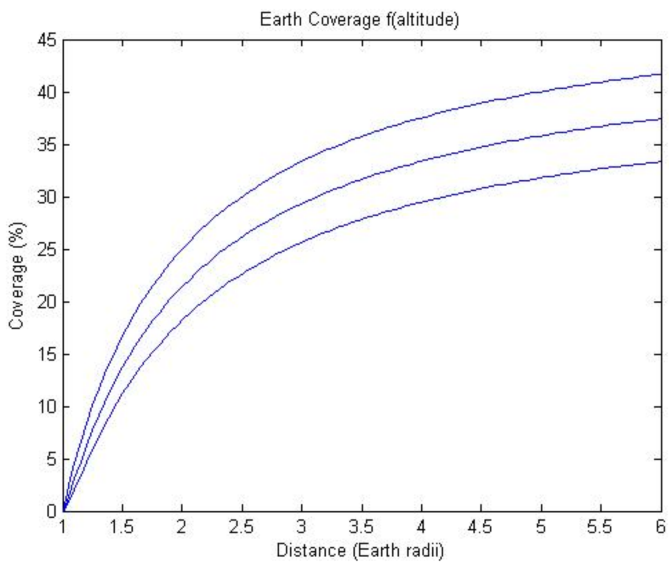
2 Ground Tracks

2.1 Earth Coverage

The ground track of a satellite orbiting Earth ($h = 260km$) is influenced by two major effects that will cause a shift in the ground track measured when the satellite is passing the equator.

1. What are the two influences causing this shift for the ground track and what is the condition for obtaining a repeating ground track?
2. Consider a S/C in LEO with 16 revolutions per day and determine whether such a satellite will cover the whole Earth in one day (hint: swath width).
3. Determine the percentage coverage of the Earth for a Geostationary satellite with a radius of 6,61 Earth radii for minimum elevation angles of 0°, 5°, 10°.



1.
The spacecraft experiences a shift $\Delta\Phi$ in the pass over the equator for succesive revolutions. The $\Delta\Phi$ is composed of two parts:

- $\Delta\Phi_1$ due to the rotation of the earth
- $\Delta\Phi_2$ due to the regression of the line of nodes

The first part can be computed with the following equation:

$$\Delta\Phi_1 = -2\pi \cdot \frac{\tau}{\tau_E}$$

τ denotes the periods for

- spacecraft revolution $= \tau$

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- a sideral day $\tau_E = 86160sec$

The second part can be computed with the equation

$$\Delta\Phi_2 = -\frac{3\pi \cdot J_2 \cdot R_E^2 \cdot \cos i}{a^2\left(1-e^2\right)^2}$$

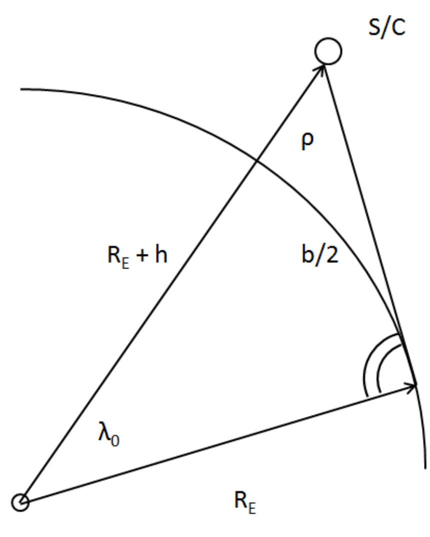
In order to obtain a repeating ground track

$$n \cdot \Delta\Phi = m \cdot 2\pi$$

has to be fulfilled. Here n denotes the number of spacecraft revolutions performed and m the number of erath revolutions performed before the identical ground track occurs.

2.
First we can compute the distance of two passes on the equator. We estimate roughly

$$b_{required} \approx \frac{40,000km}{16} = 2,500km$$



In the drawing b/2 denotes the distance on the ground that is covered by the spacecraft (swath width), h denotes the orbit height. In the drawing the elevation angle ϵ is set to zero (causing the 90° angle on the right side). This means that the direct line to the satellite would be exactly at the horizon. This case works only in theory. In reality due to atmosphere pertubations and objects (houses, trees,...) this angle does not work. A minimum elevation of about 5° is used in most cases. From the drawing we can state the following:

$$\sin\rho = \cos\lambda_0 = \frac{R_E}{R_E + h}$$

$$\lambda_{rad} \cdot R = \frac{b}{2}$$

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$$\frac{2 \cdot \lambda_0}{360°} \cdot 2\pi R_E = b$$

Using the given orbit height we find

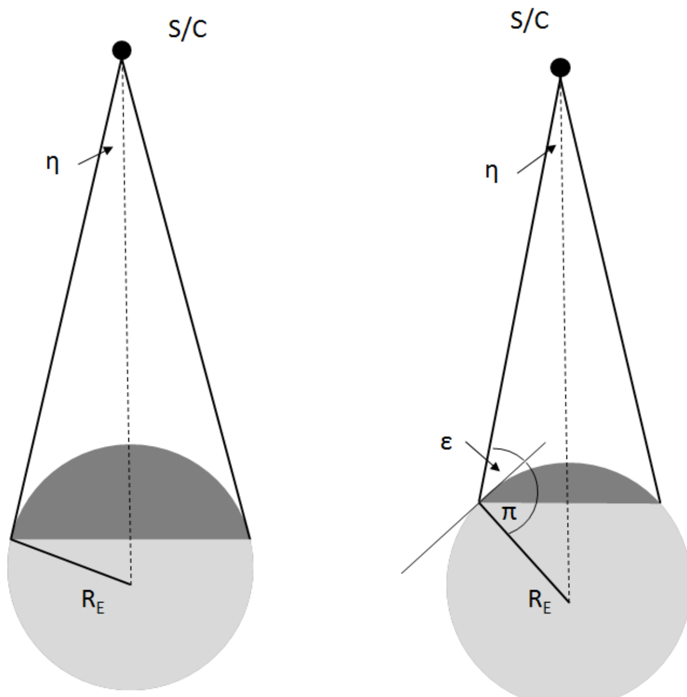
$$\cos \lambda_0 = \frac{6,378km}{6,378km + 260km} = 0.961$$

$$\lambda_0 = 16.1°$$

$$b = \frac{2 \cdot 16.1°}{360°} \cdot 2\pi \cdot 6,378km = 3,584km$$

This maximal possible footprint b is larger than the needed $b_{required}$. So in theory this ground track should cover the whole surface.

3.
For the coverge from a geostationary orbit we assume the following geometry:



From the drawing we can state:

$$r = R_E + h$$

$$\frac{r}{\sin\left(\epsilon + \frac{\pi}{2}\right)} = \frac{R_E}{\sin \eta}$$

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$$\eta = \arcsin\left(\frac{R_E}{r} \cdot \cos\epsilon\right)$$

$$\pi = \lambda + \frac{\pi}{2} + \epsilon + \eta$$

$$\lambda = \frac{\pi}{2} - \epsilon - \eta$$

$$\lambda = \arccos\left(\frac{R_E}{r} \cdot \cos\epsilon\right) - \epsilon$$

The area of an spherical cap can be computed using:

$$A_C = 2\pi \cdot R_E^2(1 - \cos\lambda)$$

So the fraction of the earth surface can be found with:

$$F_C = \frac{A_C}{A} = \frac{2\pi \cdot R_E^2 \cdot (1 - \cos\lambda)}{4\pi \cdot R_E^2} = \frac{1}{2} \cdot (1 - \cos\lambda)$$

For the given parameter ($r = 6.61 R_E = 42,158 km$) we get the following percentages:

- $\lambda_0 = 81.4°$ and $F_{C,0} = 42.4\%$
- $\lambda_5 = 76.3°$ and $F_{C,5} = 38.1\%$
- $\lambda_{10} = 71.4°$ and $F_{C,10} = 34.0\%$

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