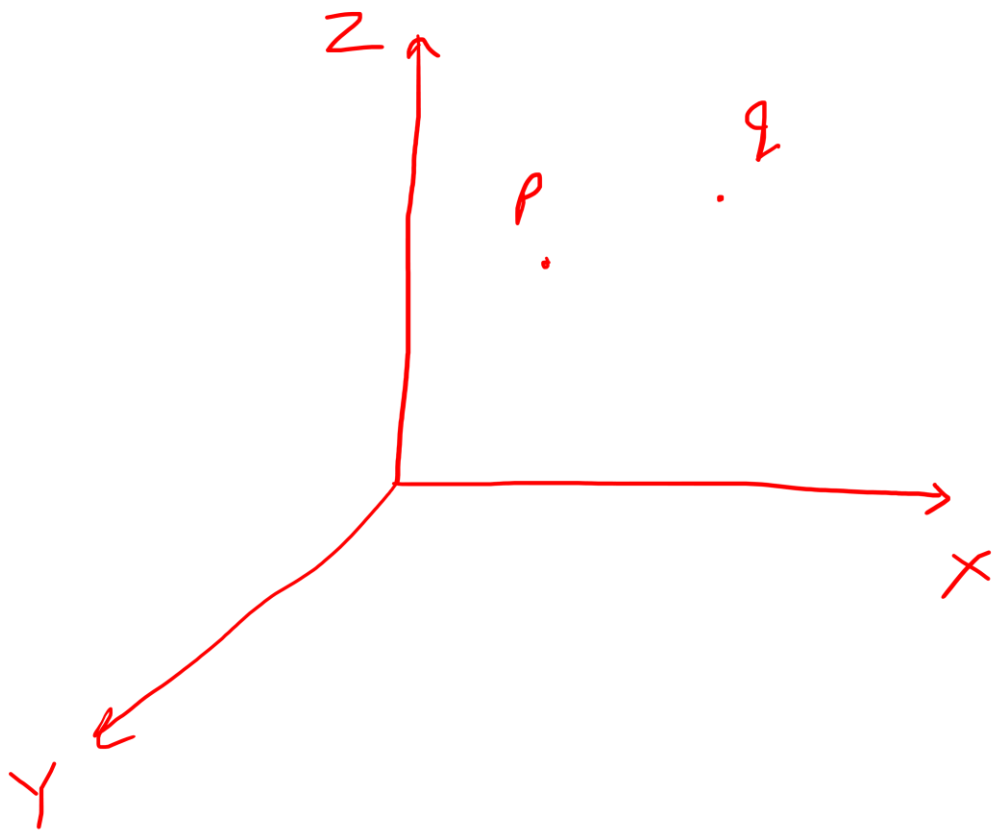
# Transformations

3D

# Translation




$$x_1 = x + x_0$$

$$y_1 = y + y_0$$

$$z_1 = z + z_0$$

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & x_0 \\ 0 & 1 & 0 & y_0 \\ 0 & 0 & 1 & z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

# Scaling

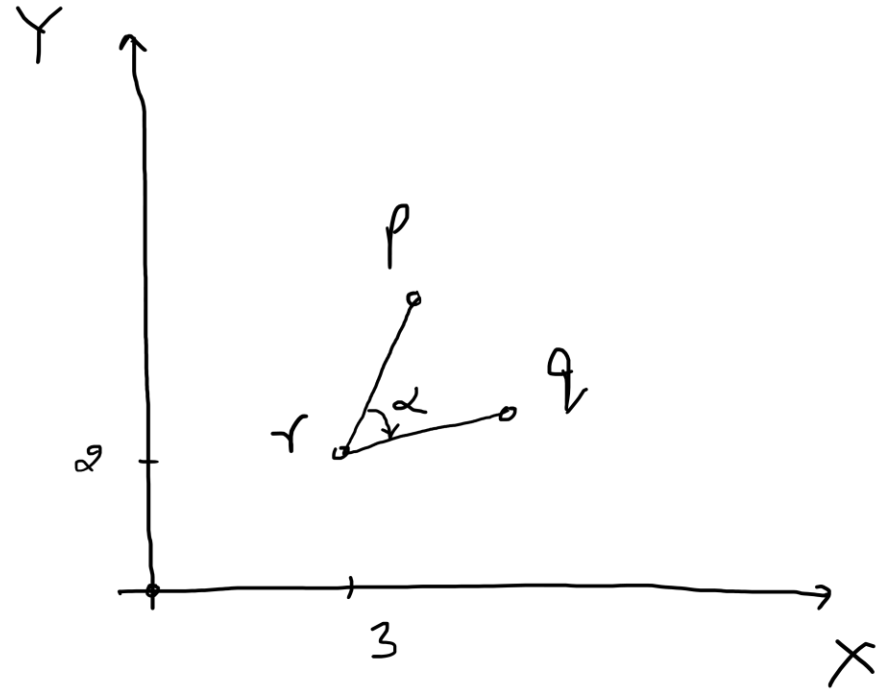
$$x_1 = c_x x$$

$$y_1 = c_y y$$

$$z_1 = c_z z$$

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} = \begin{bmatrix} c_x & 0 & 0 & 0 \\ 0 & c_y & 0 & 0 \\ 0 & 0 & c_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

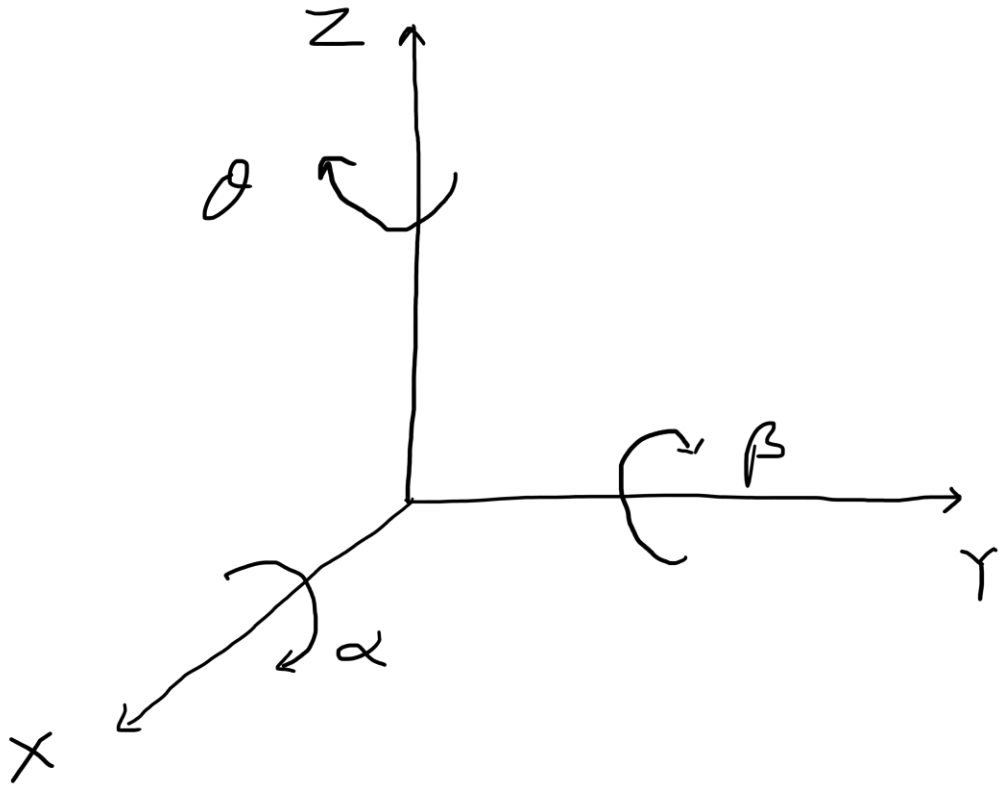
# 2D Rotation




$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$q = \left( T^{-1} \left( R_\theta \left( T(P) \right) \right) \right)$$

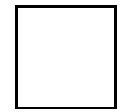
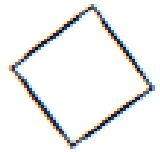
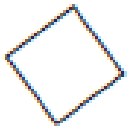
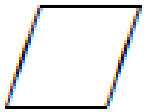
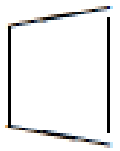
# 3D Rotation




$$R_\theta = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_\beta = \begin{bmatrix} \cos\beta & 0 & -\sin\beta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\beta & 0 & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_\alpha = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha & 0 \\ 0 & -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

| Transformation | Matrix | # DoF | Preserves | Icon |
|---|---|---|---|---|
| translation | $\begin{bmatrix} \boldsymbol{I} & \boldsymbol{t} \end{bmatrix}_{3\times4}$ | 3 | orientation |  |
| rigid (Euclidean) | $\begin{bmatrix} \boldsymbol{R} & \boldsymbol{t} \end{bmatrix}_{3\times4}$ | 6 | lengths |  |
| similarity | $\begin{bmatrix} s\boldsymbol{R} & \boldsymbol{t} \end{bmatrix}_{3\times4}$ | 7 | angles |  |
| affine | $\begin{bmatrix} \boldsymbol{A} \end{bmatrix}_{3\times4}$ | 12 | parallelism |  |
| projective | $\begin{bmatrix} \tilde{\boldsymbol{H}} \end{bmatrix}_{4\times4}$ | 15 | straight lines |  |