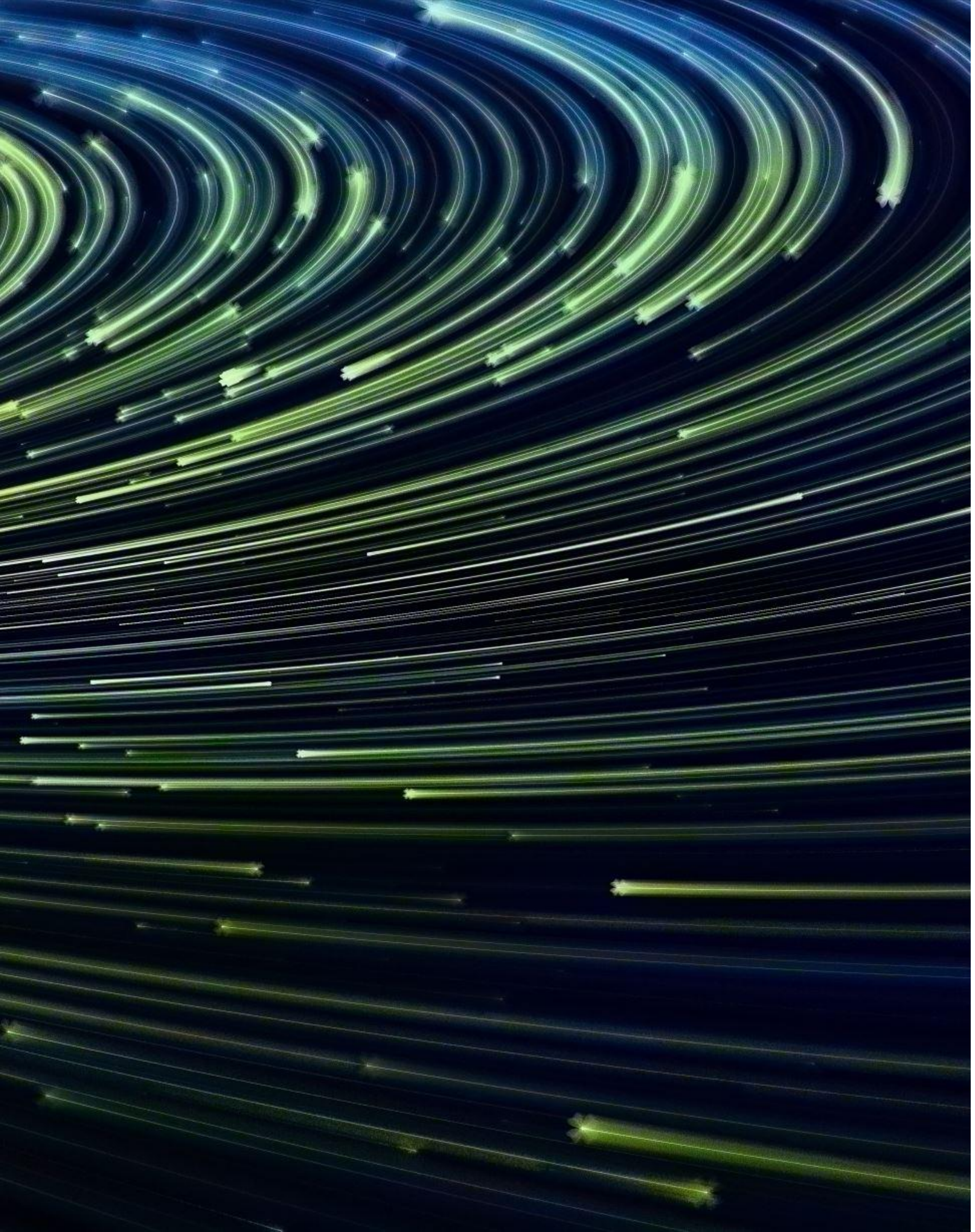

Optical Flow

Motion analysis

Horn & Schunck Optical flow

Brightness constancy assumption

$$f(x, y, t) = f(x + dx, y + dy, t + dt)$$

Taylor Series

$$f(x, y, t) = f(x, y, t) + \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial t} dt$$

$$f_x dx + f_y dy + f_t dt = 0$$

$$f_x u + f_y v + f_t = 0$$

Solving Optical flow equation

+ Pseudo inverse
+ Least squares
+ Eigen vector

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Optical flow eq

$$f_x u + f_y v = -f_t$$

Consider 3 by 3 window

$$f_{x1} u + f_{y1} v = -f_{t1}$$

$$\vdots$$

$$f_{x9} u + f_{y9} v = -f_{t9}$$

$$\begin{bmatrix} f_{x1} & f_{y1} \\ \vdots & \vdots \\ f_{x9} & f_{y9} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -f_{t1} \\ \vdots \\ -f_{t9} \end{bmatrix}$$

$$\mathbf{Au} = \mathbf{f_t}$$

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$$\mathbf{A}\mathbf{u} = \mathbf{f}_t$$

$$\mathbf{A}^T\mathbf{A}\mathbf{u} = \mathbf{A}^T\mathbf{f}_t$$

$$\mathbf{u} = \boxed{(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}}^T\mathbf{f}_t$$

Pseudo Inverse

$$\min \sum_i (f_{xi}u + f_{yi}v + f_t)^2$$

Least Squares Fit

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$$\min \sum_{i} (f_{xi}u + f_{yi}v + f_{t})^2$$

$$\Downarrow$$

$$\sum (f_{xi}u + f_{yi}v + f_{ti}) f_{xi} = 0$$

$$\sum (f_{xi}u + f_{yi}v + f_{ti}) f_{yi} = 0$$

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$$\sum (f_{xi}u + f_{yi}v + f_{ti}) f_{xi} = 0$$

$$\sum (f_{xi}u + f_{yi}v + f_{ti}) f_{yi} = 0$$

$$\sum f_{xi}^2 u + \sum f_{xi} f_{yi} v = -\sum f_{xi} f_{ti}$$
$$\sum f_{xi} f_{yi} u + \sum f_{yi}^2 v = -\sum f_{yi} f_{ti}$$

$$\begin{bmatrix} \sum f_{xi}^2 & \sum f_{xi} f_{yi} \\ \sum f_{xi} f_{yi} & \sum f_{yi}^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -\sum f_{xi} f_{ti} \\ -\sum f_{yi} f_{ti} \end{bmatrix}$$

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$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \sum f_{xi}^2 & \sum f_{xi} f_{yi} \\ \sum f_{xi} f_{yi} & \sum f_{yi}^2 \end{bmatrix}^{-1} \begin{bmatrix} -\sum f_{xi} f_{ti} \\ -\sum f_{yi} f_{ti} \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{1}{\sum f_{xi}^2 \sum f_{yi}^2 - (\sum f_{xi} f_{yi})^2} \begin{bmatrix} \sum f_{yi}^2 & -\sum f_{xi} f_{yi} \\ -\sum f_{xi} f_{yi} & \sum f_{xi}^2 \end{bmatrix} \begin{bmatrix} -\sum f_{xi} f_{ti} \\ -\sum f_{yi} f_{ti} \end{bmatrix}$$

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$$u = \frac{-\sum f_{yi}^2 \sum f_{xi} f_{ti} + \sum f_{xi} f_{yi} \sum f_{yi} f_{ti}}{\sum f_{xi}^2 \sum f_{yi}^2 - (\sum f_{xi} f_{yi})^2}$$

$$v = \frac{\sum f_{xi} f_{ti} \sum f_{xi} f_{yi} - \sum f_{xi}^2 \sum f_{yi} f_{ti}}{\sum f_{xi}^2 \sum f_{yi}^2 - (\sum f_{xi} f_{yi})^2}$$

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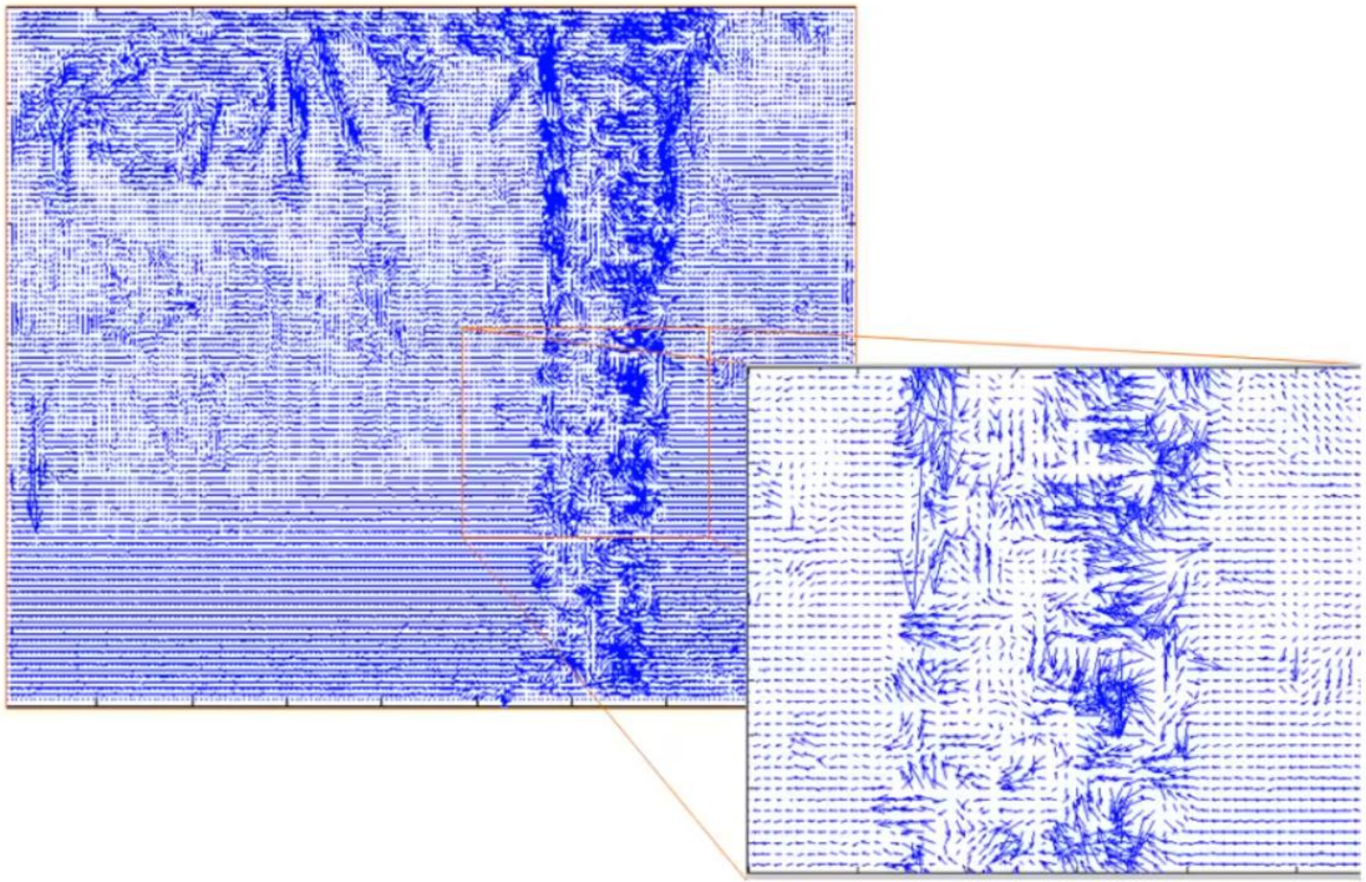
- Without Pyramid
- Fails in areas of large motion



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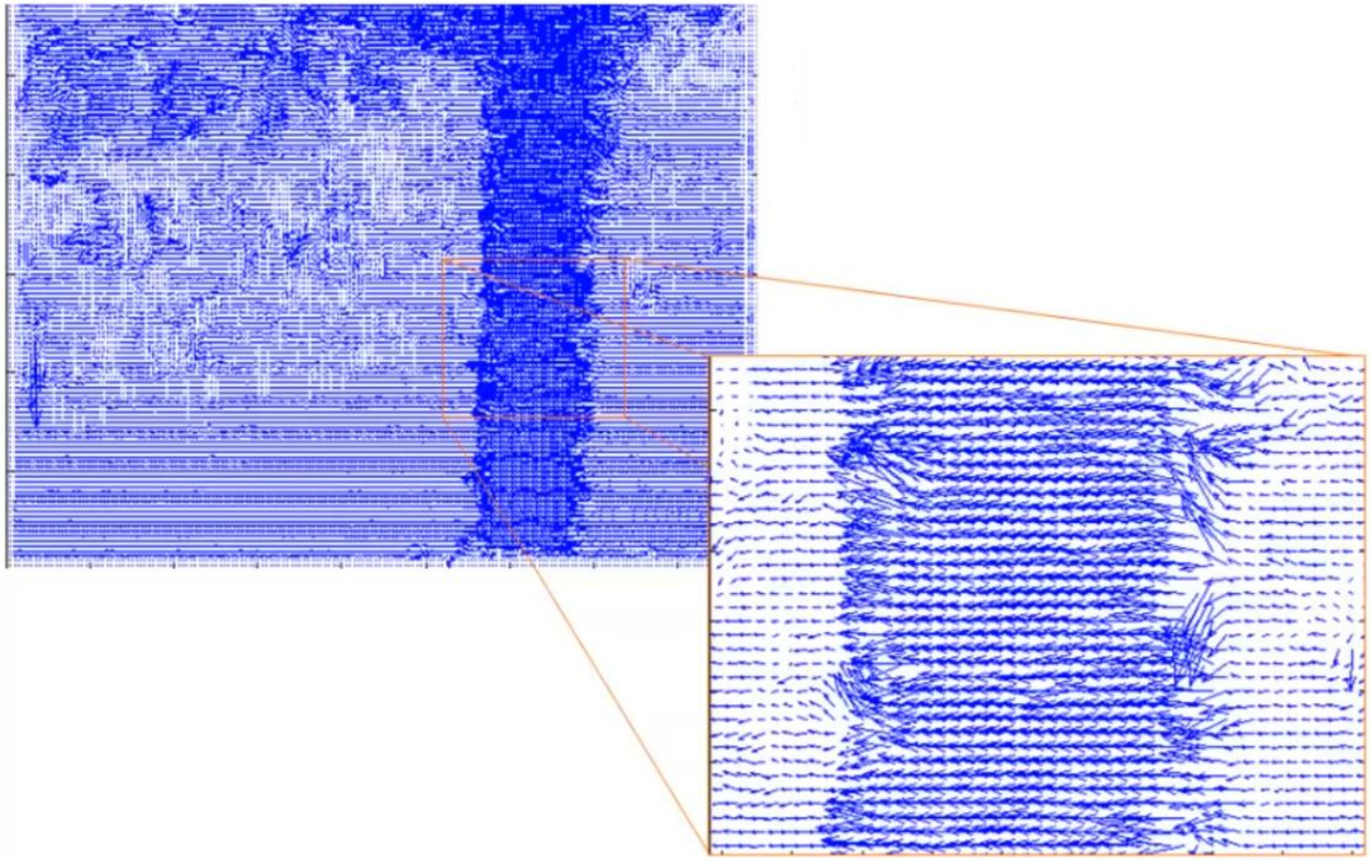
+ With Pyramid



Conclusion

+ Horn-Schunck and Lucas-Kanade optical flow works only for small motion
+ If the object moves faster then brightness changes rapidly,

  2*2 or 3*3 masks fails to estimate spatiotemporal derivatives
+ Pyramids can be used to compute large optical flow vectors