②

ML	85	90	93	65	87	71	98	68	84	87
HUR	82	88	96	72	91	80	95	72	89	84

(a) let ML be the independent variable

let $h_w = W_0 + W_1 x$

$$J(W) = \frac{1}{2m}\sum_{i=1}^{m}(h_w(x_i) - y_i)^2 \qquad m = 10$$

$$J(W) = \frac{1}{2\times 10}\sum_{i=1}^{10}\left(W_0 + W_1 x_i - y_i\right)^2$$

on solving $\frac{\partial J(w)}{\partial W_0} = 0$ and $\frac{\partial J(w)}{\partial W_1} = 0$

we get $W_0 = \frac{BC - AD}{Cm - A^2}$, $W_1 = \frac{AB - Dm}{A^2 - Cm}$

where, $A = \sum x_i$ $\quad B = \sum y_i$

$C = \sum x_i^2$ $\quad D = \sum x_i y_i$

$A = 828$ $\quad B = 849$

$C = 69662$ $\quad D = 71085$

$W_0 = 25.7936$ $\quad W_1 = 0.713846$

least square fitting line comes out to be

$$h_w(x) = 25.7936 + (0.713846)x \quad \text{—} (1)$$

(b) let HUR be the independent variable

let $h'_w(x') = w'_0 + w'_1 x'$

$$J'(w') = \frac{1}{2m} \sum_{i=1}^{m} \left(h'_w(x'_i) - y'_i \right)^2$$

$$J'(w') = \frac{1}{2m} \sum_{i=1}^{m} \left(w'_0 + w'_1 x'_i - y'_i \right)^2$$

on solving $\frac{\partial J(w')}{\partial w'_0} = 0$ and $\frac{\partial J(w')}{\partial w'_1} = 0$

we get $w'_0 = \frac{B'C' - A'D'}{C'm - (A')^2}$, $w'_1 = \frac{A'B' - D'm}{(A')^2 - C'm}$

where $A' = \sum x'_i$ $\quad B' = \sum y'_i$

$C' = \sum (x'_i)^2$ $\quad D' = \sum x'_i y'_i$

$A' = 849$ $\quad B' = 828$

$C' = 72735$ $\quad D' = 71085$

$\therefore$ $w'_0 = -19.3289$ $\quad \therefore w'_1 = 1.20293$

So, the least square fitting lines comes out to be

$$h_{w'}(x') = -19.3289 + (1.20293)x' \quad \text{—— (2)}$$

(c) Marks in ML = 96

using equation (1)

Expected marks in HUR = $h_w(96)$

$= 25.7936 + (0.713846) \times 96$

$= 94.322816$

(d) Marks in HUR = 95

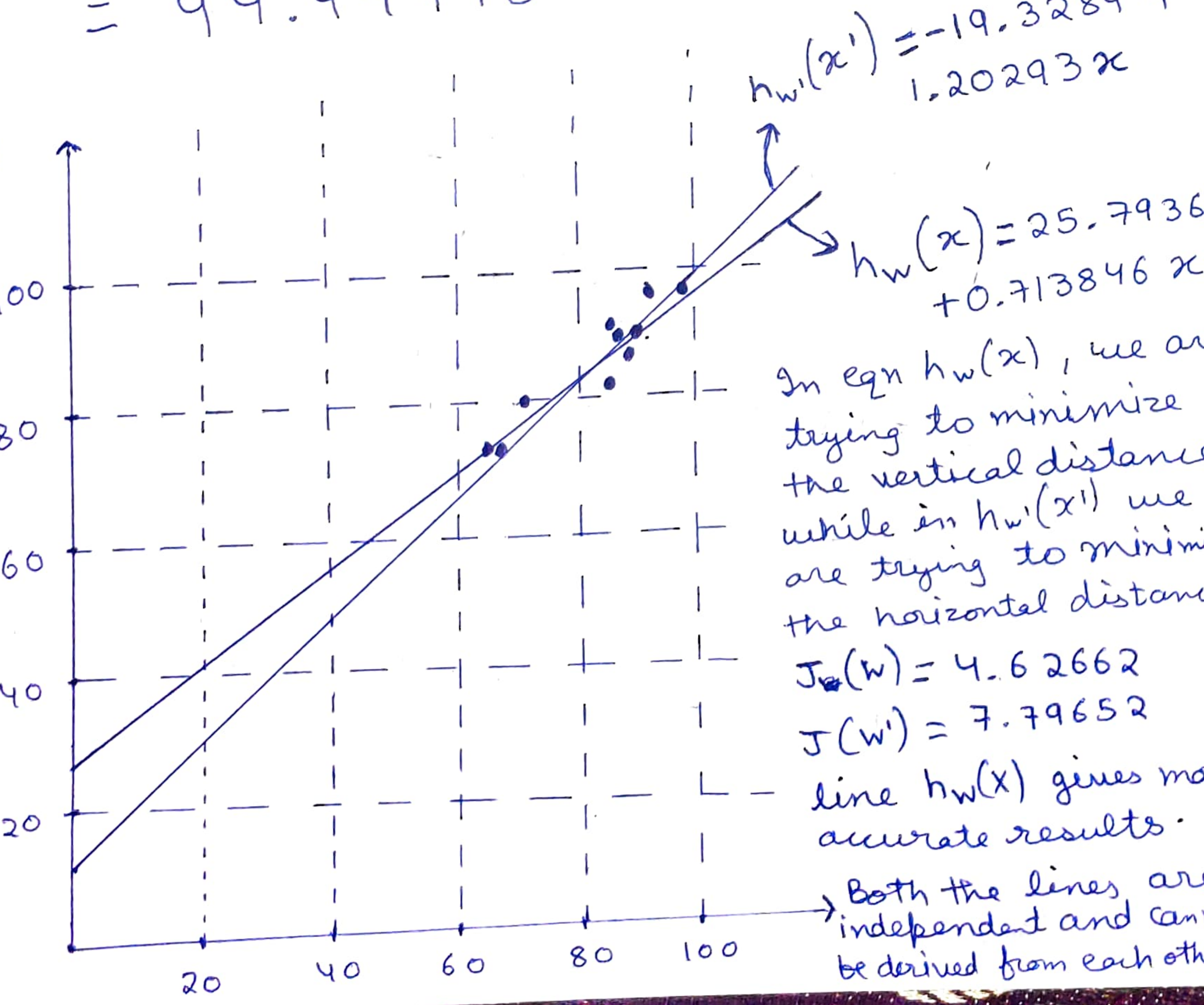
using equation (2)

Expected marks in ML = $h_{w'}(95)$

$= -19.3289 + (1.20293) \times 95$

$= 94.94945$

(e)



In eqn $h_w(x)$, we are trying to minimize the vertical distance while in $h_{w'}(x')$ we are trying to minimize the horizontal distance

$J(w) = 4.62662$

$J(w') = 7.79652$

line $h_w(x)$ gives more accurate results.

Both the lines are independent and cannot be derived from each other