Problem Set 6

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DUE DATE : 2021.10.13. time 11:00pm
submit your solution and code files on Blackboard page.

Question 1. Solving a Discrete-Continuous Problem without Uncertainty
An agent is endowed with initial asset a_0 and may work until retirement $d_t = 0$. Retirement is an absorbing state, so once retired, an agent can not go back to work. In this question, you are asked to solve for the policy function (consumption plan) $c_t(a_t, d_t)$ and value function $v_t(a_t, d_t)$ conditional on work decision d_t.
The flow utility function is as follows. Working incurs a disutility cost δ.

$$u(c, d) = \log(c) - \delta \cdot 1(d = 1) \tag{0.1}$$

The structural parameter values are as follows :

Parameter	Description	Value
T	Lifetime	20
r	interest rate	0
β	time discount	0.98
a_0	initial asset	0
$\underline{c}$	minimum consumption	10^-5
$\overline{y}$	per-period income	20
δ	work disutility	1

When you set an asset grid, use age-specific grid and use an unequal grid point generated from log transformation and use the $N_A = 1000$ gridpoints.
When you need to interpolate any function, use "linear" interpolation.

(a) Compute $c_t(a_t, d_t)$ using the value function iteration. Show the policy function for worker $c_t(a_t, d_t = 1)$ for $t = 18, t = 15, t = 10$. Discuss how the number of discontinuities increases as $t \to 1$, using the notion of primary and secondary kink points.

(b) Compute the marginal utility of consumption $MU_t(c_t(a_t))$ as a function of asset. Plot the $MU_t(c_t(a_t))$ for $t = 18, t = 15, t = 10$. Using the plot, discuss why the Euler equation may not provide a sufficient condition for

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optimality.

(c) Compute the unconditional value function $V_t(a_t)$ and conditional value function $V_{t,d=1}(a_t), V_{t,d=0}(a_t)$, conditional on work status. Plot the unconditional value function and conditional value function for t=20. Find the primary kink point. Discuss the how the unconditional value function looks like around the primary kink point.

[Bonus Question, +15 points] Solve the Question 1 using the Iskhakov et al. (2017)'s endogenous grid method.