

1

$$\sin^2 \alpha + \cos^2 \alpha = 1 \quad (1.1)$$

$$1 + \tan^2 \alpha = \sec^2 \alpha \quad (1.2)$$

$$1 + \cot^2 \alpha = \csc^2 \alpha \quad (1.3)$$

$$\arctan x + \arctan \frac{1}{x} = \begin{cases} \frac{\pi}{2}, & x > 0 \\ -\frac{\pi}{2}, & x < 0 \end{cases} \quad (1.4)$$

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta \quad (1.5)$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta \quad (1.6)$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta} \quad (1.7)$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha \quad (1.8)$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = 2 \cos^2 \alpha - 1 = 1 - 2 \sin^2 \alpha \quad (1.9)$$

$$\sin \frac{\alpha}{2} = \sqrt{\frac{1 - \cos \alpha}{2}} \quad (1.10)$$

$$\cos \frac{\alpha}{2} = \sqrt{\frac{1 + \cos \alpha}{2}} \quad (1.11)$$

$$\tan \frac{\alpha}{2} = \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}} = \frac{\sin \alpha}{1 + \cos \alpha} = \frac{1 - \cos \alpha}{\sin \alpha} \quad (1.12)$$

$$\sin \alpha = \frac{2 \tan \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}} \quad (1.13)$$

$$\cos \alpha = \frac{1 - \tan^2 \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}} \quad (1.14)$$

2

$$(C)' = 0 \quad (2.1)$$

$$(x^\mu)' = \mu x^{\mu-1} \quad (2.2)$$

$$(a^x)' = a^x \ln a \quad (2.3)$$

$$(e^x)' = e^x \quad (2.4)$$

$$(\log_a x)' = \frac{1}{x \ln a} \quad (2.5)$$

$$(\ln x)' = \frac{1}{x} \quad (2.6)$$

$$(\sin x)' = \cos x \quad (2.7)$$

$$(\cos x)' = -\sin x \quad (2.8)$$

$$(\tan x)' = \frac{1}{\cos^2 x} = \sec^2 x \quad (2.9)$$

$$(\cot x)' = -\frac{1}{\sin^2 x} = -\csc^2 x \quad (2.10)$$

$$(\sec x)' = \sec x \tan x \quad (2.11)$$

$$(\csc x)' = -\csc x \cot x \quad (2.12)$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (2.13)$$

$$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (2.14)$$

$$(\arctan x)' = \frac{1}{1+x^2} \quad (2.15)$$

$$(\operatorname{arccot} x)' = -\frac{1}{1+x^2} \quad (2.16)$$

3

$$\sin x = x - \frac{x^3}{6} + o(x^4) \quad (3.1)$$

$$\arcsin x = x + \frac{x^3}{6} + o(x^4) \quad (3.2)$$

$$\tan x = x + \frac{x^3}{3} + o(x^4) \quad (3.3)$$

$$\arctan x = x - \frac{x^3}{3} + o(x^4) \quad (3.4)$$

4

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + o(x^n) \quad (4.1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^m \frac{x^{2m+1}}{(2m+1)!} + o(x^{2m+1}) \quad (4.2)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^m \frac{x^{2m}}{(2m)!} + o(x^{2m}) \quad (4.3)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{n-1} \frac{x^n}{n} + o(x^n) \quad (4.4)$$

$$\frac{1}{1+x} = 1 - x + x^2 - \cdots + (-1)^n x^n + o(x^n) \quad (4.5)$$

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots + x^n + o(x^n) \quad (4.6)$$

$$(1+x)^\mu = 1 + \mu x + \frac{\mu(\mu-1)}{2!} x^2 + \cdots + \frac{\mu(\mu-1) \cdots (\mu-n+1)}{n!} x^n + o(x^n) \quad (4.7)$$

5

$$k = \frac{|y''|}{(1+y'^2)^{\frac{3}{2}}} \quad (5.1)$$

$$k = \frac{1}{R} \quad (5.2)$$

$$\begin{cases} \xi = x - \frac{y'(1+y'^2)}{y''} \\ \eta = y + \frac{(1+y'^2)}{y''} \end{cases} \quad (5.3)$$

6

$$\int 0 \, dx = C \quad (6.1)$$

$$\int 1 \, dx = x + C \quad (6.2)$$

$$\int x^\mu \, dx = \frac{1}{\mu+1} x^{\mu+1} + C \quad (6.3)$$

$$\int \frac{1}{x} \, dx = \ln|x| + C \quad (6.4)$$

$$\int a^x \, dx = \frac{a^x}{\ln a} + C \quad (6.5)$$

$$\int e^x \, dx = e^x + C \quad (6.6)$$

$$\int \sin x \, dx = -\cos x + C \quad (6.7)$$

$$\int \cos x \, dx = \sin x + C \quad (6.8)$$

$$\int \sec^2 x \, dx = \int \frac{1}{\cos^2 x} \, dx = \tan x + C \quad (6.9)$$

$$\int \csc^2 x \, dx = \int \frac{1}{\sin^2 x} \, dx = -\cot x + C \quad (6.10)$$

$$\int \sec x \tan x \, dx = \sec x + C \quad (6.11)$$

$$\int \csc x \cot x \, dx = -\csc x + C \quad (6.12)$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C \quad (6.13)$$

$$\int \frac{dx}{1+x^2} = \arctan x + C \quad (6.14)$$

7

$$\int \tan x \, dx = -\ln|\cos x| + C \quad (7.1)$$

$$\int \cot x \, dx = \ln|\sin x| + C \quad (7.2)$$

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C \quad (7.3)$$

$$\int \csc x \, dx = \ln|\csc x - \tan x| + C \quad (7.4)$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a} \arctan \frac{x}{a} + C \quad (7.5)$$

$$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C \quad (7.6)$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C \quad (7.7)$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left| x + \sqrt{x^2 \pm a^2} \right| + C \quad (7.8)$$

$$\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \arcsin \frac{x}{a} + C \quad (7.9)$$

$$\int \sqrt{x^2 \pm a^2} dx = \frac{x}{2} \sqrt{x^2 \pm a^2} \pm \frac{a^2}{2} \ln \left| x + \sqrt{x^2 \pm a^2} \right| + C \quad (7.10)$$

8

当 $p \neq 1$, 敛散与 \ln 无关
等号跟着发散走

$$\int_1^{+\infty} \frac{1}{x^p} dx \begin{cases} \text{收敛,} & p > 1 \\ \text{发散,} & p \leq 1 \end{cases} \quad (8.1)$$

$$\int_0^1 \frac{1}{x^q} dx \begin{cases} \text{收敛,} & 0 < q < 1 \\ \text{发散,} & q \geq 1 \end{cases} \quad (8.2)$$

9

$$S = \int_a^b y(x) dx \quad (9.1)$$

$$S = \int_{t_1}^{t_2} y(t) x'(t) dt \quad (9.2)$$

$$S = \frac{1}{2} \int_{\alpha}^{\beta} r^2(\theta) d\theta \quad (9.3)$$

$$V = \int_a^b S(x) dx \quad (9.4)$$

$$V_x = \pi \int_a^b y^2(x) dx \quad (9.5)$$

$$V_x = \pi \int_{t_1}^{t_2} y^2(t) x'(t) dt \quad (9.6)$$

$$V_y = 2\pi \int_a^b |x| |y(x)| dx \quad (9.7)$$

$$V_y = 2\pi \int_{t_1}^{t_2} |x(t)| |y(t)| x'(t) dt \quad (9.8)$$

$$s = \int_a^b \sqrt{1 + [y'(x)]^2} dx \quad (9.9)$$

$$s = \int_{t_{\text{小}}}^{t_{\text{大}}} \sqrt{[x'(t)]^2 + [y'(t)]^2} dt \quad (9.10)$$

$$s = \int_{\alpha}^{\beta} \sqrt{[r(\theta)]^2 + [r'(\theta)]^2} d\theta \quad (9.11)$$

$$S = 2\pi \int_a^b |y(x)| \sqrt{1 + [y'(x)]^2} dx \quad (9.12)$$

$$S = 2\pi \int_{t_{\text{小}}}^{t_{\text{大}}} |y(t)| \sqrt{[x'(t)]^2 + [y'(t)]^2} dt \quad (9.13)$$

10

$$\frac{dy}{dx} + p(x)y = 0 : y = Ce^{-\int p(x) dx} \quad (10.1)$$

$$\frac{dy}{dx} + p(x)y = q(x) : y = e^{-\int p(x) dx} (C + \int q(x)e^{\int p(x) dx} dx) \quad (10.2)$$

11

若 $f^{(i)}(x_0) = 0, i = 1, 2, \dots, n-1$, 而 $f^{(n)}(x_0) \neq 0$, 则:

当 n 为奇数时, x_0 是拐点, 但不是极值点;

当 n 为偶数时, x_0 是极值点, 但不是拐点。

12

$$\int \sqrt{a^2 - x^2} dx : x = a \sin \theta \quad (12.1)$$

$$\int \sqrt{a^2 + x^2} dx : x = a \tan \theta \quad (12.2)$$

$$\int \sqrt{x^2 - a^2} dx : x = a \sec \theta \quad (12.3)$$

13

$$\int_0^{\frac{\pi}{2}} f(\sin x) dx = \int_0^{\frac{\pi}{2}} f(\cos x) dx \quad (13.1)$$

$$\int_0^{\pi} x f(\sin x) dx = \frac{\pi}{2} \int_0^{\pi} f(\sin x) dx = \pi \int_0^{\frac{\pi}{2}} f(\sin x) dx \quad (13.2)$$

$$\int_0^{\frac{\pi}{2}} \sin^n x dx = \int_0^{\frac{\pi}{2}} \cos^n x dx = \begin{cases} \frac{(n-1)(n-3)\cdots 2}{n(n-2)\cdots 3}, & n \text{ 为奇数} \\ \frac{(n-1)(n-3)\cdots 1}{n(n-2)\cdots 2} \cdot \frac{\pi}{2}, & n \text{ 为偶数} \end{cases} \quad (13.3)$$

$$\int_{-a}^a f(x) dx = \int_0^a [f(x) + f(-x)] dx \quad (13.4)$$