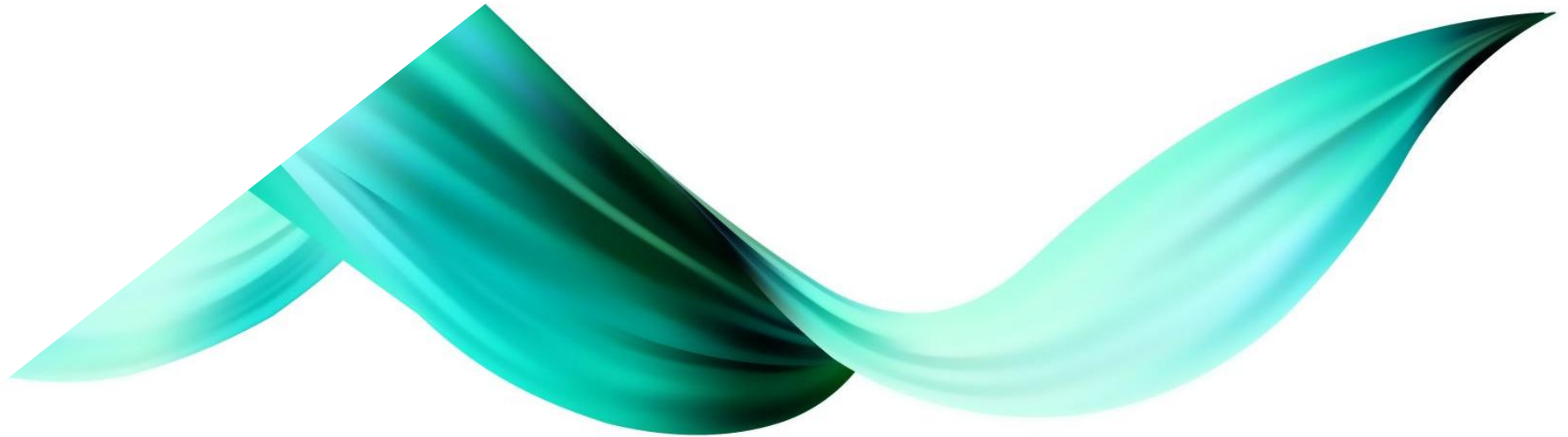
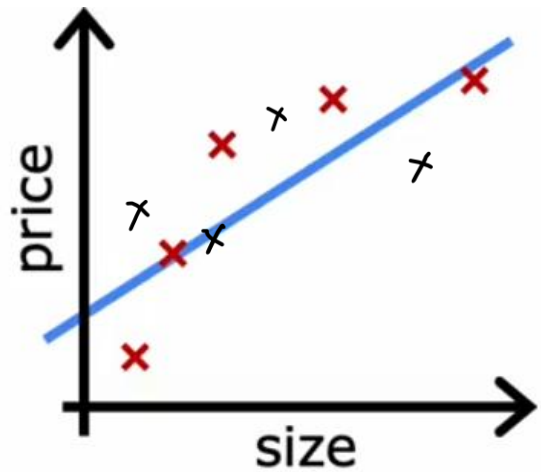


# Machine learning diagnostic

Bias & Variance



# Bias & Variance



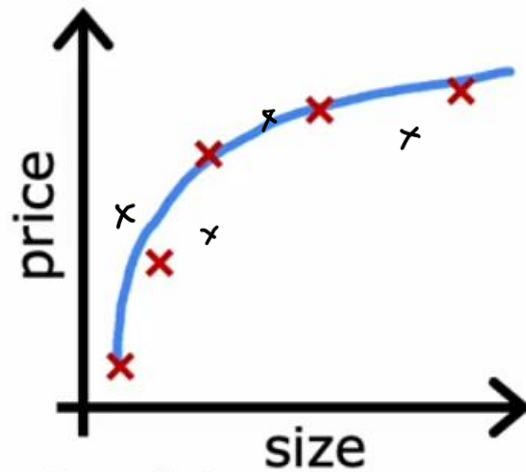
$$f_{\bar{w},b}(x) = w_1x + b$$

High bias  
(underfit)

$d=1$

$J_{\text{train}}$  high

$J_{\text{cv}}$  high



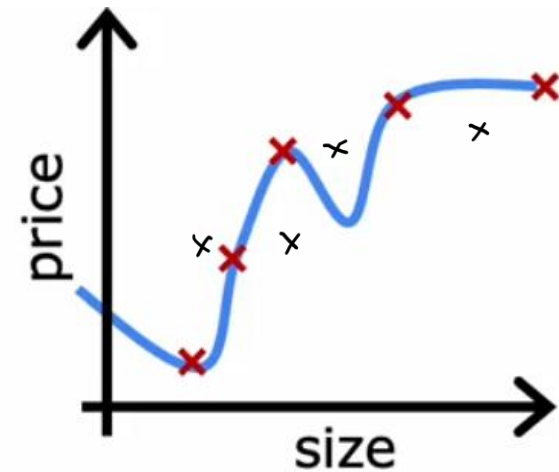
$$f_{\bar{w},b}(x) = w_1x + w_2x^2 + b$$

"Just right"

$d=2$

$J_{\text{train}}$  Low

$J_{\text{cv}}$  Low



$$f_{\bar{w},b}(x) = w_1x + w_2x^2 + w_3x^3 + w_4x^4 + b$$

High variance  
(overfit)

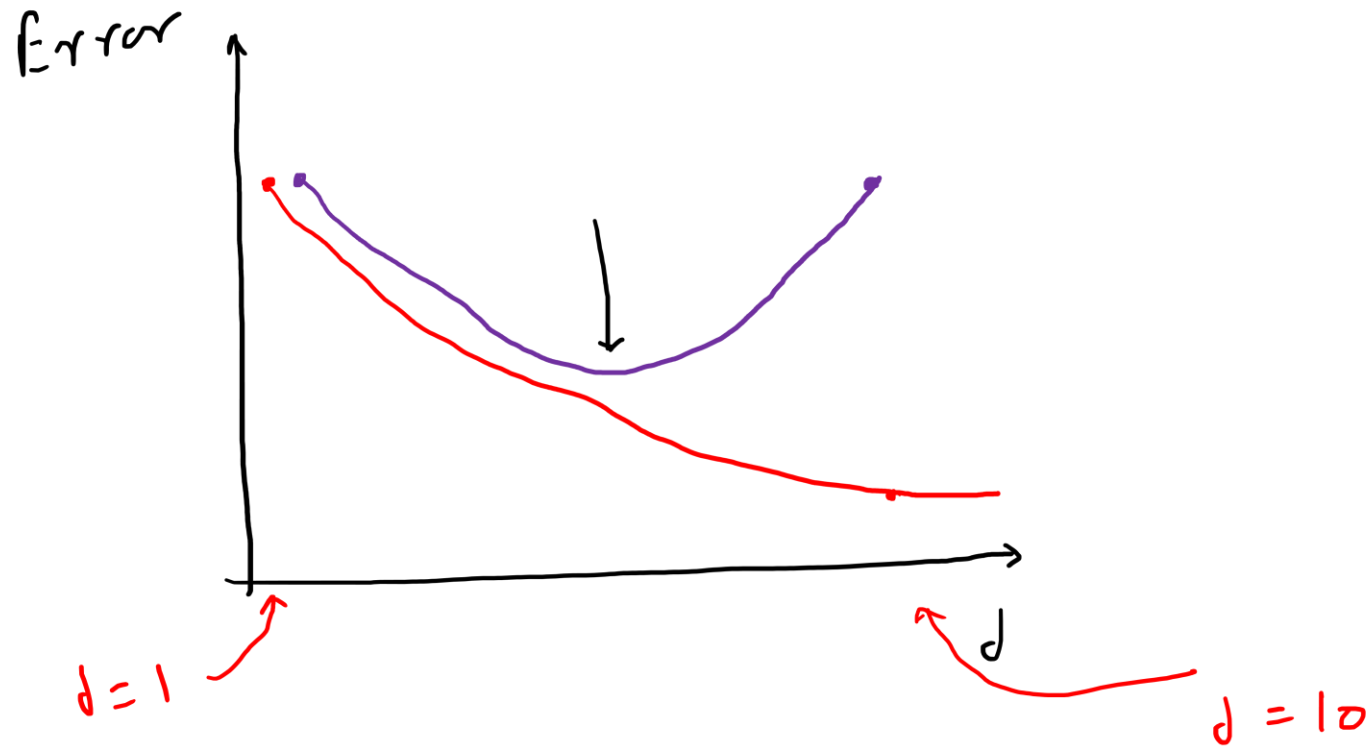
$d=4$

$J_{\text{train}}$  Low

$J_{\text{cv}}$  high

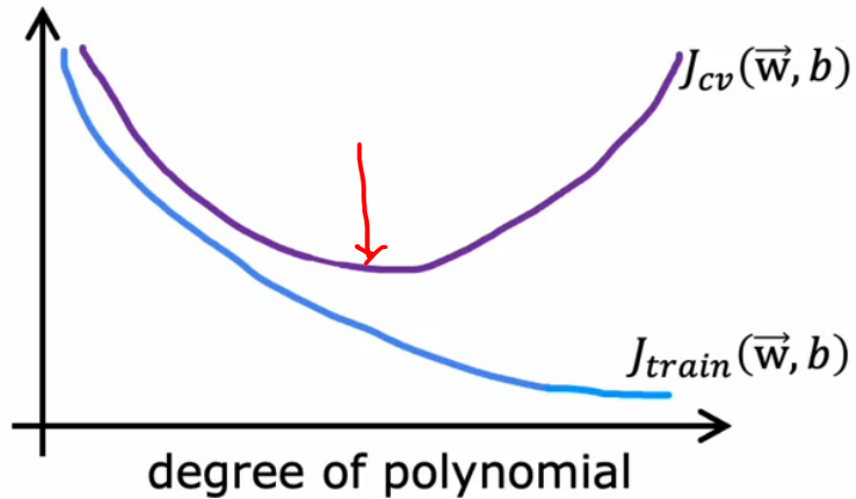
• → train  
• → cv

# Diagnosing Bias & Variance



# Diagnosing Bias & Variance

How do you tell if your algorithm has a bias or variance problem?



High bias (underfit)

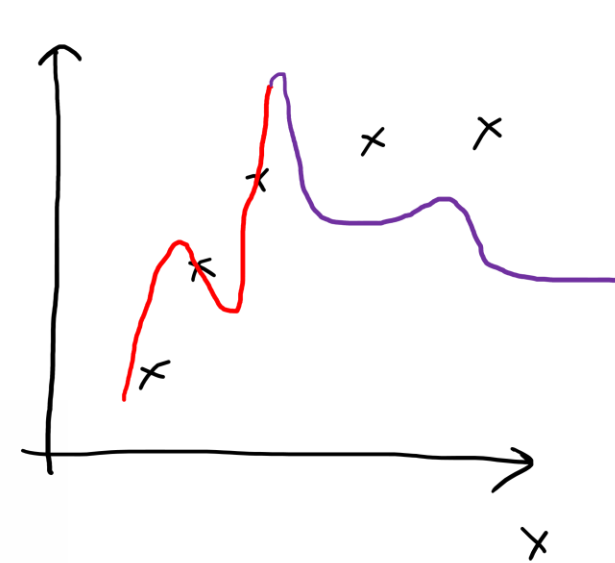
$J_{train}$  will be high  
( $J_{train} \approx J_{cv}$ )

High variance (overfit)

$J_{cv} \gg J_{train}$   
( $J_{train}$  may be low)

High bias and high variance

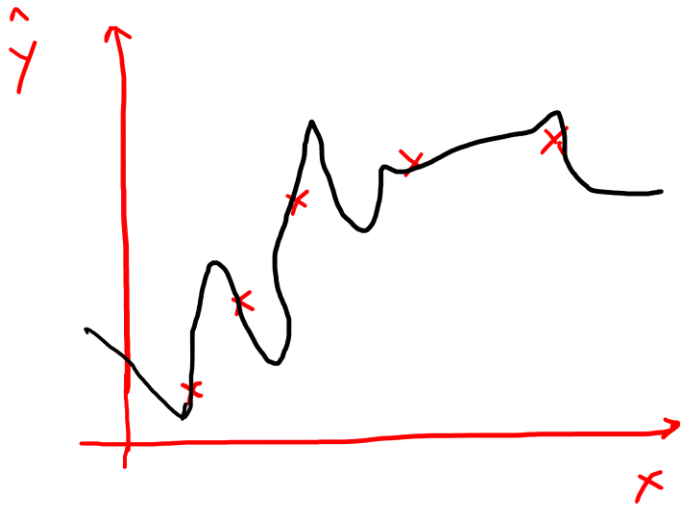
$J_{train}$  will be high  
and  $J_{cv} \gg J_{train}$



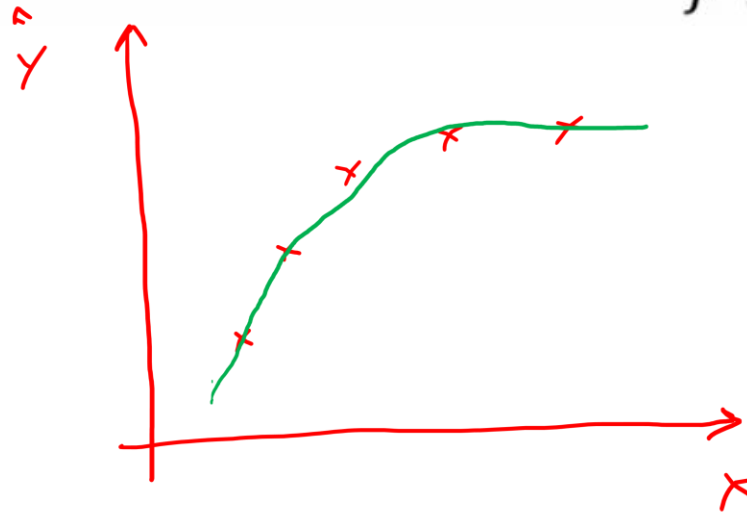
# Regularization and bias/variance

Model:  $f_{\vec{w},b}(x) = w_1x + w_2x^2 + w_3x^3 + w_4x^4 + b$

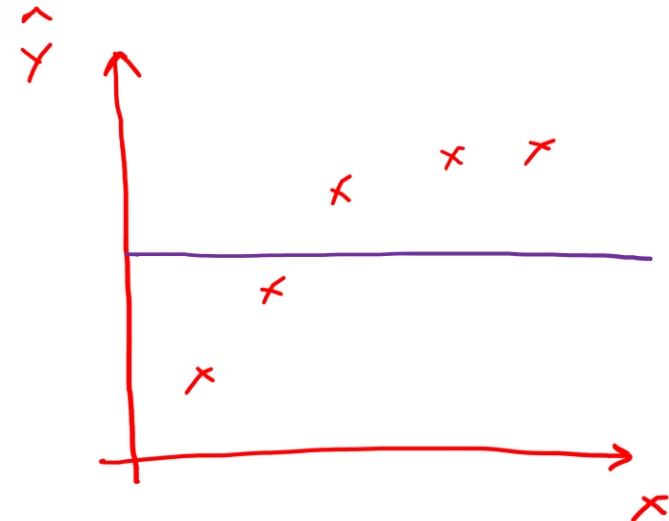
$$J(\vec{w}, b) = \frac{1}{2m} \sum_{i=1}^m (f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)})^2 + \frac{\lambda}{2m} \sum_{j=1}^n w_j^2$$



$\lambda \rightarrow 0$  High Variance



$\lambda \neq 0$   
 $\lambda \neq \text{very high.}$



$\lambda \rightarrow \infty$   
High Bias

# Regularization parameter ( $\lambda$ ) value

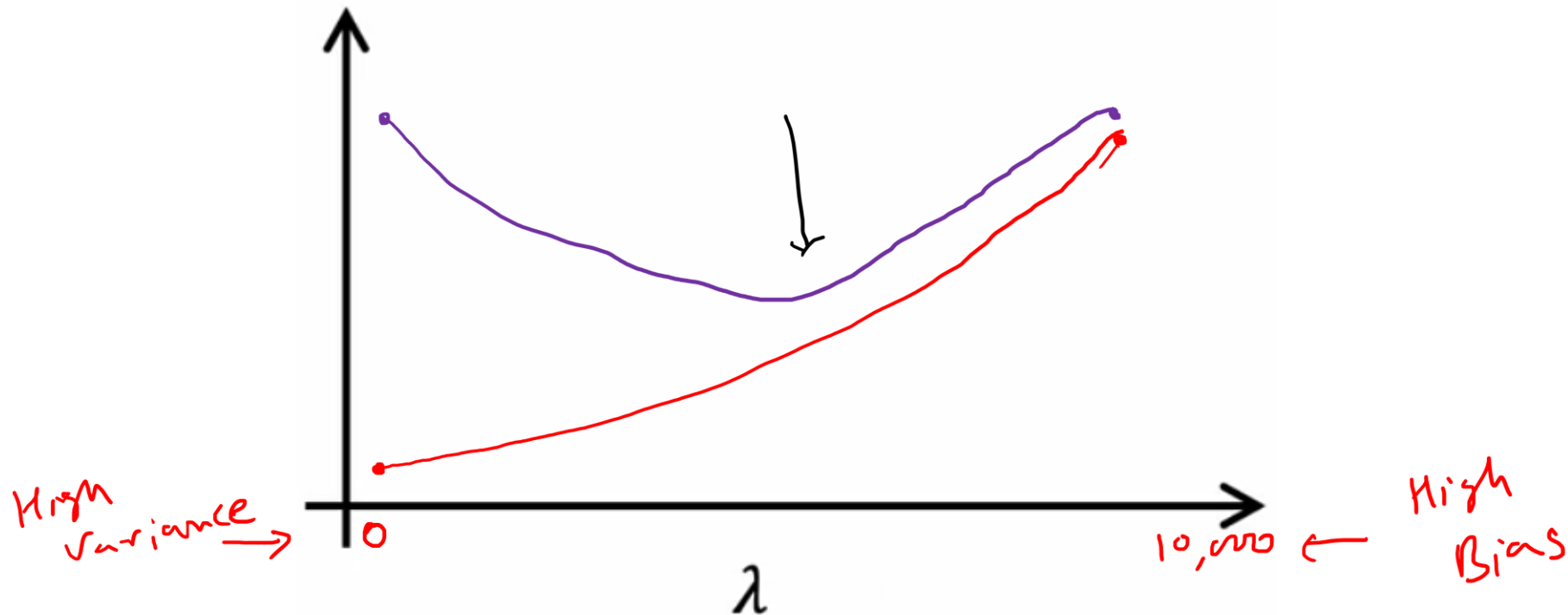
Model:  $f_{\vec{w},b}(x) = w_1x + w_2x^2 + w_3x^3 + w_4x^4 + b$

1. Try $\lambda = 0$	$\min_{\vec{w},b} J(\vec{w},b)$	$w^{<1>}, b^{<1>}$	$J_{cv}(w^{<1>}, b^{<1>})$
2. Try $\lambda = 0.01$		$w^{<2>}, b^{<2>}$	$J_{cv}(w^{<2>}, b^{<2>})$
3. Try $\lambda = 0.02$			$J_{cv}(w^{<3>}, b^{<3>})$
4. Try $\lambda = 0.04$			$J_{cv}(w^{<5>}, b^{<5>})$
5. Try $\lambda = 0.08$			
$\vdots$			
12. Try $\lambda \approx 10$		$w^{<12>}, b^{<12>}$	$J_{cv}(w^{<12>}, b^{<12>})$

- → train
- → CV

## Bias & Variance as a function of $\lambda$

$$J(\vec{w}, b) = \frac{1}{2m} \sum_{i=1}^m (f_{\vec{w}, b}(\vec{x}^{(i)}) - y^{(i)})^2 + \frac{\lambda}{2m} \sum_{j=1}^n w_j^2$$



# Speech Recognition



$$\begin{array}{rcll} J_{\text{human}} & = & 10.6\% & \\ J_{\text{train}} & = & 10.8\% & \\ J_{\text{cv}} & = & 14.8\% & \end{array} \quad \left. \begin{array}{l} \left. \begin{array}{l} ] \\ ] \end{array} \right\} \begin{array}{l} 0.2\% \\ 4\% \end{array} \right\} \text{High Variance}$$





# Establishing a baseline level of performance

What is the level of error one can reasonably hope to get to?

- Human level performance
- Competing algorithms' performance
- Guess based on experience

# Bian & Variance examples

- Baseline Performance



- Training error ( $J_{\text{train}}$ )



- Cross Validation error ( $J_{\text{cv}}$ )

10.6% } 0.2%  
10.8% }  
14.8% } 4%

High  
Variance

10.6% } 4.4%  
15.0% }  
15.5% } 0.5%

High  
Bias

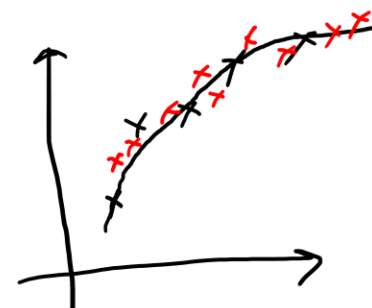
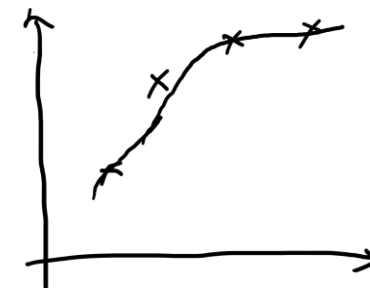
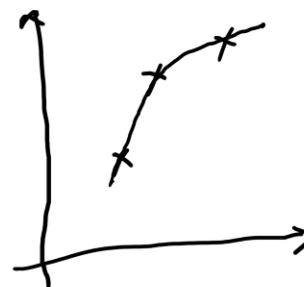
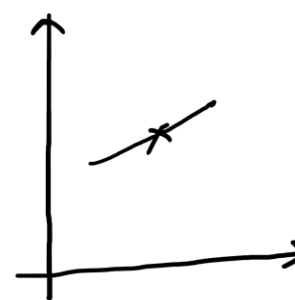
10.6% } 4.4%  
15.0% }  
19.7% } 4.7%

High Bias  
High  
Variance

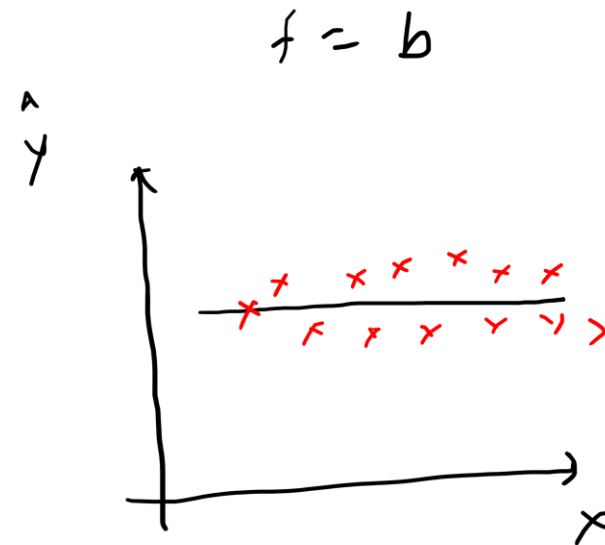
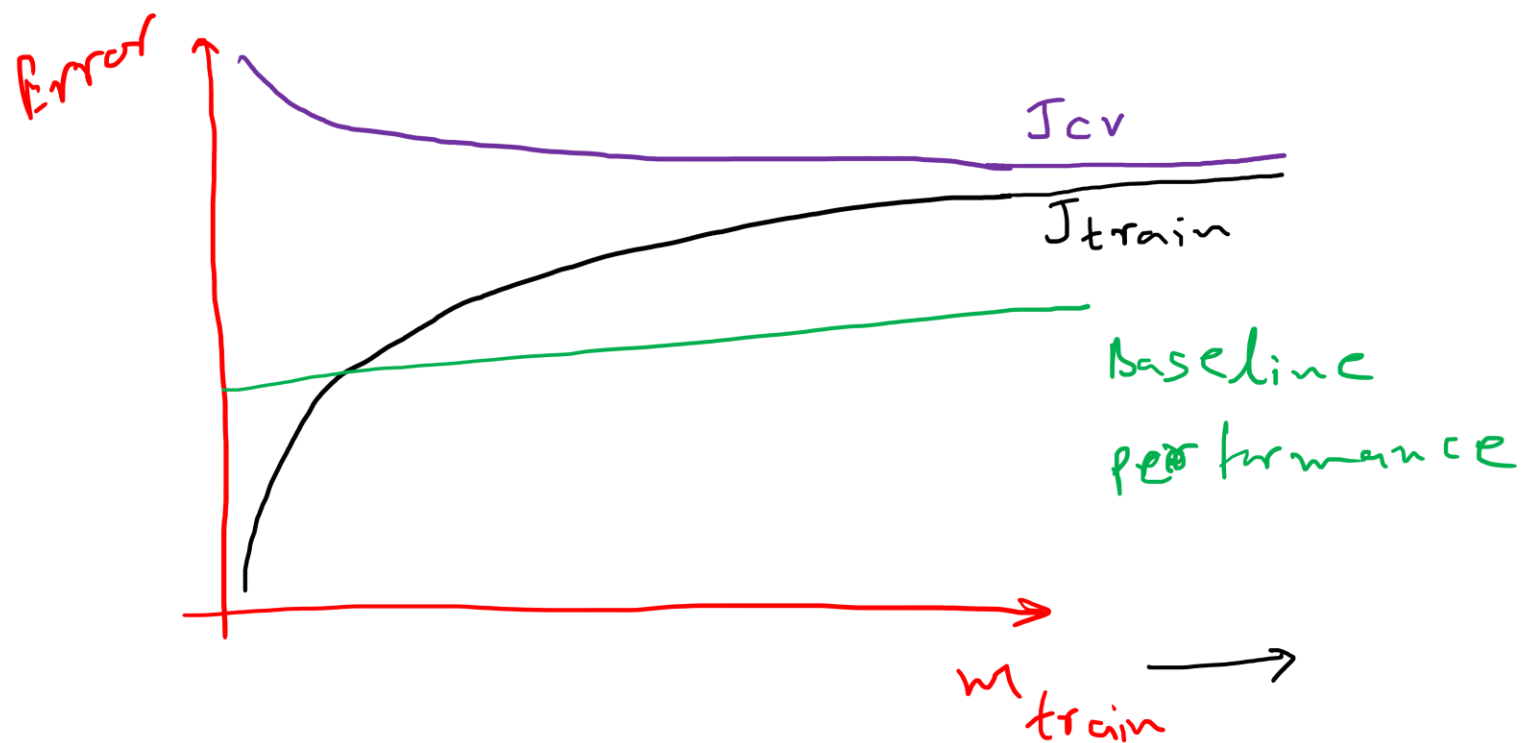
$\bullet \rightarrow J_{\text{train}}$   
 $\circ \rightarrow J_{\text{cv}}$

$$f_{\vec{w},b}(\vec{x}) = w_1 x + w_2 x^2 + b$$

## Learning curves

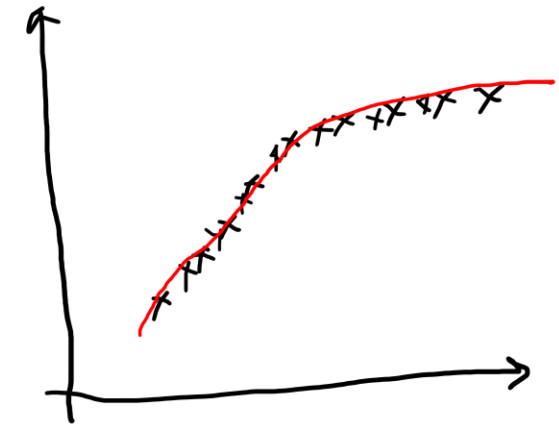
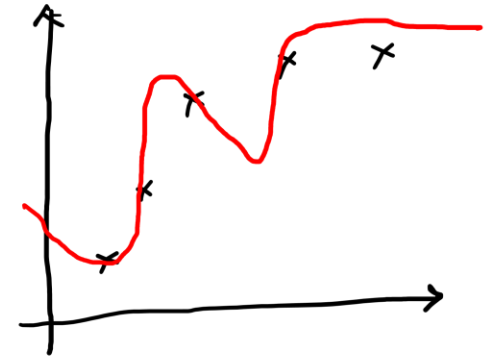
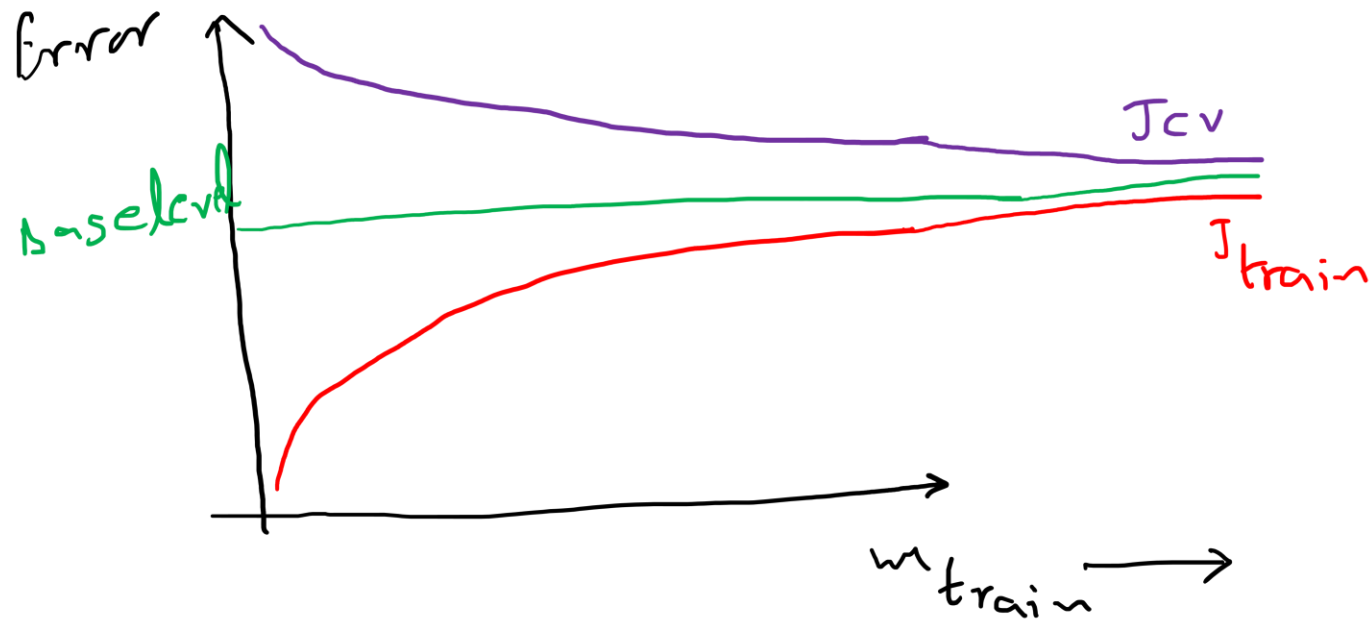


# High Bias



$$f = w_1 x + w_2 x^2 + w_3 x^3 + w_4 x^4 + b$$

## High Variance



# Debugging a learning algorithm

- Regularized linear regression

$$J(\vec{w}, b) = \frac{1}{2m} \sum_{i=1}^m (f_{\vec{w}, b}(\vec{x}^{(i)}) - y^{(i)})^2 + \frac{\lambda}{2m} \sum_{j=1}^n w_j^2$$

- Large errors in prediction
- Solutions
  1. Get more training examples
  2. Try smaller set of features
  3. Try getting additional features
  4. Try adding polynomial features
  5. Try decreasing  $\lambda$
  6. Try increasing  $\lambda$