Mathematical Operations

Arithmetic and Logical

Distance

- For pixel p, q, and z with coordinates (x,y), (s,t) and (u,v),
- D is a distance function or metric if

- $D(p,q) \geq 0$ ($D(p,q) = 0$ if and only if $p = q$)

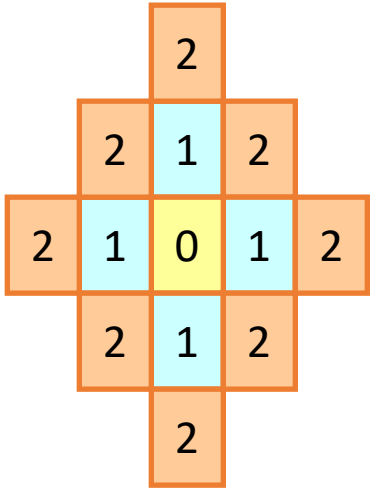
- $D(p,q) = D(q,p)$

$$D_e(p,q) = \sqrt{(x-s)^2 + (y-t)^2}$$

- $D(p,z) \leq D(p,q) + D(q,z)$

D_4-distance (city-block distance)

$$D_4(p,q) = |x-s| + |y-t|$$



Pixels with $D_4(p)$ = 1 is 4-neighbors of p.

D_8-distance (chessboard distance)

$$D_8(p,q) = \max(|x-s|, |y-t|)$$

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Pixels with $D_8(p) = 1$ is 8-neighbors of p.

Mathematical Tools Used in Digital Image Processing

- Linear operation

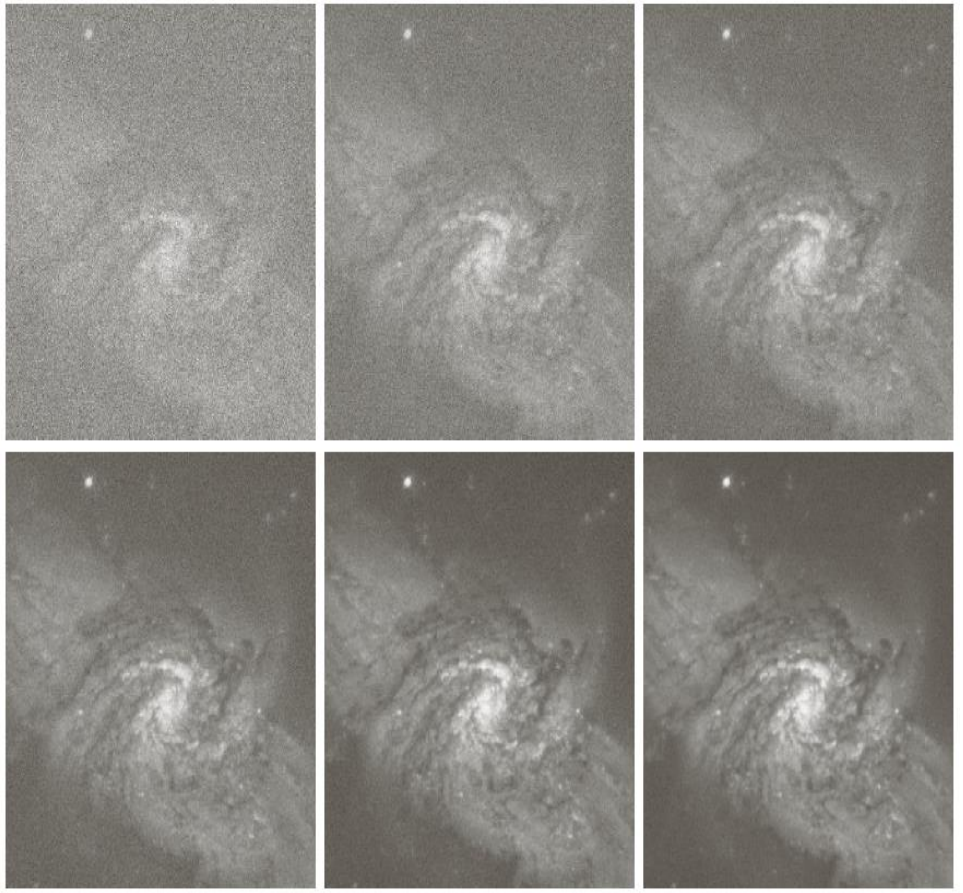
- H is said to be a linear operator if, for any two images f and g and any two scalars a and b,

$$H(af + bg) = aH(f) + bH(g)$$

Arithmetic operations

- Addition

  $s(x,y) = f(x,y)+g(x,y)$

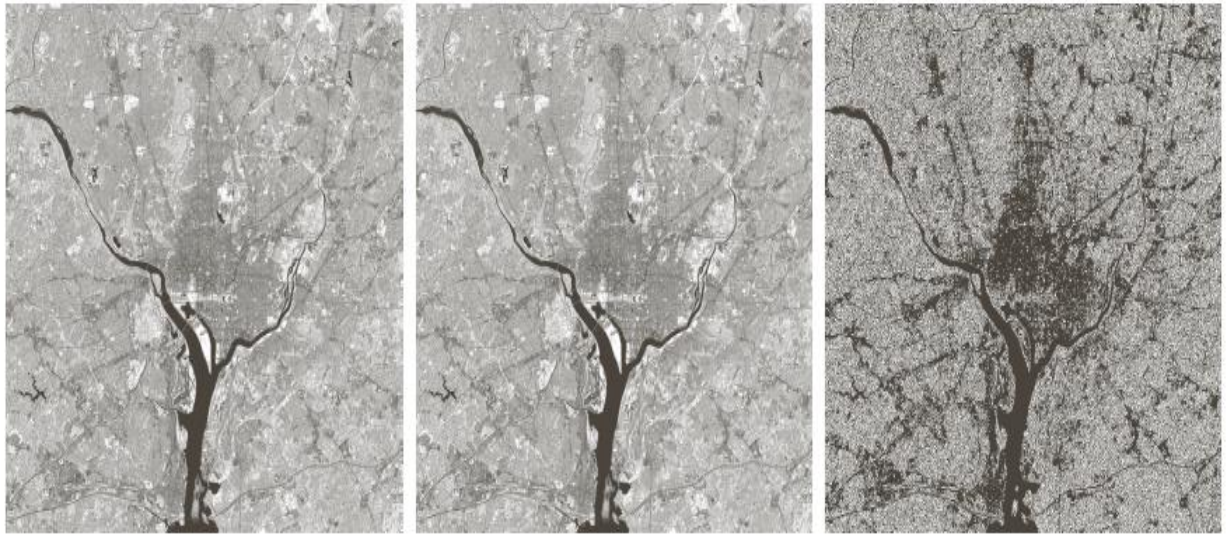


a b c
d e f

FIGURE 2.26 (a) Image of Galaxy Pair NGC 3314 corrupted by additive Gaussian noise. (b)–(f) Results of averaging 5, 10, 20, 50, and 100 noisy images, respectively. (Original image courtesy of NASA.)

Subtraction

- Subtraction to enhance differences
- s(x,y) = f(x,y)-g(x,y)



a b c

FIGURE 2.27 (a) Infrared image of the Washington, D.C. area. (b) Image obtained by setting to zero the least significant bit of every pixel in (a). (c) Difference of the two images, scaled to the range [0, 255] for clarity.

Digital subtraction angiography

a b
c d

FIGURE 2.28
Digital subtraction angiography. (a) Mask image. (b) A live image. (c) Difference between (a) and (b). (d) Enhanced difference image. (Figures (a) and (b) courtesy of The Image Sciences Institute, University Medical Center, Utrecht, The Netherlands.)

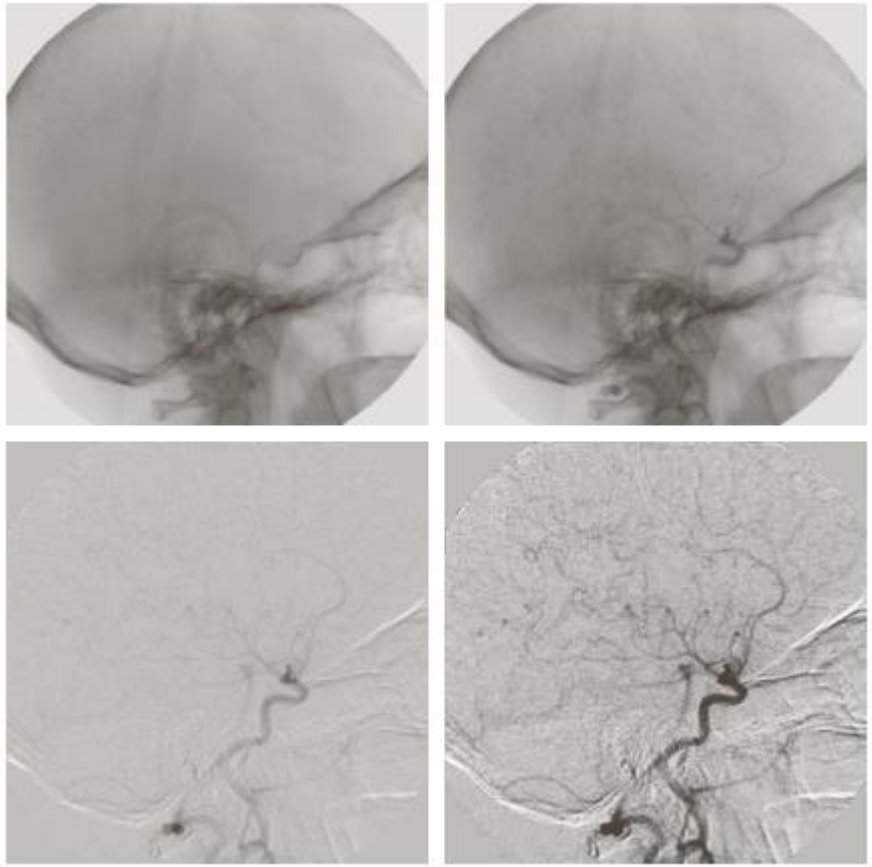


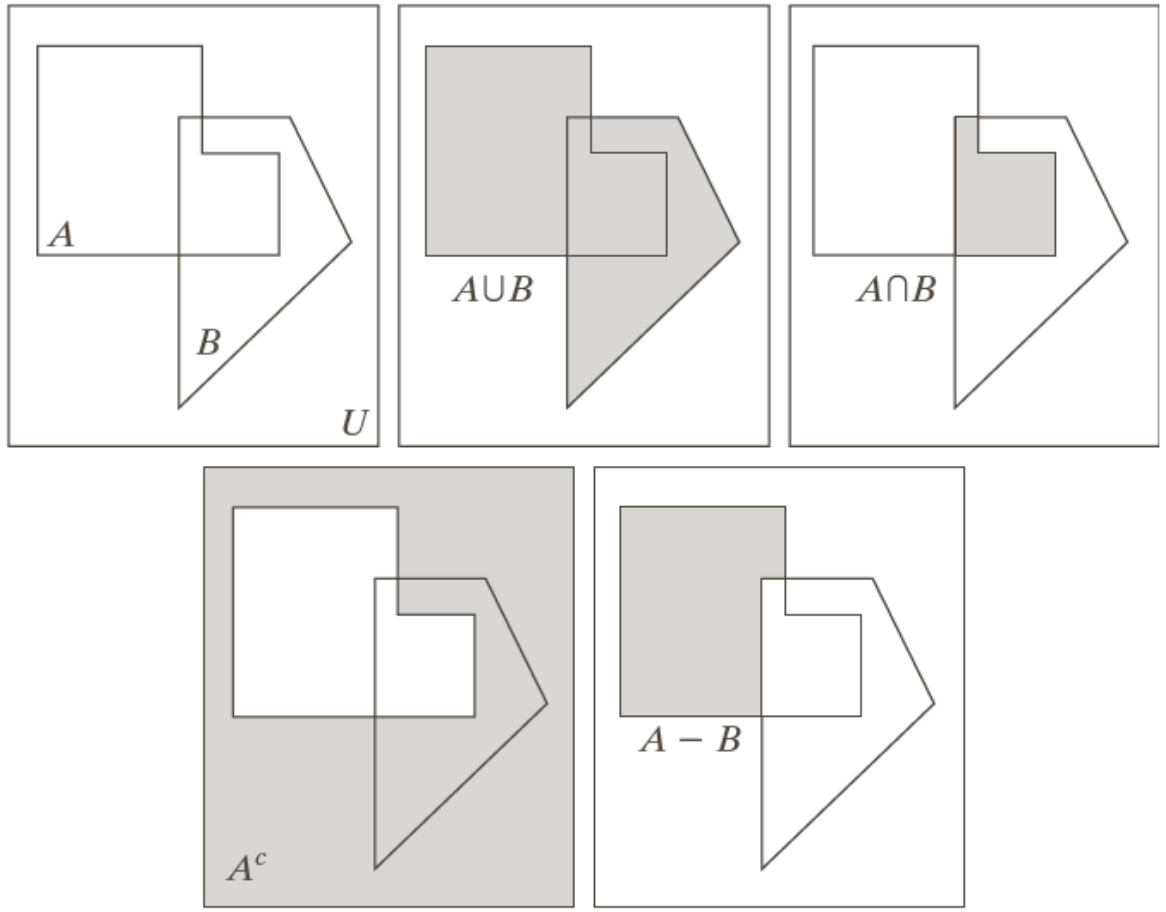
Image multiplication

- s(x,y) = f(x,y).g(x,y)

a b c

FIGURE 2.30 (a) Digital dental X-ray image. (b) ROI mask for isolating teeth with fillings (white corresponds to 1 and black corresponds to 0). (c) Product of (a) and (b).

Set operations



a b c
d e

FIGURE 2.31
(a) Two sets of coordinates, A and B, in 2-D space. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B. In (b)–(e) the shaded areas represent the member of the set operation indicated.

Logical operations

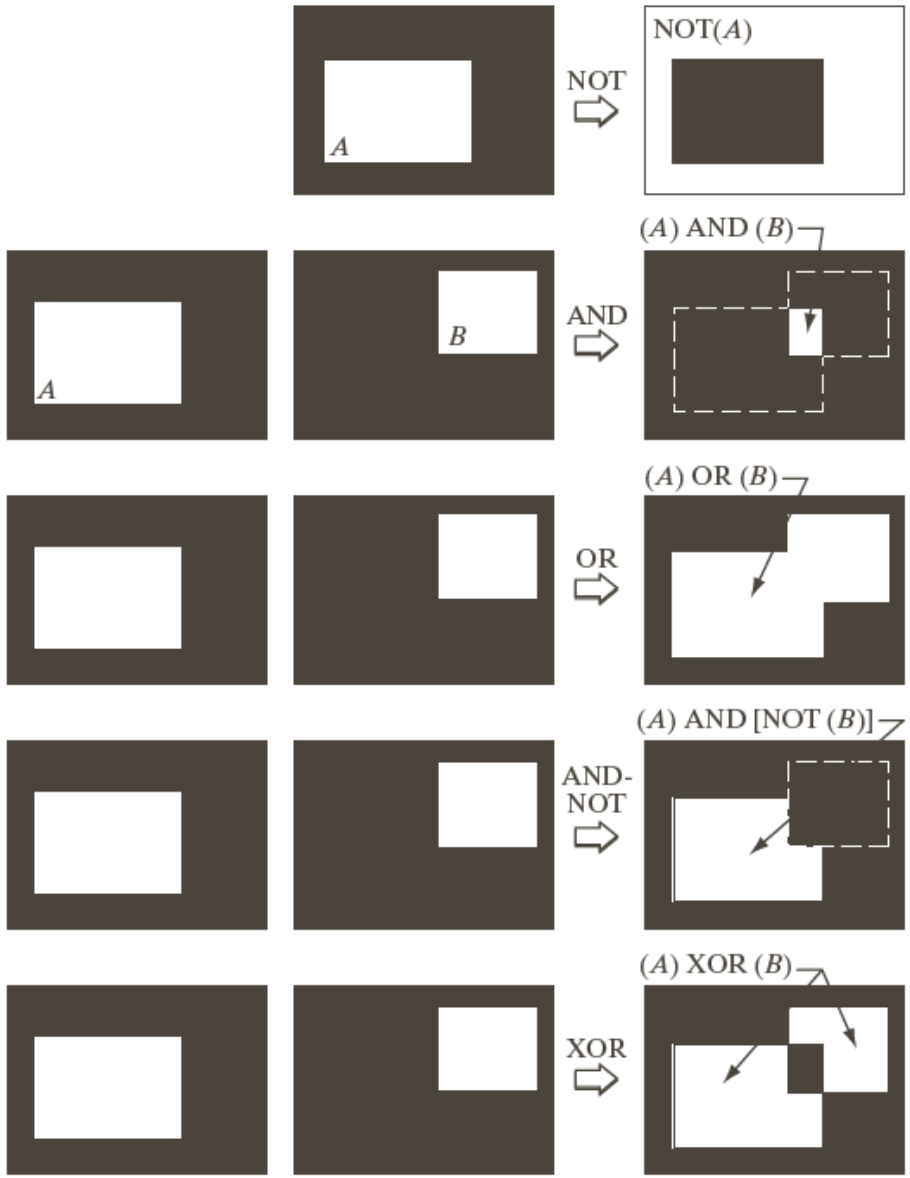


FIGURE 2.33
Illustration of logical operations involving foreground (white) pixels. Black represents binary 0s and white binary 1s. The dashed lines are shown for reference only. They are not part of the result.

Single-pixel operations

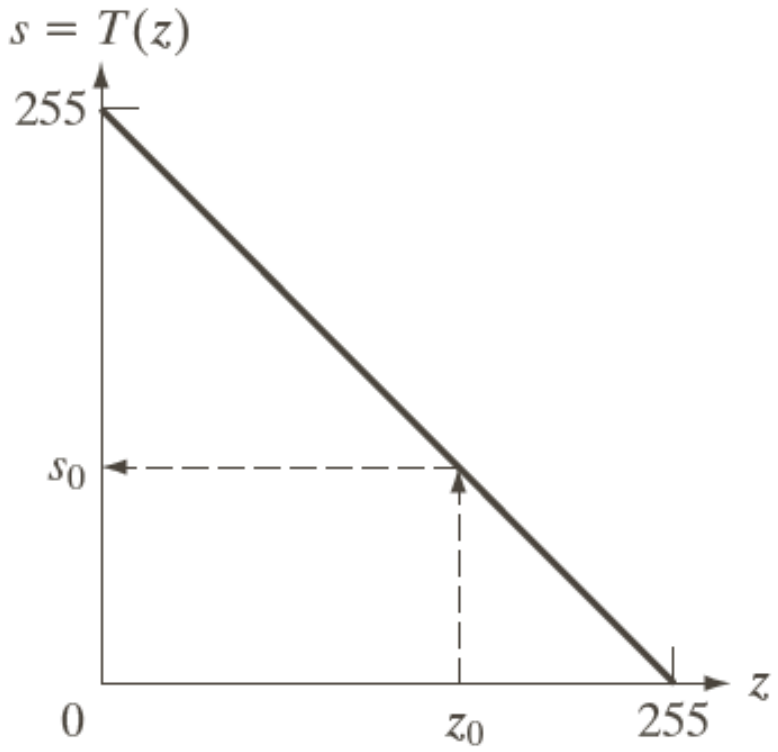


FIGURE 2.34 Intensity transformation function used to obtain the negative of an 8-bit image. The dashed arrows show transformation of an arbitrary input intensity value z_0 into its corresponding output value s_0.

Neighborhood operations

a b
c d

FIGURE 2.35
Local averaging using neighborhood processing. The procedure is illustrated in (a) and (b) for a rectangular neighborhood. (c) The aortic angiogram discussed in Section 1.3.2. (d) The result of using Eq. (2.6-21) with $m = n = 41$. The images are of size 790×686 pixels.