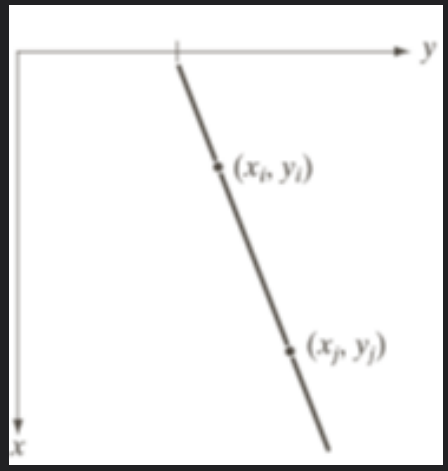
Hough Transform

Line detection

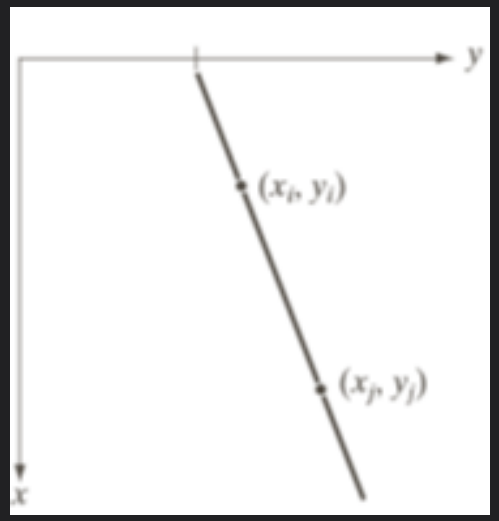
Problem under consideration

How to fit a Line

- Least square Fit (over constraint)
- RANSAC (constraint)
- Hough Transform (under constraint)



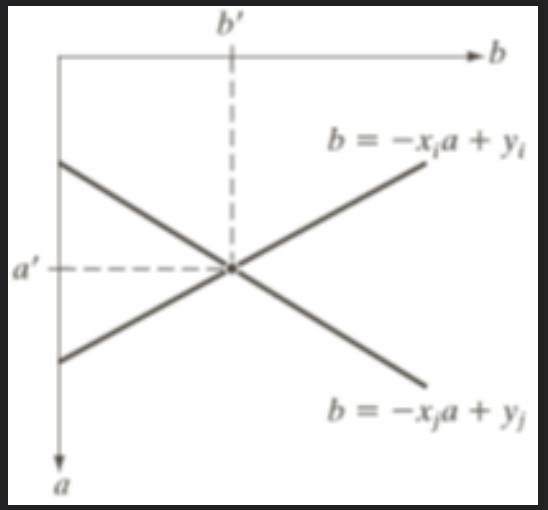
Hough Transform



$y_i = ax_i + b$

Infinitely many lines passes through (x_i, y_i)

$b = -x_i a + y_i$ (parameter space)



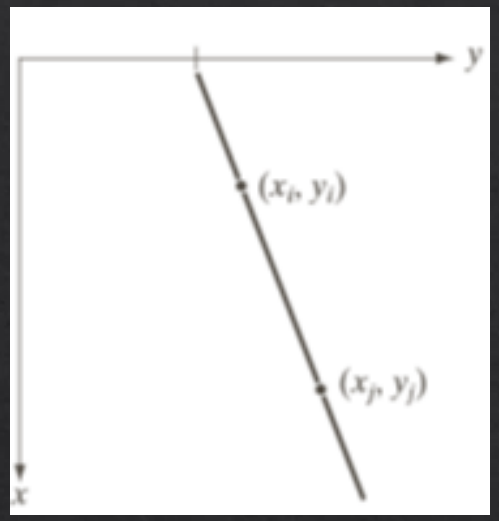
Hough Transform Algo for fitting Straight line

1. Quantize the parameter space $P[c_{min}, \ldots\ldots, c_{max}, m_{min}, \ldots\ldots, m_{max}]$.

2. For each edge point (x, y) do
   for $(m = m_{min}, m \leq m_{max}, m++)$ do
   $c = (-x)m + y,$
   $P[c, m] = P[c, m] + 1.$

3. Find the local maxima in the parameter space.

Hough Transform

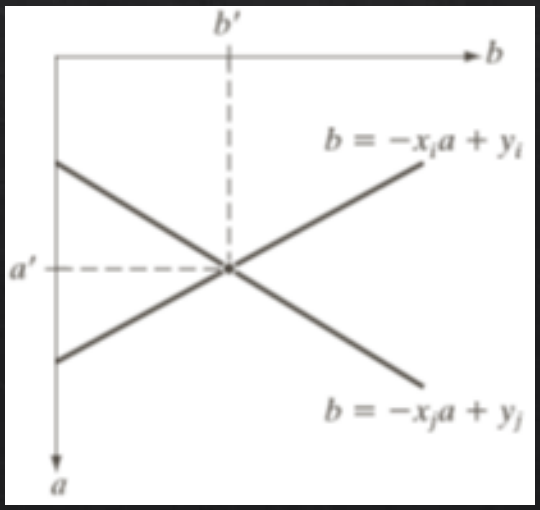


$y_i = ax_i + b$

Infinitely many lines passes through (x_i, y_i)

$b = -x_i a + y_i$ (parameter space)

Slope of the line (a) approaches infinity as the line approaches the vertical direction.



Normal (Polar) representation of Line

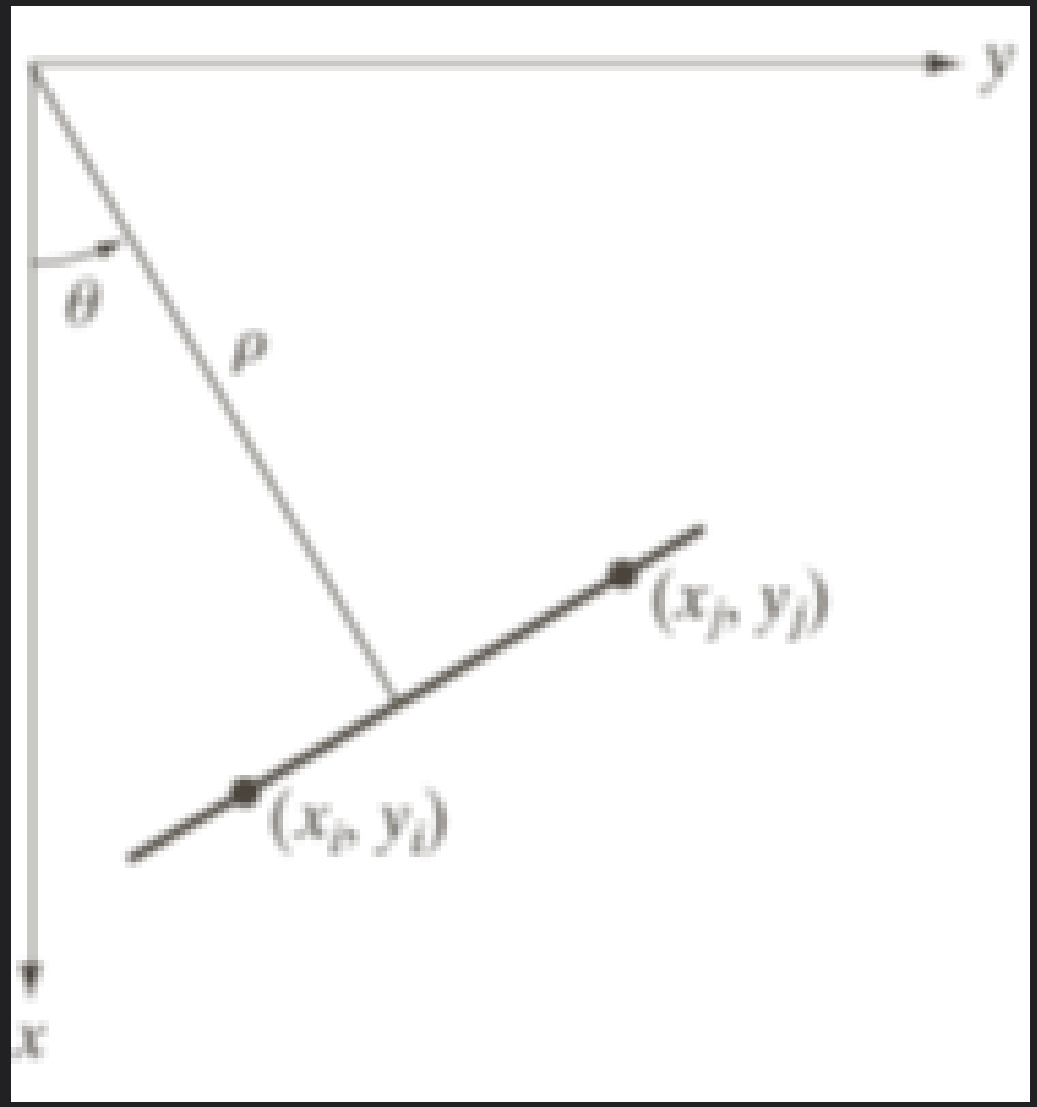
$x \cos \theta + y \sin \theta = \rho$

$-D \leq \rho \leq D$

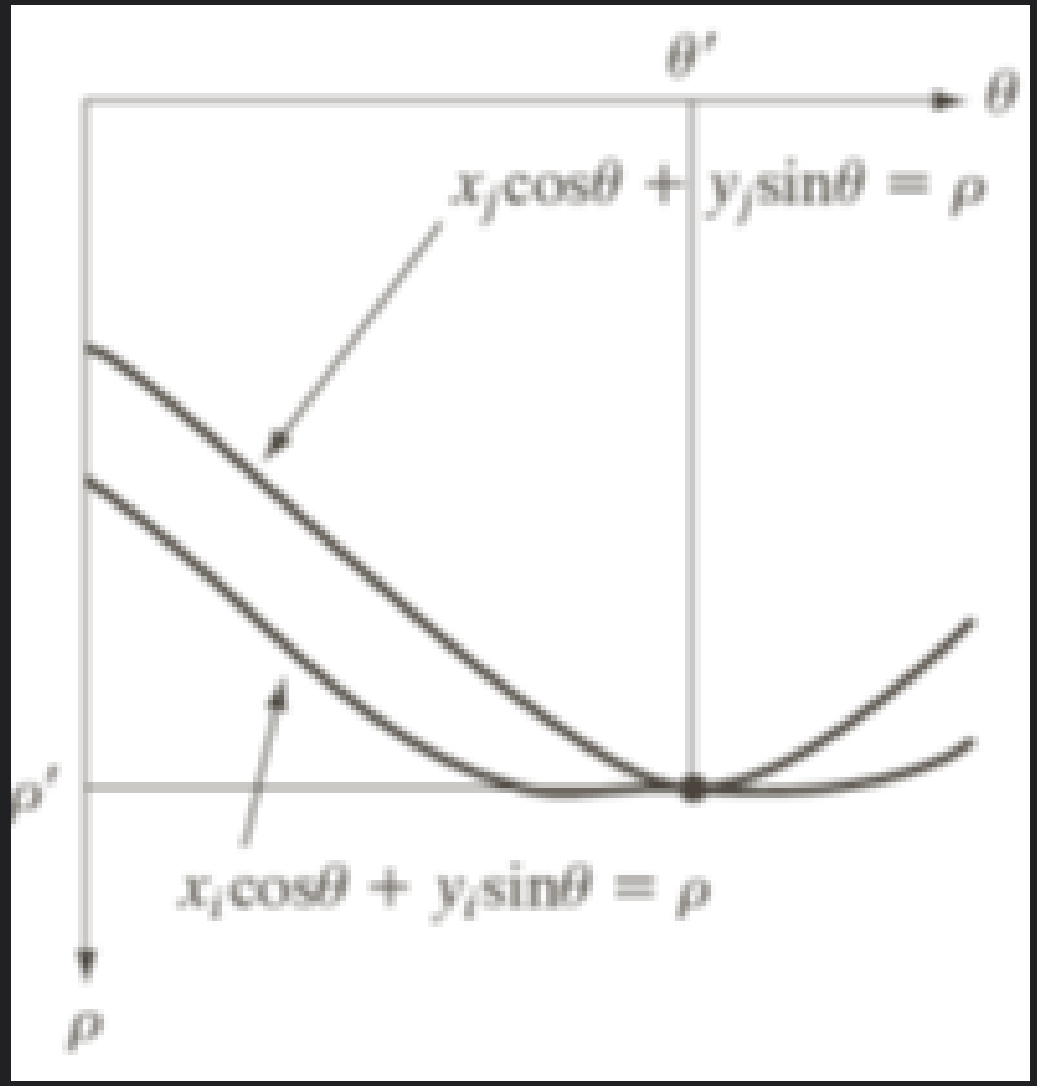
D is maximum distance between opposite corners in an image

$\theta = \tan^{-1} (g_y / g_x)$

$+90^0 \leq \theta \leq -90^0$

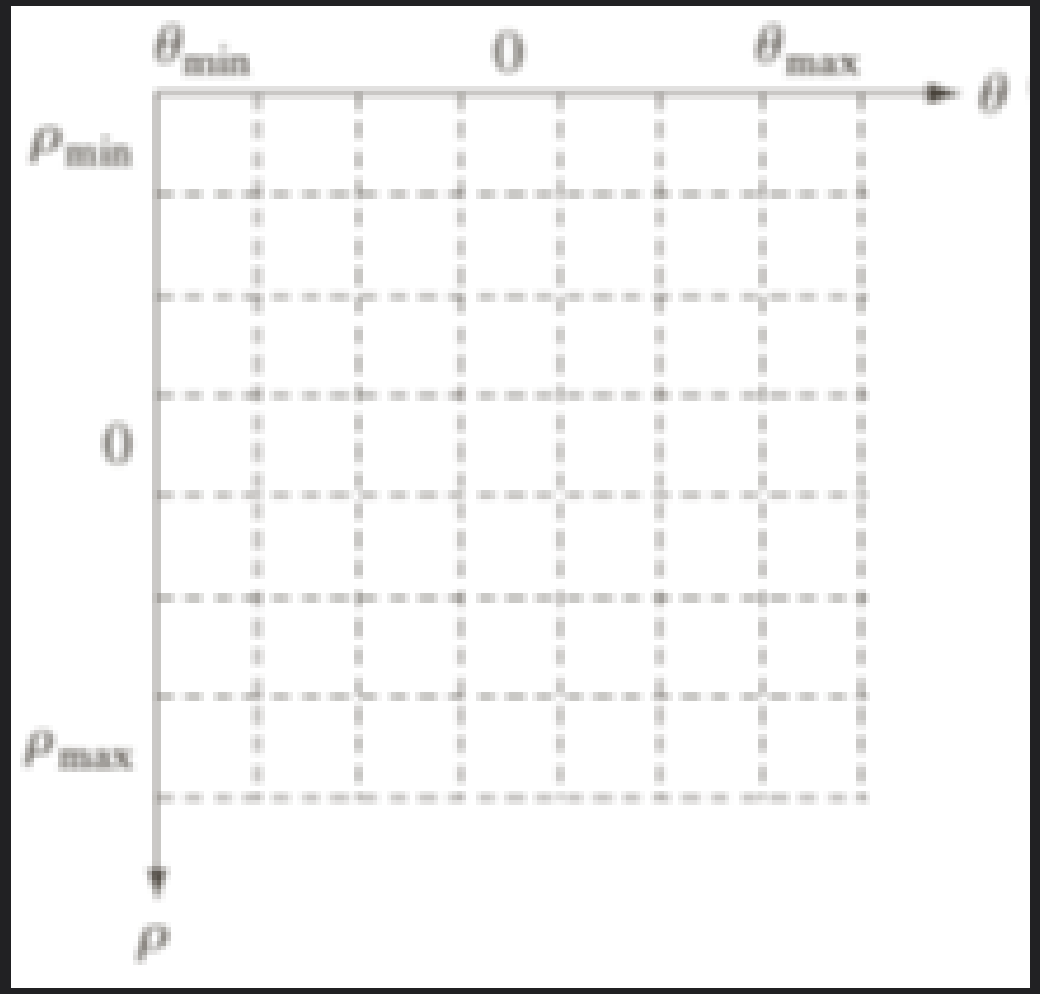


ρ and θ parameter space



Accumulator array

- A(p,q)
- A(p,q) = A(p,q) + 1



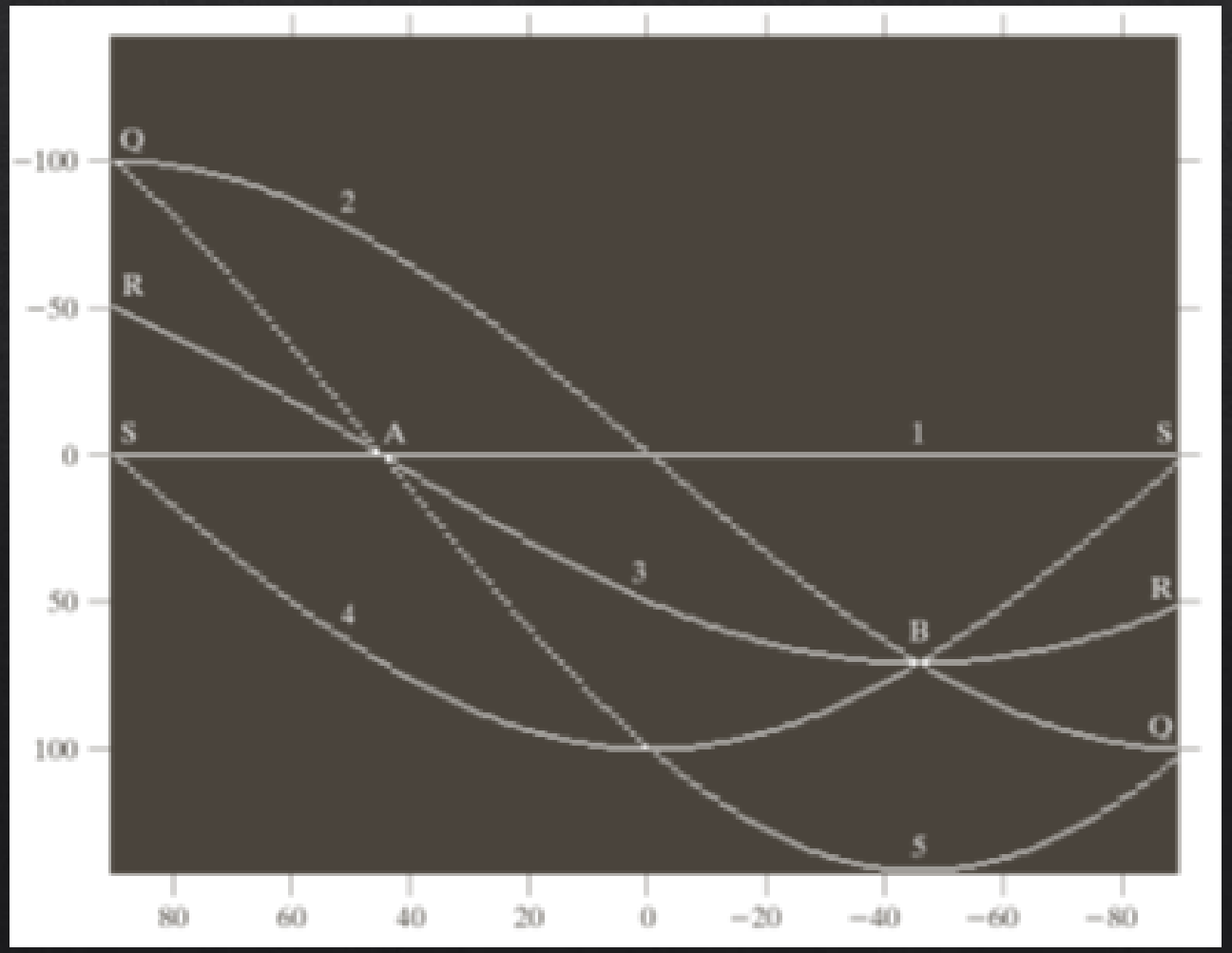
Hough Transform Algo for fitting Straight line

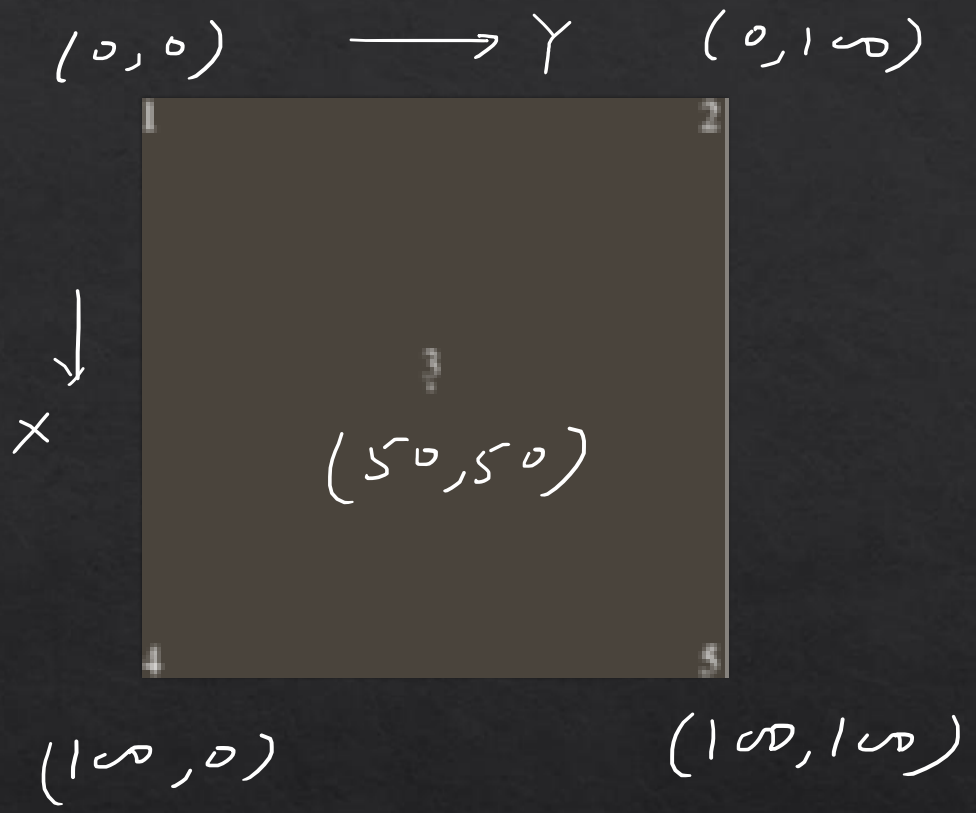
1. Quantize the parameter space $P[\theta_{min}, \ldots\ldots, \theta_{max}, p_{min}, \ldots\ldots, p_{max}]$.

2. For each edge point (x, y) do
   $p = x \cos\theta + y \sin\theta,$
   $P[\theta, p] = P[\theta, p] + 1.$

3. Find the local maxima in the parameter space.

Example





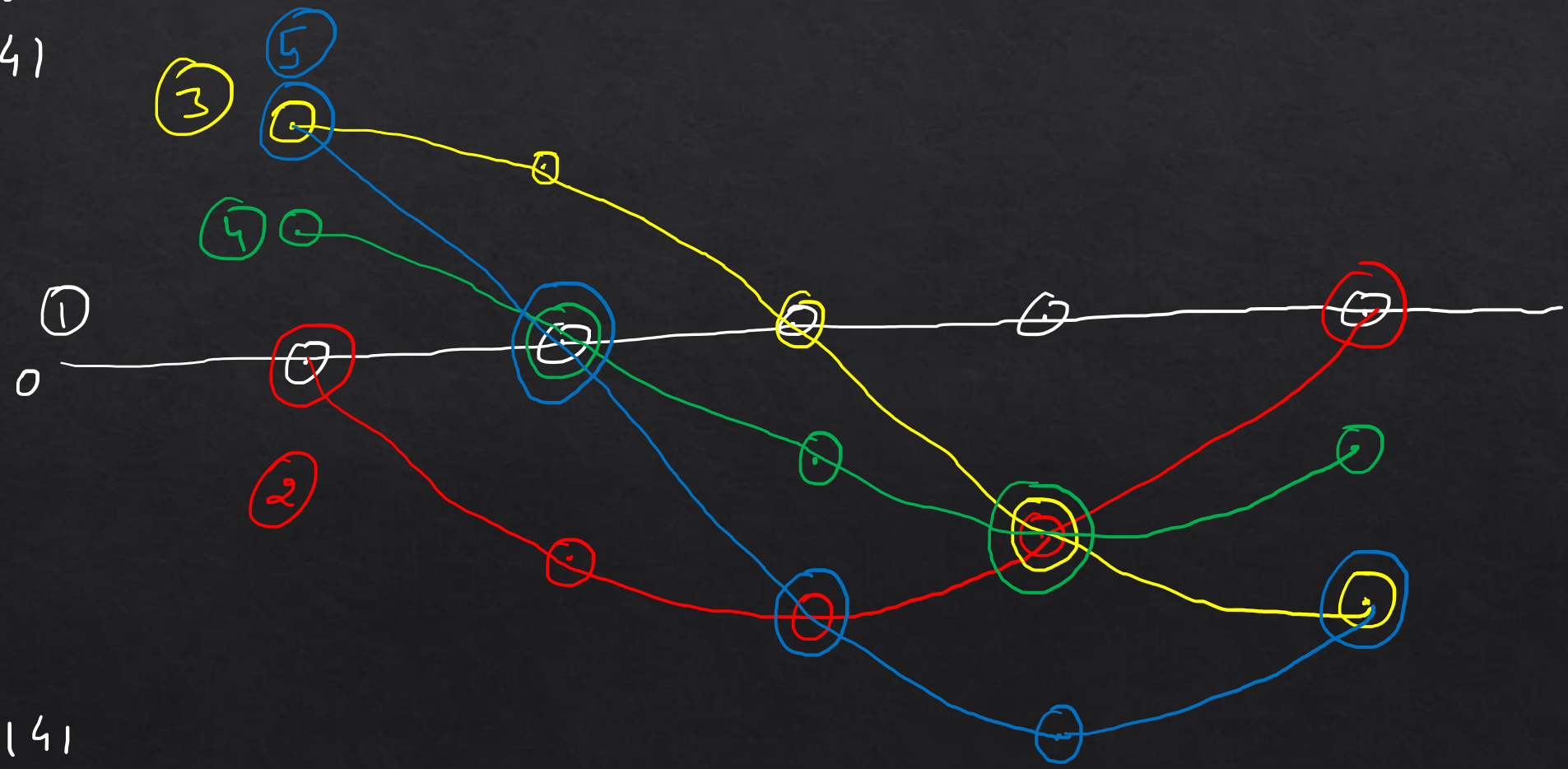
$$X\cos\theta + Y\sin\theta = \rho$$

$$x \cos\theta + y \sin\theta = \rho$$

$\theta \rightarrow$ -90° -45° 0° $+45^\circ$ $+90^\circ$

$\downarrow \rho$

-141



$+141$

$$x\cos\theta + y\sin\theta = \rho$$

① $(0, 0) \Rightarrow \rho = 0$

② $(100, 0) \Rightarrow 100\cos\theta = \rho$

$\rho = 0$ & $\theta = 0^\circ$ +1

$\rho = 71$ & $\theta = +45^\circ$ +1

③ $(0, 100) \Rightarrow 100\sin\theta = \rho$

$\rho = 0$ & $\theta = \pm 90^\circ$ +1

④ $(50, 50) \Rightarrow 50(\cos\theta + \sin\theta) = \rho$

$\rho = 0$ & $\theta = -45^\circ$ +1

$\rho = 71$ & $\theta = +45^\circ$ +1

⑤ $(100, 100) \Rightarrow 100(\cos\theta + \sin\theta) = \rho$

$\rho = 100$ & $\theta = 0^\circ$ +1

$\rho = 100$ & $\theta = +90^\circ$ +1

$\rho = -100$ & $\theta = -90^\circ$ +1

$\rho = 0$ & $\theta = -45^\circ$ +1

P.L. ↓ / θ →	-90°	-45°	0°	$+45^\circ$	$+90^\circ$
(0, 0)	0	0	0	0	0
(100, 0)	0	71	100	71	0
(0, 100)	-100	-71	0	71	100
(50, 50)	-50	0	50	71	50
(100, 100)	-100	0	100	141	100

p	θ	Count
0	+90	+1
0	-90	+1
0	-45	+2
0	0	+1
0	+45	+2
71	0	+1
100	+90	+1
100	-90	+1
-100		

All remaining cells → 0

$x \cos\theta + y \sin\theta = \rho$

$\rho = 0$ & $\theta = -45^\circ$

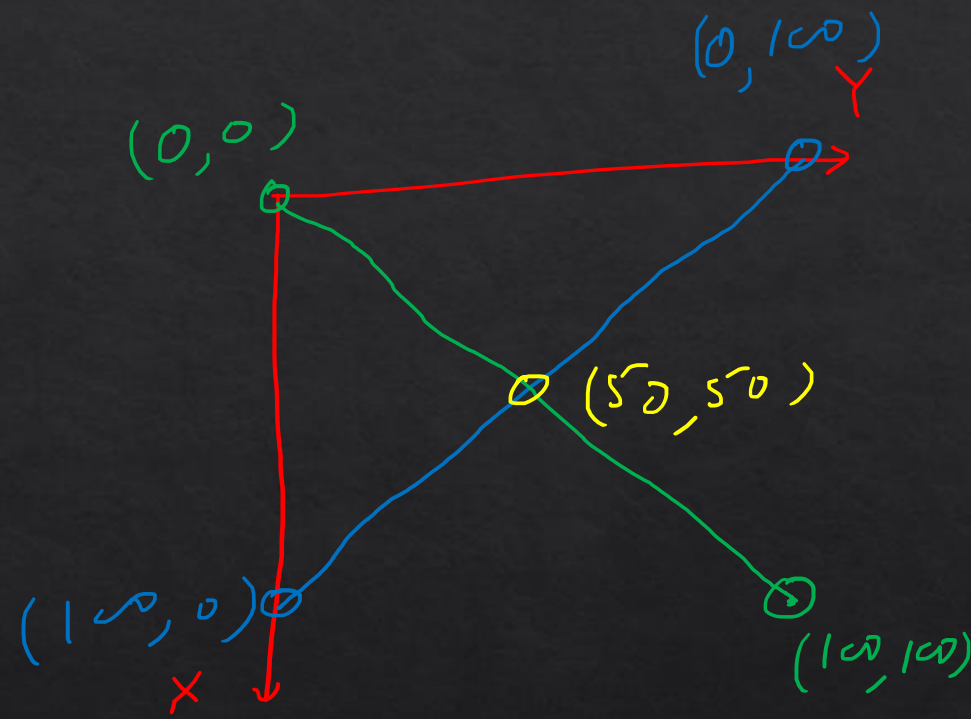
Eqn of first line

$$x - y = 0 \quad (1)$$

$\rho = 71$ & $\theta = +45^\circ$

Eqn of second line

$$x + y = 100 \quad (2)$$



Application