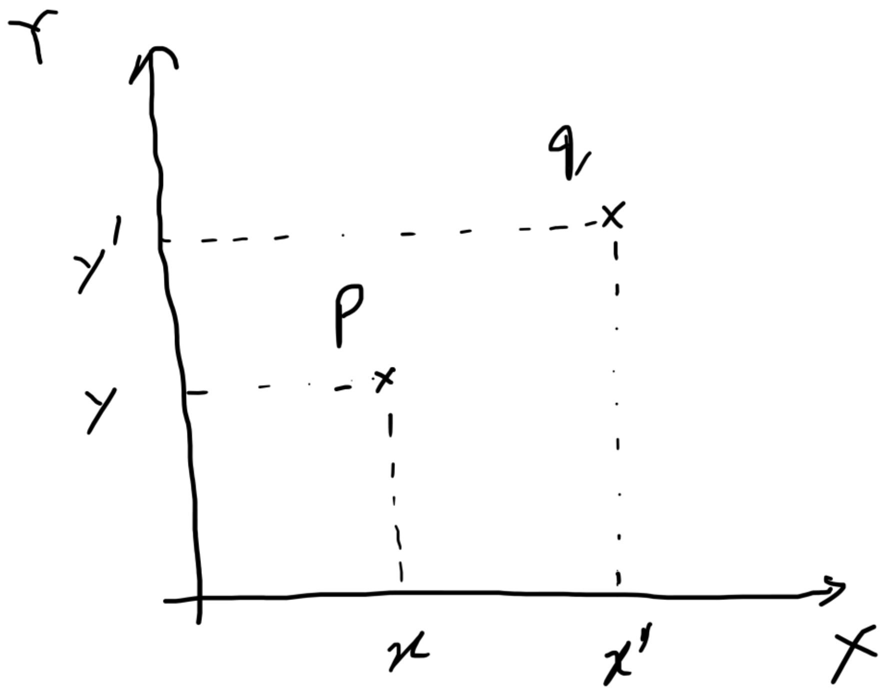
# Transformations

2D

# Translation



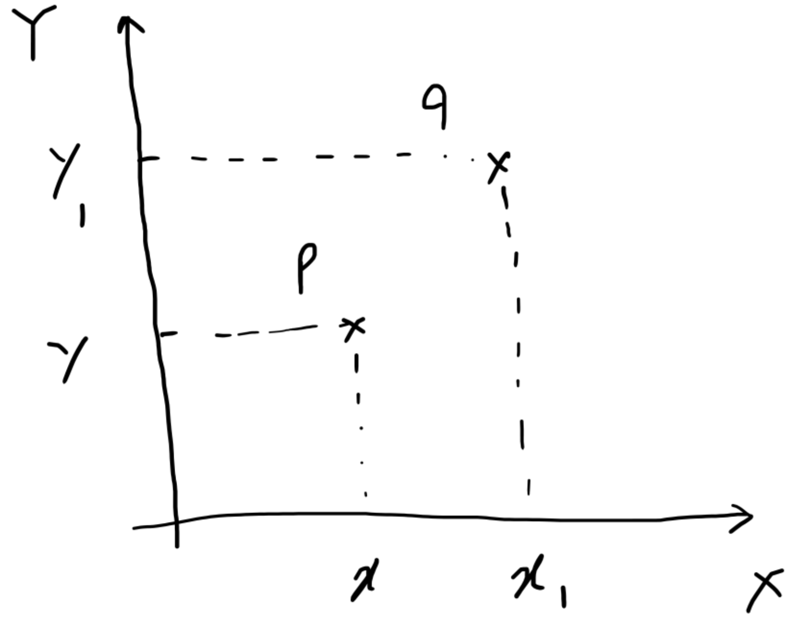

$$x' = x + t_x$$

$$y' = y + t_y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

# Scaling



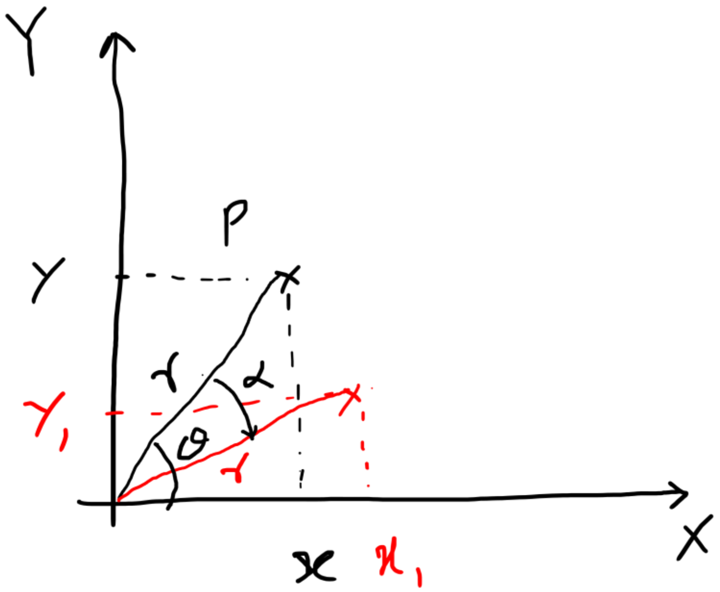

$$x_1 = C_x x$$

$$Y_1 = C_y Y$$

$$\begin{bmatrix} x_1 \\ Y_1 \end{bmatrix} = \begin{bmatrix} C_x & 0 \\ 0 & C_y \end{bmatrix} \begin{bmatrix} x \\ Y \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ Y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} C_x & 0 & 0 \\ 0 & C_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ Y \\ 1 \end{bmatrix}$$

# Rotation




$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$x_1 = r\cos(\theta - \alpha)$$

$$y_1 = r\sin(\theta - \alpha)$$

$$x_1 = r\cos\theta\cos\alpha + r\sin\theta\sin\alpha$$

$$\left.\begin{aligned} x_1 &= x\cos\alpha + y\sin\alpha \\ y_1 &= y\cos\alpha - x\sin\alpha \end{aligned}\right\}$$

$$\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

R + T ( Euclidean)

$$x' = Rx + t$$

S + R ( Similarity)

$$x' = SRx + t$$

Affine

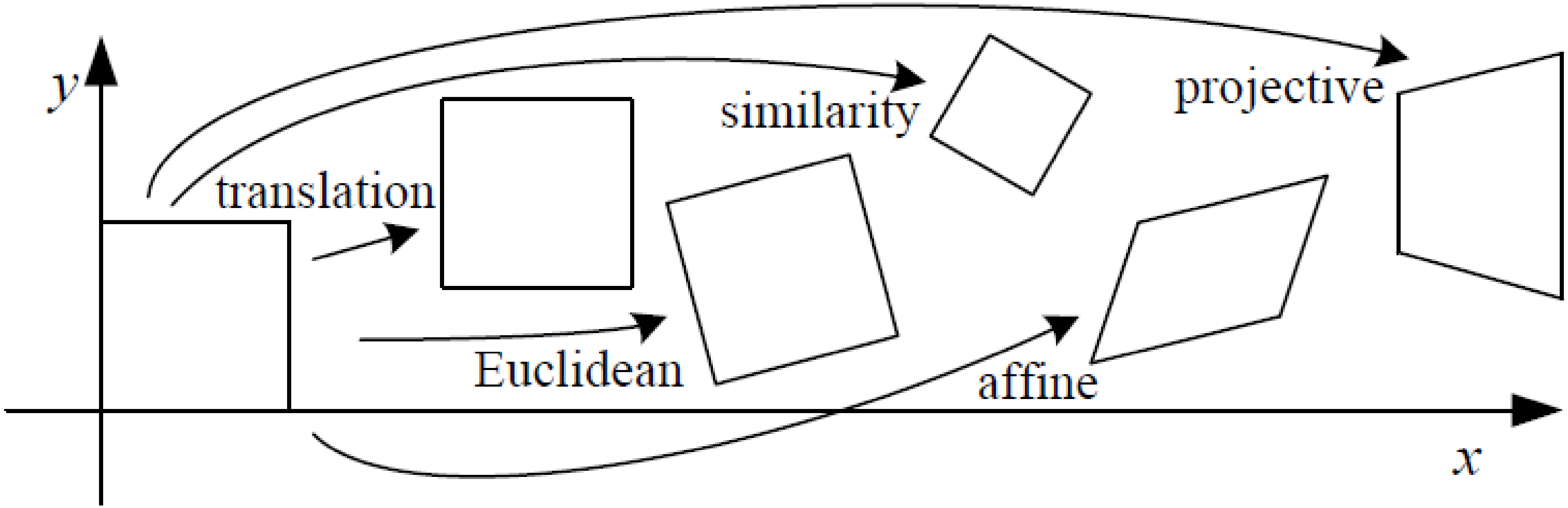
$$x' = Ax$$

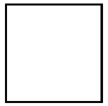
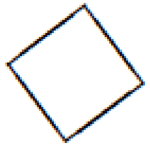
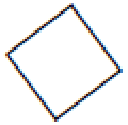
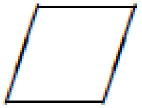
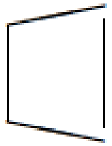
## Projective (Homography)

$$\tilde{x}' = \tilde{H}\tilde{x}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$x' = \frac{h_{00}x + h_{01}y + h_{02}}{h_{20}x + h_{21}y + h_{22}} \; ; \quad y' = \frac{h_{10}x + h_{11}y + h_{12}}{h_{20}x + h_{21}y + h_{22}}$$


y
translation
similarity
projective
Euclidean
affine
x

| Transformation | Matrix | # DoF | Preserves | Icon |
|---|---|---|---|---|
| translation | $\begin{bmatrix} \boldsymbol{I} & \vert & \boldsymbol{t} \end{bmatrix}_{2\times 3}$ | 2 | orientation |  |
| rigid (Euclidean) | $\begin{bmatrix} \boldsymbol{R} & \vert & \boldsymbol{t} \end{bmatrix}_{2\times 3}$ | 3 | lengths |  |
| similarity | $\begin{bmatrix} s\boldsymbol{R} & \vert & \boldsymbol{t} \end{bmatrix}_{2\times 3}$ | 4 | angles |  |
| affine | $\begin{bmatrix} \boldsymbol{A} \end{bmatrix}_{2\times 3}$ | 6 | parallelism |  |
| projective | $\begin{bmatrix} \tilde{\boldsymbol{H}} \end{bmatrix}_{3\times 3}$ | 8 | straight lines |  |

# Concatenation of several transforms

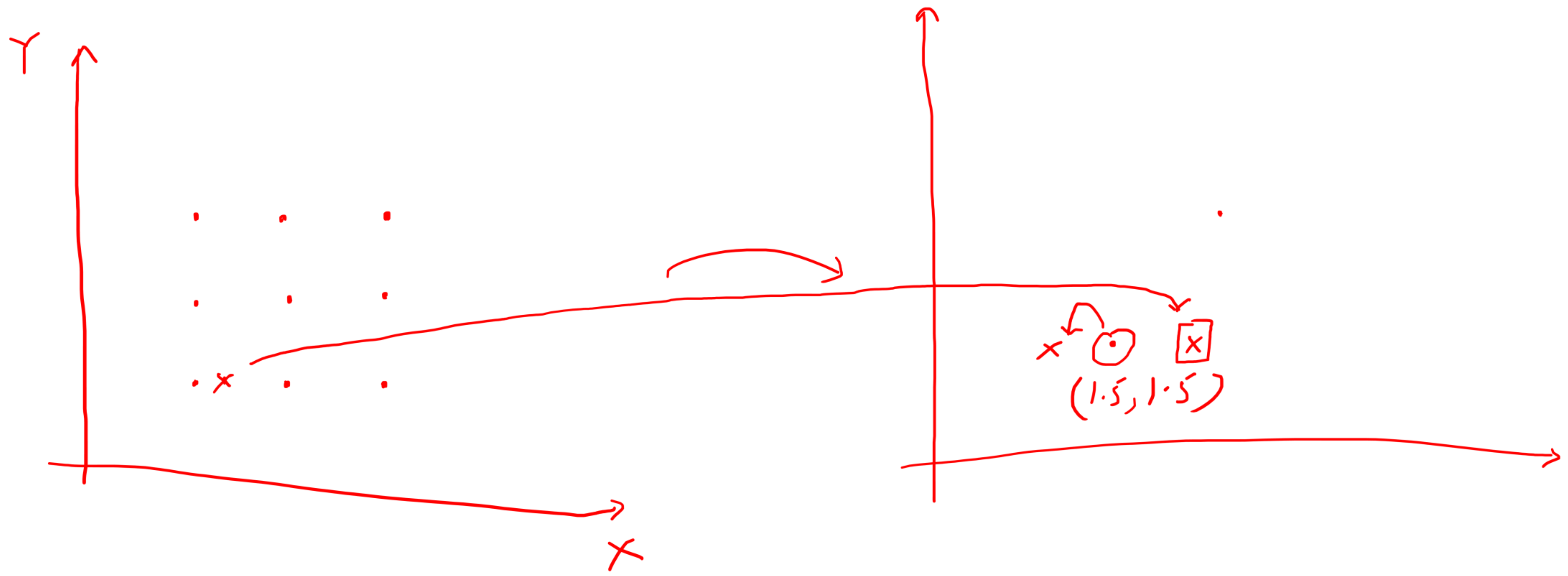
$$P(x, y) \longrightarrow q(x_1, y_1)$$

$$q = R_\theta(S(T(P)))$$

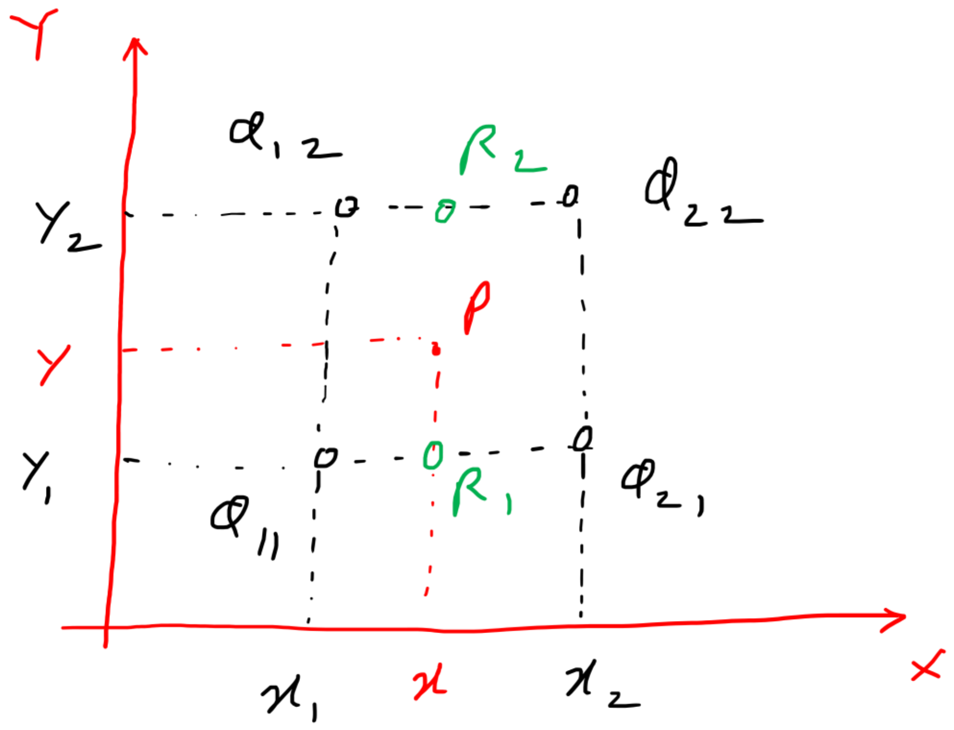
$$T \rightarrow S \rightarrow R$$

# Interpolation

$c_x = c_y = 1.5$



# Bilinear Interpolation




## Horizontal

$$f(R_1) = \frac{x_2 - x}{x_2 - x_1} Q_{11} + \frac{x - x_1}{x_2 - x_1} Q_{21}$$

$$f(R_2) = \frac{x_2 - x}{x_2 - x_1} Q_{12} + \frac{x - x_1}{x_2 - x_1} Q_{22}$$

## Vertical

$$f(P) = \frac{Y_2 - Y}{Y_2 - Y_1} f(R_1) + \frac{Y - Y_1}{Y_2 - Y_1} f(R_2)$$

$$I(x,y) = ax + bY + cxy + d$$