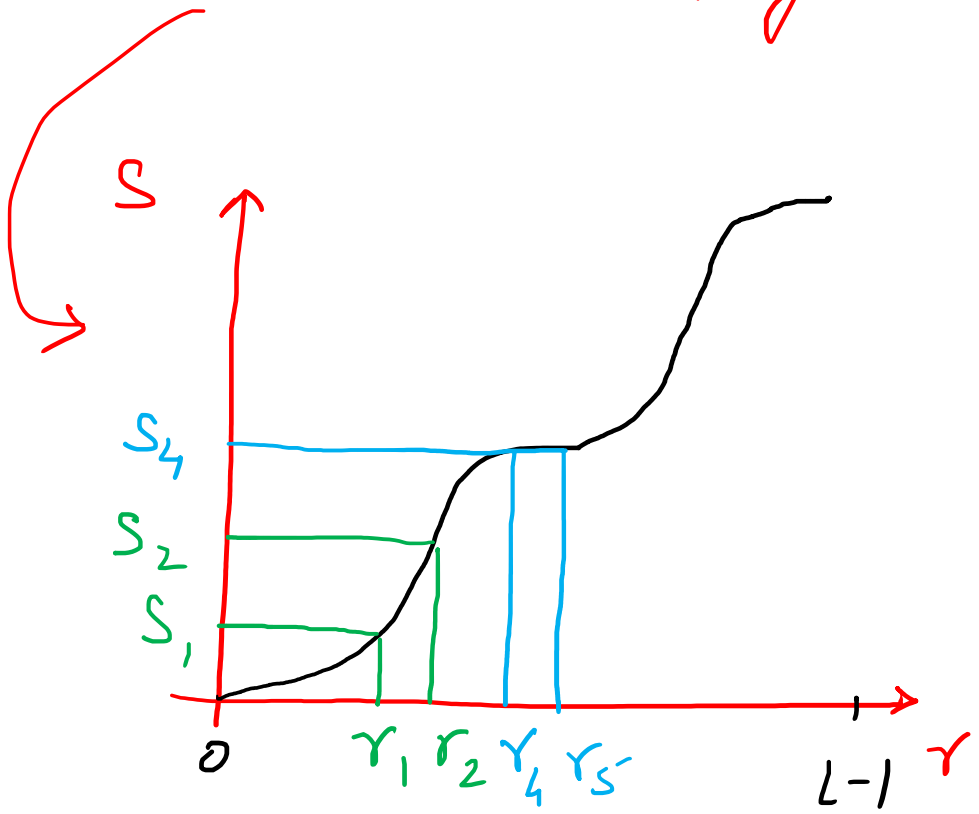
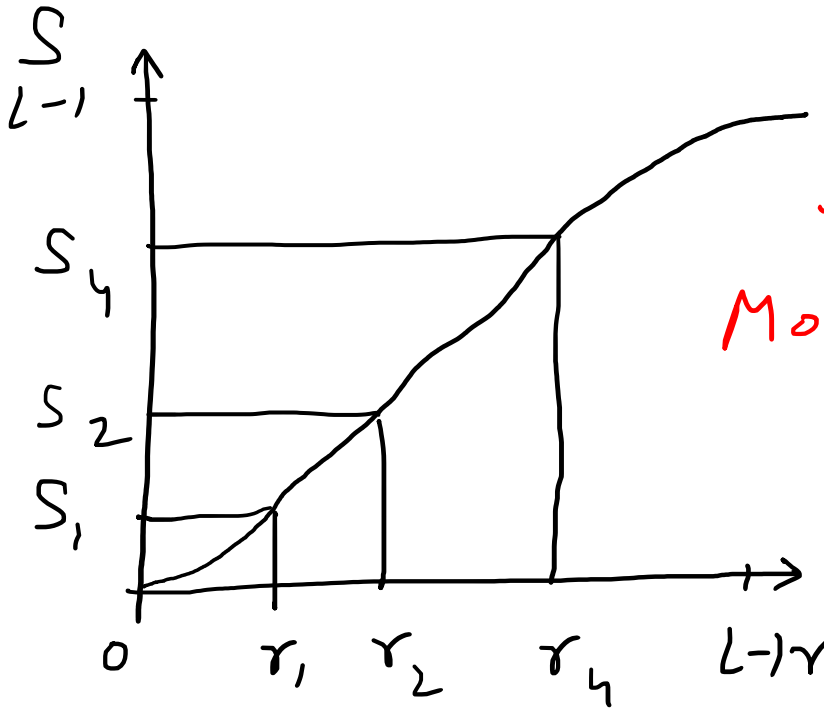
Histogram Equalization

$$S = T(r) \qquad 0 \le r \le L-1$$

a) Monotonically increasing fun^n



$0 \le r \le L-1$



Strictly Monotonically increasing fun^n

b) $0 \leq T_r \leq L-1$ for $0 \leq r \leq L-1$

c) $r = T^{-1}(s)$ $\quad 0 \leq s \leq L-1$

a') Strictly Monotonically increasing fun^{n}

Continuous Domain

PDF $\rightarrow$ $p_s(s) = p_r(r) \left| \frac{dr}{ds} \right|$ (1)

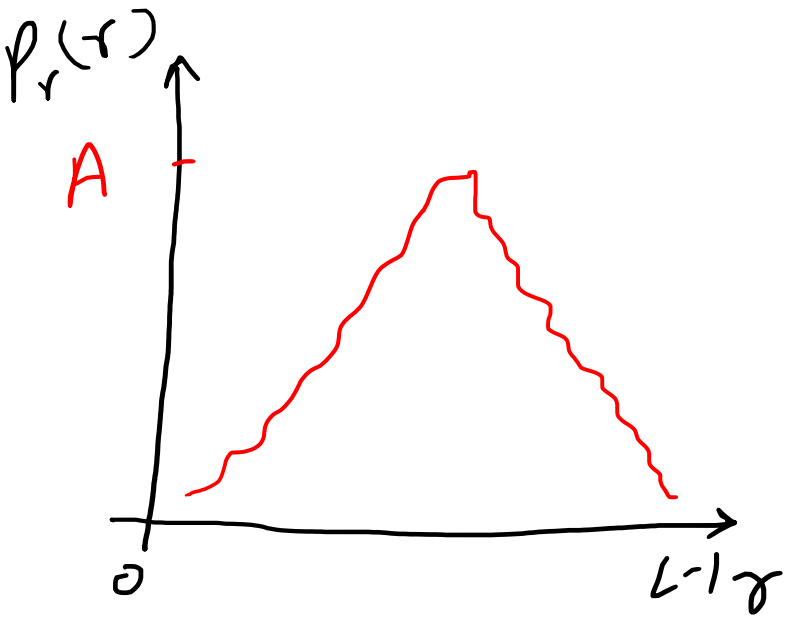
CDF $\leftarrow$ $s = T(r) = (L-1) \int_0^r p_r(w)\, dw$ (2)

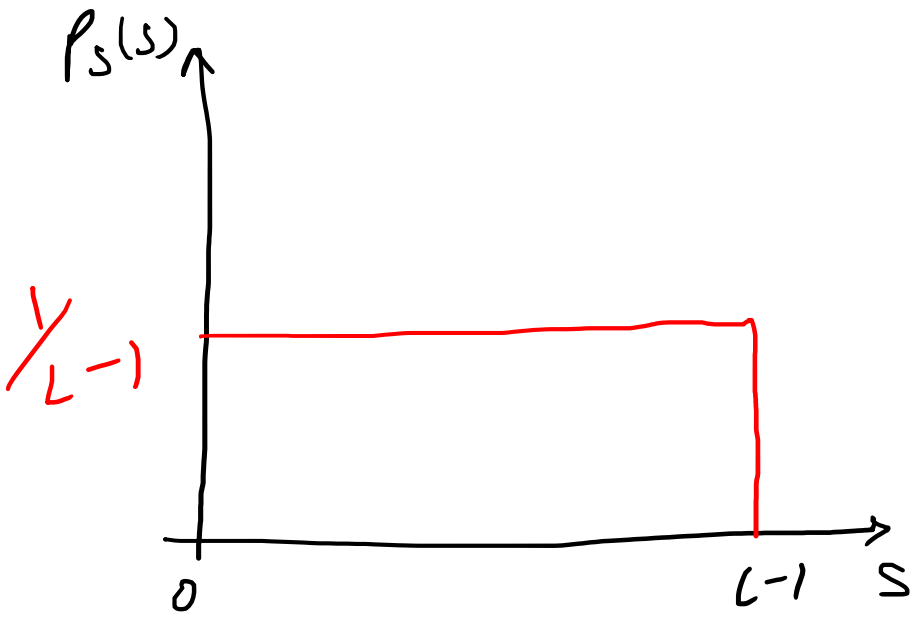
$$\frac{ds}{dr} = \frac{d}{dr}\left[(L-1) \int_0^r p_r(w)\, dw \right] = (L-1) \frac{d}{dr}\left[\int_0^r p_r(w)\, dw \right]$$

$$= (L-1)\, p_r(r)$$

$$\frac{dr}{ds} = \frac{1}{(L-1)\, p_r(r)}$$

$$p_s(s) = p_r(r) \left| \frac{dr}{ds} \right| = \left| \frac{1}{L-1} \right|$$





a) $$P_r(r) = \begin{cases} 2r/(L-1)^2 & ; \quad 0 \le r \le L-1 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

$$S = T(r) = (L-1) \int_0^r P_r(w)\, dw$$

$$= (L-1) \int_0^r \frac{2w}{(L-1)^2}\, dw$$

$$= \frac{1}{(L-1)} \int_0^r 2w\, dw = \frac{r^2}{(L-1)}$$

$$p_s(s) = p_r(r) \left| \frac{dr}{ds} \right|$$

$$s = \frac{r^2}{L-1} \Rightarrow \frac{ds}{dr} = \frac{d}{dr}\left[\frac{r^2}{L-1}\right] = \frac{2r}{(L-1)}$$

$$\Rightarrow \frac{dr}{ds} = \frac{(L-1)}{2r}$$

$$p_s(s) = \frac{2r}{(L-1)^2} \times \frac{(L-1)}{2r} \Rightarrow \boxed{p_s(s) = \frac{1}{(L-1)}}$$

Discrete Domain (Digital Images)

$$p_r(r_k) = \frac{n_k}{MN} \quad ; \quad k = 0, 1, 2, \ldots L-1$$

$$S_k = T(r_k) = (L-1) \sum_{j=0}^{k} p_r(r_j)$$

$$S_k = \frac{(L-1)}{MN} \sum_{j=0}^{k} n_j$$