# Transformations

2D

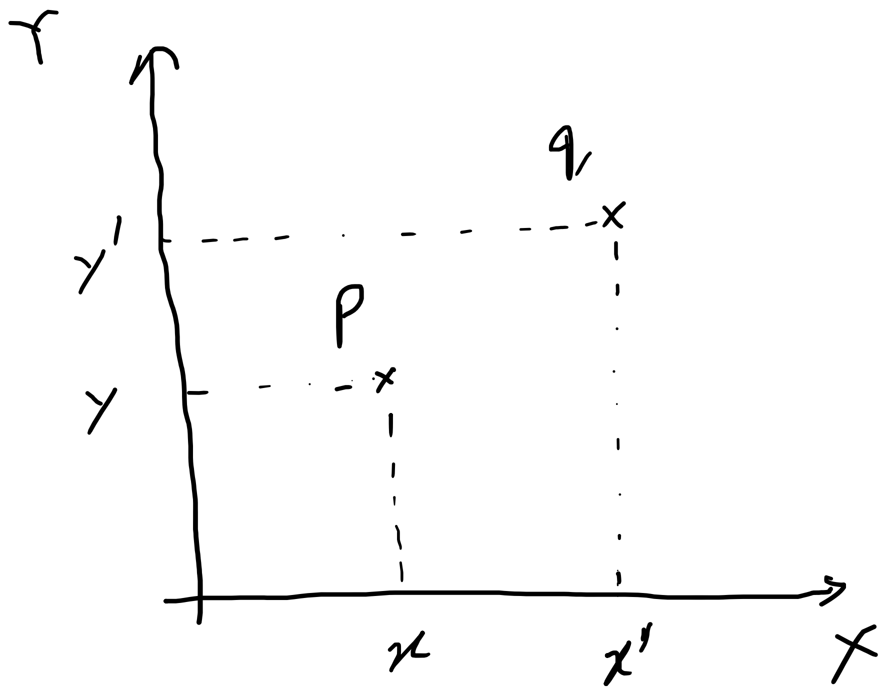
Original Image

Translated Image

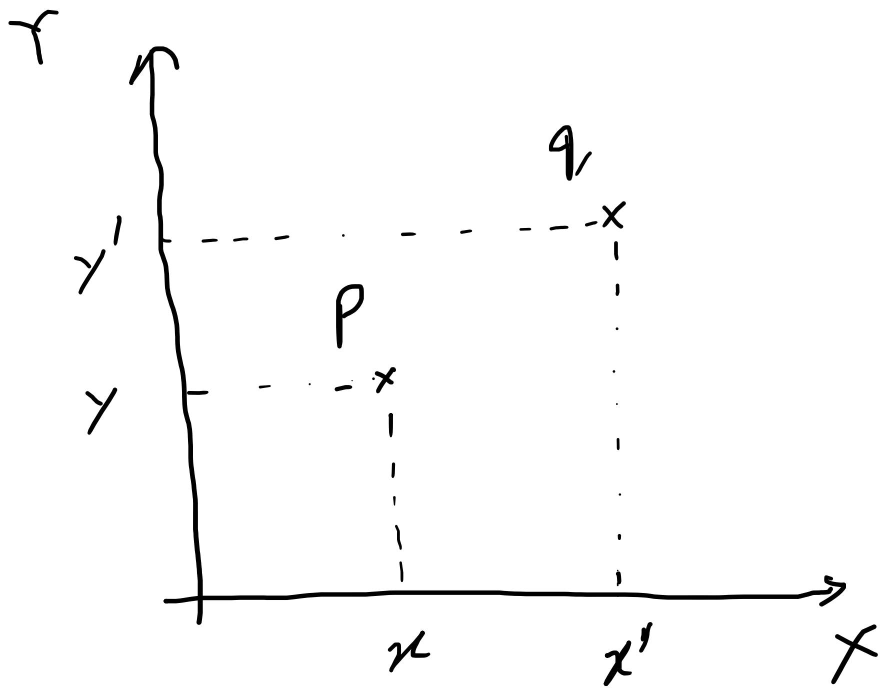
Scaled Image

Rotated Image

# Translation



# Translation



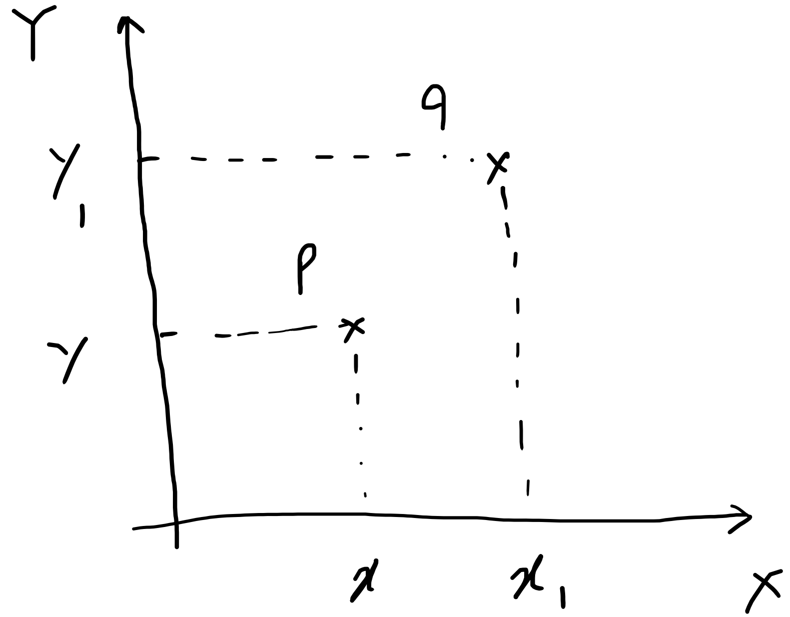

$$x' = x + t_x$$

$$y' = y + t_y$$

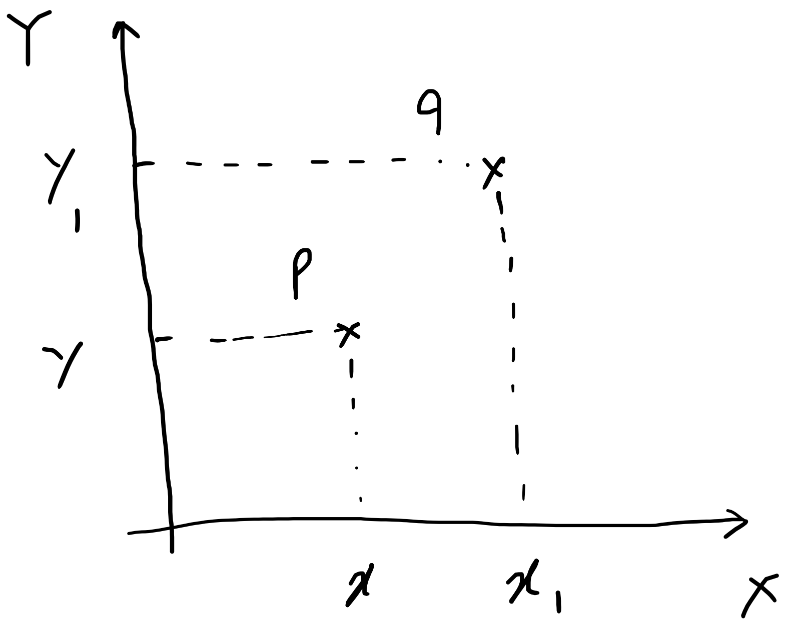
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

# Scaling



# Scaling



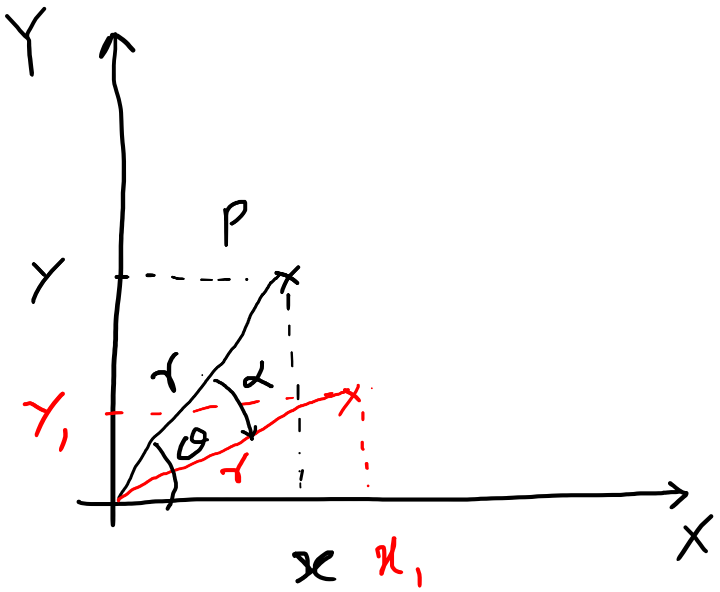

$$x_1 = C_x\, x$$

$$y_1 = C_y\, y$$

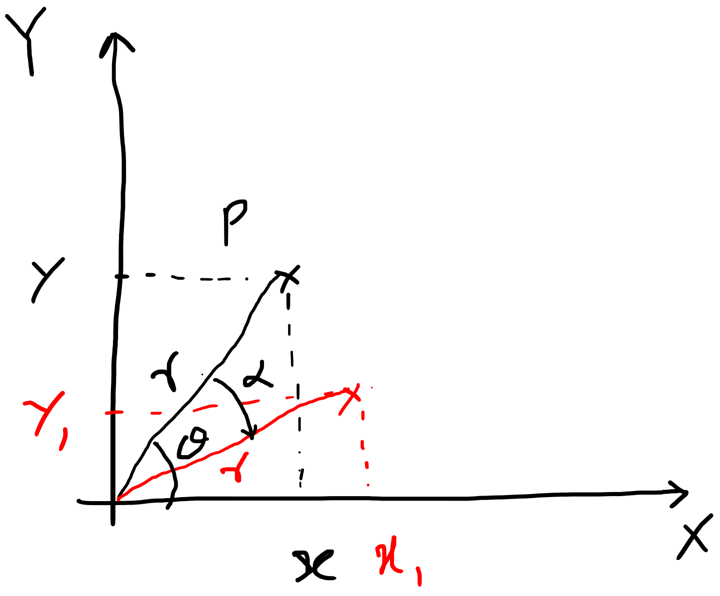
$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} C_x & 0 \\ 0 & C_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} C_x & 0 & 0 \\ 0 & C_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

# Rotation



# Rotation




$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$x_1 = r\cos(\theta-\alpha)$$

$$y_1 = r\sin(\theta-\alpha)$$

$$x_1 = r\cos\theta\cos\alpha + r\sin\theta\sin\alpha$$

$$\left.\begin{aligned} x_1 &= x\cos\alpha + y\sin\alpha \\ y_1 &= y\cos\alpha - x\sin\alpha \end{aligned}\right\}$$

$$
\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}
$$

R + T (Euclidean)

$$x' = Rx + t$$

S + R (Similarity)

$$x' = SRx + t$$

Affine

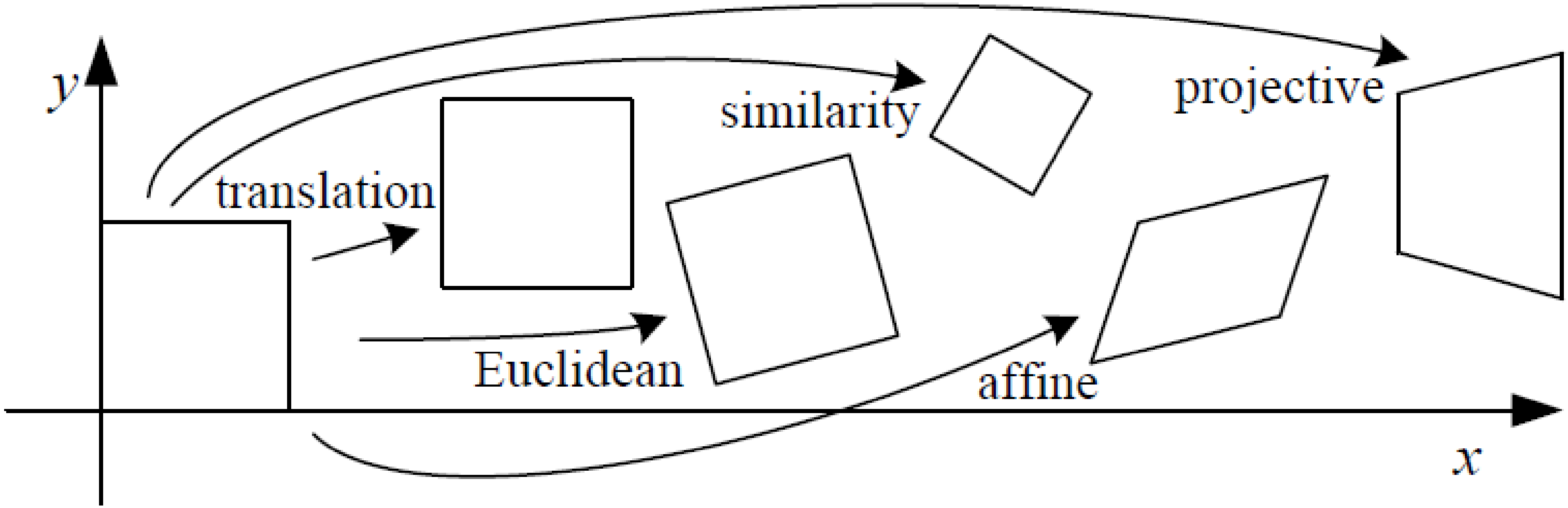
$$x' = Ax$$

## Projective (Homography)

$$\tilde{x}' = \tilde{H}\,\tilde{x}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$x' = \frac{h_{00}x + h_{01}y + h_{02}}{h_{20}x + h_{21}y + h_{22}} \; ; \quad y' = \frac{h_{10}x + h_{11}y + h_{12}}{h_{20}x + h_{21}y + h_{22}}$$


y
similarity
projective
translation
Euclidean
affine
x

# Concatenation of several transforms

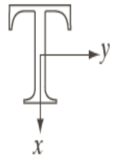
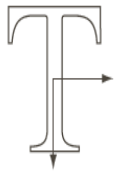
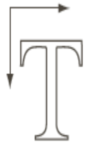
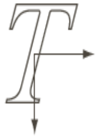
$$P(x, y) \longrightarrow q(x_1, y_1)$$

$$q = R_\theta(S(T(P)))$$

$$T \rightarrow S \rightarrow R$$

# Affine transformations

**TABLE 2.2**
Affine transformations based on Eq. (2.6.–23).

| Transformation Name | Affine Matrix, T | Coordinate Equations | Example |
|---|---|---|---|
| Identity | $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ | $x = v$ <br> $y = w$ |  |
| Scaling | $\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$ | $x = c_x v$ <br> $y = c_y w$ |  |
| Rotation | $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ | $x = v\cos\theta - w\sin\theta$ <br> $y = v\cos\theta + w\sin\theta$ |  |
| Translation | $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$ | $x = v + t_x$ <br> $y = w + t_y$ |  |
| Shear (vertical) | $\begin{bmatrix} 1 & 0 & 0 \\ s_v & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ | $x = v + s_v w$ <br> $y = w$ |  |
| Shear (horizontal) | $\begin{bmatrix} 1 & s_h & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ | $x = v$ <br> $y = s_h v + w$ |  |