

Histogram Matching (Specification)

$$S = T(r) = \frac{(L-1)}{MN} \sum_{j=0}^K n_j \quad \text{--- ①}$$

$$S = T(\underline{r}) = (L-1) \int_0^r p_r(\omega) d\omega \quad \text{--- ②}$$

$$G(\underline{z}) = (L-1) \int_0^z p_z(t) dt \quad \text{--- ③}$$

$$G(z) = S = T(r) \Rightarrow z = G^{-1}(s) = G^{-1}[T(r)]$$

Given $p_r(r) = \begin{cases} 2r/(L-1)^2 & ; 0 \leq r \leq (L-1) \\ 0 & ; \text{otherwise} \end{cases}$

S = $T(r) = (L-1) \int_0^r \frac{2w}{(L-1)^2} dw = \frac{r^2}{(L-1)} \quad \text{--- (1)}$

Desired $p_z(z) = \begin{cases} 3z^2/(L-1)^3 & ; 0 \leq z \leq (L-1) \\ 0 & ; \text{otherwise} \end{cases}$

S = $G(z) = (L-1) \int_0^z \frac{3t^2}{(L-1)^3} dt = \frac{z^3}{(L-1)^2} \quad \text{--- (2)}$

$$Z = [(L-1)^2 s]^{\frac{1}{3}} = [(L-1) r^2]^{\frac{1}{3}}$$

Discrete domain

$$S_k = T(r_k) = (L-1) \sum_{j=0}^k p_r(r_j)$$

Given

$$= \frac{(L-1)}{M \Delta} \sum_{j=0}^k n_j ; \quad k = 0, 1, 2, \dots, L-1$$

Desired

$$G(z_q) = (L-1) \sum_{i=0}^q p_z(z_i) = S_k$$

$$q = 0, 1, 2, \dots, L-1$$

$$z_q = G^{-1}(S_k) = G^{-1}[T(r_k)]$$

64x64 I/p image

Intensity range (0-7)

r_k	n_k	$P_r(r_k)$
0	790	0.19
1	1023	0.25
2	850	0.21
3	656	0.16
4	329	0.08
5	245	0.06
6	122	0.03
7	81	0.02

$$S_k = \frac{(L-1)}{MN} \sum_{j=0}^k n_j$$

$$S_0 = 1.33 \rightarrow 1$$

$$S_1 = 3.08 \rightarrow 3$$

$$S_2 = 4.55 \rightarrow 5$$

$$S_3 = 5.67 \rightarrow 6$$

$$S_4 = 6.23 \rightarrow 6$$

$$S_5 = 6.65 \rightarrow 7$$

$$S_6 = 6.86 \rightarrow 7$$

$$S_7 = 7 \rightarrow 7$$

$$k = 0, 1, 2, \dots, 7$$

$$P_r(r_k) = \frac{n_k}{MN}$$

z_q	Specified $p_z(z_k)$	Calculated $p_z(z_k)$
0	0.00	0.00
1	0.00	0.00
2	0.00	0.00
3	0.15	0.19
4	0.20	0.25
5	0.30	0.21
6	0.20	0.24
7	0.15	0.11

$$G(z_q) = (L-1) \sum_{i=0}^q p_z(z_i)$$

$$G(z_0) = 0.00 \rightarrow 0$$

$$G(z_1) = 0.00 \rightarrow 0$$

$$G(z_2) = 0.00 \rightarrow 0$$

$$G(z_3) = 1.05 \rightarrow 1$$

$$G(z_4) = 2.45 \rightarrow 2$$

$$G(z_5) = 4.55 \rightarrow 5$$

$$G(z_6) = 5.95 \rightarrow 6$$

$$G(z_7) = 7 \rightarrow 7$$

z_q	$Q(z_q) = S$	r	$p_z(z_k)$
0	0	—	0.00
1	0	—	0.00
2	0	—	0.00
3	1	0	0.19
4	2	1	0.25
5	5	2	0.21
6	6	3 & 4	0.24
7	7	5, 6 & 7	0.11







