CONSTRAINT SATISFACTION PROBLEMS

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- Factored representation for each state.
- X is a set of variables, $\{X_1,\ldots,X_n\}$
- D is a set of domains, $\{D_1,\ldots,D_n\}$, one for each variable.
  - A domain, D_i, consists of a set of allowable values, $\{v_1,\ldots,v_k\}$, for variable X_i.
- C is a set of constraints that specify allowable combinations of values.
  - Each constraint C_j consists of a pair $\langle scope, rel \rangle$, where scope is a tuple of variables that participate in the constraint and relation is a relation that defines the values that those variables can take on.
  - If X_1 and X_2 both have the domain $\{1,2,3\}$, then
  - Constraint C_j = $\langle (X1,X2),\{(3,1),(3,2),(2,1)\} \rangle$ explicit or $\langle (X_1,X_2), X_1 > X_2 \rangle$ implicit

ASSIGNMENT

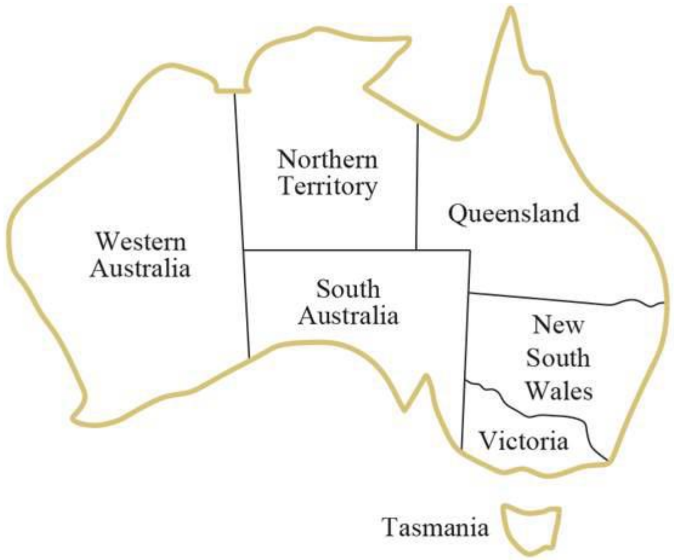
- An assignment that does not violate any constraints is called a consistent or legal assignment.
- A complete assignment is one in which every variable is assigned a value, and a solution to a CSP is a consistent, complete assignment.
- A partial assignment is one that leaves some variables unassigned, and a partial solution is a partial assignment that is consistent.

MAP COLORING

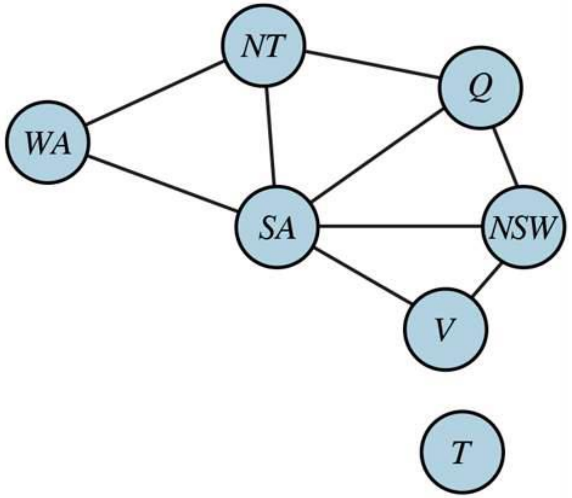
- We are given the task of coloring each region either red, green, or blue in such a way that no two neighboring regions have the same color.

X = {WA,NT,Q,NSW,V ,SA,T}

D_i = {red,green,blue}



(a)



(b)

$$C = \{SA \neq WA, SA \neq NT, SA \neq Q, SA \neq NSW, SA \neq V, WA \neq NT, NT \neq Q, Q \neq NSW, NSW \neq V\}.$$

If {SA = blue} 3^5 assignments to 2^5 assignments using CSP.

$D_i = \{0,1,2,3,\ldots,30\}$

JOB-SHOP SCHEDULING

- Car assembly, consisting of 15 tasks: install axles (front and back), affix all four wheels (right and left, front and back), tighten nuts for each wheel, affix hubcaps, and inspect the final assembly.

$$\begin{aligned} X \; = \; & \{Axle_F, Axle_B, Wheel_{RF}, Wheel_{LF}, Wheel_{RB}, Wheel_{LB}, Nuts_{RF}, \\ & Nuts_{LF}, Nuts_{RB}, Nuts_{LB}, Cap_{RF}, Cap_{LF}, Cap_{RB}, Cap_{LB}, Inspect\}. \end{aligned}$$

Precedence Constraints $T_1 + d_1 \leq T_2.$

$$\begin{aligned} & Axle_F + 10 \leq Wheel_{RF}; \quad Axle_F + 10 \leq Wheel_{LF}; \\ & Axle_B + 10 \leq Wheel_{RB}; \quad Axle_B + 10 \leq Wheel_{LB}. \end{aligned}$$

$$\begin{aligned} & Wheel_{RF} + 1 \leq Nuts_{RF}; \quad Nuts_{RF} + 2 \leq Cap_{RF}; \\ & Wheel_{LF} + 1 \leq Nuts_{LF}; \quad Nuts_{LF} + 2 \leq Cap_{LF}; \\ & Wheel_{RB} + 1 \leq Nuts_{RB}; \quad Nuts_{RB} + 2 \leq Cap_{RB}; \\ & Wheel_{LB} + 1 \leq Nuts_{LB}; \quad Nuts_{LB} + 2 \leq Cap_{LB}. \end{aligned}$$

Disjunctive Constraint $(Axle_F + 10 \leq Axle_B)$ **or** $(Axle_B + 10 \leq Axle_F)$

CONSTRAINTS

- Unary Constraint

$$\langle (SA), SA \neq green \rangle$$

- Binary Constraint

$$SA \neq NSW$$

- Higher-order Constraints

$$\langle (X,Y,Z), X < Y < Z \text{ or } X > Y > Z \rangle$$

- Global Constraint

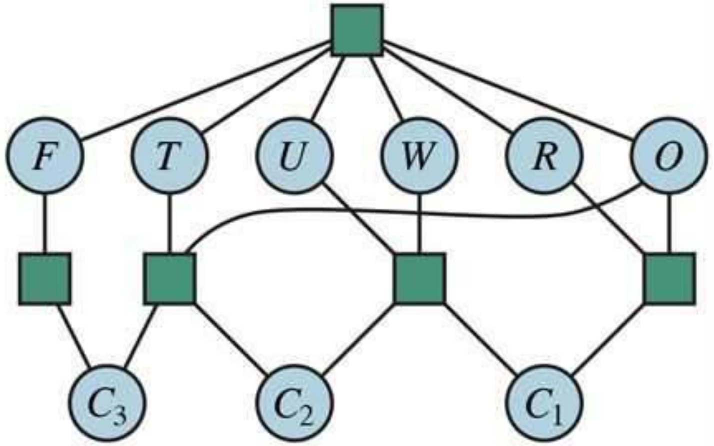
Alldif f

- Preference Constraints -> Constrained Optimization Problem (COP)

CRYPTARITHMETIC PUZZLES

$$
\begin{array}{r}
T\ W\ O \\
+\ T\ W\ O \\
\hline
F\ O\ U\ R
\end{array}
$$

(a)



(b)

$$O + O = R + 10 \cdot C_1$$
$$C_1 + W + W = U + 10 \cdot C_2$$
$$C_2 + T + T = O + 10 \cdot C_3$$
$$C_3 = F\ ,$$