AM 5340: End Semester Exam

Problem 5

Please prepare a single pdf file containing a report (handwritten and scanned is okay), figures and codes and upload in Google Classroom by May 15.

Question:
Consider the response of two components in an automobile, both modelled as oscillators, subjected to support motion acceleration modelled as mean zero stationary process with auto-PSD

$$S_{FF}(\omega) = \frac{6}{400 + \omega^2}.$$

Each oscillator is modelled as a sdof syste,

$$\ddot{X}_j(t) + 2\zeta_j\omega_j\dot{X}_j(t) + \omega_j^2 X_j(t) = F(t)$$

with $X_j(t)$ representing the displacement of the $j$the component. $\omega_1 = 10$ rad/s, $\omega_2 = 15$ rad/s, $\zeta_1 = 0.01$ and $\zeta_2 = 0.005$.

1. Let b the static clearance between the two oscillators. This gives the clearance between the two components during operating conditions as $Y = b - X_1 + X_2$. Analytically, find the stationary pdf for Y, assuming that X_1 and X_2 are independent.

2. Next, simulate the random forcing and show that the simulated forcings match the target pdfs and the target PSDs.

3. Use the simulated forcings as input and numerically integrate the equations. Plot sample time histories of $X_1(t)$ and $X_2(t)$. Show that the probabilistic characterizers match the analytically derived pdfs and PSDs.

4. Next, derive the formulation for representing the response in PCE framework. Using PCE, numerically estimate the pdf of Y. Show graphically the accuracy of your PCE based estimate with the numerically obtained estimates. Comment on the accuracy and computational costs.

5. Estimate the probability of collisions analytically, and numerically using PCE as well as through direct numerical integration. What should the value of b be such that the probability of collisoons is less than 1×10^{-3}.