Problem Set 8

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DUE DATE : 2021.10.27. time 11:00pm
submit your solution and code files on Blackboard page.

Note : I thank Professor Andrew Shephard for kindly sharing his problem set for this class. These questions came from his lecture material.

Suppose there are three types of men (I=3) and three types of women (J=3), with measures as given by the population vectors $\mathscr{M} = \mathscr{F} = [1/3, 1/3, 1/3]$. The wage of each type of man (woman) is given by $w_i = i$ ($w_j = j$). All types have the non-labour income $y = 1$. Single men have the preferences

$$U_i(c, P_i) = \log c - \beta P_i + \epsilon_{p_i}, \tag{0.1}$$

where $c = y + w_i P_i$ is consumption, P_i is an employment indicator, and $\epsilon_{P_i} \sim \mathrm{Gumbel}(0, \sigma_\epsilon)$ is a state-specific error attached to the time-allocation alternatives. The preferences for single women are defined symmetrically.

Question 1. What is the expected utility of a single type-i man? (The expectation is over the realisation of the state specific errors.)
(Note : don't forget the scale parameter σ_ϵ, which may not be necessarily 1.)

Question 2. Consider a type-ij marriage pairing. Within marriage, suppose that i) consumption is priovate, ii) the state-specific errors are public goods and vary with the joint allocation (P_i, P_j) with $\epsilon_{P_i,P_j} \sim \mathrm{Gumbel}(0, \sigma_\epsilon)$, and iii) decisions are made efficiently, as in the collective model. Finally, let λ_{ij} denote the weight on male utility in the household problem, and $1-\lambda_{ij}$ denote the weight on female utility. These assumptions imply that the household solves

$$\max_{P_i, P_j, s_{ij}} \quad \{\lambda_{ij} \times [\log[s_{ij} \cdot c] - \beta P_i] + (1-\lambda_{ij}) \times [\log[(1-s_{ij}) \cdot c] - \beta P_j] + \epsilon_{P_i,P_j}\} \tag{0.2}$$

where s_{ij} is the endogenous consumption share for the man, and the household budget constraint is given by $c = 2y + w_i P_i + w_j P_j$.

(a) Determine the male consumption share s_{ij} given the joint allocation (P_i, P_j).

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(b) What is the expected utility of a type-i man in a type-ij marriage pairing ? (Agian, the expectation is over the realisation of the state specific errors)

Question 3. Suppose that men and women match in a frictionless marriage market. Matching occurs prior to the realisation of the state-specific time allocation shocks, such that the expected values calculated in (1) and (2) above respectively correspond to the economic value of singlehood and marriage. As in Choo and Siow (2006), assume that a man g of type-i has additiev preference heterogeneity $\theta_{ij}^{i,g}$ over the type of his spouse, $j = 0, \cdots, J$, as given by $\theta_{ij}^{i,g} \sim \mathtt{Gumbel}(0, \sigma_\theta)$, where $j = 0$ indicates the singel state. The same spousal preference heterogeneity structure exists for women. Assume that $\beta = 0.8, \sigma_\epsilon = 0.4, \sigma_\theta = 0.7$. Compute the equilibrium of the marriage market. What is the equilibrium matrix of Pareto weights λ ? What is the equilibrium marriage matching function ? What is the proportion of single men within each type?

Question 4. The government decides to the tax labour income of single men at the constant marginal tax rate $\tau = 0.5$. How does this change the equilibrium distribution of Pareto ewights and the equilibrium marriage matching function ? What is the proportion of single men within each type? Discuss the differences from your answer from Question 3.