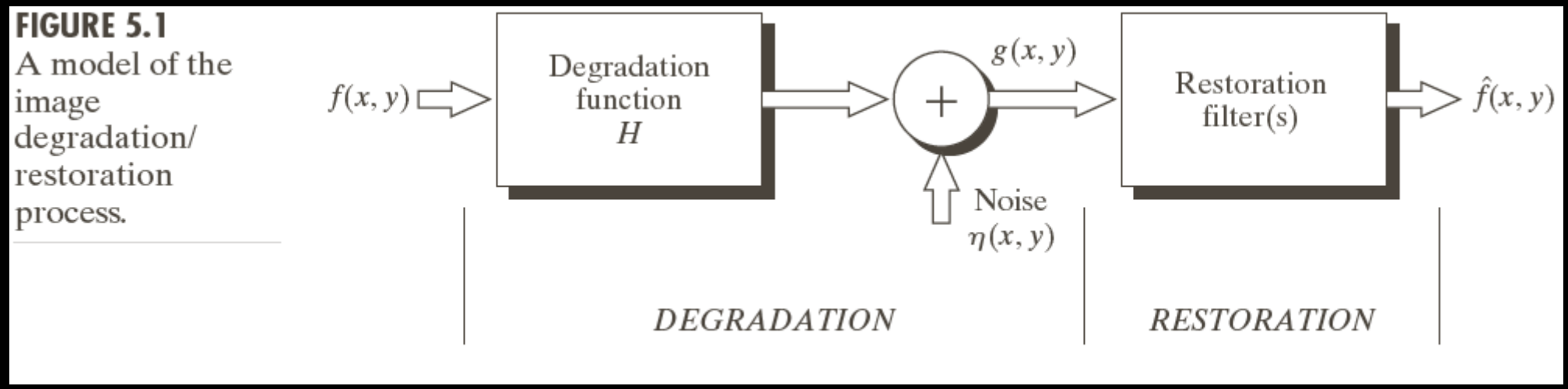
Image Restoration

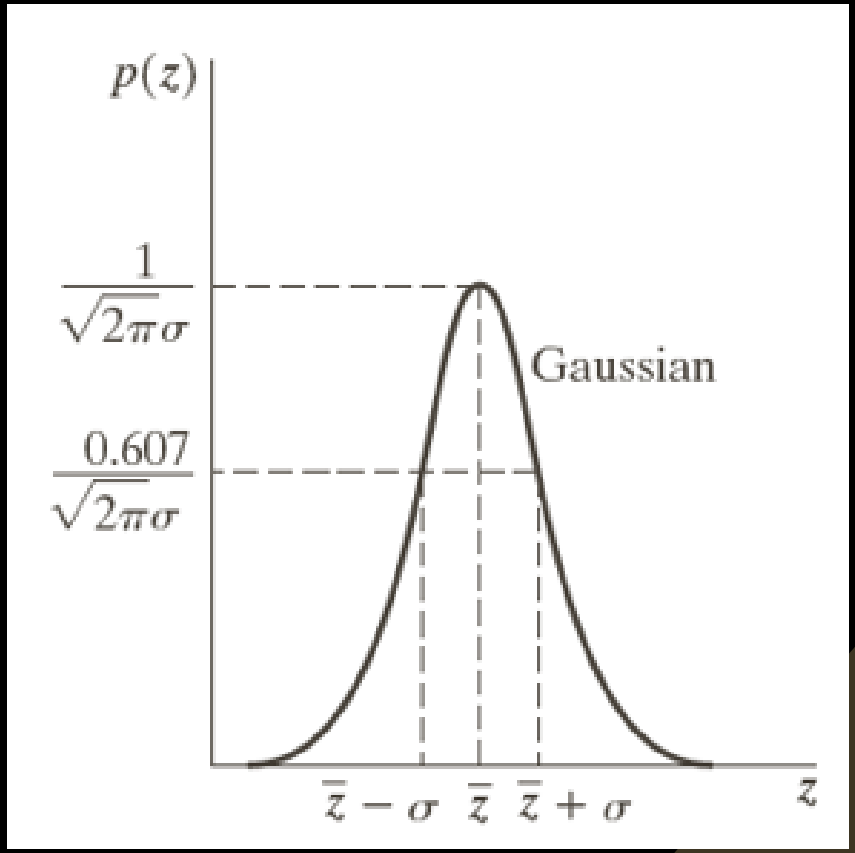
Noise profiles

Image degradation/restoration process



Gaussian Noise

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\bar{z})^2/2\sigma^2}$$

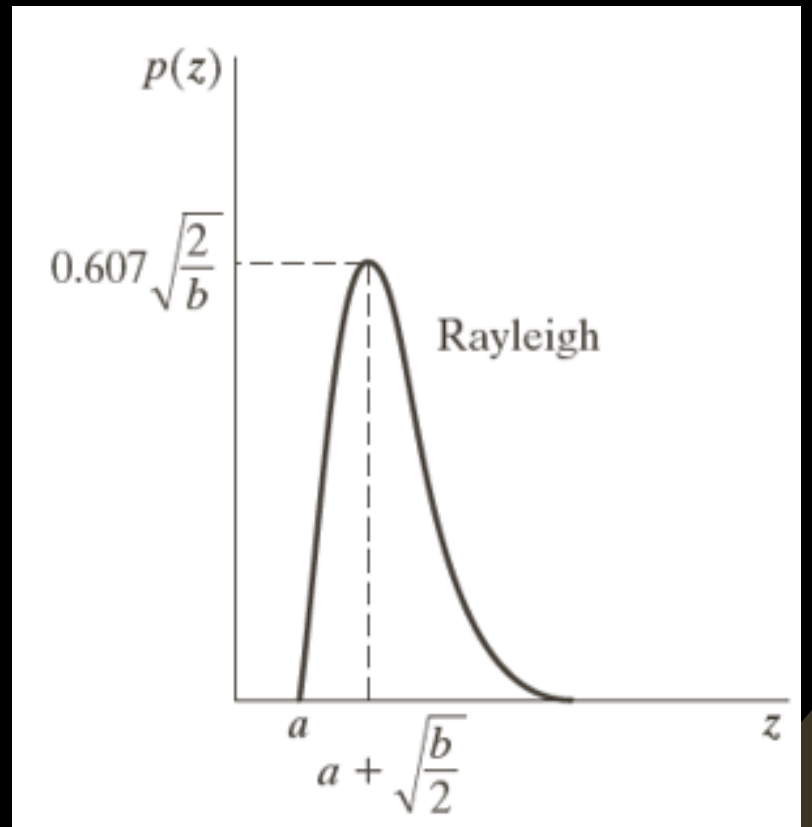


Rayleigh Noise

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & ; z \geq a \\ 0 & ; z < a \end{cases}$$

$$\bar{z} = a + \sqrt{\pi b/4}$$

$$\sigma^2 = \frac{b(4-\pi)}{4}$$

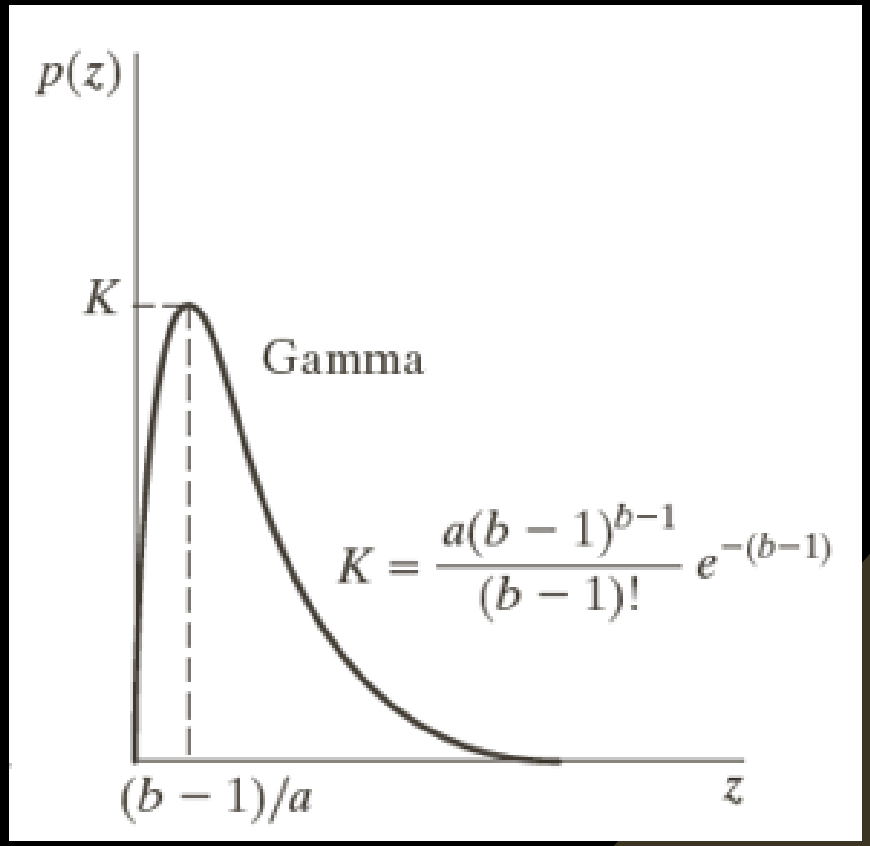


Erlang (Gamma) Noise

$$p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az} & ; \quad z \geq 0 \\ 0 & ; \quad z < 0 \end{cases}$$

$$\bar{z} = b/a$$

$$\sigma^2 = b/a^2$$

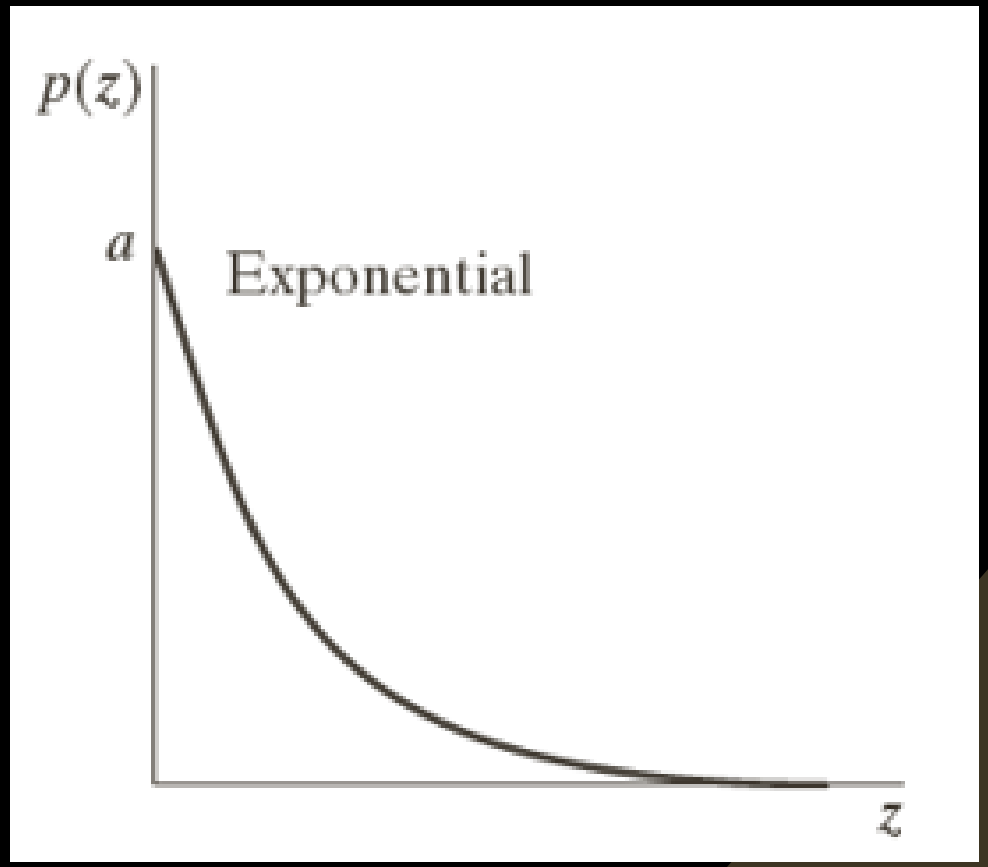


Exponential Noise

$$p(z) = \begin{cases} ae^{-az} & ; \quad z \geq 0 \\ 0 & ; \quad z < 0 \end{cases}$$

$$\bar{z} = \frac{1}{a}$$

$$\sigma^2 = \frac{1}{a^2}$$

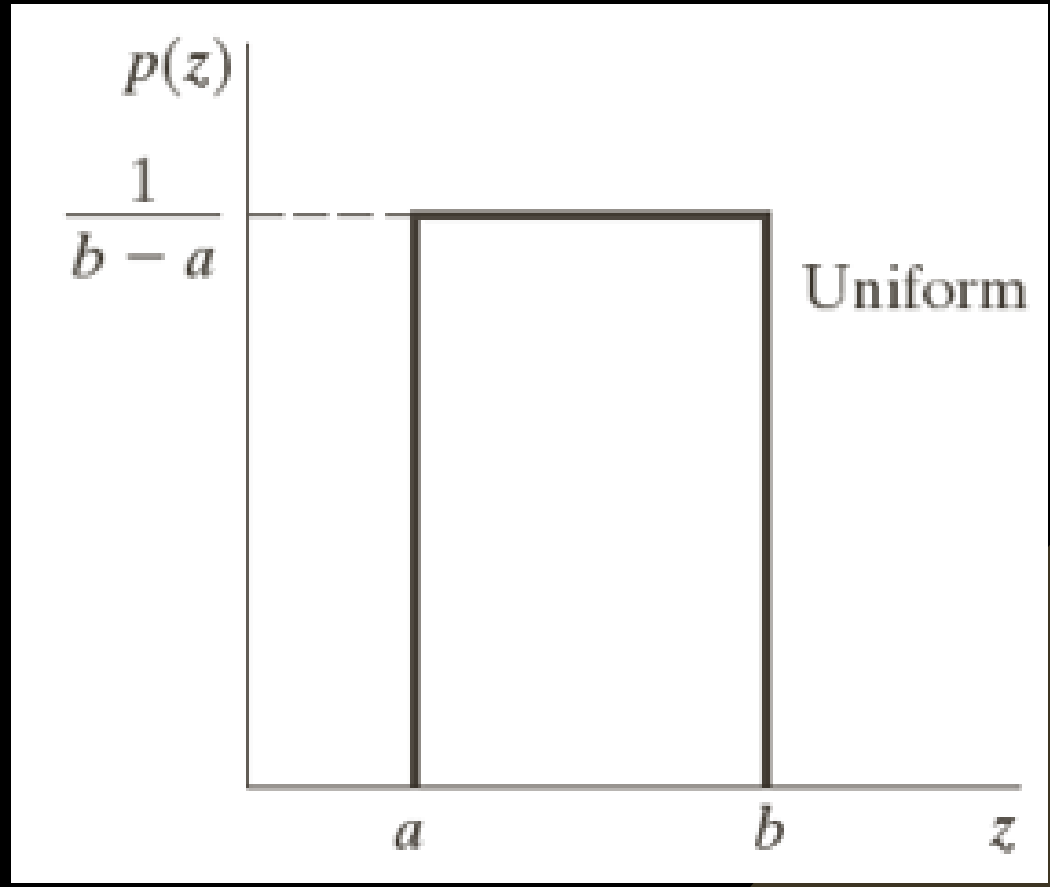


Uniform Noise

$$p(z) = \begin{cases} \frac{1}{b-a} & ; \ a \le z \le b \\ 0 & ; \ \text{otherwise} \end{cases}$$

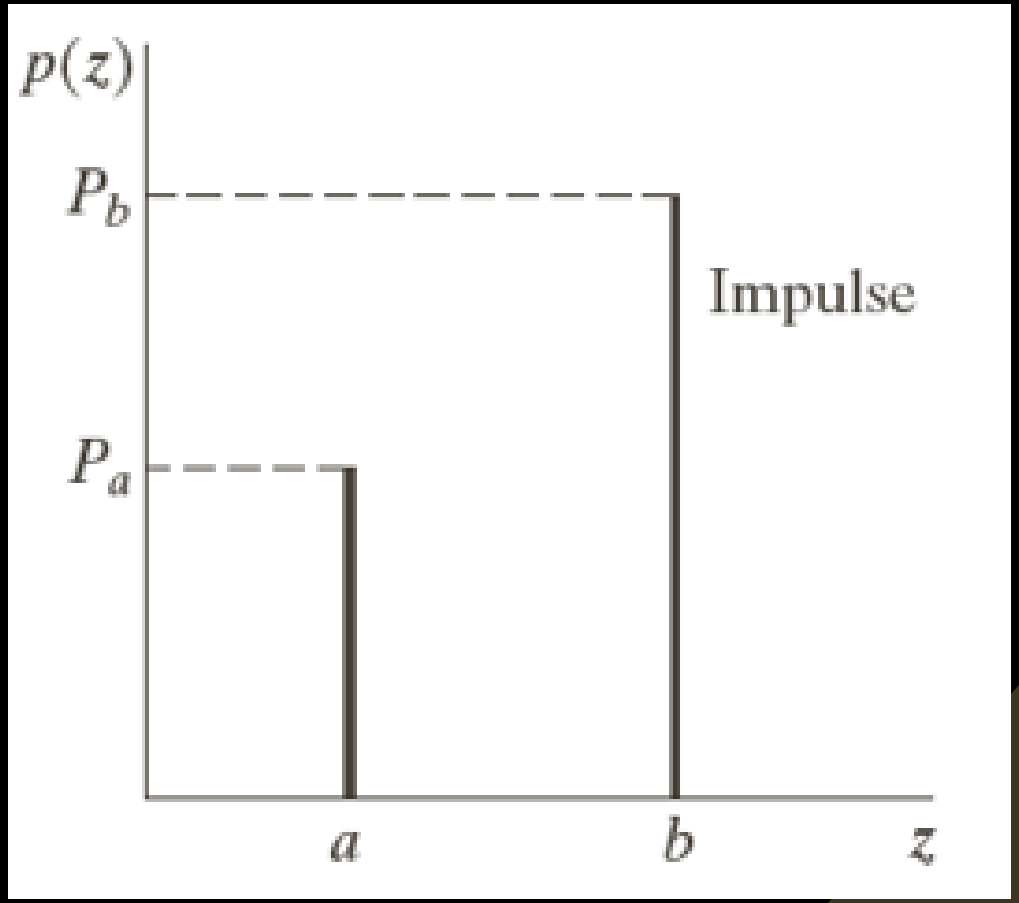
$$\bar{z} = \frac{(a+b)}{2}$$

$$\sigma^2 = \frac{(b-a)^2}{12}$$



Impulse Noise

$$P(z) = \begin{cases} P_a & ; \quad z = a \\ P_b & ; \quad z = b \\ 0 & ; \quad \text{otherwise} \end{cases}$$



Test image

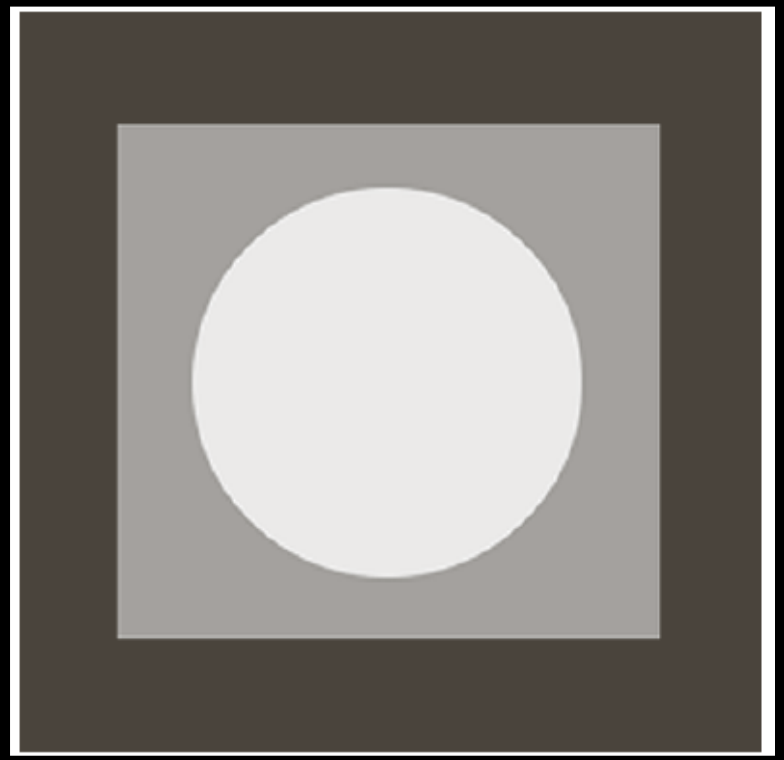
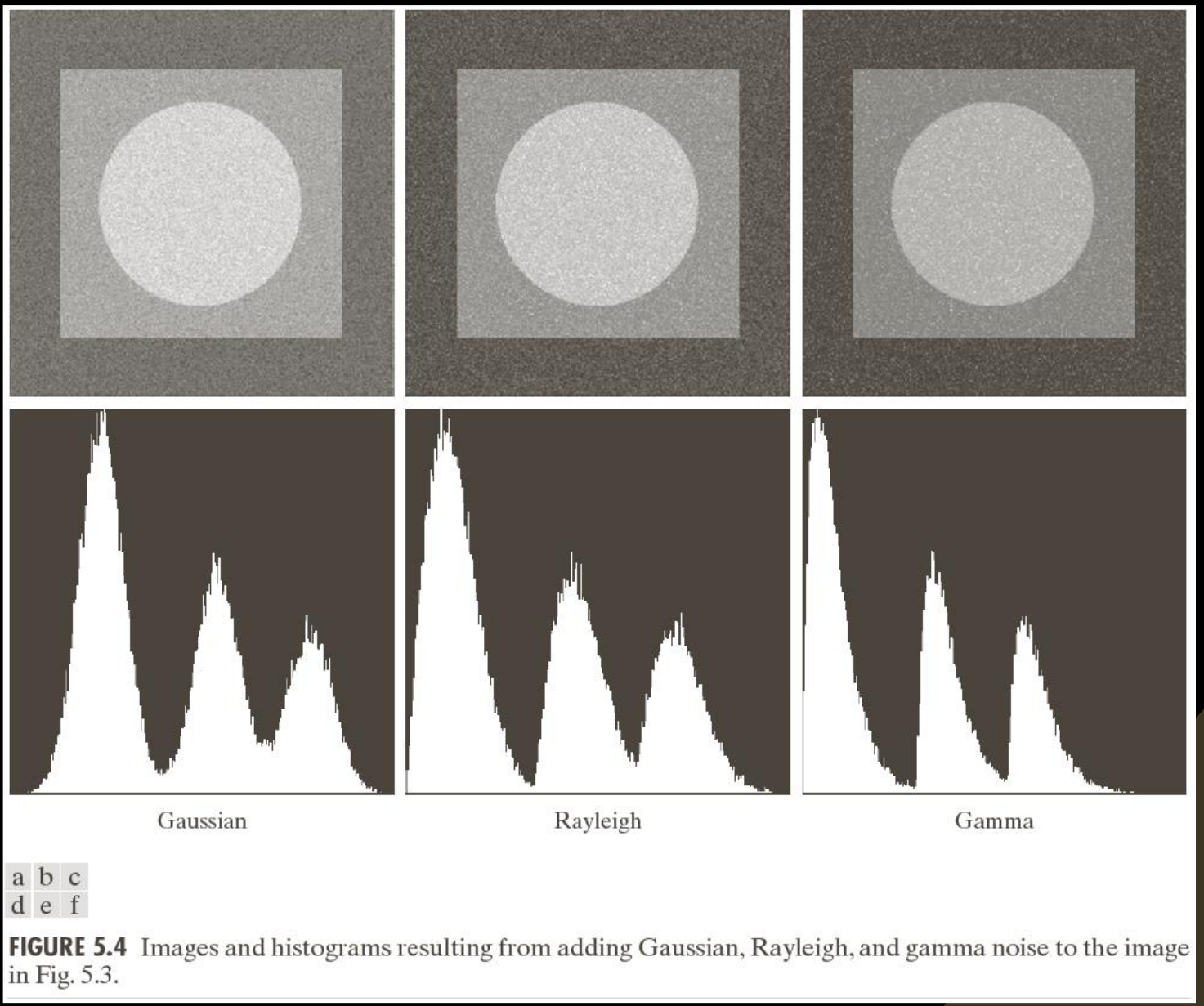


FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.

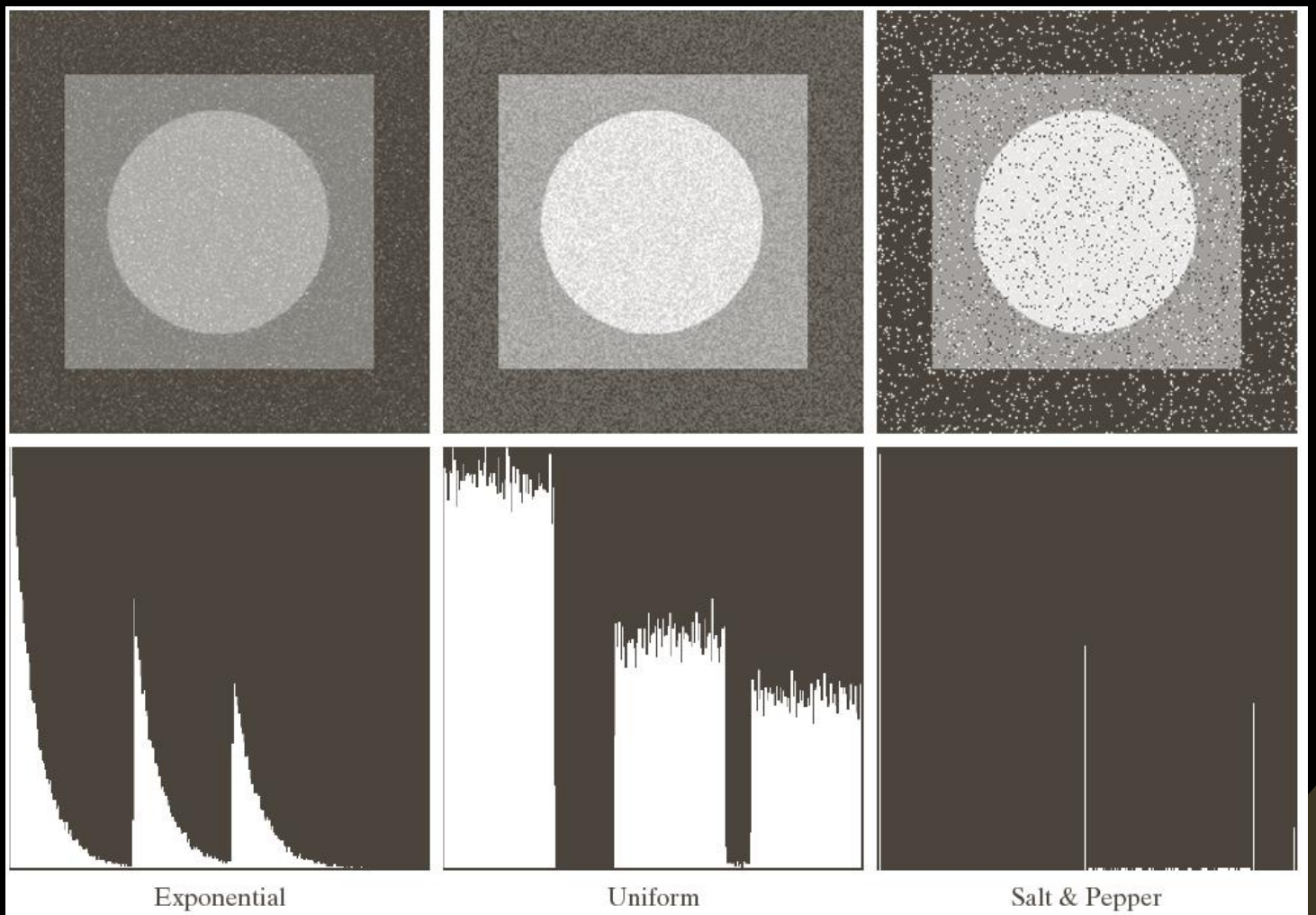
Image degradation



a b c
d e f

FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.

Image degradation



g h i
j k l

FIGURE 5.4 *(Continued)* Images and histograms resulting from adding exponential, uniform, and salt and pepper noise to the image in Fig. 5.3.

Filters

- Arithmetic Mean
- Geometric Mean
- Harmonic Mean
- Contra harmonic Mean
- Order Static
- Alpha trimmed

Arithmetic mean filter

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s,t)$$

Geometric mean filter

$$\hat{f}(x,y) = \left[\prod_{(s,t) \in S_{xy}} g(s,t) \right]^{1/mn}$$

Harmonic mean filter

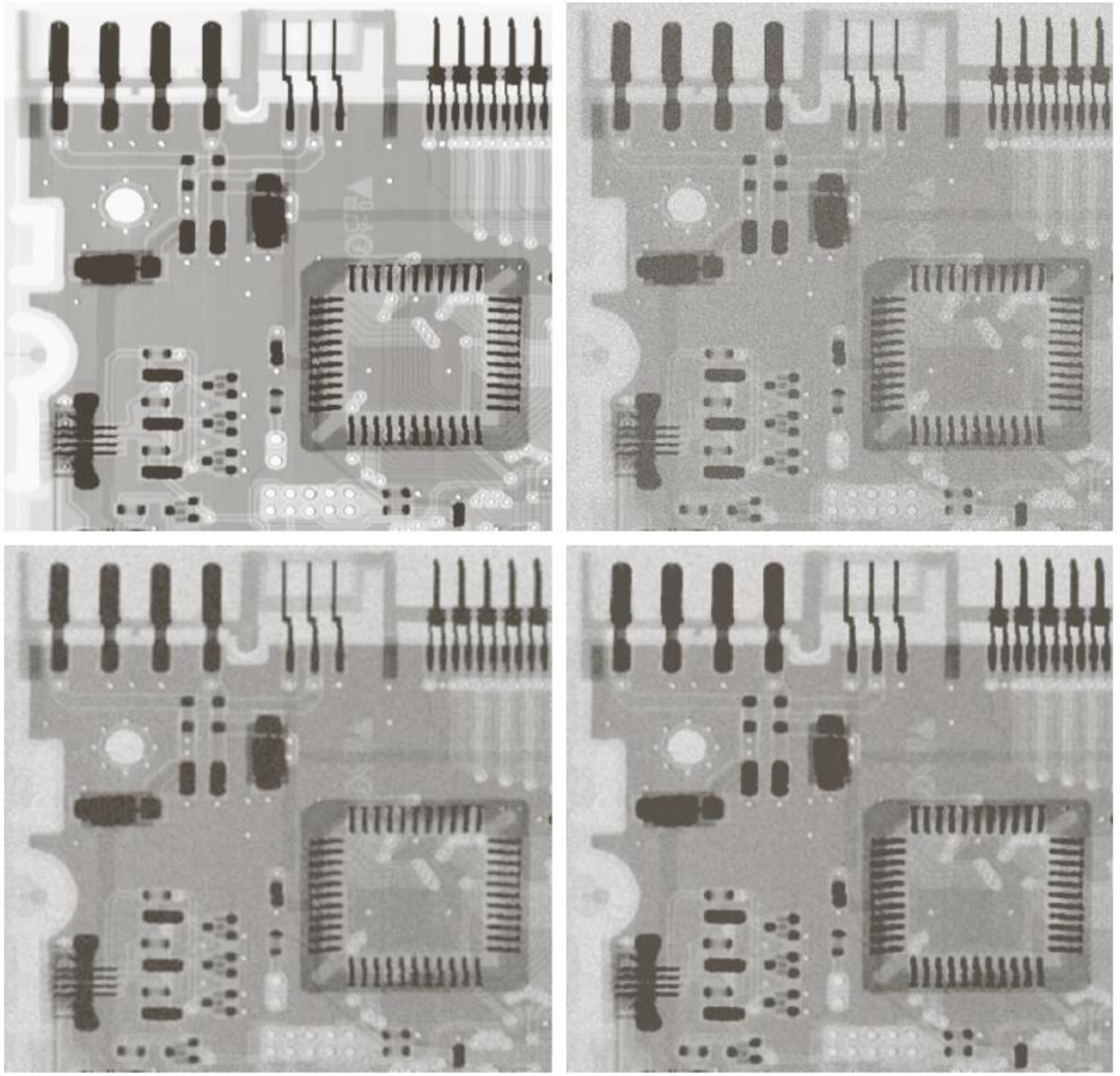
$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

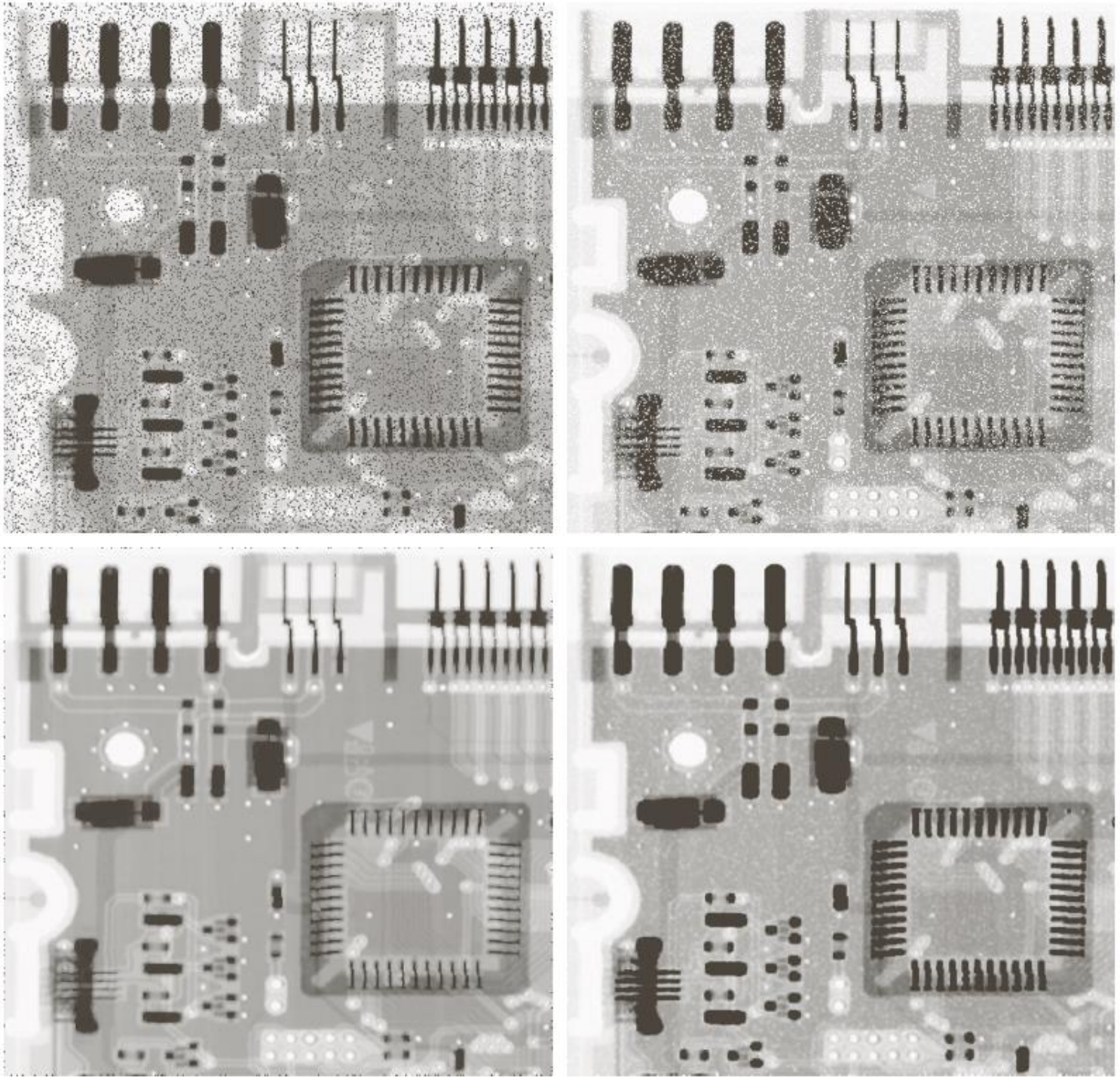
Contraharmonic mean filter

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} \{g(s,t)\}^{Q+1}}{\sum_{(s,t)\in S_{xy}} \{g(s,t)\}^{Q}}$$

a b
c d

FIGURE 5.7
(a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3×3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)



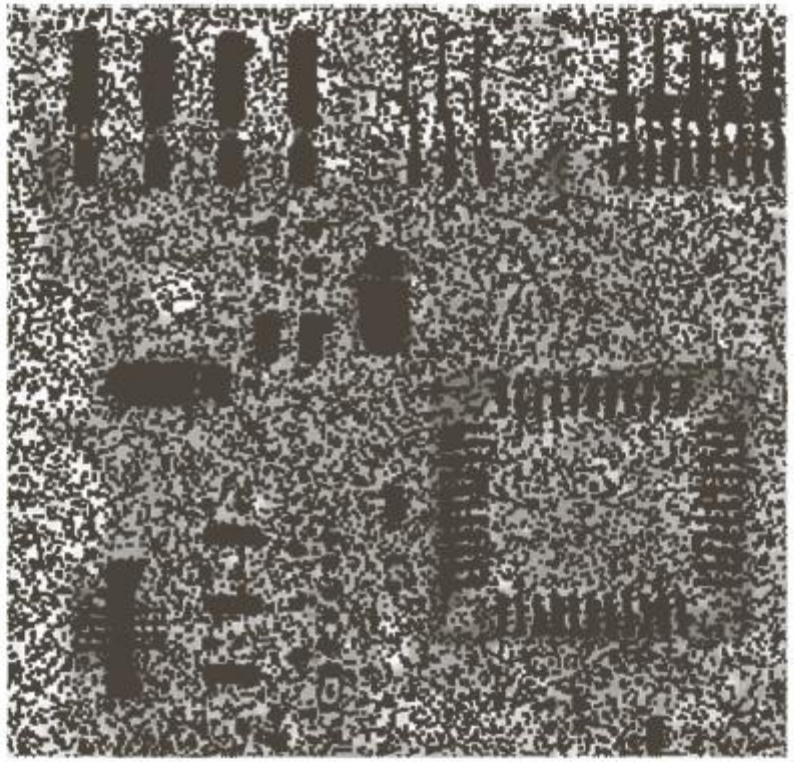


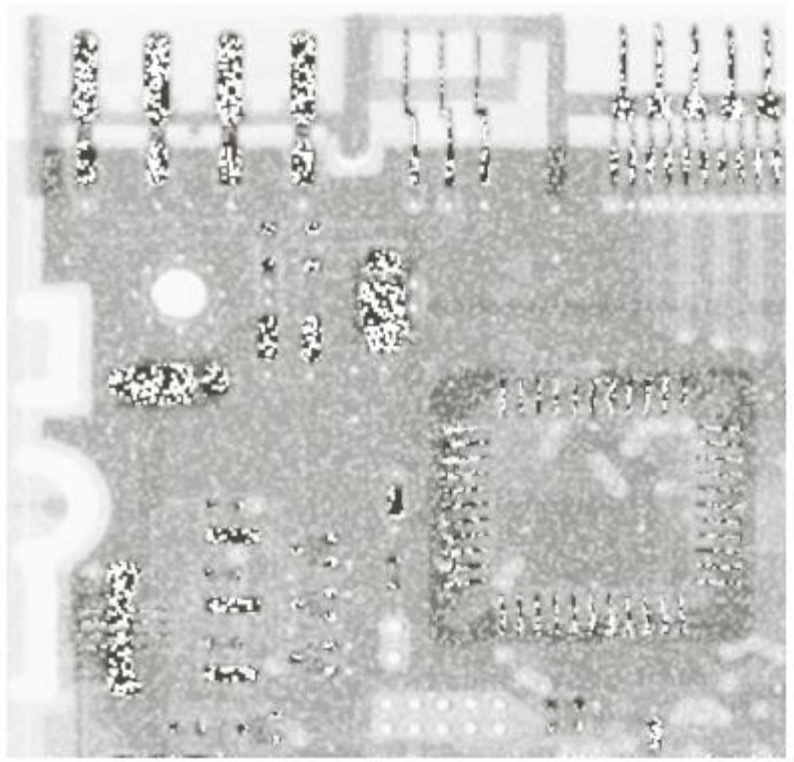
a b
c d

FIGURE 5.8
(a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3×3 contra-harmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.

a b

FIGURE 5.9
Results of selecting the wrong sign in contraharmonic filtering.
(a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3×3 and $Q = -1.5$.
(b) Result of filtering 5.8(b) with $Q = 1.5$.





Order Static filter

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{\text{median}} \left\{ g(s,t) \right\}$$

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{\min} \left\{ g(s,t) \right\}$$

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{\max} \left\{ g(s,t) \right\}$$

Midpoint filter

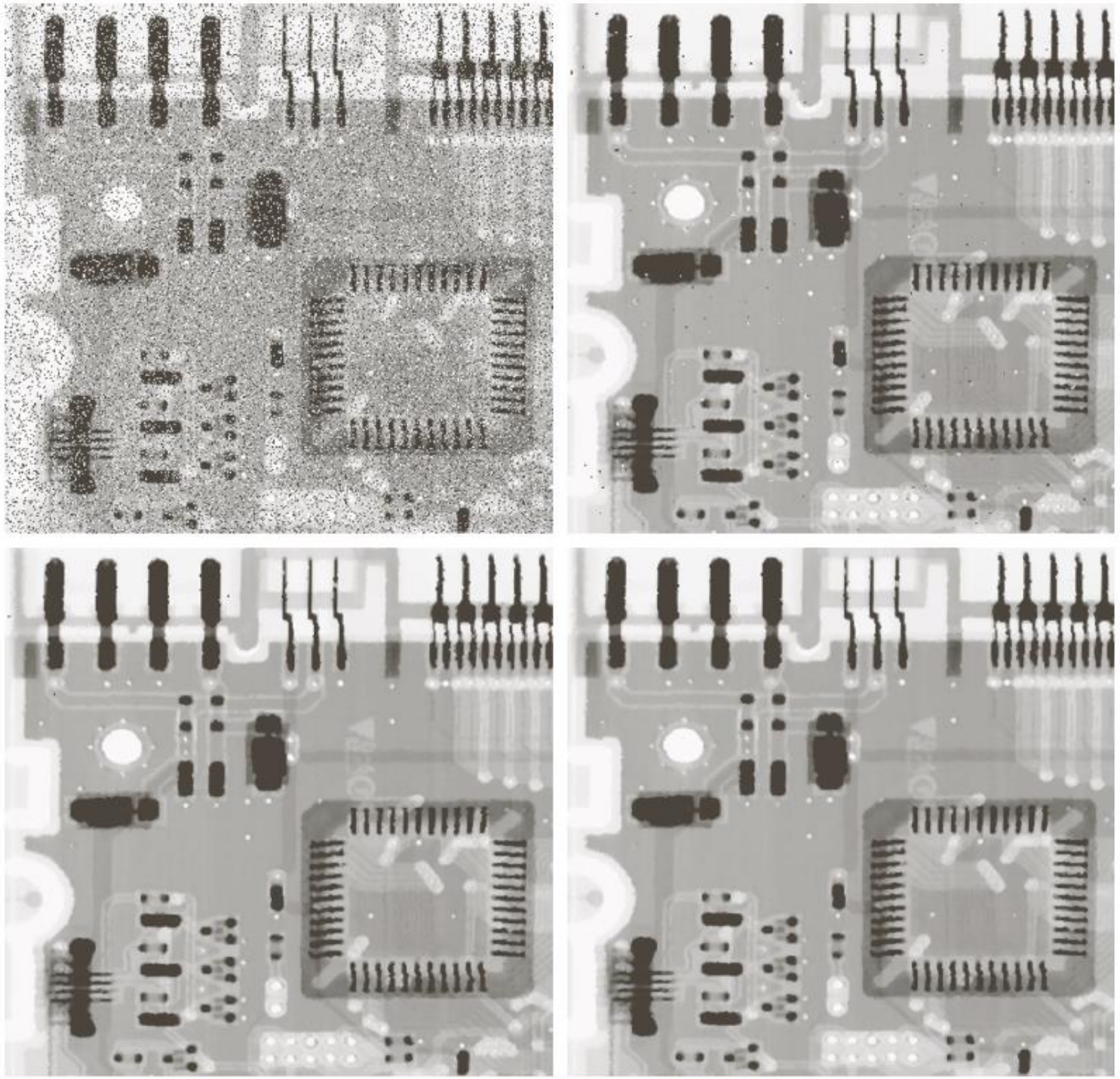
$$\hat{f}(x,y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\} + \min_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$

Alpha Trimmed filter

$$\hat{f}(x,y) = \frac{1}{mn-d}\sum_{(s,t)\in S_{xy}} g_r(s,t)$$

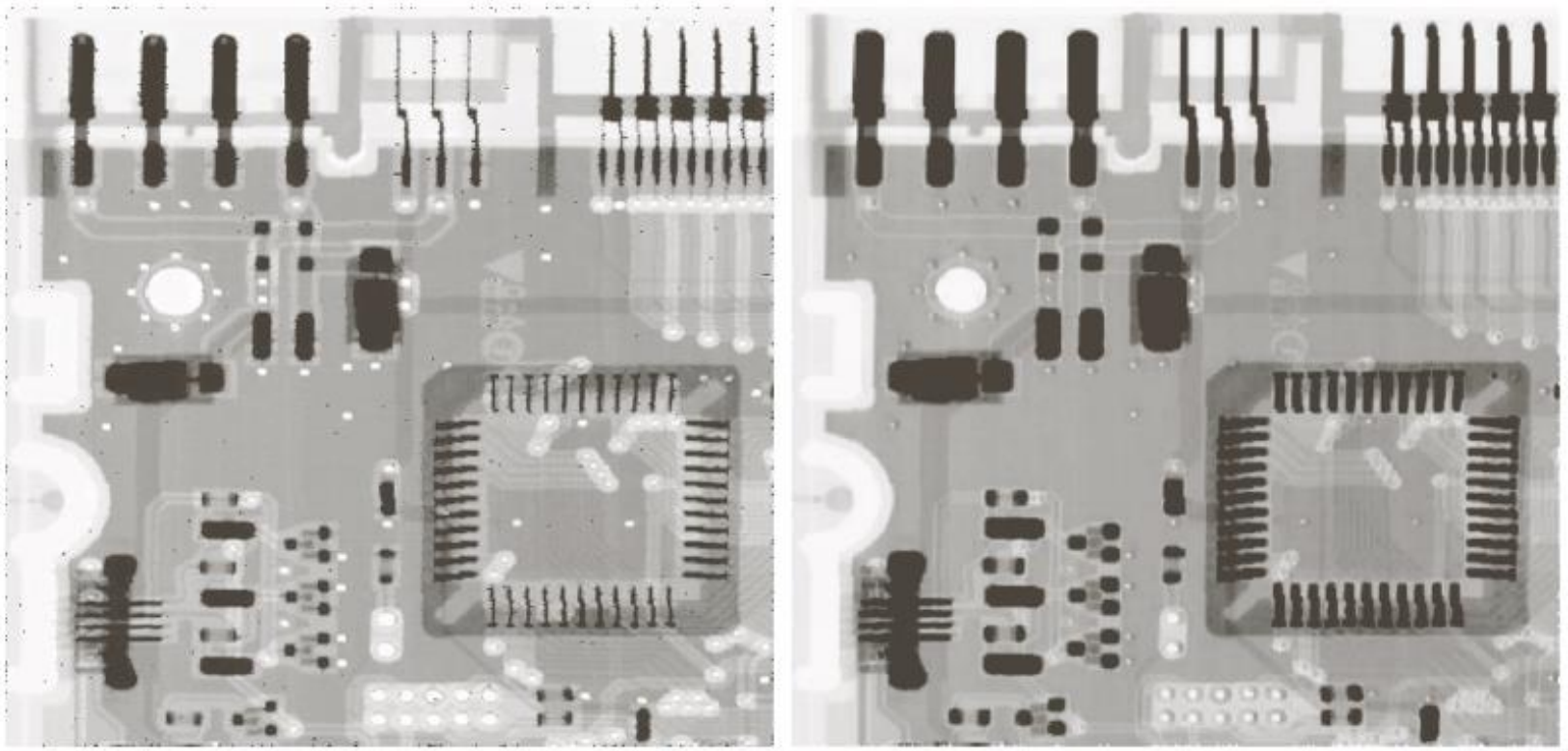
a b
c d

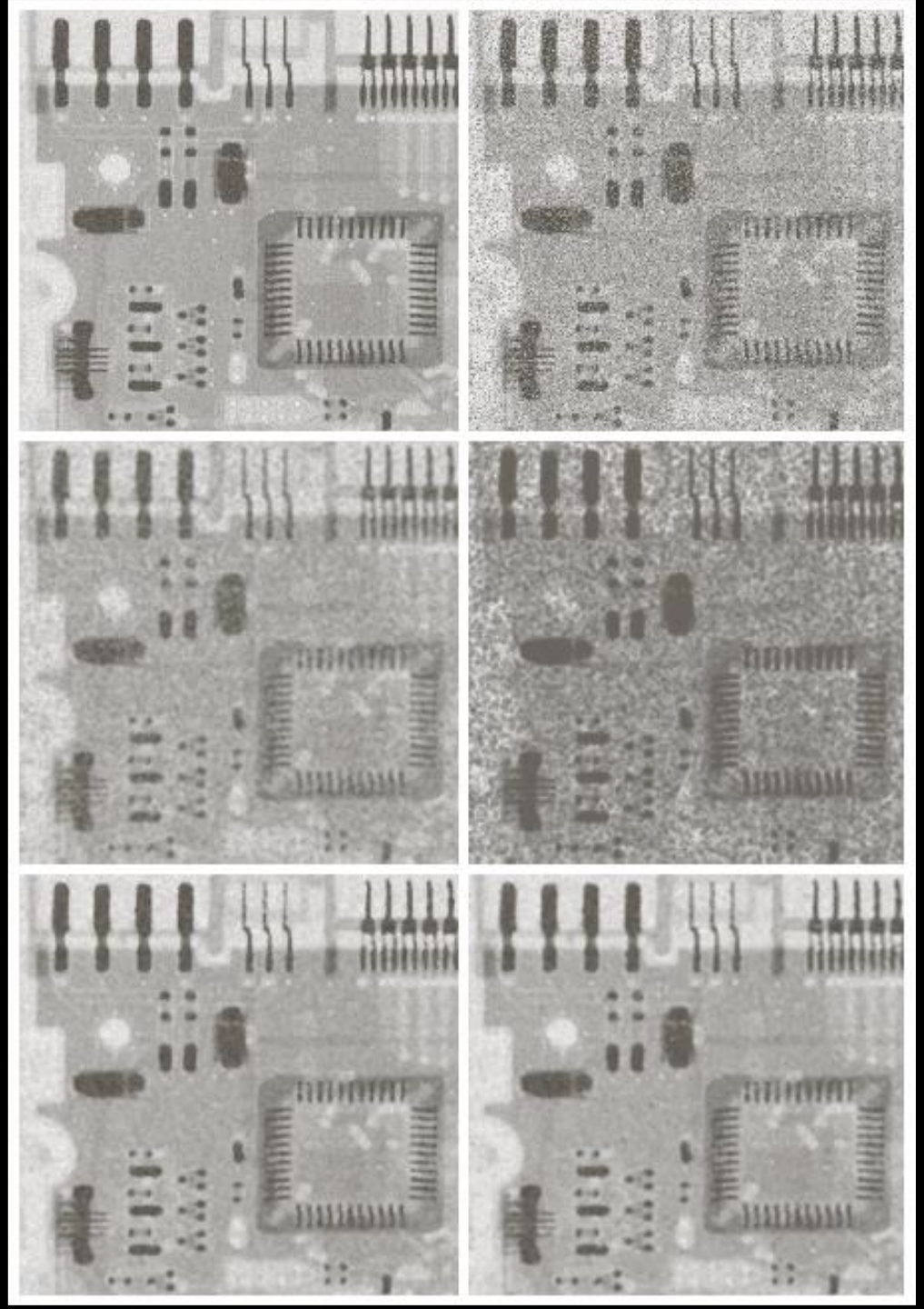
FIGURE 5.10
(a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$.
(b) Result of one pass with a median filter of size 3×3.
(c) Result of processing (b) with this filter.
(d) Result of processing (c) with the same filter.



a b

FIGURE 5.11
(a) Result of filtering Fig. 5.8(a) with a max filter of size 3×3. (b) Result of filtering 5.8(b) with a min filter of the same size.





a b
c d
e f

FIGURE 5.12
(a) Image corrupted by additive uniform noise. (b) Image additionally corrupted by additive salt-and-pepper noise. Image (b) filtered with a 5×5; (c) arithmetic mean filter; (d) geometric mean filter; (e) median filter; and (f) alpha-trimmed mean filter with $d = 5$.