Problem Set 1

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DUE DATE : 2021.9.8. time 11:00pm
submit your solution and code files on Blackboard page.

Question 1. Analytical solution for the cake-eating problem with no income flows

An agent starts life with an asset a_1 and live for T periods. Saving is allowed. Flow utility function is the CRRA utility function. Derive an equation for the optimal consumption plan. (Derive equation (2) in slide Lecture 1, page 14)

Question 2. Understanding numerical errors

Note : this question was intended to use an exogenous grid for asset. you may compare solutions using exogenous grids and an endogenous grid in the Bonus question below.

In this question, you are asked to solve for the policy function (consumption plan) $c_t(a_t)$ for the cake-eating problem with no income flow using various solution methods.

The flow utility function is CRRA.

$$u(c) = \frac{c^{1-\gamma}}{(1-\gamma)} \tag{0.1}$$

The structural parameter values are as follows :

Parameter	Description	Value
T	Lifetime	40
r	interest rate	0.03
β	time discount	0.95
γ	CRRA parameter	1.5
a_0	initial asset	1
$\underline{c}$	minimum consumption	$10^{-}5$

When you set an asset grid, use age-specific grid and use an unequal grid point generated from log transformation and use the $N_{\mathcal{A}} = 20$ gridpoints.

When you need to interpolate any function, use "linear" interpolation.

(a) Compute $c_t(a_t)$ using value function.

(b) Compute $c_t(a_t)$ using Euler equation without linear transformation of marginal utility function.

(c) Compute $c_t(a_t)$ using Euler equation with linear transformation of marginal utility function.

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Show in one graph the true analytical solution for $t = 20$ derived in Question 1 and the solution (a) - (c) for age t=20. Discuss the differences.

Question 3. Simulating consumption and asset path using the solution

Using the policy function solution (c) computed in Question 2, simulate the consumption and saving decision and plot them in the same graph.

[Bonus Question] You are not required to do this, but if you successfully solve this question, you get extra bonus points [+5 pts in your final grade].

Compare the computation time to solve Question 2 using exogenous grid points (graph (a), (b), (c) respectively) and using endogenous grid point. Add a policy function $c_t(a_t)$ computed from endogenous grid point method and discuss the differences from solutions for Question 2.