# Problem Set 2

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submit your solution and code files on Blackboard

### Question 1. discretizing the AR(1) income process

Using the parameters below, discretize the income grid and compute the numerical approximation of transition probability matrix. Every year, an agent receives an income flow $y_t$. The log of $y_t$ follows an AR (1) process with mean zero and an autocorrelation $\rho$.

$$\log y_t = \rho \log y_{t-1} + \nu_t, \quad \nu_t \sim N(0, \sigma_\nu^2) \tag{0.1}$$

| Parameter | Description | Value |
|:---:|:---:|:---:|
| $N_y$ | number of Income Grid | 5 |
| $\rho$ | AR(1) process coefficient | 0.75 |
| $\sigma_\nu$ | SD of shock | 0.25 |

### Question 2. cake-eating problem with uncertain income

In this question, you are asked to solve for the policy function (consumption plan) $c_t(a_t)$ for the cake-eating problem with income uncertainty using various solution methods. You should use the discretized income process in Question 1. Borrowing/saving is ALLOWED.

The flow utility function is CRRA.

$$u(c) = \frac{c^{1-\gamma}}{(1-\gamma)} \tag{0.2}$$

The structural parameter values are as follows :

| Parameter | Description | Value |
|:---:|:---:|:---:|
| $T$ | Lifetime | 40 |
| $r$ | interest rate | 0.03 |
| $\beta$ | time discount | 0.95 |
| $\gamma$ | CRRA parameter | 1.5 |
| $a_0$ | initial asset | 0 |
| $\underline{c}$ | minimum consumption | $10^{-5}$ |

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When you set an asset grid, use age-specific grid and use an unequal grid point generated from log transformation and use the $N_A = 20$ gridpoints.

When you need to interpolate any function, use "linear" interpolation.

(a) Compute $c_t(a_t)$ using value function.

(b) Compute $c_t(a_t)$ using Euler equation without linear transformation of marginal utility function.

(c) Compute $c_t(a_t)$ using Euler equation with linear transformation of marginal utility function.

Show in one graph the solution (a) - (c) for age t = 20 and the highest income and the lowest income. Discuss the differences.

### Question 3. computing simulated moments on consumption and asset

Using the policy function solution (c) computed in Question 2, simulate the consumption decision for 100 individuals and plot the mean asset and mean consumption over the life-cycle. Comment on the mean asset and the mean consumption path using the relative size of $\beta$ and r.

Remember to set seed for your code. Otherwise, your result will not be replicable.