase tracking is continuous and free from cycle slips, the carrier smoothing of pseudoranges with an optimal smoothing filter window-width can effectively suppress receiver noise and short-term multipath noise on pseudoranges. However, the effectiveness of the conventional range-domain carrier-smoothing filters is limited in urban areas and difficult GNSS environments because carrier-phase measurements deteriorate easily and substantially due to blockages and foliage and

suffer from phase discontinuities, cycle-slip contamination, and other measurement anomalies. This is illustrated in FIGURE 1. The figure shows that in a kinematic urban environment, frequent carrier-phase outages and anomalies occur, which cause frequent resets of the carrier-smoothing filter and hence carrier smoothing of pseudoranges suffers in robustness and effective continuous smoothing.

Doppler Frequency Shift. While carrier-phase tracking can be discontinuous in the presence of continuous pseudoranges, a receiver generates continuous Doppler-frequency-shift measurements. The Doppler measurements are immune to cycle slips. Moreover, the precision of the Doppler measurements is better than the precision of pseudoranges because the absolute multipath error of the Doppler observable is only a few centimeters. Thus, devising methods that utilize the precision of raw Doppler measurements to reduce the receiver noise and high-frequency multipath on pseudoranges may prove valuable especially in GNSS-challenged environments. FIGURE 2 shows an example of the availability and the precision of the receiver-generated Doppler measurements alongside the delta-range values derived from the C/A-code pseudoranges and from the L1 carrier-phase measurements. This figure also shows that frequent carrier-phase outages and anomalies occur while for every C/A-code pseudorange measurement there is a corresponding Doppler measure-



▲ FIGURE 2 Plots of C/A-code-pseudorange-derived delta-ranges (top), L1 carrier-phase-derived delta-ranges (middle), and L1 raw receiver-generated Doppler shifts that are transformed into delta-ranges for the satellite PRN G18 during the observation time-span when it was tracked by the receiver (bottom).



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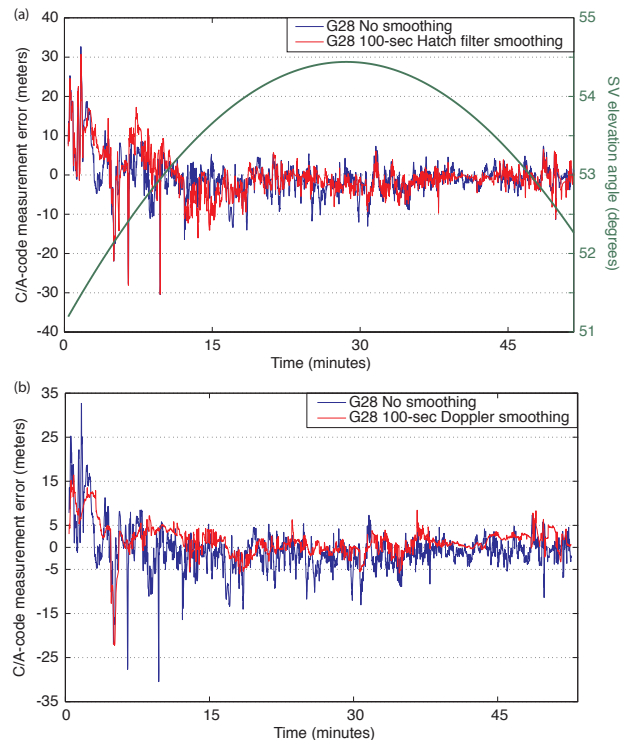
Smoothing. A rich body of literature has been published exploring aspects of carrier smoothing of pseudoranges. One factor that has not received sufficient study in the literature is utilization of Doppler measurements to smooth pseudoranges and to investigate the influence of improved pseudorange accuracy on both positioning and the integer-ambiguity resolution. Utilizing the Doppler measurements to smooth pseudoranges could be a good example of an algorithm that maximally utilizes the information redundancy and diversity provided by a GPS/GNSS receiver to improve positioning accuracy. Moreover, utilizing the Doppler measurements does not require any hardware modifications to the receiver. In fact, receivers measure Doppler frequency shifts all the time as a by-product of satellite tracking.

GNSS Doppler Measurement Overview

The Doppler effect is the apparent change in the transmission frequency of the received signal and is experienced whenever there is any relative motion between the emitter and receiver of wave signals. Theoretically, the observed Doppler frequency shift, under Einstein's Special Theory of Relativity, is approximately equal to the difference between the received and transmitted signal frequencies, which is approximately proportional to the receiver-satellite topocentric range rate.

Beat Frequency. However, the transmitted frequency is replicated locally in a GNSS receiver. Therefore, strictly speaking, the difference of the received frequency and the receiver locally generated replica of the transmitted frequency is the Doppler frequency shift that is also termed the beat frequency. If the receiver oscillator frequency is the same as the satellite oscillator frequency, the beat frequency represents the Doppler frequency shift due to the relative, line-of-sight motion between the satellite and the receiver. However, the receiver internal oscillator is far from being perfect and therefore, the receiver Doppler measurement output is the apparent Doppler frequency shift (that includes local oscillator effects). The Doppler frequency shift is also subject to satellite-oscillator frequency bias and other disturbing effects such as atmospheric effects on the signal propagation.

To estimate the range rate, a receiver typically forms an average of the delta-range by simply integrating the Doppler over a very short period of time (for example, 0.1 second) and then dividing it by the duration of the integration interval. Since the integration of frequency over time gives the phase of the signal over that time interval, the procedure continuously forms the carrier-phase observable that is the integrated Doppler over time. Therefore, Doppler frequency shift can also be estimated by time differencing carrier-phase measurements. The carrier-phase-derived Doppler is computed over a longer time span, leading to smoother Doppler measurements, whereas direct loop filter output is an



▲ FIGURE 3 Smoothed pseudorange errors (observed range deviations) using the traditional Hatch carrier-smoothing filter (a) and the Doppler-smoothing filter (b). Smoothing filter length in the experiments for both filters was set to 100 seconds. Satellite PRN G28 was chosen to represent a satellite at medium-to-high elevation angle.

instantaneous measure produced over a short time interval.

Doppler frequency shift is routinely used to determine the satellite or user velocity vector. Apart from velocity determination, it is worth mentioning that Doppler frequency shifts are also exploited for coarse GPS positioning. Moreover, the user velocity vector obtained from the raw Doppler frequency shift can be and has been applied by a number of researchers to instantaneous RTK applications to constrain the float solution and hence improve the integer-ambiguity-resolution success rates in kinematic surveying. In this article, a simple combination procedure of the noisy pseudorange measurements and the receiver-generated Doppler measurements is suggested and its benefits are examined.

Doppler-Smoothing Algorithm Description

Motivated both by the continual availability and the centimeter-level precision of receiver-generated (raw) Doppler measurements, even in urban canyons, a method has been introduced by the authors that utilizes the precision of raw Doppler measurements to reduce the receiver noise and high-frequency multipath on code pseudoranges. For more detail on the Doppler-smoothing technique, see Further Reading. The objective is to smooth the pseudoranges and push the accuracy of the code-based or both code- and carrier-based positioning applications in GNSS-challenged environments.

Previous work on Doppler-aided velocity/position algorithms is mainly in the position domain. In those approaches,

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the improvement in the quality of positioning is gained mainly by integrating the kinematic velocities and accelerations derived from the Doppler measurement in a loosely coupled extended Kalman filter or its variations such as the complementary Kalman filter. Essentially, these techniques utilize the well-known ability of the Kalman filter to use independent velocity estimates to reduce the noise of positioning solutions and improve positioning accuracy. The main difference among these position-domain filters is that different receiver dynamic models are used.

The proposed method combines centimeter-level precision receiver-generated Doppler measurements with pseudorange measurements in a combined pseudorange measurement that retains the significant information content of each.

Two-Stage Process. The proposed Doppler-smoothing process has two stages: (1) the prediction or initialization stage and (2) the filtering stage. In the prediction stage, a new estimated smoothed value of the pseudorange measurement for the Doppler-smoothing starting epoch is obtained. In this stage, for a fixed number of epochs, a set of estimated pseudoranges for the starting epoch is obtained from the subsequent pseudorange and Doppler measurements. The estimated pseudoranges are then averaged to obtain a good estimated starting point for the smoothing process. The number of epochs used in the prediction stage is the averaging window-width or Doppler-smoothing-filter length. In the filtering stage, the smoothed pseudorange profile is constructed using the estimated smoothed starting pseudorange and the integrated Doppler measurements over time. The Doppler-smoothing procedures outlined here can be performed successively epoch-by-epoch (that is, in a moving filter), where the estimated initial pseudorange (the averaged pseudorange) is updated from epoch to epoch. Alternatively, an efficient and elegant implementation of the measurement-domain Doppler-smoothing method is in terms of a Kalman filter, where it can run as a continuous process in the receiver from the first epoch (or in post-processing software, but then without the real-time advantage). This filter allows real-time operation of the Doppler-smoothing approach.

In the experiments described in this article, a short filter window-width is used. The larger the window width used in the averaging filter process, the more precise the averaged pseudorange becomes. However, this filter is also susceptible to the ionospheric divergence phenomenon because of the opposite signs of the ionospheric contribution in the pseudorange and Doppler observables. Therefore, the ionospheric divergence effect between pseudoranges and Doppler observables increases with averaging window-width and the introduced bias in the averaged pseudoranges become apparent for longer filter lengths.

Using the propagation of variance law, it can be shown that the precision of the delta-range calculated with the integrated Doppler measurements over time depends on both

the Doppler-measurement epoch interval and the precision of the Doppler measurements, assuming that noise/errors on the measurements are uncorrelated.

Experimental Results

To validate the improvement in the performance and availability of single-frequency instantaneous RTK in urban areas, the proposed Doppler-aided instantaneous RTK technique has been investigated using actual GPS data collected in both static and kinematic pedestrian trials in central London. In this article, we only focus on the static results and the kinematic trial results are omitted. It is remarked, however, that the data collected in the static mode were post-processed in an epoch-by-epoch approach to simulate RTK processing.

In the static testing, GPS test data were collected with a measurement rate of 1 Hz. At the rover station, a consumer-grade receiver with a patch antenna was used. This is a single-frequency 16-channel receiver that, in addition to the C/A-code pseudoranges, is capable of logging carrier-phase measurements and raw Doppler measurements. Reference station data were obtained from the Ordnance Survey continuously operating GNSS network. Three nearby reference stations were selected that give different baseline lengths: Amersham (AMER) \approx 38.3 kilometers away, Teddington (TEDD) \approx 20.8 kilometers away, and Stratford (STRA) \approx 7.1 kilometers away. In addition, a virtual reference station (VRS) was also generated in the vicinity (60 meters away) of the rover receiver.

Doppler-Smoothing. Before we present the improvement in the performance of instantaneous RTK positioning, the effect of the Doppler-smoothing of the pseudoranges in the measurement domain and comparison with carrier-phase smoothing of pseudoranges is given. To do this, we computed the C/A-code measurement errors or observed range deviations (the differences between the expected and measured pseudoranges) in the static mode (with surveyed known coordinates) using raw, Doppler-smoothed and carrier-smoothed pseudoranges. **FIGURE 3a** illustrates the effect of 100-second Hatch-filter carrier smoothing and **FIGURE 3b** shows a 100-second Doppler-smoothing of the pseudoranges for satellite PRN G28 (RINEX satellite designator) with medium-to-high elevation angle. The raw observed pseudorange deviations (in blue) are also given as reference. The quasi-sinusoidal oscillations are characteristic of multipath. Comparing the Doppler-smoothing in Figure 3b to the Hatch carrier-smoothing in Figure 3a, it can be seen that Doppler-smoothing of pseudoranges offers a modest improvement and is more robust and effective than that of the traditional Hatch filter in difficult environments.

FIGURE 4a illustrates carrier-phase Hatch-filter smoothing for low-elevation angle satellite PRN G18. In this figure, the Hatch carrier-smoothing filter reset is indicated. It can be seen that due to the frequent carrier-phase discontinuities

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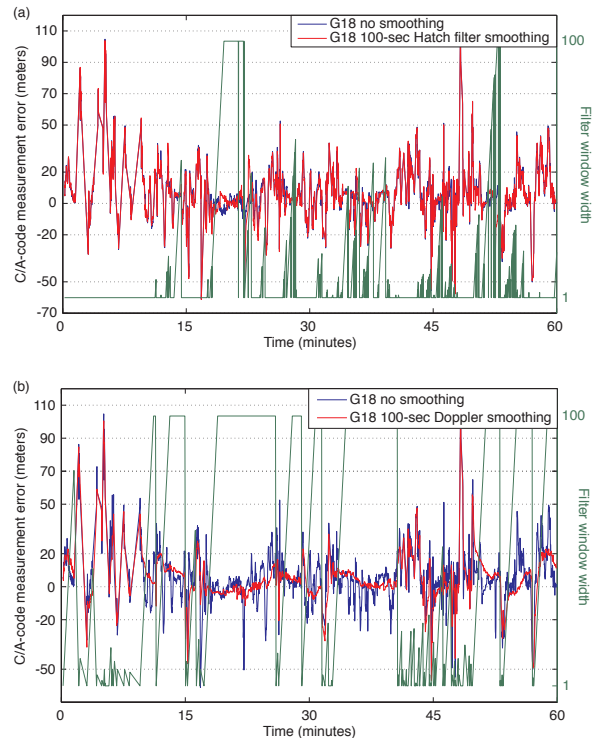
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and cycle slips, the smoothing has to be reset and restarted from the beginning and hardly reaches its full potential. In contrast, Doppler smoothing for PRN G18 shown in **FIGURE 4b** had few filter resets and managed effectively to smooth the very noisy pseudorange in some sections of the data.

Considering RTK in this analysis, we can demonstrate the increase in the success rate of the Doppler-aided integer ambiguity resolution (and hence the RTK availability) by comparison of the obtained integer ambiguity vectors from the conventional LAMBDA (Least-squares AMBiguity Decorrelation Adjustment) ambiguity resolution method using Doppler-smoothed pseudoranges with those obtained without Doppler-aiding in post-processed mode. The performance of ambiguity resolution was evaluated based on the number of epochs where the ambiguity validation passed the discrimination/ratio test. The ambiguity validation ratio test was set to the fixed critical threshold of 2.5 in all the experiments. In addition to the ratio test, the fixed solutions obtained using the fixed integer ambiguity vectors that passed the ratio test were compared against the true position of the surveyed point to make sure that indeed the correct set of integer ambiguities were estimated.

The overall performance of the single-epoch single-frequency integer ambiguity resolution obtained by the conventional LAMBDA ambiguity resolution method without Doppler-aiding is shown in **FIGURE 5** for baselines from 60



▲ **FIGURE 4** Smoothed pseudorange errors (observed range deviations) and filter resets and filter length (window width) using the traditional Hatch carrier-smoothing filter (a) and the Doppler-smoothing (b). Smoothing filter length in the experiments for both filters was set to 100 seconds. Satellite PRN G18 was chosen to represent a satellite at low elevation angle as it rises from 10 to 30 degrees.

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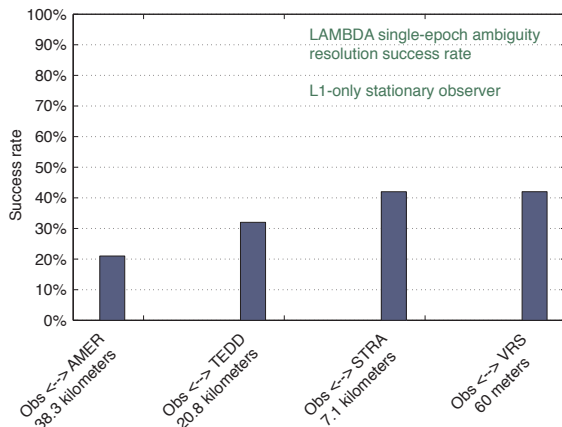
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▲ **FIGURE 5** Single-epoch single-frequency integer ambiguity resolution success rate obtained by the conventional LAMBDA ambiguity resolution method without Doppler-aiding.

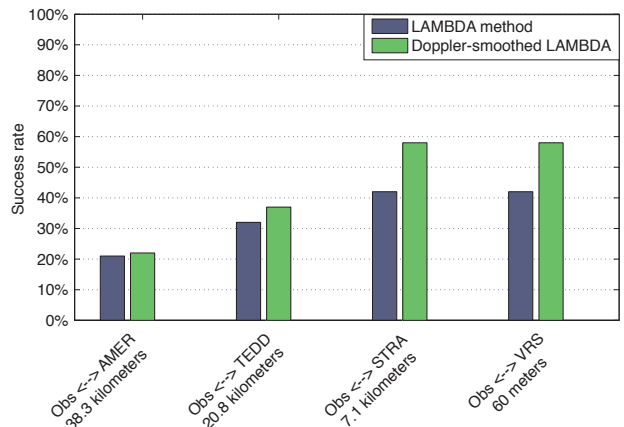
meters up to 38 kilometers in length. In comparison, the performance of the single-epoch single-frequency integer ambiguity resolution from the LAMBDA method using Doppler-smoothed pseudoranges are shown in **FIGURE 6** for those baselines and they are compared with integer ambiguity resolution success rates of the conventional LAMBDA ambiguity resolution method without Doppler-aiding. Figure 6 shows that using Doppler-smoothed pseudoranges enhances the probability of identifying the correct set of integer ambiguities and hence increases the success rate of the integer ambiguity resolution process in instantaneous RTK, providing higher availability. This is more evident for shorter baselines. For long baselines, the residual of satellite-ephemeris error and atmospheric-delay residuals that do not cancel in double differencing potentially limits the effectiveness of the Doppler-smoothing approach. It is well understood that those residuals for long baselines strongly degrade the performance of ambiguity resolution. Relative kinematic positioning with single frequency mass-market receivers in urban areas using VRS has also shown improvement.

Conclusion

In urban areas, the proposed Doppler-smoothing technique is more robust and effective than traditional carrier smoothing of pseudoranges. Static and kinematic trials confirm this technique improves the accuracy of the pseudorange-based absolute and relative positioning in urban areas characteristically by the order of 40 to 50 percent.

Doppler-smoothed pseudoranges are then used to aid the integer ambiguity resolution process to enhance the probability of identifying the correct set of integer ambiguities. This approach shows modest improvement in the ambiguity resolution success rate in instantaneous RTK where the probability of fixing ambiguities to correct integer values is dominated by the relatively imprecise pseudorange measurements.

The importance of resolving the integer ambiguities cor-



▲ **FIGURE 6** Plots of integer ambiguity resolution success rates: single-epoch single-frequency integer ambiguity resolution success rate obtained by the conventional LAMBDA ambiguity resolution method without Doppler-aiding (in blue) and using Doppler-smoothed pseudoranges (in green).

rectly must be emphasized. Therefore, devising innovative and robust methods to maximize the success rate and hence reliability and availability of single-frequency, single-epoch integer ambiguity resolution in the presence of biased and noisy observations is of great practical importance especially in GNSS-challenged environments.

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The data for the trial discussed in this article were obtained from a **u-blox AG** (www.u-blox.com) AEK-4T receiver with a u-blox ANN-MS-0-005 patch antenna.

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