3 AOCS

3.1 Spin stabilization

Consider a spin stabilized satellite after its deployment from the launcher. The angular velocity is given in the body-fixed coordinate system and its values are:

$$\omega_0 = (\omega_x, \omega_y, \omega_z)^T = (50, 150, 500)^T [\deg / s]$$

The moments of inertia of the satellite are also known:

$$I_x = I_y = 60kgm^2$$

$$I_z = 200kgm^2$$

1. Compute the absolute value of the spin vector L, the nutation frequency s_N and the nutation angle θ_N
2. The nutation will be decreased with the help of a damping mechanism (damping constant $T_D = 800s$). Compute the time that will be necessary to damp the nutation below a an angle of 0,5°/s.

To compute the spin stabilized satellite we have to calculate the angular momentum vector which is defined by:

$$\underline{L} = \underline{\underline{I}} \cdot \underline{\omega} = \begin{pmatrix} 60 & 0 & 0 \\ 0 & 60 & 0 \\ 0 & 0 & 200 \end{pmatrix} \cdot \begin{pmatrix} 0.87 \\ 2.618 \\ 8.762 \end{pmatrix} = \begin{pmatrix} 52.36 \\ 157.08 \\ 1745.33 \end{pmatrix} [\frac{kg \cdot m^2}{s}]$$

The absolute value of the angular momentum becomes

$$|\underline{L}| = \sqrt{L_x^2 + L_y^2 + L_z^2} = 1753.16 \frac{kg \cdot m^2}{s}$$

The formulary provides with an equation to compute the nutation angular velocity:

$$\omega_N = \frac{I_Z - I_X}{I_X} \cdot \omega_Z = \frac{200 - 60}{60} \cdot 8.726 \frac{rad}{s} = 20.36 \frac{rad}{s}$$

The nutation frequency can be found via

$$s_N = \frac{\omega_N}{2\pi} = 3.24 \frac{1}{s}$$

As the for a body with a nutation movement the vector for the angular momentum is constant which does not neccessarily have to coincide with the rotational vector of the motien. The body-fixed Z-axis is performing an coneshaped motion around the vector of angular momentumg. This cone, being called nutation cone, can be calculated using the equation:

$$\underline{L} \cdot \underline{z} = |\underline{L}| \cdot |\underline{z}| \cdot \cos \Theta_N$$

Using the calculated vectors

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$$\underline{L} = \begin{pmatrix} 52.36 \\ 157.08 \\ 1745.33 \end{pmatrix} \frac{kg \cdot m^2}{s}$$

and

$$\underline{z} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

we get

$$\Theta_N = 5.42\,\mathrm{deg}$$

b)
Now we want to compute the time needed until the nutation is damped down using a dissipative damper. This slowly "converts" the kinetic energy of the nutation/tumbling motion into waste heat. One equation to approximate this damping is using an exponential decrease due to the fact that the dissipating force is proportional to the velocity.

$$\Theta_N\left(t\right) = \Theta_{N,0} \cdot \exp^{-\frac{t}{T_D}}$$

Here T_D is the damping constant that determines how quick the damping takes place. It is set to $T_D = 800s$. We use the starting value for the nutation angle which is $\Theta_{N,0} = 5.42\,\mathrm{deg}$. We rearange the equation as we are looking for the time t until a nutation angle of less than $0.5\,\mathrm{deg}$ is achieved.

$$t = -T_D \cdot \ln \frac{\Theta_N}{\Theta_{N,0}}$$

$$t = 1879 sec$$

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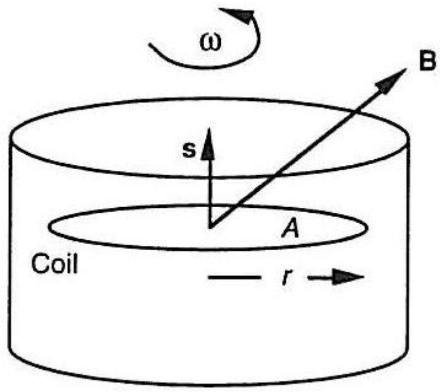
3.2 Magneto Torquer

A technique for attitude control that can be used for near Earth orbiting spacecrafts is to use the reaction between Earth's magnetic field and an internally generated magnet-ic field in the S/C. The technique of magnetic torquing has the major advantage of no moving parts. Let us consider a cylindrical spacecraft with a coil with area A and radius r centered in the middle of the structure shown below.



Cylindrical Spacecraft

where **s** is the normal vector to the coil, **B** is the external magnetic field of planet Earth of $3.0 \cdot 10 - 6$ and ω is the spin rate of the spacecraft itself.

1. Compute the maximum torque **T** acting on the spacecraft, due to the magnetic moment **m**, by leading a current of 1 ampere in the coil consisting of 160 turns of wire and an area of $0.5m^2$.

2. When the coil is positioned in a spinning spacecraft with its spinning axis parallel to the **s** direction the spacecraft will precess as **m** tends to line up with **B**. Calculate the maximum precession rate when the moment of inertia of the spacecraft about its spin axis is $15kgm^2$ and the spin rate is 12 rpm.

Since the dimensions of the coil will be restricted by the spacecraft size and power supply it is desirable to investigate how the torque **T** is related to the choice of material for the coil.

3. Let R be the coil resistance, a the cross-sectional area of the wire, σ its resistivity, ρ its density and M the mass of the coil, the power dissipated in the coil is P, the current through the coil is I and V is the voltage across the coil. Determine the relative torque for an Aluminum coil compared to the torque produced by a Copper coil due to the different material properties.

$$(\sigma\rho)_{Al} = 0.8 \cdot 10^{-4} kg\Omega m^{-2}$$

$$(\sigma\rho)_{Co} = 1.8 \cdot 10^{-4} kg\Omega m^{-2}$$

1. The earths external magnetic field is defined as $B = 3.0 \cdot 10^{-6}$. We are looking for the magentic torque $T_{mag} = m_s \cdot B$. We alreday know B, now we have to find the magnetic moment m_s.

$$m_s = I \cdot A = 1A \cdot 160 \cdot 0.5m^2 = 80Am^2$$

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$$T_{mag} = 3 \cdot 10^{-6} \cdot 80 = \underline{2.4 \cdot 10^{-4} Nm}$$

2. The precession rate is defined as $\omega_p = \frac{L \times T}{L^2}$. We already know T, which is the magnetic torque from above.

$$T = T_{mag} = 2.4 \cdot 10^{-4} Nm$$

The angular momentunm L is defined as:

$$L = I \times \omega_s = 15kgm^2 \cdot 12rpm = 15kgm^2 \cdot \frac{12 \cdot 2\pi}{60s} = 18.8495 \frac{kgm^2}{s}$$

Using both values in the defined formula we get:

$$\omega_p = \frac{L \times T}{L^2} = \frac{18.8495 \cdot 2.4 \cdot 10^{-4}}{18.8495^2} = 1.27324 \cdot 10^{-5} = 0.0007295°/s$$

3. In order to compare both materials we have to find a way to calculate the torque dependant on σ and ρ in the formula $T_{mag} = m_s \cdot B$

Coil resistance

$$R = \frac{2 \cdot \pi \cdot r \cdot N \cdot \sigma}{a} \rightarrow N = \frac{a \cdot R}{2 \cdot \pi \cdot r \cdot \sigma}$$

Mass of coil

$$M = 2 \cdot \pi \cdot r \cdot N \cdot a\rho \rightarrow N = \frac{M}{2 \cdot \pi \cdot r \cdot a\rho}$$

Power

$$P = U \cdot I = I^2 \cdot R \rightarrow I = \sqrt{\frac{P}{R}}$$

Max torque

$$T = N \cdot I \cdot A \cdot B = N \cdot \sqrt{\frac{P}{R}}$$

For the magentic torque we can writedown:

$$T_{mag} = m_s \cdot B$$

Rearranging the equation for given values (resistivity and density)

$$T_{mag} = m_s \cdot B = I \cdot A \cdot N \cdot B = N \cdot \underbrace{\sqrt{\frac{P}{R}}}_{=I} \cdot A \cdot B = \sqrt{\frac{P \cdot N^2}{R}} \cdot A \cdot B$$

$$= \sqrt{\frac{P}{R} \cdot \underbrace{\frac{a \cdot R}{2 \cdot \pi \cdot r \cdot \sigma}}_{=N} \cdot \underbrace{\frac{M}{2 \cdot \pi \cdot r \cdot a \cdot \rho}}_{=N}} \cdot \pi \cdot r^2 \cdot B = \frac{B \cdot r}{2} \cdot \sqrt{\frac{P \cdot M}{\sigma\rho}}$$

hence

$$T_{Al} = \frac{B \cdot r}{2} \cdot \sqrt{\frac{P \cdot M}{(\sigma\rho)_{Al}}}$$

$$T_{Co} = \frac{B \cdot r}{2} \cdot \sqrt{\frac{P \cdot M}{(\sigma\rho)_{Co}}}$$

Finally we can compare the torque produced by the aluminum coil with the torque from the copper coil:

$$\frac{T_{Al}}{T_{Co}} = \sqrt{\frac{(\sigma\rho)_{Co}}{(\sigma\rho)_{Al}}} = \sqrt{\frac{1.8}{0.8}} = \underline{1.5}$$

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