ELC 3035, Lecture 7: Observer design

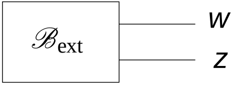
Ivan Markovsky

- Observers
- Observer design by pole placement
- Duality between observer and controller design
- Pole placement by output feedback

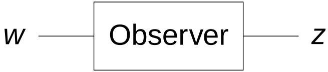
General observer design problem

Given dynamical system $\mathscr{B}_{\text{ext}}$ with two types of external variables:

- observed variables w
- to-be-estimated variables z



find system (called observer) accepting w and producing z



We will consider the case: $\mathscr{B} = \mathscr{B}_{\text{i/s/o}}(A, B, C, D)$, $w = (u, y)$, $z = x$.

Lecture 6: nonrecursive, feedforward observer for the initial state $x(0)$

Now our goal is recursive feedback observer for the current state $x(t)$

Output feedback control
separation and certainty principles

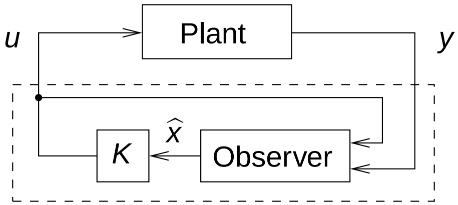
We can extend a given state-feedback controller

$$u = Kx$$

to output feedback controller

$$u = K\widehat{x}$$

by using the observer state estimate $\widehat{x}$ in place of x.



Output feedback controller

Internal model and feedback principles

The observer design is based on the following principles:

1. Internal model: the model run by u, gives an estimate $\widehat{x}$ for x
2. Feedback: correct the estimate $\widehat{x}$, so that the error

$$x(t) - \widehat{x}(t) =: e(t) \to 0 \qquad \text{as} \quad t \to \infty$$

Let the feedback be a linear function of the output error

$$\text{feedback correction} = L(y - \widehat{y})$$

Then the observer for the model $\mathscr{B}_{\mathrm{i/s/o}}(A,B,C,D)$ is

$$\begin{aligned} \sigma\widehat{x} &= A\widehat{x} + Bu - L(y - \widehat{y}) \\ \widehat{y} &= C\widehat{x} + Du \end{aligned}$$

Error dynamics

Our goal is to choose L, so that the state error $e(t) \to 0$ as $t \to \infty$.

The dynamics of e is

$$\begin{aligned}\sigma e &= \sigma(x - \widehat{x}) \\ &= Ax + Bu - A\widehat{x} - Bu + L(y - \widehat{y}) \\ &= A(x - \widehat{x}) + LC(x - \widehat{x}) \\ &= \underbrace{(A + LC)}_{A_o} e\end{aligned}$$

i.e., $e \in \mathscr{B}_{ss}(A_o)$ — an autonomous LTI system.

Therefore, $e(t) \to 0$ as $t \to \infty$ is equivalent to stability of $\mathscr{B}_{ss}(A_o)$.

Comparison with state-feedback stabilization

In the state feedback stabilization problem we have

$$\sigma x = Ax + Bu \qquad \text{and} \qquad u = Kx$$

which gives an autonomous LTI closed loop system

$$\sigma x = \underbrace{(A + BK)}_{A_\mathrm{c}} x$$

and the aim is to choose K, so that $\mathscr{B}_{\mathrm{ss}}(A_\mathrm{c})$ is stable.

Observer design by pole placement

The condition $e(t) \to 0$ as $t \to \infty$ is a minimum requirement.

In fact we want $e(t) \to 0$ fast

(possibly in a finite (small) number of steps $\rightsquigarrow$ deadbeat observer)

The error dynamics is governed by the poles of the matrix

$$A_o := A + LC$$

so for desired error dynamics we can

select desired pole locations of A_o and choose L to achieve them.

Duality of the observer PP and controller PP problems

Observer PP problem: Choose L, so that

$$\det\big(zI - (A + LC)\big) = p_{\text{des}}(z)$$

Controller PP problem: Choose K, so that

$$\det\big(zI - (A + BK)\big) = p_{\text{des}}(z)$$

Observer PP is not a new problem:

$$\begin{aligned}\det\big(zI - (A + LC)\big) &= \det\Big(\big(zI - (A + LC)\big)^\top\Big)\\ &= \det\Big(zI - (A^\top + C^\top L^\top)\Big)\\ &= \det\Big(zI - (\widetilde{A} + \widetilde{B}\widetilde{K})\Big)\end{aligned}$$

$\Longrightarrow$ observer PP is controller PP for the dual system.

The results for state feedback PP can be restated for observer PP:

Theorem: The eigenvalues of $A+LC$ can be assigned choosing L to any locations in $\mathbb{C}$ if and only if A, C is observable.

Observer canonical form $\quad \leftrightarrow \quad$ Controller canonical form

Lemma:

- Let A, c and A', c' be two observable pairs and
- assume that A and A' have the same char. polynomials.

Then there is a unique similarity transformation given by the matrix

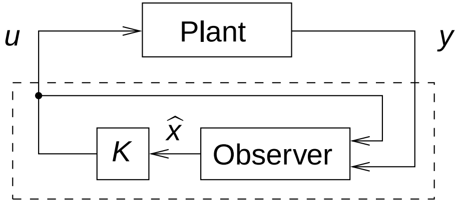
$$T := \left(\mathscr{O}(A', c')\right)^{-1} \mathscr{O}(A, c)$$

such that

$$T^{-1} A T = A' \qquad \text{and} \qquad cT = c'.$$

Closed-loop system with output feedback controller

Consider the closed loop system



Output feedback controller

where

Plant: $\sigma x = Ax + Bu, \quad y = Cx + Du$

Observer: $\sigma \widehat{x} = A\widehat{x} + Bu - L(y - C\widehat{x} - Du)$

State feedback controller: $u = K\widehat{x}$

Feedback controller:

$$\begin{aligned} \sigma\widehat{x} &= (A+LC)\widehat{x} + (B+LD)u - Ly, \qquad u = K\widehat{x} \\ &= (A+LC+BK+LDK)\widehat{x} - Ly \end{aligned}$$

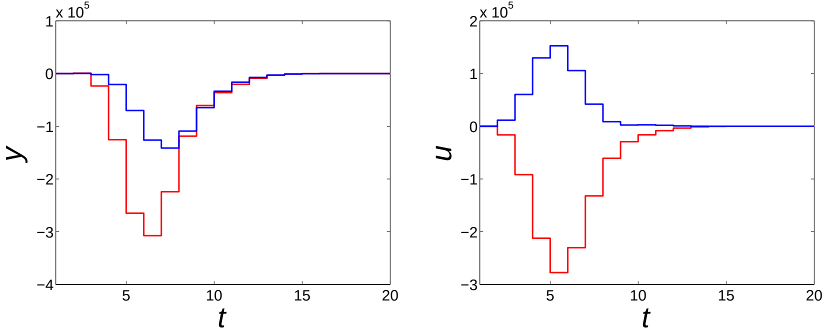
Note: the feedback controller is a dynamical system

Closed-loop system: $$\sigma \begin{bmatrix} x \\ \widehat{x} \end{bmatrix} = \begin{bmatrix} A & BK \\ -LC & A+LC+BK \end{bmatrix} \begin{bmatrix} x \\ \widehat{x} \end{bmatrix}$$

Note: closed-loop system order $=$ plant order + controller order

Error equation: $$\sigma \begin{bmatrix} x \\ e \end{bmatrix} = \begin{bmatrix} A+BK & -BK \\ 0 & A+LC \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix}$$

Example: output feedback deadbeat control



15th order single-input open-loop system, 30 order closed-loop system

(The same system as the one used in the example of Lecture 1)