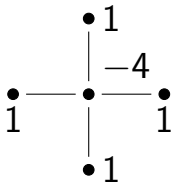
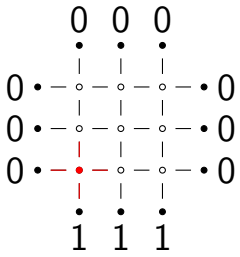
We are going to study simple solutions of the Poisson equation. A rectangular region should be discretized using finite differences (FD) on an equidistant grid ($\Delta x = \Delta y = 1$) using a 5-point-stencil. We define dirichlet boundary conditions which are 1 on the $y = 0$ boundary and 0 everywhere else.





For example the assembled system matrix A as well as the right-hand-side vector b looks like

$$\begin{bmatrix} -4. & 1. & 0. & 1. & 0. & 0. & 0. & 0. & 0. \\ 1. & -4. & 1. & 0. & 1. & 0. & 0. & 0. & 0. \\ 0. & 1. & -4. & 0. & 0. & 1. & 0. & 0. & 0. \\ 1. & 0. & 0. & -4. & 1. & 0. & 1. & 0. & 0. \\ 0. & 1. & 0. & 1. & -4. & 1. & 0. & 1. & 0. \\ 0. & 0. & 1. & 0. & 1. & -4. & 0. & 0. & 1. \\ 0. & 0. & 0. & 1. & 0. & 0. & -4. & 1. & 0. \\ 0. & 0. & 0. & 0. & 1. & 0. & 1. & -4. & 1. \\ 0. & 0. & 0. & 0. & 0. & 1. & 0. & 1. & -4. \end{bmatrix} \cdot \mathrm{u} = \begin{bmatrix} -1. \\ -1. \\ -1. \\ 0. \\ 0. \\ 0. \\ 0. \\ 0. \\ 0. \end{bmatrix}$$

The following steps need to be performed to solve the problem

- define 2D grid (vertex numbering)
- setup 2D Laplacian
- define boundary conditions / sources
- solve linear system using `scipy` solvers
- visualize the 2D solution: