LU Factorization

We are going to study the LU factorization as a method to directly solve a system of linear equations.

- Write a code which performs an LU factorization on a general matrix A (without pivoting) and solve the corresponding linear system $A \cdot x = b$ (directly using available LU libray functions is not allows this time).
- Apply your method to the following test system

$$\begin{bmatrix} 7 & 3 & -1 & 2 \\ 3 & 8 & 1 & -4 \\ -1 & 1 & 4 & -1 \\ 2 & -4 & -1 & 6 \end{bmatrix} \cdot x = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

- Perform benchmarks for some random matrices of different size. How does the algorithm scale with the system size? Visualize the results using a log-log plot.
- Are there any linear systems which cannot be solved using your algorithm? Why? Try e.g.:

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$