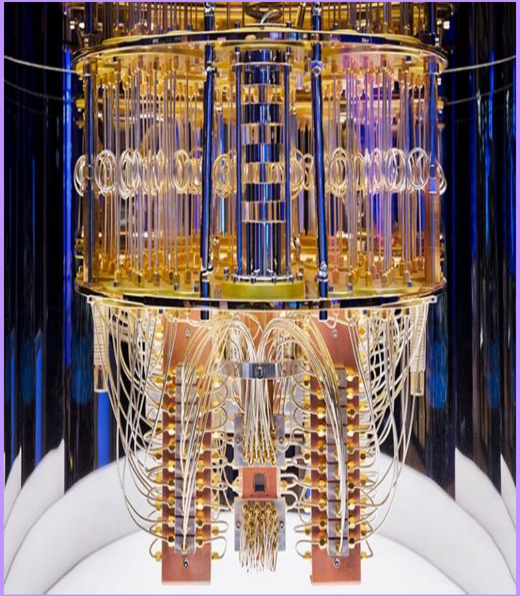
Introduction to Quantum Computing

- Pradeep

What is a Quantum Computer ?

- Fundamental Building Blocks contains Qubits

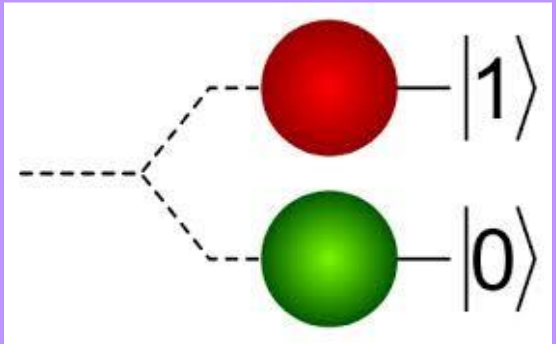
- Computation is done through Applying Quantum Gates



What is a Qubit ?

- Any two-level Quantum System can be considered as qubits

- Ex: Two level atoms, Polarization of photons, Spins of electrons



Mathematical Representation of a Qubit

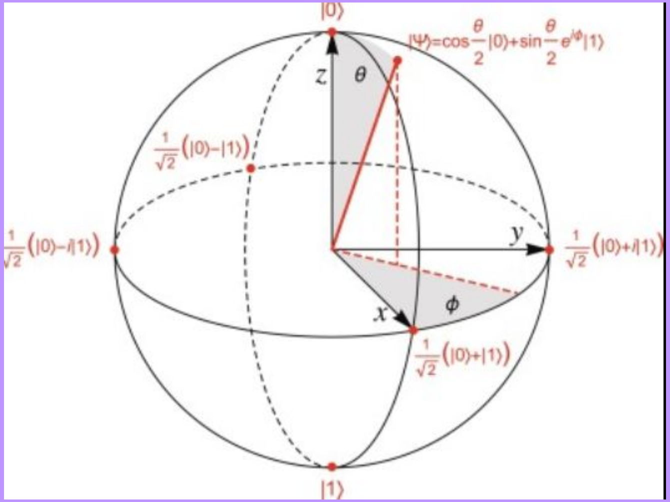
$$\begin{pmatrix}1\\0\end{pmatrix} = |0\rangle \quad \text{and} \quad \begin{pmatrix}0\\1\end{pmatrix} = |1\rangle,$$

- It is denoted by 2-orthonormal vectors, but a state of qubit can be in superposition these two states

$$|\varphi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

The Bloch Sphere

- The theta and Phi angles represents the direction of qubit vector in the Bloch sphere



Mathematical Representation of Many-qubits

- Many-qubit states are represented by taking tensor products of single qubit vectors

The tensor product of $|0\rangle$ and $|0\rangle$ to generate $|00\rangle$ is:

$$|00\rangle = |0\rangle \otimes |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1\begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ 0\begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

The tensor product of $|0\rangle$ and $|1\rangle$ to generate $|01\rangle$ is:

$$|01\rangle = |0\rangle \otimes |1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1\begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ 0\begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

Similarly you can find the vector representation of $|10\rangle$ and $|11\rangle$

Quantum Gates

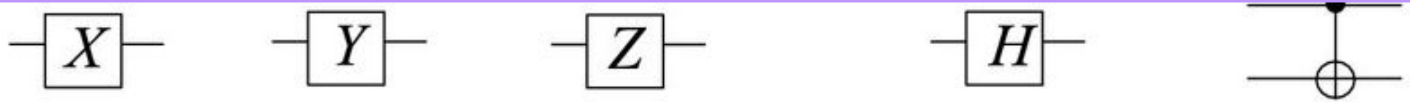
- We can manipulate qubits using quantum gates.
- Quantum Gates map qubits vectors to other qubit vectors thereby giving us a way to perform computation them.

Mathematical Representation of Quantum Gates

- Quantum Gates are represented by square matrices which act on the qubits vectors.

- Gates can be considered as rotations on the Bloch sphere

Mathematical Representation of Quantum Gates



$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Mathematical Representation of Quantum Gates

- The action of all the gates can be done by multiplying the matrix on the vector.

- We can have gates acting on single qubits as well as 2 qubits.

Mathematical Representation of Quantum Gates

So Z operates on $|0\rangle$ and $|1\rangle$ as:

$$Z|0\rangle = |0\rangle$$
$$Z|1\rangle = -|1\rangle$$

So X operates on $|0\rangle$ and $|1\rangle$ as:

$$X|0\rangle = |1\rangle$$
$$X|1\rangle = |0\rangle$$

So H operates on $|0\rangle$ and $|1\rangle$ as:

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$
$$H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

So CNOT gate operates as follows:

$$\mathrm{CNOT}|00\rangle = |00\rangle$$
$$\mathrm{CNOT}|01\rangle = |01\rangle$$
$$\mathrm{CNOT}|10\rangle = |11\rangle$$
$$\mathrm{CNOT}|11\rangle = |10\rangle$$

Mathematical Representation of Quantum Gates

Pauli-X matrix

The Pauli-X matrix (X), also known as the quantum NOT gate, is:

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Action on $|0\rangle$ and $|1\rangle$:

- For $|0\rangle$:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$X|0\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$$

- For $|1\rangle$:

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$X|1\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle$$

Mathematical Representation of Quantum Gates

Hadamard matrix:

The Hadamard matrix H is defined as:

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

The state $|0\rangle$ and $|1\rangle$ are:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

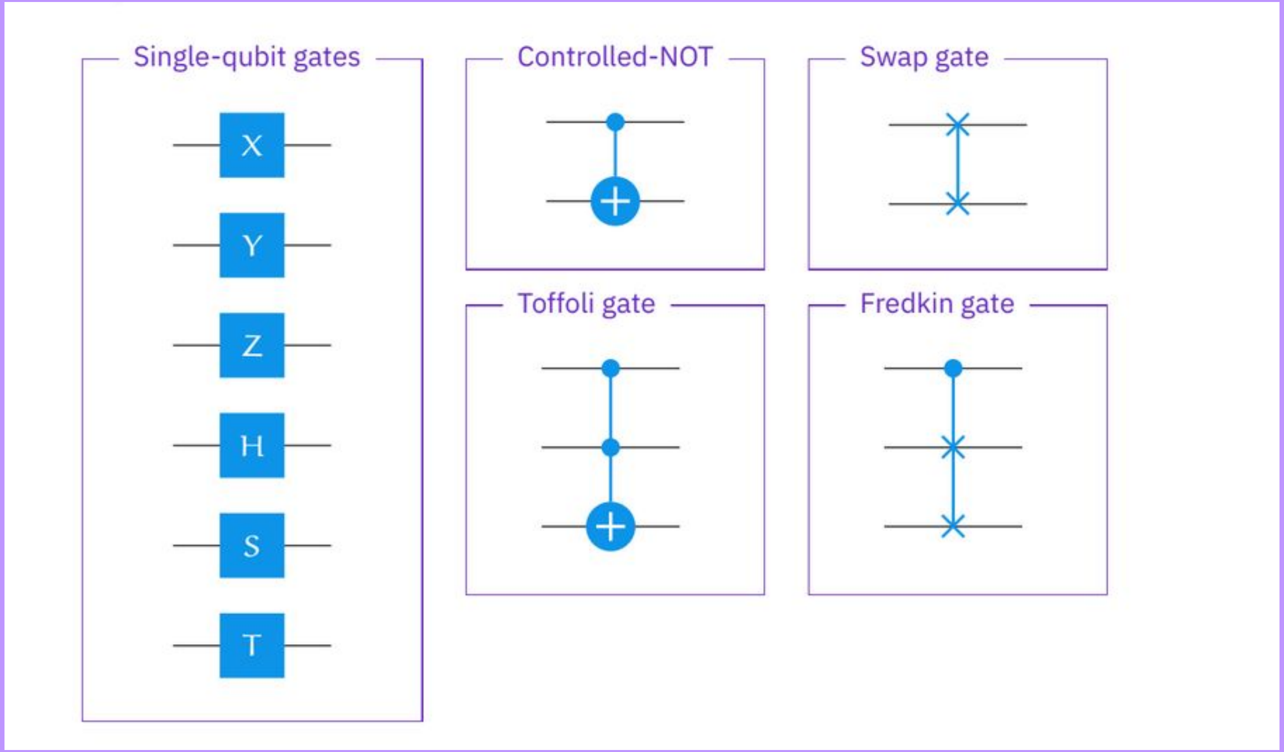
Applying the Hadamard matrix on $|0\rangle$:

$$H|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

Applying the Hadamard matrix on $|1\rangle$:

$$H|1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

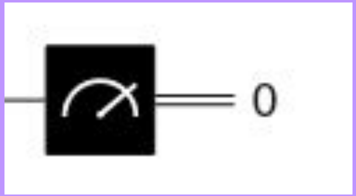
Quantum Gates

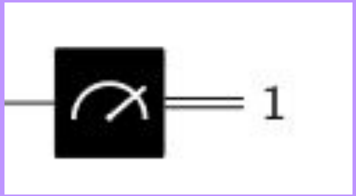


Quantum Measurement

- Measuring a quantum state collapses the wavefunction to either |0> or the |1> state.

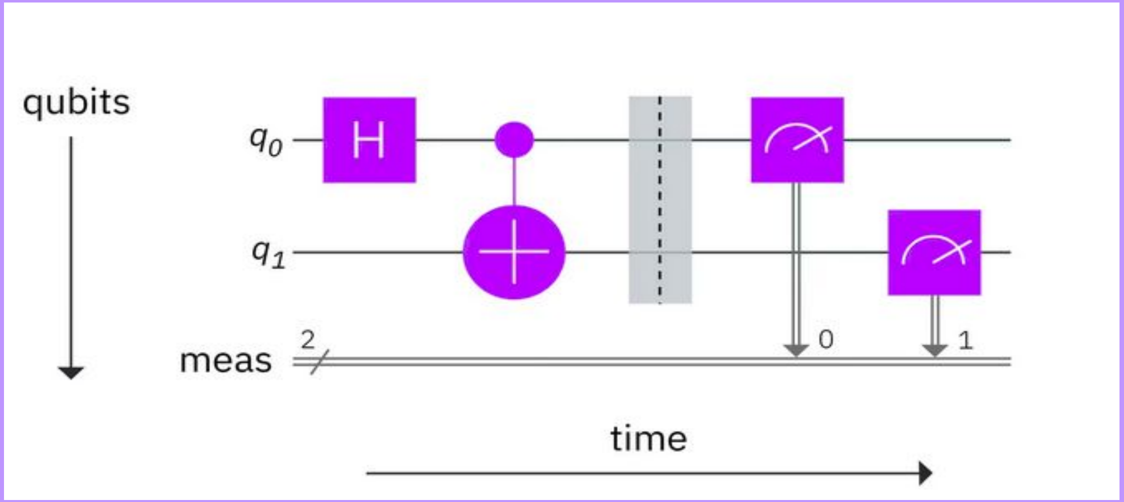
- The probability distribution would depend upon the coefficients - theta and phi





Quantum Circuits

- Combining all elements we have learnt till now we get what is called us a Quantum Circuit



Quantum Circuits