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- They are assumed to be in different locations.
- The specific roles they play must be clarified in different situations.
- Additional characters (e.g., Charlie, Diane, Eve, and Mallory) may be introduced as needed.

# Remarks on entanglement

example of an *entangled state* of two qubits:

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

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$|\psi_{AB}\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$ → NOT Entangled

$|\psi_{AB}\rangle \neq |\psi_A\rangle \otimes |\psi_B\rangle$ → Entangled ✓

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It is typical in the study of quantum information and computation that we view entanglement as a *resource* that can be used to accomplish different tasks.

When we do this we view the state $|\phi^+\rangle$ as representing one unit of entanglement called an *e-bit.*

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Terminology

To say that Alice and Bob *share an e-bit* means that Alice has a qubit $\mathsf{A}$, Bob has a qubit $\mathsf{B}$, and together the pair $(\mathsf{A}, \mathsf{B})$ is in the state $|\phi^+\rangle$.

# Teleportation set-up

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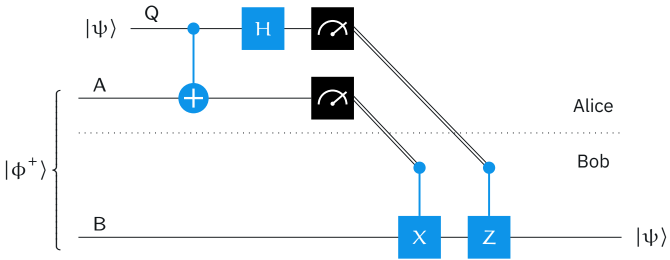
- The state of Q is “unknown” to both Alice and Bob.
- Correlations (including entanglement) between Q and other systems must be preserved by the transmission.
- The *no-cloning theorem* implies that if Bob receives the transmission, Alice must no longer have the qubit in its original state.

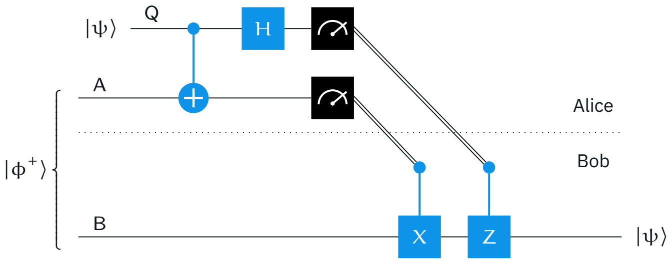
# Teleportation protocol

Circuit description



# Teleportation protocol

## Circuit description




## Initial conditions

Alice and Bob share one e-bit: Alice has a qubit A, Bob has a qubit B, and $(A, B)$ is in the state $|\phi^+\rangle$.

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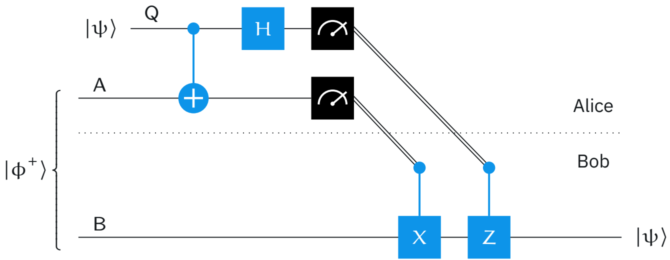


## Initial conditions

Alice and Bob share one e-bit: Alice has a qubit A, Bob has a qubit B, and $(\mathsf{A}, \mathsf{B})$ is in the state $|\phi^+\rangle$.

Alice also has a qubit Q that she wishes to transmit to Bob.

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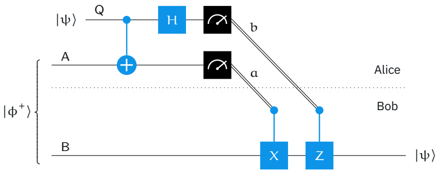
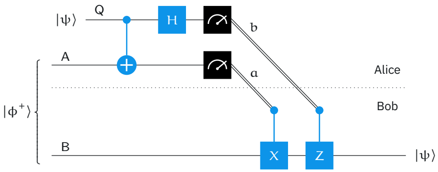



## Protocol

1. Alice performs a controlled-NOT operation, where Q is the control and A is the target.

# Teleportation protocol

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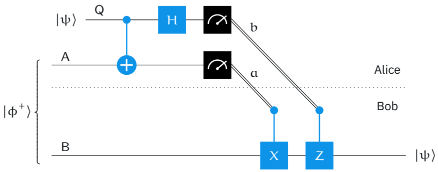


## Protocol

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3. Alice measures A and Q, obtaining binary outcomes $a$ and $b$, respectively.

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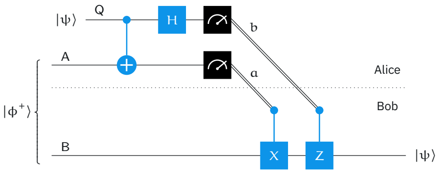


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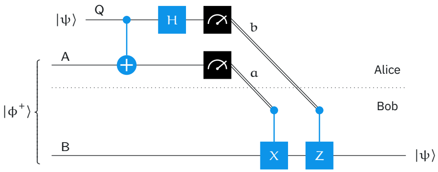


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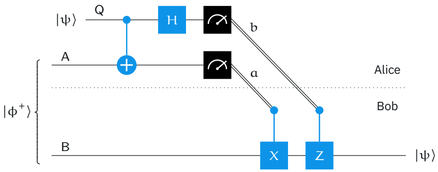
5. Bob performs these two steps:

   5.1 If $a = 1$, then Bob applies an $X$ operation to the qubit B.

   5.2 If $b = 1$, then Bob applies a $Z$ operation to the qubit B.

# Teleportation protocol




## Operation performed by Bob

| | |
|---|---|
| $\mathbb{1}$ | if $ab = 00$ |
| $Z$ | if $ab = 01$ |
| $X$ | if $ab = 10$ |
| $ZX$ | if $ab = 11$ |

## Protocol

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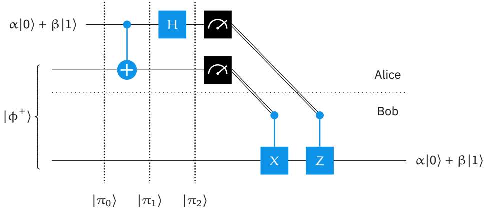
# Teleportation analysis




$$|\pi_0\rangle = |\phi^+\rangle \otimes [\alpha|0\rangle + \beta|1\rangle]$$

$$= \left[\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle\right] \otimes [\alpha|0\rangle + \beta|1\rangle]$$

$$= \frac{\alpha|000\rangle + \alpha|110\rangle + \beta|001\rangle + \beta|111\rangle}{\sqrt{2}}$$

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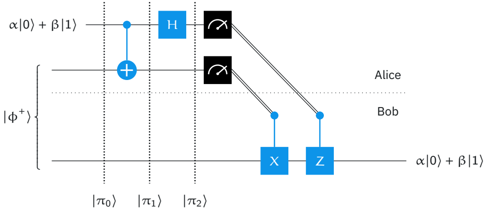


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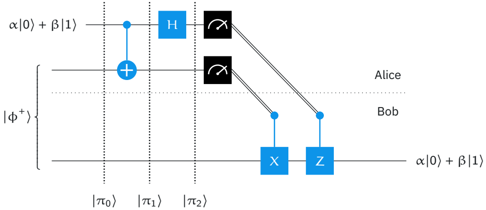


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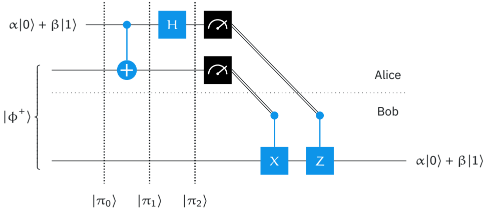


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$$|\pi_2\rangle = \frac{\alpha|00\rangle|+\rangle + \alpha|11\rangle|+\rangle + \beta|01\rangle|-\rangle + \beta|10\rangle|-\rangle}{\sqrt{2}}$$

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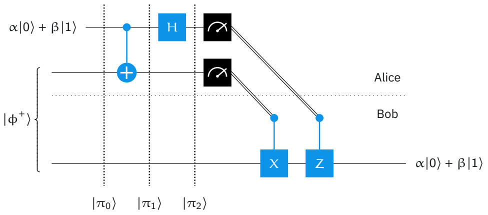


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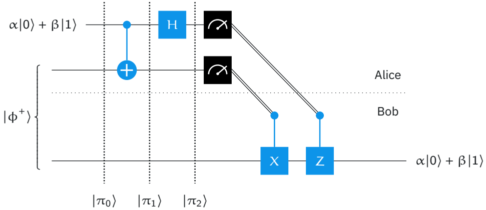


$$
\begin{aligned}
|\pi_2\rangle &= \frac{\alpha|00\rangle|+\rangle + \alpha|11\rangle|+\rangle + \beta|01\rangle|-\rangle + \beta|10\rangle|-\rangle}{\sqrt{2}} \\
&= \frac{\alpha|00\rangle(|0\rangle + |1\rangle) + \alpha|11\rangle(|0\rangle + |1\rangle) + \beta|01\rangle(|0\rangle - |1\rangle) + \beta|10\rangle(|0\rangle - |1\rangle)}{2}
\end{aligned}
$$

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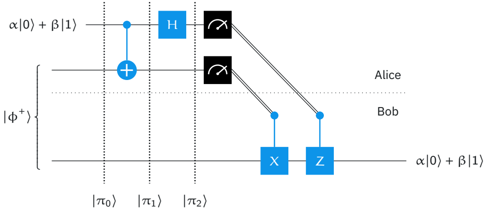


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\begin{aligned}
|\pi_2\rangle &= \frac{\alpha|00\rangle|+\rangle + \alpha|11\rangle|+\rangle + \beta|01\rangle|-\rangle + \beta|10\rangle|-\rangle}{\sqrt{2}} \\
&= \frac{\alpha|00\rangle\big(|0\rangle + |1\rangle\big) + \alpha|11\rangle\big(|0\rangle + |1\rangle\big) + \beta|01\rangle\big(|0\rangle - |1\rangle\big) + \beta|10\rangle\big(|0\rangle - |1\rangle\big)}{2} \\
&= \frac{\alpha|000\rangle + \alpha|001\rangle + \alpha|110\rangle + \alpha|111\rangle + \beta|010\rangle - \beta|011\rangle + \beta|100\rangle - \beta|101\rangle}{2}
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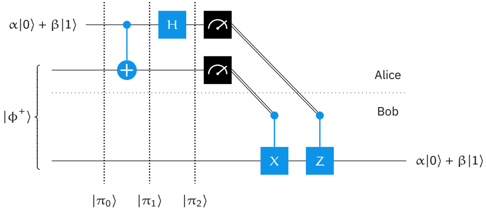


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|\pi_2\rangle &= \frac{\alpha|00\rangle|+\rangle + \alpha|11\rangle|+\rangle + \beta|01\rangle|-\rangle + \beta|10\rangle|-\rangle}{\sqrt{2}} \\
&= \frac{\alpha|00\rangle\big(|0\rangle + |1\rangle\big) + \alpha|11\rangle\big(|0\rangle + |1\rangle\big) + \beta|01\rangle\big(|0\rangle - |1\rangle\big) + \beta|10\rangle\big(|0\rangle - |1\rangle\big)}{2} \\
&= \frac{\alpha|000\rangle + \alpha|001\rangle + \alpha|110\rangle + \alpha|111\rangle + \beta|010\rangle - \beta|011\rangle + \beta|100\rangle - \beta|101\rangle}{2} \\
&= \frac{1}{2}\big(\alpha|0\rangle + \beta|1\rangle\big)|00\rangle + \frac{1}{2}\big(\alpha|0\rangle - \beta|1\rangle\big)|01\rangle + \frac{1}{2}\big(\alpha|1\rangle + \beta|0\rangle\big)|10\rangle + \frac{1}{2}\big(\alpha|1\rangle - \beta|0\rangle\big)|11\rangle
\end{aligned}
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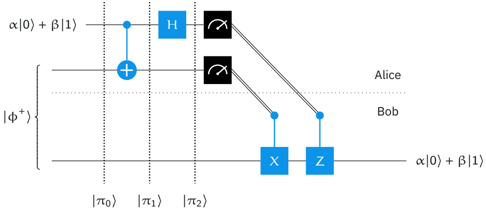


$$|\pi_2\rangle = \frac{1}{2}(\alpha|0\rangle + \beta|1\rangle)\,|00\rangle + \frac{1}{2}(\alpha|0\rangle - \beta|1\rangle)\,|01\rangle + \frac{1}{2}(\alpha|1\rangle + \beta|0\rangle)\,|10\rangle + \frac{1}{2}(\alpha|1\rangle - \beta|0\rangle)\,|11\rangle$$

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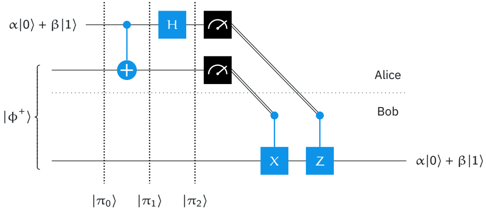


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$$\Pr(ab = 00) = \frac{1}{4}\|\alpha|0\rangle + \beta|1\rangle\|^2 = \frac{1}{4}$$

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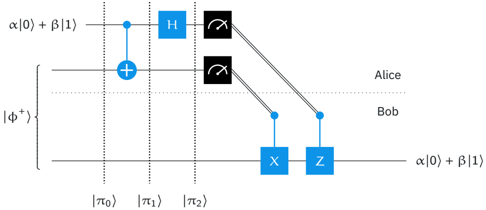


$$|\pi_2\rangle = \frac{1}{2}(\alpha|0\rangle + \beta|1\rangle)|00\rangle + \frac{1}{2}(\alpha|0\rangle - \beta|1\rangle)|01\rangle + \frac{1}{2}(\alpha|1\rangle + \beta|0\rangle)|10\rangle + \frac{1}{2}(\alpha|1\rangle - \beta|0\rangle)|11\rangle$$

$$\Pr(ab = 00) = \frac{1}{4}\|\alpha|0\rangle + \beta|1\rangle\|^2 = \frac{1}{4}$$

$$\Pr(ab = 01) = \frac{1}{4}\|\alpha|0\rangle - \beta|1\rangle\|^2 = \frac{1}{4}$$

$$\Pr(ab = 10) = \frac{1}{4}\|\alpha|1\rangle + \beta|0\rangle\|^2 = \frac{1}{4}$$

$$\Pr(ab = 11) = \frac{1}{4}\|\alpha|1\rangle - \beta|0\rangle\|^2 = \frac{1}{4}$$

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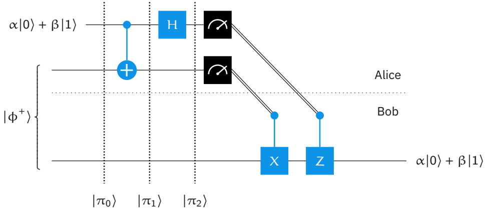


$$|\pi_2\rangle = \frac{1}{2}\big(\alpha|0\rangle + \beta|1\rangle\big)\,|00\rangle + \frac{1}{2}\big(\alpha|0\rangle - \beta|1\rangle\big)\,|01\rangle + \frac{1}{2}\big(\alpha|1\rangle + \beta|0\rangle\big)\,|10\rangle + \frac{1}{2}\big(\alpha|1\rangle - \beta|0\rangle\big)\,|11\rangle$$

| $a\,b$ | Probability | Conditional state of $(\mathsf{B}, \mathsf{A}, \mathsf{Q})$ | Operation on $\mathsf{B}$ | Final state of $\mathsf{B}$ |
|---|---|---|---|---|
| 00 | $\frac{1}{4}$ | $(\alpha|0\rangle + \beta|1\rangle)|00\rangle$ | $\mathbb{1}$ | $\alpha|0\rangle + \beta|1\rangle$ |
| 01 | $\frac{1}{4}$ | $(\alpha|0\rangle - \beta|1\rangle)|01\rangle$ | $Z$ | $\alpha|0\rangle + \beta|1\rangle$ |
| 10 | $\frac{1}{4}$ | $(\alpha|1\rangle + \beta|0\rangle)|10\rangle$ | $X$ | $\alpha|0\rangle + \beta|1\rangle$ |
| 11 | $\frac{1}{4}$ | $(\alpha|1\rangle - \beta|0\rangle)|11\rangle$ | $ZX$ | $\alpha|0\rangle + \beta|1\rangle$ |

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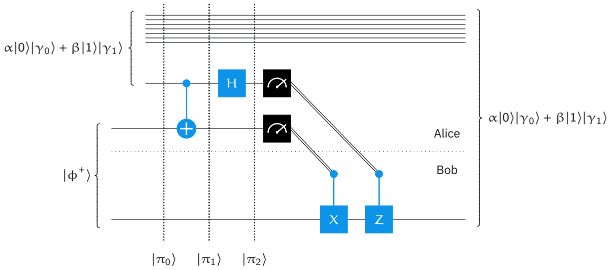
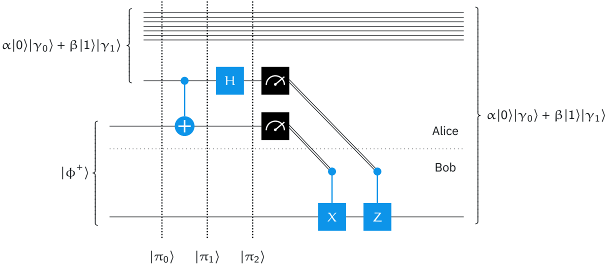



$$|\pi_0\rangle = \frac{1}{\sqrt{2}}\Big(\alpha|00\rangle|0\rangle|\gamma_0\rangle + \alpha|11\rangle|0\rangle|\gamma_0\rangle + \beta|00\rangle|1\rangle|\gamma_1\rangle + \beta|11\rangle|1\rangle|\gamma_1\rangle\Big)$$

# Teleportation analysis

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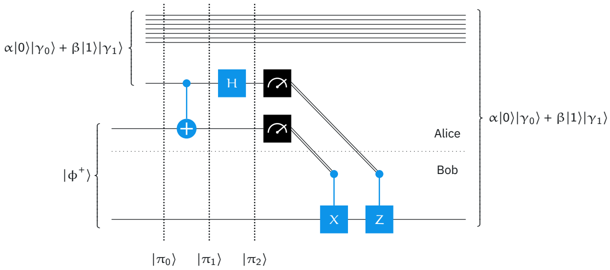
$$|\pi_1\rangle = \frac{1}{\sqrt{2}}\Big(\alpha|00\rangle|0\rangle|\gamma_0\rangle + \alpha|11\rangle|0\rangle|\gamma_0\rangle + \beta|01\rangle|1\rangle|\gamma_1\rangle + \beta|10\rangle|1\rangle|\gamma_1\rangle\Big)$$

# Teleportation analysis




$$|\pi_0\rangle = \frac{1}{\sqrt{2}}\Big(\alpha|00\rangle|0\rangle|\gamma_0\rangle + \alpha|11\rangle|0\rangle|\gamma_0\rangle + \beta|00\rangle|1\rangle|\gamma_1\rangle + \beta|11\rangle|1\rangle|\gamma_1\rangle\Big)$$

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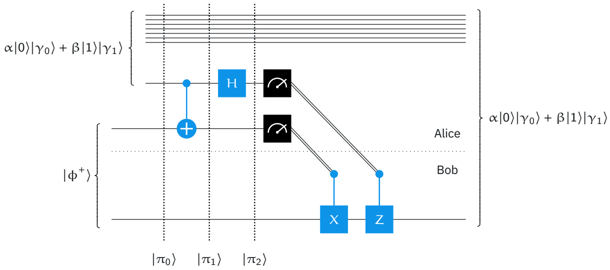


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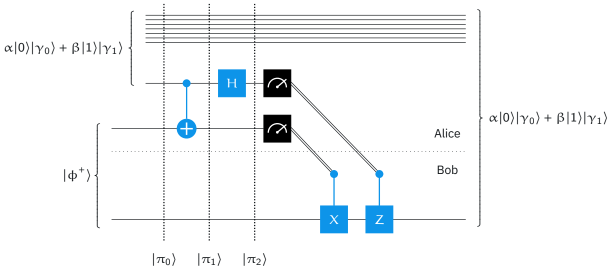
# Teleportation analysis




$$\begin{aligned}|\pi_2\rangle = \tfrac{1}{2}\Big(&\alpha|0\rangle|00\rangle|\gamma_0\rangle + \alpha|0\rangle|01\rangle|\gamma_0\rangle + \alpha|1\rangle|10\rangle|\gamma_0\rangle + \alpha|1\rangle|11\rangle|\gamma_0\rangle\\ &+\beta|1\rangle|00\rangle|\gamma_1\rangle - \beta|1\rangle|01\rangle|\gamma_1\rangle + \beta|0\rangle|10\rangle|\gamma_1\rangle - \beta|0\rangle|11\rangle|\gamma_1\rangle\Big)\end{aligned}$$

| $ab$ | Probability | Conditional state of $(\mathsf{B},\mathsf{R},\mathsf{A},\mathsf{Q})$ | Operation on B | Final state of $(\mathsf{B},\mathsf{R})$ |
|---|---|---|---|---|
| 00 | $\frac{1}{4}$ | $(\alpha|0\rangle|\gamma_0\rangle + \beta|1\rangle|\gamma_1\rangle)|00\rangle$ | $\mathbb{1}$ | $\alpha|0\rangle|\gamma_0\rangle + \beta|1\rangle|\gamma_1\rangle$ |
| 01 | $\frac{1}{4}$ | $(\alpha|0\rangle|\gamma_0\rangle - \beta|1\rangle|\gamma_1\rangle)|01\rangle$ | $Z$ | $\alpha|0\rangle|\gamma_0\rangle + \beta|1\rangle|\gamma_1\rangle$ |
| 10 | $\frac{1}{4}$ | $(\alpha|1\rangle|\gamma_0\rangle + \beta|0\rangle|\gamma_1\rangle)|10\rangle$ | $X$ | $\alpha|0\rangle|\gamma_0\rangle + \beta|1\rangle|\gamma_1\rangle$ |
| 11 | $\frac{1}{4}$ | $(\alpha|1\rangle|\gamma_0\rangle - \beta|0\rangle|\gamma_1\rangle)|11\rangle$ | $ZX$ | $\alpha|0\rangle|\gamma_0\rangle + \beta|1\rangle|\gamma_1\rangle$ |

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- Teleportation motivates *entanglement distillation* as a means to reliable quantum communication.
- Beyond its potential for communication, teleportation also has fundamental importance in the study of quantum information and computation.