Laplacian of Gaussian

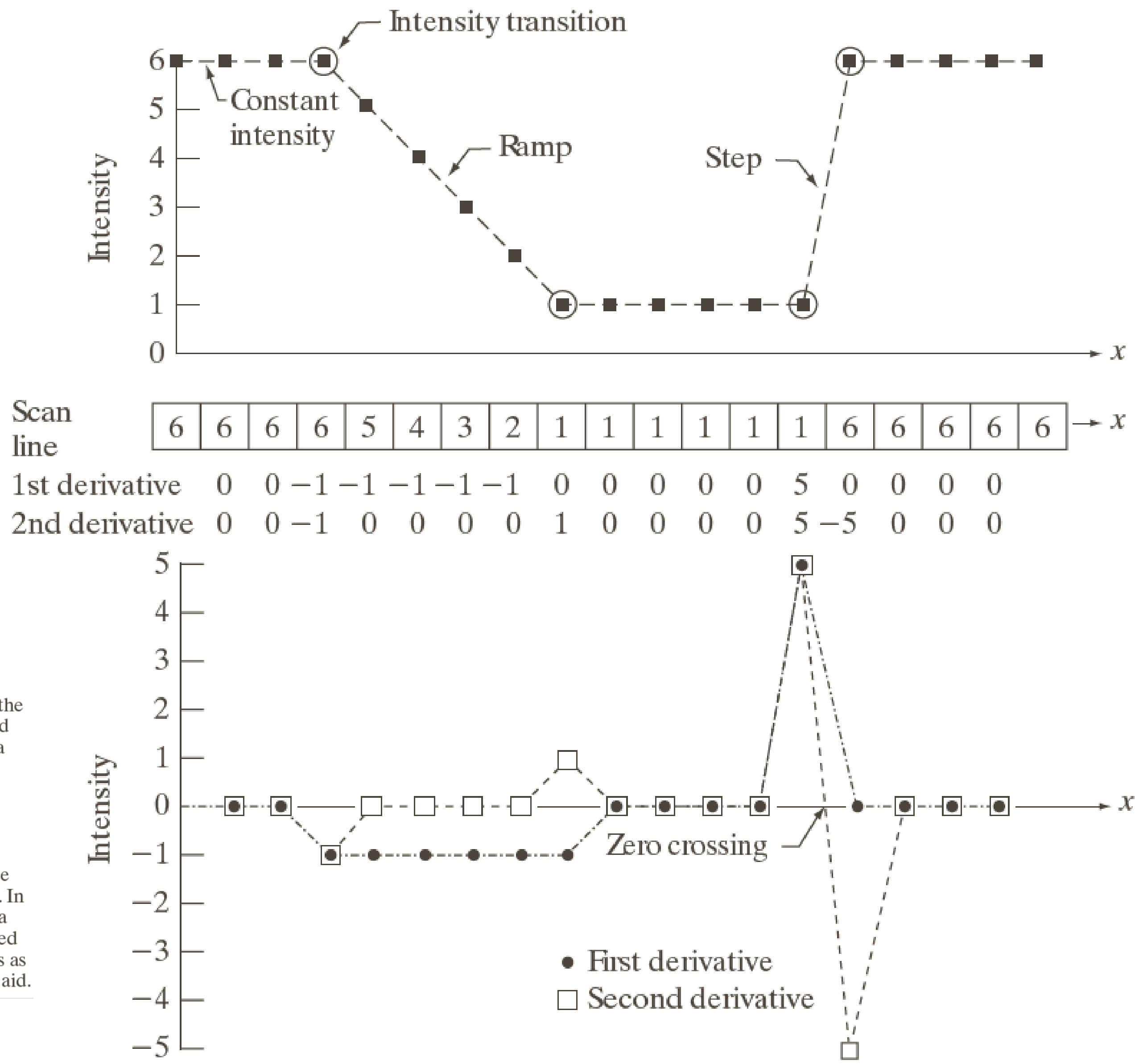
Edge Detection

Argument

- Intensity changes are not independent of image scale and so their detection requires the use of operators of different sizes.

- Sudden intensity change will give rise to a peak or trough in the first derivative or, equivalently, to a zero crossing in the second derivative.

Performance of derivatives



a
b
c

FIGURE 3.36 Illustration of the first and second derivatives of a 1-D digital function representing a section of a horizontal intensity profile from an image. In (a) and (c) data points are joined by dashed lines as a visualization aid.

Salient features of the operator

- Differential operator capable of computing a digital approximation of the first or second derivative at every point in the image.

- Capable of being “tuned” to act at any desired scale, so that large operators can be used to detect blurry edges and small operators to detect sharply focused fine detail.

Laplacian of Gaussian (LoG) $\nabla^2 G$

$$G(x, y) = e^{-\frac{x^2+y^2}{2\sigma^2}}$$

99.7% of the volume under a 2-D Gaussian surface lies between $\pm 3\sigma$ about the mean. Thus, as a rule of thumb, the size of an n*n LoG discrete filter should be such that n is the smallest odd integer greater than or equal to 6σ

$$\nabla^2 G(x, y) = \frac{\partial^2 G(x, y)}{\partial x^2} + \frac{\partial^2 G(x, y)}{\partial y^2}$$

Laplacian of Gaussian (LoG)

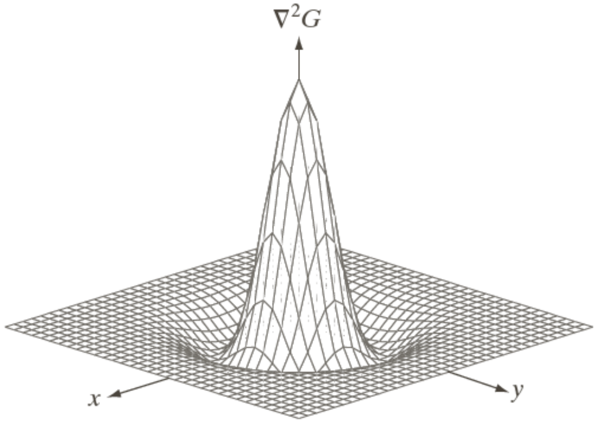
$$\nabla^2 G(x, y) = \frac{\partial^2 G(x, y)}{\partial x^2} + \frac{\partial^2 G(x, y)}{\partial y^2}$$

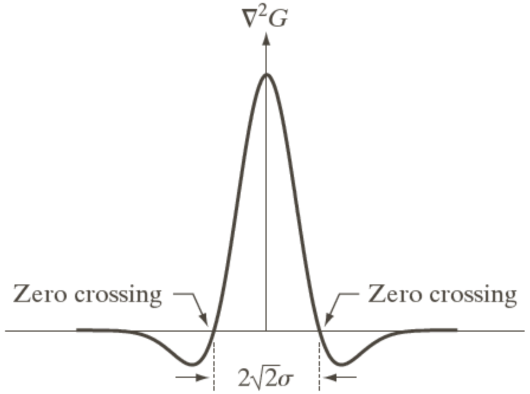
$$= \frac{\partial}{\partial x}\left[\frac{-x}{\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}\right] + \frac{\partial}{\partial y}\left[\frac{-y}{\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}\right]$$

$$= \left[\frac{x^2}{\sigma^4} - \frac{1}{\sigma^2}\right] e^{-\frac{x^2+y^2}{2\sigma^2}} + \left[\frac{y^2}{\sigma^4} - \frac{1}{\sigma^2}\right] e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$\nabla^2 G(x, y) = \left[\frac{x^2 + y^2 - 2\sigma^2}{\sigma^4}\right] e^{-\frac{x^2+y^2}{2\sigma^2}}$$

Laplacian of Gaussian (LoG)





0	0	−1	0	0
0	−1	−2	−1	0
−1	−2	16	−2	−1
0	−1	−2	−1	0
0	0	−1	0	0

Laplacian of Gaussian (LoG) implementation

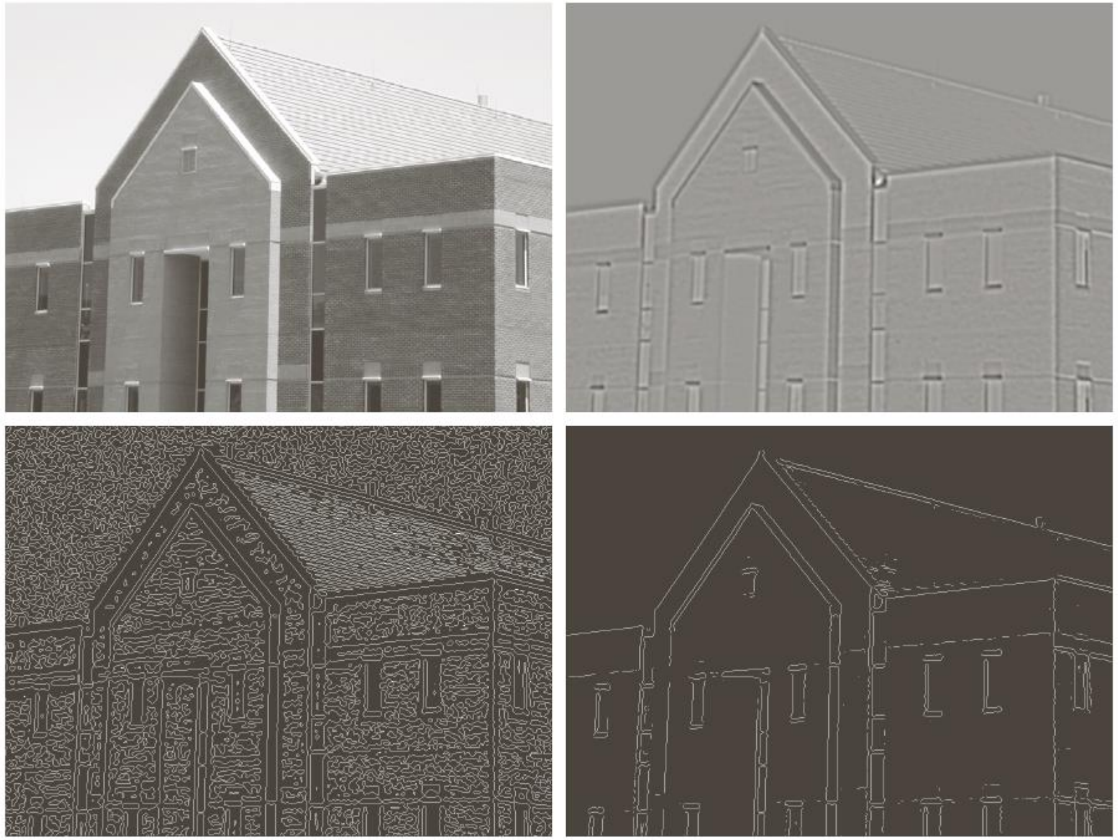
$$g(x, y) = [\nabla^2 G(x, y)] \bigstar f(x, y)$$

$$g(x, y) = \nabla^2 [G(x, y) \bigstar f(x, y)]$$

Laplacian of Gaussian (LoG) implementation

1. Filter the input image with an n*n Gaussian lowpass filter obtained by sampling.
2. Compute the Laplacian of the image resulting from Step 1 using, for example, the 3*3 mask.
3. Find the zero crossings of the image from Step 2.

Laplacian of Gaussian (LoG)



a b
c d

FIGURE 10.22 (a) Original image of size 834×1114 pixels, with intensity values scaled to the range [0, 1]. (b) Results of Steps 1 and 2 of the Marr-Hildreth algorithm using $\sigma = 4$ and $n = 25$. (c) Zero crossings of (b) using a threshold of 0 (note the closed-loop edges). (d) Zero crossings found using a threshold equal to 4% of the maximum value of the image in (b). Note the thin edges.