1

$$\mathrm{P}(A\cup B\cup C)=\mathrm{P}(A)+\mathrm{P}(B)+\mathrm{P}(C)-\mathrm{P}(AB)-\mathrm{P}(AC)-\mathrm{P}(BC)+\mathrm{P}(ABC) \tag{1.1}$$

$$\mathrm{P}(A\overline{B})=\mathrm{P}(A)-\mathrm{P}(AB) \tag{1.2}$$

2

$$\mathrm{P}(A|B)=\frac{\mathrm{P}(AB)}{\mathrm{P}(A)} \tag{2.1}$$

$$\mathrm{P}(A_1A_2A_3)=\mathrm{P}(A_1)\,\mathrm{P}(A_2|A_1)\,\mathrm{P}(A_3|A_1A_2) \tag{2.2}$$

$$\mathrm{P}(B)=\sum_{i=1}^{n}\mathrm{P}(A_i)\,\mathrm{P}(B|A_i) \tag{2.3}$$

$$\mathrm{P}(A_i|B)=\frac{\mathrm{P}(A_i)\,\mathrm{P}(B|A_i)}{\sum_{j=1}^{n}\mathrm{P}(A_j)\,\mathrm{P}(B|A_j)}, A_i\text{互不相容}, B\subset\sum A_i \tag{2.4}$$

$$\mathrm{P}(AB)=\mathrm{P}(A)\,\mathrm{P}(B)\iff A\perp\!\!\!\perp B \tag{2.5}$$

3 $\mathrm{N}(\mu,\sigma^2)$

$$\Phi(x)=F(x), X\sim\mathrm{N}(0,1) \tag{3.1}$$

$$F(x)=\Phi(\frac{x-\mu}{\sigma}) \tag{3.2}$$

3.1 $\mathrm{N}(\mu_1,\mu_2;\sigma_1,\sigma_2;\rho)$

$$X\sim\mathrm{N}(\mu_1,\sigma_1); Y\sim\mathrm{N}(\mu_2,\sigma_2) \tag{3.3}$$

4

$$f(x)=F^{'}(x) \tag{4.1}$$

$$F(x)=\int_{-\infty}^{x}f(t)\,\mathrm{d}t \tag{4.2}$$

4.1

$y=g(x)$ 严格单调，$x=h(y)$ 为 $g(x)$ 的反函数，则

$$f_Y(y)=f_X(h(y))\left|h^{'}(y)\right| \tag{4.3}$$

5

$$f(x,y)=\frac{\partial^2F(x,y)}{\partial x\partial y} \tag{5.1}$$

$$F(x,y)=\int_{-\infty}^{x}\int_{-\infty}^{y}f(u,v)\,\mathrm{d}u\,\mathrm{d}v \tag{5.2}$$

$$F_X(x)=F(x,+\infty); F_Y(y)=F(+\infty,y) \tag{5.3}$$

$$f_X(x)=\int_{-\infty}^{+\infty}f(x,y)\,\mathrm{d}y; f_Y(y)=\int_{-\infty}^{+\infty}f(x,y)\,\mathrm{d}x \tag{5.4}$$

$$F(x,y)=F_X(x)F_Y(y), X\perp\!\!\!\perp Y \tag{5.5}$$

$$f(x,y)=f_X(x)f_Y(y), X\perp\!\!\!\perp Y \tag{5.6}$$

6

$$f_{X|Y}(x|y) = \frac{f(x,y)}{f_Y(y)}; f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)} \tag{6.1}$$

7

7.1 $Z = X + Y$

$$f_Z(z) = \int_{-\infty}^{+\infty} f(x, z-x)\,\mathrm{d}x = \int_{-\infty}^{+\infty} f(z-y, y)\,\mathrm{d}y \tag{7.1}$$

$$f_Z(z) = \int_{-\infty}^{+\infty} f_X(x) f_Y(z-x)\,\mathrm{d}x = \int_{-\infty}^{+\infty} f_X(z-y) f_Y(y)\,\mathrm{d}y, X \perp\!\!\!\perp Y \tag{7.2}$$

7.2 $Z = \max\{X, Y\}$

$$F_Z(z) = \iint\limits_{x \le z, y \le z} f(x,y)\,\mathrm{d}x\,\mathrm{d}y \tag{7.3}$$

$$F_Z(z) = F_X(z) F_Y(z), X \perp\!\!\!\perp Y \tag{7.4}$$

7.3 $Z = \min\{X, Y\}$

$$F_Z(z) = 1 - \iint\limits_{x > z, y > z} f(x,y)\,\mathrm{d}x\,\mathrm{d}y \tag{7.5}$$

$$F_Z(z) = 1 - [1 - F_X(z)][1 - F_Y(z)], X \perp\!\!\!\perp Y \tag{7.6}$$

8

分布	记号	分布列/概率密度	期望	方差
二项分布	$\mathrm{B}(n, p)$	$\mathrm{P}(X=k) = C_n^k p^k q^{n-k}$, $p + q = 1$	np	npq
泊松分布	$\mathrm{P}(\lambda)$	$\mathrm{P}(X=k) = \frac{\lambda^k}{k!} e^{-\lambda}$, $\lambda > 0$	λ	λ
几何分布	$\mathrm{G}(p)$	$\mathrm{P}(X=k) = q^{k-1} p$, $p + q = 1$	$\frac{1}{p}$	$\frac{q}{p^2}$
均匀分布	$\mathrm{U}[a, b]$	$f(x) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \\ 0, & \text{otherwise} \end{cases}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
指数分布	$\mathrm{E}(\lambda)$	$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0, \\ 0, & x \le 0, \end{cases}$ $\lambda > 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
正态分布	$\mathrm{N}(\mu, \sigma^2)$	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	μ	σ^2

9

$$\mathrm{E}(X) = \int_{-\infty}^{+\infty} x f(x) \,\mathrm{d}x \tag{9.1}$$

$$\mathrm{E}(X) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} x f(x, y) \,\mathrm{d}x \,\mathrm{d}y ; \mathrm{E}(Y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} y f(x, y) \,\mathrm{d}x \,\mathrm{d}y \tag{9.2}$$

$$\mathrm{E}(g(X)) = \int_{-\infty}^{+\infty} g(x) f(x) \,\mathrm{d}x \tag{9.3}$$

$$\mathrm{E}(C) = C \tag{9.4}$$

$$\mathrm{E}(CX) = C \,\mathrm{E}(X) \tag{9.5}$$

$$\mathrm{E}(X \pm Y) = \mathrm{E}(X) \pm \mathrm{E}(Y) \tag{9.6}$$

$$\mathrm{E}(XY) = \mathrm{E}(X) \,\mathrm{E}(Y) + \mathrm{Cov}(X, Y) \tag{9.7}$$

$$\mathrm{E}(XY) = \mathrm{E}(X) \,\mathrm{E}(Y) \iff X, Y\text{不相关} \tag{9.8}$$

10

$$\mathrm{D}(X) = E([x - \mathrm{E}(x)]^2) = \int_{-\infty}^{+\infty} [x - \mathrm{E}(x)]^2 f(x) \,\mathrm{d}x \tag{10.1}$$

$$\mathrm{D}(C) = 0 \tag{10.2}$$

$$\mathrm{D}(CX) = C^2 \,\mathrm{D}(X) \tag{10.3}$$

$$\mathrm{D}(X) = \mathrm{E}(X^2) - \mathrm{E}^2(X) \tag{10.4}$$

$$\mathrm{D}(X \pm Y) = \mathrm{D}(X) + \mathrm{D}(Y) \pm 2 \,\mathrm{Cov}(X, Y) \tag{10.5}$$

$$\mathrm{D}(X \pm Y) = \mathrm{D}(X) + \mathrm{D}(Y) \iff X, Y\text{不相关} \tag{10.6}$$

$$\mathrm{D}(XY) = \mathrm{D}(X) \,\mathrm{D}(Y) + \mathrm{E}^2(Y) \,\mathrm{D}(X) + \mathrm{E}^2(X) \,\mathrm{D}(Y), X \perp\!\!\!\perp Y \tag{10.7}$$

11

$$\mathrm{Cov}(X, Y) = \mathrm{E}(XY) - \mathrm{E}(X) \,\mathrm{E}(Y) \tag{11.1}$$

$$\mathrm{Cov}(X, Y) = 0 \iff X, Y\text{不相关} \tag{11.2}$$

$$\mathrm{Cov}(X, X) = \mathrm{D}(X) \tag{11.3}$$

$$\mathrm{Cov}(X, Y) = \rho_{XY} \sqrt{\mathrm{D}(X)} \sqrt{\mathrm{D}(Y)} \tag{11.4}$$

$$\mathrm{Cov}(aX + b, cY + d) = ac \,\mathrm{Cov}(X, Y) \tag{11.5}$$

$$\mathrm{Cov}(X_1 \pm X_2, Y) = \mathrm{Cov}(X_1, Y) \pm \mathrm{Cov}(X_2, Y) ; \mathrm{Cov}(X, Y_1 \pm Y_2) = \mathrm{Cov}(X, Y_1) \pm \mathrm{Cov}(X, Y_2) \tag{11.6}$$

12

$$\rho_{XY} = \frac{\mathrm{Cov}(X, Y)}{\sqrt{\mathrm{D}(X)} \sqrt{\mathrm{D}(Y)}} \tag{12.1}$$

$$\rho_{XY} = 0 \iff X, Y\text{不相关} \tag{12.2}$$

13

$$P(|X - E(X)| \geq \varepsilon) \leq \frac{D(X)}{\varepsilon^2} \tag{13.1}$$

$$P(|X - E(X)| < \varepsilon) \geq 1 - \frac{D(X)}{\varepsilon^2} \tag{13.2}$$

14

14.1 χ^2 分布

$$\sum_{i=1}^{n} X_i^2 \sim \chi^2(n), X_i \sim N(0,1), X_i \perp\!\!\!\perp X_j \tag{14.1}$$

14.2 t 分布

$$\frac{X}{\sqrt{\frac{Y}{n}}} \sim t(n), X \sim N(0,1), Y \sim \chi^2(n) \tag{14.2}$$

14.3 F 分布

$$\frac{\frac{X}{n_1}}{\frac{Y}{n_2}} \sim F(n_1, n_2), X \sim \chi^2(n_1), Y \sim \chi^2(n_2) \tag{14.3}$$

15

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \tag{15.1}$$

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2 = \frac{1}{n-1}(\sum_{i=1}^{n} X_i^2 - n\overline{X}^2) \tag{15.2}$$

$$S = \sqrt{S^2} \tag{15.3}$$

16 $X_1, X_2, \cdots, X_n$ 为总体 $N(\mu, \sigma^2)$ 的样本

$$\overline{X} \sim N(\mu, \frac{\sigma^2}{n}) \tag{16.1}$$

$$\frac{n-1}{\sigma^2}S^2 \sim \chi^2(n-1) \tag{16.2}$$

$$u = \frac{\overline{X} - \mu}{\sigma}\sqrt{n} \sim N(0,1) \tag{16.3}$$

$$t = \frac{\overline{X} - \mu}{S}\sqrt{n} \sim t(n-1) \tag{16.4}$$

$$\text{置信水平} = 1 - \alpha \tag{16.5}$$

16.1 σ^2 已知，求 μ 的置信区间

$$(\overline{X} - u_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}, \overline{X} + u_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}), \Phi(u_{\frac{\alpha}{2}}) = 1 - \frac{\alpha}{2} \tag{16.6}$$

16.2 σ^2 未知，求 μ 的置信区间

$$(\overline{X} - t_{\frac{\alpha}{2}}(n-1)\frac{S}{\sqrt{n}}, \overline{X} + t_{\frac{\alpha}{2}}(n-1)\frac{S}{\sqrt{n}}) \tag{16.7}$$

16.3 求 σ^2 的置信区间

$$(S\sqrt{\frac{n-1}{\chi^2_{\frac{\alpha}{2}}(n-1)}}, S\sqrt{\frac{n-1}{\chi^2_{1-\frac{\alpha}{2}}(n-1)}}) \tag{16.8}$$

17

17.1 矩估计

1. 求 $E(X), E(X_2), \cdots, E(X_m)$，$m$ 为未知参数个数
2. 用 $E(X_i)$ 表示 θ_j
3. 代入样本的 $E(X_i)$，θ_j 写成 $\hat{\theta}_j$

17.2 最大似然估计

17.2.1 方法一

1. 求 $L(X_1, X_2, \cdots, X_n; \theta) = \prod\limits_{i=1}^{n} f(X_i)$
2. 求 $\ln L$
3. 求 $\frac{\mathrm{d} \ln L}{\mathrm{d}\theta}$
4. 解方程 $\frac{\mathrm{d} \ln L}{\mathrm{d}\theta} = 0$

17.2.2 方法二

1. 求 $f(X_1), f(X_2), \cdots, f(X_n)$
2. 求 $\ln f(X_i)$
3. 求 $\frac{\mathrm{d} \ln f(X_i)}{\mathrm{d}\theta}$
4. 解方程 $\sum\limits_{i=1}^{n} \frac{\mathrm{d} \ln f(X_i)}{\mathrm{d}\theta} = 0$

18

18.1 无偏性

$$\mathrm{E}(\hat{\theta}) = \theta \tag{18.1}$$

18.2 有效性

$$\mathrm{D}(\hat{\theta}) \le \mathrm{D}(\hat{\theta}'), \text{至少对某个}\theta_0\text{小于号成立} \tag{18.2}$$

19

19.1

19.1.1 σ 已知，检验 $H_0 : \mu = \mu_0, H_1 : \mu \neq \mu_0$

$$|u| \geq u_{\frac{\alpha}{2}}, \text{拒绝} H_0 \tag{19.1}$$

19.1.2 σ 已知，检验 $H_0 : \mu \leq \mu_0, H_1 : \mu > \mu_0$

$$u \geq u_\alpha, \text{拒绝} H_0 \tag{19.2}$$

19.1.3 σ 已知，检验 $H_0 : \mu \geq \mu_0, H_1 : \mu < \mu_0$

$$u \leq -u_\alpha, \text{拒绝} H_0 \tag{19.3}$$

19.2

19.2.1 σ 未知，检验 $H_0 : \mu = \mu_0, H_1 : \mu \neq \mu_0$

$$|t| \geq t_{\frac{\alpha}{2}}(n-1), \text{拒绝} H_0 \tag{19.4}$$

19.2.2 σ 未知，检验 $H_0 : \mu \leq \mu_0, H_1 : \mu > \mu_0$

$$t \geq t_\alpha(n-1), \text{拒绝} H_0 \tag{19.5}$$

19.2.3 σ 未知，检验 $H_0 : \mu \geq \mu_0, H_1 : \mu < \mu_0$

$$t \leq -t_\alpha(n-1), \text{拒绝} H_0 \tag{19.6}$$

19.3

19.3.1 检验 $H_0 : \sigma^2 = \sigma_0^2, H_1 : \sigma^2 \neq \sigma_0^2$

$$\chi^2 \geq \chi^2_{\frac{\alpha}{2}}(n-1) \text{或} \chi^2 \leq \chi^2_{1-\frac{\alpha}{2}}(n-1), \text{拒绝} H_0 \tag{19.7}$$

19.3.2 检验 $H_0 : \sigma^2 \leq \sigma_0^2, H_1 : \sigma^2 > \sigma_0^2$

$$\chi^2 \geq \chi^2_\alpha(n-1), \text{拒绝} H_0 \tag{19.8}$$

19.3.3 检验 $H_0 : \sigma^2 \geq \sigma_0^2, H_1 : \sigma^2 < \sigma_0^2$

$$\chi^2 \leq \chi^2_{1-\alpha}(n-1), \text{拒绝} H_0 \tag{19.9}$$

20

$$\begin{aligned} \Gamma(s) &= \int_0^{+\infty} x^{s-1} e^{-x} \, \mathrm{d}x \\ \Gamma(1) &= 1, \Gamma(\frac{1}{2}) = \sqrt{\pi} \\ \Gamma(s+1) &= s\Gamma(s), \Gamma(n) = (n-1)! \end{aligned} \tag{20.1}$$