Kalman Filtering for Linear Time-Delayed Continuous-Time Systems with Stochastic Multiplicative Noises

Huanshui Zhang, Xiao Lu, Weihai Zhang, and Wei Wang

Abstract: The paper deals with the Kalman stochastic filtering problem for linear continuous-time systems with both instantaneous and time-delayed measurements. Different from the standard linear system, the system state is corrupted by multiplicative white noise, and the instantaneous measurement and the delayed measurement are also corrupted by multiplicative white noise. A new approach to the problem is presented by using projection formulation and *re-organized innovation analysis*. More importantly, the proposed approach in the paper can be applied to solve many complicated problems such as stochastic H_∞ estimation, H_∞ control stochastic system with preview and so on.

Keywords: Delay systems, filtering, innovation analysis, Riccati differential equations.

1. INTRODUCTION

In recent years, stochastic filtering has become a popular research topic, and it has been extensively developed [6,8,14,20,21]. The stochastic filtering problem as well as the stochastic control problem have wide application in areas such as nuclear fission and heat transfer, population models and immunology, see [2,5] and references therein. There are mainly two performances under which the filtering problem has been considered, one is the $H2$ index, [10,12,13,16], and the other one is H_∞ performance, see [6,8, 9,14,15]. The $H2$ filtering which is also termed as Kalman filtering is to address the minimization of the filtering error covariance. The so-called H_∞ stochastic filtering is to design an estimator to estimate the unknown state combination via output measurement, which guarantees the L_2 gain (from the external disturbance to the estimation error) to be less than a prescribed level $\gamma > 0$.

[13] presented a robust recursive Kalman filtering algorithm that addresses estimation problems arising in linear time-varying systems with stochastic parametric uncertainties. In [8], the discrete-time H_∞ stochastic filtering is considered and a filter of Luenberger-type structure is given. A modified-Riccati recursion which guarantees a given H_∞ filtering level to minimize an upper-bound on the covariance of the estimation error is presented. However, dimension of the resulting modified Riccati equation is larger than the origin system. In [15], the stationary discrete-time linear stochastic filtering is considered, where the stochastic uncertainty appears in the dynamic, input and measurement matrices. A filter which is based on the stochastic bounded real lemma (BRL) of [9] is given in terms of LMIs. However, the condition is only sufficient. In [14], continuous-time stochastic linear plants which are controlled by dynamic output feedback and subjected to both deterministic and stochastic perturbations are considered. A bounded real lemma is given and the necessary and sufficient conditions for the existence of a stabilizing compensator were obtained. Based on the *BRL* proposed in [14], [4] gives the continuous-time H_∞ stochastic filter, and the sufficient and necessary condition for the existence of the solutions in terms of LMIs is presented.

The above mentioned works are normal systems without delay. For the case of time-delay, retarded type linear systems robust L_2 filtering problem was considered in [19], where delay-independent conditions were derived in the form of LMIs. A more general neutral type of systems and a less conservative delay-dependent filter is given in [11]. Robust H_∞ filtering of uncertain systems with state delays has been considered in [17,18], only sufficient conditions have been derived. For the above mentioned works, the systems aren't corrupted by multiple white noise and delayless measurement can be used.

Manuscript received June 5, 2006; revised June 5, 2007; accepted June 18, 2007. Recommended by Editor Jae Weon Choi. This work was supported by the National Nature Science Foundation of China (60574016, 60474013, 6047 4058, 60534010).

Huanshui Zhang is with Shandong University, Jinan 250001, P. R. China (e-mail: huanshuizhang@hotmail. com).

Xiao Lu and Weihai Zhang are with Shandong University of Science and Technology, Qingdao 266510, P. R. China (e-mail: {luxiao98, w_hzhang}@163.com).

Wei Wang and Xiao Lu are with Research Center of Information and Control, Dalian University of Technology, Dalian 116023, P. R. China (e-mail: wangwei@dlut.edu.cn).

In this paper, we consider the finite horizon robust Kalman stochastic filtering problem for linear time-varying continuous-time system with delayed measurement, assuming that the system state is corrupted by white noise, and the measurement including the delayed measurement are also corrupted by white noise. Different from the works for the robust stochastic filter [13], delayed measurement is introduced in this paper. By using re-organized innovation analysis based on the knowledge of Krein space [1,21-24], a new approach to the Kalman stochastic filtering is proposed. With the proposed new approach, the stochastic filter for time delayed system is obtained. The solution is given in terms of Riccati equation which has the same dimension as the origin system.

The rest of the paper is organized as follows. The system under consideration and the problem statement are given in Section 2. Main results concluding the Riccati equation and the optimal stochastic filter are presented in Section 3. Some concluding remarks are made in Section 4 as sensor fusion [7] and network systems. The key is the re-organization of the measurements and innovation.

2. PROBLEM STATEMENT

Consider the following stochastic differential systems

$$\dot{\mathrm{x}}(t) = A(t)\,\mathrm{x}(t) + B(t)\,\mathrm{x}(t)\,\mathrm{w}(t) + \mathrm{u}(t), \tag{1}$$

$$y_0(t) = C_0(t)\,\mathrm{x}(t) + D_0(t)\,\mathrm{x}(t)\,\mathrm{w}(t) + \mathrm{v}_0(t), \tag{2}$$

$$y_1(t) = C_1(t)\,\mathrm{x}(t) + D_1(t)\,\mathrm{x}(t)\,\mathrm{w}(t) + \mathrm{v}_1(t), \tag{3}$$

where $t_l = t - l, t \le T$, with T is the horizon, $\mathrm{x}(t) \in R^n$ is the state, $y_0(t) \in R^{P_0}$ is measurement, and $y_1(t) \in R^{p_1}$ is time-delayed measurements. $\mathrm{u}(t) \in R^n$ is input sequence, $\mathrm{w}(t) \in R^1$ $\mathrm{v}_i(t) \in R^{P_l}$ are white noises of zero means. $A(t) \in R^{n\times n}$, $B(t) \in R^{n\times n}$, $C_i(t) \in R^{p_i\times n}$ and $D_i(t) \in R^{p_i\times n}$ are known time-varying matrices. $\mathrm{x}(t)\big|_{t=0} = \mathrm{x}_0$ and $\langle u(t), u(s)\rangle = Q(t)\delta_{t,s}$ $\langle \mathrm{v}_i(t), \mathrm{v}_j(t)\rangle = R_i(t)\delta_{i,j}\delta_{t,s}$, $\langle \mathrm{w}(t), \mathrm{w}(s)\rangle = M(t)\delta_{t,s}$ and $\langle \mathrm{x}_0, \mathrm{x}_0\rangle = \prod_0$.

The stochastic filtering problem considered in the paper for the system model (1)-(3) can be stated as

Problem P: Given the observation $\{y_0(\tau)|_{0\le\tau\le t}, y_1(t)|_{l\le\tau\le t}\}$, find the optimal estimation of $\mathbf{x}$(t), denoted $\hat{\mathrm{x}}(t|t)$, such that

$$\min \varepsilon\left\{\left[\mathrm{x}(t) - \hat{\mathrm{x}}(t|t)\right]'\left[\mathrm{x}(t) - \hat{\mathrm{x}}(t|t)\right]\right\}, \tag{4}$$

where ε denotes the mathematical expectation.

Remark 1: The stochastic Kalman filtering problem for discrete-time system has been considered in [13], where delayless measurement is available. However, in practice, it is inevitable that delay exists in system, then we give the system model (1)-(3). Such problem has important applications in many engineering problems such as sensor fusion [7] and network systems. The key is the re-organization of the measurements and innovation.

Let *y(t)* denote the observation of the system (4) at time t and $v(t)$ is the related observation noise at time then we have

$$y(t) = \begin{cases} y_0(t), & 0 \le t \le l \\ \begin{bmatrix} y_0(t) \\ y_1(t) \end{bmatrix}, & t \ge l, \end{cases} \tag{5}$$

$$v(t) = \begin{cases} v_0(t), & 0 \le t \le l \\ \begin{bmatrix} v_0(t) \\ v_1(t) \end{bmatrix}, & t \ge l. \end{cases} \tag{6}$$

Remark 2: For $t < l$, the measurement is mainly from one channel $y_0(t)$, and when $t \ge l$, the measurement is two channel $y_0(t)$ and $y_1(t)$.

Remark 3: In our paper, the main idea is to reorganize the delayed measurements (that is $y_0(t)$, $y_1(t)$) into delay-free measurements from different systems (y(t)), thus (5) and (6) can be easily introduced. In this way, the system with delayed measurement has been changed into a system with two different measurements without delay.

For the convenience of discussion, we will consider the case of $t \ge l$, the case of $t < l$ can be studied in the same way.

3. MAIN RESULTS

In this section, we will introduce the re-organized innovation in terms of which, the new stochastic filter will be given [22].

3.1. Re-organized measurements

As is well known, given the measurement sequence $\{y(s)|_{0\le s\le t}\}$, the optimal state filter $\hat{\mathrm{x}}(t|t)$ is the projection of $\mathbf{x}(t)$ onto the linear space spanned by the measurement sequence, denoted by $\mathscr{L}\{y(s)|_{0\le s\le t}\}$ [3]. In the following part, we shall re-organize the measurement sequence and give a new observation sequence which is delayless.

Lemma 1: The linear space of $\mathscr{L}\{y(s)|_{0\le s\le t}\}$ is equivalent to

$$\mathscr{L}\left\{\mathscr{Y}_2(s)\Big|_{0\le s\le t};\mathscr{Y}_1(s)\Big|_{t_l\le s\le t}\right\}, \tag{7}$$

where

$$\mathscr{Y}_2(s)=\begin{bmatrix} \mathrm{y}_0(s)\\ \mathrm{y}_1(s+l)\end{bmatrix},\qquad \mathscr{Y}_1(s)=\mathrm{y}_0(s) \tag{8}$$

satisfies that

$$\mathscr{Y}_1(s)=\mathscr{C}_1(s)\,\mathrm{x}(s)+\mathscr{D}_1(s)\,\mathrm{x}(s)\,\mathrm{w}(s)+\mathscr{V}_1(s), \tag{9}$$

$$\mathscr{Y}_2(s)=\mathscr{C}_2(s)\,\mathrm{x}(s)+\mathscr{D}_2(s)x(s)w(s)+\mathscr{V}_2(s) \tag{10}$$

with

$$\mathscr{C}_2(s)=\begin{bmatrix} C_0(s)\\ C_1(s)+l\end{bmatrix},\mathscr{D}_2(s)=\begin{bmatrix} D_0(s)\,\mathrm{w}(s)\\ D_1(s)\,\mathrm{w}(s+l)\end{bmatrix},$$

$$\mathscr{V}_2(s)=\begin{bmatrix} \mathrm{v}_0(s)\\ \mathrm{v}_1(s+l)\end{bmatrix},\mathscr{C}_1(s)=C_0(s), \tag{11}$$

$$\mathscr{D}_1(s)=D_0(s),\mathscr{V}_1(s)=\mathrm{v}_0(s).$$

Moreover, $\mathscr{V}_i(s)$ is a white noise of zero means and covariance matrices

$$Q_{\mathscr{V}_2}(s)=diag\left\{R_0(s),R_1(s+l)\right\},\qquad Q_{\mathscr{V}_1}(s)=R_0(s). \tag{12}$$

Proof: It is straightforward by re-organizing the delayed observations.

Remark 4: In the above lemma, by re-organizing all the measurements, the system with l-delay measurements has been changed into a normal system with measurement of (9) which does not involve delays.

3.2. Re-organized innovation sequence

Before presenting the re-organized innovation, we will give the following definitions first.

Definition 1: Given time instant t, the estimator $\hat{\xi}(s,1)$ for $s>t_l$ is the optimal estimation of $\xi(s)$ given the observation

$$\mathscr{L}\{\mathscr{Y}_2(\tau)\Big|_{0\le\tau\le t_l};\mathscr{Y}_1(\tau)\Big|_{t_l\le\tau\le s}\}. \tag{13}$$

For $s = t_l$, $\hat{\xi}(s,1)$ is the optimal estimation of $\xi(s)$ given the observation

$$\mathscr{L}\{\mathscr{Y}_2(\tau)\Big|_{0\le\tau\le t_l}\}. \tag{14}$$

Definition 2: Given time instant t, the estimator $\hat{\xi}(s,2)$ for $s< t_l$, $\hat{\xi}(s,2)$ is the optimal estimation of $\xi(s)$ given the observation

$$\mathscr{L}\{\mathscr{Y}_2(\tau)\Big|_{0\le\tau\le s}\}. \tag{15}$$

Remark 5: ξ in $\hat{\xi}(s,i)$ $(i=1,2)$ in the above two definitions may be x or $\mathscr{Y}_1$, $\mathscr{Y}_2$.

Definition 3: For i= 2 and any $s\ge 0$

$$\mathscr{W}_2(s)\triangleq\mathscr{Y}_2(s)-\hat{\mathscr{Y}}_2(s,2), \tag{16}$$

and for i=1 and any s>0

$$\mathscr{W}_1(t_l+s)\triangleq\mathscr{Y}_1(t_l+s)-\hat{\mathscr{Y}}_1(t_l+s,1), \tag{17}$$

where $\hat{\mathscr{Y}}_i(s,i)$ is as defined in Definitions 2 and 3.

Thus we have the following lemma.

Lemma 2: The sequence $\mathscr{W}$ defined in the above is mutually uncorrelated and

$$\{\mathscr{W}_2(s)\Big|_{0\le s\le l};\mathscr{W}_1(t+s)\Big|_{0\le s\le l};\}, \tag{18}$$

spans the same linear space as $\mathscr{L}\{\mathrm{y}(s)\big|_{0\le s\le t}\}$.

Proof: By recalling [22,23] where the case for the linear system without multiplicative noise is considered, the proof can be established by applying a similar argument as in [22,23].

Remark 6: It will be seen in the following discussions, the re-organized innovation presented in the subsection will play an important role in deriving the Kalman filer for the system with measurement delays.

From the definition (16)-(17), it is easy to observe the relationships as

$$\mathscr{W}_2(s)=\mathscr{C}_2(s)\,\mathrm{e}(s)+\mathscr{D}_2(s)\,\mathrm{x}(s)+\mathscr{V}_2(s), \tag{19}$$

$$\begin{aligned}\mathscr{W}_2(t_l+s)=\mathscr{C}_1(t_l+s)\,\mathrm{e}_1(t_l+s)&\\ +\mathscr{D}_1(t_l+s)\,\mathrm{x}(t_l+s)\,\mathrm{w}(t_l+s)&\\ +\mathscr{V}_1(t_l+s),s>0,&\end{aligned} \tag{20}$$

where

$$\mathrm{e}_2(s)=\mathrm{x}(s)-\hat{\mathrm{x}}(s,2), \tag{21}$$

$$\mathrm{e}_1(t_l+s)=\mathrm{x}(t_l+s)-\hat{\mathrm{x}}(t_l+s,1). \tag{22}$$

As in the standard Kalman filtering formulation, $\mathscr{P}_2(s)$ and $\mathscr{P}_1(t_l+s)$ play key role for computing the filter for the system with measurement delays, it is obvious

$$\mathscr{P}_2(s)\triangleq\left\langle \mathrm{e}_2(s),\mathrm{e}_2(s)\right\rangle,s\ge 0, \tag{23}$$

$$\mathscr{P}_1(t_l+s)\triangleq\left\langle \mathrm{e}_1(t_l+s),\mathrm{e}_1(t_l+s)\right\rangle, \tag{24}$$

$\mathrm{e}_2(s)$ and $\mathrm{e}_1(t_l+s)$ are as in (21) and (22), respectively. $\prod(s)$ is the state covariance matrix and is defined as

$$\prod(s)\triangleq\left\langle \mathrm{x}(s),\mathrm{x}(s)\right\rangle,s\ge 0. \tag{25}$$

Then we have the following theorem for computing $\mathscr{P}_2(s)$ and $\mathscr{P}_1(t_l+s)$.

Theorem 1: The one-step state predictive covariance matrices defined in the above are calculated by:

1) $\mathscr{P}_2(s)$ is the solution to the following Riccati differential equation

$$\begin{aligned}\frac{d\mathscr{P}_2(s)}{ds} &= A(s)\mathscr{P}_2(s)+\mathscr{P}_2(s)A'(s)\\ &+B(s)\prod(s)B'(s)M(s)\\ &-\mathscr{K}_2(s)Q_{w_2}(s)\mathscr{K}_2'(s)\\ &+Q(s),\mathscr{P}_2(0)=\prod{}_0,\end{aligned} \tag{26}$$

$$\begin{aligned}\frac{d\prod(s)}{ds} &= A(s)\prod(s)+\prod(s)A'(S)\\ &+B(s)\prod(s)B'(s)M(s)\\ &+Q(s),\prod(s)=\prod{}_0,\end{aligned} \tag{27}$$

where

$$\begin{aligned}\mathscr{K}_2(s) &= \left[\mathscr{P}_2(s)C_0'(S)+B(S)\prod(s)D_0'(s)\right.\\ &\left.+\mathscr{P}_2(s)C_1'(s+l)\right]\times Q_{w_2}^{-1}(s)\end{aligned} \tag{28}$$

and

$$\begin{aligned}Q_{w_2}(s) &= diag\{D_0(s)\prod(s)M(s)D_0'(s)+R_0(s),\\ &D_1(s+l)\prod(s)M(s+l)D_1'(s+l)+R_1(s+l)\}.\end{aligned} \tag{29}$$

2) $\mathscr{P}_1(t_l+s)$ is the solution to the following Riccati differential equation

$$\begin{aligned}\frac{d\mathscr{P}_1(t_l+s)}{ds} &= A(t_l+s)\mathscr{P}_1(t_l+s)+\mathscr{P}_1(t_l+s)A'(t_l+s)\\ &+B(t_l+s)\prod(t_l+s)B'(t_l+s)M(t_l+s)\\ &-\mathscr{K}_1(t_l+s)Q_{w_1}(t_l+s)\mathscr{K}_1'(t_l+s)\\ &+Q(t_l+s),\mathscr{P}_1(t_l)=\mathscr{P}_2(t_l),\end{aligned} \tag{30}$$

$$\begin{aligned}\frac{d\prod(t_l+s)}{ds} &= A(t_l+s)\prod(t_l+s)+\prod(t_l+s)A'(t_l+s)\\ &+B(t_l+s)\prod(t_l+s)B'(t_l+s)M(t_l+s)\\ &+Q(t_l+s),\end{aligned} \tag{31}$$

where

$$\begin{aligned}\mathscr{K}_1(t_l+s) &= [\mathscr{P}_1(t_l+s)\mathscr{C}_1'(t_l+s)+B(t_l+s)\\ &\times\prod(t_l+s)\mathscr{D}_1'(t_l+s)M(t_l+s)]\\ &\times Q_{w_1}^{-1}(t_l+s)\end{aligned} \tag{32}$$

with

$$\begin{aligned}Q_{w_1}(t_l+s) &= \mathscr{D}_1(t_l+s)\prod(t_l+s)M(t_l+s)\mathscr{D}_1'(t_l+s)\\ &+Q_{\mathrm{v}}(t_l+s).\end{aligned} \tag{33}$$

Proof: 1) The proof of (26)-(28):

By using Lemma 1 state-space model (1)-(3) can be approximated as

$$\begin{aligned}\frac{\mathrm{x}(i\Delta+\Delta)-\mathrm{x}(i\Delta)}{\Delta} &= A(i\Delta)\,\mathrm{x}(i\Delta)+\mathrm{u}(i\Delta)\\ &+B(i\Delta)\,\mathrm{x}(i\Delta)\,\mathrm{w}(i\Delta),\end{aligned} \tag{34}$$

$$\mathscr{Y}_2(i\Delta)=\mathscr{C}_2(i\Delta)\,\mathrm{x}(i\Delta)+\mathscr{D}_2(i\Delta)\,\mathrm{x}(i\Delta)+\mathscr{V}_2(i\Delta), \tag{35}$$

where

$$\begin{aligned}\mathscr{C}_2(i\Delta) &= \begin{bmatrix}C_0(i\Delta)\\ C_1(i\Delta+\frac{l}{t}i\Delta)\end{bmatrix},\\ \mathscr{D}_2(i\Delta) &= \begin{bmatrix}D_0(i\Delta)\,\mathrm{w}(i\Delta)\\ D_0(i\Delta+\frac{l}{t}i\Delta)\,\mathrm{w}(i\Delta+l\Delta)\end{bmatrix},\\ \mathscr{V}_2(i\Delta) &= \begin{bmatrix}v_0(i\Delta)\\ v_1(i\Delta+\frac{l}{t}i\Delta)\end{bmatrix}.\end{aligned} \tag{36}$$

Equivalently, (34) and (35) can be rewritten as

$$\begin{aligned}\mathrm{x}(i\Delta+\Delta) &= [I+A(i\Delta)\Delta]\,\mathrm{x}(i\Delta)\\ &+\Delta[B(i\Delta)\,\mathrm{x}(i\Delta)\,\mathrm{w}(i\Delta)+\mathrm{u}(i\Delta)],\end{aligned} \tag{37}$$

$$\mathscr{Y}_2(i\Delta)=\mathscr{C}_2(i\Delta)\,\mathrm{x}(i\Delta)+\mathscr{D}_2(i\Delta)\,\mathrm{x}(i\Delta)+\mathscr{V}_2(i\Delta) \tag{38}$$

with

$$\begin{aligned}&\left\langle \mathrm{u}(i\Delta),\mathrm{u}(i\Delta)\right\rangle=\frac{Q(i\Delta)}{\Delta}\delta_{i,j},\\ &\left\langle \mathscr{V}_2(i\Delta),\mathscr{V}_2(i\Delta)\right\rangle=\frac{Q_{\mathscr{V}_2(i\Delta)}}{\Delta}\delta_{i,j},\\ &\left\langle \mathrm{w}(i\Delta),\mathrm{w}(i\Delta)\right\rangle=\frac{M(i\Delta)}{\Delta}\delta_{i,j},\end{aligned}$$

and $\left\langle \mathrm{x}_0,\mathrm{x}_0\right\rangle=\prod_0$.

Applying discrete-time Kalman filtering formulation to the above system, it follows that

$$\begin{aligned}\hat{\mathrm{x}}(i\Delta+\Delta,2) &= \left[I+A(i\Delta)\Delta\right]\hat{\mathrm{x}}(i\Delta,2)\\ &+\mathscr{K}_{p_2}(i\Delta)\mathscr{W}_2(i\Delta),\\ \hat{\mathrm{x}}(0,2) &= 0,\end{aligned} \tag{39}$$

where $\mathscr{W}_2(i\Delta)$ is the innovation of process $\mathscr{Y}_2(i\Delta)$ which is given by

$$\begin{aligned}\mathscr{W}_2(i\Delta) &\triangleq \mathscr{Y}_2(i\Delta)-\hat{\mathscr{Y}}_2(i\Delta,2)\\ &= \mathscr{Y}_2(i\Delta)-\mathscr{C}_2(i\Delta)\,\hat{\mathrm{x}}(i\Delta,2)\\ &= \mathscr{C}_2(i\Delta)\,\mathrm{e}_2(i\Delta)+\mathscr{D}_2(i\Delta)\,\mathrm{x}(i\Delta)\\ &+\mathscr{V}_2(i\Delta),\end{aligned} \tag{40}$$

where $\hat{\mathscr{Y}}_2(i\Delta,2)$ and $\hat{\mathrm{x}}(i\Delta,2)$ are respectively the optimal estimation of $\hat{\mathscr{Y}}_2(i\Delta,2)$ and $\mathrm{x}(i\Delta)$ given observation $\hat{\mathscr{Y}}_2(s\Delta)(s=i-1,i-2,\ldots,0)$ associated with u(s), w(s) and $\mathscr{V}_2(s\Delta)$ for $s=i-1,i-2,\ldots,0$, and

$$\mathrm{e}_2(i\Delta) \triangleq \mathrm{x}(i\Delta)-\hat{\mathrm{x}}(i\Delta,2).$$

Define

$$\Omega(i\Delta)=diag\{D_0(i\Delta)\prod(i\Delta)M(i\Delta)D_0'(i\Delta),$$
$$D_1(i\Delta+\frac{l}{t}i\Delta)\prod(i\Delta)M(i\Delta+\frac{l}{t}i\Delta)D_1'(i\Delta+\frac{l}{t}i\Delta)\}$$

then from (40), it can be easily seen

$$\begin{aligned}&\langle\mathscr{W}_2(i\Delta),\mathscr{W}_2(i\Delta)\rangle_{u,w,v}\\&=\left[\mathscr{C}_2(i\Delta)\mathscr{P}_2(i\Delta)\mathscr{C}_2'(i\Delta)+\frac{\Omega(i\Delta)}{\Delta}+\frac{Q_{\mathscr{V}_2}(i\Delta)}{\Delta}\right]\\&=\frac{R_{w_2}(i\Delta)}{\Delta},\end{aligned}\tag{41}$$

that is

$$\begin{aligned}R_{\mathrm{w}_2}(i\Delta)&=\Delta\mathscr{C}_2(i\Delta)\mathscr{P}_2(i\Delta)\mathscr{C}_2'(i\Delta)\\&\quad+\Omega(i\Delta)+Q_{\mathscr{V}_2}(i\Delta),\end{aligned}\tag{42}$$

where $\mathscr{P}_2(i\Delta)\triangleq\langle\mathrm{e}_2(i\Delta),\mathrm{e}_2(i\Delta)\rangle_{u,w,v}$

$$\lim_{\Delta\to0,i\Delta\to s}R(i\Delta)=\Omega(s)+Q_{\mathscr{V}_2}(s)\triangleq Q_{\mathrm{w}_2}(s).\tag{43}$$

That is

$$\begin{aligned}Q_{\mathrm{W}_2}(s)&=diag\{D_0(s)\prod(s)M(s)D_0{}'(s)\\&\quad+R_0(s),\ D_1(s+l)\prod(s)M(s+l)\\&\quad\times D'(s+l)+R_1(s+l)\},\end{aligned}\tag{44}$$

which is (29).

From (37), (40), and (42), we know

$$\begin{aligned}\mathscr{K}_{p_2}(i\Delta)&=\langle\mathrm{x}(i\Delta+\Delta),\mathscr{W}_2(i\Delta)\rangle\left[\frac{R_{\mathrm{W}_2}(i\Delta)}{\Delta}\right]^{-1}\\&=\langle[I+A(i\Delta)]\mathrm{x}(i\Delta),\ \mathscr{W}_2(i\Delta)\rangle\left[\frac{R_{\mathrm{W}_2}(i\Delta)}{\Delta}\right]^{-1}\\&\quad+\langle\Delta B(i\Delta)\mathrm{x}(i\Delta)\mathrm{w}(i\Delta),\mathscr{W}_2(i\Delta)\rangle\left[\frac{R_{\mathrm{W}_2}(i\Delta)}{\Delta}\right]^{-1}\\&\quad+\langle\Delta\mathrm{u}(i\Delta),\mathscr{W}_2(i\Delta)\rangle\left[\frac{R_{\mathrm{W}_2}(i\Delta)}{\Delta}\right]^{-1}\\&=[I+A(i\Delta)\Delta]\mathscr{P}_2(i\Delta)\mathscr{C}_2'(i\Delta)\left[\frac{R_{\mathrm{W}_2}(i\Delta)}{\Delta}\right]^{-1}\\&\quad+B(i\Delta)\prod(i\Delta)[D_0'(i\Delta)M(i\Delta),0]\left[\frac{R_{\mathrm{W}_2}(i\Delta)}{\Delta}\right]^{-1},\end{aligned}\tag{45}$$

thus

$$\begin{aligned}\mathscr{K}_2(s)&\triangleq\lim_{\Delta\to0}\frac{\mathscr{K}_{p_2}(i\Delta)}{\Delta}\\&=[\mathscr{K}_{2,11}(s),\mathscr{K}_{2,12}(s)]Q_{w_2}^{-1}(s),\end{aligned}\tag{46}$$

where $\mathscr{K}_{2,11}(s)=\mathscr{P}_2(s)C_0'(s)+B(s)\prod(s)D_0'(s)M(s)$ and $\mathscr{K}(s)=\mathscr{A}(s)C'(s+l)$.

Combining (40) with (39) yields

$$\begin{aligned}\hat{\mathrm{x}}(i\Delta+\Delta,2)&=[I+A(i\Delta)]\hat{\mathrm{x}}(i\Delta,2)+\mathscr{K}_{p_2}(i\Delta)\\&\quad\times[\mathscr{Y}_2(i\Delta)-\mathscr{C}_2(i\Delta)\hat{\mathrm{x}}(i\Delta,2)],\end{aligned}\tag{47}$$

which can be rewritten as

$$\begin{aligned}&\frac{\hat{\mathrm{x}}(i\Delta+\Delta,2)-\mathrm{x}(i\Delta,2)}{\Delta}\\&=A(i\Delta)\hat{\mathrm{x}}(i\Delta,2)\\&\quad+\frac{\mathscr{K}_{p_2}(i\Delta)}{\Delta}[\mathscr{Y}_2(i\Delta)-\mathscr{C}_2(i\Delta)\hat{\mathrm{x}}(i\Delta,2)]\end{aligned}\tag{48}$$

$\Delta\to0,i\Delta\to s$ yields

$$\begin{aligned}\frac{d\,\hat{\mathrm{x}}(s,2)}{ds}&=A(s)\hat{\mathrm{x}}(s,2)\\&\quad+\mathscr{K}_2(s)[\mathscr{Y}_2(s)-\mathscr{C}_2(s)\hat{\mathrm{x}}(s,2)].\end{aligned}\tag{49}$$

Note that

$$\mathscr{W}_2(i\Delta)=\mathscr{Y}_2(i\Delta)-\mathscr{C}_2(i\Delta)\hat{\mathrm{x}}(i\Delta,2).\tag{50}$$

Similarly, we have

$$\mathscr{W}_2(s)=\mathscr{Y}_2(s)-\mathscr{C}_2(s)\hat{\mathrm{x}}(s,2).\tag{51}$$

We shall compute $\mathscr{P}_2(s)$ and $\prod(s)$ as follow. Combining (37) with (39) yields

$$\begin{aligned}\mathrm{e}_2(i\Delta+\Delta)&\triangleq\mathrm{x}(i\Delta+\Delta)-\hat{\mathrm{x}}(i\Delta+\Delta,2)\\&=[I+A(i\Delta)\Delta]\mathrm{e}_2(i\Delta)\\&\quad+\Delta B(i\Delta)\mathrm{x}(i\Delta)\mathrm{w}(i\Delta)\\&\quad+\Delta\mathrm{u}(i\Delta)-\mathscr{K}_{p_2}(i\Delta)\mathscr{W}_2(i\Delta).\end{aligned}\tag{52}$$

Then $\mathscr{P}_2(i\Delta)=\langle\mathrm{e}_2(i\Delta),\mathrm{e}_2(i\Delta)\rangle$ can be given as

$$\begin{aligned}&\mathscr{P}_2(i\Delta+\Delta)\\&=\mathscr{P}_2(i\Delta)+\Delta\mathscr{P}_2(i\Delta)A'(i\Delta)\\&\quad+\Delta A(i\Delta)\mathscr{P}_2(i\Delta)+A(i\Delta)\mathscr{P}_2(i\Delta)A'(i\Delta)\Delta^2\\&\quad+\Delta B(i\Delta)\prod(i\Delta)B'(i\Delta)M(i\Delta)+\Delta Q(i\Delta)\\&\quad-\mathscr{K}_{P_2}(i\Delta)R_{w_2}(i\Delta)\mathscr{K}_{p_2}'(i\Delta)/\Delta.\end{aligned}\tag{53}$$

Thus

$$\frac{\mathscr{P}_2(i\Delta+\Delta)-\mathscr{P}_2(i\Delta)}{\Delta}$$

$$=\mathscr{P}_2(i\Delta)A'(i\Delta)+A(i\Delta)\mathscr{P}_2(i\Delta)$$
$$+B(i\Delta)\prod(i\Delta)B'(i\Delta)M(i\Delta)+Q(i\Delta) \quad (54)$$
$$-\mathscr{K}_{p_2}(i\Delta)R_{w_2}(i\Delta)\mathscr{K}'_{p_2}(i\Delta)/\Delta^2$$

$\Delta\to 0, i\Delta\to s$ yields

$$\frac{d\mathscr{P}_2(s)}{ds}=A(s)\mathscr{P}_2(s)+\mathscr{P}_2(s)A'(s)$$
$$+B(s)\prod(s)B'(s)M(s)+Q(s) \quad (55)$$
$$-\mathscr{K}_2(s)Q_{w_2}(s)\mathscr{K}_2'(s),$$

which is (26).

According to (37),we have

$$\prod(i\Delta+\Delta)=\prod(i\Delta)+\Delta\prod(i\Delta)A'(i\Delta)+\Delta A(i\Delta)\prod(i\Delta)$$
$$+A(i\Delta)\prod(i\Delta)A'(i\Delta)\Delta^2+\Delta Q(i\Delta)$$
$$+\Delta B(i\Delta)\prod(i\Delta)B'(i\Delta)M(i\Delta), \quad (56)$$

then

$$\frac{\prod(i\Delta+\Delta)-\prod(i\Delta)}{\Delta}=\prod(i\Delta)A'(i\Delta)+A(i\Delta)\prod(i\Delta)$$
$$+B(i\Delta)\prod(i\Delta)B'(i\Delta)M(i\Delta)$$
$$+Q(i\Delta), \quad (57)$$

which is (27),this the first part is over.

2) The proof of (30)-(32)

Similar to the proof of part 1), it can be omitted.

3.3. Stochastic optimal filter

Now we are in the position to present the main result in our paper, the following Kalman filtering formulation for the measurement delayed system can be easily given from the proof of Theorem 1.

Theorem 2: Consider the system (1)-(3), optimal estimate $\hat{x}(t|t)$ is given by

$$\hat{x}(t|t)=\hat{x}(t,1), \quad (58)$$

where $\hat{x}(t,1)$ is as defined in (13), which is computed in the following steps:

Step 1: Calculate $\hat{x}(s,2)$ for $s=t_l$:

$$\frac{d\,\hat{x}(s,2)}{ds}=A(s)\,\hat{x}(s,2)$$
$$+\mathscr{K}_2(s)[\mathscr{Y}_2(s)-\mathscr{C}_2(s)\,\hat{x}(s,2)], \quad (59)$$
$$\hat{x}(0,2)=0,$$

where $\mathscr{K}_2(s)$ is as in (28).

Step 2: Compute $\hat{x}(t_l+s,1)$ for $0\le s\le l$ with the initial value $\hat{x}(t_l,2)$

$$\frac{d\,\hat{x}(t_l+s,1)}{ds}=A(t_l+s)\,\hat{x}(t_l+s,1) \quad (60)$$
$$+\mathscr{K}_1(t_l+s)[\mathscr{Y}_1(t_l+s)$$
$$-\mathscr{C}_1(t_l+s)\,\hat{x}(t_l+s,1)],$$
$$\hat{x}(t_l,1)=\hat{x}(t_l,2),$$

where $\mathscr{K}_1(t_l+s)$ is as in (32).

Step 3: The estimator $\hat{x}(t,1)$ is computed from Step 2 when s=l.

Proof: (58) is from (13), (59) is just (49), and (60) is from Theorem 1.

Remark 7: It can be easily seen that, the new derived stochastic filter is different from the filter from the system without delayed measurement. The new filter involves two parts, for each part, two Riccati equations will be used, but the dimension of the Ricatti equations is the same as the origin system.

Remark 8: It should be noted that the proposed approach has given the stochastic optimal filter for continuous-time systems with delayed measurements by the technique of *re-organized innovation analysis.* The advantage of the proposed approach is mainly the computation burden, since it has the same dimension as the origin system. However, the performance of the optimal filter is same as traditional approach.

4. NUMERICAL EXAMPLES

In this section, an example will be given to show the efficiency of the proposed approach. Consider the system (1)-(3) with $l = 0.4s$, and

$$A(t)=\begin{pmatrix}-4 & -3\\ 1 & -1\end{pmatrix},\ B(t)=\begin{pmatrix}0.5 & 2\\ 1 & 1\end{pmatrix},$$
$$C_0(t)=[0.5t\quad t],\ C_1(t)=[t\quad 3t],$$
$$D_0(t)=[1\quad 1],\ D_1(t)=[0.5\quad 2].$$

The initial state value $x(0)=[0.5\ \ 1]$, $\hat{x}(0)=[0\ \ 0]'$ and $u(t), w(t), v_0(t)$, and $v_1(t)$ are uncorrelated white noises with zero means and unity covariance matrices, i.e.,

$$P_0(t)=I_2,\ Q(t)=I_2.M(t)=1,\ R_0(t)=R_1(t)=1.$$

Our aim is to calculate the optimal estimate $\hat{x}(t|t)$ of the signal $\mathbf{x}(t)$ based on observation $\{y_0(\tau)\big|_{0\le\tau\le t,}\ y_1(\tau)\big|_{l\le\tau\le t}\}$ In this example, we assume that $t\ge 0.4s$. When $0\le\tau\le 0.4s$, the problem becomes a standard Kalman filtering for the system without the delayed measurement. To compute the filter, we re-organize the linear space $\mathscr{L}\{\{y_0(\tau)\big|_{0\le\tau\le t,}\ y_1(\tau)\big|_{l\le\tau\le t}\}\}$ equivalently as the following linear space

$$\mathscr{L}\{\mathscr{Y}_2(\tau)\big|_{0\le\tau\le t_1};\mathscr{Y}_1(\tau)\big|_{t_1\le\tau\le t}\}, \quad (61)$$

where $t_1 = t - 0.4$, and the new observation $\mathscr{Y}_i(\tau)$ $(i = 1, 2)$ are

$$\begin{aligned} \mathscr{Y}_2(\tau) &= [\mathrm{y}_0^T(\tau) \quad \mathrm{y}_1^T(\tau + 0.4)]^T, \\ \mathscr{Y}_1(\tau) &= \mathrm{y}_0(\tau), \end{aligned} \tag{62}$$

satisfying

$$\mathscr{Y}_1(\tau) = \mathscr{C}_1(\tau)\mathrm{x}(\tau) + \mathscr{D}_1(\tau)\mathrm{x}(\tau)\mathrm{w}(\tau) + \mathscr{V}_1(\tau), \tag{63}$$

$$\mathscr{Y}_2(\tau) = \mathscr{C}_2(\tau)\mathrm{x}(\tau) + \mathscr{D}_2(\tau)\mathrm{x}(\tau) + \mathscr{V}_2(\tau), \tag{64}$$

with

$$\begin{aligned} &\mathscr{C}_2(\tau) = \begin{pmatrix} 0.5\tau & \tau \\ \tau & 3\tau \end{pmatrix}, \quad \mathscr{C}_1(\tau) = \begin{bmatrix} 0.5\tau & \tau \end{bmatrix}, \\ &\mathscr{D}_2(\tau) = \begin{pmatrix} \mathrm{w}(\tau) & \mathrm{w}(\tau) \\ 0.5\,\mathrm{w}(\tau + 0.4) & 2\,\mathrm{w}(\tau + 0.4) \end{pmatrix}, \\ &\mathscr{D}_1(\tau) = [1 \quad 1], \quad \mathscr{V}_2(\tau) = \begin{bmatrix} \mathrm{v}_0(\tau) \\ \mathrm{v}_1(\tau + 0.4) \end{bmatrix}, \\ &\mathscr{V}_1(\tau) = \mathrm{v}_0(\tau), \end{aligned} \tag{65}$$

and $\mathscr{V}_i(t)(i = 1, 2)$ are white noises with zero means and unity covariance $Q_{\mathscr{V}_i}(t) = I_i$.

Having obtained the re-organized observation sequence, the computation of the optimal filter $\hat{\mathrm{x}}(t|t)$ can be summarized by three steps below:

Step 1: Compute $\mathscr{P}_2^{\tau}$ by (26) with l = 0.4s and initial value $\mathscr{P}^3 = 0$. Compute $\hat{\mathrm{x}}(\tau, 2)$ by (60) with initial value $\hat{\mathrm{x}}(0, 2) = 0$.

Step 2: Note $\mathscr{P}_1^{t_1} \equiv \mathscr{P}_2^{t_1}$ and $\hat{\mathrm{x}}(t_l, 1) \equiv \hat{\mathrm{x}}(t_l, 2)$. $\mathscr{P}_1^{t_l+\tau}$ and $\hat{\mathrm{x}}(t_l + \tau, 1)$ are calculated by (30) and (61), respectively, with i=1 and initial values $\mathscr{P}_1^{t_1} \equiv \mathscr{P}_2^{t_1}$ and $\hat{\mathrm{x}}(t_l, 1) \equiv \hat{\mathrm{x}}(t_l, 2)$.

Step 3: Calculate the optimal filter $\hat{\mathrm{x}}(t|t)$ in (58).

In the simulation, the sampling period is T= 0.02s Fig. 1. The estimate for $\mathrm{x}_1(t)$ with delayed measurement and time horizon is T= 4s.

We denote $\hat{\mathrm{x}}(t|t) = [\hat{\mathrm{x}}_1(t|t), \hat{\mathrm{x}}_2(t|t)]'$. The tracking performance of the filter $\hat{\mathrm{x}}(t|t)$ with the time delayed measurement is given in Fig. 1 whereas $\hat{\mathrm{x}}(t|t)$ is given in Fig. 2. In Fig. 3, the comparison of the sum of error variance between the cases for the system with delay-free measurement and time-delayed measurement is given. In Figs. 1 and 2, the solid line denotes the true signal and the dashed line is its estimate. In Fig. 3, the solid line denotes the sum of error variance for the case when only the delay free measurement is used and the dashed line is the sum of error variance for the case when the delay free

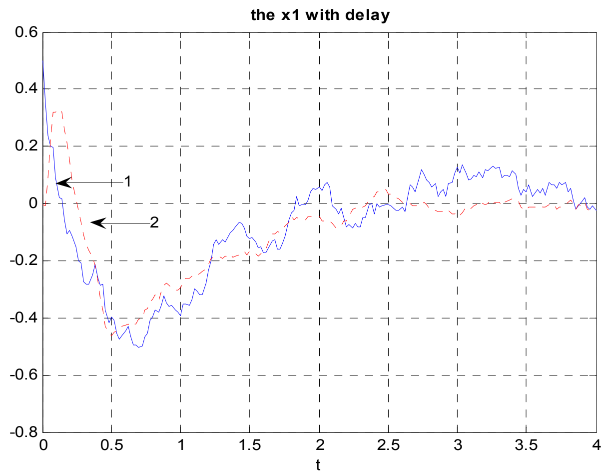

Fig. 1. The estimate for $\mathrm{x}_1(t)$ with delayed, measurement where 1 is the original state and 2 is the estimate.

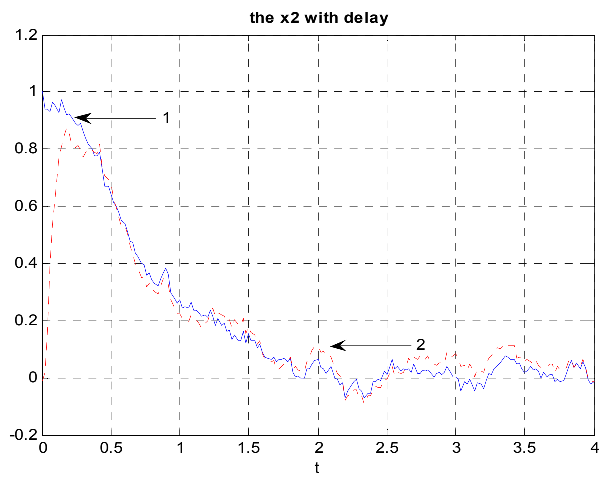

Fig. 2. The estimate for $\mathrm{x}_2(t)$ with delayed measurement where 1 is the original state and 2 is the estimate.

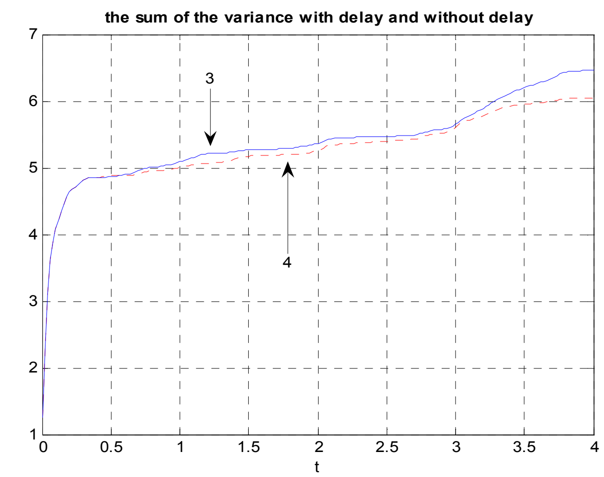

Fig. 3. The comparison for the sum of error covariance where 3 is for without delay and 4 is for with delayed measurement.

measurement and the time-delayed measurement is used. It can be observed that with additional delayed measurement, the filtering performance is significantly improved as expected.

5. CONCLUSION

The paper has studied the stochastic Kalman filtering problem for a linear continuous-time system with both instantaneous and time-delayed measurements. Multiplicative white noise is introduced in the state-space model. By using *re-organized innovation analysis,* the new stochastic filter is presented. The solution to the derived Kalman filter is given in terms of Riccati differential equations. More importantly, the proposed approach in the paper can be applied to solve many complicated problems such as stochastic H_∞ estimation [20], H_∞ control stochastic system with preview and so on.

REFERENCES

[1] B. Hassibi, A. H. Sayed, and T. Kailath, *Indefinite-Quadratic Estimation and Control A unified Approach to H_2 and H_∞ Theories*, Prentice-Hall, Englewood. Cliffs, NJ, 1999.

[2] O. L. V. Costa and C. S. Kubrusly, "State-feedback H_∞ control for discrete-time infinite-dimensional stochastic bilinear systems," *Journal of Mathematical Systems, Estimation and Control,* vol. 6, pp. 1-32, 1996.

[3] B. D. O. Anderson and J. B. Moore, *Optimal Filtering*, Prentice-Hall, Englewood, Cliffs, NJ, 1979.

[4] E. Gershon, J. N. Limebeer, U. Shaked, and I. Yaesh, "H_∞ filtering of continuous-time linear systems with stochastic parameter uncertainties," *IEEE Trans. on Automatic Control*, vol. 46, no. 11, pp. 1788-1793, 2001.

[5] M. M. Mohler and W. J. Kolodziej, "An overview of stochastic bilinear control process," *IEEE Trans. on System, Man and Cybernetics*, vol. 10, no. 12, pp. 913-919, 1980.

[6] V. Dragan and A. Stoica, "A γ -attenuation problem for discrete-time time-varying stochastic systems with multiplicative noise," *Reprint Series of the Institute of Mathematics of Romanian Academy*, no. 10, 1998.

[7] L. A. Klein, *Sensor and Data Fusion Concepts and Applications*, SPIE Press, 1999.

[8] E. Gershon, U. Shaked, and I. Yaesh, "H_∞ control and filtering of discrete-time stochastic systems with multiplicative noise," *Automatica,* vol. 37, pp. 409-417, 2001.

[9] A. E. Bouhtouri, D. Hinriechsen, and A. J. Pritchard, "H_∞ type control for discrete-time stochastic system," *Int. J. Robust Nonlinear Control*, vol. 9, pp. 923-948, 1999.

[10] R. M. Oisiovici and S. L. Cruz, "State estimation of batch distillation columns using an extended Kalman filter," *Chemical Engineering Science*, vol. 55, pp. 4667-4680, 2000.

[11] E. Fridman, U. Skaked, and L. H. Xie, "Robust *H*2 *f*iltering of linear systems with time delays," *Proc. of the 41st IEEE Conf. Decision Contr.*, USA, Dec. 2002.

[12] Y. Chen and B. S. Chen, "Minimax robust deconvolution filters under stochastic parametric and noise uncertainties," *IEEE Trans. on Signal Processing*, vol. 42, pp. 32-45, 1994.

[13] F. Wang and V. Balakrishnan, "Robust Kalman filters for linear time-varying systems with stochastic parametric uncertainties," *IEEE Trans. on Signal Processing*, vol. 50, no. 4, pp. 803-813, 2002.

[14] D. Hinrichsen and A. J. Pritchard, "Stochastic H_∞," *SIAM J. Control Optim.,* vol. 36, pp. 1504-1538, 1998.

[15] E. Gershon, U. Shaked, and I. Yaesh, "Robust H_∞ estimation of stationary discrete-time linear processes with stochastic uncertainties," *Systems and Control Letters*, vol. 45, pp. 257-269, 2002.

[16] L. H. Xie, Y. Soh, and C. E. de Souza, "Robust Kalman filtering for uncertain discrete-time systems," *IEEE Trans. on Automatic Control,* vol. 39, pp. 1310-1314, 1994.

[17] C. E. De Souza and L. H. Xie, "Delay-dependent robust H_∞ control of uncertain linear state-delayed systems," *Automatica*, vol. 35, pp. 1313-1321, 1999.

[18] C. E. De Souza, R. Palhares, and P. Peres, "Robust H_∞ filtering for uncertain linear continuous-time systems with multiple time-varying state delays: An LMI approach," *Proc. of the 38th CDC,* Phoenix, Arizona, USA, Dec. 1999.

[19] M. Mahmound, N. Muthairi, and S. Bingulac, "Robust Kalman filtering for continuous time-lag systems," *Systems and Control Letters*, vol. 38, pp. 309-319, 1999.

[20] W. Zhang, B. Chen, and C. Tseng, "Robust H_∞ filtering for nonlinear stochastic systems," *IEEE Trans. on Signal Processing*, vol. 53, no. 2, pp. 589-598, 2005.

[21] X. Lu, L. Xie, H. Zhang, and W. Wang, "Robust Kalman filtering for discrete-time systems with measurement delay," *IEEE Trans. on Circuits and Systems Part II*, vol. 54, no. 6, pp. 522-526, 2007.

[22] H. Zhang and D. Zhang, "Finite horizon H_∞ fixed-lag smoothing for time-varying continuous systems," *IEEE Trans. on Circuits and Systems Part II*, vol. 51, no. 9, pp. 496-499, 2004.

[23] H. Zhang, X. Lu, and D. Cheng, "Optimal estimation for continuous-time systems with delayed measurements," *IEEE Trans. on Automatic Control*, vol. 51, no. 5, pp. 823-827, 2006.

[24] X. Lu, H. Zhang, W. Wang, and K. L. Teo, "Kalman filtering for multiple time-delay systems," *Automatica*, vol. 41, no. 8, pp. 1455-1461, 2005.

Huanshui Zhang graduated in Mathematics from the Qufu Normal University in 1986 and received the M.Sc. and Ph.D. degrees in Control Theory and Signal Processing from the Heilongjiang University, P. R. China, and Northeastern University, P. R. China, in 1991 and 1997, respectively. He worked as a Postdoctoral Fellow at the Nanyang Technological University from 1998 to 2001 and Research Fellow at Hong Kong Polytechnic University from 2001 to 2003. He joined Shandong Taishan College in 1986 as an Assistant Professor and became an Associate Professor in 1994. He joined Shandong University in 1999 as a Professor. Currently he is a Professor of Shandong University. His interests include optimal estimation, robust filtering and control, time delay systems, singular systems, wireless communication and signal processing.

Xiao Lu received the B.S. degree from College of Electron and Information, Dalian Jiaotong University in 1998. He is now a Teacher in Shandong University of Science and Technology, and he is toward a Ph.D. degree of Dalian University of Technology. His interests include optimal estimation and control, robust filtering and control, time-delay systems.

Weihai Zhang received the M.S. degree from Hangzhou University, and the Ph.D. degree from Zhejiang University, Hangzhou, China, in 1994 and 1998, respectively. From August 1998 to May 2001, he worked at Shandong Institute of Light Industry as an Associate Professor. He was a Postdoctoral Researcher from May 2001 to July 2003 at National Tsing Hua University, Hsinchu, Taiwan. From 2004 to 2005, he did research work as a Research Fellow at City University of Hong Kong and Shenzhen Graduate School of Harbin Institute of Technology, respectively. He is currently a Professor of Shandong University of Science and Technology. His research interests include linear and nonlinear stochastic control, robust filtering and stochastic stability.

Wei Wang obtained the Bachelor, Master Degree and Ph.D. in Industrial Automation from Northeastern University, China, in 1982, 1986, and 1988 respectively. He is presently a Professor and Director of Research Center of Information and Control, Dalian University of Technology, China. Previously he was a Post-doctor at the Division of Engineering Cybernetics, Norwegian Science and Technology University (1990-1992), Professor and Vice Director of Research Center of Automation, Northeastern University, China (1995-2000). His research interests are in non-linear control, predictive control, robotics, computer integrated manufacturing systems, and computer control of industrial process. He was awarded the National Distinguished Young Fund of the National Natural Science Foundations of China in 1998. He has published over 100 papers in international and domestic journals. He is currently the Chairman of IFAC Technical Committee (4.4) of Cost Oriented Automation (2005-2008).