**Kinematics Analysis of Free-Floating Redundant Space Manipulator based on Momentum Conservation**

**Mingming Wang**(1)
(1)*Institute of Astronautics, TU Muenchen, Boltzmannstr. 15, D-85748, Garching, Germany, +49-8928-91-5996, M.Wang@lrt.mw.tum.de*

***Abstract:*** *This paper focuses on the kinematics analysis of free-floating space redundant manipulator at velocity level based on the momentum conservation law. Firstly, a general solution for inverse kinematics is proposed based on the concept of task priority. Secondly, in terms of pseudo-inverse and null-space components of the inertia matrix, the manipulator Jacobian matrix and the generalized Jacobian matrix, four different solutions of inverse kinematics at velocity level are derived when only the base orientation is taken into account. Thirdly, using these solutions, a number of space tasks can be completed by using corresponding inverse kinematics solution, which can give us a deeper understanding of the complex system's motion in space environment. Finally, fixed attitude restricted (FAR) motion and fixed end-effector restricted (FER) motion are proposed to complete the specific space mission by using the redundancy of the manipulator. Through singular-value decomposition, the dexterity measures of FAR and FER motions are derived. The proposed inverse kinematics formulations will be of great value for the future space applications.*

***Keywords:*** *Free-Floating, Task Priority, Generalized Jacobian Matrix, Momentum Conservation.*

## 1. Introduction

One of the main reasons for the development of space manipulators is to replace astronauts to perform tasks that involving long, repetitive operations and unhealthy, hazardous environment. Due to the particular harsh environment of space and the increasing demands of satellite maintenance, on-orbit refueling and assembly etc., the application of space robot has received significant attention. Some research has been done in the area of satellite servicing task, such as "Engineering Test Satellite VII (ETS-VII)" project [1], and "Orbital Express (OE)" program [2] etc.

Except the great value of space robot, however, there are some special properties of space robot that differ from fixed base robot, which make the application of space robot more challenging and complicated. One of these special properties is the dynamic coupling between the mounted manipulator and the spacecraft. This dynamic coupling is highly not welcomed since it yields a disturbance to the motion of the end-effector. Correspondingly, the workspace of the space robot will be reduced as expressed in [3]. In fact, additional actuators mounted on the spacecraft, such as thruster jets or momentum wheels can be utilized to compensate for these disturbances induced by the dynamic coupling. But their extensive use severely limits the system's useful life span. In order to overcome this drawback and conserve fuel and energy of the spacecraft, operation in a free-floating mode has been proposed [3][4][5], which indicates that the position and orientation of the spacecraft are not actively controlled during the operation of space manipulator. The primary idea of this approach is to widely utilize the motion of the manipulator itself, while regulate the position and orientation of the base simultaneously.

Many investigations have been made in the field of space robot kinematics analysis since two decades ago. Vafa and Dubowsky [3] have firstly proposed virtual manipulator method to analyze, design and develop the kinematics and dynamics of space manipulator system. Umetani and Yoshida [9] have derived the generalized Jacobian matrix (GJM) to express the relationship between end-effector motion and the whole system dynamics. Nakamura, Hanafusa and Yoshikawa [14] have proposed a concept of task priority in relation to the inverse kinematic problem of redundant manipulators. In terms of pseudo-inverse and null-space components of Jacobian matrix and inertia matrix, three alternative solutions of inverse kinematics at velocity level have been presented by Nenchev [8][10] to track a pre-defined path of the end-effector. These solutions are of great value for space operations, however, not complete. The aforementioned literature provokes our interests to explore a more general method to solve the inverse kinematics problem for the redundant space robot.

The main objective of this paper is to present a general inverse kinematics solution for free-floating space robot at velocity level, while the task priority is considered. Therefore the following contributions are made:
(1). It combines the task priority and inverse kinematics, and proposes a general solution for the inverse kinematics when many space tasks are considered;
(2). It provides new space motions to accomplish various space tasks.

The rest of this paper is organized as follows. Section 2 derives the general inverse kinematics solution based on the task priority and pseudo-inverse and null-space components. Section 3 analyzes the feasibility of the aforementioned solutions applying to the free-floating space robot. Four alternative solutions are listed to implement various space tasks. Section 4 proposes different motions for space robot to meet different requirements, and Section 5 concludes the paper.

## 2. Background

### 2.1. Assumptions

An $N$ degree-of-freedom (DOF) manipulator with $n$ rotational joints mounted on a free-floating spacecraft (base) as shown in Fig. 1 is considered. Suppose each rotational joint has $n_i$ DOF, therefore $N=\sum_{i=1}^{n} n_i$. The whole mechanical chain is composed of $n+1$ rigid bodies, and has $N+6$ generalized coordinates. When each rotational joint has only one single axis, then $N=n$. On condition of free-floating, an initial state of spacecraft position and orientation are well-known in reference to the inertia coordinate frame. No mechanical constraints nor external forces or torques are applied to the $N$ DOF mechanical chain. Only internal forces or torques from joint actuators are considered. Therefore, the momentum conservation and the equilibrium of forces and torques can be strictly held.

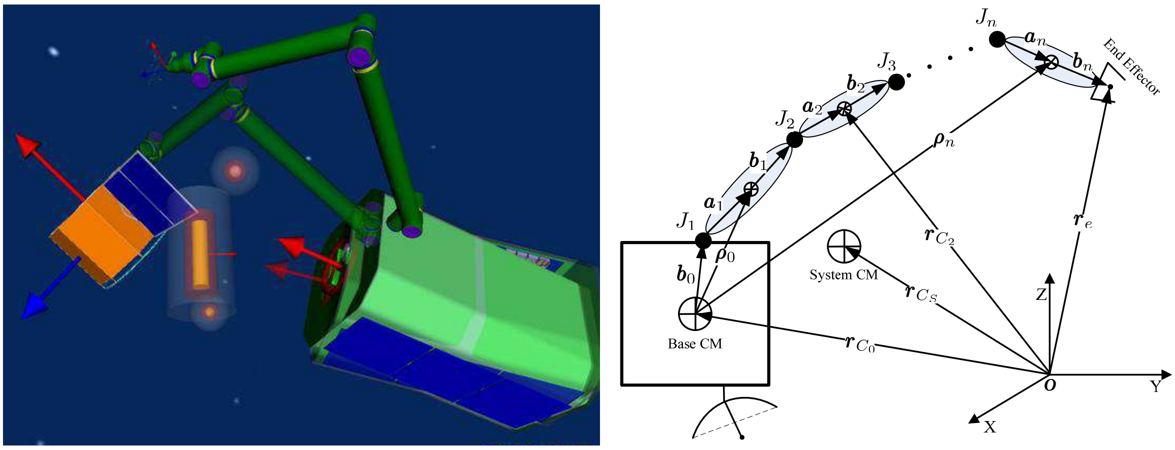



**Figure 1. Schematic diagram of free-floating space robot**

### 2.2. Redundancy and Task Priority

The degree-of-redundancy (DOR) for fixed base robot has been defined as the difference between the DOF of the manipulator and the number of end-effector task variables. In general, 6 DOF manipulator can meet the end-effector task space motion requirements. However, in the real situation, there are some other tasks as well as end-effector task to fulfill synchronously, such as joint range limits, singularity and collision avoidance, etc. When considering free-floating space manipulator, base motion control by using manipulator's redundancy can be regarded as another manipulator's task. These additional tasks call for system redundancy to meet the aforementioned requirements.

Supposing a general case at velocity level is taken into account. If there are $m$ tasks required for the manipulator, each task needs $m_i$ task variables, then the total number of task variables $M = \sum_{i=1}^{m} m_i$. According to the definition of DOR, three cases of redundancy can be distinguished in reference to the total number of task variables $M$ and the number of manipulator DOF $N$. The first case can be characterized by $N < min_{i \in [1,m]}\ m_i$, which indicates that, there are not sufficient DOF to satisfy even the simplest task with the minimum number of task variables, without mentioning residual redundancy available for the other tasks. This case is highly unusual, and primarily of theoretical interest. The second case can be denoted as $N \geq M$. In this case, besides the required task variables, there will be some additional redundancy left to satisfy certain criteria. For the third case, $min_{i \in [1,m]}\ m_i \leq N < M$, the available DOR will not suffice to accomplish all the tasks concurrently, although the manipulator may contain some redundancy with reference to one single task.

As before-mentioned the third case, if it is impossible to accomplish all the tasks completely, since the shortage of DOF, then it would be reasonable to perform the most significant task preferentially and the less significant tasks as much as possible using the remaining DOR. Different tasks required can be categorized into a list of subtasks with different level of significance, and this was firstly defined as tasks with the order of priority by Nakamura in [14]. Even for a 6 DOF manipulator, the subtasks decomposition between position and orientation is highly favorable. It will enlarge the workspace of the primary subtask by allowing incompleteness for the secondary subtask. Redundancy analysis and application based on the concept of tasks with the order of priority will be discussed in the following section.

### 2.3. General Solution for Inverse Kinematics

Let $\dot{\boldsymbol{\theta}} = (\dot{\theta}_1, \cdots, \dot{\theta}_n)^T \in \mathbb{R}^{n\times 1}$ and $\dot{\boldsymbol{x}} = (\dot{\boldsymbol{x}}_1, \cdots, \dot{\boldsymbol{x}}_m)^T \in \mathbb{R}^{M\times 1}$ denote the joint space and task space of space manipulator, the relationship between these two spaces can be established by using Jacobian matrix, and hence can be expressed by the following formula:

$$\begin{cases} \boldsymbol{J}_1\dot{\boldsymbol{\theta}} = \dot{\boldsymbol{x}}_1 \\ \boldsymbol{J}_2\dot{\boldsymbol{\theta}} = \dot{\boldsymbol{x}}_2 \\ \quad\cdots \\ \boldsymbol{J}_m\dot{\boldsymbol{\theta}} = \dot{\boldsymbol{x}}_m \end{cases} \tag{1}$$

where $\boldsymbol{J}_i \in \mathbb{R}^{m_i\times n}$ is the Jacobian matrix of the $i^{th}$ manipulator task. If task 1 is chosen as the primary task, the other tasks will be treated as the secondary tasks. Generally, the primary should be ensured to accomplish, and thereby holds the highest priority. Based on the significance of tasks, total $m$ tasks can thus form a top-down hierarchy. For an arbitrary task $m_i$, suppose that there are sufficient DOF to perform this task, the solutions can be described as follows:

$$\dot{\boldsymbol{\theta}} = \boldsymbol{J}_i^+\dot{\boldsymbol{x}}_i + (\boldsymbol{E}_n - \boldsymbol{J}_i^+\boldsymbol{J}_i)\boldsymbol{h}_i \tag{2}$$

where $\boldsymbol{J}_i^+ \in \mathbb{R}^{n\times m_i}$ is the pseudo-inverse of $\boldsymbol{J}_i$, $\boldsymbol{E}_n \in \mathbb{R}^{n\times n}$ is an identity matrix, $(\boldsymbol{E}_n - \boldsymbol{J}_i^+\boldsymbol{J}_i)$ is the null-space of $\boldsymbol{J}_i$, and $\boldsymbol{h}_i \in \mathbb{R}^{n\times 1}$ is an arbitrary vector. If the priority of the tasks is taken into account, in order to fulfill all the $m$ tasks expressed in Eq. (1), a recursive algorithm can be adopted to solve the priority based inverse kinematics problem at velocity level. In accordance with Eq. (2), enabling $i = 1$ and plugging this equation into the second task $\boldsymbol{J}_2\dot{\boldsymbol{\theta}} = \dot{\boldsymbol{x}}_2$ gives the solution of $\boldsymbol{h}_1$:

$$\boldsymbol{h}_1 = [\boldsymbol{J}_2(\boldsymbol{E}_n - \boldsymbol{J}_1^+\boldsymbol{J}_1)]^+(\dot{\boldsymbol{x}}_2 - \boldsymbol{J}_2\boldsymbol{J}_1^+\dot{\boldsymbol{x}}_1) + \left\{\boldsymbol{E}_n - [\boldsymbol{J}_2(\boldsymbol{E}_n - \boldsymbol{J}_1^+\boldsymbol{J}_1)]^+[\boldsymbol{J}_2(\boldsymbol{E}_n - \boldsymbol{J}_1^+\boldsymbol{J}_1)]\right\}\boldsymbol{h}_2 \tag{3}$$

Accordingly, the solution sufficing to coordinate exactly task 1 and task 2 in joint space can be denoted by:

$$\begin{aligned} \dot{\boldsymbol{\theta}} =& \boldsymbol{J}_1^+\dot{\boldsymbol{x}}_1 + [\boldsymbol{J}_2(\boldsymbol{E}_n - \boldsymbol{J}_1^+\boldsymbol{J}_1)]^+(\dot{\boldsymbol{x}}_2 - \boldsymbol{J}_2\boldsymbol{J}_1^+\dot{\boldsymbol{x}}_1) \\ &+ (\boldsymbol{E}_n - \boldsymbol{J}_1^+\boldsymbol{J}_1)\left\{\boldsymbol{E}_n - [\boldsymbol{J}_2(\boldsymbol{E}_n - \boldsymbol{J}_1^+\boldsymbol{J}_1)]^+[\boldsymbol{J}_2(\boldsymbol{E}_n - \boldsymbol{J}_1^+\boldsymbol{J}_1)]\right\}\boldsymbol{h}_2 \end{aligned} \tag{4}$$

Substituting the obtained solutions from upper tasks into the successive task sequentially, the solution meeting total $m$ tasks can be represented as follows:

$$
\begin{aligned}
\dot{\boldsymbol{\theta}} =& \boldsymbol{J}_1^{+}\dot{\boldsymbol{x}}_1 \\
&+[\boldsymbol{J}_2(\boldsymbol{E}_n-\boldsymbol{J}_1^{+}\boldsymbol{J}_1)]^{+}(\dot{\boldsymbol{x}}_2-\boldsymbol{J}_2\boldsymbol{J}_1^{+}\dot{\boldsymbol{x}}_1) \\
&+\Big\{\boldsymbol{J}_3(\boldsymbol{E}_n-\boldsymbol{J}_1^{+}\boldsymbol{J}_1)\big\{\boldsymbol{E}_n-[\boldsymbol{J}_2(\boldsymbol{E}_n-\boldsymbol{J}_1^{+}\boldsymbol{J}_1)]^{+}[\boldsymbol{J}_2(\boldsymbol{E}_n-\boldsymbol{J}_1^{+}\boldsymbol{J}_1)]\big\}\Big\}^{+} \\
&\quad\big(\dot{\boldsymbol{x}}_3-\boldsymbol{J}_3\boldsymbol{J}_1^{+}\dot{\boldsymbol{x}}_1-\boldsymbol{J}_3[\boldsymbol{J}_2(\boldsymbol{E}_n-\boldsymbol{J}_1^{+}\boldsymbol{J}_1)]^{+}(\dot{\boldsymbol{x}}_2-\boldsymbol{J}_2\boldsymbol{J}_1^{+}\dot{\boldsymbol{x}}_1)\big) \\
&+\cdots+\big\{\boldsymbol{E}_n-(\bullet)^{+}(\bullet)\big\}\boldsymbol{h}_m
\end{aligned} \tag{5}
$$

The last term $\boldsymbol{E}_n-(\bullet)^{+}(\bullet)$ in Eq. (5) is the projection matrix with respect to $\boldsymbol{J}_1,\cdots,\boldsymbol{J}_m$, which is symmetric and idempotent. $\boldsymbol{h}_m \in \mathbb{R}^{n\times 1}$ is an arbitrary vector. A schematic diagram of this algorithm is shown in Fig. 2. Eq. (5) reveals the inverse kinematics solution considering the priority of subtasks. This solution will be utilized to free-floating space robot in the following section.

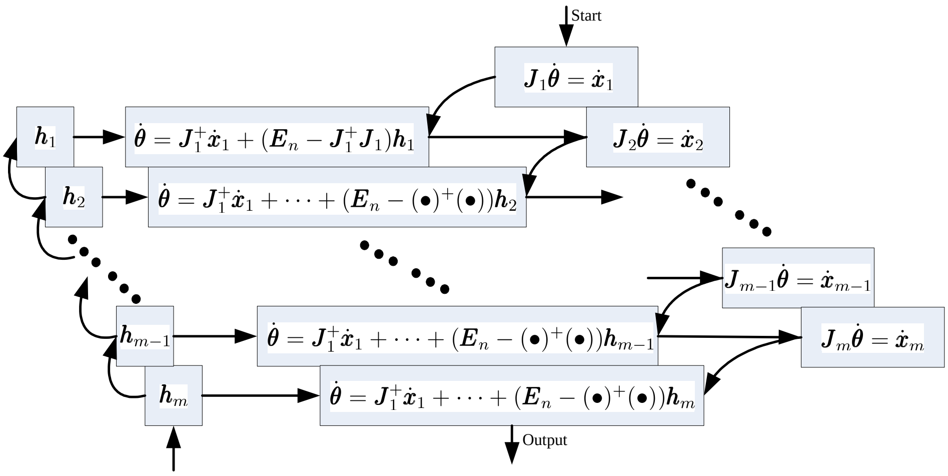
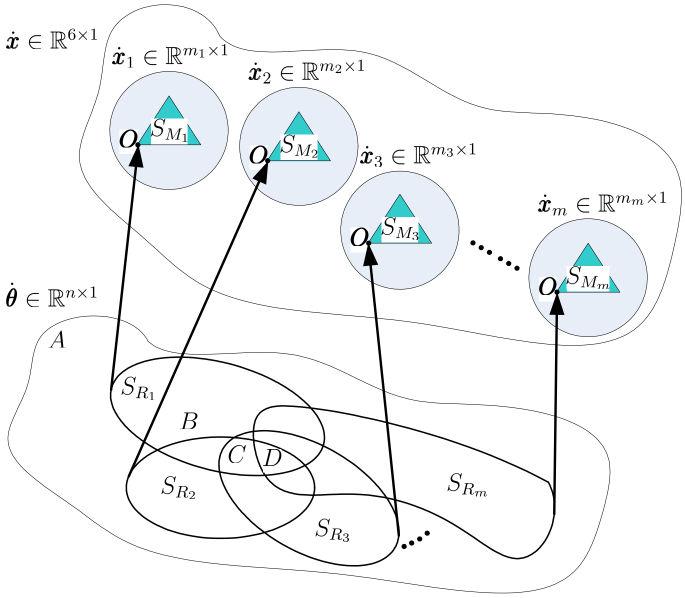



**Figure 2. Schematic diagram of inverse kinematics based on task priority**

Let the range space of the Jacobian matrix $\mathcal{R}(\boldsymbol{J})$ be the manipulable space, and the null-space of the Jacobian matrix $\mathcal{N}(J)$ be the redundant space. The general relationship between the manipulable and redundant spaces for different task variables is displayed in Fig. 3.

Using $S_{M_i}=\mathcal{R}(\boldsymbol{J}_i)$ and $S_{R_i}=\mathcal{N}(\boldsymbol{J}_i)$ as a description of manipulable space and redundant space, respectively. The subspace $A,B,C,D$ can be denoted by $S_{R_1}^{\perp}$, $S_{R_1}\cap(S_{R_1}\cap S_{R_2})^{\perp}$, $S_{R_1}\cap S_{R_2}\cap(S_{R_1}\cap S_{R_2}\cap S_{R_3})^{\perp}$, $S_{R_1}\cap S_{R_2}\cap\cdots\cap S_{R_m}$. Where $S_*^{\perp}$ represents the orthogonal complement of subspace $S_*$. The first term in the right side of Eq. (5) $\boldsymbol{J}_1^{+}\dot{\boldsymbol{x}}_1$ is the least-squares mapping $\dot{\boldsymbol{x}}_1$ onto subspace $A$. The second term represents the least-squares mapping $\dot{\boldsymbol{x}}_2-\boldsymbol{J}_2\boldsymbol{J}_1^{+}\dot{\boldsymbol{x}}_1$ onto subspace $B$, where $\dot{\boldsymbol{x}}_2-\boldsymbol{J}_2\boldsymbol{J}_1^{+}\dot{\boldsymbol{x}}_1$ is the modified command of the second task variables due to the effect of the first task variables applied on the second task. The third term implies the least-squares mapping $\dot{\boldsymbol{x}}_3-\boldsymbol{J}_3\boldsymbol{J}_1^{+}\dot{\boldsymbol{x}}_1-\boldsymbol{J}_3[\boldsymbol{J}_2(\boldsymbol{E}_n-\boldsymbol{J}_1^{+}\boldsymbol{J}_1)]^{+}(\dot{\boldsymbol{x}}_2-\boldsymbol{J}_2\boldsymbol{J}_1^{+}\dot{\boldsymbol{x}}_1)$ onto subspace $C$. It is also a modified command for the third task. The last term is the orthogonal projection of the arbitrary vector $\boldsymbol{h}_m$ onto subspace $D$. As a result, the solutions of inverse kinematics problem are obtained based on the concept of tasks with the order of priority.




$$\begin{cases} S_{M_1} = \mathcal{R}(\boldsymbol{J}_1) & S_{R_1} = \mathcal{N}(\boldsymbol{J}_1) \\ S_{M_2} = \mathcal{R}(\boldsymbol{J}_2) & S_{R_2} = \mathcal{N}(\boldsymbol{J}_2) \\ \cdots & \cdots \\ S_{M_m} = \mathcal{R}(\boldsymbol{J}_m) & S_{R_m} = \mathcal{N}(\boldsymbol{J}_m) \end{cases} \begin{cases} Subspace A = S_{R_1}^{\perp} \\ Subspace B = S_{R_1} \cap (S_{R_1} \cap S_{R_2})^{\perp} \\ Subspace C = S_{R_1} \cap S_{R_2} \cap (S_{R_1} \cap S_{R_2} \cap S_{R_3})^{\perp} \\ Subspace D = S_{R_1} \cap S_{R_2} \cap \cdots \cap S_{R_m} \end{cases}$$

**Figure 3. Manipulable and Redundant Space of Various Tasks**

## 3. Inverse Kinematics Solution for Space Robot

This section illustrates basic dynamic equations of space robot. Based on the definition of task priority, various space tasks at velocity level are introduced, their corresponding solutions are given in this section.

### 3.1. General Formulations

Refer to [7][9], for a space based robot, the general dynamics equation of an $n$ DOF space manipulator mounted on a 6 DOF spacecraft can be described by the following expression:

$$\begin{bmatrix} \boldsymbol{H}_b & \boldsymbol{H}_{bm} \\ \boldsymbol{H}_{bm}^T & \boldsymbol{H}_m \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{x}}_b \\ \ddot{\boldsymbol{\theta}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{c}_b \\ \boldsymbol{c}_m \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_b \\ \boldsymbol{\tau} \end{bmatrix} + \begin{bmatrix} \boldsymbol{J}_b^T \\ \boldsymbol{J}_e^T \end{bmatrix} \boldsymbol{f}_e \tag{6}$$

where $\boldsymbol{H}_b$, $\boldsymbol{H}_{bm}$ and $\boldsymbol{H}_m$ are the base inertia matrix, dynamic coupling matrix and manipulator's inertia matrix, respectively, $\dot{\boldsymbol{x}}_b = [\dot{\boldsymbol{r}}_{C0}^T, \boldsymbol{\omega}_0^T]^T \in \mathbb{R}^{6\times 1}$ stands for the base translational and angular velocity, $\boldsymbol{c}_b \in \mathbb{R}^{6\times 1}$ and $\boldsymbol{c}_m \in \mathbb{R}^{n\times 1}$ are the Coriolis and centrifugal force vectors of the base and manipulator, respectively, $\boldsymbol{\tau} \in \mathbb{R}^{n\times 1}$ are the torques of the joint actuators, $\boldsymbol{f}_b \in \mathbb{R}^{6\times 1}$ and $\boldsymbol{f}_e \in \mathbb{R}^{6\times 1}$ are the generalized forces applied on the base and end-effector, respectively. $\boldsymbol{J}_b \in \mathbb{R}^{6\times 6}$ and $\boldsymbol{J}_e \in \mathbb{R}^{6\times n}$ are the

Jacobian matrix of the base and end-effector. When $\boldsymbol{f}_b$ is actively generated (e.g. jet thrusters or reaction wheels etc.), the system is termed a free-flying space robot, if there are no active actuators are applied on the base, then the system is called a free-floating space robot. In this paper, the later case, i.e. free-floating space robot is considered.

Unlike ground fixed base manipulator, the tasks of space robot can be divided into three categories: base/spacecraft motion task, end-effector motion task and other constraints tasks (e.g. collision and singularity avoidance, joint range limits, etc.). In this paper, the first two categories are taken into account. Denote by $m_1$ and $m_2$ the number of task variables for base motion and end-effector motion, respectively. Hence, $m_1 \leq 6$ and $m_2 \leq 6$, which indicates that, at least 12 DOF manipulator has to be used to completely perform the base and end-effector tasks. It would be unrealistic to expect one manipulator with adequate DOR to perform all the space tasks concurrently. Therefore, a strategy of space tasks with the order of priority would be adopted to meet the aforementioned requirements.

The total momentum around the mass center of the base is conserved considering free-floating space robot. As expressed in Eq. (6), the total momentum $\boldsymbol{\mathcal{M}}_0$ composed of translational and rotational momentum can be expressed by:

$$\boldsymbol{\mathcal{M}}_0 = \begin{bmatrix} \boldsymbol{P}_0 \\ \boldsymbol{L}_0 \end{bmatrix} = \boldsymbol{H}_b \dot{\boldsymbol{x}}_b + \boldsymbol{H}_{bm} \dot{\boldsymbol{\theta}} \tag{7}$$

where $\boldsymbol{P}_0$ and $\boldsymbol{L}_0$ stand for translational and rotational momentum, respectively. From Eq. (7), since the base inertia matrix is invertible, the base motion expressed by joint motion rate $\dot{\boldsymbol{\theta}}$ and total momentum $\boldsymbol{\mathcal{M}}_0$ can be denoted as:

$$\dot{\boldsymbol{x}}_b = \boldsymbol{H}_b^{-1} \boldsymbol{\mathcal{M}}_0 - \boldsymbol{H}_b^{-1} \boldsymbol{H}_{bm} \dot{\boldsymbol{\theta}} \tag{8}$$

By substituting Eq. (8) into the kinematic mapping of the end-effector $\dot{\boldsymbol{x}}_E = \boldsymbol{J}_b \dot{\boldsymbol{x}}_b + \boldsymbol{J}_e \dot{\boldsymbol{\theta}}$, the motion of the end-effector is given as follows:

$$\dot{\boldsymbol{x}}_E = \boldsymbol{J}_b \boldsymbol{H}_b^{-1} \boldsymbol{\mathcal{M}}_0 + \boldsymbol{J}_G \dot{\boldsymbol{\theta}} \tag{9}$$

where $\dot{\boldsymbol{x}}_E \in \mathbb{R}^{6\times 1}$ is the total task variables of end-effector, $\boldsymbol{J}_G = \boldsymbol{J}_e - \boldsymbol{J}_b \boldsymbol{H}_b^{-1} \boldsymbol{H}_{bm} \in \mathbb{R}^{6\times n}$ is termed the generalized Jacobian matrix (GJM) which is proposed by Umetani and Yoshida in [9]. From Eq. (8) and (9), one can see that, the motion of the base and the end-effector can be regulated only utilizing the motion of joints since the existence of dynamic coupling between the base and the manipulator.

Generally, motions of manipulator without base attitude disturbance will be of prime interest. From Eq. (7), the following equation can be obtained:

$$\boldsymbol{I}_s \boldsymbol{\omega}_0 + \boldsymbol{I}_{bm} \dot{\boldsymbol{\theta}} = \boldsymbol{L}_0 \tag{10}$$

where $\boldsymbol{I}_s \in \mathbb{R}^{3\times 3}$ is an inertia matrix of the whole system, $\boldsymbol{I}_{bm} \in \mathbb{R}^{3\times n}$ is the coupling inertia matrix of the manipulator. On the other hand, for an end-effector task, the following expression can be given:

$$\dot{\boldsymbol{x}}_e = \dot{\boldsymbol{x}}_p + \boldsymbol{J}_0\boldsymbol{\omega}_0 + \boldsymbol{J}_m\dot{\boldsymbol{\theta}} \tag{11}$$

where $\dot{\boldsymbol{x}}_e \in \mathbb{R}^{m_2\times 1}$ is the end-effector task variables, $\dot{\boldsymbol{x}}_p \in \mathbb{R}^{m_2\times 1}$ is the initial motion rate vector of the base, $\boldsymbol{J}_0 \in \mathbb{R}^{m_2\times 3}$ and $\boldsymbol{J}_m \in \mathbb{R}^{m_2\times n}$ are the Jacobian matrix of the base and manipulator, respectively. Substitute expression $\boldsymbol{\omega}_0$ from Eq. (10) into Eq. (11), we can obtain:

$$\dot{\boldsymbol{x}}_e = \boldsymbol{J}_g\dot{\boldsymbol{\theta}} + \dot{\boldsymbol{x}}_p + \boldsymbol{J}_0\boldsymbol{I}_s^{-1}\boldsymbol{L}_0 \tag{12}$$

where $\boldsymbol{J}_g = \boldsymbol{J}_m - \boldsymbol{J}_0\boldsymbol{I}_s^{-1}\boldsymbol{I}_{bm} \in \mathbb{R}^{m_2\times n}$ is also termed generalized Jacobian matrix which can be used to deal the base attitude control problem.

### 3.2. Alternative Solutions of Inverse Kinematics

As introduced in Section 2.1, we suppose each rotational joint has one single axis and $N = n$, without considering external forces or torques, only manipulator joint active control is considered. We would like to coordinate the motion of end-effector $\dot{\boldsymbol{x}}_e$ and spacecraft attitude motion $\boldsymbol{\omega}_0$. Assuming there are available DOR, which means $n \geq m_1 + m_2$, then four cases can be obtained:

$$\begin{cases} (1)\ \boldsymbol{I}_s\boldsymbol{\omega}_0 + \boldsymbol{I}_{bm}\dot{\boldsymbol{\theta}} = \boldsymbol{L}_0;\ (2)\ \dot{\boldsymbol{x}}_p + \boldsymbol{J}_0\boldsymbol{\omega}_0 + \boldsymbol{J}_e\dot{\boldsymbol{\theta}} = \dot{\boldsymbol{x}}_e & case\ 1 \\ (1)\ \boldsymbol{I}_s\boldsymbol{\omega}_0 + \boldsymbol{I}_{bm}\dot{\boldsymbol{\theta}} = \boldsymbol{L}_0;\ (2)\ \dot{\boldsymbol{x}}_p + \boldsymbol{J}_0\boldsymbol{I}_s^{-1}\boldsymbol{L}_0 + \boldsymbol{J}_g\dot{\boldsymbol{\theta}} = \dot{\boldsymbol{x}}_e & case\ 2 \\ (1)\ \dot{\boldsymbol{x}}_p + \boldsymbol{J}_0\boldsymbol{\omega}_0 + \boldsymbol{J}_e\dot{\boldsymbol{\theta}} = \dot{\boldsymbol{x}}_e;\ (2)\ \boldsymbol{I}_s\boldsymbol{\omega}_0 + \boldsymbol{I}_{bm}\dot{\boldsymbol{\theta}} = \boldsymbol{L}_0 & case\ 3 \\ (1)\ \dot{\boldsymbol{x}}_p + \boldsymbol{J}_0\boldsymbol{I}_s^{-1}\boldsymbol{L}_0 + \boldsymbol{J}_g\dot{\boldsymbol{\theta}} = \dot{\boldsymbol{x}}_e;\ (2)\ \boldsymbol{I}_s\boldsymbol{\omega}_0 + \boldsymbol{I}_{bm}\dot{\boldsymbol{\theta}} = \boldsymbol{L}_0 & case\ 4 \end{cases} \tag{13}$$

In these four cases, the differences between them are which space task is chosen as the primary task. Refer to the general solutions of inverse kinematics based on the concept of task priority derived in Section 2.3, one can see that, the higher priority task of case 1 and case 2 is the base attitude motion control task, since the rotational momentum equilibrium Eq. (10) is firstly used and the base angular velocity $\boldsymbol{\omega}_0$ will be contained in the first term $\boldsymbol{I}_{bm}^{+}(\boldsymbol{L}_0 - \boldsymbol{I}_s\boldsymbol{\omega}_0)$ as the minimum-norm component of $\dot{\boldsymbol{\theta}}$. The solutions for case 1 and case 2 can be denoted by:

$$\begin{cases} \begin{aligned} \dot{\boldsymbol{\theta}} &= \boldsymbol{I}_{bm}^{+}(\boldsymbol{L}_0 - \boldsymbol{I}_s\boldsymbol{\omega}_0) \\ &\quad + \left[\boldsymbol{J}_m(\boldsymbol{E}_n - \boldsymbol{I}_{bm}^{+}\boldsymbol{I}_{bm})\right]^{+}\left[\dot{\boldsymbol{x}}_e - \dot{\boldsymbol{x}}_p - (\boldsymbol{J}_0 - \boldsymbol{J}_m\boldsymbol{I}_{bm}^{+}\boldsymbol{I}_s)\boldsymbol{\omega}_0\right] \\ &\quad + (\boldsymbol{E}_n - \boldsymbol{I}_{bm}^{+}\boldsymbol{I}_{bm})\Big\{\boldsymbol{E}_n - \left[\boldsymbol{J}_m(\boldsymbol{E}_n - \boldsymbol{I}_{bm}^{+}\boldsymbol{I}_{bm})\right]^{+}\left[\boldsymbol{J}_m(\boldsymbol{E}_n - \boldsymbol{I}_{bm}^{+}\boldsymbol{I}_{bm})\right]\Big\}\boldsymbol{h} \end{aligned} & case\ 1 \\ \\ \begin{aligned} \dot{\boldsymbol{\theta}} &= \boldsymbol{I}_{bm}^{+}(\boldsymbol{L}_0 - \boldsymbol{I}_s\boldsymbol{\omega}_0) \\ &\quad + \left[\boldsymbol{J}_g(\boldsymbol{E}_n - \boldsymbol{I}_{bm}^{+}\boldsymbol{I}_{bm})\right]^{+}\left[\dot{\boldsymbol{x}}_e - \dot{\boldsymbol{x}}_p - \boldsymbol{J}_0\boldsymbol{I}_s^{-1}\boldsymbol{L}_0 - \boldsymbol{J}_g\boldsymbol{I}_{bm}^{+}(\boldsymbol{L}_0 - \boldsymbol{I}_s\boldsymbol{\omega}_0)\right] \\ &\quad + (\boldsymbol{E}_n - \boldsymbol{I}_{bm}^{+}\boldsymbol{I}_{bm})\Big\{\boldsymbol{E}_n - \left[\boldsymbol{J}_g(\boldsymbol{E}_n - \boldsymbol{I}_{bm}^{+}\boldsymbol{I}_{bm})\right]^{+}\left[\boldsymbol{J}_g(\boldsymbol{E}_n - \boldsymbol{I}_{bm}^{+}\boldsymbol{I}_{bm})\right]\Big\}\boldsymbol{h} \end{aligned} & case\ 2 \end{cases} \tag{14}$$

The second and its subsequent terms in the expression $\dot{\boldsymbol{\theta}}$ for case 1 and 2 are the self-motion of the manipulator. These terms are obtained through null-space component. Hence, they have no impact on the base attitude motion. For case 3 and case 4, the higher priority task is the end-effector motion task. Since the kinematic mapping of the end-effector is firstly utilized and the end-effector motion $\dot{\boldsymbol{x}}_e$ is included in the minimum-norm component of $\dot{\boldsymbol{\theta}}$. The null-space components of $\boldsymbol{J}_m$ and $\boldsymbol{J}_g$, will have no impact on the end-effector's motion. The solutions for case 3 and case 4 can be expressed by:

$$\begin{cases} \dot{\boldsymbol{\theta}} = \boldsymbol{J}_m^+(\dot{\boldsymbol{x}}_e - \dot{\boldsymbol{x}}_p - \boldsymbol{J}_0\boldsymbol{\omega}_0) \\ \quad + \left[\boldsymbol{I}_{bm}(\boldsymbol{E}_n - \boldsymbol{J}_m^+\boldsymbol{J}_m)\right]^+\left[\boldsymbol{L}_0 - \boldsymbol{I}_s\boldsymbol{\omega}_0 - \boldsymbol{I}_{bm}\boldsymbol{J}_m^+(\dot{\boldsymbol{x}}_e - \dot{\boldsymbol{x}}_p - \boldsymbol{J}_0\boldsymbol{\omega}_0)\right] & case\ 3 \\ \quad + (\boldsymbol{E}_n - \boldsymbol{J}_m^+\boldsymbol{J}_m)\left\{\boldsymbol{E}_n - \left[\boldsymbol{I}_{bm}(\boldsymbol{E}_n - \boldsymbol{J}_m^+\boldsymbol{J}_m)\right]^+\left[\boldsymbol{I}_{bm}(\boldsymbol{E}_n - \boldsymbol{J}_m^+\boldsymbol{J}_m)\right]\right\}\boldsymbol{h} \\ \\ \dot{\boldsymbol{\theta}} = \boldsymbol{J}_g^+(\dot{\boldsymbol{x}}_e - \dot{\boldsymbol{x}}_p - \boldsymbol{J}_0\boldsymbol{I}_s^{-1}\boldsymbol{L}_0) \\ \quad + \left[\boldsymbol{I}_{bm}(\boldsymbol{E}_n - \boldsymbol{J}_g^+\boldsymbol{J}_g)\right]^+\left[\boldsymbol{L}_0 - \boldsymbol{I}_s\boldsymbol{\omega}_0 - \boldsymbol{I}_{bm}\boldsymbol{J}_g^+(\dot{\boldsymbol{x}}_e - \dot{\boldsymbol{x}}_p - \boldsymbol{J}_0\boldsymbol{I}_s^{-1}\boldsymbol{L}_0)\right] & case\ 4 \\ \quad + (\boldsymbol{E}_n - \boldsymbol{J}_g^+\boldsymbol{J}_g)\left\{\boldsymbol{E}_n - \left[\boldsymbol{I}_{bm}(\boldsymbol{E}_n - \boldsymbol{J}_g^+\boldsymbol{J}_g)\right]^+\left[\boldsymbol{I}_{bm}(\boldsymbol{E}_n - \boldsymbol{J}_g^+\boldsymbol{J}_g)\right]\right\}\boldsymbol{h} \end{cases} \tag{15}$$

It would be noted that above four alternative solutions are based on the condition that $n \geq m_1 + m_2$, when there is no sufficient DOR ($max(m_1, m_2) \leq n < m_1 + m_2$), only one subtask can be fulfilled while the other will be performed as much as possible. No common solutions can be obtained for each of the case expressed in (13) since the lacking of DOR.

## 4. Inverse kinematics Application

This section illustrates various space tasks based on the aforementioned inverse kinematics solutions. Three special space tasks are presented and the dexterity measure of related motion is given.

### 4.1. Different Space Tasks

As mentioned in [10], on the assumption for sufficient DOR ($n \geq m_1 + m_2$), to simply the expression of the solutions, let $\boldsymbol{\Phi}_i(\bullet)$ represent the third term in the expression $\dot{\boldsymbol{\theta}}$ of case $i$ for all four solutions. The following space tasks can be obtained assuming no initial motion $\boldsymbol{L}_0 = 0$ and $\dot{\boldsymbol{x}}_p = 0$.

Task I. End-effector path tracking with simultaneous base attitude maintenance. By setting $\boldsymbol{\omega}_0 = 0$, the solutions in Eq. (14) and (15) can be given by

$$\dot{\boldsymbol{\theta}} = \begin{cases} \left[\boldsymbol{J}_m(\boldsymbol{E}_n - \boldsymbol{I}_{bm}^+\boldsymbol{I}_{bm})\right]^+\dot{\boldsymbol{x}}_e + \boldsymbol{\Phi}_1(\bullet)\boldsymbol{h} & case\ 1 \\ \left[\boldsymbol{J}_g(\boldsymbol{E}_n - \boldsymbol{I}_{bm}^+\boldsymbol{I}_{bm})\right]^+\dot{\boldsymbol{x}}_e + \boldsymbol{\Phi}_2(\bullet)\boldsymbol{h} & case\ 2 \\ \boldsymbol{J}_m^+\dot{\boldsymbol{x}}_e + \left[\boldsymbol{I}_{bm}(\boldsymbol{E}_n - \boldsymbol{J}_m^+\boldsymbol{J}_m)\right]^+\left[-\boldsymbol{I}_{bm}\boldsymbol{J}_m^+\dot{\boldsymbol{x}}_e\right] + \boldsymbol{\Phi}_3(\bullet)\boldsymbol{h} & case\ 3 \\ \boldsymbol{J}_g^+\dot{\boldsymbol{x}}_e + \left[\boldsymbol{I}_{bm}(\boldsymbol{E}_n - \boldsymbol{J}_g^+\boldsymbol{J}_g)\right]^+\left[-\boldsymbol{I}_{bm}\boldsymbol{J}_g^+\dot{\boldsymbol{x}}_e\right] + \boldsymbol{\Phi}_4(\bullet)\boldsymbol{h} & case\ 4 \end{cases} \tag{16}$$

Task II. Regulating base attitude whilst keeping fixed position/orientation of the end-effector with reference to inertia coordinate frame. By setting $\dot{\boldsymbol{x}}_e = 0$, the solutions in Eq. (14) and (15) can be given by

$$\dot{\boldsymbol{\theta}} = \begin{cases} -\boldsymbol{I}_{bm}^{+}\boldsymbol{I}_s\boldsymbol{\omega}_0 + \left[\boldsymbol{J}_m(\boldsymbol{E}_n - \boldsymbol{I}_{bm}^{+}\boldsymbol{I}_{bm})\right]^{+}\left[-(\boldsymbol{J}_0 - \boldsymbol{J}_m\boldsymbol{I}_{bm}^{+}\boldsymbol{I}_s)\boldsymbol{\omega}_0\right] + \boldsymbol{\Phi}_1(\bullet)\boldsymbol{h} & case\ 1 \\ -\boldsymbol{I}_{bm}^{+}\boldsymbol{I}_s\boldsymbol{\omega}_0 + \left[\boldsymbol{J}_g(\boldsymbol{E}_n - \boldsymbol{I}_{bm}^{+}\boldsymbol{I}_{bm})\right]^{+}\left[\boldsymbol{J}_g\boldsymbol{I}_{bm}^{+}\boldsymbol{I}_s\boldsymbol{\omega}_0\right] + \boldsymbol{\Phi}_2(\bullet)\boldsymbol{h} & case\ 2 \\ -\boldsymbol{J}_m^{+}\boldsymbol{J}_0\boldsymbol{\omega}_0 + \left[\boldsymbol{I}_{bm}(\boldsymbol{E}_n - \boldsymbol{J}_m^{+}\boldsymbol{J}_m)\right]^{+}\left[-\boldsymbol{I}_s\boldsymbol{\omega}_0 + \boldsymbol{I}_{bm}\boldsymbol{J}_m^{+}\boldsymbol{J}_0\boldsymbol{\omega}_0\right] + \boldsymbol{\Phi}_3(\bullet)\boldsymbol{h} & case\ 3 \\ \left[\boldsymbol{I}_{bm}(\boldsymbol{E}_n - \boldsymbol{J}_g^{+}\boldsymbol{J}_g)\right]^{+}\left[-\boldsymbol{I}_s\boldsymbol{\omega}_0\right] + \boldsymbol{\Phi}_4(\bullet)\boldsymbol{h} & case\ 4 \end{cases} \tag{17}$$

Task III. Regulating base attitude whilst keeping fixed postion/orientation of the end-effector with reference to relative coordinate frame. By setting $\dot{\boldsymbol{x}}_e - \boldsymbol{J}_0\boldsymbol{\omega}_0 = 0$, the solutions in Eq. (14) and (15) can be given by

$$\dot{\boldsymbol{\theta}} = \begin{cases} -\boldsymbol{I}_{bm}^{+}\boldsymbol{I}_s\boldsymbol{\omega}_0 + \left[\boldsymbol{J}_m(\boldsymbol{E}_n - \boldsymbol{I}_{bm}^{+}\boldsymbol{I}_{bm})\right]^{+}\left[\boldsymbol{J}_m\boldsymbol{I}_{bm}^{+}\boldsymbol{I}_s\boldsymbol{\omega}_0\right] + \boldsymbol{\Phi}_1(\bullet)\boldsymbol{h} & case\ 1 \\ \left[\boldsymbol{I}_{bm}(\boldsymbol{E}_n - \boldsymbol{J}_m^{+}\boldsymbol{J}_m)\right]^{+}\left[-\boldsymbol{I}_s\boldsymbol{\omega}_0\right] + \boldsymbol{\Phi}_3(\bullet)\boldsymbol{h} & case\ 3 \end{cases} \tag{18}$$

Note that case 2 and case 4 are not considered since there is not explicit term $\dot{\boldsymbol{x}}_e - \boldsymbol{J}_0\boldsymbol{\omega}_0$ in these related solutions.

### 4.2. Special Space Tasks

Among above space tasks, some special space tasks can be extracted to implement very special space tasks. Fixed-attitude-restricted (FAR) motion based on Task I was first defined by Nenchev in [10]. This motion can be denoted by:

$$\dot{\boldsymbol{x}}_{FAR} = \begin{cases} \boldsymbol{J}_m(\boldsymbol{E}_n - \boldsymbol{I}_{bm}^{+}\boldsymbol{I}_{bm})\boldsymbol{h} \\ \boldsymbol{J}_g(\boldsymbol{E}_n - \boldsymbol{I}_{bm}^{+}\boldsymbol{I}_{bm})\boldsymbol{h} \end{cases} \tag{19}$$

$\boldsymbol{J}_m(\boldsymbol{E}_n - \boldsymbol{I}_{bm}^{+}\boldsymbol{I}_{bm})$ and $\boldsymbol{J}_g(\boldsymbol{E}_n - \boldsymbol{I}_{bm}^{+}\boldsymbol{I}_{bm})$ both can be used as FAR motion Jacobian matrix, this motion will not affect the attitude velocity of the spacecraft. We can use this motion to maintain the base attitude whilst regulate the position or orientation of the end-effector.

Similarly, we define a fixed-end-effector-restricted (FER) motion here, which can be expressed by:

$$\dot{\boldsymbol{x}}_{FER} = \begin{cases} \boldsymbol{I}_{bm}(\boldsymbol{E}_n - \boldsymbol{J}_m^{+}\boldsymbol{J}_m)\boldsymbol{h} \\ \boldsymbol{I}_{bm}(\boldsymbol{E}_n - \boldsymbol{J}_g^{+}\boldsymbol{J}_g)\boldsymbol{h} \end{cases} \tag{20}$$

$\boldsymbol{I}_{bm}(\boldsymbol{E}_n - \boldsymbol{J}_m^{+}\boldsymbol{J}_m)$ and $\boldsymbol{I}_{bm}(\boldsymbol{E}_n - \boldsymbol{J}_g^{+}\boldsymbol{J}_g)$ both can be used as FER motion Jacobian matrix. This motion can be used to keep the end-effector steady and regulate the base attitude simultaneously.

If we have sufficient DOR, the third term in the expression $\dot{\boldsymbol{\theta}}$ for the four alternative solutions will exist. These terms are the self-motion of the manipulator induced by the null-space of the Jacobian matrix and dynamic coupling matrix. Therefore, these motions have no impact on the motion of both the base and the end-effector, which implies, they can be utilized to adjust the configuration of the manipulator to avoid the singularities or collisions, or the other constraints in joint space. We thereby define the following joint velocity motions as fixed-attitude-end-effector-restricted (FAER) motion:

$$\dot{\boldsymbol{\theta}} = \begin{cases} (\boldsymbol{E}_n - \boldsymbol{I}_{bm}^{+}\boldsymbol{I}_{bm})\Big\{\boldsymbol{E}_n - \big[\boldsymbol{J}_m(\boldsymbol{E}_n - \boldsymbol{I}_{bm}^{+}\boldsymbol{I}_{bm})\big]^{+}\big[\boldsymbol{J}_m(\boldsymbol{E}_n - \boldsymbol{I}_{bm}^{+}\boldsymbol{I}_{bm})\big]\Big\}\boldsymbol{h} \\ (\boldsymbol{E}_n - \boldsymbol{I}_{bm}^{+}\boldsymbol{I}_{bm})\Big\{\boldsymbol{E}_n - \big[\boldsymbol{J}_g(\boldsymbol{E}_n - \boldsymbol{I}_{bm}^{+}\boldsymbol{I}_{bm})\big]^{+}\big[\boldsymbol{J}_g(\boldsymbol{E}_n - \boldsymbol{I}_{bm}^{+}\boldsymbol{I}_{bm})\big]\Big\}\boldsymbol{h} \\ (\boldsymbol{E}_n - \boldsymbol{J}_{m}^{+}\boldsymbol{J}_{m})\Big\{\boldsymbol{E}_n - \big[\boldsymbol{I}_{bm}(\boldsymbol{E}_n - \boldsymbol{J}_{m}^{+}\boldsymbol{J}_{m})\big]^{+}\big[\boldsymbol{I}_{bm}(\boldsymbol{E}_n - \boldsymbol{J}_{m}^{+}\boldsymbol{J}_{m})\big]\Big\}\boldsymbol{h} \\ (\boldsymbol{E}_n - \boldsymbol{J}_{g}^{+}\boldsymbol{J}_{g})\Big\{\boldsymbol{E}_n - \big[\boldsymbol{I}_{bm}(\boldsymbol{E}_n - \boldsymbol{J}_{g}^{+}\boldsymbol{J}_{g})\big]^{+}\big[\boldsymbol{I}_{bm}(\boldsymbol{E}_n - \boldsymbol{J}_{g}^{+}\boldsymbol{J}_{g})\big]\Big\}\boldsymbol{h} \end{cases} \tag{21}$$

### 4.3. Dexterity Measure

The concept of manipulability was first proposed by Yoshikawa in [11]. Subsequently, Klein and Blaho [12] have presented variety of dexterity measures for redundant fixed base manipulator, such as condition number, minimum singular value, etc.

The FAR and FER manipulability measure $\omega_{FAR}$ and $\omega_{FER}$ can be defined as

$$\omega_{FAR} = \begin{cases} \sqrt{det\big[\boldsymbol{J}_m(\boldsymbol{E}_n - \boldsymbol{I}_{bm}^{+}\boldsymbol{I}_{bm})\boldsymbol{J}_m^T\big]} & \boldsymbol{J}_m \ is \ used \\ \sqrt{det\big[\boldsymbol{J}_g(\boldsymbol{E}_n - \boldsymbol{I}_{bm}^{+}\boldsymbol{I}_{bm})\boldsymbol{J}_g^T\big]} & \boldsymbol{J}_g \ is \ used \end{cases} \tag{22}$$

$$\omega_{FER} = \begin{cases} \sqrt{det\big[\boldsymbol{I}_{bm}(\boldsymbol{E}_n - \boldsymbol{J}_{m}^{+}\boldsymbol{J}_{m})\boldsymbol{I}_{bm}^T\big]} & \boldsymbol{J}_m \ is \ used \\ \sqrt{det\big[\boldsymbol{I}_{bm}(\boldsymbol{E}_n - \boldsymbol{J}_{g}^{+}\boldsymbol{J}_{g})\boldsymbol{I}_{bm}^T\big]} & \boldsymbol{J}_g \ is \ used \end{cases} \tag{23}$$

The singular value decomposition (SVD) of matrix $\boldsymbol{J}$ is given by:

$$\boldsymbol{J} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^T \tag{24}$$

where $\boldsymbol{U} \in \mathbb{R}^{m_2 \times m_2}$ and $\boldsymbol{V} \in \mathbb{R}^{n \times n}$ are orthogonal matrices and $\boldsymbol{\Sigma} \in \mathbb{R}^{m_2 \times n}$ is given by

$$\boldsymbol{\Sigma} = (\boldsymbol{D} \mid \boldsymbol{0}) \qquad \boldsymbol{D} = diag\{\sigma_1, \sigma_2, \cdots, \sigma_{m_2}\} \tag{25}$$

with singular value $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_{m_2} \geq 0$. The number of non-zero singular values is equal to the rank of the matrix $\boldsymbol{J}$. If $rank(\boldsymbol{J}) < m_2$, then a singularity happens, which indicates that one or more singular values are zero. The manipulability of FAR and FER can be denoted by:

$$\omega_{(\bullet)} = \sigma_1\sigma_2\cdots\sigma_{m_2} \tag{26}$$

where $(\bullet)$ denotes FAR or FER here. This quantity can be utilized to design and analyze the coordinated motion between base and manipulator, from the viewpoint of minimum impact on the motion of spacecraft or end-effector.

## 5. Conclusions

A general method of inverse kinematics at velocity level for a redundant free-floating space robot has been proposed based on the concept of task priority and the pseudo-inverse redundancy resolution. Unlike fixed base manipulator, free-floating space robot can be used to coordinate the motion of spacecraft and the end-effector without using external devices, such as jet thrusters or reaction wheels.

Four alternative solutions for space robot joint velocities under different level of DOR have been derived based on the expressions of momentum conservation and the kinematic mapping. Three space tasks coordinating base/manipulator motion have been presented. Well-planned manipulator joint velocity control can certainly save the fuel and energy on board, therefore extend the system’s useful life span. Future work based on these solutions of inverse kinematics includes integration with an optimal controller or a model predictive controller, which could accomplish the closed-loop control of a free-floating space manipulator to track and capture the servicing target satellite dynamically.

## 6. Acknowledgements

The author wishes to acknowledge Prof. Dr. Ulrich Walter for his comments and support during the preparation of this manuscript.

## 7. References

[1]. Yoshida, K., “Engineering Test Satellite VII Flight Experiments for Space Robot Dynamics and Control: Theories on Laboratory Test Beds Ten Years Ago, Now in Orbit,” International Journal of Robotics Research, Vol. 22, No. 5, 2003, pp. 321–335.

[2]. Weismuller, T., Leinz, M., “GN&C Technology Demonstrated by the Orbital Express Autonomous Rendezvous and Capture Sensor System,” In 29th Annual AAS Guidance and Control Conference, AAS06-016, 2006.

[3]. Vafa, Z., Dubowsky, S., “The Kinematics and Dynamics of Space Manipulators: The Virtual Manipulator Approach,” International Journal of Robotics Research, Vol. 9, No. 4, 1990, pp. 3–21.

[4]. Xu, Y., Kanade, T., “Space Robotics: Dynamics and Control,” Kluwer, Boston, 1993.

[5]. Papadopoulos, E., Moosavian, S. Ali A., “Dynamics and control of space free-flyers with multiple arms,” Journal Advanced Robotics, Vol. 9, No. 6 1995, pp. 603–624.

[6]. Featherstone, R., “Robot dynamics algorithms”, 2nd ed, Springer, London, 2008.

[7]. Lilly, K. W., Bonaventura, C. S., “A generalized formulation for simulation of space robot constrained motion,” In Proc. IEEE Int. Conf. on Robot. and Automat., 1995, pp. 2835–2840.

[8]. Nenchev, D., Yoshida, K., Umetani, Y., “Introduction of Redundant Arms for Manipulation in Space,” In Proc. IEEE Int. Workshop Intelligent Robots Syst., 1988, pp. 679-684.

[9]. Umetani, Y., Yoshida, K., “Resolved motion rate control of space manipulators with generalized Jacobian matrix,” IEEE Trans. Robot. Automat., Vol. 5, No. 3, 1989, pp. 303–314.

[10]. Nenchev, D., Umetani, Y., Yoshida, K., “Analysis of a Redundant Free-Flying Spacecraft/Manipulator System,” IEEE Trans. Robot. Automat., Vol. 8, No. 1, 1992, pp. 1-6.

[11]. Yoshikawa, T., “Analysis and control of robot manipulators with redundancy,” In 1$^{st}$ Int. Symp. Robotics Res., M. Brady and R. Paul, Eds. Cambridge, MA: PIT Press, 1984, pp. 735-747.

[12]. Klein, C. A., Blaho, B. E., “Dexterity measures for the design and control of kinematically redundant manipulators,” International Journal of Robotics Research, Vol. 6, No. 2, 1987, pp. 72-83.

[13]. Maciejewski, A. A., Klein, C. A., “Obstacle Avoidance for Kinematically Redundant Manipulators in Dynamically Varying Environments,” International Journal of Robotics Research, Vol. 4, No. 3, 1985, pp. 109-117.

[14]. Nakamura, Y., Hanafusa, H., Yoshikawa, T., “Task-Priority Based Redundancy Control of Robot Manipulators,” International Journal of Robotics Research, Vol. 6, No. 2, 1987, pp. 3-15.