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Years	2006	2008	2009	2011	2013	2014	2015	2016	2017	2018	2019
Rev. in Billion Ruppees	100.2	98.3	87.1	89.2	88.9	83.5	89.1	84	92.3	96	97

(a) let $h_w(x) = W_0 + W_1 x$

$$J(W_0, W_1) = \sum_{i=1}^{m} \frac{1}{2m} \left(h_w(x_i) - y_i \right)^2, \quad m = 11$$

$$J(W) = \frac{1}{2m} \sum_{i=1}^{m} \left(W_0 + W_1 x_i - y_i \right)^2$$

[We have to minimize the $J(W)$]

$$= \frac{1}{2m} \left((W_0 + W_1 x_1 - y_1)^2 + \cdots \cdots + (W_0 + W_1 x_m - y_m)^2 \right)$$

finding the derivatives wrt W_0, W_1

$$\frac{\partial J}{\partial W_0} = \frac{1}{2m} \left(2(W_0 + W_1 x_1 - y_1)(1 + 0 - 0) + \cdots + 2(W_0 + W_1 x_m - y_m)(1) \right)$$

$$\frac{\partial J}{\partial W_1} = \frac{1}{2m} \left(2(W_0 + W_1 x_1 - y_1)(x_1) + \cdots + 2(W_0 + W_1 x_m - y_m)(x_m) \right)$$

simplifying

$$\frac{\partial J}{\partial W_0} = \frac{1}{m} \left(W_0 \cdot m + W_1 \left(\sum_{i=1}^{m} x_i \right) - \sum_{i=1}^{m} y_i \right)$$

$$\frac{\partial J}{\partial W_1} = \frac{1}{m} \left(W_0 \cdot \sum_{i=1}^{m} x_i + W_1 \left(\sum_{i=1}^{m} x_i^2 \right) - \sum_{i=1}^{m} (x_i y_i) \right)$$

$\left[\sum x_i = 22146, \ \sum x_i^2 = 44586122, \ \sum y_i = 1005.6 \right]$

putting $\frac{\partial J}{\partial w_0} = 0$ and $\frac{\partial J}{\partial w_1} = 0$

$\left(\frac{\partial J}{\partial w_0}\right)$: $\frac{1}{m}\left(W_0 m + w_1 \sum x_i - \sum y_i\right) = 0$

$\Rightarrow W_0 m + W_1 (22446) - 1005.6 = 0$

$\Rightarrow W_0(11) + W_1(22446) - 1005.6 = 0 \quad (1)$

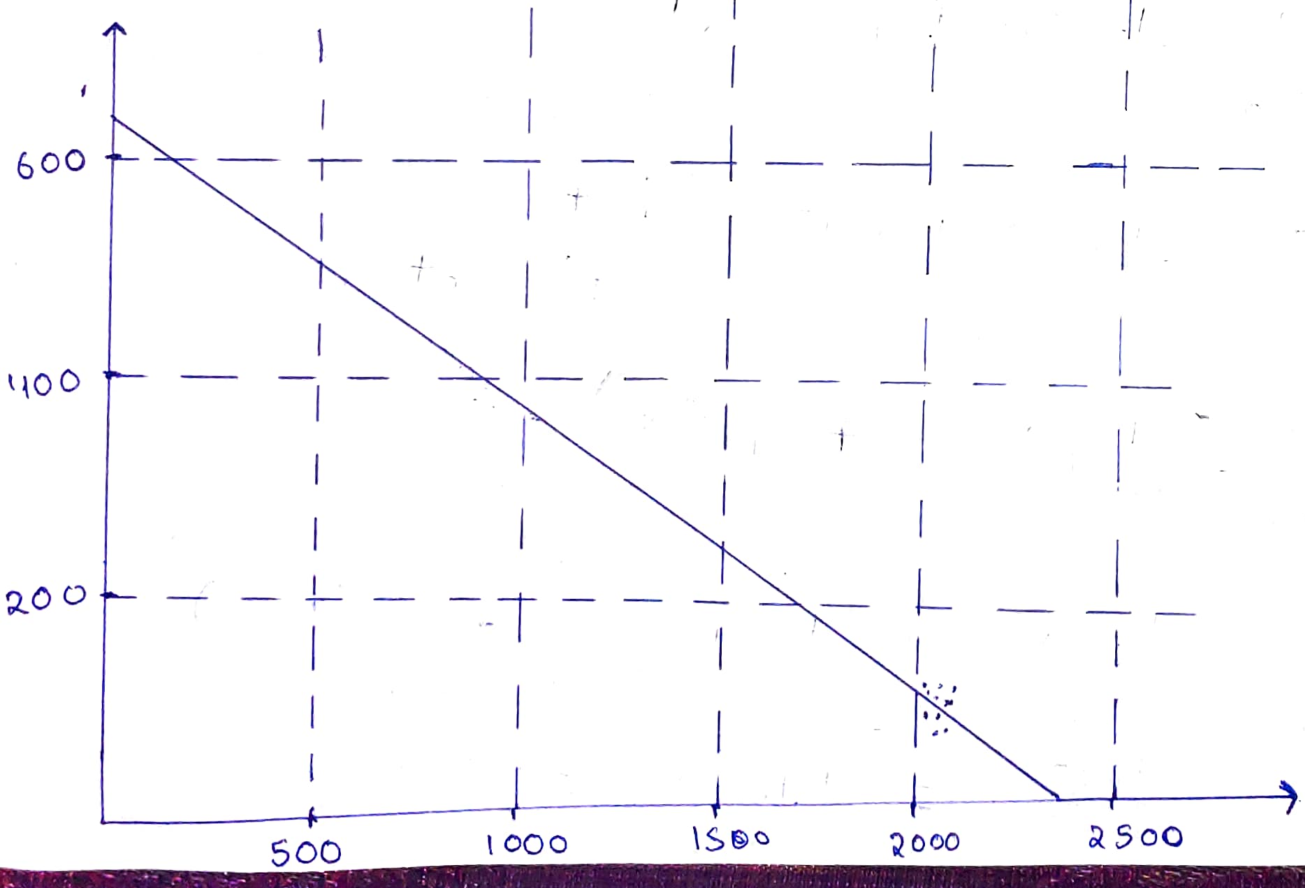
$\left(\frac{\partial J}{\partial w_1}\right)$: $\frac{1}{m}\left(W_0(22446) + W_1(44586122) - 2024498\right) = 0$

$\Rightarrow (22446) W_0 + W_1 (44586122) - 2024498 = 0 \quad (2)$

solving the eqns (1) & (2) we get

$W_0 = 627.63 \qquad W_1 = -0.26634$

$h_w(x) = 627.63 - 0.26634x$



(b) Expected value for 2021 is

$$h_w(2021) = 627.63 + (-0.26634)(2021)$$

$$= 89.35 \text{ billion ruppees}$$

(c) Error $= J(w)$

$$= \frac{1}{2m}\sum\left(h_w(x_i) - y_i\right)^2$$

$$= \frac{1}{2m}\sum\left(w_0 + w_1 x_i - y_i\right)^2$$

$$= \frac{1}{2m}\sum\left(w_0^2 + \omega_1^2 x_i^2 + y_i^2 - 2y_i w_0 - 2\omega_1 x_i y_i + 2\omega_0\omega_1 x_i\right)$$

$$= \frac{1}{2m}\left(m\omega_0^2 + \omega_1^2\sum x_i^2 + \sum y_i^2 - 2\omega_0\sum y_i - 2\omega_1\sum x_i y_i + 2\omega_0\omega_1\sum x_i\right)$$

$$= \frac{1}{2m}\left((393919.41)\times m + (0.0709)(44586122) + 92260.54 - 2\times 627.63\times(1005.6) + 2(0.26634)(2024498) - 2(627.63)(0.26634)(22146)\right)$$

$$= \frac{1}{22}\times\left(4333113.51 + 3161156.04 + 92260.54 - 1262289.456 + 1078409.594 - 7403982.45\right)$$

$$\simeq 14.4251$$

②

ML	85	90	93	65	87	71	98	68	84	87
HUR	82	88	96	72	91	80	95	72	89	84

(a) let ML be the independent variable

let $h_w = W_0 + W_1 x$

$$J(W) = \frac{1}{2m} \sum_{i=1}^{m} (h_w(x_i) - y_i)^2 \qquad m = 10$$

$$J(W) = \frac{1}{2 \times 10} \sum_{i=1}^{10} \left(W_0 + W_1 x_i - y_i \right)^2$$

on solving $\frac{\partial J(w)}{\partial W_0} = 0$ and $\frac{\partial J(w)}{\partial W_1} = 0$

we get $W_0 = \frac{BC - AD}{Cm - A^2}$, $W_1 = \frac{AB - Dm}{A^2 - Cm}$

where, $A = \sum x_i$ $\quad B = \sum y_i$

$C = \sum x_i^2$ $\quad D = \sum x_i y_i$

$A = 828$ $\quad B = 849$

$C = 69662$ $\quad D = 71085$

$W_0 = 25.7936$ $\quad W_1 = 0.713846$

least square fitting line comes out to be

$$h_w(x) = 25.7936 + (0.713846)x \quad \text{— (1)}$$

(b) let HUR be the independent variable

let $h'_w(x') = w'_0 + w'_1 x'$

$$J'(w') = \frac{1}{2m} \sum_{i=1}^{m} \left(h'_w(x'_i) - y'_i \right)^2$$

$$J'(w') = \frac{1}{2m} \sum_{i=1}^{m} \left(w'_0 + w'_1 x'_i - y'_i \right)^2$$

on solving $\frac{\partial J(w')}{\partial w'_0} = 0$ and $\frac{\partial J(w')}{\partial w'_1} = 0$

we get $w'_0 = \frac{B'C' - A'D'}{C'm - (A')^2}$, $w'_1 = \frac{A'B' - D'm}{(A')^2 - C'm}$

where $A' = \sum x'_i$ $\quad B' = \sum y'_i$

$C' = \sum (x'_i)^2$ $\quad D' = \sum x'_i y'_i$

$A' = 849$ $\quad B' = 828$

$C' = 72735.$ $\quad D' = 71085$

$\therefore$

$w'_0 = -19.3289$ $\quad \therefore w'_1 = 1.20293$

So, the least square fitting lines comes out to be

$$h_{w'}(x') = -19.3289 + (1.20293)x' \quad \text{--- (2)}$$

(c) Marks in ML = 96

using equation (1)

Expected marks in HUR = $h_w(96)$

$= 25.7936 + (0.713846) \times 96$

$= 94.322816$

(d) Marks in HUR = 95

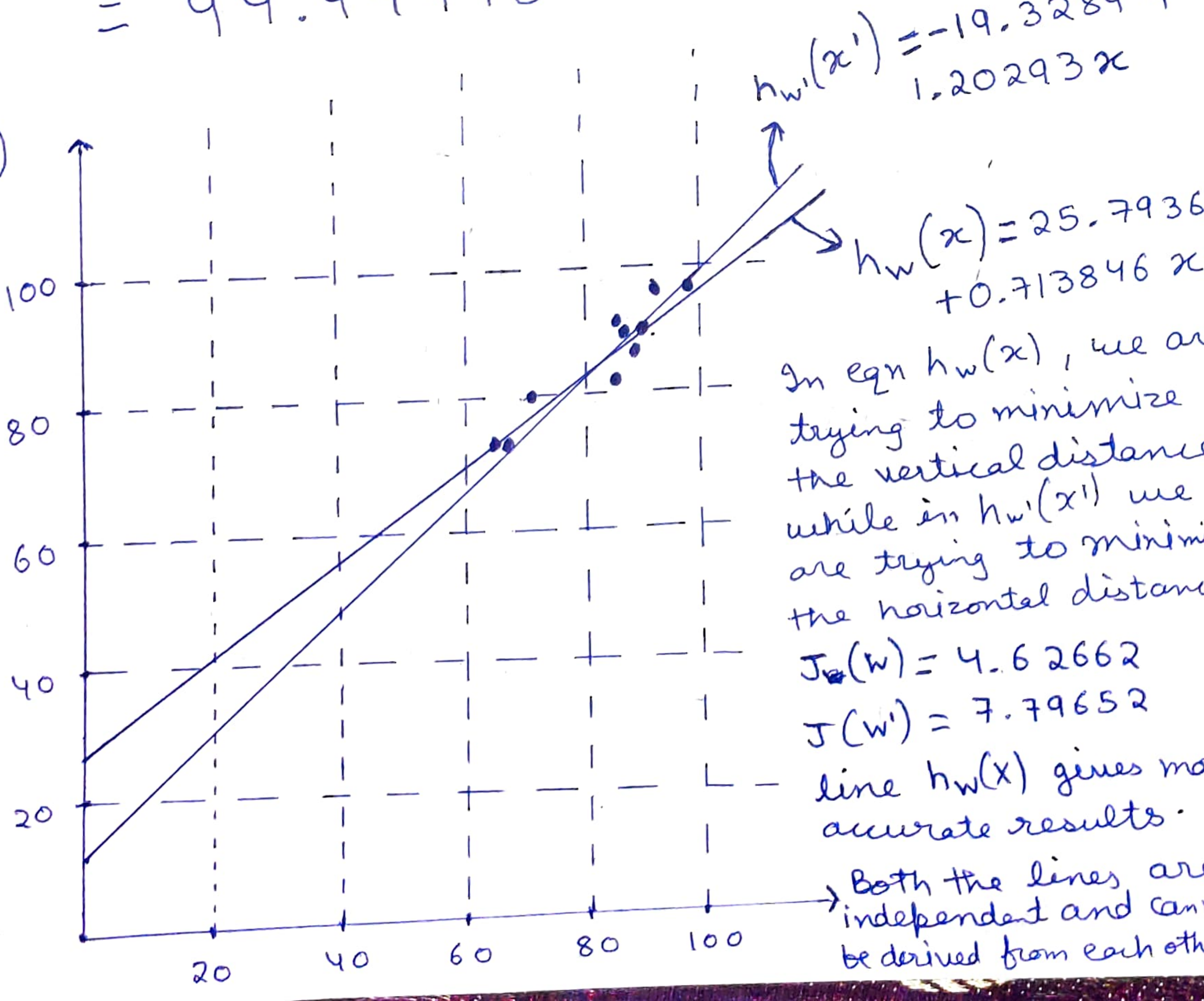
using equation (2)

Expected marks in ML = $h_{w'}(95)$

$= -19.3289 + (1.20293) \times 95$

$= 94.94945$

(e)



In eqn $h_w(x)$, we are trying to minimize the vertical distance while in $h_{w'}(x')$ we are trying to minimize the horizontal distance

$J(w) = 4.62662$

$J(w') = 7.79652$

line $h_w(x)$ gives more accurate results.

Both the lines are independent and cannot be derived from each other

(3)

v	54.3	61.8	72.4	88.7	118.6	194
p	61.2	49.5	37.5	28.4	19.2	10.1

given : $P V^n = c$

(a) taking log on both sides

$$\log(P V^n) = \log(c) = K \text{ (constant)}$$

$$\log(P) + n\log(V) = K$$

$$\log(P) = K - n\log(V)$$

so, let $h(V) = K - n\log(V)$

let $\log V = V'$

$$h(V') = K - nV'$$

comparing with $W_0 + W_1 x$

$$J = \frac{1}{2m}\sum_{i=1}^{m}\left(K + n'V' - P'\right)^2$$

where $P' = \log P$

$V' = \log V$

$n' = -n$

on solving $\frac{\partial J}{\partial K} = 0$ and $\frac{\partial J}{\partial n'} = 0$

we get, $K = 9.67592 \Rightarrow \log(c) = 9.67592$

$n' = -1.40371 \Rightarrow n = 1.40371$

$\Rightarrow n = 1.40371$, $c = 15929.3723241$

(b) So, $P \cdot V^{1.40371} = 15929.3723241$ —(1)

is the equation connecting P & V

(c) given $V = 100$

using equation 1

$$P = \frac{15929.3723241}{(100)^{1.40371}} \simeq 24.81867$$

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X	0	1	2	3	4	5	6
Y	2.4	2.1	3.2	5.6	9.3	14.6	21.9

$$Y = W_0 + W_1 x + W_2 x^2$$

$$J = \frac{1}{2m} \sum_{i=1}^{m} \left(\omega_0 + \omega_1 x_i + \omega_2 x_i^2 - y_i \right)^2$$

$$\frac{\partial J}{\partial \omega_0} = \frac{1}{2m} \sum_{i=1}^{m} 2\left(\omega_0 + \omega_1 x_i + \omega_2 x_i^2 - y_i\right)(1)$$

$$\frac{\partial J}{\partial \omega_1} = \frac{1}{2m} \sum_{i=1}^{m} 2\left(\omega_0 + \omega_1 x_i + \omega_2 x_i^2 - y_i\right)(x_i)$$

$$\frac{\partial J}{\partial \omega_2} = \frac{1}{2m} \sum_{i=1}^{m} 2\left(\omega_0 + \omega_1 x_i + \omega_2 x_i^2 - y_i\right)(x_i^2)$$

Solving $\frac{\partial J}{\partial \omega_0} = 0$, $\frac{\partial J}{\partial \omega_1} = 0$, $\frac{\partial J}{\partial \omega_2} = 0$

$$\frac{\partial J}{\partial \omega_0} = \frac{1}{m}\left(\omega_0 m + \omega_1 \sum x_i + \omega_2 \sum x_i^2 - \sum y_i\right) = 0$$

$$\frac{\partial J}{\partial \omega_1} = \frac{1}{m}\left(\omega_0 \sum x_i + \omega_1 \sum x_i^2 + \omega_2 \sum x_i^3 - \sum y_i x_i\right) = 0$$

$$\frac{\partial J}{\partial \omega_2} = \frac{1}{m}\left(\omega_0 \sum x_i^2 + \omega_1 \sum x_i^3 + \omega_2 \sum x_i^4 - \sum y_i x_i^2\right) = 0$$

$$\omega_0 \cdot (7) + \omega_1 \cdot (21) + \omega_2 (91) - 59.1 = 0 \quad \text{—(1)}$$

$$\omega_0 (21) + \omega_1 (91) + \omega_2 (441) - 266.9 = 0 \quad \text{—(2)}$$

$$\omega_0 (91) + \omega_1 (441) + \omega_2 (2275) - 1367.5 = 0 \quad \text{—(3)}$$

~~so~~ solving equations (1), (2) and (3), we get

$\omega_0 = 2.5095$, $\omega_1 = -1.2$, $z = 0.73334$

equation comes out to be

$$2.5095 - 1.2x + 0.73334x^2 = 0$$