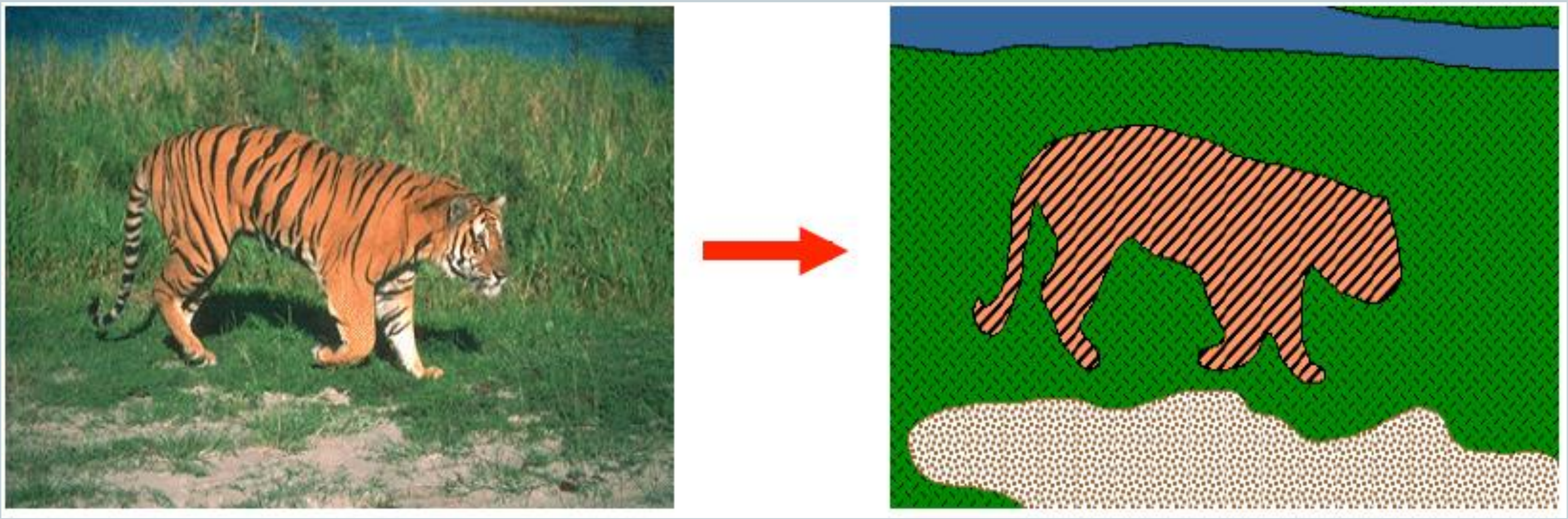
ELEC 474 Machine Vision

K-MEANS CLUSTERING

Edith Deretey, Zia Ud Din, Michael Greenspan

Review: Segmentation

- Identifying groups of pixels that go together



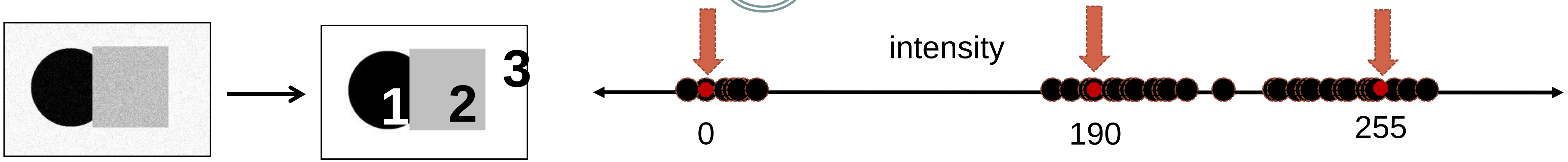
Clustering

- Clustering: grouping together similar connected points and representing them with a single token
- Criteria for forming a cluster depends on the application
- Some examples:
  - Intensity
  - Colour
  - Distance
  - Texture

K-Means Clustering

- K-means clustering is one of the simplest unsupervised learning algorithms.
- Supervised Learning
  - Knowledge of output
    - Learning in the presence of an expert
    - Training data is labelled
  - Goal: Predict labels of unseen data
  - Examples: Support Vector Machines (SVM), Decision Trees, Neural Networks,
- Unsupervised Learning
  - No knowledge of output
  - Goal: Discover patterns/groupings in data
  - Examples: Clustering, Genetic Algorithms, Autoencoders

K-Means Clustering

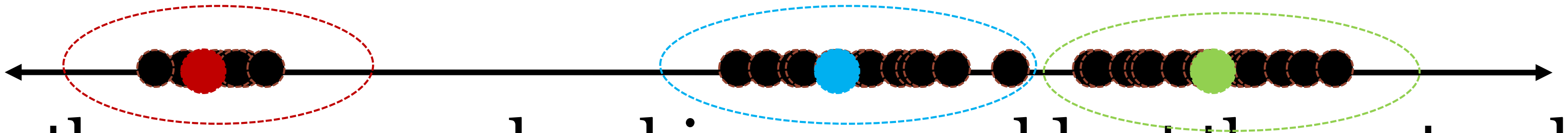


- Approach: choose three "centers" as the representative intensities
  - Label every pixel according to which of these centers it is nearest to
- Best cluster centers are those that minimize Sum of Square Distance (SSD) between all points x and their nearest cluster center c_i

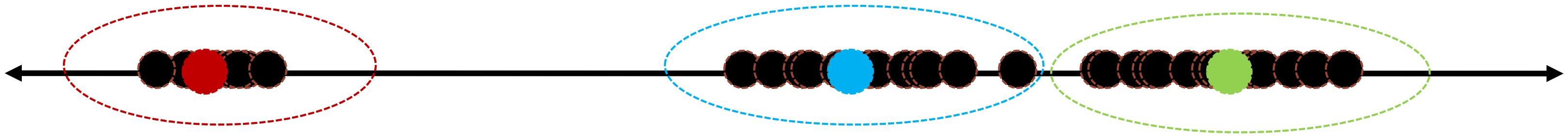
$$SSD = \sum_{clusters\ i} \sum_{x \in cluster\ i} (x - c_i)^2$$

K-Means Clustering

- "chicken and egg problem"
- If we knew the cluster centers, we could allocate points to groups by assigning each to its closest center



- If we knew the group memberships, we could get the centers by computing the mean per group

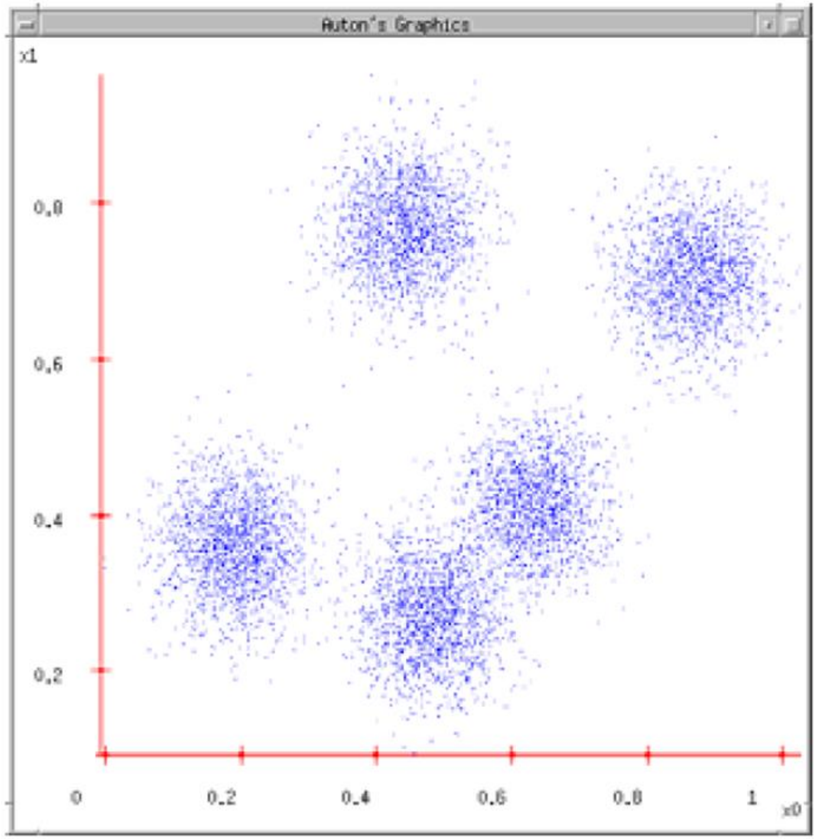


K-Means Clustering

Algorithm:

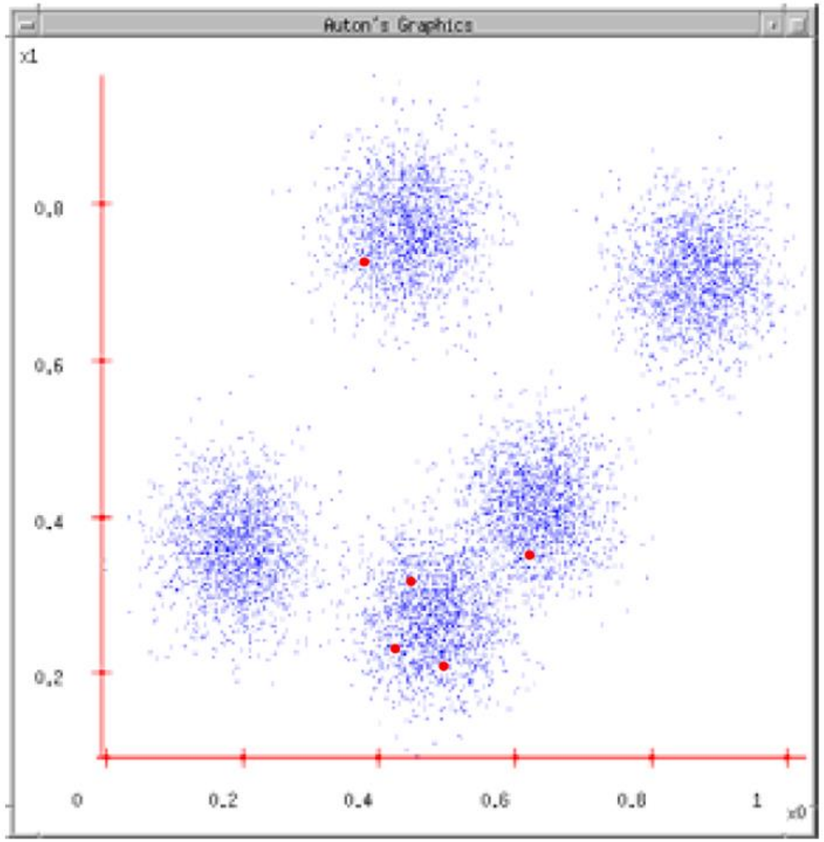
1. Randomly initialize k points to act as cluster centers (c_1, …, c_k)
2. Given cluster centers, determine points in each cluster
   - point x is in cluster i iff $|x-c_i| \leq |x-c_j|$ for all i, j in $\{1..k\}$
3. Given points in each cluster, solve for c_i
4. If c_i have changed, repeat from Step 2

Example: K-Means Clustering



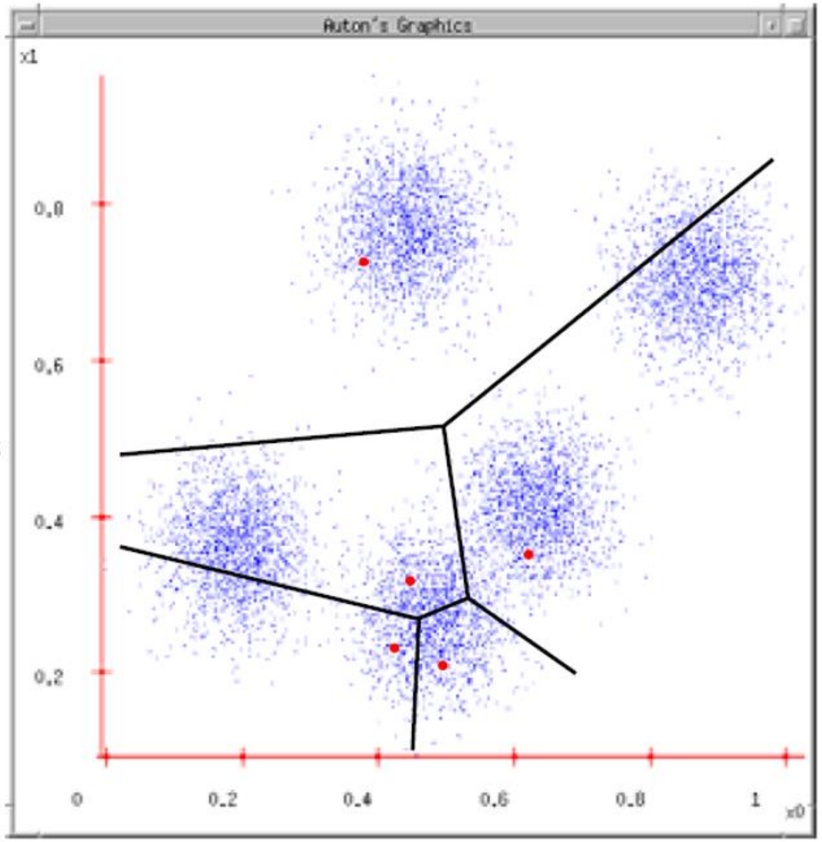
Example: K-Means Clustering

- Step 1: Randomly initialize *k* points to act as cluster centers



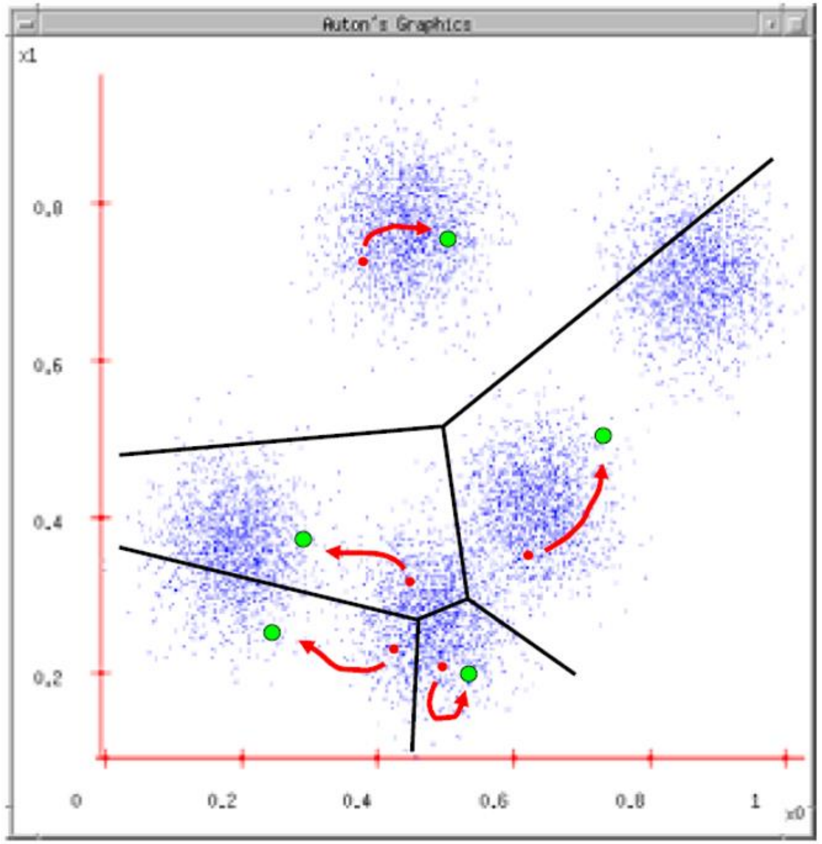
Example: K-Means Clustering

- Step 2: Given cluster centers, determine points in each cluster



Example: K-Means Clustering

- Step 3: Given points in each cluster, solve for c_i



Step 4: If c_i have changed, repeat from Step 2

K-Means Clustering

- Online demo:
  - https://www.naftaliharris.com/blog/visualizing-k-means-clustering/

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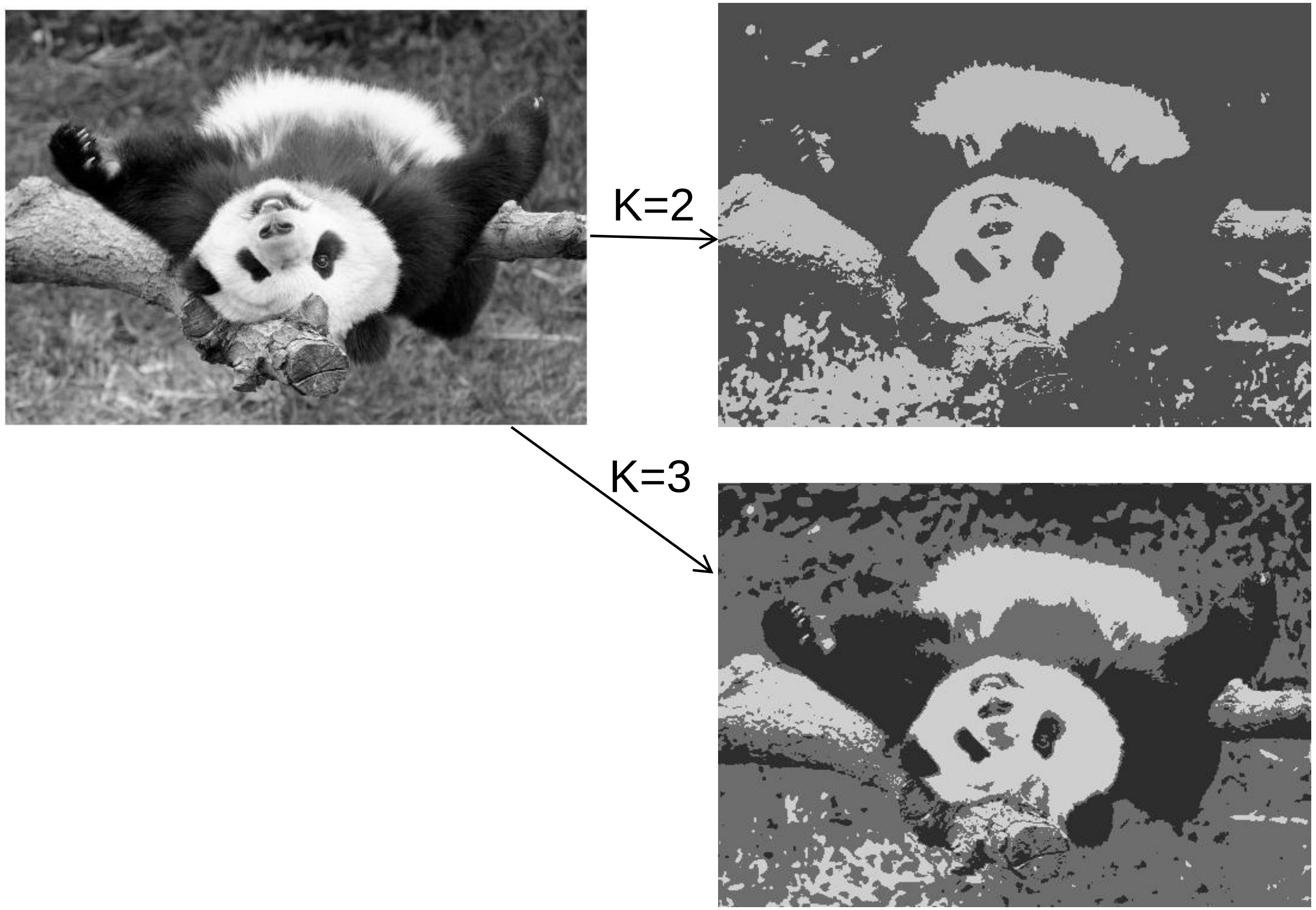
Segmentation as Clustering

- Depending on what we choose as the feature space, the pixels can be grouped in different ways
- Grouping pixels based on **intensity** similarity:

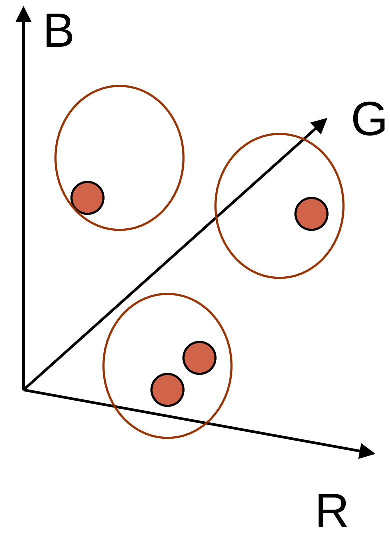
Feature space: intensity value (1-D)

Segmentation as Clustering

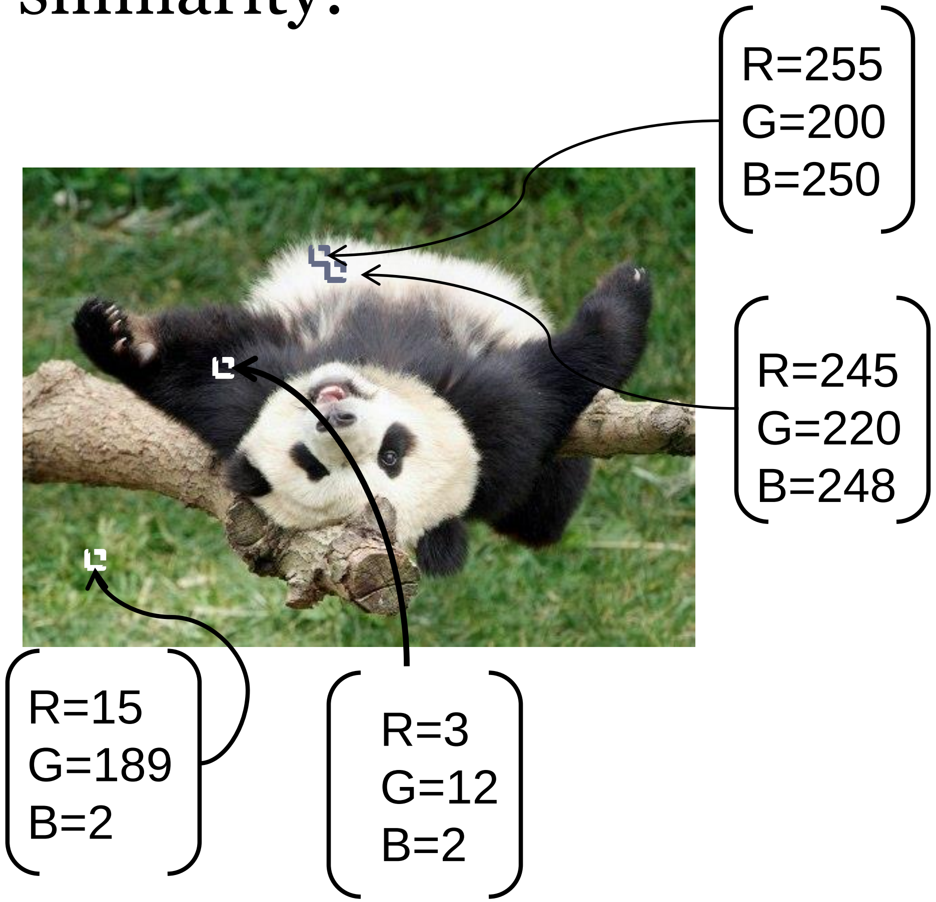


Segmentation as Clustering

- Grouping pixels based on **colour** similarity:

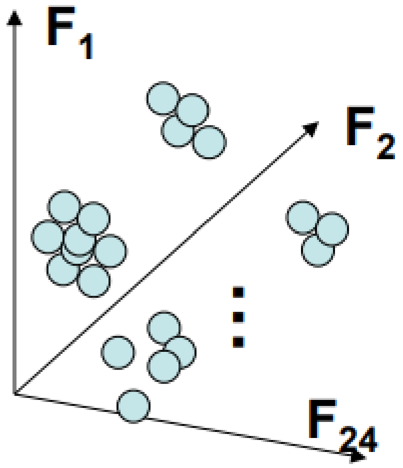


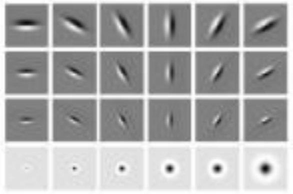
Feature space: intensity value (3-D)



Segmentation as Clustering

- Grouping pixels based on **texture** similarity





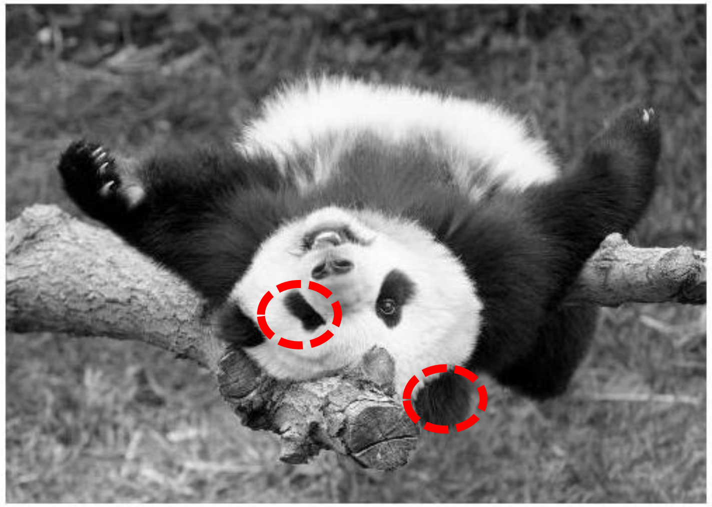
Filter bank of
24 filters

Feature space: filter bank responses (e.g., 24-D)

Segmentation as Clustering

- Returning to grouping pixels based on **intensity** similarity
- In what case is intensity not enough for segmentation?

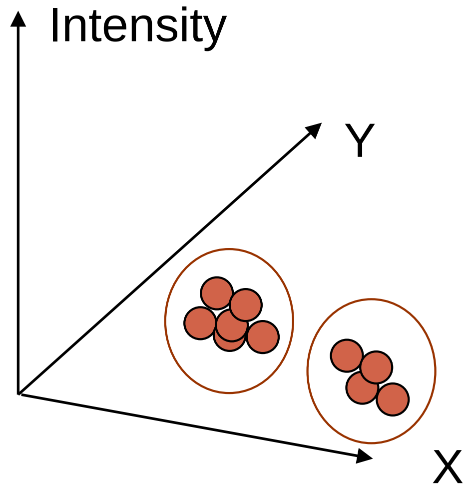
Feature space: intensity value (1-D)

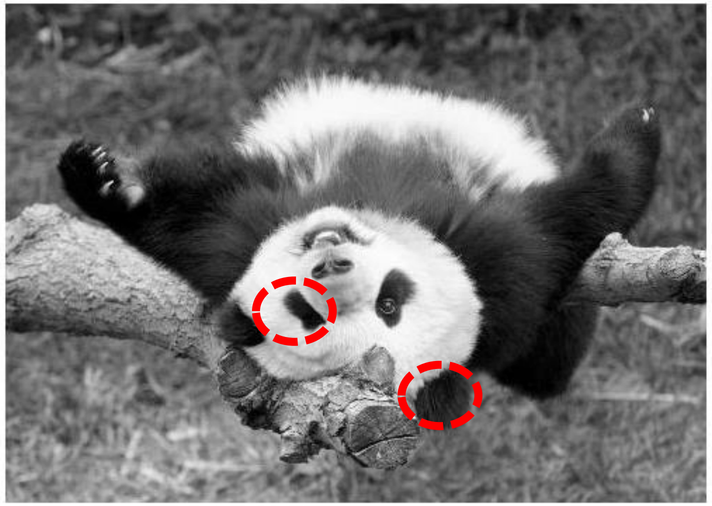


- Clusters based on intensity similarity don’t have to be spatially coherent

Segmentation as Clustering

- Grouping pixels based on **intensity & position** similarity:





- Both regions are black
- Including **position (*x*,*y*)** information allows us to group the regions into two distinct segments

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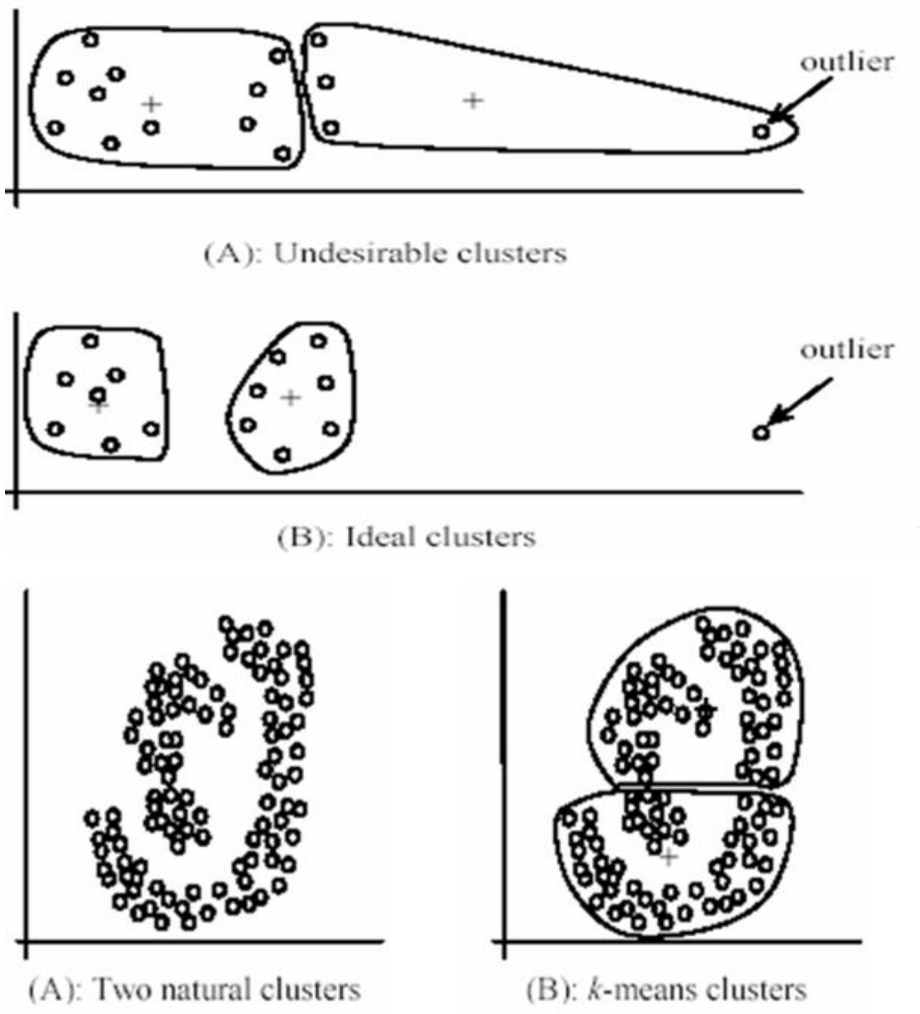
K-Means Clustering

- **Pros:**
  - Simple and fast
  - Easy to implement
  - Converges to local minimum of within-cluster squared error

- **Cons:**
  - Specifying *k*
  - Sensitive to initial centers
  - Sensitive to outliers
  - Detects spherical clusters only



K-Means: Choosing K

- The most common approach is to use elbow method
  - Try different values of K
  - Plot the K-means objective versus K
  - Look at the "elbow-point" in the plot
    - K value below which SSD explodes
- For the above plot, K = 6 is the elbow point

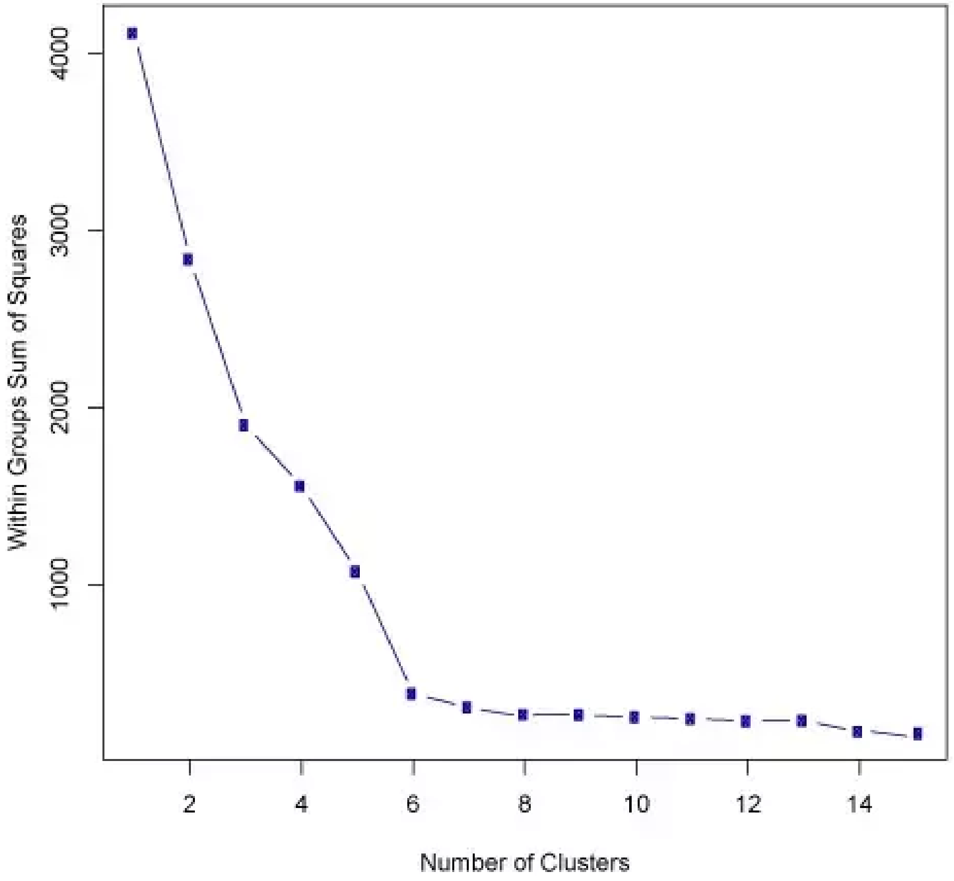


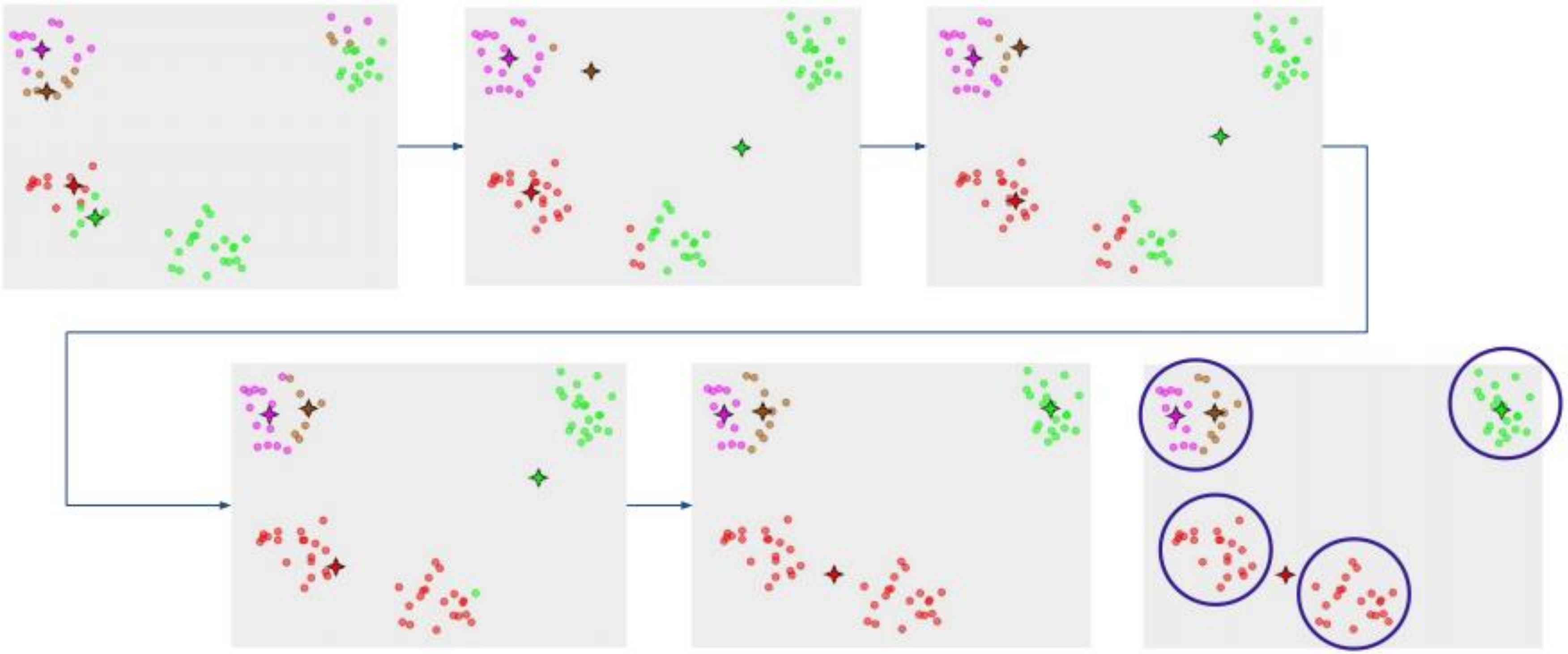
Image source: https://www.quora.com/How-can-we-choose-a-good-K-for-K-means-clustering

K-Means: Initialization

- Initialization = where to distribute the initial c_i
- K-means is extremely sensitive to initialization
- Bad initialization can lead to:
  - Poor convergence speed
  - Bad overall clustering
- How to initialize?
  - Randomly from data
  - Try to find K “spread-out” points
    - Furthest point algorithm
    - K-means ++
- Safeguarding measure:
  - Try multiple initializations and choose the best

K-Means: Initialization

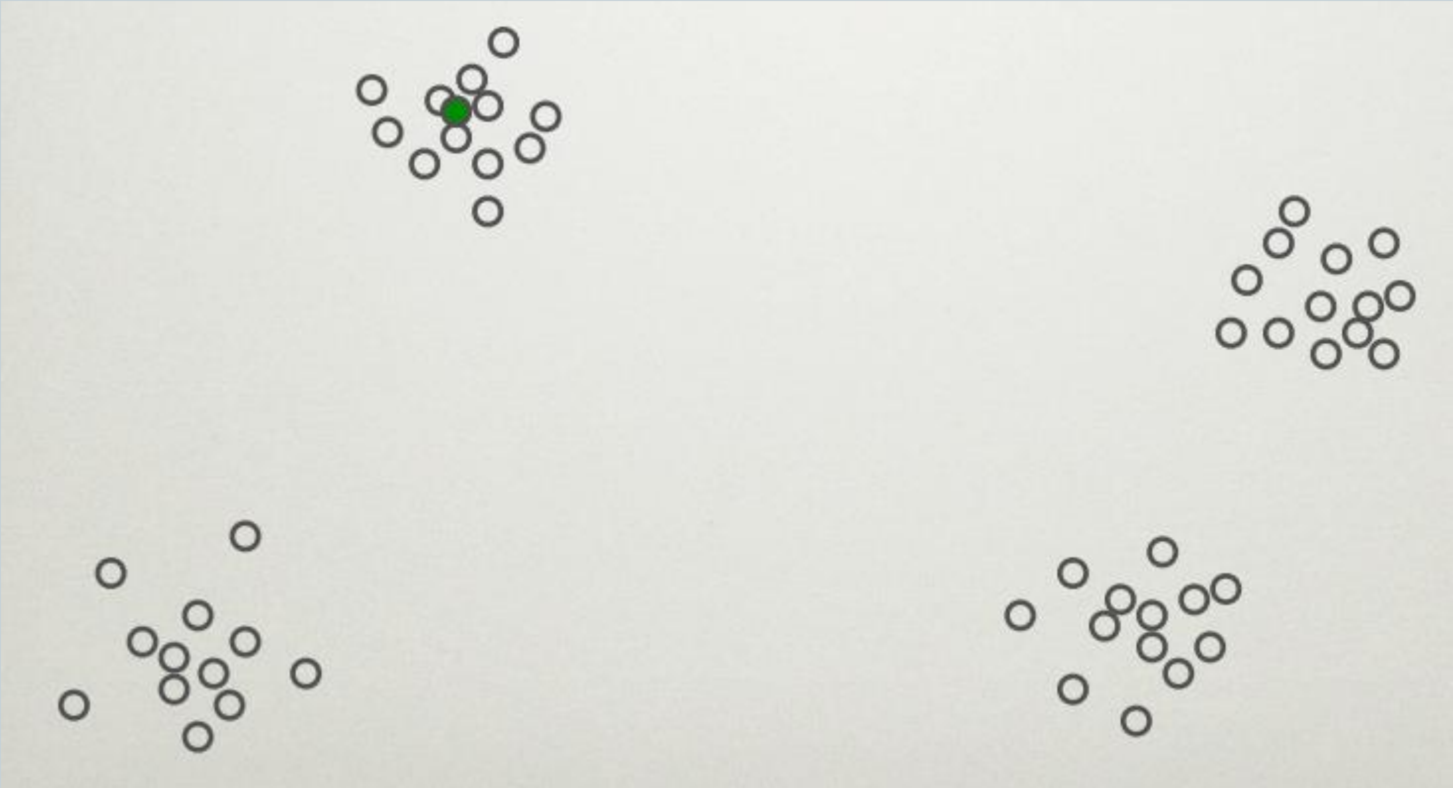
- Bad initialization can lead to:
  - Poor convergence speed
  - Bad overall clustering



Furthest Point Algorithm

- For better initialization of cluster centers
- Algorithm:
  - Pick first center randomly
  - Next center is the point furthest from the first center
  - Next center is the point furthest from both previous centers
  - In general: next center is $\arg\max_x \min_c d(x,c)$

Furthest Point Algorithm



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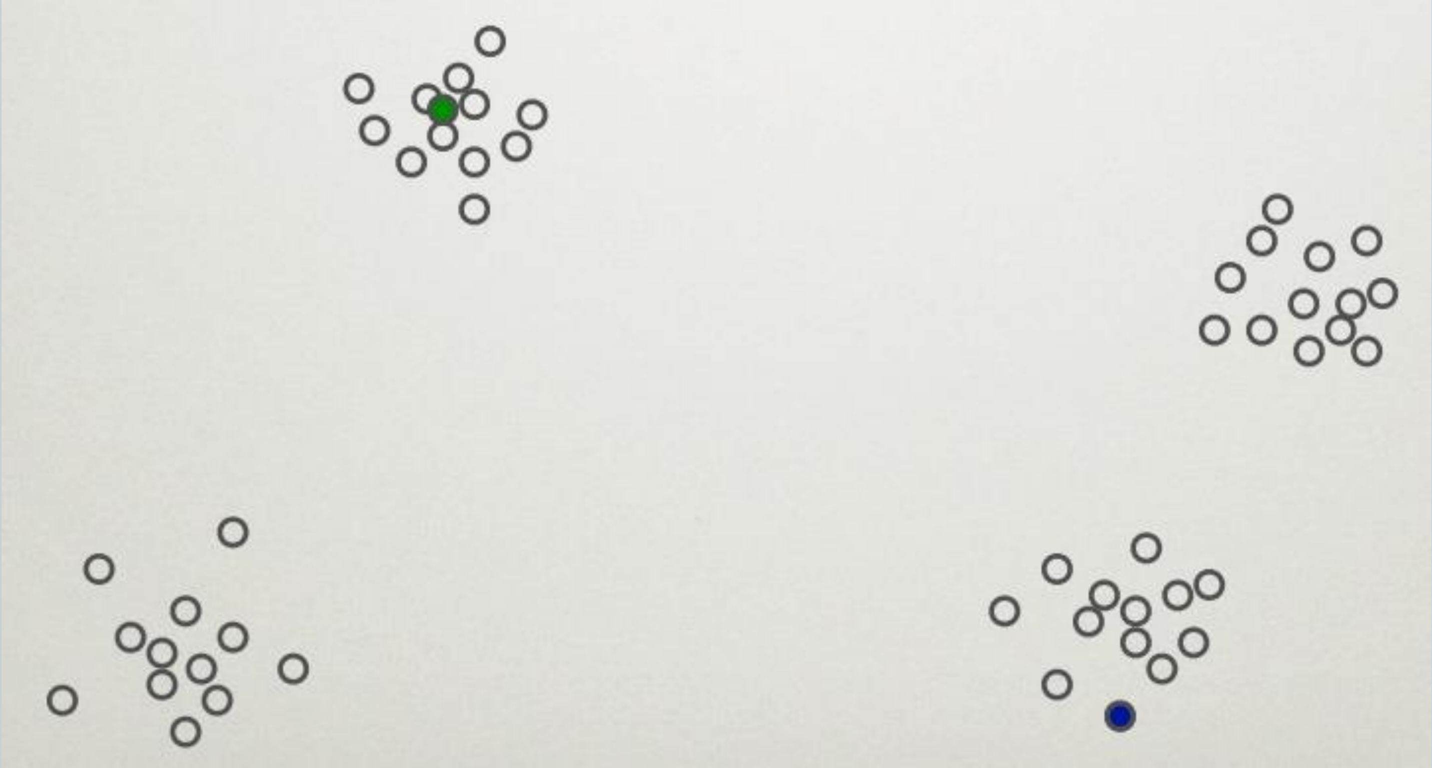
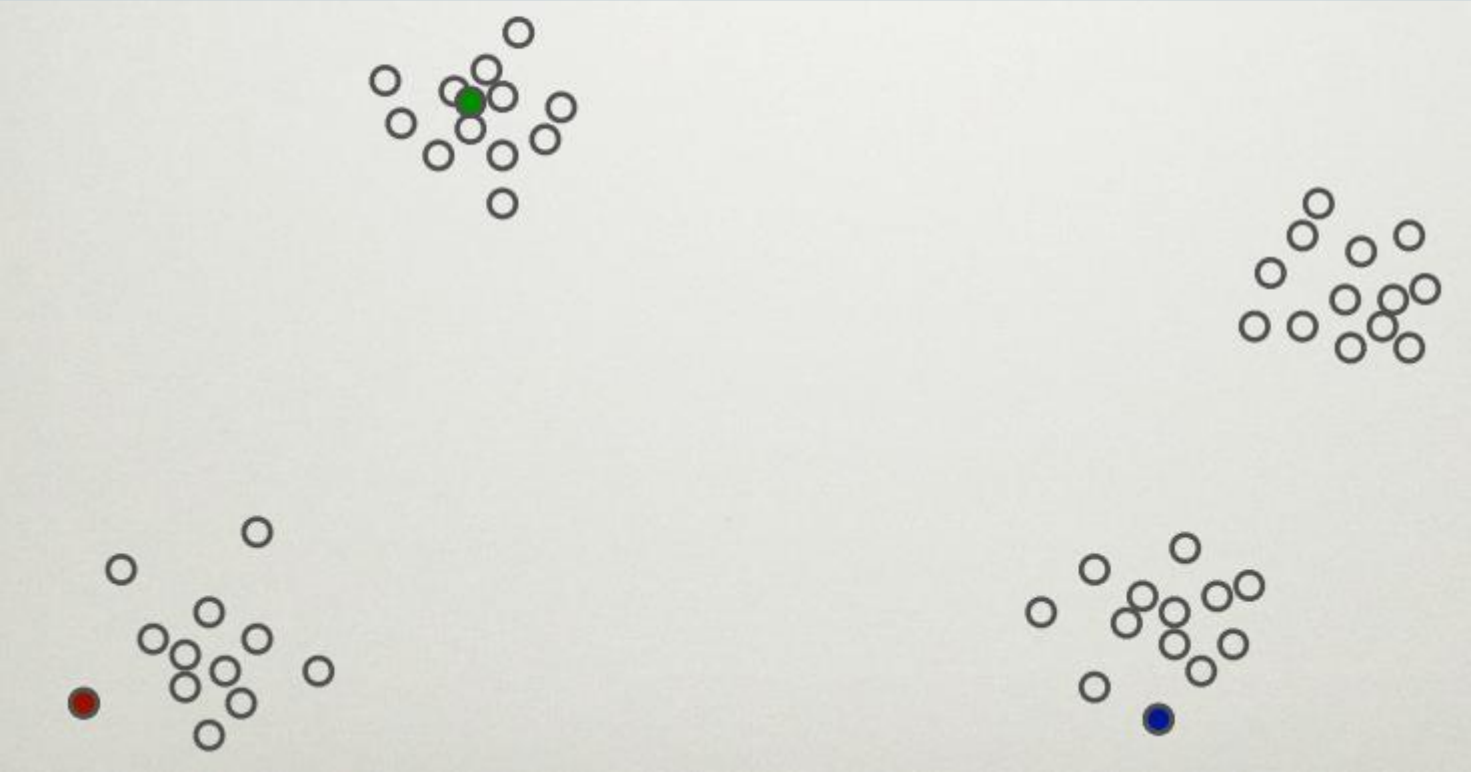


Furthest Point Algorithm



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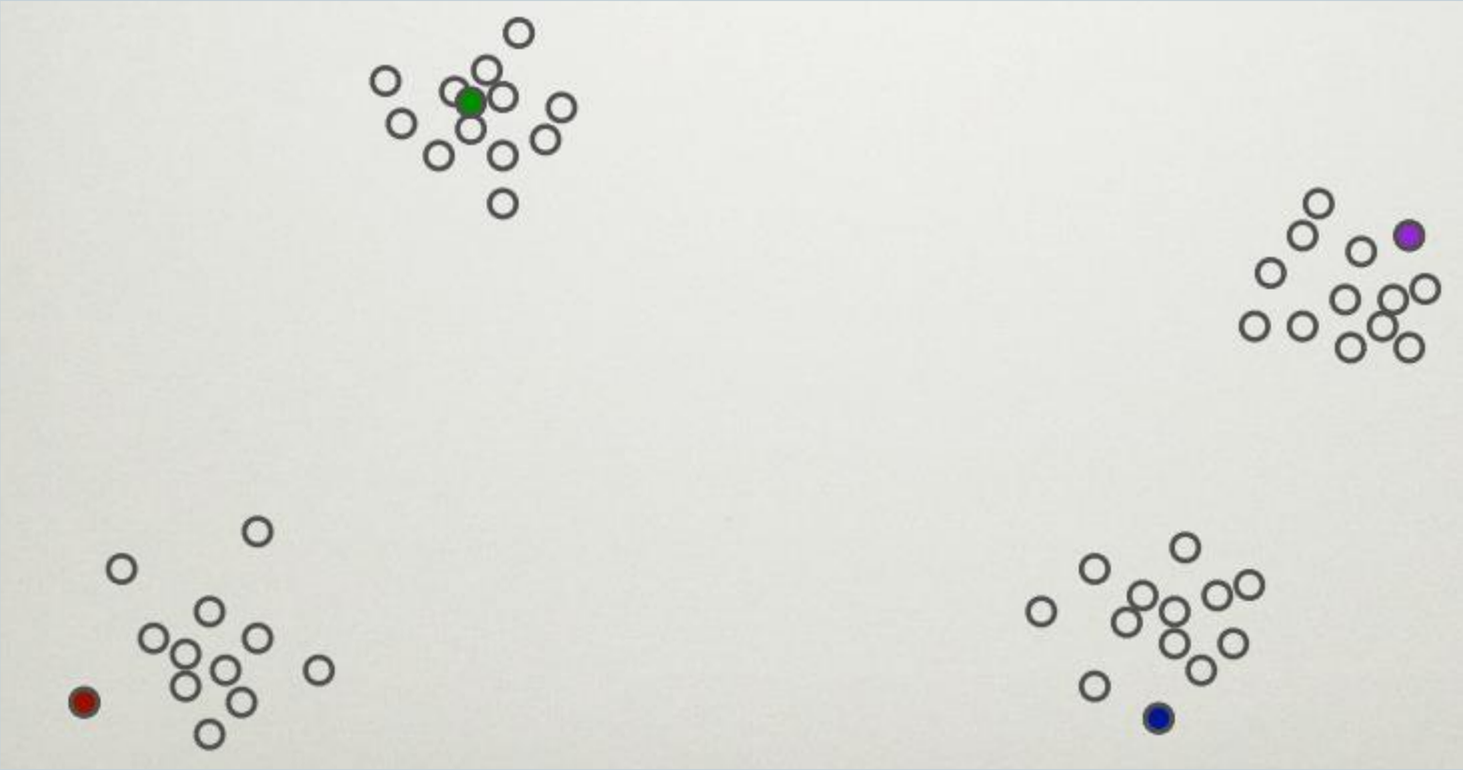

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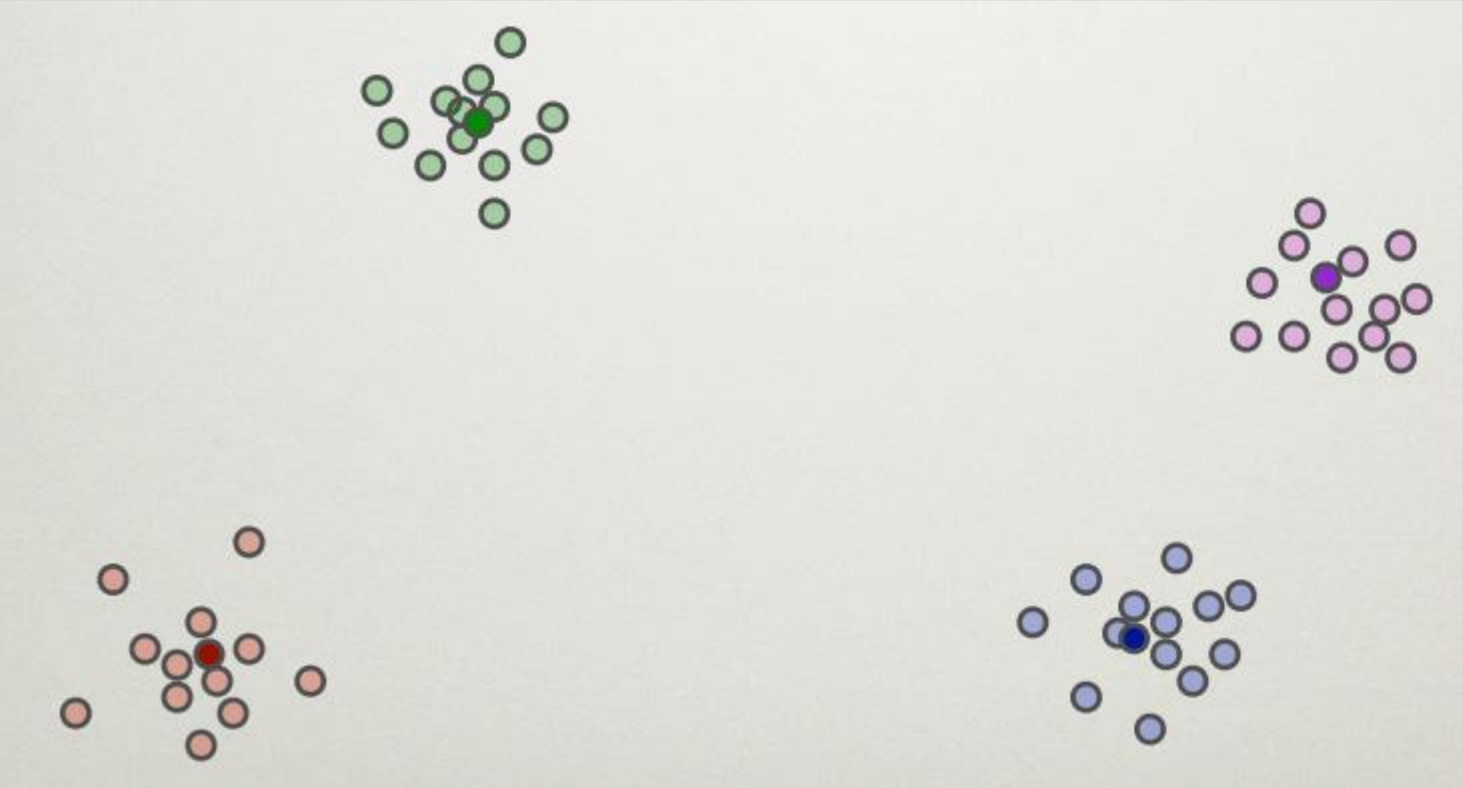
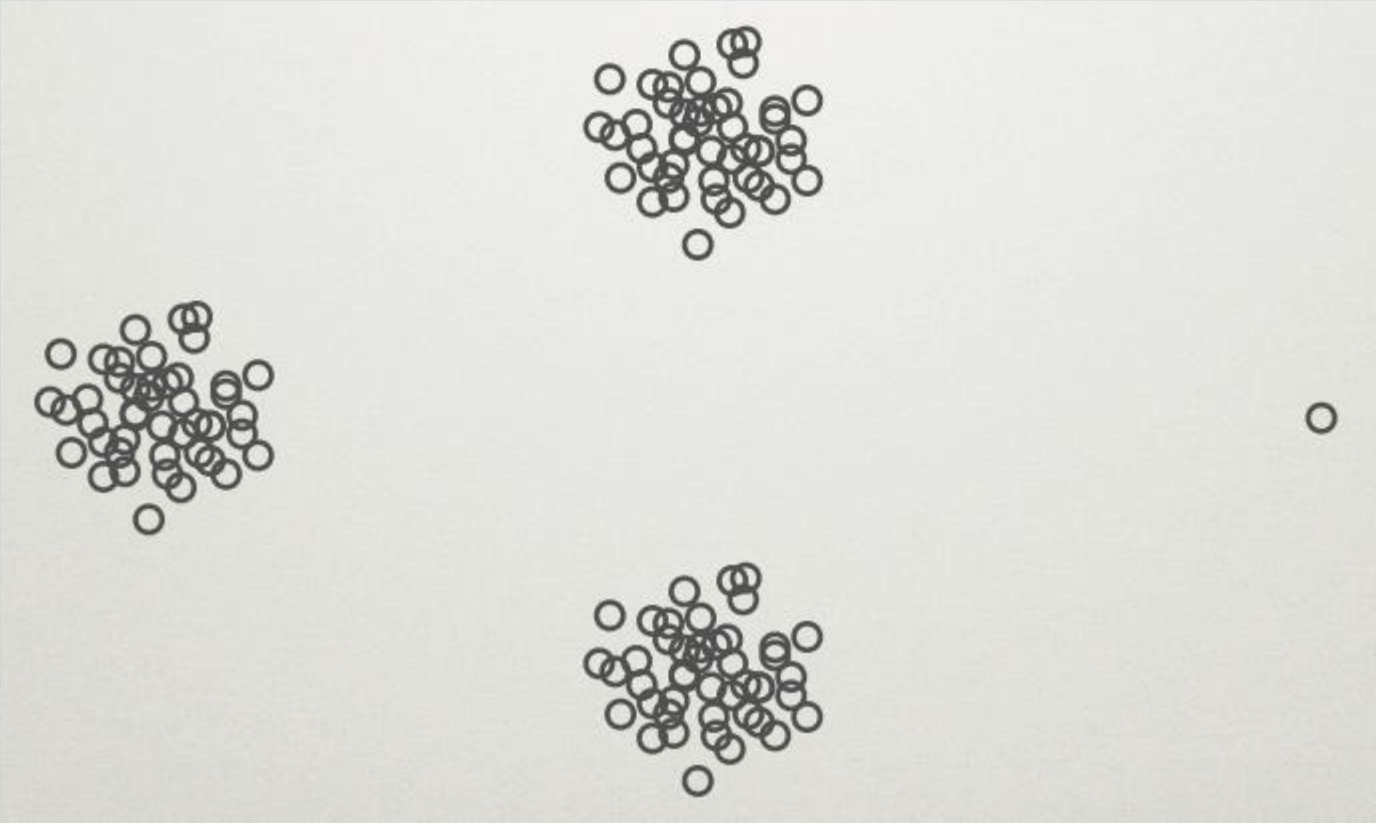


Sensitivity to Outliers



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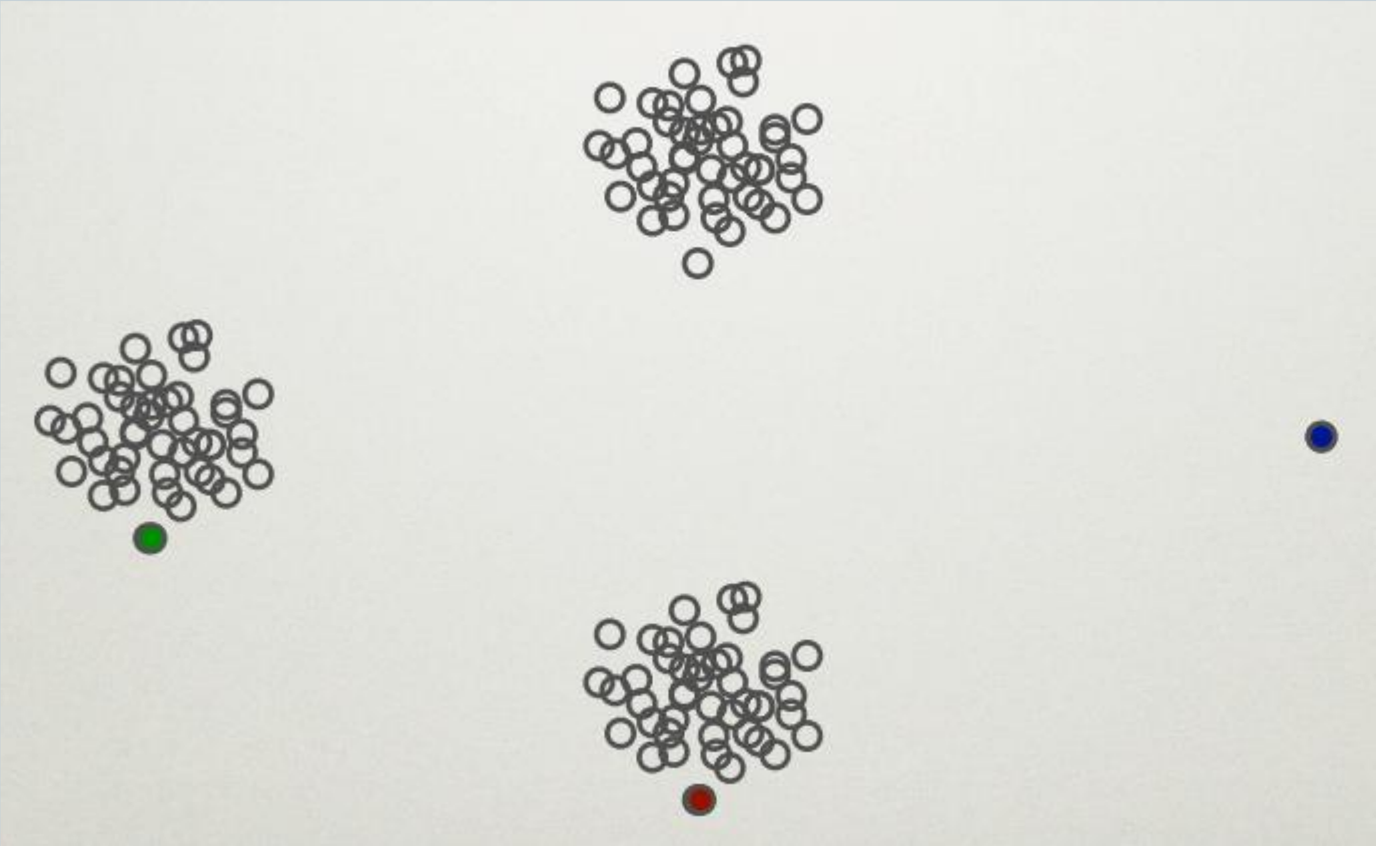
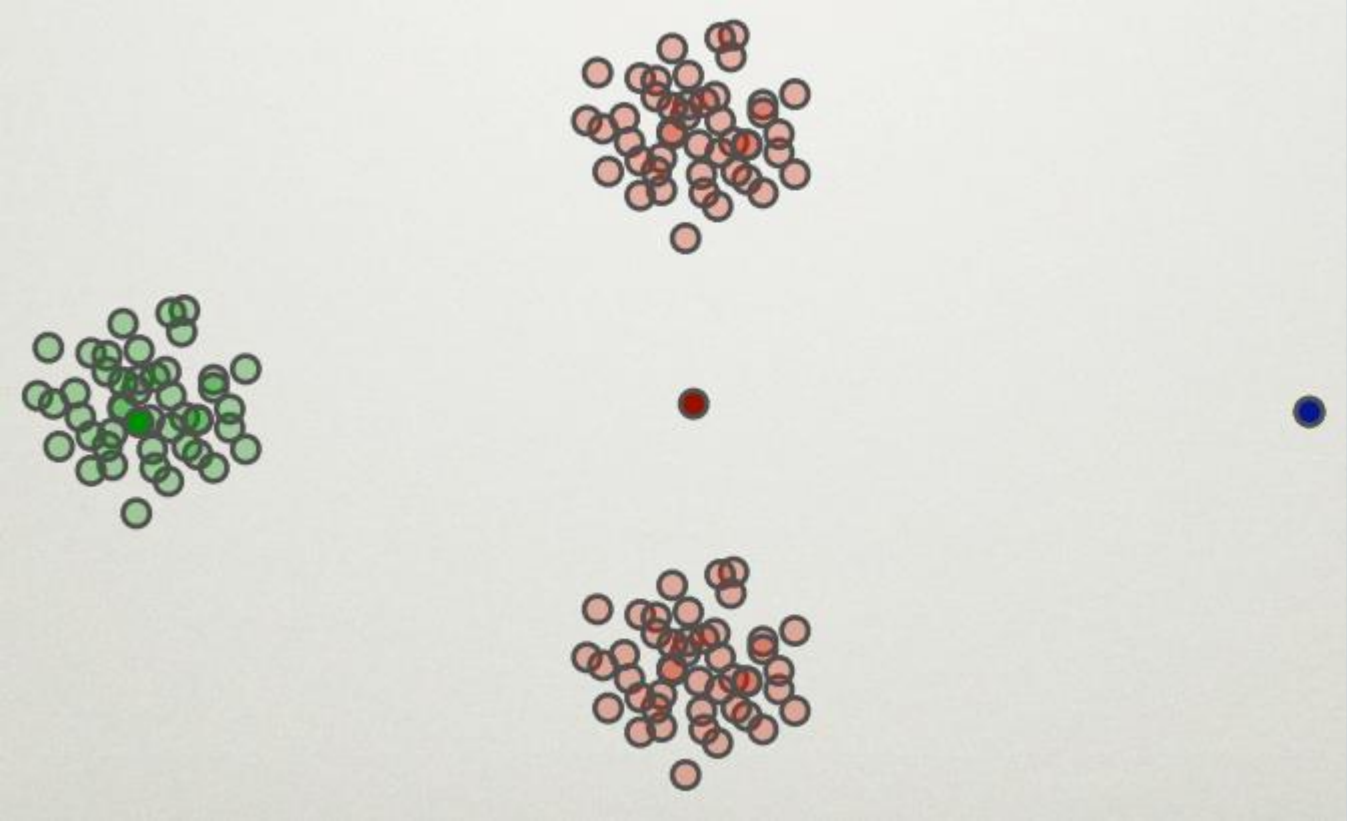


Sensitivity to Outliers



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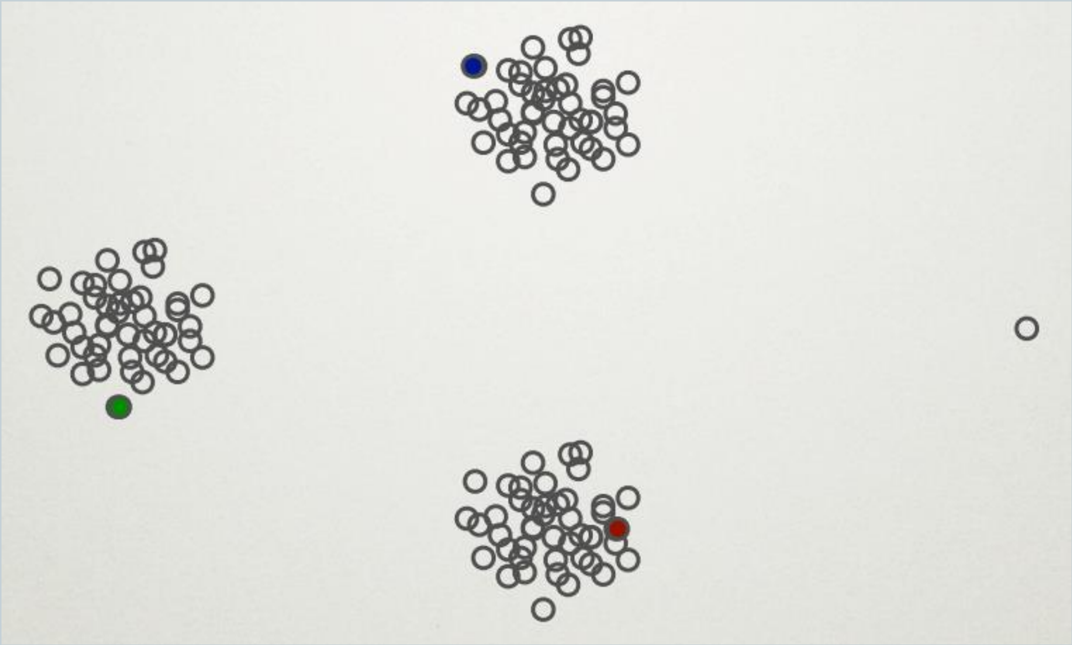
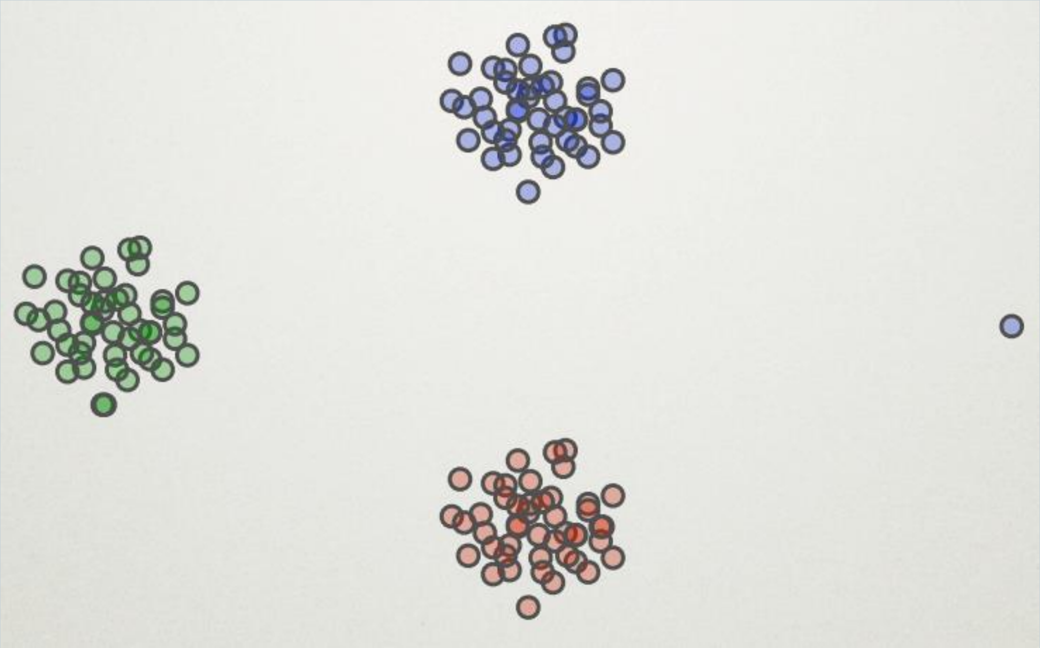
K-Means ++

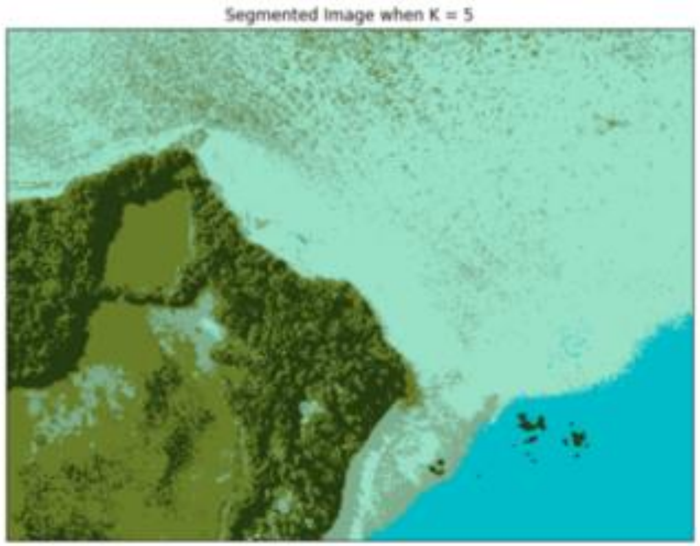
- Provides better initialization as well as better sensitivity to outliers
- Algorithm:
  - Randomly choose first center
  - Pick new center with probability proportional to $(x - c_i)^2$
  - Repeat for K centers

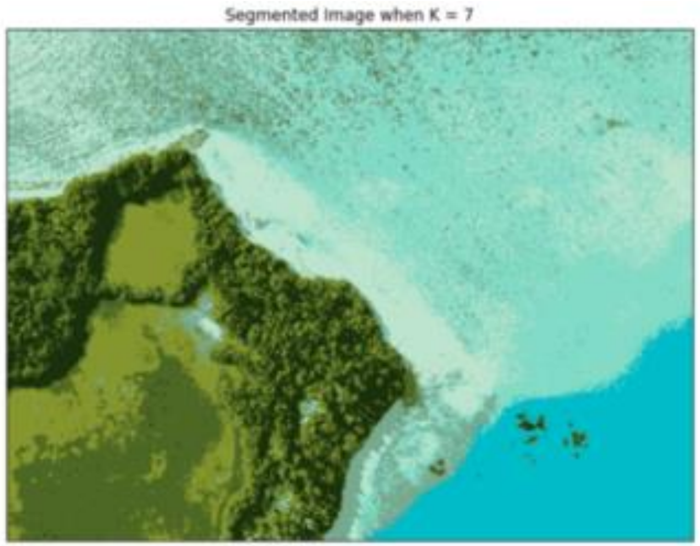
K-Means ++



K-Means ++







https://towardsdatascience.com/introduction-to-image-segmentation-with-k-means-clustering-83fd0a9e2fc3

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References and Resources

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- Slide sources
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  - Sergei Vassilvitskii and David Arthur (http://theory.stanford.edu/~sergei/slides/BATS-Means.pdf)
  - http://shabal.in/visuals/kmeans/KMeansPlusPlus.pdf