

We are going to study the Conjugate Gradient(CG) Method as a method to iteratively solve a system of linear equations and compare it to a naive steepest decent method.

- Write a two codes which performs the CG and the steepest decent iteration, respectively (directly using available library functions is not allowed).
- Apply your method to the following test system

$$\begin{bmatrix} 7 & 3 & -1 & 2 \\ 3 & 8 & 1 & -4 \\ -1 & 1 & 4 & -1 \\ 2 & -4 & -1 & 6 \end{bmatrix} \cdot x = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

- Perform benchmarks for matrices of different size (use e.g. the matrices of the 2D Laplacian of the last exercise). Visualize the scaling of the algorithm using a log-log plot and compare it with the results of the LU solvers.
- Are there any linear systems which cannot be solved using your algorithm? Why?