Problem Set 1 – Review of Basic Concepts in Probability

Problem 1:

n fair coins are flipped independently (where n is a given natural number).

1. Find the probability of the following events:

- A = "head" came out **at least** once.
- B = "head" came out **exactly** once.
- C = "head" came out **at most** once.
- D = both "head" and "tail" came out **at least** once each.

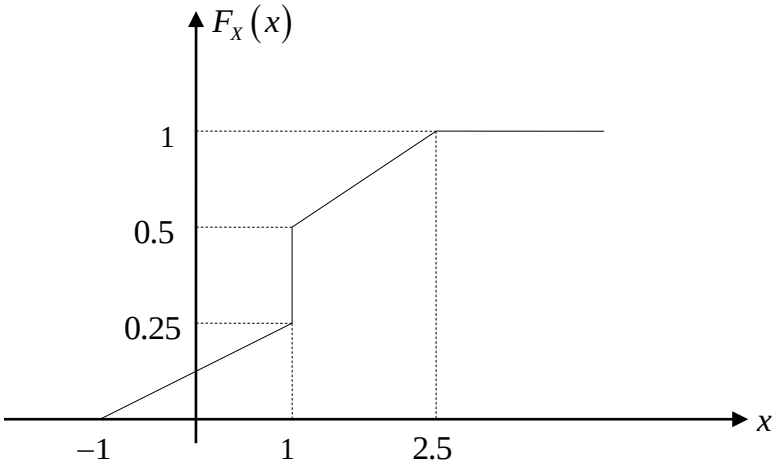
2. For which values of n are events C and D independent?

Problem 2:

The CDF of random variable X is given by $F_X(x)$ (see figure).

Find the following:

1. $P(X \le 0)$
2. $P(X < 0)$
3. $P(X \le 1)$
4. $P(X < 1)$
5. $P(0 \le X < 1)$
6. $P(0 < X \le 1.75)$
7. $P(X > 1)$
8. $P(X \ge 2.5)$



Problem 3:

Consider a discrete random variable X, defined by:

$$X = \begin{cases} -2, & w.p. = 0.1 \\ -1, & w.p. = 0.2 \\ +1, & w.p. = 0.3 \\ +2, & w.p. = 0.4 \end{cases}$$

Let us denote $Y = sign(X)$. Find the PDF and CDF of the random variable Y.

Problem 4:

Given a continuous random variable X with known PDF, $f_X(x)$, and known CDF, $F_X(x)$; find the PDF and CDF of the following random variables:

1. Y=X+17
2. Y= –19X
3. Y= max(X,0)
4. Y=|X|

Problem 5:

Consider a random variable X with the following PDF:

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

This is a standard Gaussian variable and we will come upon it frequently throughout the course.

The $Q(x)$ function is defined as:

$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t^2}{2}\right) dt .$$

1. Express the following probabilities using the $Q(x)$ function:
    a. P(X>2)
    b. P(X<7)
    c. P(X=3)
2. What are $\lim_{x\to-\infty} Q(x), Q(0), \lim_{x\to\infty} Q(x)$ equal to?
3. Denoting Y=aX+b, where a,b are constants and $a \neq 0$, what is the PDF of Y?