

Problem Set 5

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DUE DATE : 2021.10.6. time 11:00pm
submit your solution and code files on Blackboard page.

Download Pset5DataQ1.csv to solve Question 1. The data includes a balanced panel of 500 buses observed for 20 periods. The data includes bus mileage x ranging from 0 to 9, and a replacement history d . The data will be used to estimate Rust (1987) bus engine problem. The model was simplified by assuming a deterministic mileage change. When the engine is replaced, the next period mileage changes to 0. When the engine is not replaced, the next period mileage increases by 1 deterministically until hitting the maximum mileage M . That is,

$$\begin{aligned} x_{it} &= x_{it-1} + 1 & \text{if } x_{it-1} < M \\ x_{it} &= x_{it-1} & \text{if } x_{it-1} = M \end{aligned}$$

The per-period maintenance cost depends on the bus mileage x_{it} and equals to $\theta_1 x_{it}$. If the engine is replaced, it incurs the fixed cost θ_2 . The flow payoff formula is as follows :

$$u(x_{it}, j_{it}) = -\theta_1 x_{it} - \theta_2 1(j_{it} = 1) \quad (0.1)$$

Apart from the information above, set the external parameters as follows. Assume you know the time discount parameter β .

Parameter	Description	Value
β	time discount	0.95
M	maximum bus mileage	9
γ	Euler constant	0.5772
N	number of buses	500
T	number of periods per bus	20

Question 1. [Rust (1987) Bus Engine Replacement, fully-observable state space]

(a) Estimate the parameters using the Rust NFXP algorithm (full-solution method) and report the standard errors.

(b) Estimate the parameters using the Aguirregabiria and Mira 1-step NPL estimator using the Hotz and Miller inversion. Report the standard errors.

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(c) Estimate the parameters using the Aguirregabiria and Mira 2-step NPL estimator using the Hotz and Miller inversion. Report the standard errors.

Question 2. Derive the equation (16) in Lecture 5 (the first order condition of maximum likelihood function).

$$\max_{\theta, \{\pi_k\}} \log L = \sum_n \log \left[\sum_{s=1}^S (\pi_s \Pi_{t=1}^T \mathcal{L}_{nst}(\theta)) \right]$$

first-order condition for θ becomes

$$0 = \sum_n \sum_s \frac{\pi_s \Pi_{t=1}^T \mathcal{L}_{nst}(\theta)}{\underbrace{\sum_{s'=1}^S \pi_{s'} \Pi_{t=1}^T \mathcal{L}_{ns't}(\theta)}_{=q_{ns} \text{prob of ind } n \text{ of being type } s}} \left(\frac{\sum_{t'} \partial \ln \mathcal{L}_{nst'}}{\partial \theta} \right) \quad (0.2)$$

Question 3. [Extension of Rust (1987) Bus Engine Replacement, including fixed unobservable state variable]

Download Pset5DataQ3.csv to solve Question 1. The data includes a balanced panel of 5000 buses observed for 50 periods. The data includes bus mileage x ranging from 0 to 7, and a replacement history d .

The transition rule for mileage x is same as in Question 1. That is, an increment of 1 unit until hitting the threshold $M=7$ takes place every period unless the bus engine is replaced. When the bus engine is replaced, the next period bus mileage becomes 0.

The flow utility now includes the unobservable state s_i . Every bus has an unobservable fixed characteristic $s_i \in \{1, 2\}$. The bus of type $s_i = 2$ incurs higher maintenance cost given each mileage x_{it} . The bus type s_i is not observable.

$$u(x_{it}, j_{it}) = -\theta_1 x_{it} s_i - \theta_2 1(j_{it} = 1) \quad (0.3)$$

Set the external parameters as follows.

Parameter	Description	Value
β	time discount	0.95
M	maximum bus mileage	7
γ	Euler constant	0.5772
N	number of buses	5000
T	number of periods per bus	50
S	number of unobserved type	2
tol	EM algorithm tolerance	10^{-3}

Estimate the parameters using Arcidiacono and Jones (2003) 's EM algorithm in two-steps. **Report the standard errors computed from the second stage ML estimation.**

[Bonus Question 1, +5pt] If you complete this bonus question successfully, you will receive +5 points for total grade in this course.

Bootstrap Rust NFXP estimator, AM's 1-step and 2-step NPL estimators in Question 1 for 100 times. Compute the standard errors of your estimate and compare the standard errors with the ones found in Question 1, using the asymptotic variance formula. Discuss the differences.

[Bonus Question 2, +5pt] If you complete this bonus question successfully, you will receive +5 points for total grade in this course.

Bootstrap Arcidiacono and Jones (2003) estimator in Question 3 for 100 times. Compute the standard errors of your estimate and compare this standard error with the one found in Question 3. Discuss the differences.