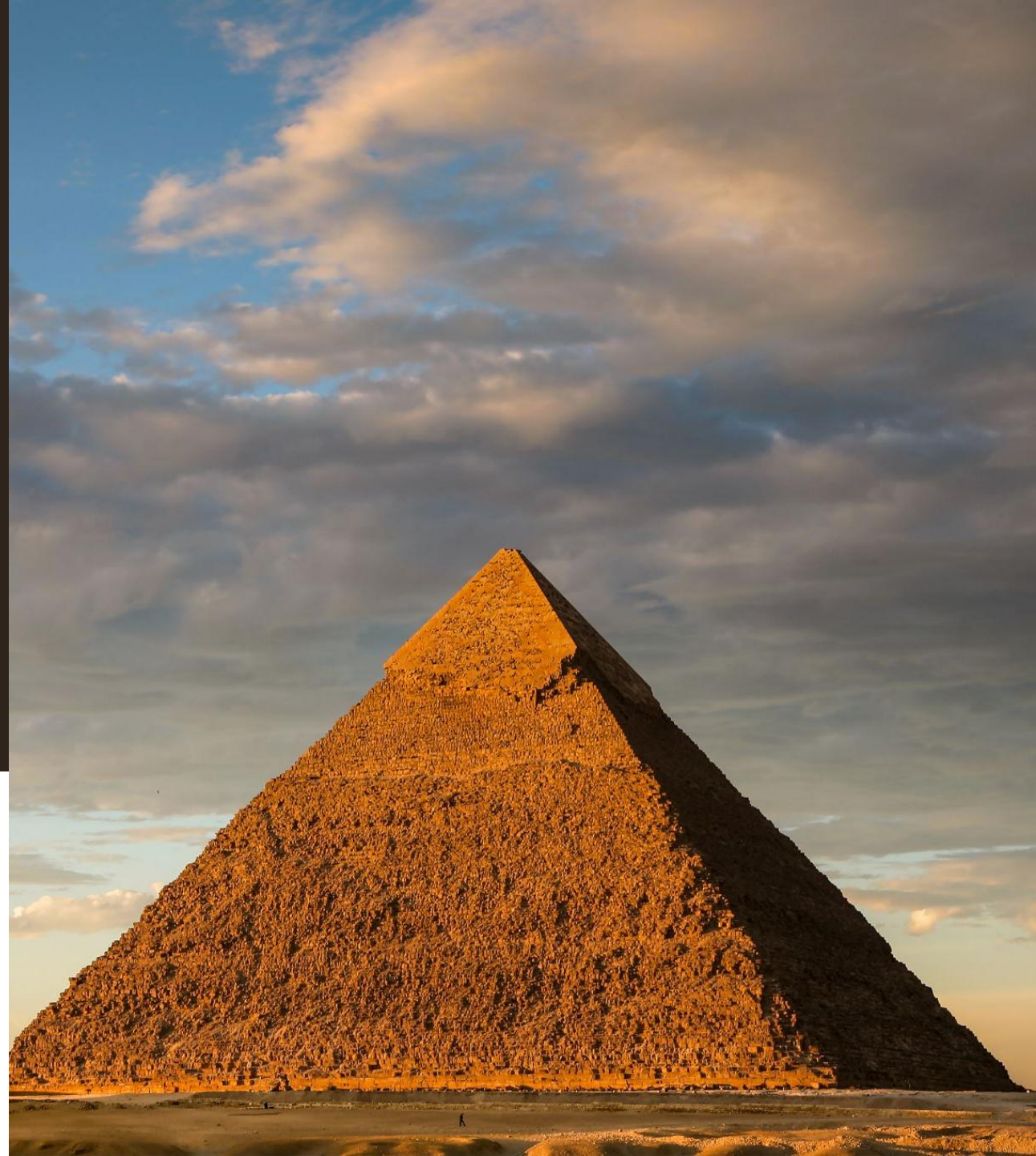
Pyramid

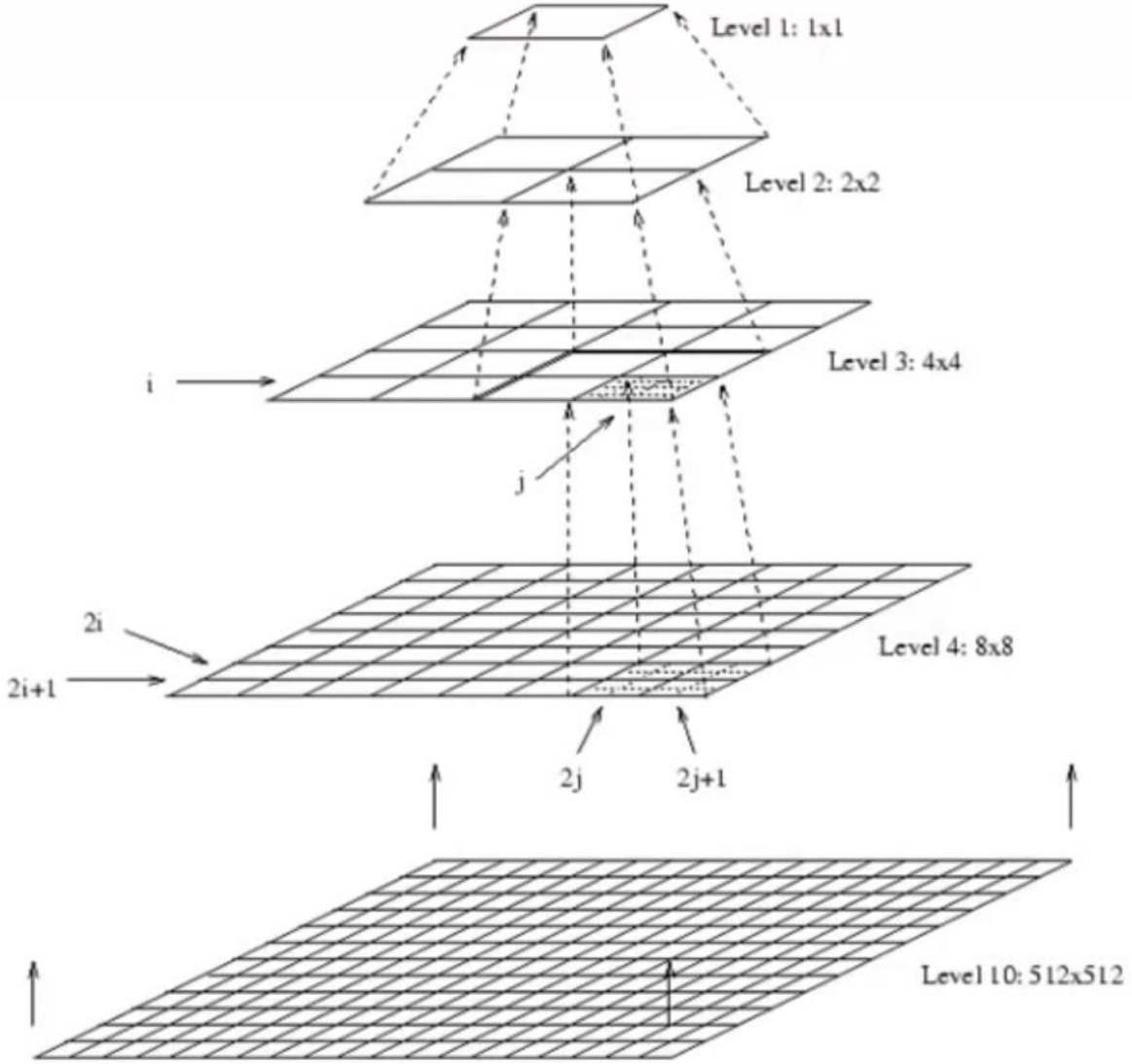
Gaussian



Applications of Pyramid

1. Representing image at different Scales
2. Reduce or Expand the resolution
3. Image Compression
4. Image Compositing
5. Optical flow using Pyramids

Gaussian Pyramid



Gaussian Pyramid (Reduce)

$$g_l(i,j) = \sum_{m=-2}^{2} \sum_{n=-2}^{2} w(m,n)\, g_{l-1}(2i+m, 2j+n)$$

$$g_l = \mathrm{REDUCE}\left[g_{l-1}\right]$$

Gaussian Pyramid (Reduce)

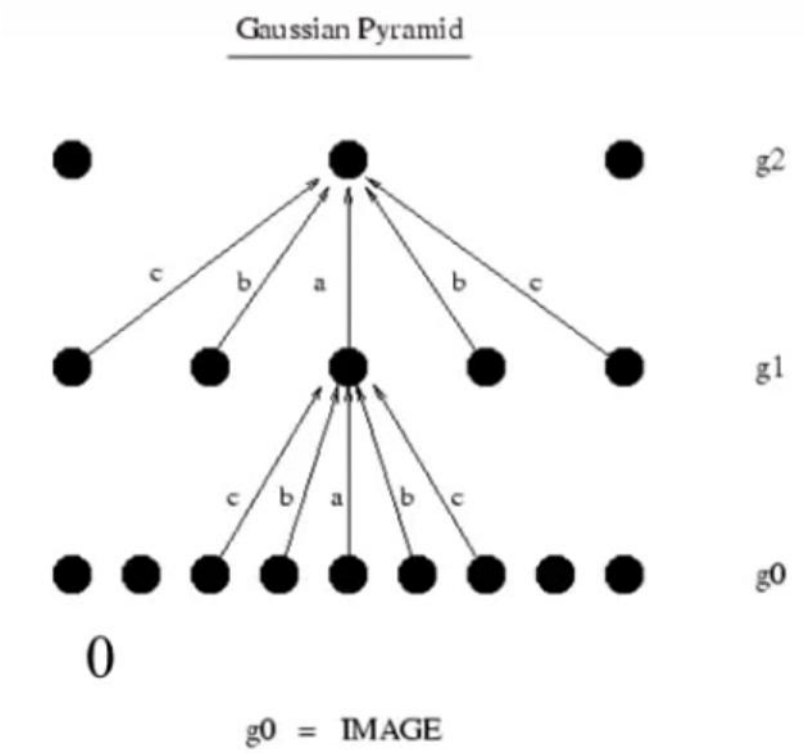
$$g_l(i) = \sum_{m=-2}^{2} \hat{w}(m)\, g_{l-1}(2i+m)$$

$i = 1$

$$g_l(1) = \hat{w}(-2)\, g_{l-1}(0) + \hat{w}(-1)\, g_{l-1}(1) + \hat{w}(0)\, g_{l-1}(2)$$

$$+ \hat{w}(1)\, g_{l-1}(3) + \hat{w}(2)\, g_{l-1}(4)$$

Gaussian Pyramid (Reduce)



Gaussian Pyramid (Expand)

$$g_{l,n}(i,j) = \sum_{p=-2}^{2} \sum_{q=-2}^{2} w(p,q)\, g_{l,n-1}\left(\frac{i-p}{2}, \frac{j-q}{2}\right)$$

$$g_{l,n} = \text{EXPAND}\left[g_{l,n-1}\right]$$

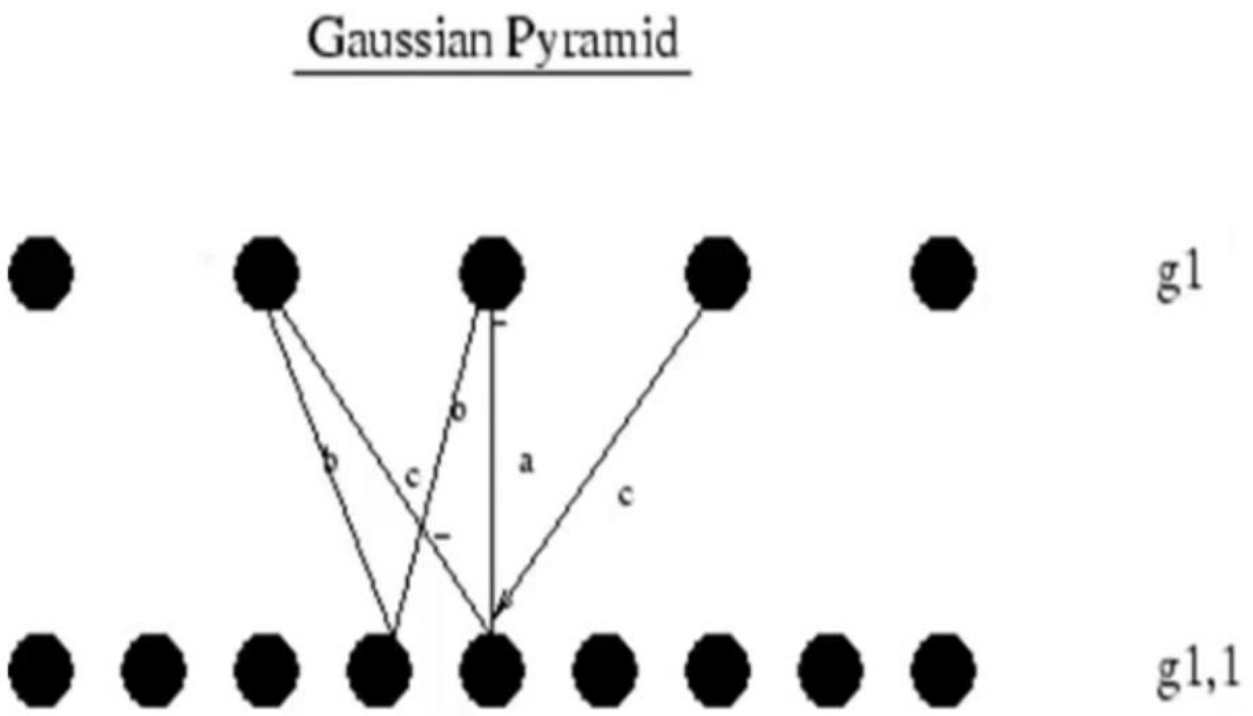
Gaussian Pyramid (Expand)

$$g_{l,n}(i) = \sum_{p=-2}^{2} \hat{w}(p)\, g_{l,n-1}\left(\frac{i-p}{2}\right)$$

$$g_{l,n}(3) = \hat{w}(-2)\, g_{l,n-1}(5/2) + \hat{w}(-1)\, g_{l,n-1}(2) + \hat{w}(0)\, g_{l,n-1}(3/2) + \hat{w}(1)\, g_{l,n-1}(1) + \hat{w}(2)\, g_{l,n-1}(1/2)$$

$$g_{l,n}(4) = \hat{w}(-2)\, g_{l,n-1}(3) + \hat{w}(-1)\, g_{l,n-1}(5/2) + \hat{w}(0)\, g_{l,n-1}(2) + \hat{w}(1)\, g_{l,n-1}(3/2) + \hat{w}(2)\, g_{l,n-1}(1)$$

Gaussian Pyramid (Expand)



Convolution mask properties

$$[\, w(-2),\ w(-1),\ w(0),\ w(1),\ w(2) \,]$$

$$[\, c \quad b \quad a \quad b \quad c \,]$$

Separable

$$w(m,n) = \hat{w}(m)\,\hat{w}(n)$$

Symmetric

$$\hat{w}(i) = \hat{w}(-i)$$

Convolution mask properties

The sum of mask should be 1.

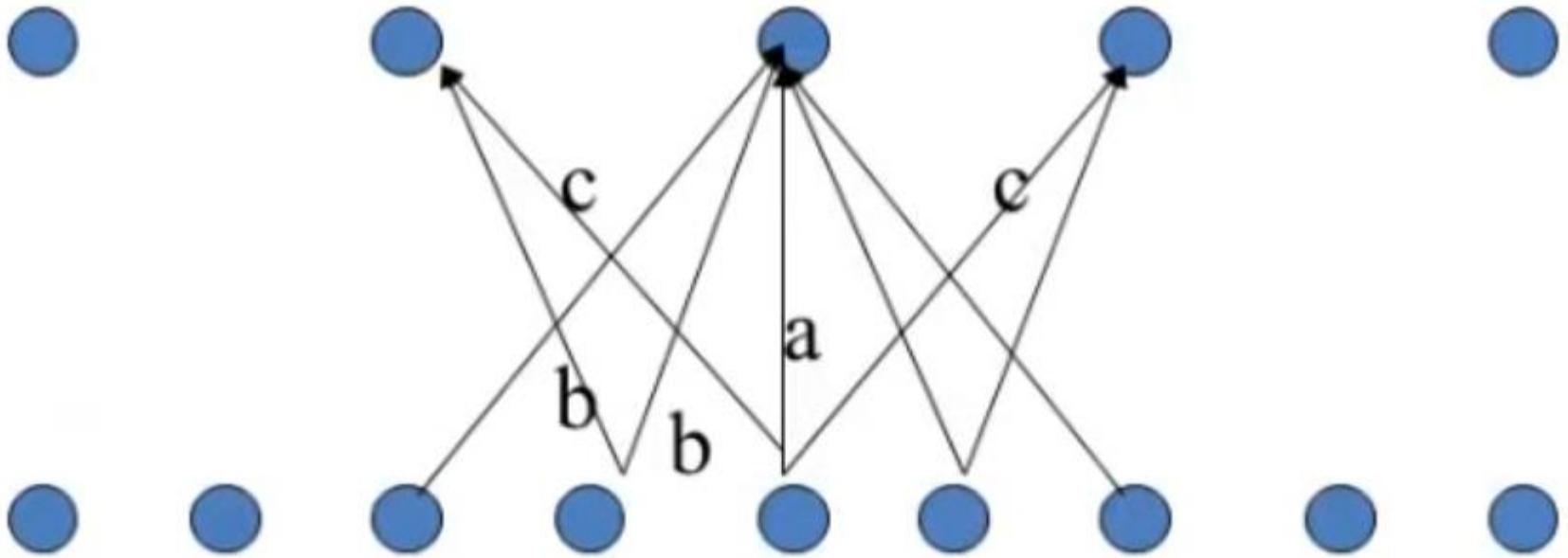
$$a + 2b + 2c = 1$$

All nodes at a given level must contribute the same total weight to the nodes at the next higher level.

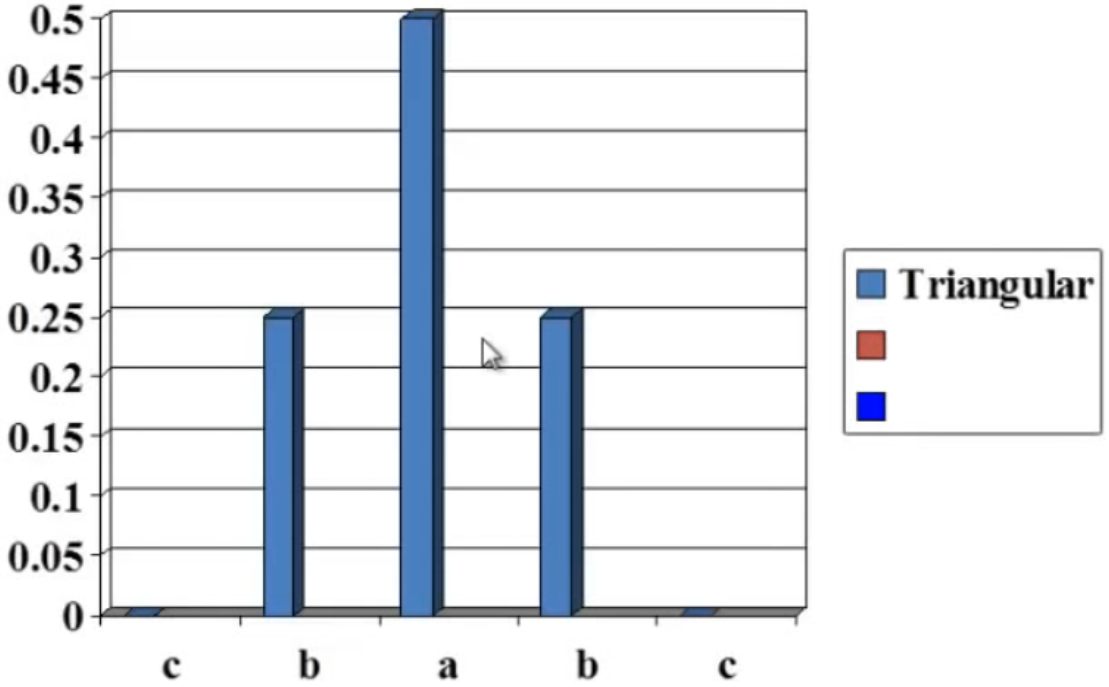
$$a + 2c = 2b$$

$$\Rightarrow \quad b = 1/4 \quad ; \quad c = 1/4 - a/2$$

Convolution mask properties

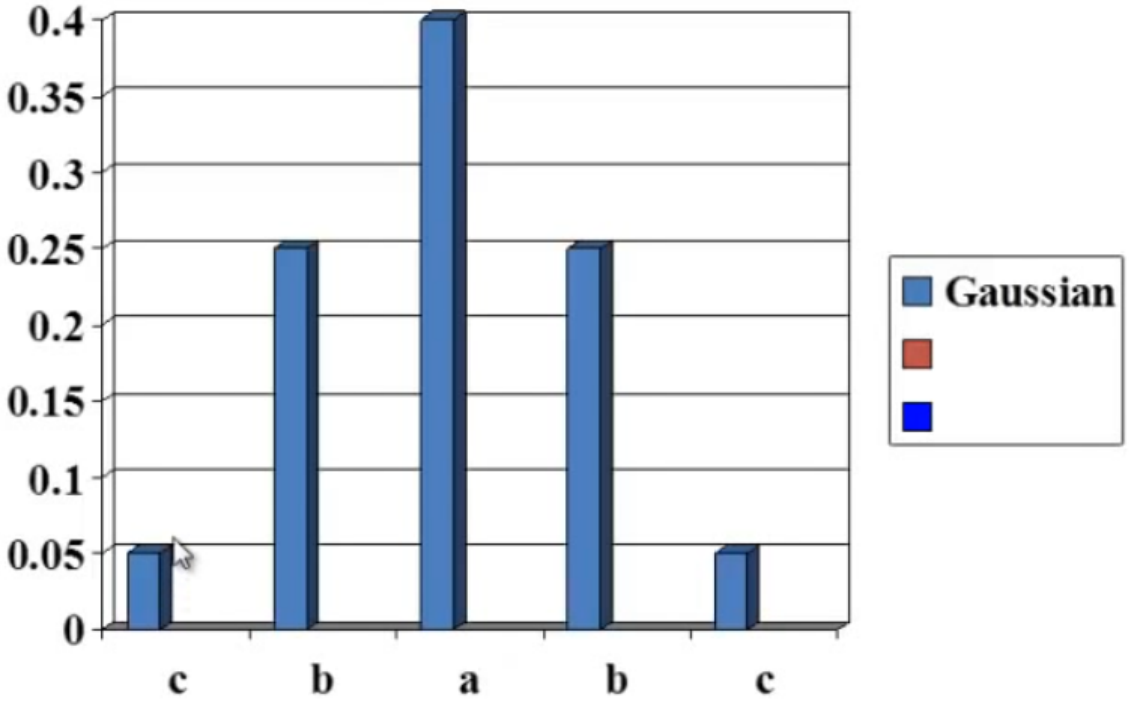


Convolution mask (Triangular)



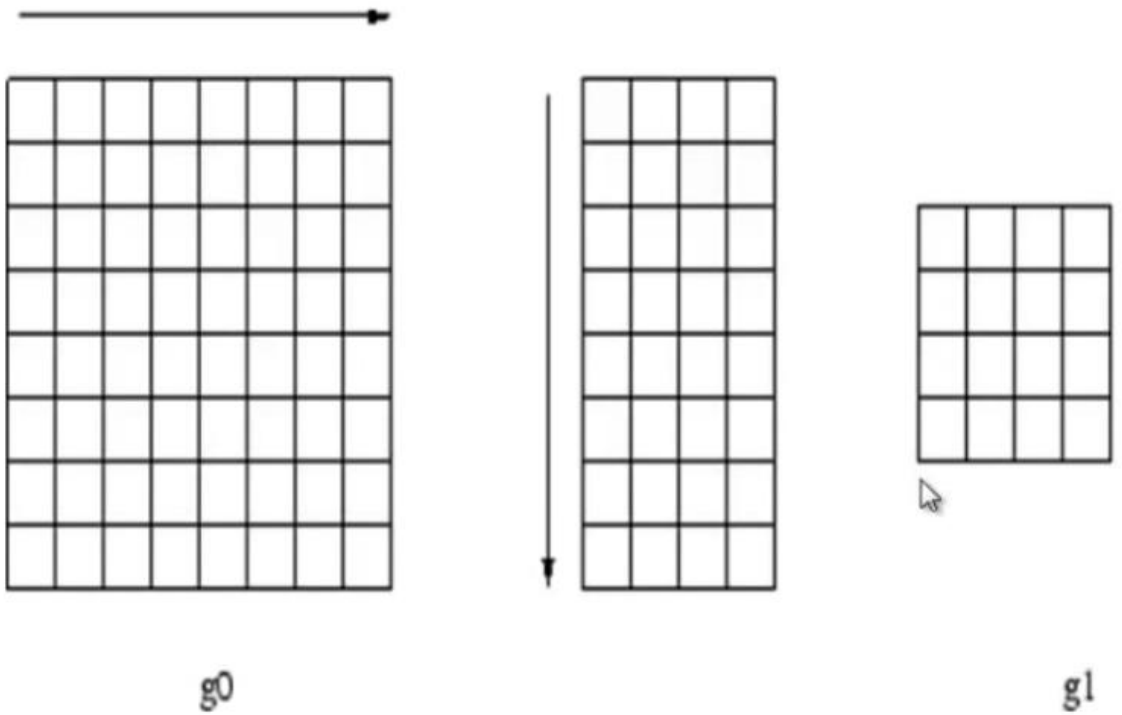
$a = 0.5$

Convolution mask (Gaussian)



$a = 0.4$

Separability



Algorithm

1. Apply 1-D mask to alternate pixels along each row of image.
2. Apply 1-D mask to each pixel along alternate columns of the resultant image from previous step.