

Here is the Title

Summary

A traditional bathtub cannot be reheated by itself, so users have to add hot water from time to time. Our goal is to establish a model of the temperature of bath water in space and time. Then we are expected to propose an optimal strategy for users to keep the temperature even and close to initial temperature and decrease water consumption.

Keywords: Heat transfer, Thermodynamic system, CFD, Energy conservation

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1 Introduction

1.1 Background

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1.2 Literature Review

A traditional bathtub cannot be reheated by itself, so users have to add hot water from time to time. Our goal is to establish a model of the temperature of bath water in space and time. Then we are expected to propose an optimal strategy for users to keep the temperature even and close to the initial temperature and decrease water consumption. According to Kim [1], He derived a relational equation based on the basic theory of heat transfer to evaluate the performance of bath tubes. The major heat loss was found to be due to evaporation. Moreover, he found out that the speed of heat loss depends more on the humidity of the bathroom than the temperature of water contained in the bathtub. So, it is best to maintain the temperature of bathtub water to be between 41 to 45°C and the humidity of bathroom to be 95%. Traditional bath systems have significant limitations in temperature control. To address this, we introduce heat transfer formulas as discussed ([2, p. 123]).

1.3 Restatement of the Problem

We are required to establish a model to determine the change of water temperature in space and time. Then we are expected to propose the best strategy for the person in the bathtub to keep the water temperature close to initial temperature and even throughout the tub. Reduction of waste of water is also needed. In addition, we have to consider the impact of different conditions on our model, such as different shapes and volumes of the bathtub, etc.

In order to solve those problems, we will proceed as follows:

- **Stating assumptions.** By stating our assumptions, we will narrow the focus of our approach towards the problems and provide some insight into bathtub water temperature issues.
- **Making notations.** We will give some notations which are important for us to clarify our models.
- **Presenting our model.** In order to investigate the problem deeper, we divide our model into two sub-models. One is a steady convection heat transfer sub-model in which hot water is added constantly. The other one is an unsteady convection heat transfer sub-model where hot water is added discontinuously.

2 Assumptions and Justification

To simplify the problem and make it convenient for us to simulate real-life conditions, we make the following basic assumptions, each of which is properly justified.

- **The bath water is incompressible Non-Newtonian fluid.** The incompressible Non-Newtonian fluid is the basis of Navier–Stokes equations which are introduced to simulate the flow of bath water.

- **All the physical properties of bath water, bathtub and air are assumed to be stable.** The change of those properties like specific heat, thermal conductivity and density is rather small according to some studies. It is complicated and unnecessary to consider these little change so we ignore them.

3 Notations

| Symbols | Description | Unit |
|-----------|--------------------------------------|--|
| h | Convection heat transfer coefficient | $\text{W}/(\text{m}^2 \cdot \text{K})$ |
| k | Thermal conductivity | $\text{W}/(\text{m} \cdot \text{K})$ |
| c_p | Specific heat | $\text{J}/(\text{kg} \cdot \text{K})$ |
| ρ | Density | kg/m^3 |
| δ | Thickness | m |
| t | Temperature | $^{\circ}\text{C}, \text{K}$ |
| τ | Time | s, min, h |
| q_m | Mass flow | kg/s |
| Φ | Heat transfer power | W |
| T | A period of time | s, min, h |
| V | Volume | m^3, L |
| M, m | Mass | kg |
| A | Area | m^2 |
| a, b, c | The size of a bathtub | m^3 |

where we define the main parameters while specific value of those parameters will be given later.

4 Model Overview

To simplify the modeling process, we firstly assume there is no person in the bathtub. We regard the whole bathtub as a thermodynamic system and introduce heat transfer formulas. We establish two sub-models: adding water constantly and discontinuously. For the former sub-model, we define the mean temperature of bath water and introduce Newton's cooling formula to determine the heat transfer capacity. After deriving the value of parameters, we deduce formulas to derive results and simulate the change of temperature field via CFD, as described by Anderson, Tannehill, and Pletcher [3].

In our basic model, we aim at three goals: keeping the temperature as even as possible, making it close to the initial temperature and decreasing the water consumption.

5 Sub-model I : Adding Water Continuously

As for the second sub-model, we define an iteration consisting of two processes: heating and standby. According to the energy conservation law, we obtain the relationship of time and total heat dissipating capacity. Then we determine the mass flow and the time of adding hot water. We also use CFD to simulate the temperature field in the second sub-model, following the techniques outlined by Thompson [4].

We first establish the sub-model based on the condition that a person add water continuously to reheat the bathing water. Then we use Computational Fluid Dynamics (CFD) to simulate the change of water temperature in the bathtub. At last, we evaluate the model with the criteria which have been defined before.

5.1 Model Establishment

Since we try to keep the temperature of the hot water in bathtub to be even, we have to derive the amount of inflow water and the energy dissipated by the hot water into the air.

5.1.1 Control Equations and Boundary Conditions

According to thermodynamics knowledge, we recall on basic convection heat transfer control equations in rectangular coordinate system. Those equations show the relationship of the temperature of the bathtub water in space.

We assume the hot water in the bathtub as a cube. Then we put it into a rectangular coordinate system. The length, width, and height of it is a , b and c .

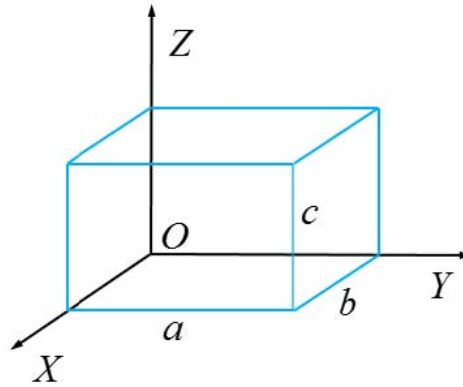


Figure 1: Modeling process

In the basis of this, we introduce the following equations:

- **Continuity equation:**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

where the first component is the change of fluid mass along the X -ray. The second component is the change of fluid mass along the Y -ray. And the third component is the change of fluid mass along the Z -ray. The sum of the change in mass along those three directions is zero.

- **Moment differential equation (N-S equations):**

$$\begin{cases} \rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial x} + \eta \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \\ \rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = -\frac{\partial p}{\partial y} + \eta \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \\ \rho \left(u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = -g - \frac{\partial p}{\partial z} + \eta \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \end{cases} \quad (2)$$

- **Energy differential equation:**

$$\rho c_p \left(u \frac{\partial t}{\partial x} + v \frac{\partial t}{\partial y} + w \frac{\partial t}{\partial z} \right) = \lambda \left(\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} \right) \quad (3)$$

where the left three components are convection terms while the right three components are conduction terms.

By Equation (3), we have

.....

On the right surface in Fig. 1, the water also transfers heat firstly with bathtub inner surfaces and then the heat comes into air. The boundary condition here is

5.1.2 Definition of the Mean Temperature

.....

5.1.3 Determination of Heat Transfer Capacity

.....

.....

5.1.4 Calculating Results

Putting the above value of parameters into the equations we derived before, we can get the some data as follows:

From Table 1,

.....

Table 1: The calculating results

| Variables | Values | Unit |
|-----------|--------|-------|
| A_1 | 1.05 | m^2 |
| A_2 | 2.24 | m^2 |
| Φ_1 | 189.00 | W |
| Φ_2 | 43.47 | W |
| Φ | 232.47 | W |
| q_m | 0.014 | g/s |

6 Model Analysis and Sensitivity Analysis

In consideration of evaporation, we correct the results of sub-models referring to studies. We define two evaluation criteria and compare the two sub-models. Adding water constantly is found to keep the temperature of bath water even and avoid wasting too much water, so it is recommended by us. We also conduct sensitivity analysis to determine the influence of factors such as radiation heat transfer, the shape and volume of the tub, the shape/volume/temperature/motions of the person, and the bubbles made from bubble bath additives, as discussed in ([5]; [6]).

6.1 The Influence of Different Bathtubs

Definitely, the difference in shape and volume of the tub affects the convection heat transfer. Examining the relationship between them can help people choose optimal bathtubs.

6.1.1 Different Volumes of Bathtubs

In reality, a cup of water will be cooled down rapidly. However, it takes quite long time for a bucket of water to become cool. That is because their volume is different and the specific heat of water is very large. So that the decrease of temperature is not obvious if the volume of water is huge. That also explains why it takes 45 min for 320 L water to be cooled by 1°C .

In order to examine the influence of volume, we analyze our sub-models by conducting sensitivity Analysis to them.

We assume the initial volume to be 280 L and change it by $\pm 5\%$, $\pm 8\%$, $\pm 12\%$ and $\pm 15\%$. With the aid of sub-models we established before, the variation of some parameters turns out to be as follows

Table 2: Variation of some parameters

| V | A_1 | A_2 | T_2 | q_{m1} | q_{m2} | Φ_q |
|---------|--------|--------|---------|----------|----------|----------|
| -15.00% | -5.06% | -9.31% | -12.67% | -2.67% | -14.14% | -5.80% |
| -12.00% | -4.04% | -7.43% | -10.09% | -2.13% | -11.31% | -4.63% |
| -8.00% | -2.68% | -4.94% | -6.68% | -1.41% | -7.54% | -3.07% |
| -8.00% | -2.68% | -4.94% | -6.68% | -1.41% | -7.54% | -3.07% |
| -8.00% | -2.68% | -4.94% | -6.68% | -1.41% | -7.54% | -3.07% |

7 Strength and Weakness

7.1 Strength

- We analyze the problem based on thermodynamic formulas and laws, so that the model we established is of great validity.
- Our model is fairly robust due to our careful corrections in consideration of real-life situations and detailed sensitivity analysis.
- Via Fluent software, we simulate the time field of different areas throughout the bathtub. The outcome is vivid for us to understand the changing process.

7.2 Weakness

- Having knowing the range of some parameters from others' essays, we choose a value from them to apply in our model. Those values may not be reasonable in reality.
- Although we investigate a lot in the influence of personal motions, they are so complicated that need to be studied further.
- Limited to time, we do not conduct sensitivity analysis for the influence of personal surface area.

8 Further Discussion

Based on our model analysis and conclusions, we propose the optimal strategy for the user in a bathtub and explain the reason for the uneven temperature throughout the bathtub. In addition, we make improvements for applying our model in real life, as suggested by the patent Wilson [7].

References

- [1] Gi-Beum Kim. "Change of the Warm Water Temperature for the Development of Smart Healthcare Bathing System". In: *Hwahak konghak* 44.3 (2006), pp. 270–276.
- [2] J. P. Holman. "Heat Transfer (9th ed.)" In: *McGraw-Hill* (2002).
- [3] D. A. Anderson, J. C. Tannehill, and R. H. Pletcher. *Computational Fluid Dynamics and Heat Transfer*. 2nd. Taylor & Francis, 2006.
- [4] H. Thompson. *Advanced CFD Techniques for Bath Water Simulation*. <https://www.example.com/advanced-cfd>. Accessed: 2024-10-01.
- [5] J. Smith. "Evaporation Effects in Bath Water Temperature Regulation". In: *Journal of Thermal Science and Engineering* 30.4 (2018), pp. 456–467.
- [6] D. Brown. "Thermal Modeling of Bath Water Systems". PhD thesis. City: University of Technology, 2015.
- [7] G. Wilson. "Smart Bath Water Temperature Control System". US Patent 1234567. 2023.

Dear, Mr. Alpha Chiang

In addition, your report must include a letter to the Chief Financial Officer (CFO) of the Goodgrant Foundation, Mr. Alpha Chiang, that describes the optimal investment strategy, your modeling approach and major results, and a brief discussion of your proposed concept of a return-on-investment (ROI). This letter should be no more than two pages in length.

Sincerely yours,

Your friends

Appendices

Appendix A First appendix

Here are simulation programmes we used in our model as follow .

Input matlab source:

```
function [t,seat,aisle]=OI6Sim(n,target,seated)
pab=rand(1,n);
for i=1:n
    if pab(i)<0.4
        aisleTime(i)=0;
    else
        aisleTime(i)=trirnd(3.2,7.1,38.7);
    end
end
end
```

Appendix B Second appendix

some more text **Input C++ source:**

```
//=====
// Name      : Sudoku.cpp
// Author    : wzlf11
// Version   : a.0
// Copyright  : Your copyright notice
// Description : Sudoku in C++.
//=====

#include <iostream>
#include <cstdlib>
#include <ctime>

using namespace std;

int table[9][9];

int main() {

    for(int i = 0; i < 9; i++){
        table[0][i] = i + 1;
    }

    srand((unsigned int)time(NULL));

    shuffle((int *)&table[0], 9);

    while(!put_line(1))
    {
        shuffle((int *)&table[0], 9);
```

```
    }  
  
    for(int x = 0; x < 9; x++){  
        for(int y = 0; y < 9; y++){  
            cout << table[x][y] << " ";  
        }  
  
        cout << endl;  
    }  
  
    return 0;  
}
```

Report on Use of AI

1. Bing AI

Query1: write a sort algorithm

Output: Sure, I can help you with that. Here is a simple implementation of the selection sort algorithm in Python:

```
def selection_sort(arr):
    for i in range(len(arr)):
        min_idx = i
        for j in range(i + 1, len(arr)):
            if arr[min_idx] > arr[j]:
                min_idx = j
        arr[i], arr[min_idx] = arr[min_idx], arr[i]
    return arr
```

This algorithm sorts an array by repeatedly finding the minimum element from the unsorted part of the array and putting it at the beginning. The time complexity of this algorithm is $O(n^2)$.

There are many other sorting algorithms available such as bubble sort, insertion sort, merge sort, quick sort, heap sort, and more. Each of these algorithms has its own advantages and disadvantages. You can learn more about them on [GeeksforGeeks](#) and [Programiz](#).

I hope this helps!

2. OpenAI Enie (Nov 5, 2023 version, Ernie 4.0)

Query1: <insert the exact wording of any subsequent input into the AI tool>

Output: <insert the complete output from the second query>

3. Github CoPilot (Feb 3, 2024 version)

Query1: <insert the exact wording you input into the AI tool>

Output: <insert the complete output from the AI tool>

4. Google Bard (Feb 2, 2024 version)

Query: <insert the exact wording of your query>

Output: <insert the complete output from the AI tool>

5. Bing AI

Query: write a FFT algorithm

Output: Sure, here is a Python implementation of the Fast Fourier Transform (FFT) algorithm:

```
import numpy as np

def fft(x):
    n = len(x)
    if n == 1:
        return x
    even = fft(x[0::2])
    odd = fft(x[1::2])
    T = [np.exp(-2j * np.pi * k / n) * odd[k] for k in range(n // 2)]
    return [even[k] + T[k] for k in range(n // 2)] + [even[k] - T[k] for k
    ↪ in range(n // 2)]
```

The FFT is a widely used algorithm for computing the **Discrete Fourier Transform (DFT)** of a sequence. It reduces the number of computations needed from $O(n^2)$ to $O(n \log n)$ 12. The basic idea of the FFT is to apply divide and conquer. We divide the coefficient vector of the polynomial into two vectors, recursively compute the DFT for each of them, and combine the results to compute the DFT of the complete polynomial 3.

I hope this helps!