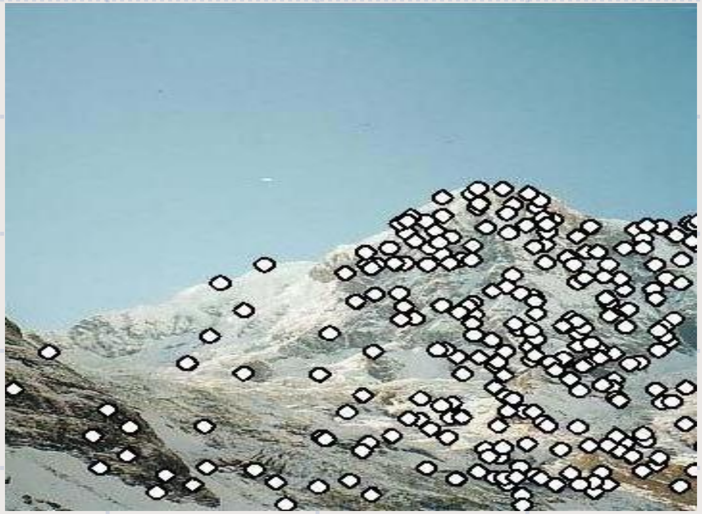
Harris Corner Detector

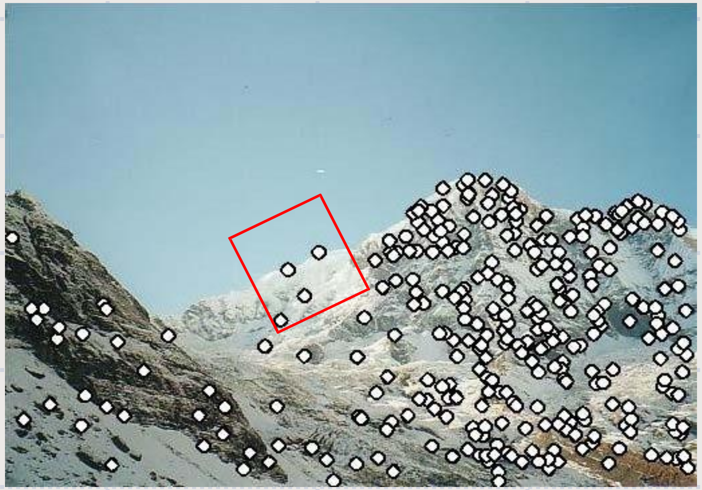
Interest Point Detection

Objective

Local features: main components

- Detection
- Description
- Matching





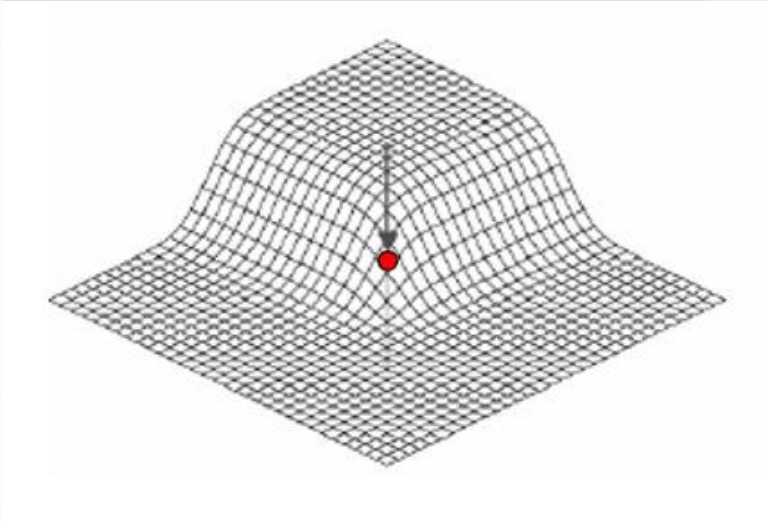
Application areas

- Automate object tracking
- Point matching for computing disparity
- Stereo calibration
- Motion based segmentation
- Recognition
- 3D object reconstruction
- Robot navigation
- Image retrieval and indexing

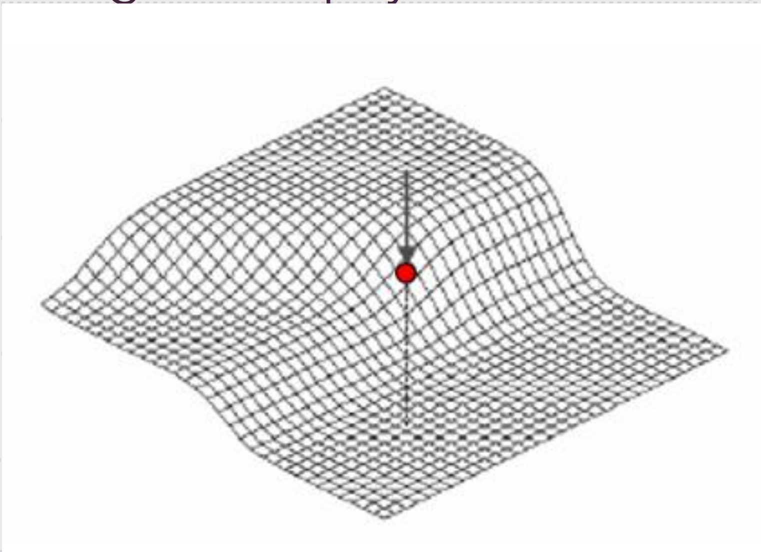
Goal: interest operator repeatability

We want to detect (at least some of) the same points in both images

What is an interest point



- Expressive texture
- The point at which the direction of the boundary of object changes abruptly
- Intersection point between two or more edge segments

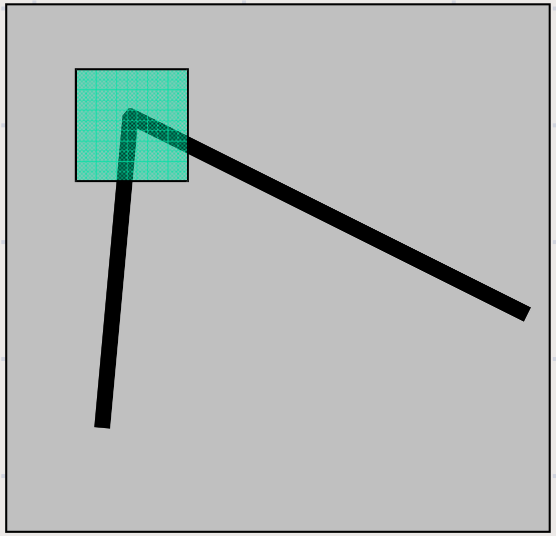


Properties of Interest Point Detectors

- Detect all (or most) true interest points
- No false interest points
- Well localized
- Robust with respect to noise
- Efficient detection

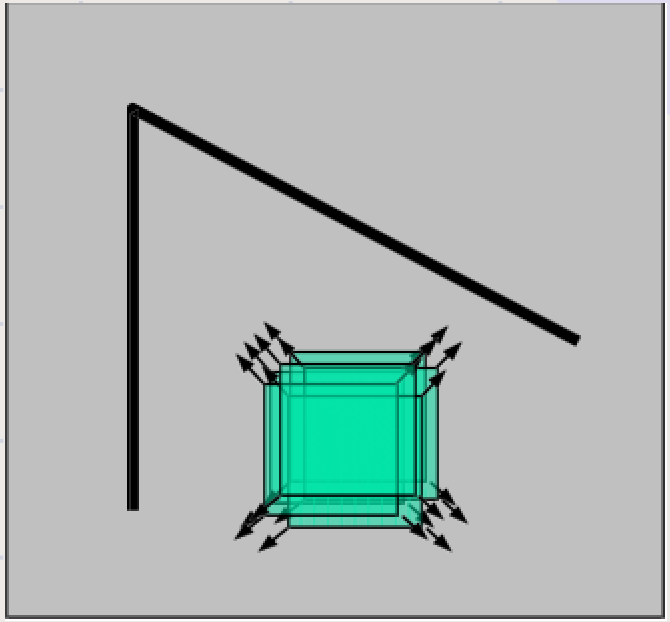
Harris Corner Detector

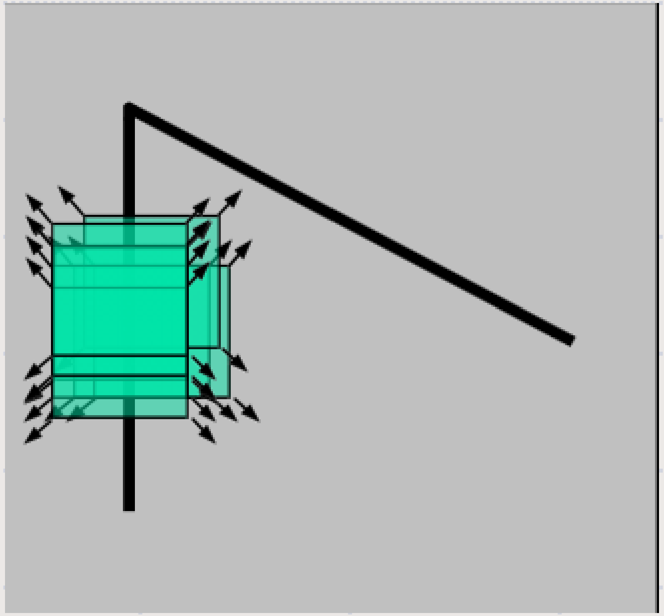
- Corner point can be recognized in a window
- Shifting a window in any direction should give a large change in intensity

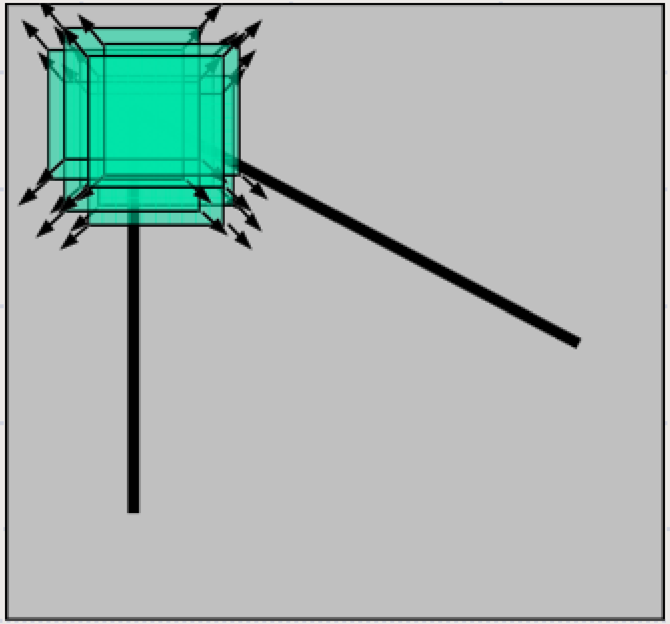


The thought!

- Flat
- Edge
- Corner

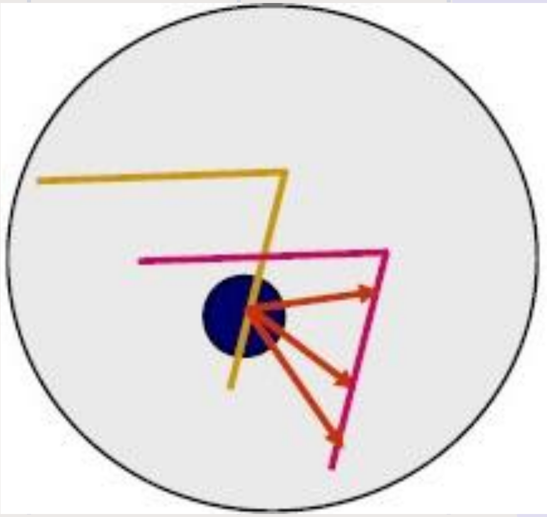


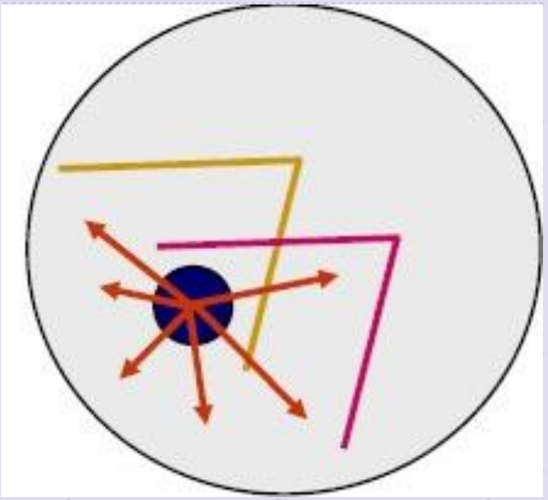


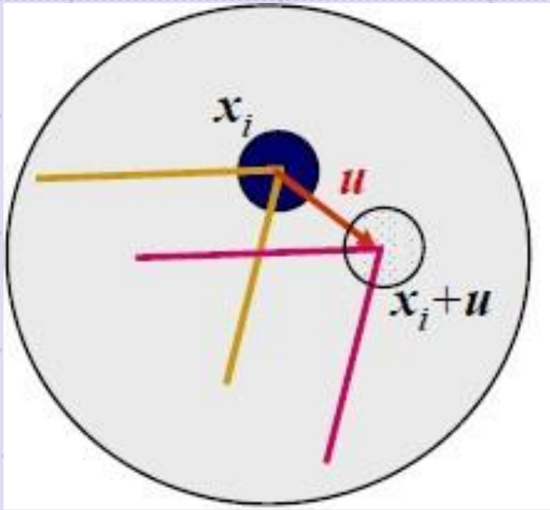


Aperture Problem

- Flat
- Edge
- Corner







Correlation

- f = image
- h = Kernal

$$f \otimes h = \sum_{k} \sum_{l} f(k,l)\,h(i+k, j+l)$$

$$f = \begin{array}{|c|c|c|} \hline f_1 & f_2 & f_3 \\ \hline f_4 & f_5 & f_6 \\ \hline f_7 & f_8 & f_9 \\ \hline \end{array} \otimes h = \begin{array}{|c|c|c|} \hline h_1 & h_2 & h_3 \\ \hline h_4 & h_5 & h_6 \\ \hline h_7 & h_8 & h_9 \\ \hline \end{array} \longrightarrow$$

$$\begin{aligned} f * h = {} & f_1h_1 + f_2h_2 + f_3h_3 \\ & + f_4h_4 + f_5h_5 + f_6h_6 \\ & + f_7h_7 + f_8h_8 + f_9h_9 \end{aligned}$$

Correlation

- Cross correlation

$$f \otimes h = \sum_k \sum_l f(k,l)h(i+k, j+l)$$

- Auto correlation

$$f \otimes f = \sum_k \sum_l f(k,l)f(i+k, j+l)$$

Correlation v/s SSD

$$SSD = \sum_k \sum_l \left(f(k,l) - h(k,l)\right)^2$$

Sum of Squares Difference

minimize

$$SSD = \sum_k \sum_l \left(\cancel{f(k,l)^2} - 2h(k,l)f(k,l) + \cancel{h(k,l)^2}\right)$$

minimize

$$SSD = \sum_k \sum_l \left(-2h(k,l)f(k,l)\right)$$

These terms do not depend on correlation

$$SSD = \sum_k \sum_l \left(\cancel{2}h(k,l)f(k,l)\right)$$

maximize

$$Correlation = \sum_k \sum_l \left(h(k,l)f(k,l)\right)$$

maximize

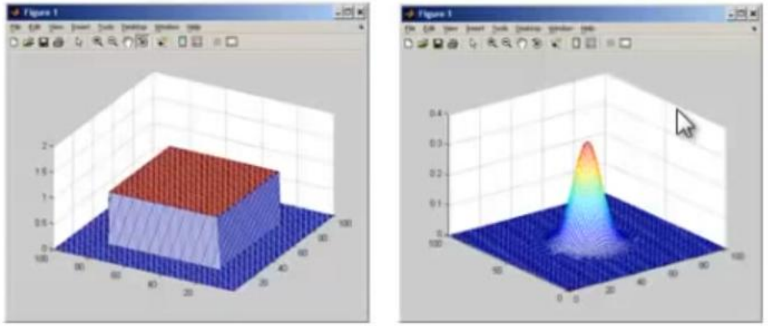
Harris Corner Detector

- Change of intensity for shift (u,v)

$$E(u,v) = \sum_{x,y} \underbrace{w(x,y)}_{\text{window function}} \, [\underbrace{I(x+u, y+v)}_{\text{shifted intensity}} - \underbrace{I(x,y)}_{\text{intensity}}]^2$$

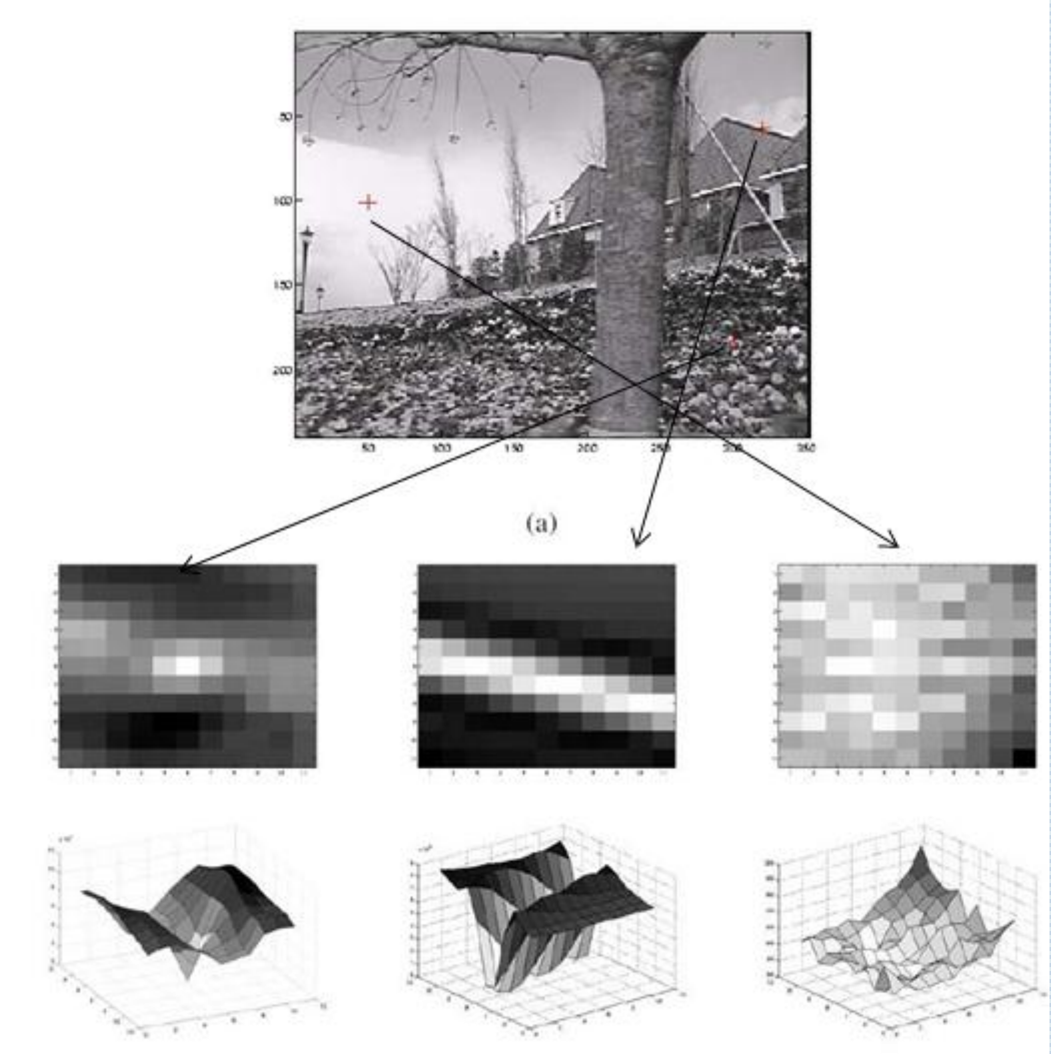
Auto-correlation

Window functions →



UNIFORM GAUSSIAN

Auto-Coorelation



Taylor Series

f(x) can be represented at point *a* in terms of its derivatives

$$f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \cdots.$$

Harris Corner Detector

$$E(u,v)=\sum_{x,y}\underbrace{w(x,y)}_{\text{window function}}\,[\underbrace{I(x+u,y+v)}_{\text{shifted intensity}}-\underbrace{I(x,y)}_{\text{intensity}}]^2$$

$$E(u,v)=\sum_{x,y}\underbrace{w(x,y)}_{\text{window function}}\,[\underbrace{I(x,y)+uI_x+vI_y}_{\text{shifted intensity}}-\underbrace{I(x,y)}_{\text{intensity}}]^2$$

Taylor Series

$$E(u,v)=\sum_{x,y}w(x,y)[\,uI_x+vI_y\,]^2$$

$$E(u,v)=\sum_{x,y}w(x,y)\left[\begin{pmatrix}u & v\end{pmatrix}\begin{pmatrix}I_x\\ I_y\end{pmatrix}\right]^2$$

$$E(u,v)=\sum_{x,y}w(x,y)\begin{pmatrix}u & v\end{pmatrix}\begin{pmatrix}I_x\\ I_y\end{pmatrix}\begin{pmatrix}I_x & I_y\end{pmatrix}\begin{pmatrix}u\\ v\end{pmatrix}$$

$$E(u,v)=\begin{pmatrix}u & v\end{pmatrix}\left[\sum_{x,y}w(x,y)\begin{pmatrix}I_x\\ I_y\end{pmatrix}\begin{pmatrix}I_x & I_y\end{pmatrix}\right]\begin{pmatrix}u\\ v\end{pmatrix}$$

$$E(u,v)=\begin{pmatrix}u & v\end{pmatrix}M\begin{pmatrix}u\\ v\end{pmatrix}$$

Harris Corner Detector

$$E(u,v) = \begin{pmatrix} u & v \end{pmatrix} M \begin{pmatrix} u \\ v \end{pmatrix}$$

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x I_x & I_x I_y \\ I_x I_y & I_y I_y \end{bmatrix}$$

E(u,v) is an equation of an ellipse

Let λ_1 and λ_2 be eigenvalues of M

Eigen Vector and Eigen Values

The eigen vector, x, of a matrix A is a special vector, with the following property

$$Ax=\lambda x$$

Where λ is called eigen value

To find eigen values of a matrix A first find the roots of:

$$\det(A-\lambda I) = 0$$

Then solve the following linear system for each eigen value to find corresponding eigen vector

$$(A-\lambda I)x = 0$$

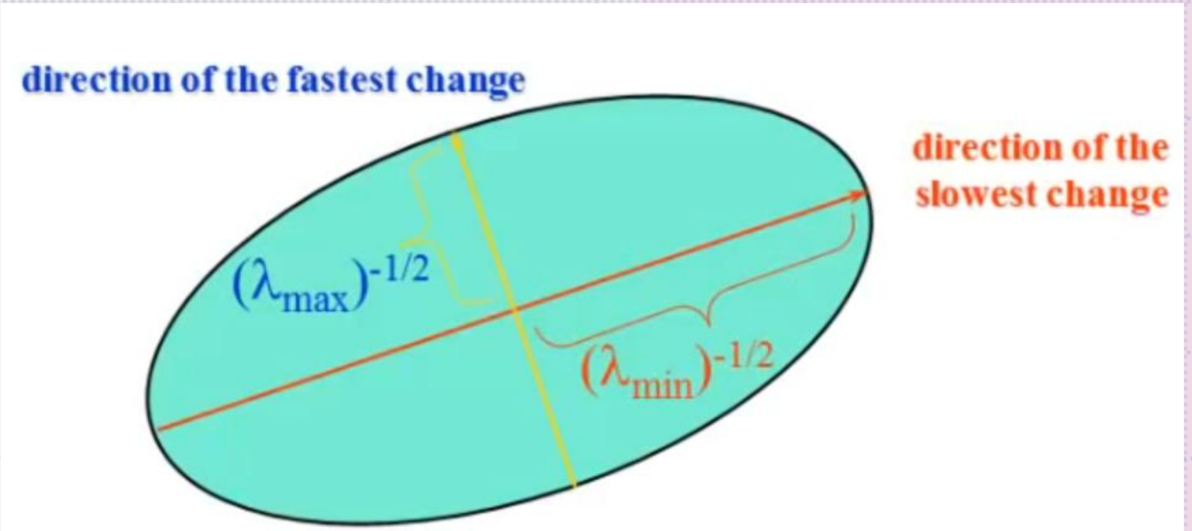
Harris Corner Detector

$$E(u,v) = \begin{pmatrix} u & v \end{pmatrix} M \begin{pmatrix} u \\ v \end{pmatrix}$$

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x I_x & I_x I_y \\ I_x I_y & I_y I_y \end{bmatrix}$$

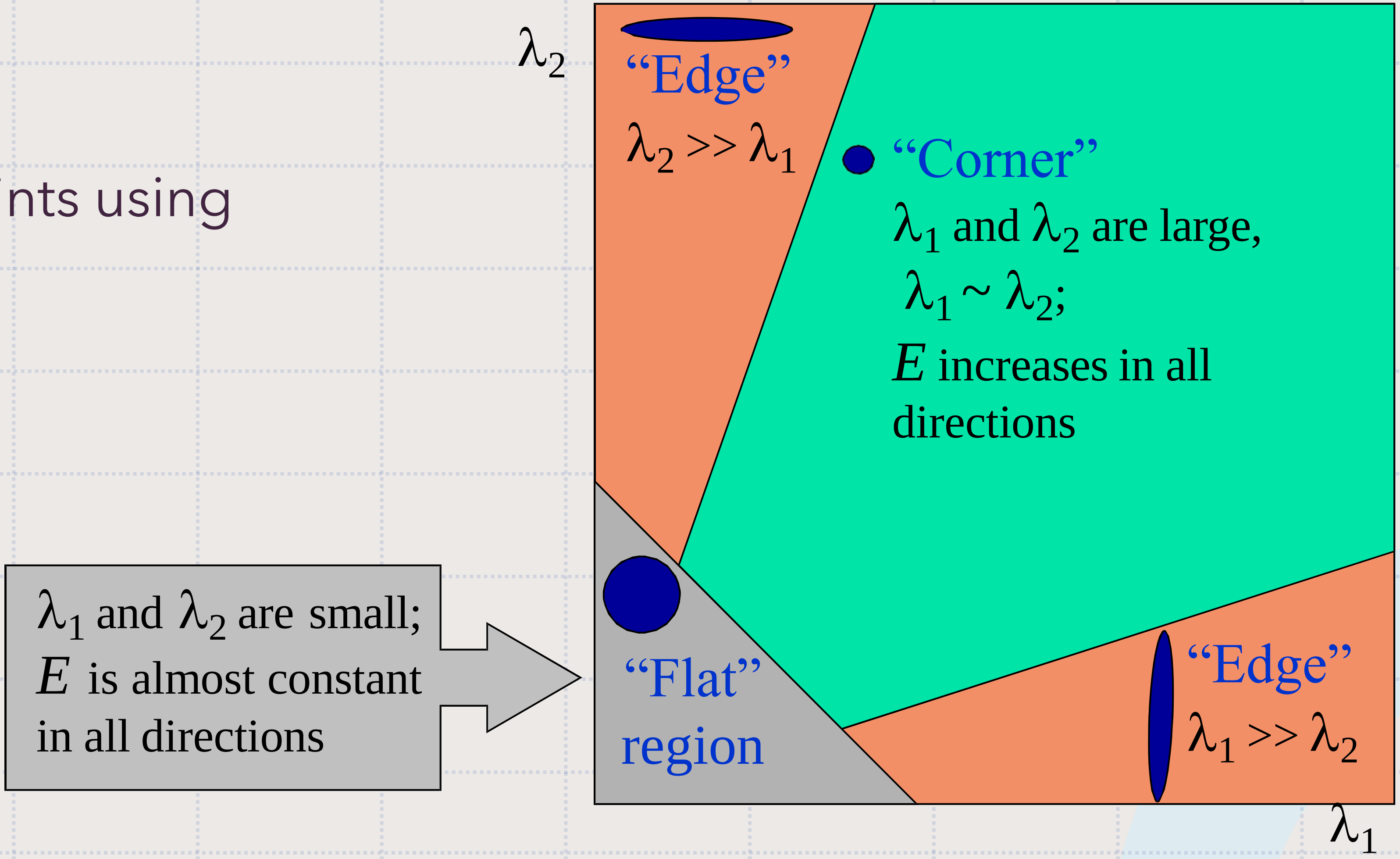
E(u,v) is an equation of an ellipse

Let λ_1 and λ_2 be eigenvalues of M



Harris Corner Detector

Classification of image points using eigenvalues of M



Harris Corner Detector

Measure of cornerness in terms of λ1 and λ2

$$R = \det D - k(trace\ D)^2$$

$$R = \lambda_1\lambda_2 - k(\lambda_1 + \lambda_2)^2$$

$$M = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

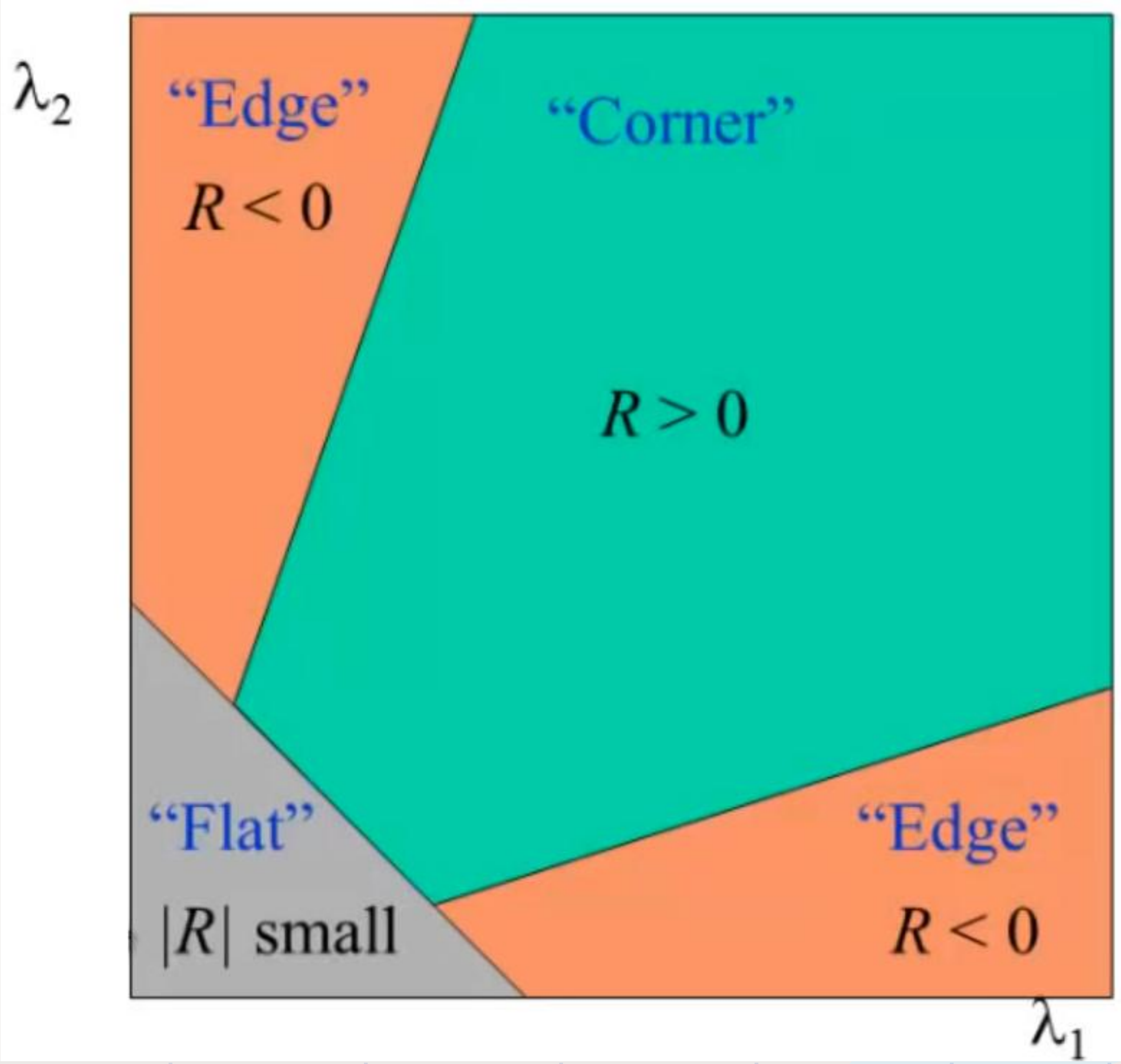
Harris Corner Detector

R depends only on eigenvalues of M

R is large for a corner

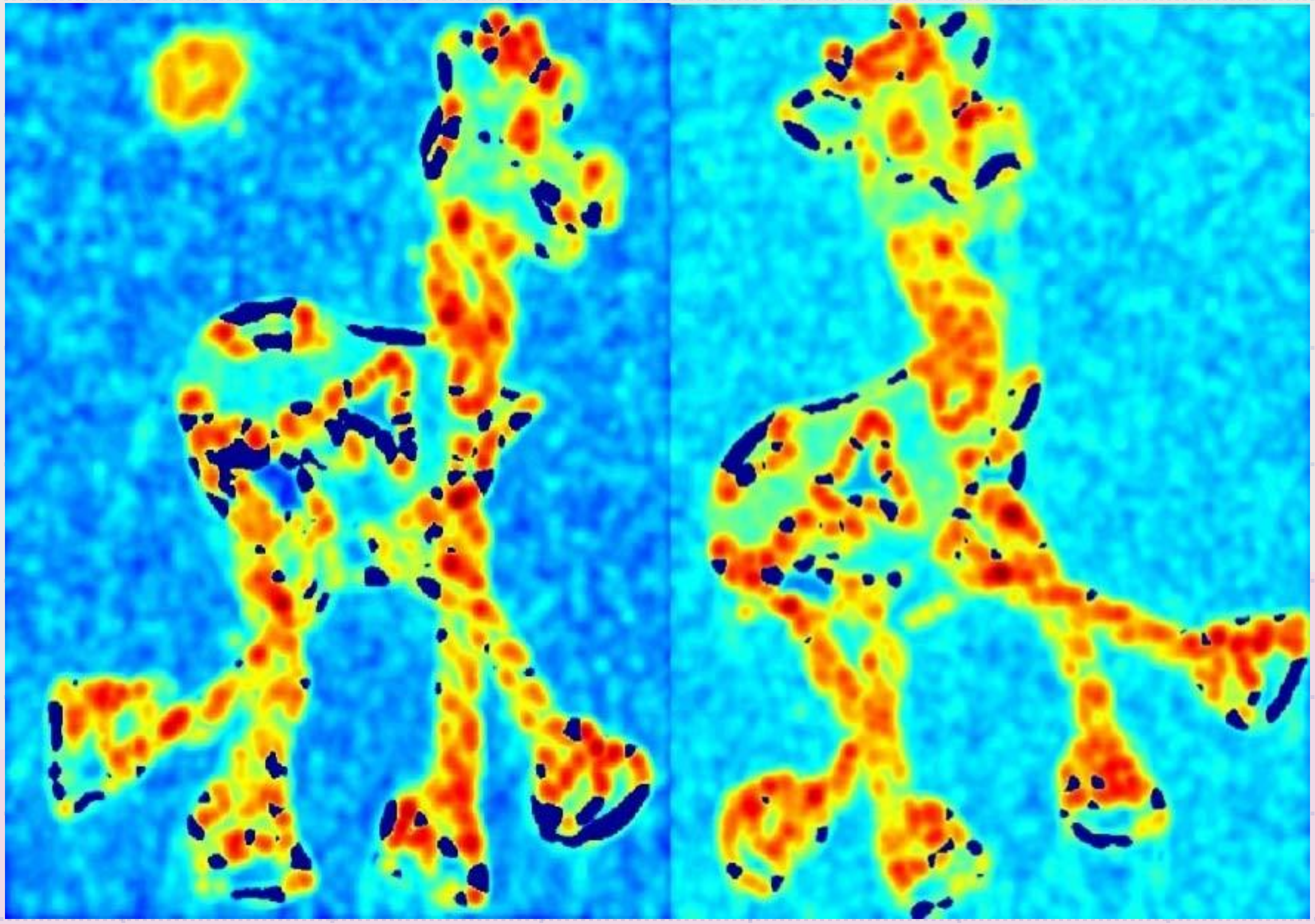
R is negative with large magnitude for an edge

|R| is small for a flat region



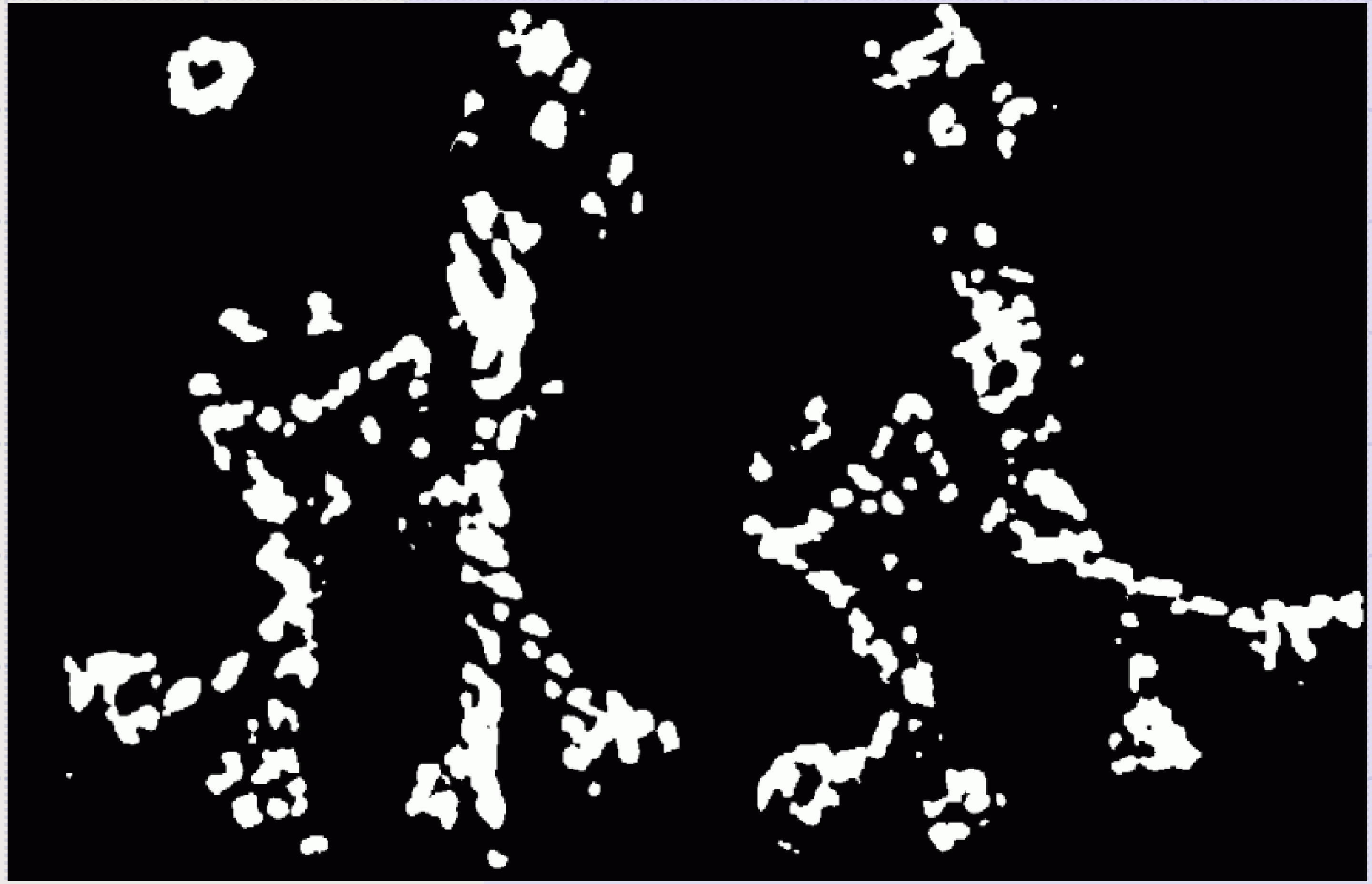
Harris Corner Detector

Harris Corner Detector



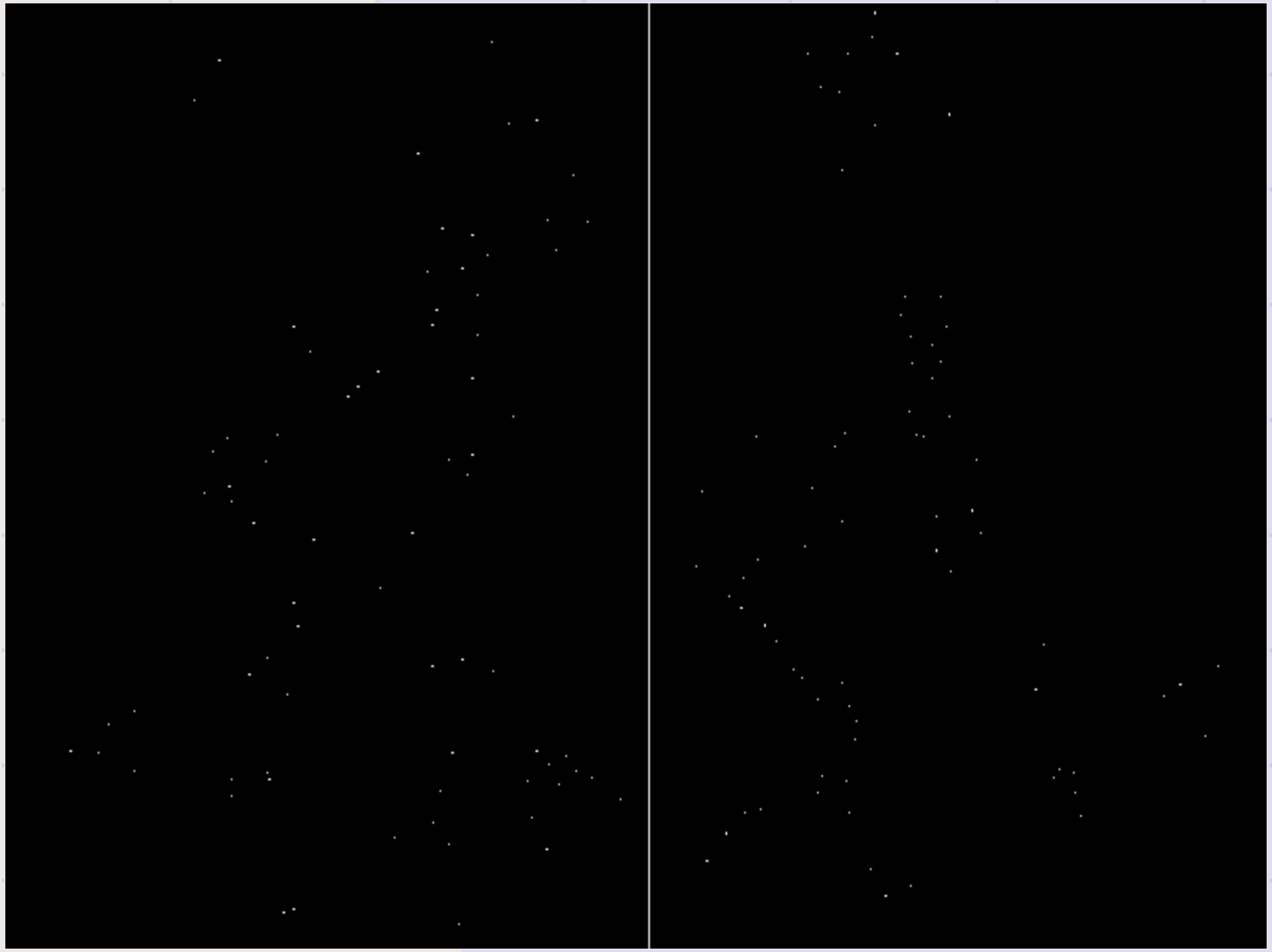
Harris Corner Detector

Find points with large corner response: R> threshold



Harris Corner Detector

Take only the points of local maxima of R



Harris Corner Detector

Other Version of Harris Detectors

$$R = \lambda_1 - \alpha\lambda_2$$

Triggs

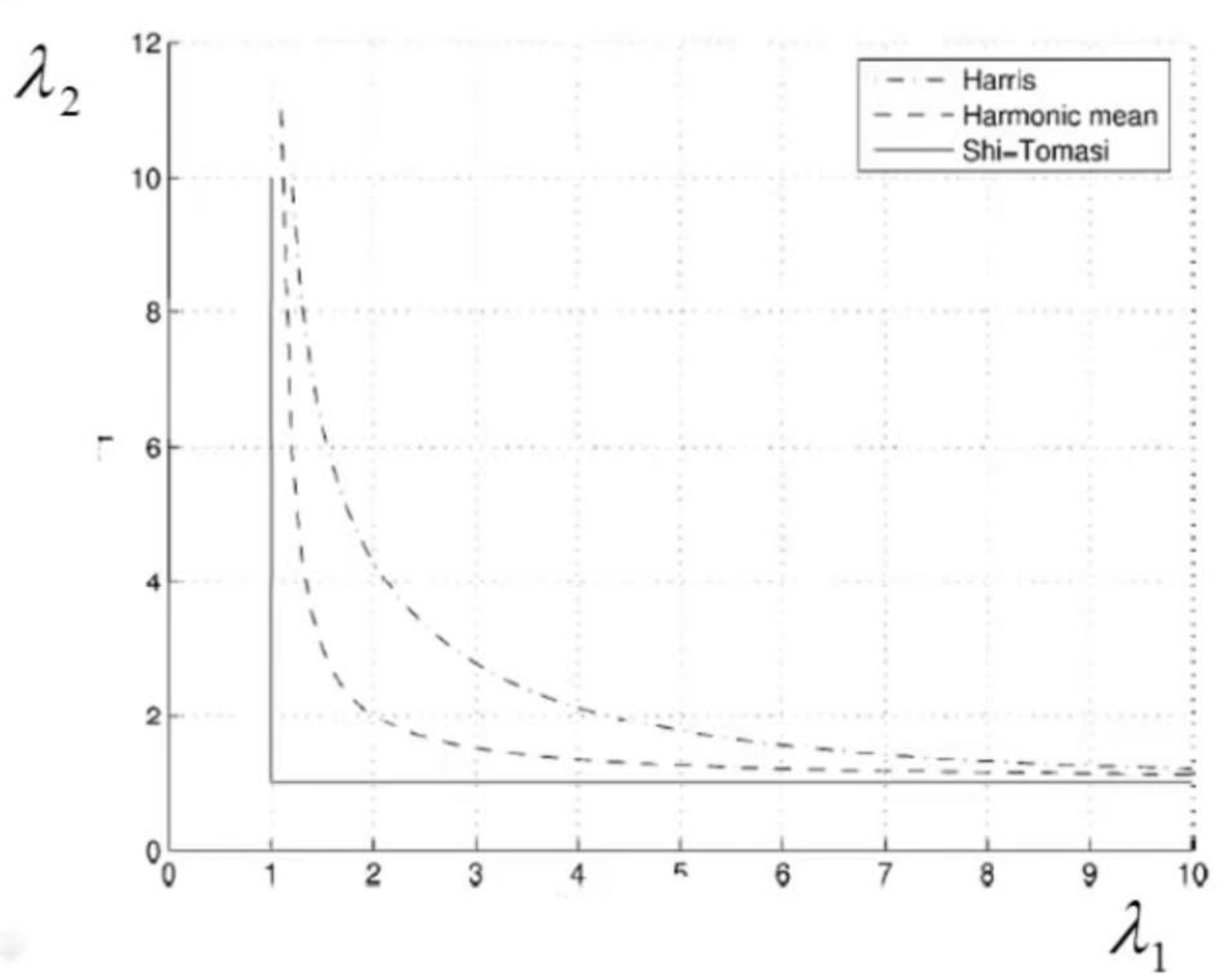
$$R = \frac{\det(D)}{trace(D)} = \frac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2}$$

Szeliski (Harmonic mean)

$$R = \lambda_1$$

Shi-Tomasi

Performance analysis



Algorithm

- Compute horizontal and vertical derivatives of image I_x and I_y.
- Compute three images corresponding to three terms in matrix *M.*
- Convolve these three images with a larger Gaussian (window).
- Compute scalar cornerness value using one of the *R* measures.
- Find local maxima above some threshold as detected interest points.

Practical

Write a program for Corner detection using:

1. Harris
2. Triggs
3. Szeliski
4. Shi-Tomasi