1

$$\sin^2\alpha + \cos^2\alpha = 1 \tag{1.1}$$

$$1 + \tan^2\alpha = \sec^2\alpha \tag{1.2}$$

$$1 + \cot^2\alpha = \csc^2\alpha \tag{1.3}$$

$$\arctan x + \arctan\frac{1}{x} = \begin{cases} \frac{\pi}{2}, & x > 0 \\ -\frac{\pi}{2}, & x < 0 \end{cases} \tag{1.4}$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \tag{1.5}$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta \tag{1.6}$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta} \tag{1.7}$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha \tag{1.8}$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha \tag{1.9}$$

$$\sin\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{2}} \tag{1.10}$$

$$\cos\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{2}} \tag{1.11}$$

$$\tan\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} = \frac{\sin\alpha}{1+\cos\alpha} = \frac{1-\cos\alpha}{\sin\alpha} \tag{1.12}$$

$$\sin\alpha = \frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} \tag{1.13}$$

$$\cos\alpha = \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} \tag{1.14}$$

2

$$(C)' = 0 \tag{2.1}$$

$$(x^\mu)' = \mu x^{\mu-1} \tag{2.2}$$

$$(a^x)' = a^x \ln a \tag{2.3}$$

$$(e^x)' = e^x \tag{2.4}$$

$$(\log_a x)' = \frac{1}{x\ln a} \tag{2.5}$$

$$(\ln x)' = \frac{1}{x} \tag{2.6}$$

$$(\sin x)' = \cos x \tag{2.7}$$

$$(\cos x)' = -\sin x \tag{2.8}$$

$$(\tan x)' = \frac{1}{\cos^2 x} = \sec^2 x \tag{2.9}$$

$$(\cot x)' = -\frac{1}{\sin^2 x} = -\csc^2 x \tag{2.10}$$

$$(\sec x)' = \sec x\tan x \tag{2.11}$$

$$(\csc x)' = -\csc x\cot x \tag{2.12}$$

$$(\arcsin x)^{'} = \frac{1}{\sqrt{1-x^2}} \tag{2.13}$$

$$(\arccos x)^{'} = -\frac{1}{\sqrt{1-x^2}} \tag{2.14}$$

$$(\arctan x)^{'} = \frac{1}{1+x^2} \tag{2.15}$$

$$(\operatorname{arccot} x)^{'} = -\frac{1}{1+x^2} \tag{2.16}$$

3

$$\sin x = x - \frac{x^3}{6} + o(x^5) \tag{3.1}$$

$$\arcsin x = x + \frac{x^3}{6} + o(x^5) \tag{3.2}$$

$$\tan x = x + \frac{x^3}{3} + o(x^5) \tag{3.3}$$

$$\arctan x = x - \frac{x^3}{3} + o(x^5) \tag{3.4}$$

4

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + o(x^n) \tag{4.1}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^m \frac{x^{2m+1}}{(2m+1)!} + o(x^{2m+1}) \tag{4.2}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^m \frac{x^{2m}}{(2m)!} + o(x^{2m}) \tag{4.3}$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{n-1} \frac{x^n}{n} + o(x^n) \tag{4.4}$$

$$\frac{1}{1+x} = 1 - x + x^2 + \cdots + (-1)^n x^n + o(x^n) \tag{4.5}$$

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots + x^n + o(x^n) \tag{4.6}$$

$$(1+x)^\mu = 1 + \mu x + \frac{\mu(\mu-1)}{2!} x^2 + \cdots + \frac{\mu(\mu-1)\cdots(\mu-n+1)}{n!} x^n + o(x^n) \tag{4.7}$$

5

$$k = \frac{\left|y^{''}\right|}{(1+y'^2)^{\frac{3}{2}}} \tag{5.1}$$

$$k = \frac{1}{R} \tag{5.2}$$

$$\begin{cases} \xi = x - \frac{y^{'}(1+y^{'2})}{y^{''}} \\ \eta = y + \frac{(1+y^{'2})}{y^{''}} \end{cases} \tag{5.3}$$

6

$$\int 0 \, \mathrm{d}x = C \tag{6.1}$$

$$\int 1 \, \mathrm{d}x = x + C \tag{6.2}$$

$$\int x^{\mu} \, \mathrm{d}x = \frac{1}{\mu + 1} x^{\mu+1} + C \tag{6.3}$$

$$\int \frac{1}{x} \, \mathrm{d}x = \ln|x| + C \tag{6.4}$$

$$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C \tag{6.5}$$

$$\int e^x \, \mathrm{d}x = e^x + C \tag{6.6}$$

$$\int \sin x \, \mathrm{d}x = -\cos x + C \tag{6.7}$$

$$\int \cos x \, \mathrm{d}x = \sin x + C \tag{6.8}$$

$$\int \sec^2 x \, \mathrm{d}x = \int \frac{1}{\cos^2 x} \, \mathrm{d}x = \tan x + C \tag{6.9}$$

$$\int \csc^2 x \, \mathrm{d}x = \int \frac{1}{\sin^2 x} \, \mathrm{d}x = -\cot x + C \tag{6.10}$$

$$\int \sec x \tan x \, \mathrm{d}x = \sec x + C \tag{6.11}$$

$$\int \csc x \cot x \, \mathrm{d}x = -\csc x + C \tag{6.12}$$

$$\int \frac{\mathrm{d}x}{\sqrt{1 - x^2}} = \arcsin x + C \tag{6.13}$$

$$\int \frac{dx}{1 + x^2} = \arctan x + C \tag{6.14}$$

7

$$\int \tan x \, \mathrm{d}x = -\ln|\cos x| + C \tag{7.1}$$

$$\int \cot x \, \mathrm{d}x = \ln|\sin x| + C \tag{7.2}$$

$$\int \sec x \, \mathrm{d}x = \ln|\sec x + \tan x| + C \tag{7.3}$$

$$\int \csc x \, \mathrm{d}x = \ln|\csc x - \tan x| + C \tag{7.4}$$

$$\int \frac{\mathrm{d}x}{x^2 + a^2} = \frac{1}{a} \arctan \frac{x}{a} + C \tag{7.5}$$

$$\int \frac{\mathrm{d}x}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x - a}{x + a} \right| + C \tag{7.6}$$

$$\int \frac{\mathrm{d}x}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C \tag{7.7}$$

$$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C \tag{7.8}$$

$$\int \sqrt{a^2 - x^2}\,\mathrm{d}x = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\arcsin\frac{x}{a} + C \tag{7.9}$$

$$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{x}{2}\sqrt{x^2 \pm a^2} \pm \frac{a^2}{2}\ln\left|x + \sqrt{x^2 \pm a^2}\right| + C \tag{7.10}$$

8

当 $p \neq 1$，敛散与 ln 无关
等号跟着发散走

$$\int_1^{+\infty} \frac{1}{x^p}\,\mathrm{d}x \begin{cases} 收敛, & p > 1 \\ 发散, & p \leq 1 \end{cases} \tag{8.1}$$

$$\int_0^1 \frac{1}{x^q}\,\mathrm{d}x \begin{cases} 收敛, & 0 < q < 1 \\ 发散, & q \geq 1 \end{cases} \tag{8.2}$$

9

$$S = \int_a^b y(x)\,\mathrm{d}x \tag{9.1}$$

$$S = \int_{t_1}^{t_2} y(t)x^{'}(t)\,\mathrm{d}t \tag{9.2}$$

$$S = \frac{1}{2}\int_\alpha^\beta r^2(\theta)\,\mathrm{d}\theta \tag{9.3}$$

$$V = \int_a^b S(x)\,\mathrm{d}x \tag{9.4}$$

$$V_x = \pi\int_a^b y^2(x)\,\mathrm{d}x \tag{9.5}$$

$$V_x = \pi\int_{t_1}^{t_2} y^2(t)x^{'}(t)\,\mathrm{d}t \tag{9.6}$$

$$V_y = 2\pi\int_a^b |x|\,|y(x)|\,\mathrm{d}x \tag{9.7}$$

$$V_y = 2\pi\int_{t_1}^{t_2} |x(t)|\,|y(t)|\,x^{'}(t)\,\mathrm{d}t \tag{9.8}$$

$$s = \int_a^b \sqrt{1 + [y'(x)]^2}\,\mathrm{d}x \tag{9.9}$$

$$s = \int_{t_{小}}^{t_{大}} \sqrt{[x'(t)]^2 + [y'(t)]^2}\,\mathrm{d}t \tag{9.10}$$

$$s = \int_\alpha^\beta \sqrt{[r(\theta)]^2 + [r'(\theta)]^2}\,\mathrm{d}\theta \tag{9.11}$$

$$S = 2\pi\int_a^b |y(x)|\sqrt{1 + [y'(x)]^2}\,\mathrm{d}x \tag{9.12}$$

$$S = 2\pi\int_{t_{小}}^{t_{大}} |y(t)|\sqrt{[x'(t)]^2 + [y'(t)]^2}\,\mathrm{d}t \tag{9.13}$$

10

$$\frac{\mathrm{d}y}{\mathrm{d}x}+p(x)y=0:\ \ y=Ce^{-\int p(x)\,\mathrm{d}x} \tag{10.1}$$

$$\frac{\mathrm{d}y}{\mathrm{d}x}+p(x)y=q(x):\ \ y=e^{-\int p(x)\,\mathrm{d}x}(C+\int q(x)e^{\int p(x)\,\mathrm{d}x}\,\mathrm{d}x) \tag{10.2}$$

11

若 $f^{(i)}(x_0)=0,i=1,2,\cdots,n-1$，而 $f^{(n)}(x_0)\neq 0$，则：
当 n 为奇数时，x_0 是拐点，但不是极值点；
当 n 为偶数时，x_0 是极值点，但不是拐点。

12

$$\int\sqrt{a^2-x^2}\,\mathrm{d}x:\ \ x=a\sin\theta \tag{12.1}$$

$$\int\sqrt{a^2+x^2}\,\mathrm{d}x:\ \ x=a\tan\theta \tag{12.2}$$

$$\int\sqrt{x^2-a^2}\,\mathrm{d}x:\ \ x=a\sec\theta \tag{12.3}$$

13

$$\int_0^{\frac{\pi}{2}}f(\sin x)\,\mathrm{d}x=\int_0^{\frac{\pi}{2}}f(\cos x)\,\mathrm{d}x \tag{13.1}$$

$$\int_0^{\pi}xf(\sin x)\,\mathrm{d}x=\frac{\pi}{2}\int_0^{\pi}f(\sin x)\,\mathrm{d}x=\pi\int_0^{\frac{\pi}{2}}f(\sin x)\,\mathrm{d}x \tag{13.2}$$

$$\int_0^{\frac{\pi}{2}}\sin^n x\,\mathrm{d}x=\int_0^{\frac{\pi}{2}}\cos^n x\,\mathrm{d}x=\begin{cases}\frac{(n-1)(n-3)\cdots 2}{n(n-2)\cdots 3}, & n\text{为奇数}\\ \frac{(n-1)(n-3)\cdots 1}{n(n-2)\cdots 2}\cdot\frac{\pi}{2}, & n\text{为偶数}\end{cases} \tag{13.3}$$

$$\int_{-a}^{a}f(x)\,\mathrm{d}x=\int_0^{a}[f(x)+f(-x)]\,\mathrm{d}x \tag{13.4}$$