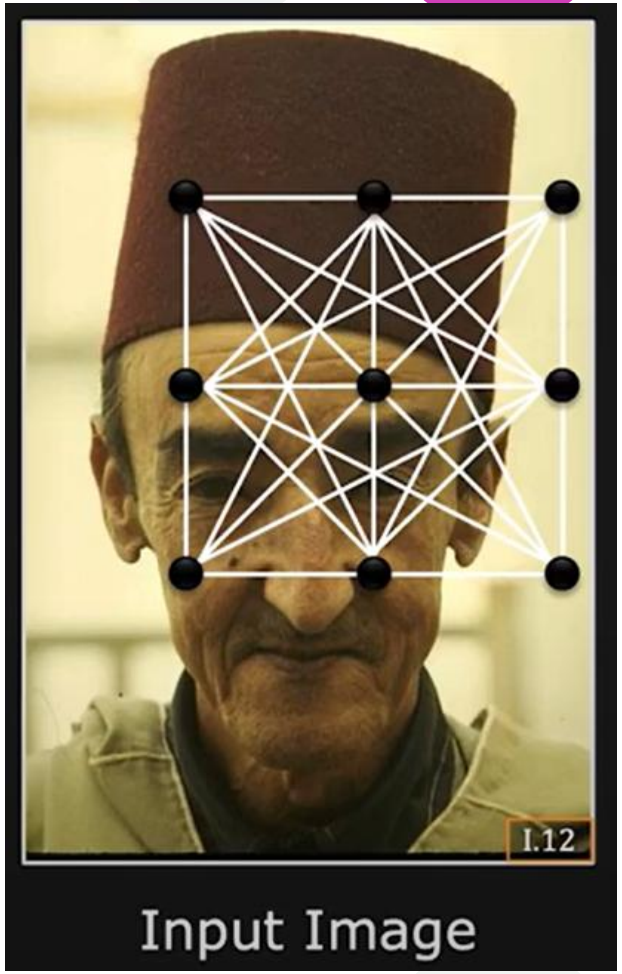
Graph based Segmentation

Images as Graph

- A vertex for each pixel
- An edge between each pair of pixels
- G = (V, E) where V are the vertex and E are the edges.
- Each edge is weighted by the affinity or similarity between its two vertices



Affinity

- Let i and j be two pixels whose features are f_i and f_j

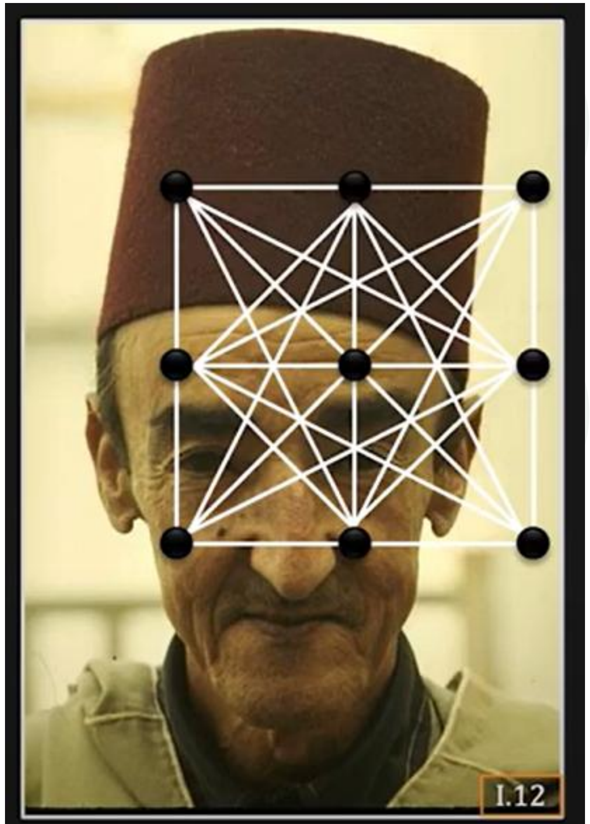
Pixel Dissimilarity:

$$S(\mathbf{f}_i, \mathbf{f}_j) = \sqrt{\left(\sum_k (f_{ik} - f_{jk})^2\right)}$$

Pixel Affinity:

$$w(i,j) = A(\mathbf{f}_i, \mathbf{f}_j) = e^{\left\{\frac{-1}{2\sigma^2} S(\mathbf{f}_i, \mathbf{f}_j)\right\}}$$

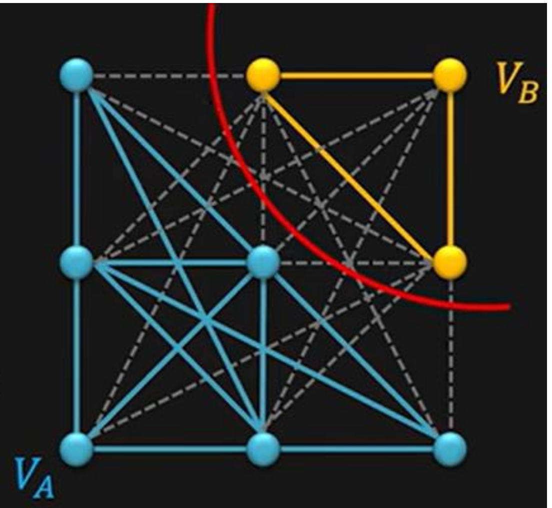
- Smaller the Dissimilarity, larger the Affinity



Input Image

Graph Cut

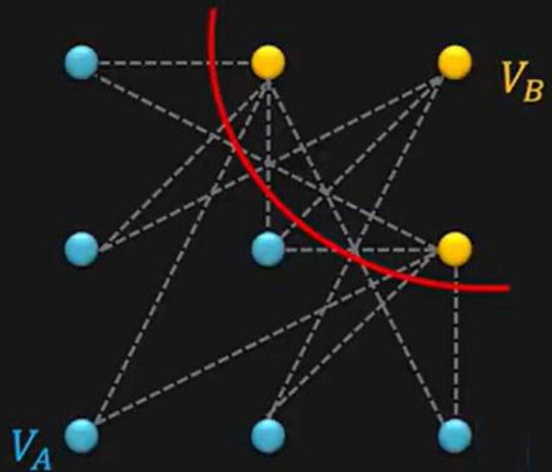
Cut $C = (V_A, V_B)$ is a partition of vertices V of a graph $G = (V, E)$ into two disjoint subsets V_A and V_B.



Cut-Set: Set of edges whose vertices are in different subsets of partition.

Cost of Cut: Sum of weights of cut-set edges.

$$\boxed{cut(V_A, V_B) = \sum_{u \in V_A, v \in V_B} w(u, v)}$$

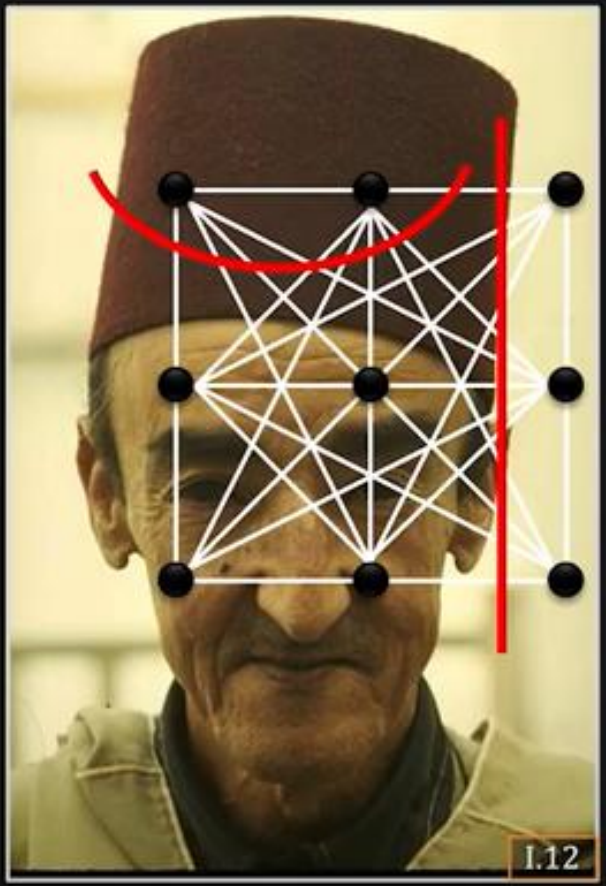


Graph cut Segmentation

Criteria for Graph Cut:

- A pair of vertices (pixels) within a subgraph have high affinity.
- A pair of vertices from two different subgraphs have low affinity.

That is, minimize the cost of cut.
Also called Min-Cut.



Input Image

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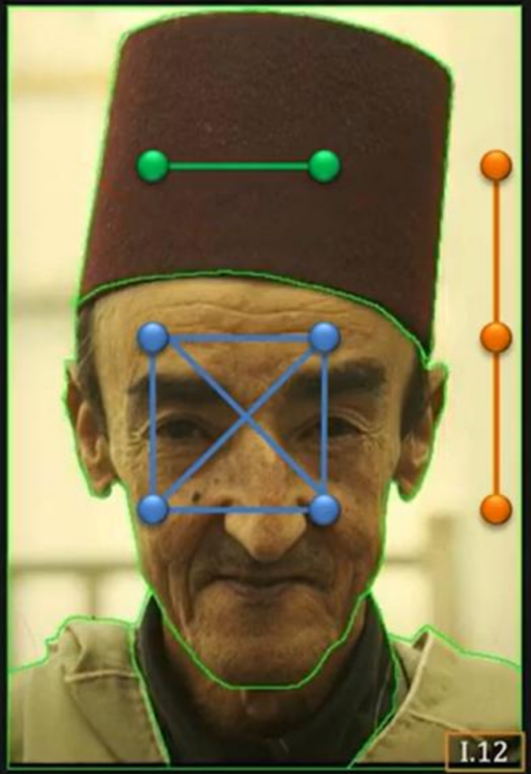
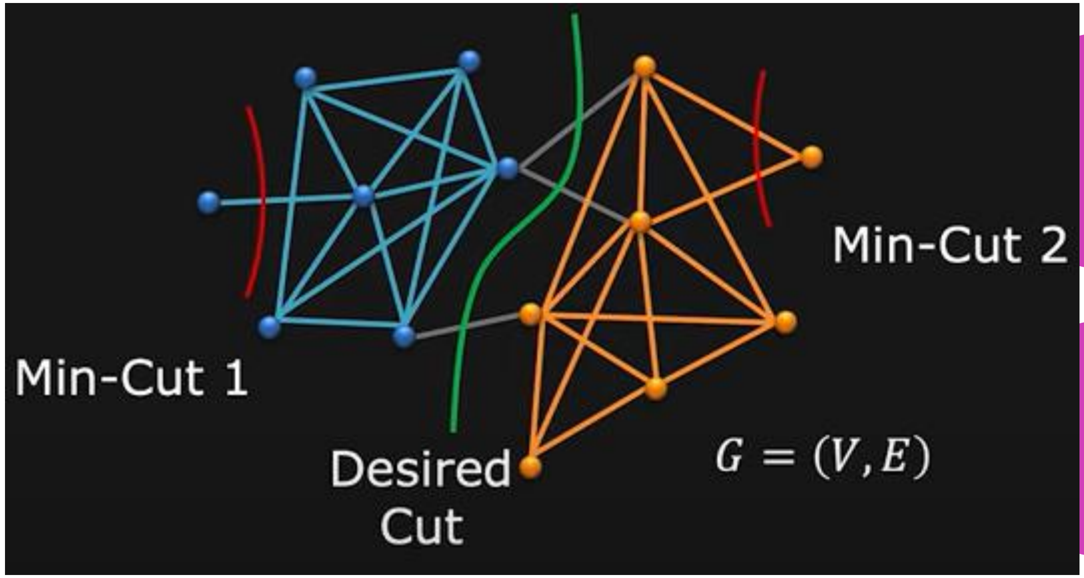


Input Image

Each subgraph is an image segment.

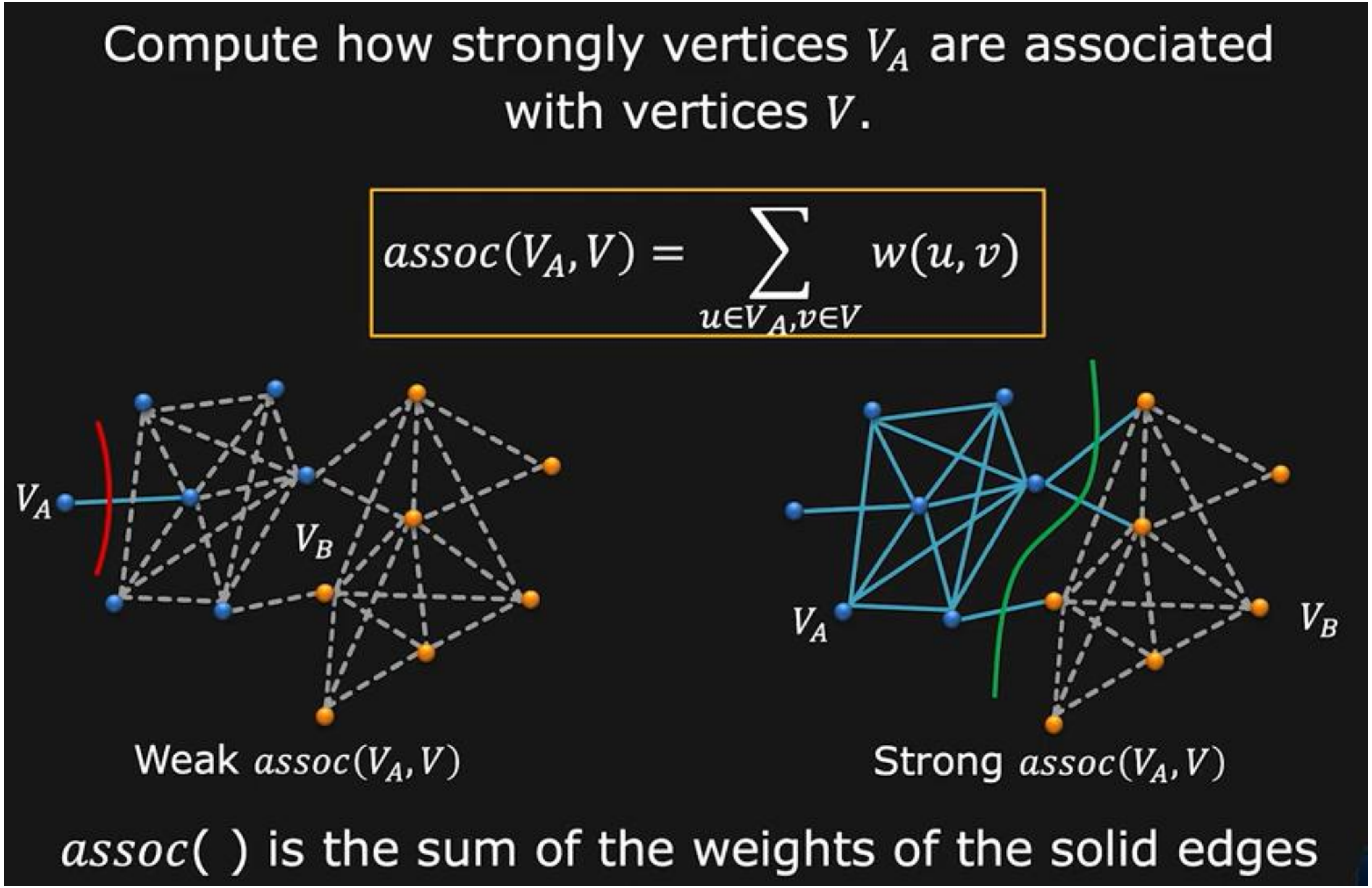
Challenge with Min-cut

- There is a bias to cut small, isolated segments.



- Solution: Normalize Cut to favour larger subgraphs.

Measure of Subgraph Size



Normalized Cut (NCut)

Minimize Cost of Normalized Cut during Partition

$$\boxed{NCut(V_A, V_B) = \frac{cut(V_A, V_B)}{assoc(V_A, V)} + \frac{cut(V_A, V_B)}{assoc(V_B, V)}}$$

NCut Segmentation Results