Image Stitching

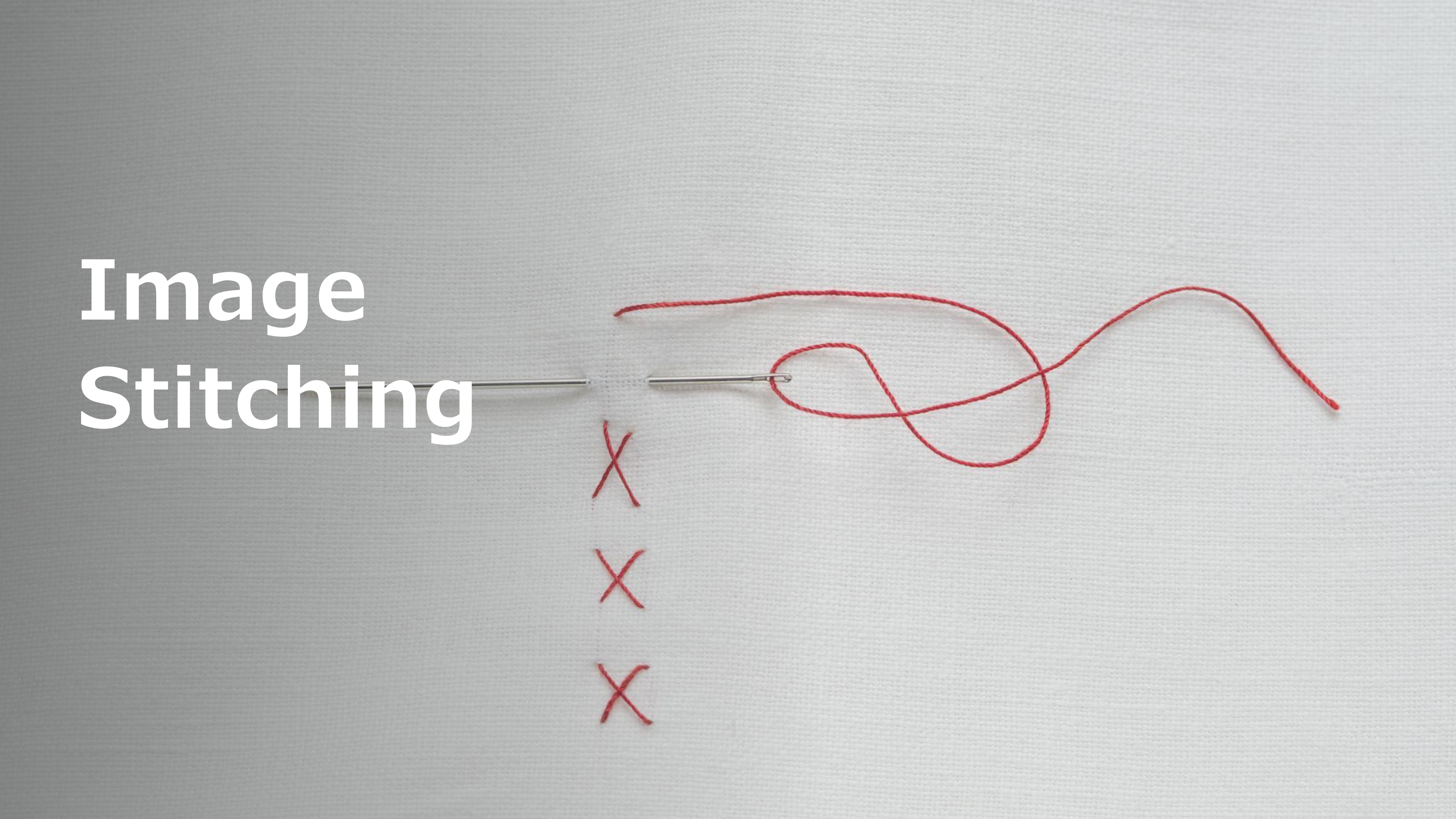


Image Stitching (Panorama)

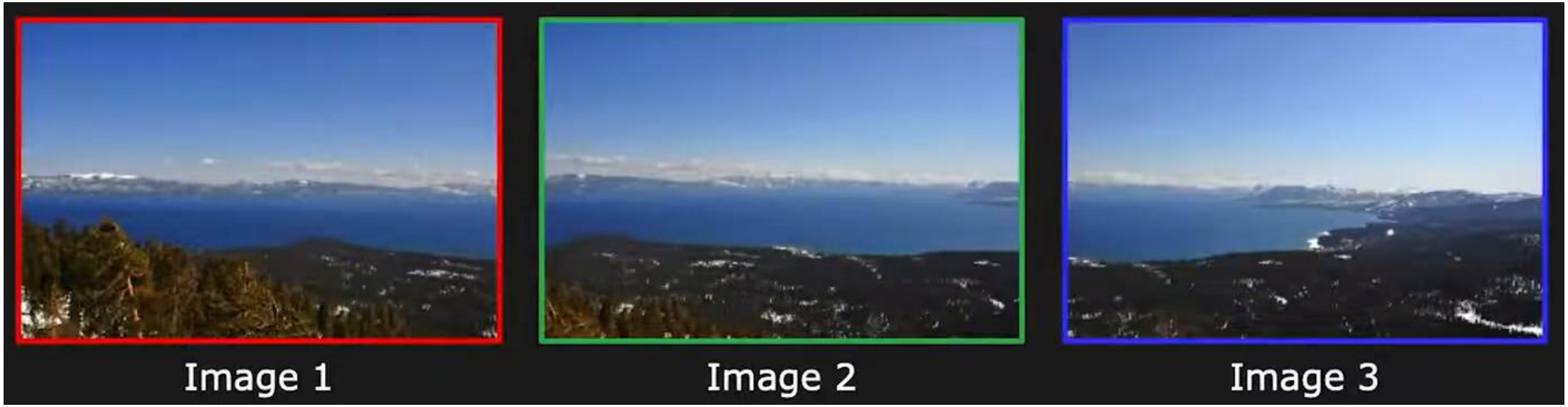


Image Stitching: SIFT detector

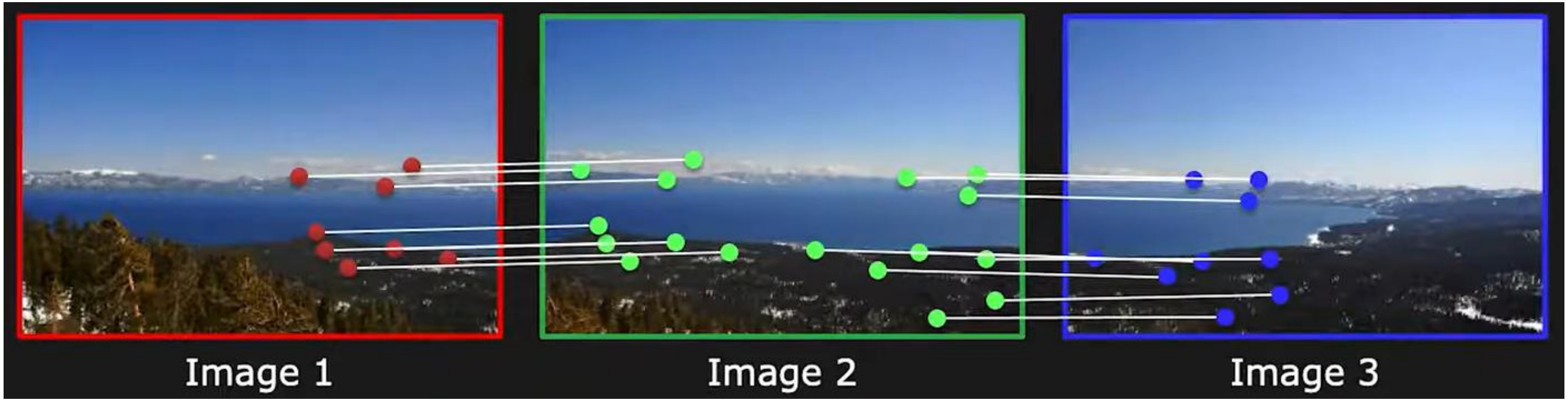


Image Stitching: Geometric relationship

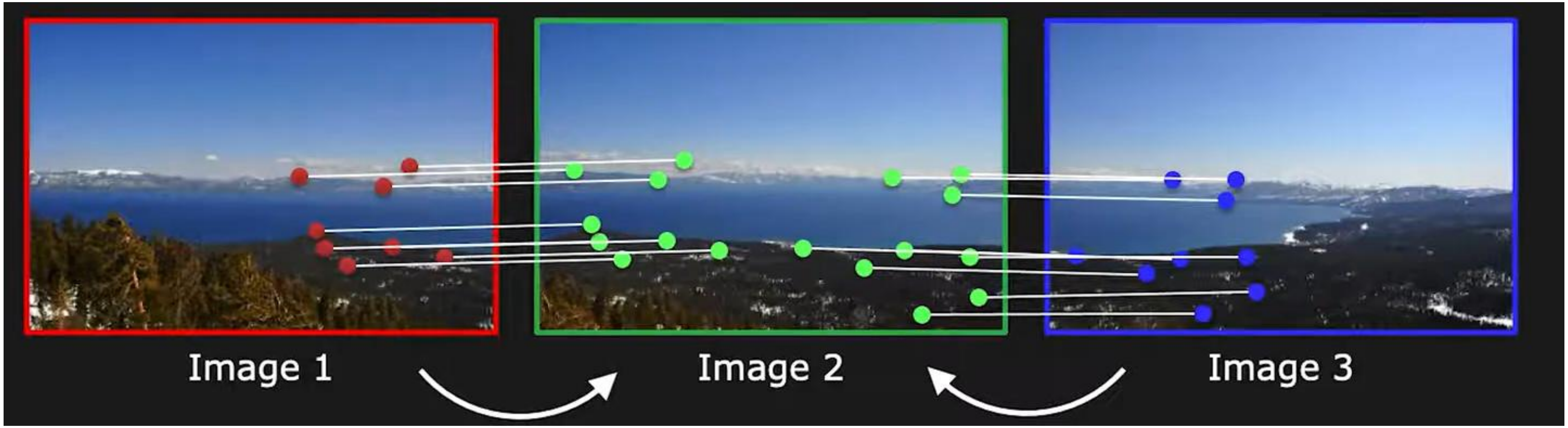


Image Stitching: Warping

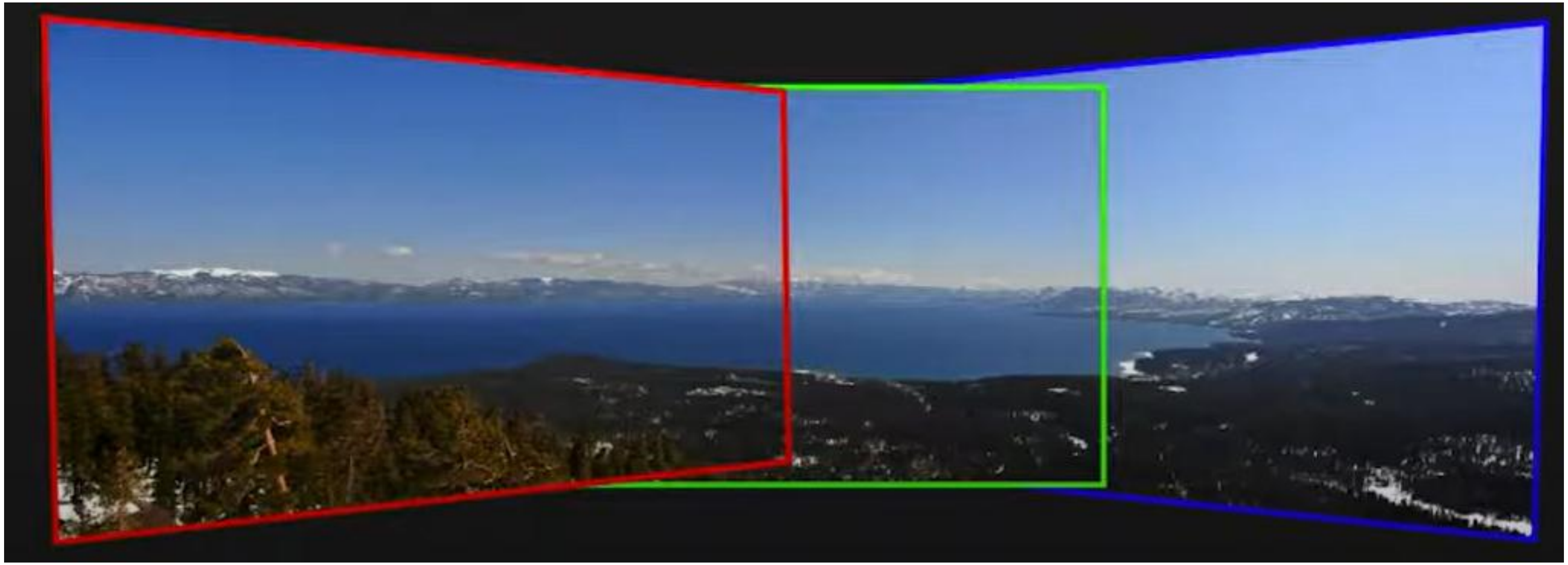


Image Stitching

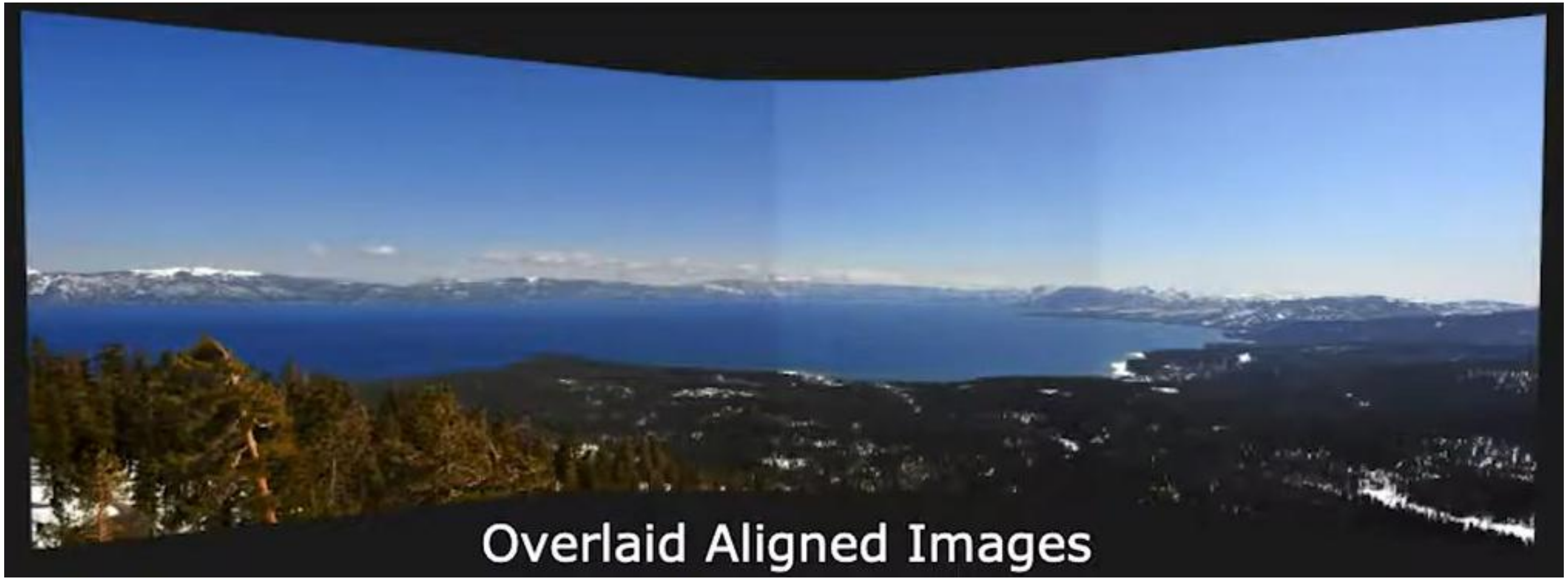


Image Stitching: Blending

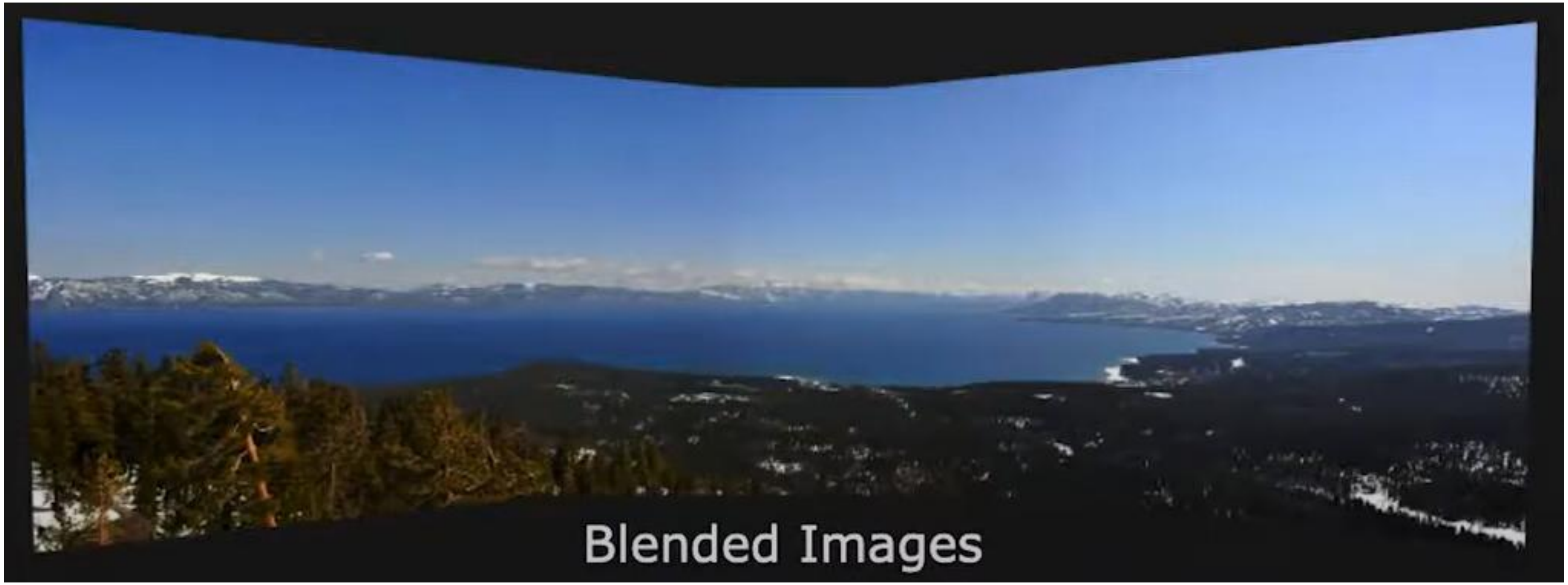
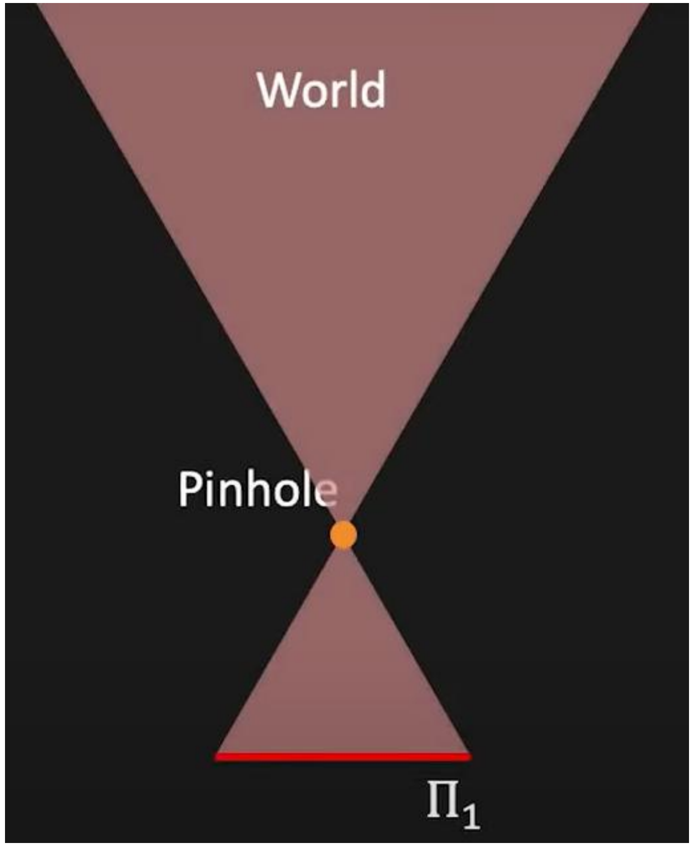


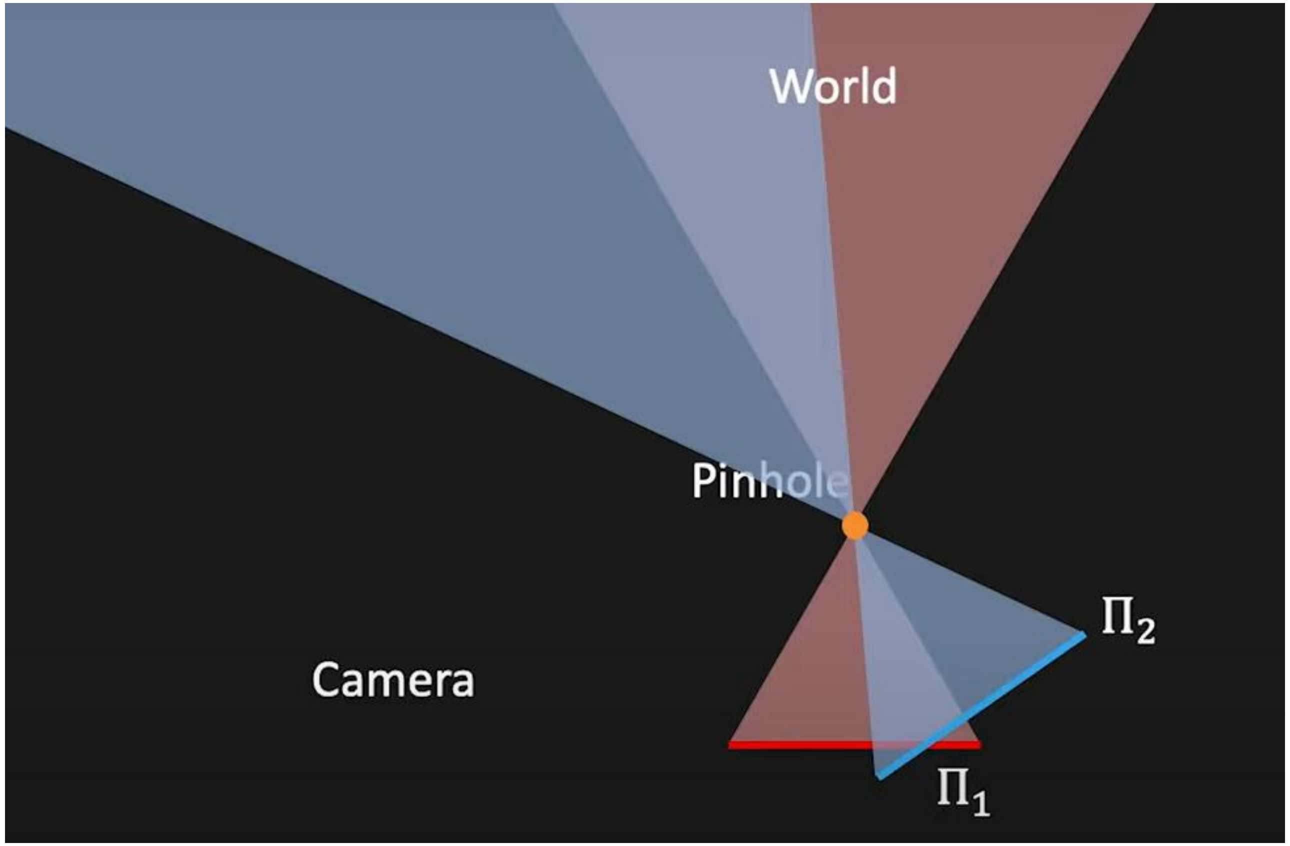
Image Stitching

1. Perform Transformations (Projective)
2. Computing Homography
3. Dealing with Outliers (RANSAC)
4. Warping and Blending images

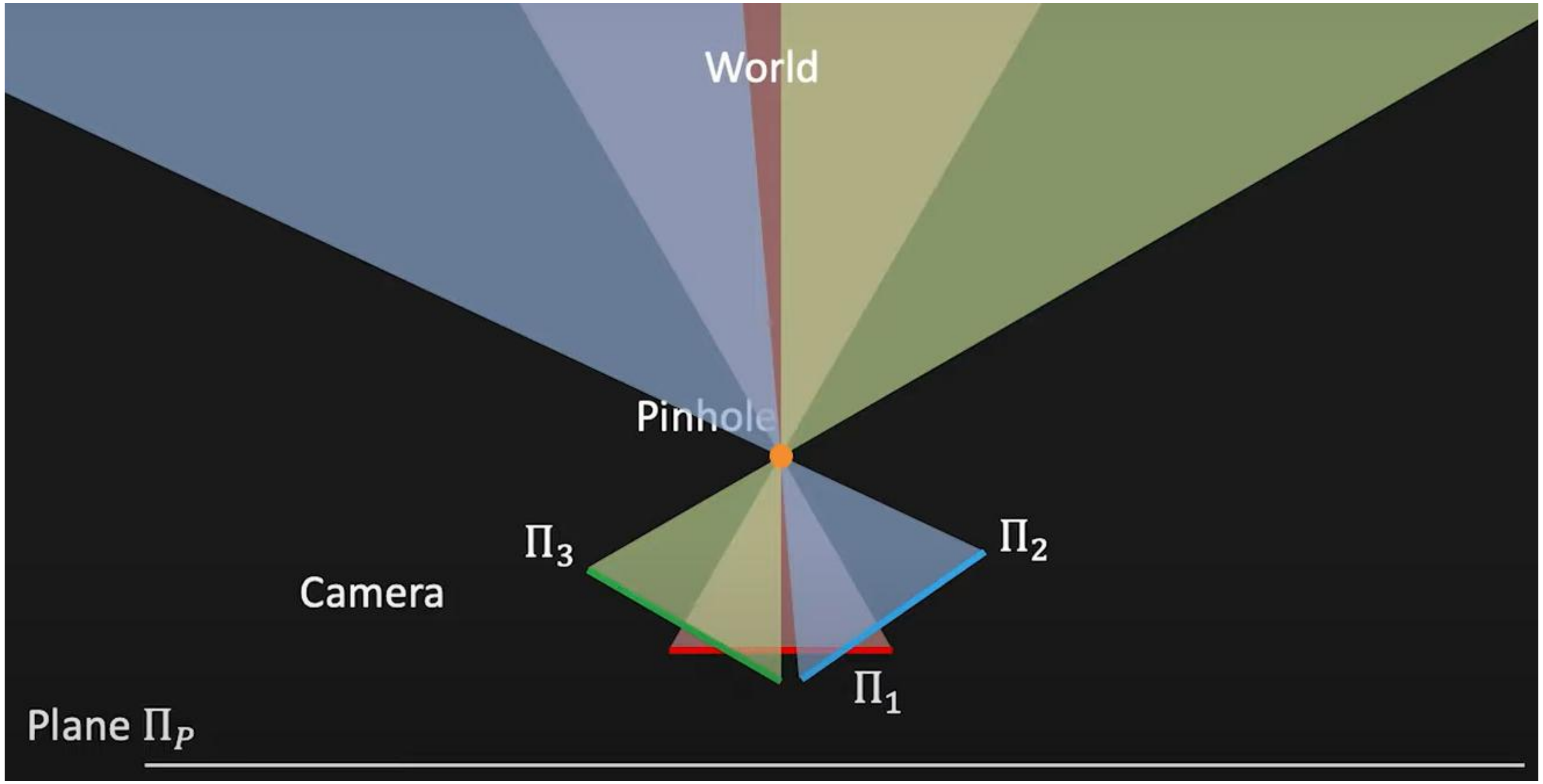
Homography Composition



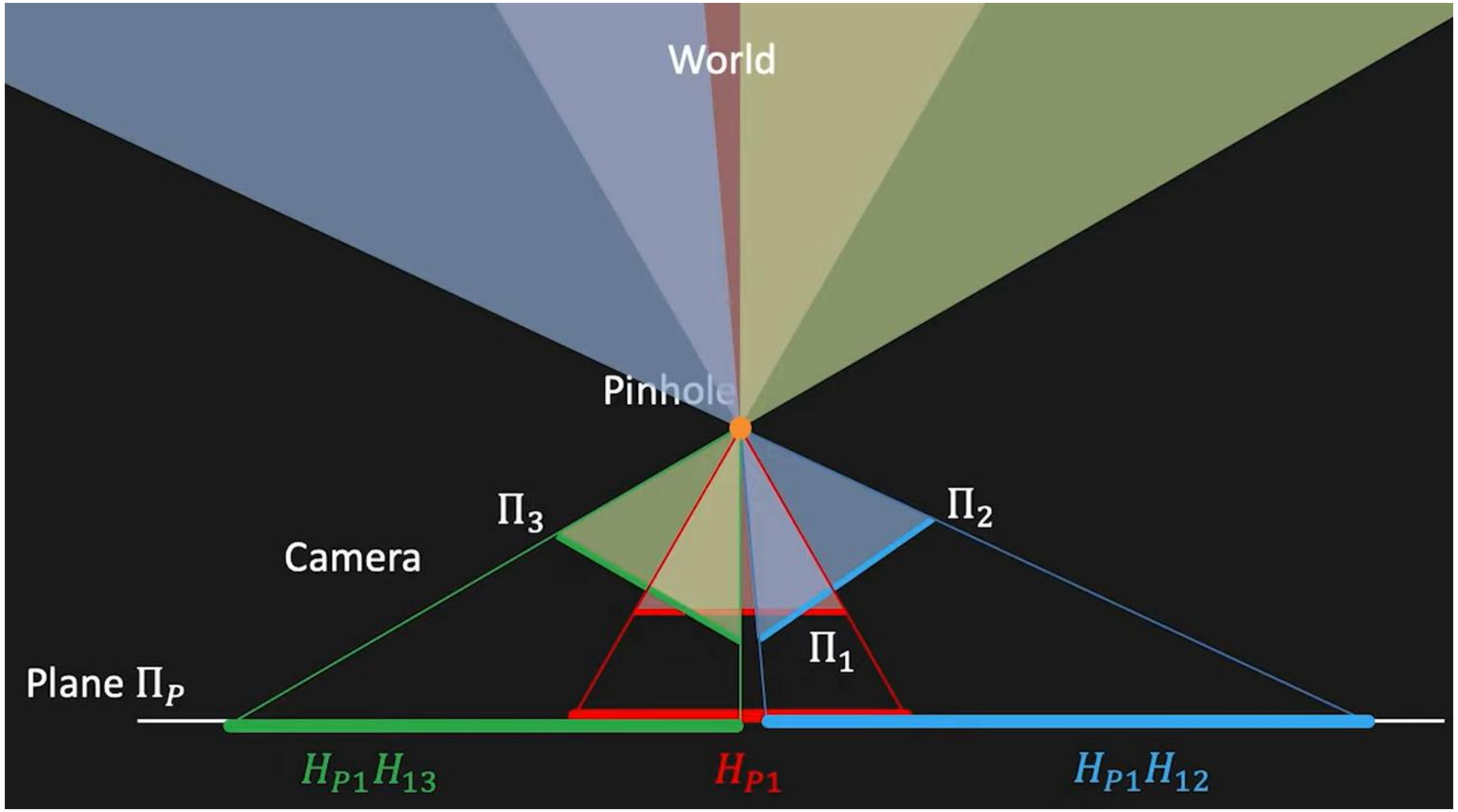
Homography Composition



Homography Composition

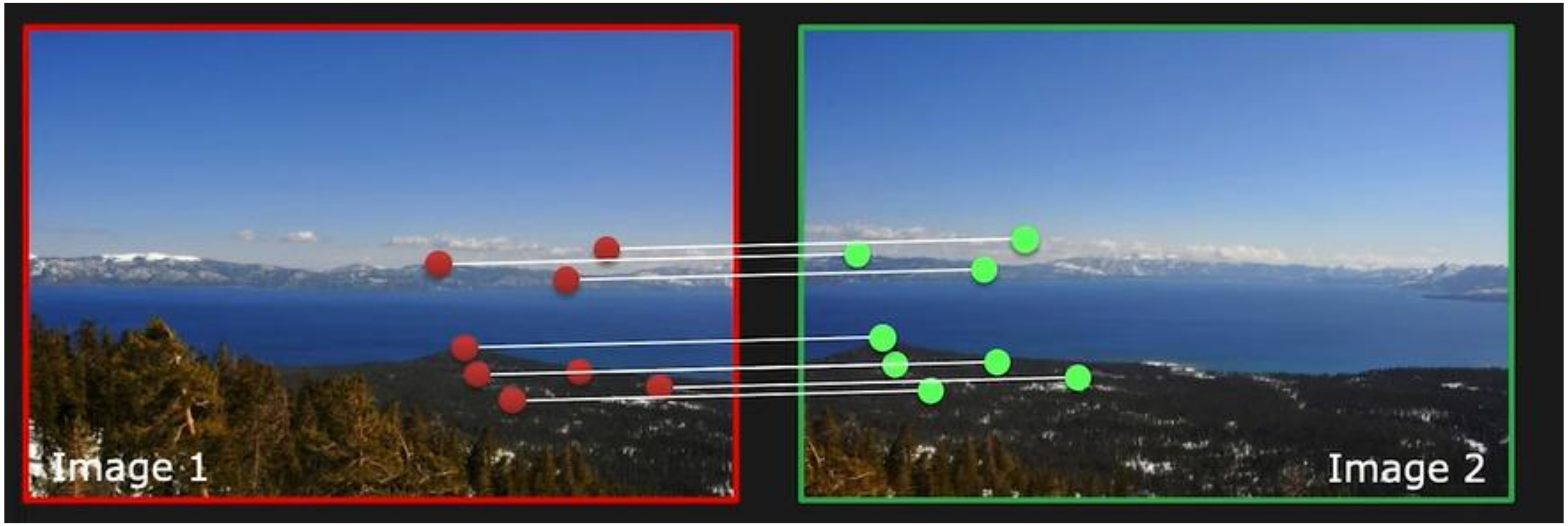


Homography Composition



Computing Homography

- Homography H that best "agrees" with the matches



Homography (conditions)

- Scene of 3D world from single view point

Or

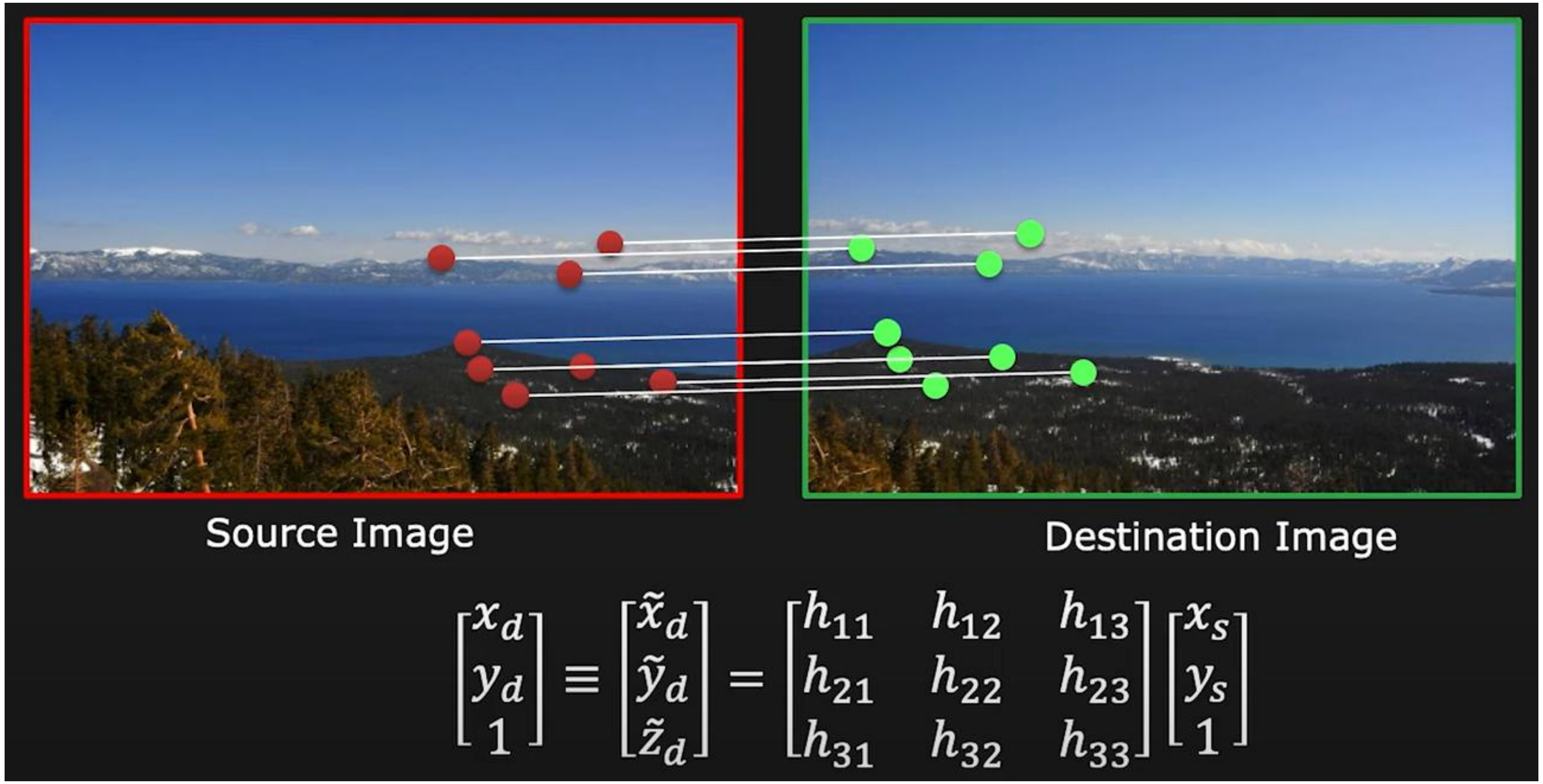
- Plane in 3D world from different view point

Or

- Scene is far away (Plane at infinity)

Computing Homography

- 8 degree of freedom
- 4 pairs of matching points (minimum)



Computing Homography

For a given pair i of corresponding points:

$$x_d{}^{(i)} = \frac{\tilde{x}_d{}^{(i)}}{\tilde{z}_d{}^{(i)}} = \frac{h_{11}x_s^{(i)} + h_{12}y_s^{(i)} + h_{13}}{h_{31}x_s^{(i)} + h_{32}y_s^{(i)} + h_{33}}$$

$$y_d{}^{(i)} = \frac{\tilde{y}_d{}^{(i)}}{\tilde{z}_d{}^{(i)}} = \frac{h_{21}x_s^{(i)} + h_{22}y_s^{(i)} + h_{23}}{h_{31}x_s^{(i)} + h_{32}y_s^{(i)} + h_{33}}$$

Rearranging the terms:

$$x_d{}^{(i)}\left(h_{31}x_s^{(i)} + h_{32}y_s^{(i)} + h_{33}\right) = h_{11}x_s^{(i)} + h_{12}y_s^{(i)} + h_{13}$$

$$y_d{}^{(i)}\left(h_{31}x_s^{(i)} + h_{32}y_s^{(i)} + h_{33}\right) = h_{21}x_s^{(i)} + h_{22}y_s^{(i)} + h_{23}$$

Computing Homography

$$x_d^{(i)}\left(h_{31}x_s^{(i)} + h_{32}y_s^{(i)} + h_{33}\right) = h_{11}x_s^{(i)} + h_{12}y_s^{(i)} + h_{13}$$

$$y_d^{(i)}\left(h_{31}x_s^{(i)} + h_{32}y_s^{(i)} + h_{33}\right) = h_{21}x_s^{(i)} + h_{22}y_s^{(i)} + h_{23}$$

Rearranging the terms and writing as linear equation:

$$\underbrace{\boxed{\begin{bmatrix} x_s^{(i)} & y_s^{(i)} & 1 & 0 & 0 & 0 & -x_d^{(i)}x_s^{(i)} & -x_d^{(i)}y_s^{(i)} & -x_d^{(i)} \\ 0 & 0 & 0 & x_s^{(i)} & y_s^{(i)} & 1 & -y_d^{(i)}x_s^{(i)} & -y_d^{(i)}y_s^{(i)} & -y_d^{(i)} \end{bmatrix}}}_{\text{(Known)}} \underbrace{\boxed{\begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix}}}_{\substack{\mathbf{h} \\ \text{(Unknown)}}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Computing Homography

$$\underbrace{\begin{bmatrix} x_s^{(1)} & y_s^{(1)} & 1 & 0 & 0 & 0 & -x_d^{(1)}x_s^{(1)} & -x_d^{(1)}y_s^{(1)} & -x_d^{(1)} \\ 0 & 0 & 0 & x_s^{(1)} & y_s^{(1)} & 1 & -y_d^{(1)}x_s^{(1)} & -y_d^{(1)}y_s^{(1)} & -y_d^{(1)} \\ & & & & \vdots & & & & \\ x_s^{(i)} & y_s^{(i)} & 1 & 0 & 0 & 0 & -x_d^{(i)}x_s^{(i)} & -x_d^{(i)}y_s^{(i)} & -x_d^{(i)} \\ 0 & 0 & 0 & x_s^{(i)} & y_s^{(i)} & 1 & -y_d^{(i)}x_s^{(i)} & -y_d^{(i)}y_s^{(i)} & -y_d^{(i)} \\ & & & & \vdots & & & & \\ x_s^{(n)} & y_s^{(n)} & 1 & 0 & 0 & 0 & -x_d^{(n)}x_s^{(n)} & -x_d^{(n)}y_s^{(n)} & -x_d^{(n)} \\ 0 & 0 & 0 & x_s^{(n)} & y_s^{(n)} & 1 & -y_d^{(n)}x_s^{(n)} & -y_d^{(n)}y_s^{(n)} & -y_d^{(n)} \end{bmatrix}}_{\substack{A \\ \text{(Known)}}} \underbrace{\begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix}}_{\substack{\mathbf{h} \\ \text{(Unknown)}}} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

Solve for $\mathbf{h}$: $\boxed{A\,\mathbf{h} = \mathbf{0}}$ such that $\|\mathbf{h}\|^2 = 1$

Constrained Least Squares

Solve for $\mathbf{h}$: $\boxed{A\,\mathbf{h} = \mathbf{0}}$ such that $\|\mathbf{h}\|^2 = 1$

Define least squares problem:

$$\min_{\mathbf{h}} \|A\mathbf{h}\|^2 \quad \text{such that } \|\mathbf{h}\|^2 = 1$$

We know that:

$$\|A\mathbf{h}\|^2 = (A\mathbf{h})^T(A\mathbf{h}) = \mathbf{h}^T A^T A\mathbf{h} \quad \text{and} \quad \|\mathbf{h}\|^2 = \mathbf{h}^T\mathbf{h} = 1$$

$$\boxed{\min_{\mathbf{h}}(\mathbf{h}^T A^T A\mathbf{h}) \quad \text{such that } \mathbf{h}^T\mathbf{h} = 1}$$

Constrained Least Squares

$$\min_{\mathbf{h}}(\mathbf{h}^T A^T A\mathbf{h}) \text{ such that } \mathbf{h}^T\mathbf{h} = 1$$

Define Loss function $L(\mathbf{h}, \lambda)$:

$$L(\mathbf{h}, \lambda) = \mathbf{h}^T A^T A\mathbf{h} - \lambda(\mathbf{h}^T\mathbf{h} - 1)$$

Taking derivatives of $L(\mathbf{h}, \lambda)$ w.r.t $\mathbf{h}$: $2A^T A\mathbf{h} - 2\lambda\mathbf{h} = \mathbf{0}$

$$\boxed{A^T A\mathbf{h} = \lambda\mathbf{h}}$$ Eigenvalue Problem

Eigenvector $\mathbf{h}$ with smallest eigenvalue λ of matrix $A^T A$ minimizes the loss function $L(\mathbf{h})$.