Analysis of a Redundant Free-Flying Spacecraft/Manipulator System

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***Abstract*— This paper presents an anlysis based on the momentum conservation equations of a redundant free-flying spacecraft/manipulator system, acting in zero-gravity environment. In order to follow a predefined end-effector path, the inverse kinematics at velocity level is considered. The redundancy i solved alternatively in terms of pseudoinverses and null-space components of the manipulator inertia matrix, the manipulator Jacobian matrix, and the generalized Jacobian matrix. A general manipulation task is defined as end-effector continuous path tracking with simultaneous attitude control of the spacecraft. Three subtasks of the general task are considered: 1) end-effector continuous path tracking with simultaneous attitude maintenance; 2) and 3) changing the attitude of the satellite while keeping fixed position/orientation of the end-effector with respect to either the orbit-fixed coordinate frame or the satellite. The case of manipulator motions that yield no spacecraft attitude disturbance is analyzed in more detail and a special "fixed-attitude-restricted" (FAR) Jacobian is defined. Through singular-value decomposition of this Jacobian, corresponding FAR dexterity measures (FAR manipulability and FAR condition number) are derived.**

I. Introduction

Recently, interest toward free-flying space robots has rapidly increased. A particular problem studied by several researchers is the disturbance of the position and attitude of the spacecraft by induced reaction forces and moments from the manipualtor arm. This disturbance is highly undersirable since it yields a disturbed end-effector motion, and conventional path planning methods developed for ground-fixed robots will not work in space. The manipulator work space will be reduced as well [1].

There are two main approaches to solve these problems. First, some means may be involved to correct the spacecraft disturbance. For instance, special devises such as reaction wheels and/or jet thrusters can be operated either simultaneously [2], [3] or intermittently [4] to manipulator motion. Another alternative for this approach is to use manipulator motion itself to correct the position and attitude of the spacecraft. This can be accomplished through small cyclic motions in joint space [1]. Second, new path planning methods can be utilized based on proper modeling of the spacecraft-manipulator dynamic interaction [1], [5]–[9]. The end-effector is able thereby to track some desired path while the spacecraft floats freely, i.e., the spacecraft/end-effector motion is noncoordinated. This might be undersirable in some cases.

Other formulations that guarantee coordinated motion of the spacecraft and the end-effector without using special compensating devices have been reported for the case of an n-DOF manipulator arm able to simultaneously control three-dimensional spacecraft attitude [10], [11].

The purpose of this paper is to propose a method for analyzing a redundant free-flying spacecraft/manipulator system and to discuss some control tasks for coordinated spacecraft/manipulator motion. Motions of the manipulator arm that do not disturb the attitude of the spacecraft will be of prime interest. Some dexterity measures will be defined for these motions as well.

II. Background and Notations

We use assumptions and fundamental equations for the free-flying system as presented in a previous work [6].

A. Assumptions

We consider an n-DOF manipulator arm with rotational joints mounted on a free-flying spacecraft. Manipulator links and spacecraft are regarded as rigid bodies. Thus, we obtain a mechanical chain of $n+1$ rigid bodies, acting under zero-gravity conditions. This system has $n+6$ generalized coordinates, n of them representing the generalized coordinates of the manipulator, and the other six defining the position and orientation of the spacecraft with respect to an inertial coordinate system. We assume that, at an initial state and while moving, the position and orientation of the spacecraft are well known from the inertial coordinate system. We assume no mechanical restrictions nor external forces or torques about the mass center, so that momemtum convervation and the equilibrium of forces and moments hold strictly true. There are no special attitude control devices such as reaction wheels or thrusters, and internal forces are generated only by joint motors.

B. Basic Equations and Relationships

The momemtum conservation equations are given as follows:

$$\sum_{i=0}^{n} m_i \dot{\boldsymbol{r}}_i = \text{const.} \tag{1}$$

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stands for translational momentum, and

$$\sum_{i=0}^{n} (\boldsymbol{I}_i \omega_i + m_i \boldsymbol{r}_i \times \dot{\boldsymbol{r}}_i) = \text{const.} \tag{2}$$

represents rotational momentum. In these equations, m_i is the mass of body i, $\boldsymbol{r}_i$ is the radius vector of its mass center, ω_i is its angular velocity, and $\boldsymbol{I}_i$ is its inertia matrix with respect to the mass center. Vectors and matrices are expressed with reference to the inertial coordinate system.

Further on, we use a geometric and a kinematic model of the free-flying system as given in [6]. It has been assumed that attitude control of the spacecraft is essential, whereas its position control can be neglected. Therefore, the models in 3-D space comprise a total of $n + 3$ generalized coordinates. Spacecraft attitude is described by means of yaw-pitch-roll notation.

From the geometric and kinematic models of the free-flying system, expressions for $\boldsymbol{r}_i$, $\dot{\boldsymbol{r}}_i$, and ω_i can be derived. If these expressions are substituted into the momentum conservation equation (2), the following expression can be obtained:

$$\boldsymbol{I_S}\dot{\boldsymbol{\Omega}} + \boldsymbol{I_M}\dot{\boldsymbol{\Theta}} = \boldsymbol{L_o} \tag{3}$$

where $\boldsymbol{L_o}$ is an initial momentum, $\boldsymbol{I_S}$ is the 3×3 inertia matrix of the spacecraft, $\boldsymbol{I_M}$ is the $3 \times n$ inertia matrix of manipulator links, $\dot{\boldsymbol{\Omega}}$ is the 3×1 velocity vector, and $\dot{\boldsymbol{\Theta}}$ is the $n \times 1$ joint velocity vector. Through (3), the spacecraft rotational momentum $\boldsymbol{I_S}\dot{\boldsymbol{\Omega}}$ can be distinguished from the manipulator momentum $\boldsymbol{I_M}\dot{\boldsymbol{\Theta}}$.

On the other hand, the relationship between motion rates can be presented as

$$\dot{\boldsymbol{x}} = \boldsymbol{J_S}\dot{\boldsymbol{\Omega}} + \boldsymbol{J_M}\dot{\boldsymbol{\Theta}} + \dot{\boldsymbol{x}}_{\mathbf{o}} \tag{4}$$

where $\dot{\boldsymbol{x}}_{\mathbf{o}}$ is an $M \times 1$ constant initial motion rate vector of the free-flying system, m being the number of end-effector task variables, $\boldsymbol{J_S}$ is an $m \times 3$ spacecraft Jacobian matrix, $\boldsymbol{J_M}$ is an $m \times n$ manipulator Jacobian matrix, and $\dot{\boldsymbol{x}}$ stands for the $m \times 1$ end-effector velocity vector. Through (4), the spacecraft-motion-dependent end-effector velocity $\boldsymbol{J_S}\dot{\boldsymbol{\Omega}}$ can be distinguished from the manipulator-motion-dependent velocity $\boldsymbol{J_M}\dot{\boldsymbol{\Theta}}$.

Expressions for matrices $\boldsymbol{I_S}$, $\boldsymbol{I_M}$, $\boldsymbol{J_S}$, and $\boldsymbol{J_M}$ can be found in [6].

Now, from (3) and (4) we can eliminate the unknown attitude variables $\dot{\boldsymbol{\Omega}}$, thus obtaining a joint/end-effector rate relationship. From (3) we have

$$\dot{\boldsymbol{\Omega}} = -\boldsymbol{I_S^{-1}}\boldsymbol{I_M}\dot{\boldsymbol{\Theta}} + \boldsymbol{I_S^{-1}}L_0. \tag{5}$$

Substitute $\dot{\boldsymbol{\Omega}}$ in (4) to obtain

$$\dot{\boldsymbol{x}} = \left(\boldsymbol{J_M} - \boldsymbol{J_S}\boldsymbol{I_S^{-1}}\boldsymbol{I_M}\right)\dot{\boldsymbol{\Theta}} + \dot{\boldsymbol{x}}_0 \tag{6}$$

where $\dot{\boldsymbol{x}}_0 = \dot{\boldsymbol{x}}_0 + \boldsymbol{J}_S\boldsymbol{I}_S^{-1}L_0$. The expression

$$\boldsymbol{J_G} \equiv \boldsymbol{J_M} - \boldsymbol{J_S}\boldsymbol{I_S^{-1}}\boldsymbol{I_M} \tag{7}$$

is said to be the *generalized Jacobian matrix* for space manipulators, mounted on a free-flying spacecraft [5], [6].

Summarizing, two important results with the generalized Jacobian approach should be pointed out: 1) this method has been based on a general description of a free-flying spacecraft/manipulator system, comprised of an n-DOF manipulator arm in 3-D space; and 2) the reaction effect of manipulator motion on the spacecraft has been described in terms of motion rates and momenta; no explicit expressions for accelerations and forces/torques have been utilized.

III. Inverse Kinematic Solutions of the Spacecraft/Manipulation System

A. Redundancies in the Free-Flying System

The degree of redundancy (DOR) for ground-fixed robots has been defined as the difference between the DOF of the manipulator arm and the number of end-effector task variables [12]. According to this definition, a free-flying robot with autonomous motion control of the spacecraft always will be redundant.

Let us consider now a free-flying system characterized by active control of manipulator joints only. This control has to solve the problem of coordinated spacecraft/manipulator motion. We can make a straightforward representation of the problem by two tasks: the spacecraft motion control task and the end-effector motion control task. These tasks determine the motion of the spacecraft and the end-effector with respect to the inertial coordinate system. Denote by m_1 and m_2 the number of task variables for the spacecraft task and the end-effector task, respectively. Then three cases of redundancy can be distinguished in relation to the number of manipulator joints n and the total number of task variables m_1+m_2. The first two cases are characterized by $n \geq m_1 + m_2$, and the manipulator arm will comprise some redundancy with respect to both the spacecraft and the end-effector tasks. This redundancy can be utilized to coordinate the motion of the spacecraft and the end-effector. For the first case, however $(n > m_1+m_2)$, there will also be some additional redundancy available, which can be resolved in order to satisfy certain criteria (for example, joint limit, singularity, or obstacle avoidance). For the second case $(n = m_1 + m_2)$, the available redundancy can be utilized only to coordinate the end-effector-spacecraft motion. The third case is characterized by $n < m_1 + m_2$. Although the manipulator arm may comprise some redundancy with respect to the spacecraft task $(n > m_1)$ or the end-effector task $(n > m_2)$, the DOR will not suffice to coordinate exactly spacecraft and manipulator end-effector motion.

B. Three Alternative Solutions of Inverse Kinematics

Let us consider the model of the free-flying system as described in Section II. For this system we would like to coordinate manipulator end-effector motion (i.e., velocity $\dot{\boldsymbol{x}}$) and angle maneuvers of the spacecraft (attitude velocity $\dot{\boldsymbol{\Omega}}$). Hence, the spacecraft task will be three-dimensional ($m_1 = 3$), whereas the end-effector task is generally of dimension $m_2 \leq 6$. Let us assume also that kinematic redundancy of the system is determined as $n > m_1 + m_2$. This yields the following solutions for joint rates from (3), (4), and (6):

$$\dot{\boldsymbol{\Theta}}_{\boldsymbol{MI}} = \boldsymbol{I}_{\boldsymbol{M}}^{\dagger}\left(\boldsymbol{L_o} - \boldsymbol{I_S}\dot{\boldsymbol{\Omega}}\right) + \left(\boldsymbol{I} - \boldsymbol{I}_{\boldsymbol{M}}^{\dagger}\boldsymbol{I_M}\right)\dot{\boldsymbol{\Phi}}_{\boldsymbol{MI}} \tag{8}$$

$$\dot{\Theta}_{MJ} = J_M^{\dagger}\left(\dot{x} - J_S\dot{\Omega} - \dot{x}_o\right) + \left(I - J_M^{\dagger}J_M\right)\dot{\Phi}_{MJ} \quad (9)$$

$$\dot{\Theta}_{GJ} = J_G^{\dagger}\left(\dot{x} - \hat{\dot{x}}_o\right) + \left(I - J_G^{\dagger}J_G\right)\dot{\Phi}_{GJ} \quad (10)$$

respectively. Here, I denotes the identity matrix, $(\cdot)^{\dagger}$ is a pseudoinverse of some matrix $(\cdot)$, whereas $(I - (\cdot)^{\dagger}(\cdot))$ is a projector onto the null space of this matrix and $\dot{\Phi}_{(\cdot)} \in \mathbb{R}^n$ is arbitrary. The subscripts we define as follows: MI stands for manipulator inertia, MJ means manipulator Jacobian, and GJ stands for generalized Jacobian.

Similar pseudoinverse-based formulations are well known from the analysis of ground-fixed redundant manipulators [13], [14].

It should be noted that the availability of redundancy with respect to the end-effector task ($n > m_2$) implies the existence of nonzero vectors in the null space of the Jacobian matrices J_M and J_G (similar to redundant ground-fixed robots). On the other hand, the redundancy with respect to the spacecraft task ($n > m_1$) yields nonzero vectors in the null space of the manipulator inertia matrix I_M. This is considered to be a unique feature of free-flying space robots, where the momentum equilibrium equation (see (3)) holds.

C. Defining the $\dot{\Phi}$ Vectors

The $\dot{\Phi}_{(\cdot)}$ vectors in (8)–(10) can be defined by considering some auxiliary restrictions. For the manipulator-inertia-based (8), this will be the restriction imposed on the end-effector motion (see (4)), whereas for the Jacobian-matrices-based (9) and (10), the restriction for spacecraft motion (see (3)) is to be considered. In order to guarantee the capability of spacecraft/manipulator motion coordination, it is necessary to assume redundancy distribution according to the expression $n \geq m_1 + m_2$. This assumption implies a common solution of each of the equation couples (8)/(4), (9)/(3), and (10)/(3), and hence, each of (8)–(10) will generate the same joint velocity, as long as singularities are avoided.

Methods for solving a redundant system under certain restrictions have been widely discussed for ground-fixed robots [16]. We used a constrained least squares approach [17] that avoids extension of system dimensions. According to this method, the $\dot{\Phi}_{(\cdot)}$ vectors and the respective joint velocities $\dot{\Theta}_{(\cdot)}$ can be specified as follows:

MI-Based Solution: Substitute (8) into (4) to obtain

$$\left(J_S - J_M I_M^{\dagger} I_S\right)\dot{\Omega} + J_M\left(I - I_M^{\dagger}I_M\right)\dot{\Phi}_{MI} = \dot{x} - \hat{\dot{x}}_0 \quad (11)$$

where $\hat{\dot{x}}_o = \dot{x}_o + J_M I_M^{\dagger} L_o$. Solve (11) for $\dot{\Phi}_{MI}$

$$\dot{\Phi}_{MI} = \left[J_M\left(I - I_M^{\dagger}I_M\right)\right]^{\dagger}\left[\dot{x} - \hat{\dot{x}}_o - \left(J_S - J_M I_M^{\dagger} I_S\right)\dot{\Omega}\right]. \quad (12)$$

Joint velocities can be obtained by substituting $\dot{\Phi}_{MI}$ back into (8):

$$\dot{\Theta}_{MI} = I_M^{\dagger}\left(L_o - I_S\dot{\Omega}\right) + \left[J_M\left(I - I_M^{\dagger}I_M\right)\right]^{\dagger} \cdot \left[\dot{x} - \hat{\dot{x}}_o - \left(J_S - J_M I_M^{\dagger} I_S\right)\dot{\Omega}\right] \quad (13)$$

where use has been made of $P(HP)^{\dagger} = (HP)^{\dagger}$, P being a symmetrical and indempotent projectional matrix [17].

MJ-Based Solution: Substituting (9) into (3) yields

$$I_M J_M^{\dagger}\left(\dot{x} - J_S\dot{\Omega} - \dot{x}_o\right) + I_M\left(I - J_M^{\dagger}J_M\right) \cdot \dot{\Phi}_{MJ} + I_S\dot{\Omega} = L_o. \quad (14)$$

This equation is solved for $\dot{\Phi}_{MJ}$ as follows:

$$\dot{\Phi}_{MJ} = \left[I_M\left(I - J_M^{\dagger}J_M\right)\right]^{\dagger} \cdot \left[\hat{L}_o - I_M J_M^{\dagger}\dot{x} - \left(I_S - I_M J_M^{\dagger} J_S\right)\dot{\Omega}\right] \quad (15)$$

where $\hat{L}_o = L_o + I_M J_M^{\dagger}\dot{x}_o$. Note that in the last equation the relative end-effector velocity term $\dot{x} - J_S\dot{\Omega}$ has been divided. This is possible since we assumed $n \geq m_1 + m_2$, which yields sufficient DOR to specify $\dot{x}$ and $\dot{\Omega}$ independently.

Joint velocities are obtained by substituting $\dot{\Phi}_{MJ}$ back into (9):

$$\dot{\Theta}_{MJ} = J_M^{\dagger}\left(\dot{x} - J_S\dot{\Omega} - \dot{x}_o\right) + \left[I_M\left(I - J_M^{\dagger}J_M\right)\right]^{\dagger} \cdot \left[\hat{L}_o - I_M J_M^{\dagger}\dot{x} - \left(I_S - I_M J_M^{\dagger} J_S\right)\dot{\Omega}\right] \quad (16)$$

where again the identity used in (13) has been applied.

GJ-Based Solution: Using (10) instead of (9) and following the previous procedure, we obtain a final expression for joint velocities as:

$$\dot{\Theta}_{GJ} = J_G^{\dagger}\left(\dot{x} - \hat{\dot{x}}_o\right) + \left[I_M\left(I - J_G^{\dagger}J_G\right)\right]^{\dagger} \cdot \left(\hat{\hat{L}}_o - I_M J_G^{\dagger}\dot{x} - I_S\dot{\Omega}\right) \quad (17)$$

where $\hat{\hat{L}}_o = L_o + I_M J_G^{\dagger}\hat{\dot{x}}_o$.

D. The Redundancy Distribution Case $n < m_1 + m_2$

If we assume $n < m_1 + m_2$, the alternative solutions (8)–(10) should be discussed in terms of tasks with order of priority [15].

For the manipulator-inertia-based solution (8), the higher priority task is the spacecraft motion control task, since the minimum-norm component of the $\dot{\Theta}_{MI}$ vector contains the spacecraft attitude velocity $\dot{\Omega}$. The self-motion of the arm, obtained through the null-space component, thereby has no impact on the spacecraft attitude velocity.

For the manipulator-Jacobian-based solution (9), the higher priority task is control of the end-effector velocity with respect to a coordinate frame with its origin placed at the mass center of the spacecraft/manipulator system and principle axes parallel to a spacecraft-fixed coordinate frame. This relative end-effector velocity is expressed by the term $\dot{x} - J_S\dot{\Omega}$ (from the minimum-norm component) in (9). The self-motion of the arm will have no impact on this end-effector velocity.

For the generalized-Jacobian-based solution (10), the higher priority task is the control of the end-effector motion with respect to an orbit-fixed coordinate frame, since the end-effector velocity $\dot{x}$ has been defined in terms of this frame.

The arbitrary vectors $\dot{\Phi}_{(\cdot)}$ can be specified again through the constrained least squares approach and by considering the following problems:

1) Solve the momentum equilibrium equation (3) for joint velocities $\dot{\Theta}_{MI}$ under minimization of the expression

$$\left\| \left(\dot{x} - J_S \dot{\Omega} - \dot{x}_o \right) - J_M \dot{\Theta}_{MI} \right\|^2 \tag{18}$$

and

2) solve (4) or (6) for joint velocities $\dot{\Theta}_{MJ}$ or $\dot{\Theta}_{GJ}$, respectively, under minimization of the terms

$$\left\| \left(L_o - I_S \dot{\Omega} \right) - I_M \dot{\Theta}_{(\cdot)} \right\|^2 \tag{19}$$

where $(\cdot)$ stands for MJ or GJ.

Minimization of (18) yields the end-effector velocity $J_M \dot{\Theta}_{MI}$, which is close in a least squares sense to the desired end-effector velocity $\dot{x} - J_S \dot{\Omega} - \dot{x}_o$. On the other hand, minimization of (19) yields manipulator momentums $I_M \dot{\Theta}_{(\cdot)}$ that in turn induce spacecraft rotational momentum, close in a least squares sense to the desired momentum $L_o - I_S \dot{\Omega}$.

Solutions of the above problems are given by (13), (16), and (17), respectively. However, we should note the following. Because of insufficient redundancy $(n < m_1 + m_2)$, there will be no common solution for each of the equation couples (8)/(4), (9)/(3), and (10)/(3) available, and each of (13), (16), and (17) will generate different joint rates.

IV. Manipulation Tasks for the Free-Flying Space Robot

Based on solutions (13), (16), and (17), as well as on the assumption for sufficient DOR $(n \geq m_1 + m_2)$, some taks for the free-flying spacecraft/manipulator system can be formulated that might be useful in future space missions. Here, we also assume no initial motion of the system ($L_o = 0$, $\dot{x}_o = 0$). It should be noted that the tasks asre stated in terms of motion rates.

A. Task I—Continuous Path Tracking by Manipulator End-Effector and Simultaneous Attitude Control of the Spacecraft

This is a general motion control task. It is reasonable to discuss further three other tasks that will be recognized as subcases of Task I.

B. Task II—Continuous Path Tracking with Simultaneous Spacecraft Attitude Maintenance

In some cases it is desirable to keep the orientation of the spacecraft unchanged during manipulator motion, for example, in order not to disturb the function of vehicle communication and sensing devices or the inner-vehicular micro-gravity environment. This task requires zero attitude velocity. Setting $\dot{\Omega} = 0$ in (13), (16), and (17), we obtain the three solutions

$$\dot{\Theta}_{MI} = [J_M (I - I_M^{\dagger} I_M)]^{\dagger} \dot{x} \tag{20}$$

$$\dot{\Theta}_{MJ} = J_M^{\dagger} \dot{x} - [I_M (I - J_M^{\dagger} J_M)]^{\dagger} I_M J_M^{\dagger} \dot{x} \tag{21}$$

and

$$\dot{\Theta}_{GJ} = J_G^{\dagger} \dot{x} - [I_M (I - J_G^{\dagger} J_G)^{\dagger} I_M J_G^{\dagger} \dot{x} \tag{22}$$

respectively.

C. Task III—Changing the Attitude of the Spacecraft while Keeping Fixed Position/Orientation of the End-Effector with Respect to the Inertial Coordinate Frame

This task yields a small angle maneuver for the spacecraft. It could be performed in order to adjust the work ranges of communication and sensing devices that have been disturbed by previous manipulation or other operations. Since the end-effector is fixed in terms of orbital coordinates, this task will be performed without any disturbance of the position/orientation of the target object already being grasped, relative to the orbit. The roles of the end-effector and the base (spacecraft) are actually exchanged, and therefore this task can be termed the "absolute-manipulator-inversion task."

Joint velocities for Task III can be obtained by substituting $\dot{x} = 0$ in (13), (16), and (17):

$$\dot{\Theta}_{MI} = I_M{}^{\dagger} (-I_S \dot{\Omega}) - [J_M (I - I_M^{\dagger} I_M)]^{\dagger} \cdot (J_S - J_M I_M^{\dagger} I_S) \dot{\Omega} \tag{23}$$

$$\dot{\Theta}_{MJ} = J_M^{\dagger} (-J_S \dot{\Omega}) - [I_M (I - J_M^{\dagger} J_M)]^{\dagger} \cdot (I_S - I_M J_M^{\dagger} J_S) \dot{\Omega} \tag{24}$$

and

$$\dot{\Theta}_{GJ} = -[I_M (I - J_G^{\dagger} J_G)]^{\dagger} I_S \dot{\Omega}. \tag{25}$$

D. Task IV—Changing the Attitude of the Spacecraft while Keeping Fixed Position/Orientation of the End-Effector with Respect to a Relative Coordinate Frame

This task is similar to the previous one; the only difference is the reference coordinate frame. It is convenient to choose a reference frame with its origin fixed at the mass center of the whole spacecraft/manipulator system and with coordinate axes parallel to some local spacecraft-fixed coordinate frame. Then simply set the term for the relative end-effector velocity $\dot{x} - J_S \dot{\Omega}$ in (13) and (16) to zero:

$$\dot{\Theta}_{MI} = I_M^{\dagger} (-I_S \dot{\Omega}) - [J_M (I - I_M^{\dagger} I_M)]^{\dagger} J_M I_M^{\dagger} I_S \dot{\Omega} \tag{26}$$

$$\dot{\Theta}_{MI} = -[I_M (I - J_M^{\dagger} J_M)]^{\dagger} I_S \dot{\Omega}. \tag{27}$$

Note that it is not convenient to use the generalized Jacobian formulation for this task since there is no explicit term for the relative end-effector velocity in (17).

Task IV can be termed the "relative-manipulator-inversion task."

Concluding this section, it should be noted that there is a solution for each task in terms of any of the available null spaces. The simplest solution for each task, however, is obtained through self-motion of the manipulator arm; these are solutions (20), (25), and (27) for Tasks II, III, and IV, respectively.

V. Fixed-Attitude Restricted Motion of the Free-Flying System

Among the above defined motion control tasks, Task II is of special interest since the manipulator arm will be able to move without inducing any reaction moments on the spacecraft. This type of motion is called "the fixed-attituted restricted (FAR) motion" of the free-flying system [18].

A. The FAR Jacobian Matrix

The kinematic relationship at velocity level for Task II is represented by (20). The matrix product $J_M(I - I_M^{\dagger} I_M)$ maps any joint velocity $\dot{\Phi}$ into a special end-effector velocity

$$\dot{x}_{\mathrm{FAR}} = J_M(I - I_M^{\dagger} I_M)\dot{\Phi}. \tag{28}$$

The matrix $J_{\mathrm{FAR}} \equiv J_M\left(I - I_M^{\dagger} I_M\right) \in \mathbb{R}^{m_2 \times n}$ is called the *FAR Jacobian matrix*, while the special end-effector velocity $\dot{x}_{\mathrm{FAR}}$ can be called the *FAR end-effector velocity*. If end-effector motion is planned in agreement with $\dot{x}_{\mathrm{FAR}}$, this would guarantee zero attitude velocity of the spacecraft.

The FAR Jacobian matrix provides a convenient tool for the analysis of free-flying spacecraft/manipulator systems from the view point of minimum attitude disturbance. This matrix can be used, for example, to analyze singular configurations of the system. Since J_{FAR} is represented as a two-matrix product, it can be shown that the set of singular configurations will be composed of two subsets: the set of manipulator singularities, where rank $J_M < m_2$, and the set of algorithmic singularities [19], where rank $J_M = m_2$ and rank $[J_M(I - I_M^{\dagger} I_M)] < m_2$. Note that due to the assumption $n \geq m_1 + m_2$, we always have rank $(I - I_M^{\dagger} I_M) = n - m_1 \geq m_2$.

It is worth noting that the FAR Jacobian is defined even if there is no redundancy of the manipulator arm with respect to the end-effector task. Redundancy with respect to the spacecraft task ($n > m_1$), however, is necessary. In our further discussion, we will assume $n > m_2$. An example of end-effector path planning and a control approach with fixed attitude maintenance and $n = m_2$ is presented in [18].

B. FAR Dexterity Measures

The FAR Jacobian matrix can be utilized to define some measures for free-flying space robots that correspond directly to the well-known manipulability measure, manipulability ellipsoid [20], or condition number [21] for ground-fixed robots.

The FAR manipulability measure for a given arm configuration can be defined as

$$w_{\mathrm{FAR}} = \sqrt{\det\left[J_{\mathrm{FAR}}(\Theta) J^T{}_{\mathrm{FAR}}(\Theta)\right]} \tag{29}$$

or, having in mind the notation $J_{\mathrm{FAR}}(\Theta) J_M(\Theta)[I - I_{\mathrm{M}}^{\dagger}(\Theta) I_M(\Theta)]$

$$w_{\mathrm{FAR}} = \sqrt{\det\left\{J_M(\Theta)\left[I - I_M^{\dagger}(\Theta) I_M\right] J_M^T(\Theta)\right\}}. \tag{30}$$

In (30) the symmetry and idempotency of the null space projector was utilized.

The singular-value decomposition (SVD) of the FAR Jacobian is

$$J_{\mathrm{FAR}} = U \Sigma V^T \tag{31}$$

where $U \in \mathbb{R}^{m_2 \times m_2}$ and $V \in \mathbb{R}^{n \times n}$ are orthogonal matrices, and having in mind the restriction $n \geq m_1 + m_2$ (or equivalently $n - m_1 \geq m_2$

$$\Sigma = \left[\begin{array}{cccc|c} \sigma_1 & & & & \\ & \sigma_2 & & 0 & \\ & & \ddots & & 0 \\ & 0 & & \sigma_{m_2} & \end{array}\right] \in \mathbb{R}^{m_2 \times n} \tag{32}$$

with singular values $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_{m_2} \geq 0$. In the case of algorithmic or manipulator singularities, one or more singular values will be zero. The principle axes of the FAR manipulability ellipsoid are $\sigma_i u_i$, $i = 1, \cdots, m_2$, u_i being vector columns of matrix U. the FAR manipulability measure also can be represented as

$$w_{\mathrm{FAR}} = \sigma_1 \sigma_2 \cdots \sigma_{m_2}. \tag{33}$$

Once the singular values are known, the FAR condition number can be computed as σ_{m_2}/σ_1. As another FAR dexterity measure, the minimum singular value of J_{FAR} can also be applied.

The dexterity measures thus defined can be utilized for the analysis and/or design of redundant free-flying spacecraft/manipulator systems from the view point of minimum spacecraft attitude disturbance. Thereby, the ideas developed by Klein and Blaho for application of various dexterity measures to redundant ground-fixed robots [21] can be followed, using the FAR Jacobian instead of the "conventional" manipulator Jacobian.

VI. Conclusion

A method of analyzing a redundant free-flying spacecraft/manipulator system has been presented. A manipulator arm in space comprises some kinematic redundancy, and therefore the analysis has been based on the pseudoinverse redundancy resolution technique. Along with the conventional end-effector trajectory tracking task, a second spacecraft attitude-velocity control task has been defined. This control task should be accomplished only through manipulator motion, thus avoiding use of external devices, such as reaction wheels or jet thrusters.

Three alternative solutions for manipulator joint velocities have been derived based on expressions for the manipulator link inertia matrix, the manipulator Jacobian, and the generalized Jacobian. Four tasks for coordinated spacecraft/manipulator motion have been suggested that might be useful in future space missions. The inertia-matrix-based solution can be used whenever fixed attitude of the spacecraft is desired. The manipulator-Jacobian-based solution can be applied for small-angle maneuvers of the spacecraft with no relative motion of the end-effector with respect to a mass center-spacecraft fixed coordinate frame. The generalized-Jacobian-based solution is suitable for small-angle maneuvers and no relative motion of the end-effector with respect to the inertial coordinate frame.

The case of manipulator motions yielding no disturbance of the spacecraft attitude has been discussed in more detail. A Jacobian matrix was defined for this purpose (the fixed-attitude-restricted Jacobian). Expressions for dexterity measures of the free-flying system were suggested as well.

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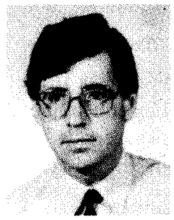

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