Analisis Algoritma

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A. Konsep Dasar Penjumlahan

1. $$\sum_{k=1}^{n}(ca_k + b_k) = c\sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$$

2. $$\sum_{k=1}^{n} k = 1 + 2 + \cdots + n = \frac{1}{2}n(n+1)$$

3. $$\sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$
$$\sum_{k=0}^{n} k^3 = \frac{n^2(n+1)^2}{4}.$$

- **Geometrik Series**

For real $x \neq 1$, the summation

$$\sum_{k=0}^{n} x^k = 1 + x + x^2 + \cdots + x^n$$

is a ***geometric*** or ***exponential series*** and has the value

$$\sum_{k=0}^{n} x^k = \frac{x^{n+1} - 1}{x - 1}. \qquad \text{(A.5)}$$

When the summation is infinite and $|x| < 1$, we have the infinite decreasing geometric series

$$\sum_{k=0}^{\infty} x^k = \frac{1}{1 - x}. \qquad \text{(A.6)}$$

A. Konsep Dasar Penjumlahan

- **Harmonik Series**

For positive integers n, the nth ***harmonic number*** is

$$
\begin{aligned}
H_n &= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n} \\
&= \sum_{k=1}^{n} \frac{1}{k} \\
&= \ln n + O(1) .
\end{aligned}
$$

- **Product**

We can write the finite product $a_1 a_2 \cdots a_n$ as

$$\prod_{k=1}^{n} a_k .$$

If $n = 0$, the value of the product is defined to be 1. We can convert a formula with a product to a formula with a summation by using the identity

$$\lg \left(\prod_{k=1}^{n} a_k \right) = \sum_{k=1}^{n} \lg a_k .$$

Rumus Deret Untuk Hitung Efisiensi

- **Deret Aritmatika**
  - **Suku pertama (a)**
  - **Suku Ke-n (U_n)**
  - **Beda (b)**
  - **Jumlah sampai suku ke-n (S_n)**

$$U_n = a + (n-1)b$$

$$b = U_n \text{-} U_{n\text{-}1}$$

$$S_n = \frac{n}{2}(a + U_n)$$

Contoh : 1 + 2 + 3 + 4 + .. + 100

a = 1, b = 1

$$U_n = a + (n-1)b = 1 + (n-1)$$

$$b = U_n \text{-} U_{n\text{-}1} = U_2 \text{-} U_1 = 2 - 1 = 1$$

$$S_n = \frac{n}{2}(a + U_n) = \frac{n}{2}(1 + 1 + (n-1)) = \frac{n}{2}(2 + (n-1)) = \frac{n}{2}(n+1)$$

Rumus Deret Untuk Hitung Efisiensi

- **Deret Geometri**
  - **Suku pertama (a)**
  - **Suku Ke-n (U_n)**
  - **Rasio (r)**
  - **Jumlah sampai suku ke-n (S_n)**

Contoh : 1 + 2 + 4 + 8 + ..
a = 1, r = 2

$$U_n = 2^{n-1}$$

$$S_n = \frac{(2^n - 1)}{2-1} = 2^n - 1$$

$$U_n = ar^{n-1}$$

$$r = \frac{U_n}{U_{n\text{-}1}}, r \neq 1$$

$$S_n = \begin{cases} \dfrac{a(r^n - 1)}{r-1} & r > 1 \\ \dfrac{a(1-r^n)}{1-r} & r < 1 \end{cases}$$

Rumus Deret Untuk Hitung Efisiensi

- **Deret Persegi**

  Contoh : $1^2 + 2^2 + 3^2 + 4^2 + .. + 100^2$

$$U_n = n^2$$

$$S_n = \frac{n(n+1)(2n+1)}{6}$$

- **Deret Kubik**

  Contoh : $1^3 + 2^3 + 3^3 + 4^3 + .. + 100^3$

$$U_n = n^3$$

$$S_n = \frac{n^2(n+1)^2}{4}$$

Latihan 1

1. **Hitunglah hasil penjumlahan operasi berikut :**
   a. **1 + 3 + 5 + 7 +…+ 999**
   b. **2 + 4 + 6 + 8 +…+ 1024**
   c. $$\sum_{i=3}^{n+1} 1; \ \sum_{i=3}^{n+1} i; \ \sum_{i=0}^{n-1} i(i+1); \ \sum_{j=1}^{n} 3^{j+1}; \ \sum_{i=1}^{n}\sum_{j=1}^{n} ij$$

2. **Hitung order of growth dari sigma penjumlahan berikut :**

$$\sum_{i=0}^{n-1} (i^2+1)^2; \ \sum_{i=2}^{n-1} \lg i^2; \ \sum_{i=1}^{n} (i+1)2^{i-1};$$

B. Analisa Algoritma

1. **Analisis Algoritma**
2. **Analisis Efisiensi Algoritma Non-Rekursif**
3. **Order Of Growth**

1. Analisis Algoritma

- **Analisis Algoritma bertujuan memeriksa efisiensi algoritma dari dua segi : waktu eksekusi dan penggunaan memori**
- **Efisiensi waktu seberapa cepat algoritma dieksekusi**
- **Efisiensi memori berapa banyak memori yang dibutuhkan untuk menjalankan algoritma**

2. Analisis Algoritma Non Rekursif

- **Best Case, Worst Case dan Average Case**
  - **Best case : ….?**
  - **Worst case : ….?**
  - **Average case : ….?**
  - **Apakah setiap algoritma selalu ada ketiga kasus tersebut ?**
  - **Kapan suatu algoritma mengandung ketiga kasus tersebut ?**

2. Analisis Algoritma Non Rekursif

Untuk apa kita mencari T(n)?

Apakah untuk mengestimasi running time algoritma?

- Tujuan utama mencari T(n) bukan mencari waktu eksak yang dibutuhkan untuk mengeksekusi sebuah algoritma
- Tetapi untuk mengetahui tingkat pertumbuhan waktu eksekusi algoritma jika ukuran input bertambah (*order of growth*)

2. Analisis Algoritma Non Rekursif

Contoh 1

Algoritma	Cost	Time
Sum = 0	C1	1
For (i=1; i <= n; i++) {	C2	n+1
sum = sum + 1;	C3	n
For (j=1; j < i; j++) {	C4	(n+1)n
Sum = sum + j;	C5	n*n
}		
}		

Total cost = $\sum Cost * Time$

$= C_1 + C_2n + C_2 + C_3n + C_4n^2 + C_4n + C_5n^2$

$= C_1 + C_2 + C_2n + C_3n + C_4n + C_4n^2 + C_5n^2$

$= A + Bn + Dn^2$

Algoritma	Cost	Time
i =1	C1	1
Sum = 0	C2	1
while (i <= n) {	C3	n+1
j = 1;	C4	n
while(j <= n) {	C5	(n+1)n
sum = sum + 1;	C6	n*n
j = j + 1;	C7	n*n
}		
i = i + 1;	C8	n
}		

2. Analisis Algoritma Non Rekursif

Contoh 2

```
For (i=1; i <= n*n; i++) {
    For (j=0; j < i; j++) {
        Proses()
    }
}
```

$$\sum_{i=1}^{n^2}\sum_{j=0}^{i} 1 = \sum_{i=1}^{n^2} i = \frac{n^2(n^2+1)}{2}$$

- **Catatan :**
- $\sum_{i=0}^{i-1} 1 = 1 - 1 + 1 = 0 = 1$
- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

2. Analisis Algoritma Non Rekursif

- **Contoh 3**

```
for (i = 0;  i ≤ n-1; i++) {
      for (j = i+1; j ≤ n-1; j ++) {
            //Stm
      }
}
```

- **Catatan :**

$$\sum_{i=0}^{n} i = \sum_{1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=m}^{n} 1 = n - m + 1$$

- **Kompleksitas Waktu**

$$\sum_{i=9}^{n-1} \sum_{j=i+1}^{n-1} 1 = \sum_{i=0}^{n-1} ((n-1) - (i+1) + 1 =$$

$$\sum_{i=0}^{n-1} ((n - 1 - i) = \sum_{i=0}^{n-1} n - \sum_{i=0}^{n-1} 1 - \sum_{i=0}^{n-1} i =$$

$$n^2 - n - \frac{(n+1)n}{2} = \frac{n^2 - n}{2}$$

Latihan 2

Tentukan Kompleksitas algoritma berikut :

1.

```
tot = 0
for (i=1; i <= n*n; i++) {
   for (j=1; j <= i; j++) {
      for (k=1; i <= 4; k++) {
         tot  = tot + 1
      }
   }
}
```

2.

```
c ← 0
for i ← 3 to n do
     for j ← 0 to i
          C ← C + 1
```

1)

```
a = 1;
while (a < b)
{
        stm;
        a = a * 2;
}
```

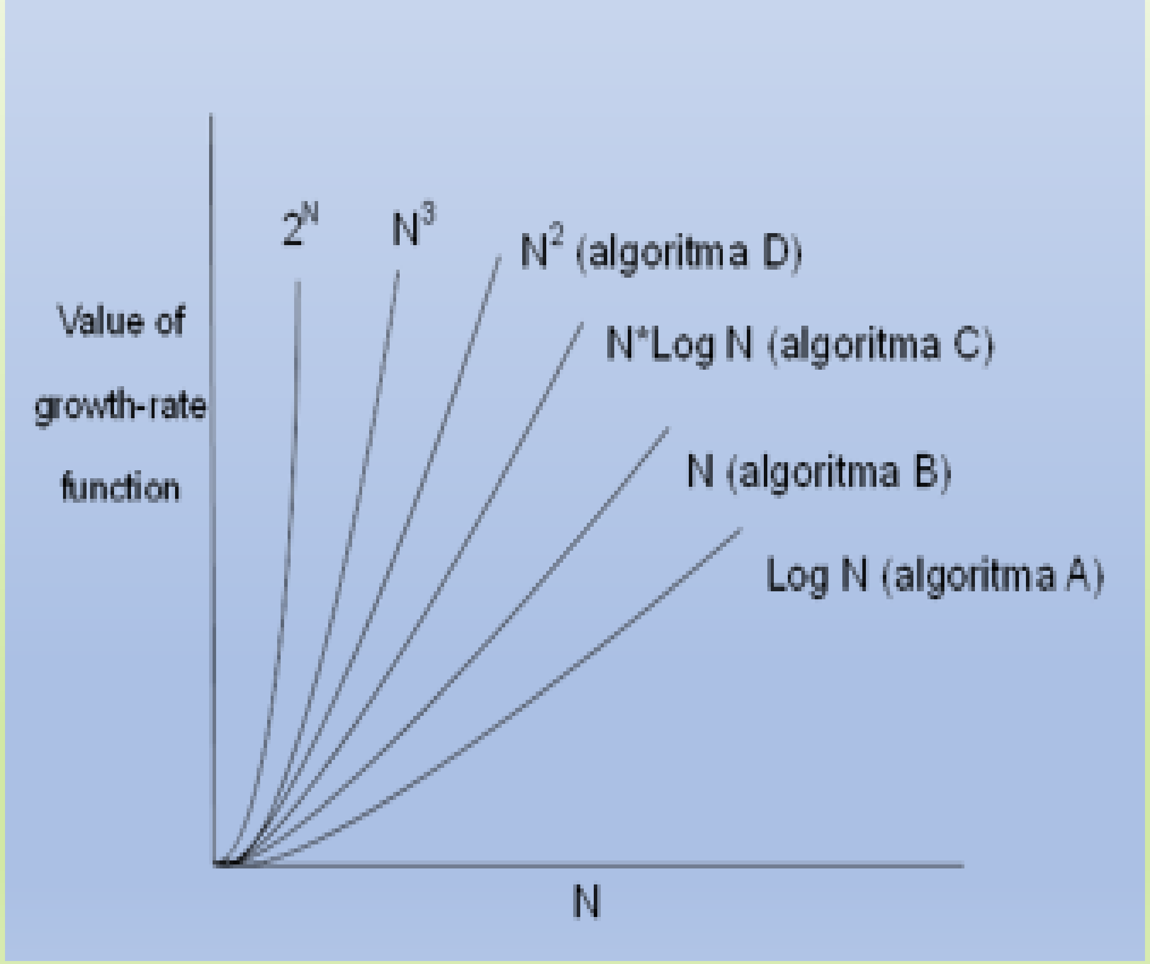
2)

```
for (i=n; i ⍰ 1; i = i/2)
{
    Stmt;
}
```

3. Orders of Growth

- ***Order of Growth*** **adalah Tingkat pertumbahan waktu eksekusi algoritma jika ukuran input bertambah**

$T_1(n) = n^2$	$T_1(10) = 100$	$T_1(100) = 10{,}000$
$T_2(n) = n^3$	$T_2(10) = 1{,}000$	$T_2(100) = 1{,}000{,}000$
$T_3(n) = n$	$T_3(10) = 10$	$T_3(100) = 100$
$T_4(n) = \log_2 n$	$T_4(10) = 3.3$	$T_4(100) = 6.6$



2. Analisis Algoritma Non Rekursif

- **Contoh 4**

INSERTION-SORT(A)	*cost*	*times*
for $j = 2$ **to** $A.length$	c_1	n
$key = A[j]$	c_2	$n - 1$
// Insert $A[j]$ into the sorted sequence $A[1 \,.\,.\, j-1]$.	0	$n - 1$
$i = j - 1$	c_4	$n - 1$
while $i > 0$ and $A[i] > key$	c_5	$\sum_{j=2}^{n} t_j$
$A[i+1] = A[i]$	c_6	$\sum_{j=2}^{n} (t_j - 1)$
$i = i - 1$	c_7	$\sum_{j=2}^{n} (t_j - 1)$
$A[i+1] = key$	c_8	$n - 1$

$$
\begin{aligned}
T(n) &= c_1 n + c_2(n-1) + c_4(n-1) + c_5 \sum_{j=2}^{n} t_j + c_6 \sum_{j=2}^{n} (t_j - 1) \\
&\quad + c_7 \sum_{j=2}^{n} (t_j - 1) + c_8(n-1) .
\end{aligned}
$$

2. Analisis Algoritma Non Rekursif

The Best Case

- the best case occurs if the array is already sorted
- For each j = 2,3, …,n, we then find that A[i] ≤ *key* in line 5 when i has its initial value of j - 1.
  - Thus $t_j = 1$ for j = 2,3, . . ., n

$$T(n) = c_1 n + c_2(n-1) + c_4(n-1) + c_5 \sum_{j=2}^{n} t_j + c_6 \sum_{j=2}^{n} (t_j - 1) + c_7 \sum_{j=2}^{n} (t_j - 1) + c_8(n-1) .$$

$T(n) = C_1 n + C_2(n-1) + C_4(n-1) + C_5(n-1) + C_8(n-1)$

$= (C_1+C_2+C_4+C_5+C_8)n + (C_2+C_4+C_5+C_8)$

$= An+B$

```
INSERTION-SORT(A)
1  for j = 2 to A.length
2      key = A[j]
3      // Insert A[j] into the sorted
           sequence A[1 .. j − 1].
4      i = j − 1
5      while i > 0 and A[i] > key
6          A[i + 1] = A[i]
7          i = i − 1
8      A[i + 1] = key
```

2. Analisis Algoritma Non Rekursif

The worst case

- If the array is in reverse sorted order
- We must compare each element A[j] with each element in the entire sorted subarray A[1. . .j -1], so $t_j = j$ for j = 2,3,. . .,n.

$$
\begin{aligned}
T(n) &= c_1 n + c_2(n-1) + c_4(n-1) + c_5 \sum_{j=2}^{n} t_j + c_6 \sum_{j=2}^{n} (t_j - 1) \\
&\quad + c_7 \sum_{j=2}^{n} (t_j - 1) + c_8(n-1) .
\end{aligned}
$$

$$\sum_{j=2}^{n} j = \frac{n(n+1)}{2} - 1$$

$$\sum_{j=2}^{n} (j-1) = \frac{n(n-1)}{2}$$

- **T(n) = An^2 + Bn + C**

```
INSERTION-SORT(A)
for j = 2 to A.length
    key = A[j]
    // Insert A[j] into the sorted
        sequence A[1 .. j - 1].
    i = j - 1
    while i > 0 and A[i] > key
        A[i + 1] = A[i]
        i = i - 1
    A[i + 1] = key
```

3. Orders of Growth

Membandingkan OOG

Algoritma *A* dan *B* merupakan algoritma untuk menyelesaikan permasalahan yang sama.

Untuk *input* berukuran *n*, waktu eksekusi algoritma *A* adalah $T_A(n)$ sedangkan waktu eksekusi algoritma *B* adalah $T_B(n)$.

***Orders of growth* mana yang paling besar?**

$$\lim_{n \to \sim} \frac{T_A(n)}{T_B(n)}$$

$$\lim_{n \to \sim} \frac{T_A(n)}{T_B(n)} \Rightarrow$$

Solusi (L'Hospital's Rule) : Penurunan Secara Berantai

- 0 maka OoG $T_A(n)$ < OoG $T_B(n)$
- C maka OoG $T_A(n)$ = OoG $T_B(n)$
- ~ maka OoG $T_A(n)$ > OoG $T_B(n)$

3. Orders of Growth

Contoh 1.

Terdapat dua algoritma yang menyelesaikan permasalahan yang sama. Untuk input berukuran n, Algoritma 1 menyelesaikan dalam

$T_1(n) = 30n^2 + 2n + 5$. Algoritma 2 dalam

$T_2(n) = n^3 + n$

- Mana yang lebih besar, OoG T_1 atau T_2? Mengapa?
- Untuk n kecil, mana yang anda pilih? Mengapa?
- Untuk n besar, mana yang anda pilih? Mengapa?

Contoh 1

- **Membandingkan OoG dari ½n(n-1) dan n^2.**

$$\lim_{n\to\infty}\frac{\frac{1}{2}n(n-1)}{n^2}=\frac{1}{2}\lim_{n\to\infty}\frac{n^2-n}{n^2}=\frac{1}{2}\lim_{n\to\infty}\left(1-\tfrac{1}{n}\right)=\frac{1}{2}$$

  - **Hasil limit = c → ½n(n-1) $\in \Theta$ (n^2)**

- **Membandingkan OoG dari $\log_2 n$ dan √n**

$$\lim_{n\to\infty}\frac{\log_2 n}{\sqrt{n}}=\lim_{n\to\infty}\frac{(\log_2 n)'}{(\sqrt{n})'}=\lim_{n\to\infty}\frac{(\log_2 e)\frac{1}{n}}{\frac{1}{2\sqrt{n}}}=2\log_2 e\lim_{n\to\infty}\frac{\sqrt{n}}{n}=0$$

  - **Hasil limit = 0 → $\log_2 n \in O(\sqrt{n})$**

Contoh 2

- **Membandingkan OoG dari n! dan 2^n.**

$$\lim_{n\to\infty}\frac{n!}{2^n}=\lim_{n\to\infty}\frac{\sqrt{2\pi n}\left(\frac{n}{e}\right)^n}{2^n}=\lim_{n\to\infty}\sqrt{2\pi n}\,\frac{n^n}{2^n e^n}=\lim_{n\to\infty}\sqrt{2\pi n}\left(\frac{n}{2e}\right)^n=\infty$$

  - **Hasil limit = ∞ → n! $\in \Omega(2^n)$**

$$\frac{n!}{2^n}=\frac{n}{2}\cdot\frac{n-1}{2}\cdot\frac{n-2}{2}\cdots\frac{3}{2}\cdot\frac{2}{2}\cdot\frac{1}{2}>\frac{n}{2}\cdot 1\cdots 1\cdot\frac{1}{2}=\frac{n}{4}$$

Latihan 3

Tentukan kelas orders of growth dari

- $T_1(n) = 2n^3 + 4n + 1$
- $T_2(n) = 0{,}5\ n! + n^{10}$
- $T_3(n) = n^3 + n \log n$
- $T_4(n) = 2^n + 4n^3 + \log n + 10$

Konsep Penting Turunan

- **Contoh-contoh turunan :**

$$y = x^n \rightarrow y' = n.x^{n-1}$$

$$y = {}^a\log x \rightarrow y' = \left({}^a\log e\right)\frac{1}{x}$$

$$y = {}^a\log x \rightarrow y' = \left(\frac{{}^e\log e}{{}^e\log a}\right)\frac{1}{x}$$

$$= \left(\frac{1}{{}^e\log a}\right)\frac{1}{x} = \frac{1}{\ln(a)x} = \frac{1}{x\ln(a)}$$

$$y = 2^n \rightarrow y' = 2^n.\ln(2)$$

$$y = \ln(x) \rightarrow y' = \frac{1}{x}$$

$$y = e^{f(x)} \rightarrow y' = e^{f(x)}.f'(x) = e^x.1 = e^x$$

$$y = u.v \rightarrow y' = u'v + uv'$$

$$y = u/v \rightarrow y' = (u'v - uv')/v^2$$

- Manipulasi operasi matematika :
  - $y = n! \rightarrow y = \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$
  - $\frac{\ln(a)}{\ln(b)} = \frac{{}^e\log a}{{}^e\log b} = {}^b\log a$
  - $e^x = \lim_{n\to\sim}\left(1 + \frac{x}{n}\right)^n$
  - ${}^a\log\left(\lim_{\Delta x\to 0}\left(1 + \frac{\Delta x}{x}\right)^{\frac{1}{\Delta x}}\right) = ...?$

$$\text{misal } n = \frac{1}{\Delta x} \rightarrow \sim \quad \text{maka} \quad \Delta x = \frac{1}{n} \rightarrow 0$$

$$= {}^a\log\left(\lim_{\Delta x\to 0}\left(1 + \frac{\Delta x}{x}\right)^{\frac{1}{\Delta x}}\right) = {}^a\log\left(\lim_{\Delta x\to 0}\left(1 + \frac{x^{-1}}{n}\right)^n\right)$$

$$= {}^a\log e^{x^{-1}} = {}^a\log e^{\frac{1}{x}} = \frac{{}^e\log e^{\frac{1}{x}}}{{}^e\log a} = \frac{1}{x}\frac{{}^e\log e}{{}^e\log a} = \frac{1}{x\ln(a)}$$

Thank You !