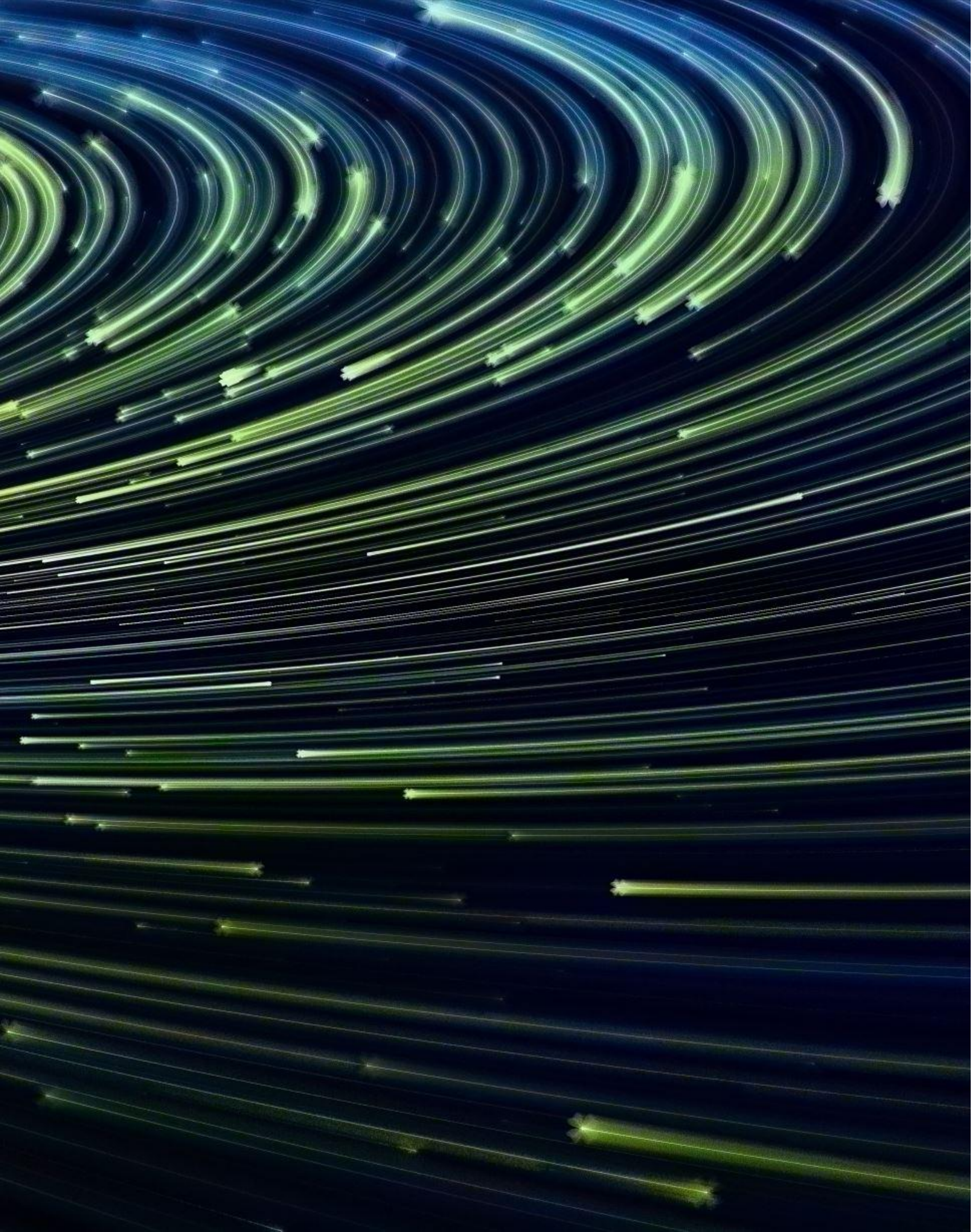
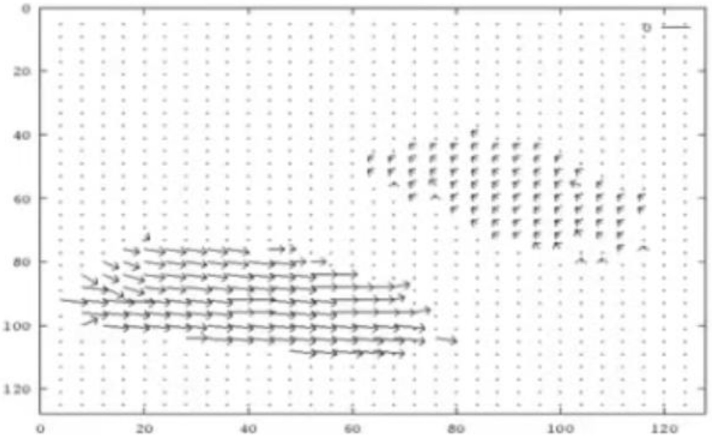


Optical Flow

Motion analysis

Optical flow

2D Displacement vector



Applications

+ Motion based segmentation
+ Structure for motion
+ Video Compression
+ Alignment (Global motion compensation)
  - Camcorder video stabilization
  - UAV video analysis

Horn & Schunck Optical flow

Brightness constancy assumption

$$f(x,y,t) = f(x+dx, y+dy, t+dt)$$

Taylor Series

$$f(x,y,t) = f(x,y,t) + \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial t}dt$$

$$f_x dx + f_y dy + f_t dt = 0$$

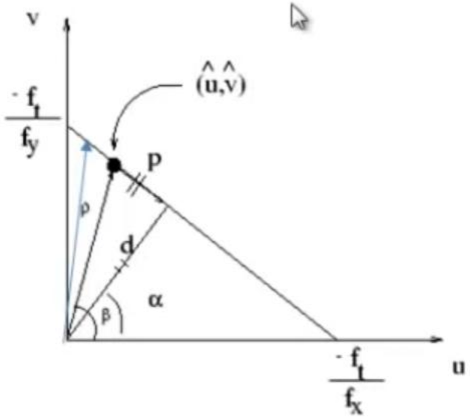
$$f_x u + f_y v + f_t = 0$$

Optical flow equation

$$f_x u + f_y v + f_t = 0$$

$$v = -\frac{f_x}{f_y}u - \frac{f_t}{f_y}$$

Equation of st.line



d=normal flow

p=parallel flow

$$d = \frac{f_t}{\sqrt{f_x^2 + f_y^2}}$$

Optical flow equation

$$\iint \{(f_x u + f_y v + f_t)^2 + \lambda(u_x^2 + u_y^2 + v_x^2 + v_y^2)\}dxdy$$

Brightness constancy

Smoothness constraint

min

$$(f_x u + f_y v + f_t)f_x + \lambda(\Delta^2 u) = 0$$

$$(f_x u + f_y v + f_t)f_y + \lambda((\Delta^2 v) = 0$$

$$\Delta^2 u = u_{xx} + u_{yy}$$

Gradient mask

$$\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \text{first image} \qquad \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} \text{first image} \qquad \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \text{first image}$$

$$\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \text{second image} \qquad \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} \text{second image} \qquad \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \text{second image}$$

$$f_x \qquad\qquad f_y \qquad\qquad f_t$$

Apply first mask to 1st image
Second mask to 2nd image
Add the responses to get f_x, f_y, f_t

Laplacian mask

$$\begin{matrix} 0 & -\frac{1}{4} & 0 \\ -\frac{1}{4} & 1 & -\frac{1}{4} \\ 0 & -\frac{1}{4} & 0 \end{matrix}$$

$$f_{xx} + f_{yy}$$

$$f_{xx} + f_{yy} = f - f_{av}$$

Optical Flow

$$\int\int\{(f_x u + f_y v + f_t)^2 + \lambda(u_x^2 + u_y^2 + v_x^2 + v_y^2)\}dxdy$$

min

$$(f_x u + f_y v + f_t)f_x + \lambda(\Delta^2 u) = 0$$
$$(f_x u + f_y v + f_t)f_y + \lambda((\Delta^2 v) = 0$$

discrete version

$$(f_x u + f_y v + f_t)f_x + \lambda(u - u_{av}) = 0$$
$$(f_x u + f_y v + f_t)f_y + \lambda((v - v_{av}) = 0$$

variational calculus

$$u = u_{av} - f_x \frac{P}{D}$$
$$v = v_{av} - f_y \frac{P}{D}$$

$$P = f_x u_{av} + f_y v_{av} + f_t$$
$$D = \lambda + f_x^2 + f_y^2$$

$$\Delta^2 u = u_{xx} + u_{yy}$$

Horn & Schunck Optical flow

- k=0
- Initialize u^K v^K
- Repeat until some error measure is satisfied (converges)

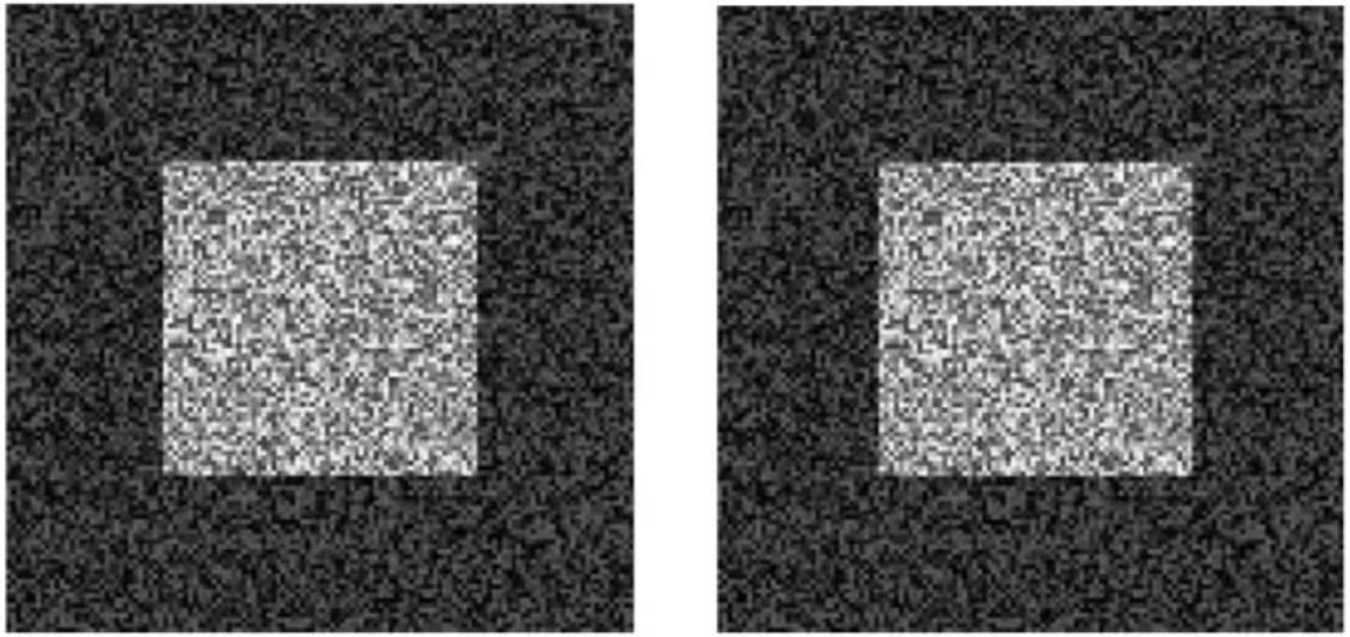
$$u = u_{av} - f_x \frac{P}{D}$$

$$v = v_{av} - f_y \frac{P}{D}$$

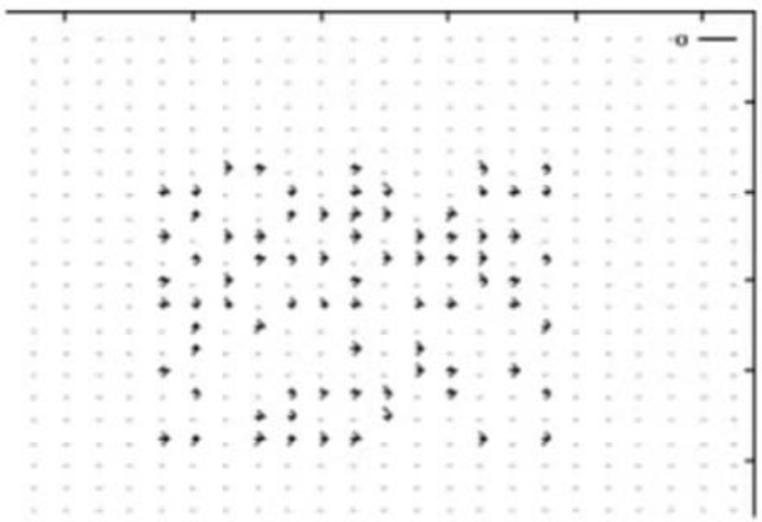
$$P = f_x u_{av} + f_y v_{av} + f_t$$

$$D = \lambda + f_x^2 + f_y^2$$

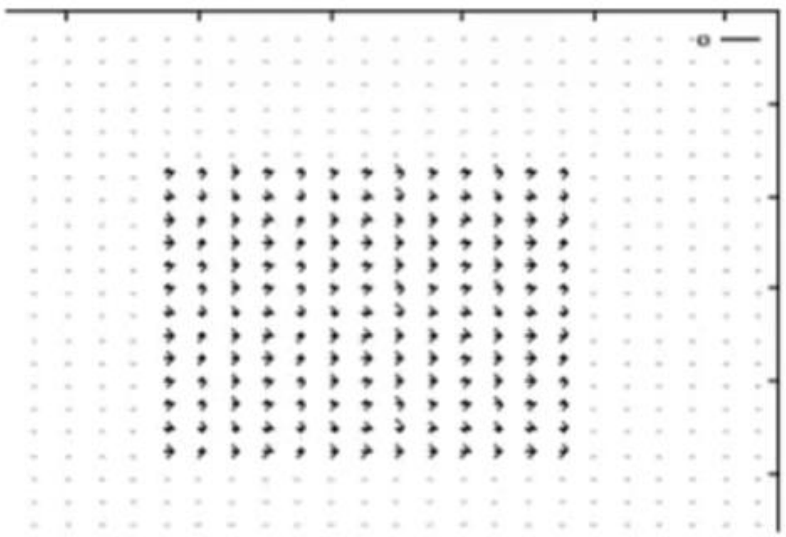
Synthetic images



Horn & Schunck Optical flow



One iteration



10 iterations