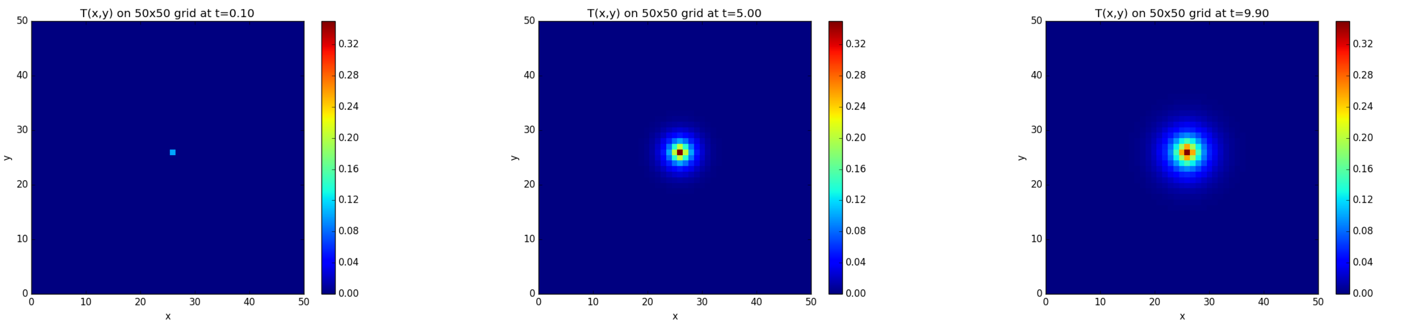
Heat-Equation

We are going to study simple solutions of the time-dependent heat-equation in 2D:

$$\frac{\partial u}{\partial t} = \alpha \Delta u + f$$

with temperature u and thermal diffusivity α. Discretizing the spacial part of the differential equations using finite differences is the same as for the stationary poisson equation from the last exercise. The resulting system of ODEs still needs to be discretises in time. For this purpose the explicit and implicit Euler method from the ODE exercise should be reused.

A rectangular region should be considered. For sake of simplicity 0-dirichlet boundary conditions should be used and a point-source $f = 1$ should be defined in the center of the region.



This time the focus of the exercise should also be an efficient implementation of the sparse operators resulting from the FD discretization. The evaluation of the Laplace operator (right-hand side of the heat equation) should be benchmarked for different system sizes. The optimal performance of the sparse matrix-vector multiplication should be $O(N)$, compared with $O(N^2)$ for dense systems, where N is the system size.

For following results should be created:

- visualization of the 2D solution for different times
- benchmark results for different system sizes (up to 150×150)