LOGISTIC REGRESSION

Cost function

1 ← j → n

	Tumor_Size X_1	Tumor_Texture X_2	Tumor_Area X_3	Tumor_Compactness x_4	Target y
1	1.600662	0.730988	1.370365	2.890064	0 (Benign)
	1.767906	2.450270	4.791903	0.000000	1 (Malignant)
	0.851814	0.000000	0.731659	3.306370	0 (Benign)
	2.244562	3.098886	2.217667	3.788212	1 (Malignant)
i	4.149168	2.124404	0.000000	5.051919	1 (Malignant)
	2.463678	0.102427	0.718578	2.826321	1 (Malignant)
	0.000000	2.465510	2.930389	3.088816	0 (Benign)
	2.872880	1.603829	1.692938	2.826498	1 (Malignant)
	1.106042	3.931754	4.882092	1.526860	0 (Benign)
m	1.119333	2.777160	3.469510	2.269244	0 (Benign)

TRAINING SET

- **Tumor_Size:** Size of the tumor in mm (continuous).
- **Tumor_Texture:** Texture (smoothness or roughness of the tumor area, continuous).
- **Tumor_Area:** The area of the tumor in mm^2 (continuous).
- **Tumor_Compactness:** How compact the tumor is (calculated as Compactness=((Perimeter)^2 / Area) − 1).

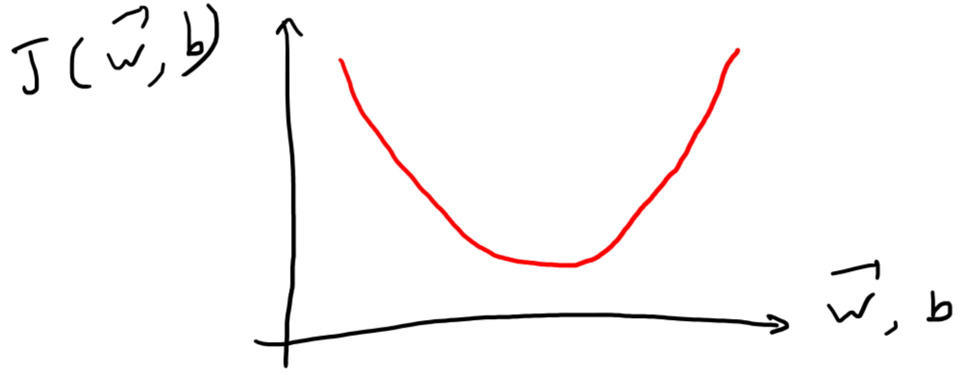
SQUARED ERROR COST

$$J(\vec{w},b) = \frac{1}{m}\sum_{i=1}^{m}\left[\frac{1}{2}\left(f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)}\right)^2\right]$$

Loss $L\left(f_{\vec{w},b}(\vec{x}^{(i)}), y^{(i)}\right)$

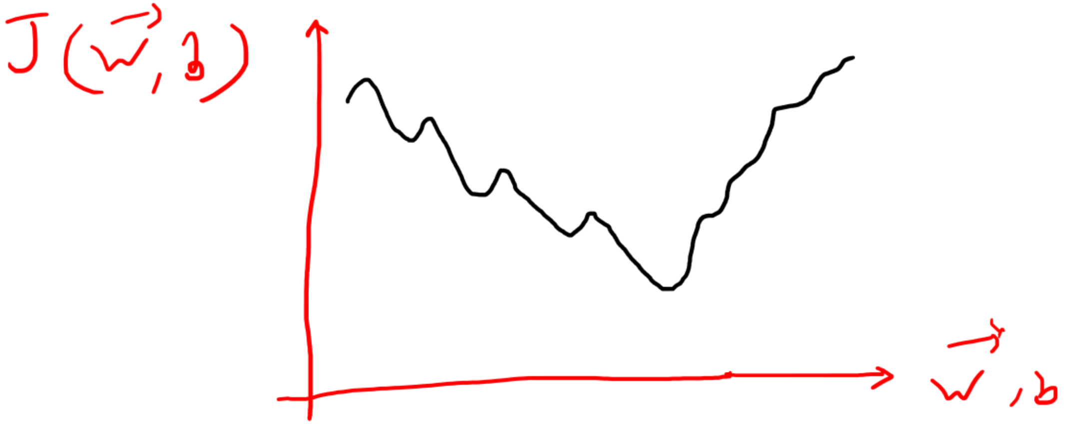
Linear Regression

$$f_{\vec{w},b} = \vec{w}\cdot\vec{x} + b$$



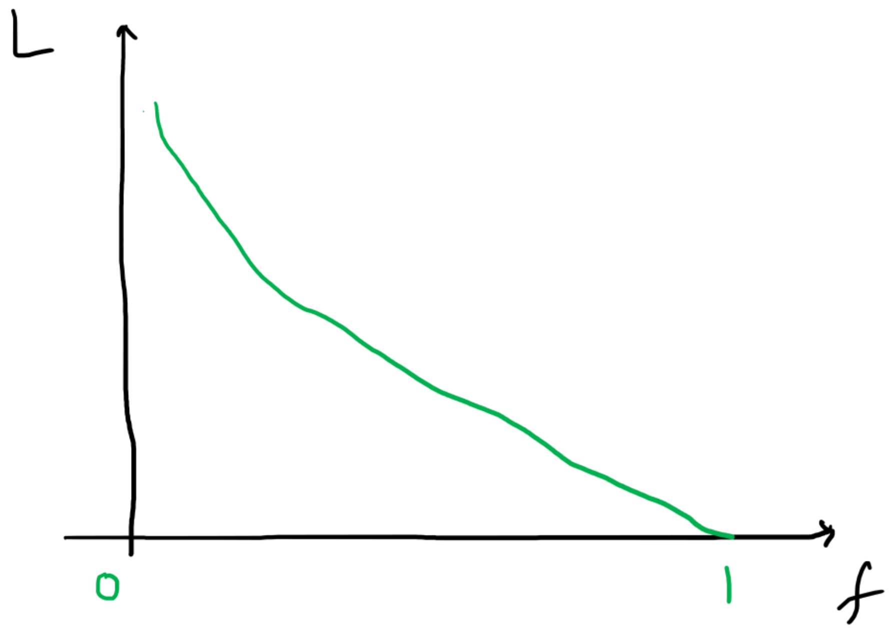
Logistic Regression

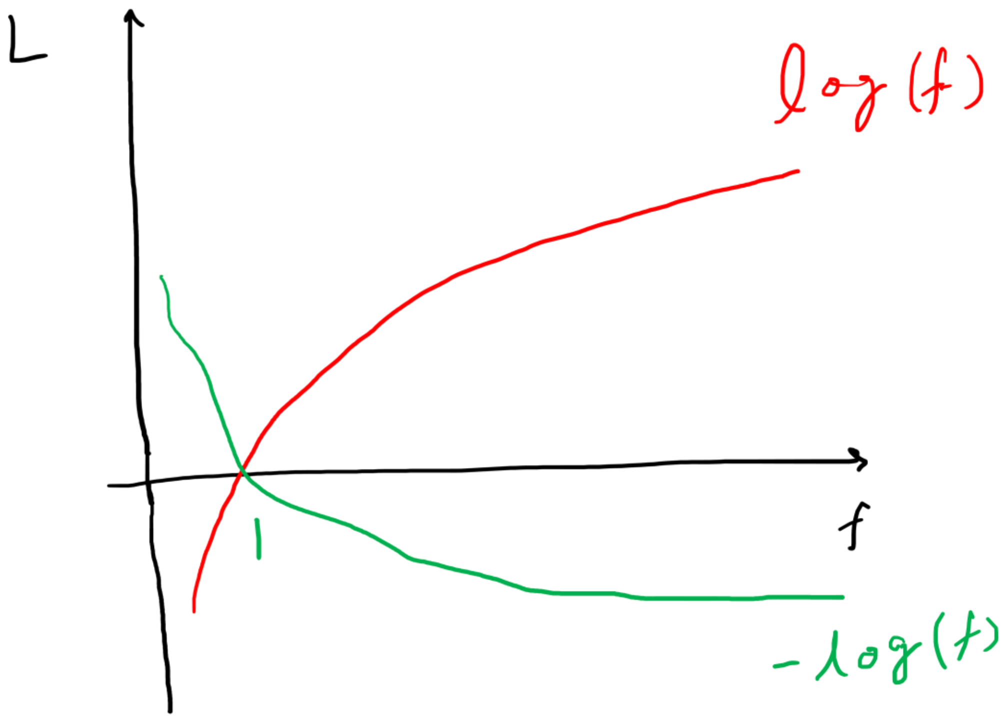
$$f_{\vec{w},b}(\vec{x}) = \frac{1}{1+e^{-\vec{w}\cdot\vec{x}+b}}$$



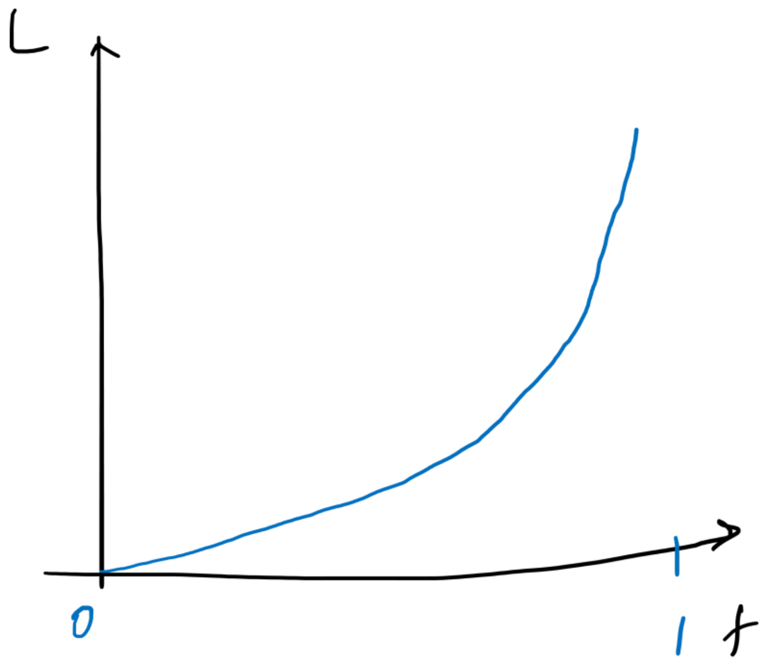
LOGISTIC LOSS FUNCTION

$$L\left(f_{\vec{w},b}(\vec{x}^{(i)}), y^{(i)}\right) = \begin{cases} -\log\left(f_{\vec{w},b}(\vec{x}^{(i)})\right); & \text{if } y^{(i)} = 1 \\ -\log\left(1 - f_{\vec{w},b}(\vec{x}^{(i)})\right); & \text{if } y^{(i)} = 0 \end{cases}$$

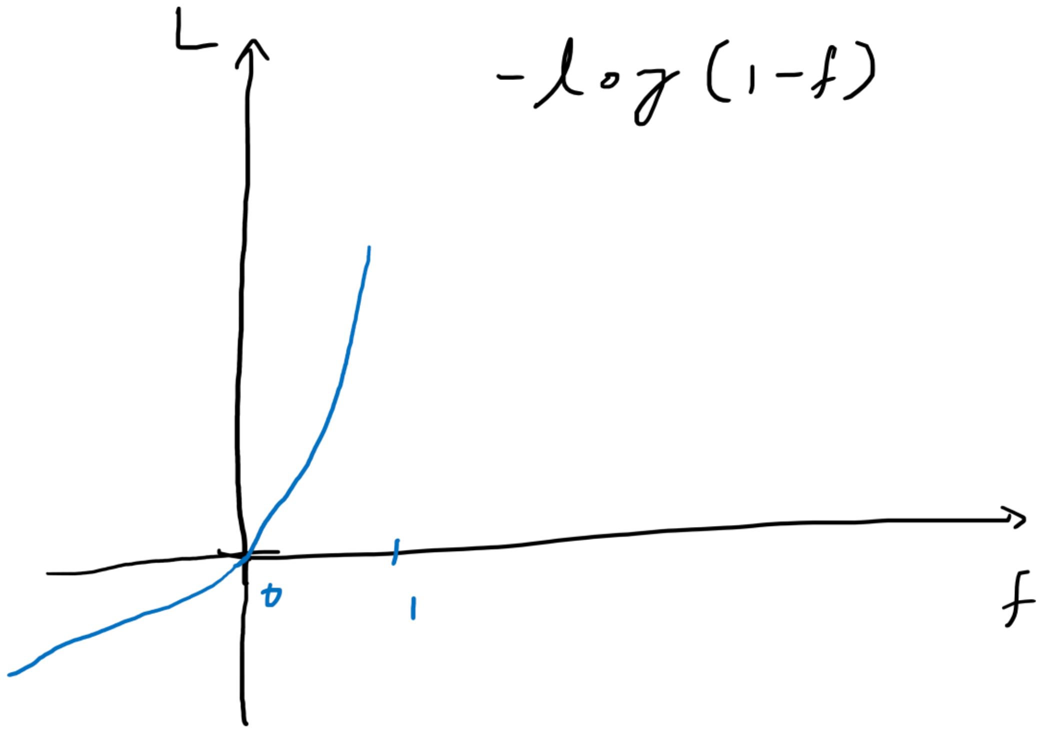




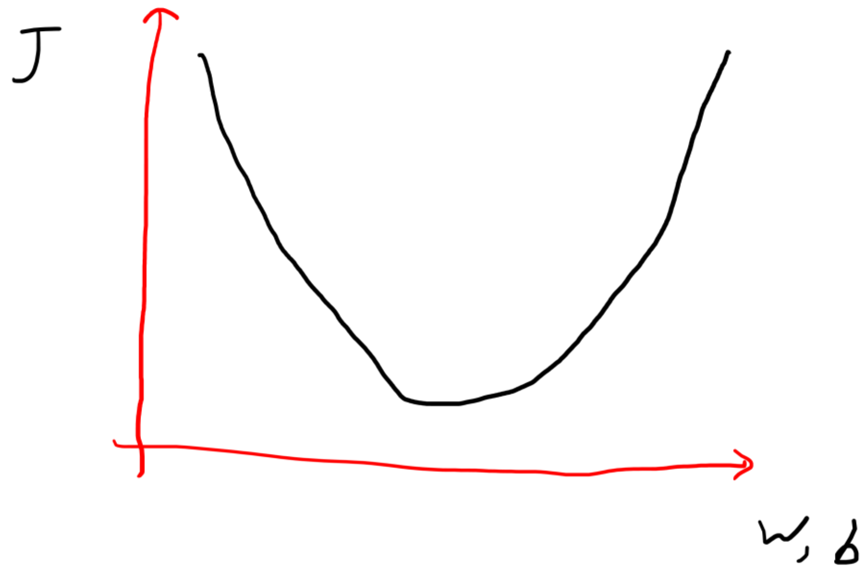
LOGISTIC LOSS FUNCTION



$$L = -\log(1-f)$$



COST



$$J(\vec{w}, b) = \frac{1}{m} \sum_{i=1}^{m} L\left(f_{\vec{w},b}(x^{(i)}), y^{(i)}\right)$$

SIMPLIFIED LOSS FUNCTION

$$L\left(f_{\vec{w},b}(\vec{x}^{(i)}), y^{(i)}\right) = -y^{(i)} \log\left(f_{\vec{w},b}(\vec{x}^{(i)})\right) - (1-y^{(i)}) \log\left(1-f_{\vec{w},b}(\vec{x}^{(i)})\right)$$

$$J(\vec{w},b) = \frac{1}{m}\sum_{i=1}^{m}\left[L\left(f_{\vec{w},b}(\vec{x}^{(i)}), y^{(i)}\right)\right]$$

$$= -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log\left(f_{\vec{w},b}(\vec{x}^{(i)})\right) + (1-y^{(i)})\log\left(1-f_{\vec{w},b}(\vec{x}^{(i)})\right)\right]$$

Maximum Likelihood

SIMPLIFIED COST FUNCTION

GRADIENT DESCENT

$j = 1, \ldots n$

Repeat {

$$w_j = w_j - \alpha \boxed{\frac{\partial}{\partial w_j} J(\vec{w}, b)} \rightarrow \frac{1}{m} \sum_{i=1}^{m} \left(f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)}\right) \underbrace{x_j^{(i)}}$$

$$b = b - \alpha \boxed{\frac{\partial}{\partial b} J(\vec{w}, b)} \rightarrow \frac{1}{m} \sum_{i=1}^{m} \left(f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)}\right)$$

}

Simultaneous update

GRADIENT DESCENT (LOGISTIC REGRESSION)

	Linear Regression	Logistic Regression
(L) Loss	S.E. $(y-\hat{y})^2$	M.L. (Log.)
(J) Cost	S.S.E. $\frac{1}{2m}\sum_{i=1}^{m}(y-\hat{y})^2$	M.L. $\frac{1}{m}\sum_{i=1}^{m}(\log)$
f	$\vec{w}\cdot\vec{x}+b$	$z=\vec{w}\cdot\vec{x}+b$ $f=g(z)=\frac{1}{1+e^{-z}}$