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Years	2006	2008	2009	2011	2013	2014	2015	2016	2017	2018	2019
Rev. in Billion Rupees	100.2	98.3	87.1	89.2	88.9	83.5	89.1	84	92.3	96	97

(a) let $h_w(x) = W_0 + W_1 x$

$$J(W_0, W_1) = \sum_{i=1}^{m} \frac{1}{2m} \left(h_w(x_i) - y_i\right)^2, \quad m = 11$$

$$J(W) = \frac{1}{2m} \sum_{i=1}^{m} \left(W_0 + W_1 x_i - y_i\right)^2$$ [we have to minimize the $J(W)$]

$$= \frac{1}{2m}\left(\left(W_0 + W_1 x_1 - y_1\right)^2 + \cdots + \left(W_0 + W_1 x_m - y_m\right)^2\right)$$

finding the derivatives wrt w_0, w_1

$$\frac{\partial J}{\partial W_0} = \frac{1}{2m}\left(2\left(W_0 + W_1 x_1 - y_1\right)(1+0-0) + \cdots + 2\left(W_0 + W_1 x_m - y_m\right)(1)\right)$$

$$\frac{\partial J}{\partial W_1} = \frac{1}{2m}\left(2\left(W_0 + W_1 x_1 - y_1\right)(x_1) + \cdots + 2\left(W_0 + W_1 x_m - y_m\right)(x_m)\right)$$

simplifying

$$\frac{\partial J}{\partial W_0} = \frac{1}{m}\left(W_0 \cdot m + W_1\left(\sum_{i=1}^{m} x_i\right) - \sum_{i=1}^{m} y_i\right)$$

$$\frac{\partial J}{\partial W_1} = \frac{1}{m}\left(W_0 \cdot \sum_{i=1}^{m} x_i + W_1\left(\sum_{i=1}^{m} x_i^2\right) - \sum_{i=1}^{m} (x_i y_i)\right)$$

[$\sum x_i = 22146$, $\sum x_i^2 = 44586122$, $\sum y_i = 1005.6$]

putting $\frac{\partial J}{\partial w_0} = 0$ and $\frac{\partial J}{\partial w_1} = 0$

$\left(\frac{\partial J}{\partial w_0}\right)$: $\frac{1}{m}\left(w_0 m + w_1 \sum x_i - \sum y_i\right) = 0$

$\Rightarrow w_0 m + w_1 (22446) - 1005.6 = 0$

$\Rightarrow w_0 (11) + w_1 (22446) - 1005.6 = 0 \quad (1)$

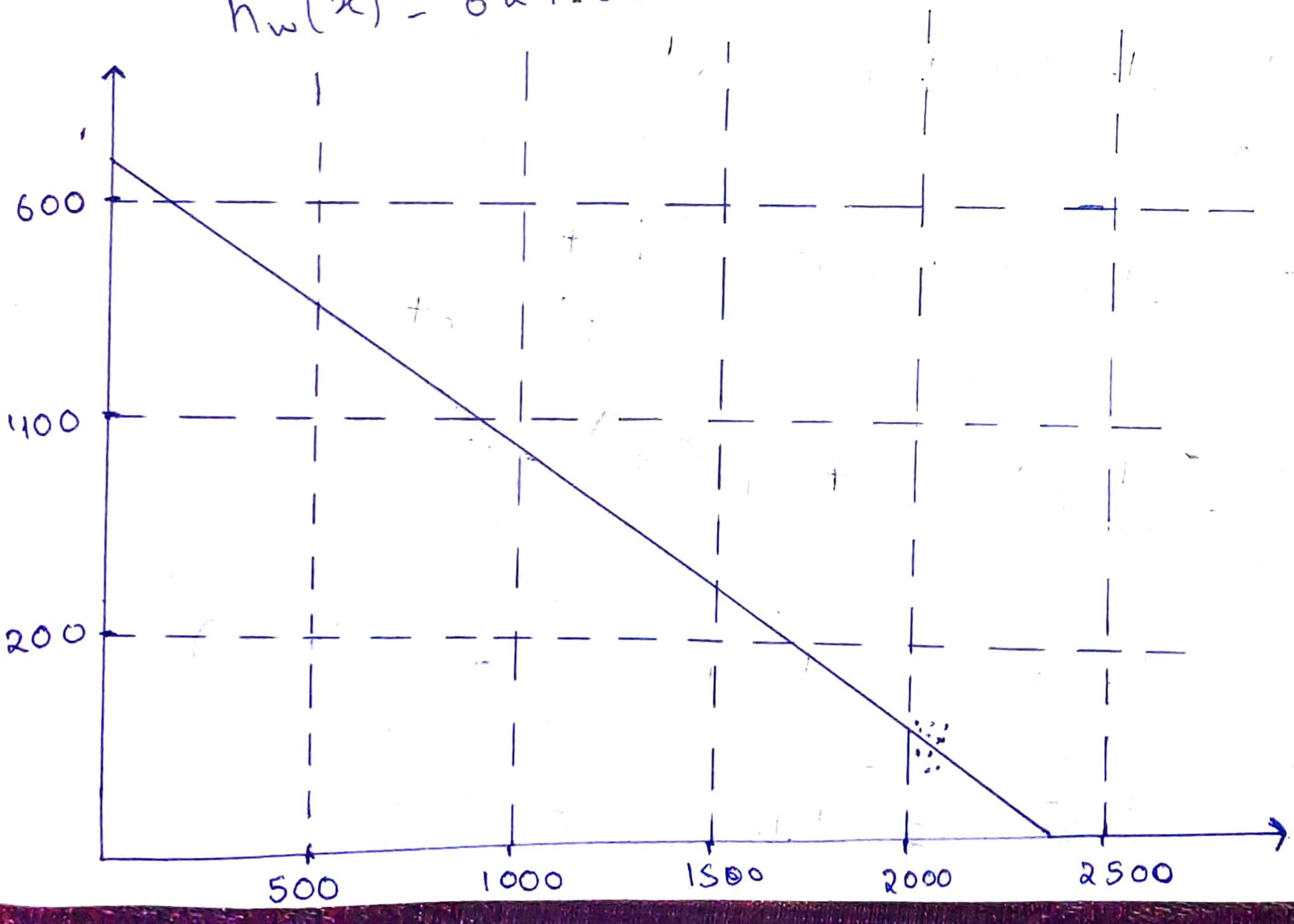
$\left(\frac{\partial J}{\partial w_1}\right)$: $\frac{1}{m}\left(w_0 (22446) + w_1 (44586122) - 2024498\right) = 0$

$\Rightarrow (22446) w_0 + w_1 (44586122) - 2024498 = 0 \quad (2)$

solving the eqns (1) & (2) we get

$w_0 = 627.63 \qquad w_1 = -0.26634$

$h_w(x) = 627.63 - 0.26634x$



(b) Expected value for 2021 is

$$h_w(2021) = 627.63 + (-0.26634)(2021)$$

$$= 89.35 \text{ billion ruppees}$$

(c) Error $= J(w)$

$$= \frac{1}{2m}\sum\left(h_w(x_i) - y_i\right)^2$$

$$= \frac{1}{2m}\sum\left(w_0 + w_1 x_i - y_i\right)^2$$

$$= \frac{1}{2m}\sum\left(w_0^2 + \omega_1^2 x_i^2 + y_i^2 - 2y_i w_0 - 2\omega_1 x_i y_i + 2\omega_0\omega_1 x_i\right)$$

$$= \frac{1}{2m}\left(m\omega_0^2 + \omega_1^2\sum x_i^2 + \sum y_i^2 - 2\omega_0\sum y_i - 2\omega_1\sum x_i y_i + 2\omega_0\omega_1\sum x_i\right)$$

$$= \frac{1}{2m}\left((393919.41)\times m + (0.0709)(44586122) + 92260.54 - 2\times 627.63\times(1005.6) + 2(0.26634)(2024498) - 2(627.63)(0.26634)(22146)\right)$$

$$= \frac{1}{22}\times\left(4333113.51 + 3161156.04 + 92260.54 - 1262289.456 + 1078409.594 - 7403982.45\right)$$

$$\approx 14.4251$$