ODEs 2 - Satellite Orbit

We are going to study the orbit of a satellite in Earth's gravitational field. Due to symmetry our problem can be solved in 2 dimensions. We start with a satellite in a geostationary orbit (distance from earth center $r = 42\,157\,\mathrm{km}$). The graviational force is given by

$$F = -G\frac{m\, m_e}{r^3}\vec{r}$$

with the gravitational constant $G = 6.674\,08 \times 10^{-11}\,\mathrm{m^3kg^{-1}s^{-2}}$, Earth mass $m_e = 5.972 \times 10^{24}\,\mathrm{kg}$, mass of the satellite m and the distance vector $\vec{r}$.

The following questions should be answered:

- the corresponding (linear) system of first order ODEs needs to be determined:

$$\begin{aligned}\dot{\mathrm{x}} &= \mathrm{f}(t, \mathrm{v})\\ \dot{\mathrm{v}} &= \mathrm{g}(t, \mathrm{x})\end{aligned}$$

- compare the explicit Euler with the following symplectic Euler method and compare the performance and stability:

$$\begin{aligned}\mathrm{x}_{n+1} &= \mathrm{x}_n + \Delta t\, \mathrm{f}(t_n, \mathrm{v}_n)\\ \mathrm{v}_{n+1} &= \mathrm{v}_n + \Delta t\, \mathrm{g}(t_n, \mathrm{x}_{n+1})\end{aligned}$$

- determine the period time T of the satellite in the geostationary orbit
- assume that in the interval $t = [2T \ldots 3T]$ the engine is fired and creates a tangential force $f = 0.01m$
- visualize the stable solution