

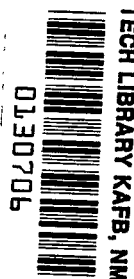
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## CHARACTERISTICS OF A SPLIT PHASE TELECOMMUNICATIONS LINK

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*Houston, Texas*



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## ABSTRACT

A split phase telecommunications link is described and its performance analyzed. A spectral analysis of the transmitted signal is performed. An analysis is made of the operation of a carrier tracking phase-lock loop receiver when its input is a split phase signal. The effects of increased loop bandwidth and of a noisy phase reference are presented. The data signal output of the loop as a function of time is obtained.

## CHARACTERISTICS OF A SPLIT PHASE

### TELECOMMUNICATIONS LINK

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Manned Spacecraft Center

#### SUMMARY

This paper presents analyses and results that are useful in describing the characteristics of a split phase telecommunications link where the split-phase-encoded data directly phase-modulate a carrier signal. The transmitted signal is analyzed, with the carrier-data power division and the power spectrum being obtained. The operation of a carrier tracking phase-lock loop receiver when its input is a split-phase-modulated signal is analyzed. Several results are presented in graphical form, which can be used in the analysis of performance of split phase telecommunications links. The analysis approach and some of the results are applicable generally to digital telecommunications links.

#### INTRODUCTION

In the design of a telecommunications link, the modulation method is a very important factor. When several different types of data (such as voice, experimental data, pulse-code-modulation (PCM) telemetry) are transmitted over one radio-frequency link, a separate subcarrier is often used for each data source. The modulated subcarriers then modulate the amplitude, phase, or frequency of the carrier. This approach minimizes the required number of transmitters and allows independent handling of data at the transmitter and receiver. In many cases, however, only one type of data (such as PCM) is to be transmitted, and direct modulation of the carrier (no subcarriers) should be considered. Basic reasons for considering direct modulation are the following: (1) Special subcarrier demodulators are not required at the receiver; and (2) the required frequency bandwidth is minimized when the link bit rates are switchable. To retain these advantages, the modulation must be compatible with existing phase-lock loop receivers and associated equipment such as that used by the National Aeronautics and Space Administration (NASA), Manned Space Flight Network (MSFN). The results can be obtained by a correctly designed split-phase-modulation method.

This paper first presents a definition of the split phase modulating signal, followed by a discussion of the division of power between the data signal and the carrier signal. The power spectrum of the split phase signal is next derived, followed by analyses of the output of a phase-lock loop receiver when the input carrier is directly modulated.

An attempt has been made to analyze the problem and present results that are of general use. However, specific examples have been given in several cases using parameters of actual hardware to give realism to the problem.

## SYMBOLS

$A$	amplitude, carrier-to-noise ratio, dB
$a$	generalized frequency, Hz
$B_i$	predetection or limiter bandwidth, Hz
$B_L$	loop noise bandwidth, Hz
$B_{LO}$	loop noise bandwidth at threshold, Hz
$E(s)$	transform of the loop data output signal
$e(t)$	transmitted signal or demodulated data signal
$f$	frequency, Hz
$f_s$	bit rate, $\frac{1}{T}$ , bits/sec
$G_1(f), G_2(f)$	Fourier transforms of $g_1(t)$ and $g_2(t)$ , respectively
$g_1(t), g_2(t)$	functions representing digital states
$j$	square root of -1
$K$	square of the loop natural frequency
$m$	index of summation
$N_O$	noise at threshold
$\left(\frac{N}{S}\right)_i$	noise-to-signal ratio in the predetection or limiter bandwidth, dB
$n$	an integer
$P_c$	carrier power, dB
$P_d$	data power, dB

$P_T$	total power in transmitted signal $e(t)$
$p, 1 - p$	state probabilities
$\frac{S}{N}$	signal-to-noise ratio, dB
$s$	Laplace transform variable
$T$	bit period, sec/bit
$t$	time, sec
$W_s(f)$	power spectral density, watts/Hz
$\alpha$	limiter suppression factor
$\alpha_O$	limiter suppression factor at threshold
$\beta$	peak phase shift or deviation, rad
$\delta(f)$	delta function
$e$	natural logarithm base
$\zeta, \zeta_O$	loop damping factor
$\theta_i(s)$	Laplace transform of $\theta_i(t)$
$\theta_i(t)$	phase modulating signal
$\tau$	translated time variable
$\theta(t)$	data signal
$\omega$	radian frequency, rad/sec
$\omega_c$	carrier frequency, rad/sec
$\omega_n$	loop natural frequency, rad/sec
$\omega_{nO}$	loop natural frequency at threshold, rad/sec

## SPLIT PHASE DEFINED

Split phase modulation is defined with the aid of figure 1. The split phase signal is also known as bi-phase-level or Manchester II + 180°. In the split phase method, a data "1" is represented by "1, 0" and a "0" is represented by "0, 1." The peak phase deviation of the carrier is  $\beta$  radians. A phase change occurs during every bit period  $T$ . The split phase modulating signal is the same as the signal obtained when the data coherently bi-phase-modulate a square-wave subcarrier whose frequency is equal to the bit rate.

Note that the carrier is phase modulated rather than frequency modulated. Phase modulation is used since coherent phase demodulation yields optimum performance and the lowest bit error probability (ref. 1). In addition, a binary phase-modulated transmitter is easily implemented, and the phase-modulated signal is compatible with existing receivers such as those used by the MSFN.

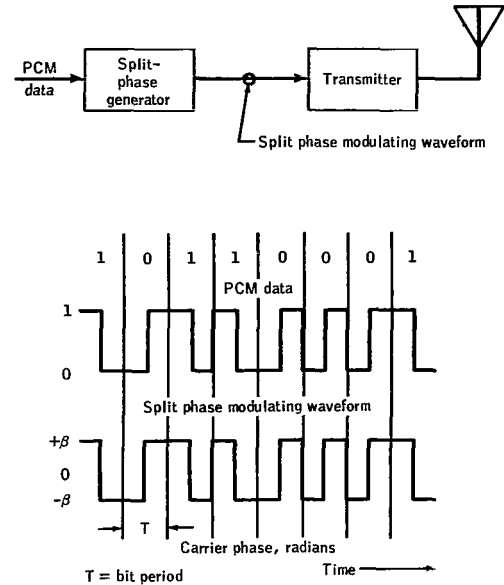


Figure 1. - Waveforms defining split phase modulation.

## POWER DIVISION BETWEEN CARRIER AND DATA SIGNAL

The process of binary phase modulation of the carrier signal causes a division of the total power. The power divides so that a portion of the power is contained in the data sidebands, and the remainder is at the carrier frequency. The exact equations for the power division are required for a detailed analysis of the performance of the telecommunications link.

The equation for the transmitted signal can be written as

$$e(t) = A \sin [\omega_c t + \beta \theta(t)] = A [\sin \omega_c t \cos \beta \theta(t) + \cos \omega_c t \sin \beta \theta(t)] \quad (1)$$

where

$A$  is the amplitude

$\omega_c$  is the carrier frequency in rad/sec

$\beta$  is the peak phase shift or deviation, rad

$\phi(t)$  is the data signal

The function  $\phi(t)$  represents a binary waveform that takes on the values  $\pm 1$ . Equation (1) can then be rewritten as

$$e(t) = A \left[ \cos \beta \sin \omega_c t + \sin \beta \phi(t) \cos \omega_c t \right] \quad (2)$$

The first term in equation (2) is an unmodulated carrier with a power of

$$P_c = \frac{A^2 \cos^2 \beta}{2} \quad (3)$$

The second term is a modulated carrier with a power of

$$P_d = \frac{A^2 \sin^2 \beta}{2} \quad (4)$$

The total power in the signal is

$$P_T = \frac{A^2}{2} \quad (5)$$

Therefore,

$$P_c = P_T \cos^2 \beta \quad (\text{carrier power}) \quad (6)$$

$$P_d = P_T \sin^2 \beta \quad (\text{data power}) \quad (7)$$

Equations (6) and (7) state that the power between carrier signal and data signal, when the modulation is binary, divides according to  $\cos^2 \beta$  and  $\sin^2 \beta$ , respectively. Note that the binary signal  $\phi(t)$  can be any binary signal and is not limited to being a split phase signal.



Equations (1) to (7) can now be used to select  $\beta$ , the modulation index. The value of  $\beta$  must be chosen so that (1) the carrier power is adequate for receiver carrier tracking purposes, and (2) the data power is adequate for the required bit error probability. As a first approximation, the following expression can be used to select  $\beta$ .

$$\frac{\sin^2 \beta}{\cos^2 \beta} = \tan^2 \beta = \frac{\left( \frac{S}{N} \text{ required in data bandwidth} \right) (\text{data bandwidth})}{\left( \frac{S}{N} \text{ required in carrier bandwidth} \right) (\text{carrier bandwidth})} \quad (8)$$

Note that the carrier power  $P_c$  is zero when  $\beta = \frac{\pi}{2}$  radians. Therefore, to place most of the power in the data, a reasonable initial example value of  $\beta$  is 1.25 radians (71.6°). The carrier power is then about 10 dB below  $P_T$ , and the data power is approximately 0.5 dB below  $P_T$ . Detailed link performance calculations can be made to determine if the modulation index is satisfactory; if not, it can be readjusted.

#### POWER SPECTRUM OF THE SPLIT PHASE SIGNAL

The power spectrum is useful in determining the bandwidth occupancy of the transmitted signal and in analyzing receiver performance. Of primary interest is the case where the PCM data bits are random with equal probability of a "1" or "0." The power spectral density of a random binary signal in which the "1" state and the "0" state are represented by arbitrary waveforms (ref. 2) is given by equation (9).

$$W_s(f) = 2f_s p(1-p) \left| G_1(f) - G_2(f) \right|^2 + f^2 \left[ pG_1(0) + (1-p)G_2(0) \right]^2 \delta(f) + 2f_s^2 \sum_{m=1}^{\infty} \left| pG_1(mf_s) + (1-p)G_2(mf_s) \right|^2 \delta(f - mf_s) \quad (9)$$

where

$W_s(f)$  is the power spectral density in watts/Hz, or similar units

$f_s$  is the bit rate or  $\frac{1}{T}$

$p$  is the probability of one state

$1 - p$  is the probability of the other state

$g_1(t)$  is the function representing one state

$g_2(t)$  is the function representing the other state

$G_1(f)$  is the Fourier transform of  $g_1(t)$

$G_2(f)$  is the Fourier transform of  $g_2(t)$

$\delta(f)$  is the delta function

For our case,

$$p = 1 - p = \frac{1}{2} \quad (10)$$

$$g_1(t) = -g_2(t) = 1 \quad (11)$$

where  $-\frac{T}{2} \leq t < 0$ , and

$$g_1(t) = -g_2(t) = -1 \quad (12)$$

where  $0 \leq t < \frac{T}{2}$ .

Therefore,

$$W_s(f) = \frac{f_s}{2} \left| G_1(f) - G_2(f) \right|^2 = 2f_s \left| G_1(f) \right|^2 \quad (13)$$

Now

$$G_1(f) = \int_{-\infty}^{\infty} g_1(t) e^{-j\omega t} dt = -\frac{4}{j\omega} \sin^2 \frac{\omega T}{4} \quad (14)$$

so that

$$W_s(f) = 2T \frac{\sin^4\left(\frac{\pi f T}{2}\right)}{\left(\frac{\pi f T}{2}\right)^2} \quad (15)$$

Equation (15) is plotted in figure 2. Note that there is no dc component; that is, there is no data component at the carrier frequency, and the "clean" carrier can be tracked by a phase-lock loop receiver. However, if the bandwidth of the receiver carrier tracking loop is wide, or if the data bit rate is low, some of the data power will be within the loop bandwidth. This power will act as noise in the loop to degrade the tracking loop performance. In addition, some degradation of the data will occur. Therefore, the amount of power within the loop bandwidth must be determined for detailed analyses. This information can be obtained by integrating the expression for the power spectral density either graphically or with a computer. Figure 2 presents the result in terms of the fraction of power passed by a rectangular low pass filter whose one-sided bandwidth is equal to  $f$ .

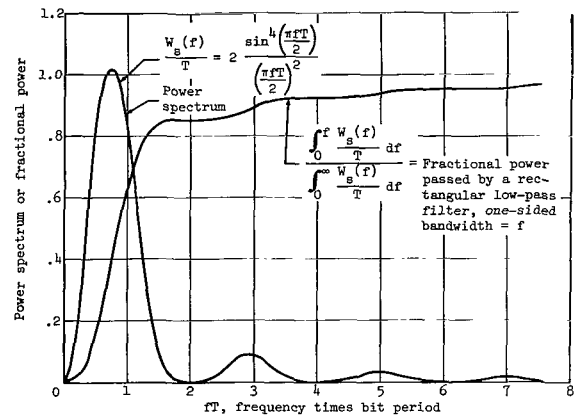
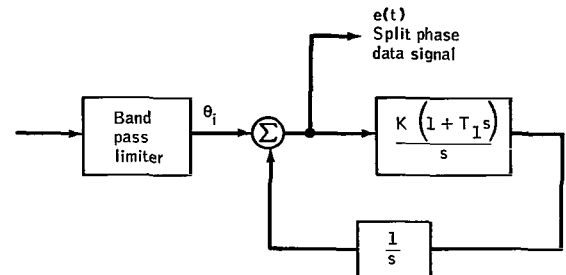


Figure 2. - Power spectrum and fractional power for a split phase signal.

## RECEIVER CARRIER TRACKING PHASE-LOCK LOOP OUTPUT

### Phase-Lock Loop Equations and Loop Bandwidth Variation

At the receiver, the split phase data signal can be recovered at the output of the carrier tracking loop phase detector, as shown in simplified form in figure 3. (Refer to references 3 to 6 for derivations of the equations describing the behavior of a phase-lock loop. Most of the equations are used here without proof.) The tracking loop noise bandwidth increases as the input signal power increases because of the action of the limiter (used to cancel the gain of the loop multiplier, which acts as a phase



Equations:

$$E(s) = \frac{s^2}{s^2 + K T_1 s + K} \theta_i(s)$$

or

$$E(s) = \frac{s^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \theta_i(s)$$

$$\text{At threshold, } \zeta = \zeta_0 = \frac{\sqrt{2}}{2}$$

$$\omega_n = \omega_{n0} = \frac{2B_{LO}}{1.06}$$

$$2B_{LO} = \text{loop noise bandwidth in Hz (at threshold)} = 50 \text{ Hz (typical)}$$

Figure 3. - Linear model of receiver carrier phase-lock loop.

detector, and thereby to maintain the loop gain constant under no-noise conditions) according to the equation

$$2B_L = \frac{2B_{LO}}{3} \left( 1 + 2 \frac{\alpha}{\alpha_O} \right) \quad (16)$$

where  $\alpha$  is the limiter suppression factor, and  $\alpha_O$  is this factor at threshold. (Threshold is usually defined as occurring when the signal-to-noise ratio in  $2B_{LO}$  is unity.) Since the bandwidth increases, some of the data power can be lost (particularly at low bit rates), or it could create interference problems in the tracking loop.

The limiter suppression factor is given by

$$\alpha = \frac{1}{\sqrt{1 + \frac{4}{\pi} \left( \frac{N}{S} \right)_i}} \quad (17)$$

where  $\left( \frac{N}{S} \right)_i$  is the noise-to-signal ratio in the predetection (or limiter) bandwidth. At threshold,  $\alpha = \alpha_O$  and the signal-to-noise ratio in  $2B_{LO}$  is unity; the noise-to-signal ratio in a predetection bandwidth of  $B_i$  is  $B_i/2B_{LO}$ . Substituting into the loop noise bandwidth equation gives

$$2B_L = \frac{2B_{LO}}{3} \left[ 1 + 2 \sqrt{\frac{1 + \frac{4}{\pi} \frac{B_i}{2B_{LO}}}{1 + \frac{4}{\pi} \left( \frac{N}{S} \right)_i}} \right] \quad (18)$$

Equation (18) is plotted in figure 4 for  $2B_{LO} = 50$  Hz and for  $B_i = 7000$  Hz.

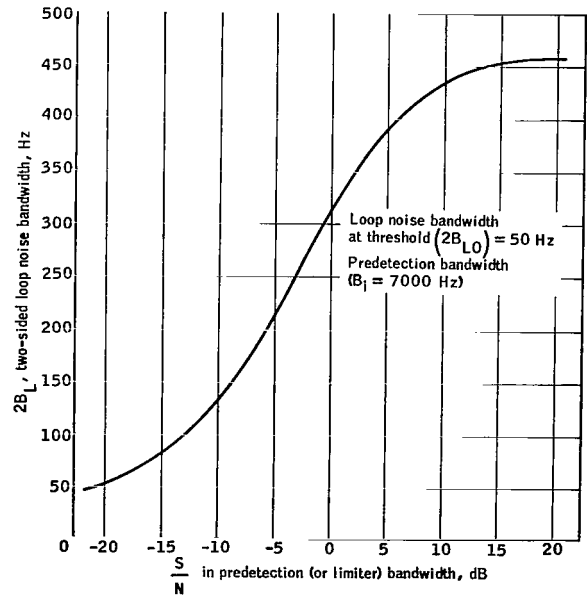


Figure 4. - Loop bandwidth versus signal-to-noise ratio.

### Ratio of Carrier Power to Data Power in $2B_L$

A quick, gross estimate of the effect of loop bandwidth on link performance is obtained by calculating the ratio of carrier power to data power in the loop bandwidth. For example, assume that the predetection signal-to-noise ratio is 5.8 dB. From figure 4, the loop noise bandwidth is  $2B_L = 400$  Hz. Assume further that the bit rate is 1000 bits/sec and that the modulation index is  $\beta = 1.25$  radians. Figure 2 shows that the ratio of data power in a loop bandwidth of 400 Hz to the total data power is about 0.02, or slightly less. Thus, the ratio of carrier power to data power in  $2B_L$  is

$$\frac{P_c}{P_d} = \left( \frac{0.1}{0.9} \right) \left( \frac{1}{0.02} \right) = 5.55 \quad (19)$$

or 7.45 dB. (The ratio of 0.1/0.9 results from the choice of modulation index, which places 10 percent of the total power in the carrier signal and 90 percent in the data signal.)

The ratio of  $P_c/P_d$  can be compared to the ratio of carrier power to thermal noise in the loop, and the effects can be estimated. Specifically, if the ratio of carrier to thermal noise power is considerably smaller than the ratio of carrier to data power, the effect of the data power in the loop bandwidth can be neglected. If desired, more detailed analysis, as presented in the following pages, can be made.

### Effect of Carrier Loop Signal-to-Noise Ratio on Bit Error Probability

Another factor which must be considered in determining the effect of the phase-lock loop on link performance is phase noise. The phase noise causes the phase-lock loop reference carrier phase to have a random variation of phase about the true carrier phase. Thus, the number of bit errors obtained in the detection process is greater than if the reference carrier were noise-free. The 'noisiness' of the reference carrier (and the bit error probability) is a function of the signal-to-noise ratio in the carrier tracking loop. Effects of the noisy reference carrier are given in references 3 and 4. The essential results are presented in figure 5. Note in particular that the bit error probability approaches an irreducible minimum, which is determined by the carrier tracking loop signal-to-noise ratio. Further, this minimum cannot be reduced by increasing the data signal-to-noise ratio.

As an example of the use of figure 5, assume that the predetection (or limiter) signal-to-noise ratio is -6 dB, which gives  $2B_L = 197.5$  Hz. Assume that the total received power is -133 dB relative to 1 milliwatt (dBm) and that the modulation index is  $\beta = 1.25$  radians so that the received carrier power is -143 dBm. The

loop carrier signal-to-noise ratio is given by

$$A = \frac{P_c}{N_O B_L} \quad (20)$$

The signal-to-noise ratio in dB becomes  $A \text{ dB} = -143 + 175 - 20 = 12 \text{ dB}$  by letting  $N_O = -175 \text{ dBm/Hz}$ .

Figure 5 shows that this value of  $A$  does not significantly affect performance for bit error probabilities greater than  $10^{-3}$ . However, if the desired bit error probability is (for example)  $10^{-5}$ , the data power must be approximately 1 dB greater than if the carrier had been noiseless.

For illustration, the preceding example uses some values that do not reflect good link design. In particular, the predetection signal-to-noise ratio would be considerably lower than -6 dB if the predetection bandwidth of 7 kHz from the previous examples had been used, resulting in a lower value of  $2B_L$  and thus a higher value of  $A$ , the carrier-to-noise ratio. This ratio would have indicated no effect on the bit error probability and, therefore, would not have been very illustrative.

The limiter input signal-to-noise ratio (and thus, the value of  $\alpha$ ) is calculated by considering the data power to add to the thermal noise power. Thus, if the total received power is -133 dBm and the modulation index is  $\beta = 1.25$  radians, the carrier power is -143 dBm and the data power is -133.5 dBm. Assume again a predetection bandwidth of 7 kHz, which passes all of the data power at low bit rates, and a noise spectral density of -175 dBm/Hz. The thermal noise power is, therefore,  $-175 + 38.5 = -136.5 \text{ dBm}$ . Hence, the effective noise power is the sum of the data and noise power, or -131.9 dBm. The "effective" limiter signal-to-noise ratio is  $-143 + 131.9 = -11.1 \text{ dB}$ .

Figure 4 shows that the loop noise bandwidth increases to only  $2B_L = 125 \text{ Hz}$ . For this value of  $2B_L$ , the carrier loop signal-to-noise ratio would be greater than 15 dB, and the carrier reference could be considered noiseless for all practical bit error probabilities.

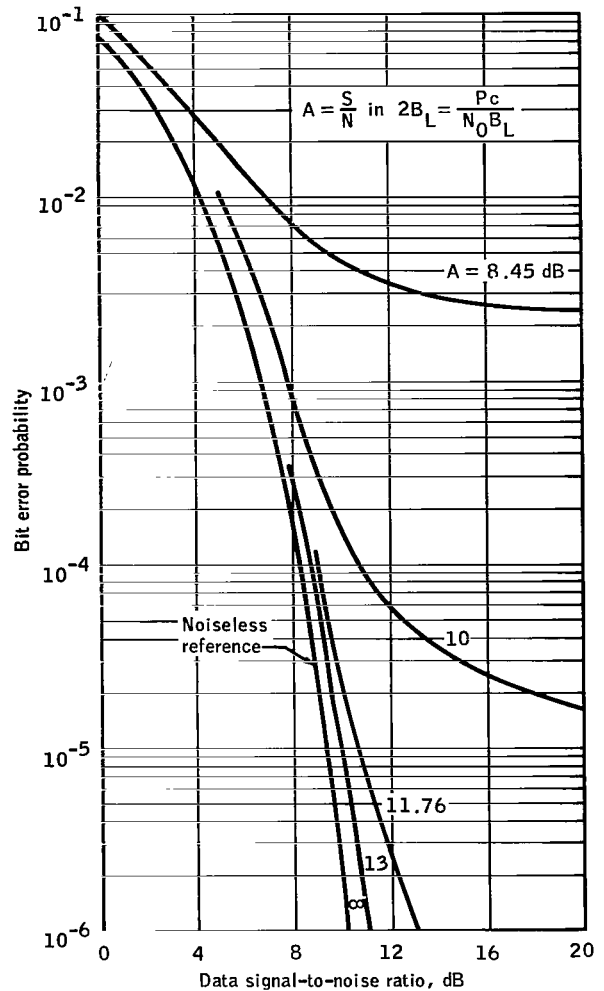


Figure 5. - Bit error probability versus data signal-to-noise ratio for several carrier loop signal-to-noise ratios.

## CONCLUDING REMARKS

The analysis of the performance of a split phase telecommunications link yields the following results of both a general and specific nature.

1. Split phase modulation of the carrier obviates the need for a subcarrier demodulator at the receiving site and yields cost savings.

2. Bit synchronization-decommutation equipment must be designed to process the demodulated data so that the performance of split phase is equivalent to that of the usual nonreturn-to-zero codes.

3. The data bit rate must be large compared to the tracking loop bandwidth so that the performance degradation due to data filtering and phase noise is minimized. If the ratio of bit rate to loop bandwidth is low, a subcarrier should be used to minimize data filtering. The effect of phase noise on bit error probability will remain unchanged.

Using a subcarrier does, of course, allow greater freedom in selecting loop bandwidths to optimize performance. Synchronizers required at the receiver for the subcarrier case are readily obtainable.

4. In a system with several selectable bit rates, split phase is conservative of bandwidth. If a subcarrier is used, the subcarrier frequency must be at least equal to the highest bit rate. This sets the required radio frequency bandwidth, which does not decrease significantly for lower bit rates. However, the bandwidth required for split phase is proportional to the bit rate (fig. 2) so that halving the bit rate halves the required bandwidth. The advantage results from the fact that the data directly modulate the carrier. Other methods of direct carrier modulation will yield a similar advantage.

5. Since split phase has a phase transition for each bit, the bit synchronization problem is simplified. Also, ac-coupled systems can be used since the split phase signal contains no dc component.

Manned Spacecraft Center  
National Aeronautics and Space Administration  
Houston, Texas, July 5, 1967  
914-40-92-99-72

## APPENDIX

### LOOP OUTPUT VERSUS TIME

To obtain further insight into link performance, the receiver time output should be examined. Of immediate concern is the case where the carrier is bi-phase modulated by a split phase modulating waveform generated by random data. However, a simpler analysis yielding valid results is obtained if the data signal, and thus the modulating signal, is periodic. A good choice for the period of the modulating signal is an "average" period. Figure 1 shows that the period of the modulating signal can vary from  $T$  to  $2T$ , depending on the data sequence. A modulating signal period of  $3T/2$  is therefore used, as shown in figure 6.

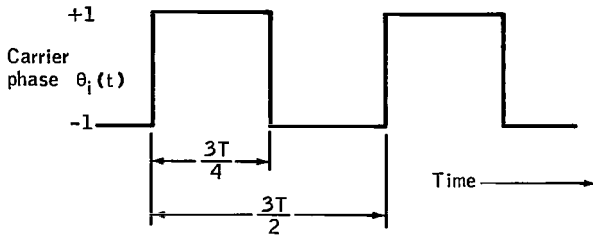


Figure 6. - Square-wave phase-modulating waveform.

From figure 3, the transform of the loop data output signal is given by

$$E(s) = \frac{s^2 \theta_i(s)}{(s + \zeta \omega_n)^2 - (\omega_n \sqrt{\zeta^2 - 1})^2} \quad (21)$$

where the loop is above threshold; that is,  $\zeta > 1$ .

The transform of the modulating signal  $\theta_i(t)$  (fig. 6) is  $\theta_i(s)$  and is given by

$$\theta_i(s) = \frac{1}{s[1 - e^{-(3T/2)s}]} \left[ 1 - 2e^{-(3T/4)s} + e^{-(3T/2)s} \right] \quad (22)$$

Thus,

$$E(s) = \frac{s}{(s + \zeta \omega_n)^2 - (\omega_n \sqrt{\zeta^2 - 1})^2} \left[ \frac{1 - 2e^{-(3T/4)s} + e^{-(3T/2)s}}{1 - e^{-(3T/2)s}} \right] \quad (23)$$

To continue the example, the values of  $\zeta$  and  $\omega_n$  must be calculated. Assume a predetection signal-to-noise ratio of -6 dB (about 15 dB above loop threshold).



Then, using

$$\zeta = \zeta_O \sqrt{\frac{\alpha}{\alpha_O}} = \frac{\sqrt{2}}{2} \sqrt{\frac{\alpha}{\alpha_O}} \quad (24)$$

$$\omega_n = \omega_{nO} \sqrt{\frac{\alpha}{\alpha_O}} \quad (25)$$

There results

$$\zeta = \frac{\sqrt{2}}{2} \sqrt{\frac{\frac{1}{1 + \frac{4}{\pi} (4)}}{\frac{1}{1 + \frac{4}{\pi} (140)}}} = 1.65 \quad (26)$$

$$\omega_n = \frac{2B_{LO}}{1.06} \sqrt{\frac{\frac{1}{1 + \frac{4}{\pi} (4)}}{\frac{1}{1 + \frac{4}{\pi} (140)}}} = 109.5 \text{ rad/sec} \quad (27)$$

Figure 4 shows that the loop noise bandwidth for the assumed signal-to-noise ratio is 197.5 Hz.

Using the previous information and performing a partial fractions expansion gives

$$E(s) = \left( \frac{1.13}{s + 324.2} - \frac{0.13}{s + 37.2} \right) \left[ \frac{1 - 2e^{-(3T/4)s} + e^{-(3T/2)s}}{1 - e^{-(3T/2)s}} \right] \quad (28)$$

The exponential terms in the right-hand term yield the same time response as the unity term except for time translation (and magnitude for the second exponential). Therefore, the exponential terms will be neglected for the time being, leaving

$$E_1(s) = \frac{1.13}{(s + 324.2) [1 - e^{-(3T/2)s}]} - \frac{0.13}{(s + 37.2) [1 - e^{-(3T/2)s}]} \quad (29)$$

The time response of a term such as

$$\frac{1}{(s + a)(1 - e^{-Ks})}$$

is given by

$$\frac{e^{-a\tau}}{e^{aK} - 1} - \frac{e^{-at}}{e^{aK} - 1}$$

$$nK < t < (n + 1)K$$

$$-K < \tau < 0$$

where

$$\tau = t - (n + 1)K \quad (30)$$

The second term is a transient term which is negligible for large  $t$ . In the present case, this means that the square wave has been present for some time. Thus, neglecting the transient terms, the time response corresponding to  $E_1(s)$  is

$$e_1(t) = \frac{1.13e^{-324.2\tau}}{e^{486T} - 1} - \frac{0.13e^{-37.2\tau}}{e^{55.8T} - 1}$$

$$-\frac{3T}{2} < \tau < 0 \quad (31)$$

Letting  $T = \frac{1}{1000}$  (bit rate = 1000 bits/sec) gives

$$e_1(t) = 1.803e^{-324.2\tau} - 2.26e^{-37.2\tau} \quad (32)$$

where  $-\frac{3T}{2} < \tau < 0$  or  $-0.0015 < \tau < 0$ .

This is the equation corresponding to the unity term for  $t$  large and for any period  $n(3T/2) < t < (n+1)3T/2$ , which corresponds to  $(-3T/2) < \tau < 0$ .

The first exponential term in the expression for  $E(s)$  gives a time response equal to  $-2$  times the unity term and displaced to the right in time by  $3T/4$ . The second exponential is the unity term displaced in time by  $3T/2$ . Since the expressions hold for any period where  $t$  is large, each expression is periodic with period equal to  $3T/2$ . Hence, the unity term and the second exponential term are combined to give  $2(1.803e^{-324.2\tau} - 2.26e^{-37.2\tau})$ ,  $-0.0015 < \tau < 0$ .

The first exponential gives the negative of the above expression displaced to the right in time by  $3T/4$ . The data signal time response is shown in figure 7. The ideal response would be an exact square wave with a peak value of 1 and a period of  $3T/2$ .

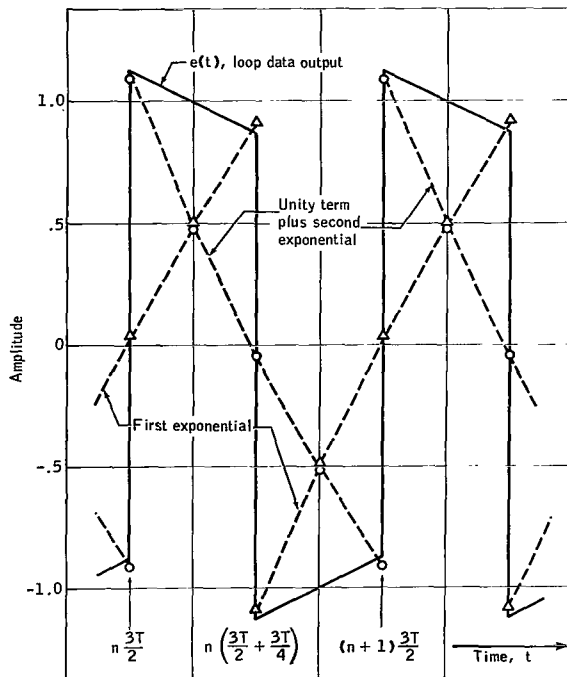


Figure 7. - Loop data output for square-wave phase-modulated input.

To recover the PCM data (zeros and ones), the loop data output signal can be passed through an integrate-and-dump detector. For the ideal signal, the detector output would be proportional to the area  $3T/4$ . The output corresponding to the actual  $e(t)$  shown in figure 7 differs from the ideal output. The actual detector output is found by integrating  $e(t)$  from  $n(3T/2)$  to  $n[(3T/2) + (3T/4)]$ . The data degradation caused by the loop is then given by the difference (or ratio, if degradation in dB is desired) between  $3T/4$  and the actual detector output. Figure 7 shows that the degradation for the example considered is very small. Detail calculations show that this degradation is less than 0.1 dB. Precise calculation can be performed if desired for other examples to find the amount of data degradation caused by the loop. Note that the degradation is very small, even when the loop noise bandwidth is a significant fraction of the square-wave frequency.

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