Logic

Logic

- Knowledge bases consist of **Sentences**.
- These sentences are expressed according to the **syntax** of the representation language.
- A logic must also define the **semantics**, or meaning, of sentences.
- The **semantics** defines the truth of each sentence with respect to each **possible world (Model)**.

Logic

- If a sentence α is true in model m, we say that m **satisfies** α or sometimes **m is a model of α**.
- Logical reasoning involves the relation of logical **entailment** between sentences—the idea that a sentence follows logically from another sentence.

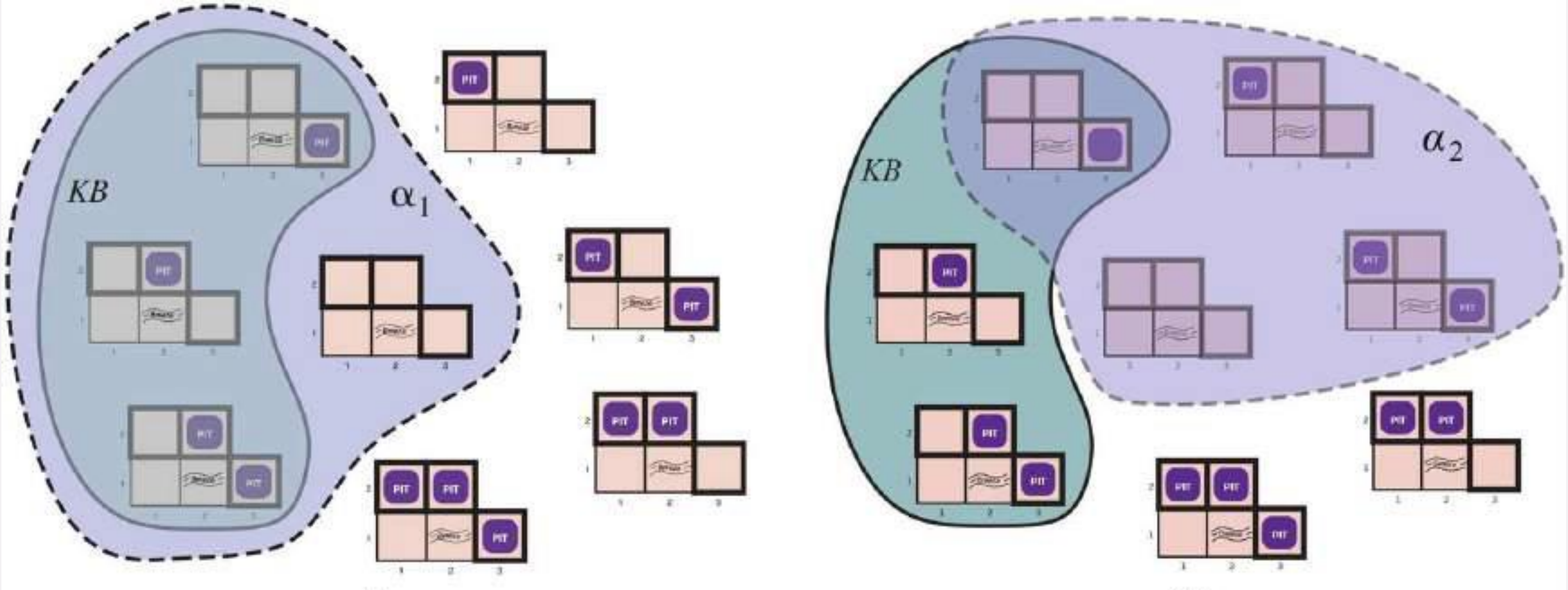
$$\alpha \models \beta$$

- The formal definition of entailment is this: $\alpha \models \beta$ if and only if, in every model in which α is true, β is also true.

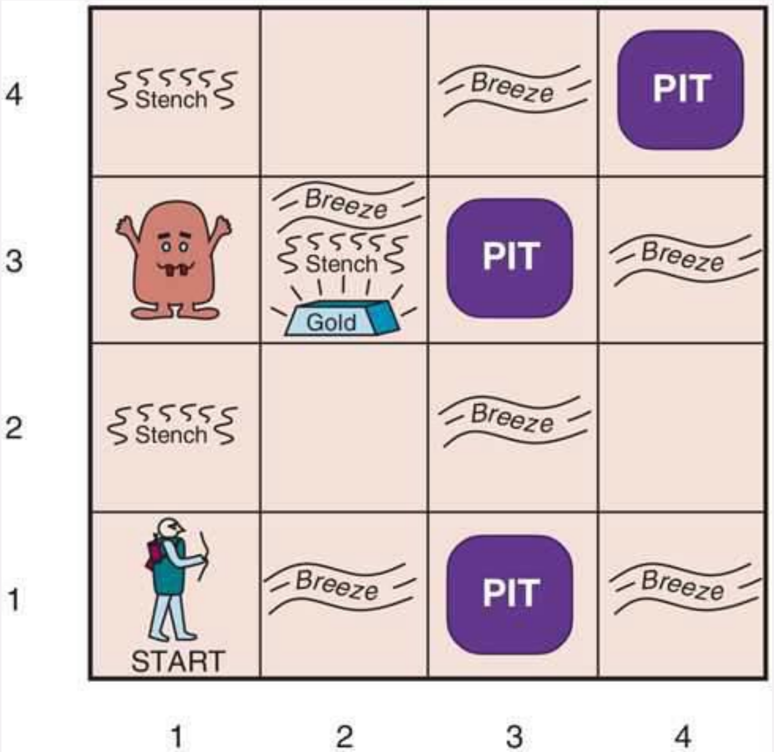
$$\alpha \models \beta \text{ if and only if } M(\alpha) \subseteq M(\beta)\,.$$

Logical Inference

- α1 = “There is no pit in [1,2].”
- α2 = “There is no pit in [2,2].”



(a) (b)



- By inspection:
- In every model in which KB is true, α1 is also true. Hence $KB \models \alpha_1$
- Conclusion: There is no pit in [1,2].
- In some models in which KB is true, α2 is false. Hence, KB does not entail α2
- Agent cannot conclude that there is no pit in [2,2].

Propositional Logic

$$
\begin{array}{rcl}
\textit{Sentence} & \rightarrow & \textit{AtomicSentence} \mid \textit{ComplexSentence} \\
\textit{AtomicSentence} & \rightarrow & \textit{True} \mid \textit{False} \mid P \mid Q \mid R \mid \ldots \\
\textit{ComplexSentence} & \rightarrow & (\,\textit{Sentence}\,) \\
 & \mid & \neg\,\textit{Sentence} \\
 & \mid & \textit{Sentence} \land \textit{Sentence} \\
 & \mid & \textit{Sentence} \lor \textit{Sentence} \\
 & \mid & \textit{Sentence} \;\Rightarrow\; \textit{Sentence} \\
 & \mid & \textit{Sentence} \;\Leftrightarrow\; \textit{Sentence} \\
 & & \\
\textsc{Operator Precedence} & : & \neg, \land, \lor, \Rightarrow, \Leftrightarrow
\end{array}
$$

Semantics

- The semantics defines the rules for determining the truth of a sentence with respect to a particular model.

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	*false*	*true*	*false*	*false*	*true*	*true*
false	*true*	*true*	*false*	*true*	*true*	*false*
true	*false*	*false*	*false*	*true*	*false*	*false*
true	*true*	*false*	*true*	*true*	*true*	*true*

- $\neg P$ is true iff P is false in m.
- $P \wedge Q$ is true iff both P and Q are true in m.
- $P \vee Q$ is true iff either P or Q is true in m.
- $P \Rightarrow Q$ is true unless P is true and Q is false in m.
- $P \Leftrightarrow Q$ is true iff P and Q are both true or both false in m.

Simple Knowledge Base

- There is no pit in [1,1]:

$$R_1 : \quad \neg P_{1,1} .$$

- A square is breezy if and only if there is a pit in a neighboring square:

$$\begin{aligned} R_2 &: \quad B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1}) . \\ R_3 &: \quad B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1}) . \end{aligned}$$

- Breeze percepts for the first two squares:

$$\begin{aligned} R_4 &: \quad \neg B_{1,1} . \\ R_5 &: \quad B_{2,1} . \end{aligned}$$

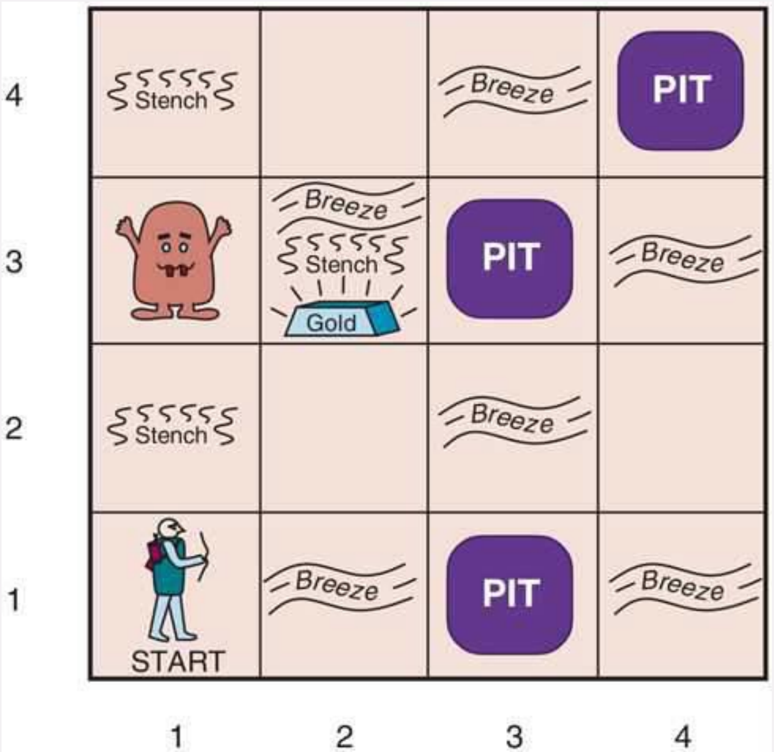
$P_{x,y}$ is true if there is a pit in $[x,y]$.

$W_{x,y}$ is true if there is a wumpus in $[x,y]$, dead or alive.

$B_{x,y}$ is true if there is a breeze in $[x,y]$.

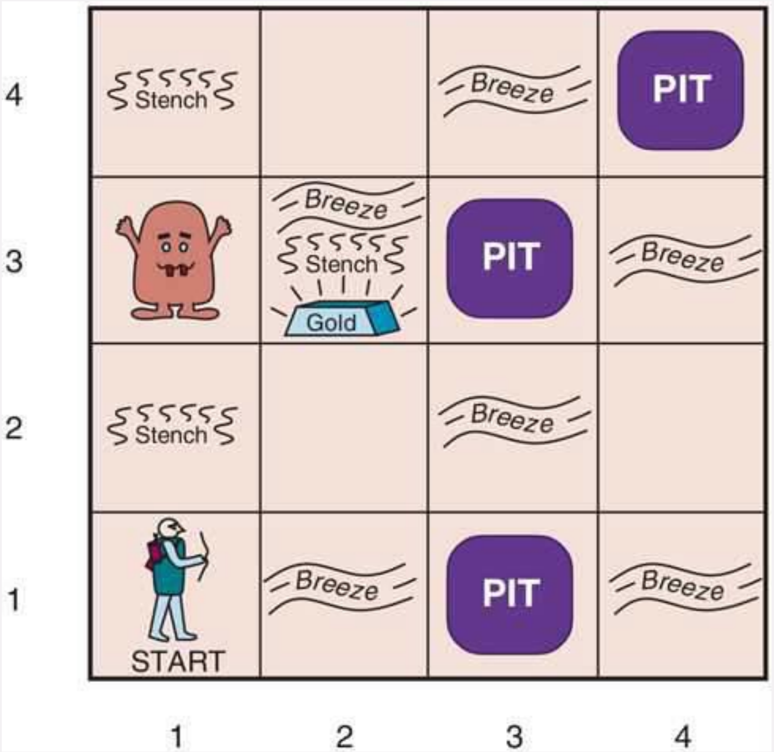
$S_{x,y}$ is true if there is a stench in $[x,y]$.

$L_{x,y}$ is true if the agent is in location $[x,y]$.



Simple Inference Procedure

- Is $\neg P_{1,2}$ entailed by our KB?



$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*