**rTextBook:**
**1.1: Scientific Method**
-Hypothesis must be testable
-Law:
-tells us **what** happens under certain circumstances. Expressed in the form of relationships between observable quantity
-Theory:
-Tells **why** something happens and explains phenomena in terms of more basic processes and relationships.
-Model:
    -Simplified conceptual representation of some phenomenon.

**1.2: Symmetry**
-Translation & Rotation Symmetry are common. Seen with an equilateral triangle, science applies this concept to testing hypotheses.
-Move an experiment to a different location and the same observation makes it symmetric translationally.

1.3: Matter and the Universe
-In the universe, we can gather physical quantities to observe it (mass, length, temp, etc.)
    -Length: SI unit is meters
-Order of Magnitude:
    -0.3 & 3 has an order of magnitude of 1
    -3 & 30 has OoM of 10
    -This concept is used in scientific notation to allow for more efficiency.
-Atoms
    -Pro, neu, elec
    -$10^{-10}$metre diameter

**1.4: Time and Change**
-Not reversible
-Principle of causality:
    -When A causes B, A must happen first.
-Law of Conservation so “change” is a difference/shift in states.

**1.5: Representation**
-Draw stuff = 5Head
-

**1.6: Physics Quantity and Units**

**Lecture**
-SigFigs:
    -Norm:
        -4.2183 = 5 sigfigs
        -80.0054 = 6
        -0.00000342 = 3 sigfigs
    -In equation
        -Answer’s sigfigs is lowest amount of sigfigs in a value in the equation
        -Ex: $2 * 3.14 * 27.3 = 171.444 -> 171$
-Scientific Notation:
    -prefixes

**Table 1.3** SI prefixes

| $10^n$ | Prefix | Abbreviation | $10^n$ | Prefix | Abbreviation |
|---|---|---|---|---|---|
| $10^0$ | — | — | | | |
| $10^3$ | kilo- | k | $10^{-3}$ | milli- | m |
| $10^6$ | mega- | M | $10^{-6}$ | micro- | $\mu$ |
| $10^9$ | giga- | G | $10^{-9}$ | nano- | n |
| $10^{12}$ | tera- | T | $10^{-12}$ | pico- | p |
| $10^{15}$ | peta- | P | $10^{-15}$ | femto- | f |
| $10^{18}$ | exa- | E | $10^{-18}$ | atto- | a |
| $10^{21}$ | zetta- | Z | $10^{-21}$ | zepto- | z |
| $10^{24}$ | yotta- | Y | $10^{-24}$ | yocto- | y |

        -Ex: 150,000,000 m = 1.5 x 10^8 m

-Units

-Length

-Meters (SI unit)

-Time

-Seconds

-Conversion

-Ex: 4.5in to mm (in cancels out)

$$4.5 \text{ in} = (4.5 \cancel{\text{in}}.)\left(\frac{25.4 \text{ mm}}{1 \cancel{\text{in}}.}\right) = 4.5 \times 25.4 \text{ mm} = 1.1 \times 10^2 \text{ mm}$$

-Order of Magnitude

-Vector

-

-Displacement

-$\Delta \overline{x}$= a vector pointing from start to end.

-Freefall

-When only force acting on motion is Gravity

-The position of an object on a plane has two parts: **its distance from a reference point and its angle measured from a reference line**. Both of these measurements are necessary to define an exact position.

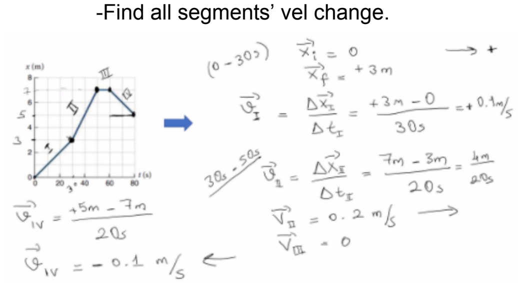
-End of calc should be SI unit. Like m/s.

-Avg vel:

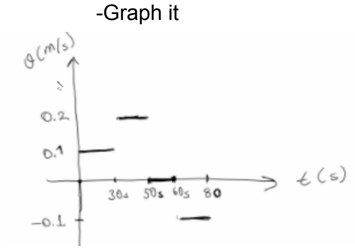
$$\vec{v}_x = \frac{\Delta \vec{x}}{\Delta t} = \frac{\vec{x}_f - \vec{x}_i}{\Delta t}$$

-Linear Pos Graph -> Vel Graph

-Find all segments' vel change.

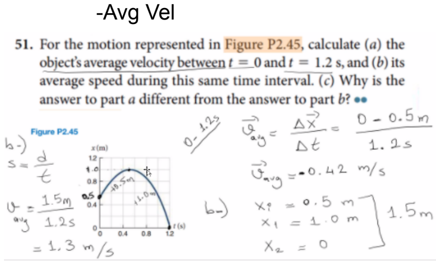



-Graph it



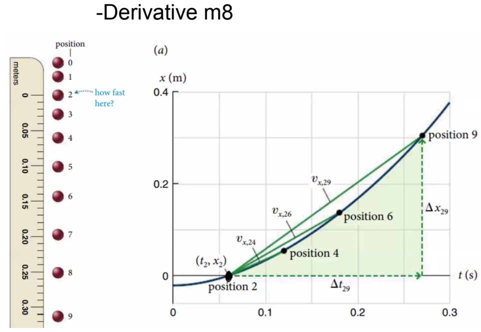

-Parabolic Pos Graph -> Vel Graph

-Avg Vel




-Instantaneous Vel:

-Derivative m8




$$\vec{v}_x \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t}.$$

$$\vec{v}_x \equiv \frac{d\vec{x}}{dt}.$$

$$\boxed{\vec{v} = \frac{d\vec{r}}{dt}.}$$

-Derivative problem @ t=1 & 4

**65.** A dragster's position as a function of time is given by $x(t) = bt^{3/2}$, where $b = 30.2\ \mathrm{m/s^{3/2}}$. Calculate the $x$ component of its velocity at 1.0 s and at 4.0 s. •

$$x(t) = \left[\left(30.2\ t^{3/2}\right)\right] \mathrm{m}$$

$$\vec{v} = \frac{d\vec{x}}{dt} = \frac{d}{dt}\left(30.2\ \frac{\mathrm{m}}{\mathrm{s}^{3/2}}\ t^{3/2}\right)$$

$$\frac{dx}{dt} = \left[\left((30.2) * \frac{3}{2}\ t^{1/2}\right)\right]\frac{\mathrm{m}}{\mathrm{s}}$$

n of #65

$$\vec{v}(t = 1\,\mathrm{s}) = \left[(30.2)\frac{3}{2}(1)^{1/2}\right]\frac{\mathrm{m}}{\mathrm{s}}$$

$$\vec{v}(t = 4\,\mathrm{s}) = \left[(30.2)\cdot\frac{3}{2}(4)^{1/2}\right]\frac{\mathrm{m}}{\mathrm{s}}$$

-Derivative problem w/ Crit Points

-Yea whatever

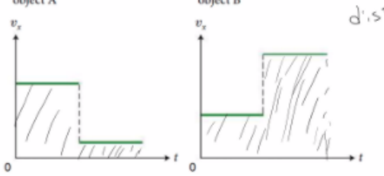
-Displacement from Vel Graph:

-If it negative, combine accordingly

**58.** Figure P2.58 shows the $x$ component of the velocity as a function of time for objects A and B. Which object has the greater displacement over the time interval shown in the graph? ••

Area under v-t
⇓
displacement

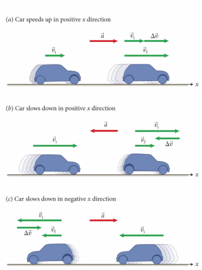
**Figure P2.58**




-Accel

**Changes in velocity**

- Whenever an object's velocity vector $\vec{v}$ and acceleration vector $\vec{a}$ point in the same direction, the object speeds up.
- If $\vec{v}$ and $\vec{a}$ point in the
- opposite direction, the object slows down.




-

-deltaV is V(final) - V(init)

-Inertia Ratio after collision

$$\frac{m_u}{m_s} \equiv -\frac{\Delta v_{sx}}{\Delta v_{ux}}$$

-

-Kinematic equations of velocity and acceleration average and what not.

**Let's review the kinematics equations**

$$\vec{v}_{x,\mathrm{av}} \equiv \frac{\Delta\vec{x}}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} \qquad \vec{a}_{x,\mathrm{av}} \equiv \frac{\Delta\vec{v}_x}{\Delta t} = \frac{\vec{v}_{x,f} - \vec{v}_{x,i}}{t_f - t_i}$$

$$\boxed{\vec{v}_{x,f} = \vec{v}_{x,i} + \vec{a}_x\Delta t \ \text{(constant acceleration)}} \qquad \vec{v}_{avg} = \frac{\vec{v}_f + \vec{v}_i}{2}$$

$$a_x = \frac{dv_x}{dt}$$

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

$$\Delta v_x = \int_{t_i}^{t_f} a_x(t)\,dt \qquad \Delta x = \int_{t_i}^{t_f} v_x(t)\,dt$$

If acceleration is constant then:

1) $\vec{x_f} = \vec{x_i} + \vec{v_{ix}}(t_f - t_i) + \frac{1}{2}\vec{a_x}(t_f - t_i)^2$

2) $\vec{v_{fx}} = \vec{v_{ix}} + \vec{a_x}(t_f - t_i)$

3) $v_{fx}^2 = v_{ix}^2 + 2\vec{a_x}(\vec{x_f} - \vec{x_i})$

4) $\vec{v_{av,x}} = \frac{1}{2}(\vec{v_{fx}} + \vec{v_{ix}}) = \frac{\Delta x}{\Delta t}$

- -Top row is average vel/accel

-Momentum

-p = mv

-Knowing conservation of momentum, **momentum init = final**: $m_1v_{11} + m_2v_{21} = m_1v_{12} + m_2v_{22}$ (left is int, right is final)

1.) Imagine pushing a crate in a straight line along a surface at a steady speed of 1 m/s. What is the time rate of change in the momentum of the crate?

$$\frac{\Delta P}{\Delta t} = m \cdot \frac{\Delta v}{\Delta t} = 0$$

-Change in momentum / Change in Time for this one.

-Inertia

## Review of concepts

Inertia: "Inertia is a measure of an object's tendency to resist any change in its velocity."

The inertia of an object is represented by the symbol m and is a scalar quantity.

Last time we have looked at collisions of carts on low-friction track. We have studied the collisions of identical carts and saw that if the velocity of one cart increases by a certain amount then the velocity of the other cart decreases by exactly same amount.

When we changed the masses, we have seen that massive objects put up more resistance to change their velocities....

$$\frac{m_u}{m_s} \equiv -\frac{\Delta v_{sx}}{\Delta v_{ux}}$$

-

Let's take a look at this graph here and calculate the ratio of the inertias of the objects A and B:

$$\frac{m_u}{m_s} = -\frac{\Delta \vec{v}_{sx}}{\Delta \vec{v}_{ux}}$$

$\frac{m_A}{m_B} = \frac{2.0\,m/s}{}$

$\vec{v}_{iB} = 0$ $\Delta\vec{v}_B = +2.0\,m/s$

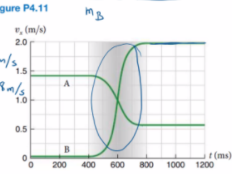
$\vec{v}_{fB} = 2.0\,m/s$ $\Delta\vec{v}_A = -0.8\,m/s$

$\vec{v}_{iA} \approx 1.4\,m/s$

$\vec{v}_{fA} = 0.6\,m/s$

$\Delta\vec{v}_A = 0.6\,m/s - (1.4\,m/s)$

Figure P4.11




-Kinetic Energy (Joules aka kg * m^2/sec^2)

$$K = \frac{1}{2}mv^2$$

-Elastic collision

-Where relative speed |v_a - v_b| of init & final are same.

-Where total kinetic energy of init & final are same.

### Classification of collisions

- **Elastic collision**: A collision in which the relative speeds before and after the collision are the same.
- **Inelastic collision**: A collision in which the relative speed after the collision is lower than before the collision.
- **Totally inelastic collision**: A special type of inelastic collision in which the two objects move together after the collision so that their relative speed is reduced to zero.

-Perfect inelastic collision

**5.4** A moving cart collides with an identical cart initially at rest on a low-friction track, and the two lock together. What fraction of the initial kinetic energy of the system remains in this totally inelastic collision?

$m_1 = m_2 = m$

$\vec{v}_{1i} = +v$

$\vec{v}_{2i} = 0$

Before collision

$(m+m) = 2m$

$\vec{v}_f = ?$

After collision

$\boxed{\vec{v}_f = +\frac{v}{2}}$

$\rightarrow +x$

$\vec{P}_i = \vec{P}_f$

$\frac{\cancel{m} v}{2} = \frac{2\cancel{m} \cdot \vec{v}_f}{2}$

**5.4** A moving cart collides with an identical cart initially at rest on a low-friction track, and the two lock together. What fraction of the initial kinetic energy of the system remains in this totally inelastic collision?

$m_1 = m_2 = m$

$\vec{v}_{1i} = +v$

$\vec{v}_{2i} = 0$

Before collision

$(m+m) = 2m$

$\vec{v}_f = ?$

After collision

$\boxed{\vec{v}_f = +\frac{v}{2}}$

$\boxed{K_f = \frac{K_i}{2}}$

$K_i = \frac{1}{2} m v^2$

$K_f = \frac{1}{2} 2m \frac{v^2}{4}$

$K_f = \frac{1}{2}\left(\frac{1}{2} m v^2\right)$

$\boxed{K_f = \frac{1}{2} K_i}$



-Where relative speed |v_a - v_b| of init & final are not same, and relative speed final is 0.

-Where kinetic energy of init & final are not same.

-Partial inelastic collision

-Partially inelastic collisions are the most common form of collisions in the real world. In this type of collision, the objects involved in the collisions do not stick, but some kinetic energy is still lost. Friction, sound and heat are some ways the kinetic energy can be lost through partial inelastic collisions.

-Where relative speed (v_a - v_b) of init & final are not same.

-Where kinetic energy of init & final are not same.

answer

Is the following collisions elastic, inelastic, or totally inelastic?

A red billiard ball moving at $v_{r\,x,i} = +2.2$ m/s hits a white billiard ball initially at rest. After the collision, the red ball is at rest and the white ball moves at $v_{w\,x,f} = +1.9$ m/s.

Before

$\vec{v}_{ri} = +2.2\,m/s$

$\vec{v}_{wi} = 0$

$\vec{v}_{(wr)i} = +2.2\,m/s - 0$

$= +2.2\,m/s$

After

$\vec{v}_{rf} = 0$

$\vec{v}_{wf} = +1.9\,m/s$

$(\vec{v}_{wr})_f = 0 - 1.9\,m/s$

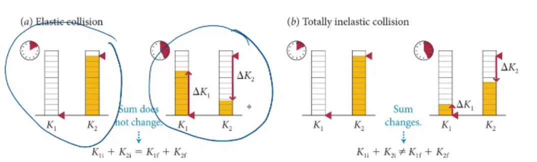
$(\vec{v}_{wr})_f = -1.9\,m/s$



-Kinetic Energy and Collisions

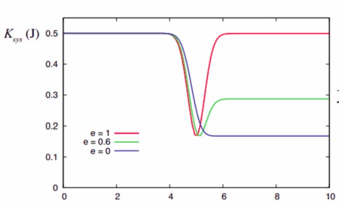
### Kinetic energy

- Because kinetic energy is a scalar extensive quantity, bar diagrams are a good way to visually represent changes in this quantity.
- The figure below shows collisions seen in the previous slides.




-e, coefficient of restitution

### Chapter 5 Review
### Kinetic energy and collisions




$$e \equiv \frac{v_{12f}}{v_{12i}}$$

← relative init & final

**Table 5.4** Coefficient of restitution for various processes

| Process | Relative speed | Coefficient of restitution |
|---|---|---|
| totally inelastic collision | $v_{12f} = 0$ | $e = 0$ |
| inelastic collision | $0 < v_{12f} < v_{12i}$ | $0 < e < 1$ |
| elastic collision | $v_{12f} = v_{12i}$ | $e = 1$ |
| explosive separation* | $v_{12f} > v_{12i}$ | $e > 1$ |

*See Section 5.8.

$E = K + E_{int}$

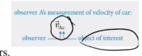
from 120

$$\text{Coefficient of restitution } (e) = \frac{|\text{Relative velocity after collision}|}{|\text{Relative velocity before collision}|}$$

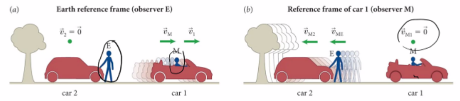
In inelastic collisions, the coefficient is related to the kinetic energy by $e = \sqrt{\frac{KE_{(\text{after collision})}}{KE_{(\text{before collision})}}}$

-Relativity in Motion

**Section 6.1: Relativity of motion** $\vec{v}_{12} = \vec{v}_2 - \vec{v}_1$




- The figure below shows two observers watching the motion of two cars.
- Observer E is at rest relative to Earth, while M is moving relative to Earth at velocity
- When Earth is the only reference frame, here the subscript $_E$ is dropped




Relative to observer E, car 2 is at rest and car 1 moves to the right.

But relative to observer M, car 1 is at rest while car 2, observer E, and Earth move to the left.

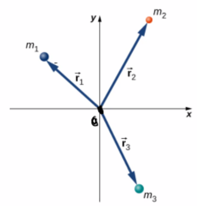
-Center of Mass & Center of Mass Velocity

Extended object of mass $M$, made of $N$ interacting particles with masses $m_j$, where j=1,2,3,…,N

Let the coordinates of $m_1$ be $(x_1,y_1,z_1)$ and so on…..Let's define the center of mass of the system as $(x_{cm}, y_{cm}, z_{cm})$

$\vec{r} = (x, y, z)$

$\vec{r} = (x, 0, 0)$




$$M = \sum_{j=1}^{N} m_j.$$

$$\boxed{\vec{r}_{cm} \equiv \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots}{m_1 + m_2 + \cdots}}$$

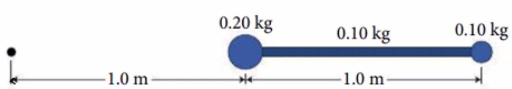
$$\vec{v}_{cm} \equiv \frac{d\vec{r}_{cm}}{dt} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots}{m_1 + m_2 + \cdots}$$

-v_cm is finding the **speed** of the center of mass. (total momentum/total mass)

-Ex)

**41.** Determine the position of the center of mass of the baton shown in Figure P6.41, taking the origin of your coordinate axis to be (*a*) the center of the larger ball, (*b*) the center of the smaller ball, and (*c*) a point 1.0 m to the left of the larger ball. How much calculation was required for each of the three parts of this problem? ••

**Figure P6.41**




a.) take origin at center large

$m_1 = 0.20$ kg

$m_2 = 0.10$ kg

$m_3 = 0.10$ kg

$$\vec{x}_{cm} = \frac{m_1\vec{x}_1 + m_2\vec{x}_2 + m_3\vec{x}_3}{m_1 + m_2 + m_3}$$

-So, add all (masses)*(distance from center) of each object/(all masses combined)

$$x_{cm} = \frac{0.20\text{kg}\cdot(0) + 0.10\text{kg}\cdot(0.5\text{m}) + 0.10\text{kg}\cdot(1.0\text{m})}{0.20\text{kg} + 0.10\text{kg} + 0.10\text{kg}}$$

$$\boxed{x_{cm} = +0.38 \text{ m}}$$

-b) would be 0.38-1 = -0.62m, since the center doesn't change (world pos) for the object.

-c) would be 1.38m, since that's how far from the center of mass.

-Acceleration ratio has inverse relation to mass ratio

$$\frac{\vec{a}_{1x}}{\vec{a}_{2x}} = -\frac{m_2}{m_1}$$

-Acceleration during interaction Exercise

**7.11** A 1000-kg compact car and a 2000-kg van, each traveling at 25 m/s, collide head-on and remain locked together after the collision, which lasts 0.20 s. (*a*) According to Eq. 7.6, their accelerations during the collision are unequal. How can this be if both initially have the same speed and the time interval during which the collision takes place is the same amount of time for both? (*b*) Calculate the average acceleration in the direction of travel experienced by each vehicle during the collision.

$m_1 = 1000$ kg

$m_2 = 2000$ kg

$|\vec{v}_1| = |\vec{v}_2| = 25$ m/s

$\Delta t = 0.20$ s

$\boxed{m_1} \to \vec{v}_1 \quad \vec{v}_2 \leftarrow \boxed{m_2}$ before

$\boxed{m_1|m_2}$ After

$$\frac{\vec{a}_{1x}}{\vec{a}_{2x}} = -\frac{m_2}{m_1}$$

$$\vec{P}_i = \vec{P}_f$$

$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{v}_f$$

$$1000\text{kg} \times 25\text{m/s} - 2000\text{kg}\cdot 25\text{m/s} = (3000\text{kg})\cdot\vec{v}_f$$

$$\vec{v}_f = -8.3 \text{ m/s}$$



-Final Vel = ? (probably going left because big inertia of m_2) (pic on right)

-Solving for Accel:

-(Delta Vel / Delta Time)

-a_1 = -167 m/s^2

-a_2 = 83.5 m/s^2

-Impulse Instantaneous (J vector)

$$\mathbf{J} = \int_{t_1}^{t_2} \mathbf{F}\,dt = \Delta\mathbf{p} = m\mathbf{v_2} - m\mathbf{v_1}$$

-Change in Momentum = Impulse

2.) Imagine that a 1.0 kg cart travelling rightward at 1.0 m/s hits a 3.0 kg cart at rest. Afterwards, the smaller cart is observed to move leftwards with a speed of 0.75 m/s. What impulse did the collision give the smallest cart at the expense of the larger?

a.) None; the larger cart was at rest and had no momentum to give.
b.) None; the lighter cart gave impulse to the more massive cart, not the other way around.
c.) 0.75 kgm/s leftward
d.) 1.75 kgm/s leftward
e.) 1.00 kgm/s leftward

$\boxed{\vec{J}_{12}} = ?$

$\vec{J} = \Delta\vec{p}$

$-1.75\ kg\,m/s = 1.0kg \cdot (-0.75\frac{m}{s}) - 1.0kg \cdot 1\frac{m}{s}$

$m_1 = 1.0\ kg$
$\vec{v}_{1i} = +1.0\ m/s$
$m_2 = 3.0\ kg$
$\vec{v}_{2i} = 0$
$\vec{v}_{1f} = -0.75\ m/s$
$\vec{v}_{2f} = ?$

-System Energy
-Elastic: Kinetic (K) -> Potential (U)
-Inelastic: System lost energy, usually becomes heat and escapes.

-Mechanical Energy / Coherent Energy
-E = **u**+**K** (where **u** is potential, and **K** is kinetic)
-E_i = E_f
-

**Mechanical Energy**

The sum of a system's kinetic energy and potential energy is called the system's **mechanical energy** or **coherent energy**.

System's mechanical energy is conserved when there are no dissipative interactions:

$$\Delta U + \Delta K = 0$$

$\boxed{E_i = E_f}$

$E = K + U$ $\quad K_i + U_i = K_f + U_f$

$\Delta U = -\Delta K$

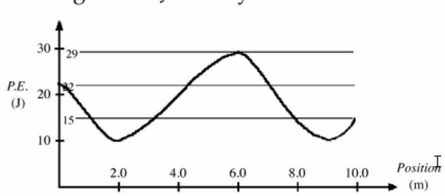
## 7.9: Potential energy near Earth's surface

- Some algebraic manipulations of previous equation gives
$$\boxed{m_b g(x_f - x_i) + \tfrac{1}{2} m_b(v_f^2 - v_i^2) = 0}$$
$$\Delta U^G + \Delta K_b = 0$$
- Therefore, the change in gravitational potential energy is
$$\Delta U^G = mg\Delta x \text{ (near Earth's surface)}$$

- We can conclude that the gravitational potential energy of the Earth-object system near Earth's surface is
$$U^G(x) = mgx$$

-m=mass
-g=9.81
-x pos at the time.
-U can be negative.
-if the reference point is "inverted", like the ground of the object is actually above the object.
-Exercises:

**Problem 3)** A 2.0 kg mass is moving along the $x$-axis. The potential energy curve as a function of position is shown in the figure. The kinetic energy of the object at the origin is 12 J. The system is conservative, and there is no friction.




(a) What will be the kinetic energy at 2.0 m along the +$x$-axis?

(b) What will be the speed of the object at 6.0 m along the +$x$-axis?

$$K(0) = 12J$$
$$E = K + U = 12J + 22J = 34J$$
$$E_i = E_f$$
$$K(2.0m) = 34J - 10J = 24J$$
$$K(6.0m) = 34J - 29J = 5J$$
$$K(6.0m) = 5J = \frac{1}{2}(2.0kg)(v)^2$$
$$v(6.0m) = +\sqrt{5}\ \frac{m}{s}, -\sqrt{5}\ \frac{m}{s}$$

-Newton's 2nd Law of Motion (in Newton (N) (kg*m/s^2))

**The vector sum of the forces exerted on an object is equal to the time rate of change in the momentum of that object.**

$$\Sigma\vec{F} = m\vec{a},$$

- Forces are manifestations of interactions.
- When an object participates in one interaction only, the **force** exerted on the object is given by the time rate of change in the object's momentum.

Let's write the above statement mathematically together:

$\vec{p} = m\,\vec{v}$

$\frac{d\vec{p}}{dt} = \frac{d}{dt}(m\,\vec{v}) = m\frac{d\vec{v}}{dt}$

$\boxed{\vec{F}_{net} = m \cdot \vec{a}}$



-ThisSlideIs the reasoning/math to get net force.

-When calculating, F_x, F_y, F_z, you have to take individual components separately.
-F direction = a direction, because Net Force is dependent on accel.

-Convertible Kinetic Energy

$$K_{conv} = \frac{1}{2}\frac{m_1 \cdot m_2}{m_1 + m_2}\left(v_2 - v_1\right)^2$$

-If asked: What percentage of the original kinetic energy is convertible to internal energy? Then $\frac{K_{conv}}{K}$

-The convertible kinetic energy is the amount of kinetic energy that can be converted to another form without violating conservation of momentum.

-Unit Vector Notation

Figure 1 shows the definition of components of a vector $\vec{\mathbf{V}}$. The angles $\alpha$, $\beta$ and $\gamma$ are the angles that the vector makes to the three coordinate axes $x$, $y$ and $z$ respectively. The dashed lines are perpendiculars drawn from the tip of the vector to the three coordinate axis. The components $V_x$, $V_y$ and $V_z$ of $\vec{\mathbf{V}}$ are defined to be the segments of the coordinate axes marked out by these perpendiculars as shown. Hence, using the definition of cosine for the three right-angle triangles, we find,

$$V_x = V\cos\alpha, \quad V_y = V\cos\beta, \quad V_z = V\cos\gamma,$$

where $V$ (short for $|\vec{\mathbf{V}}|$) is the magnitude of the vector $\vec{\mathbf{V}}$. We also define **unit vectors** (vectors of magnitude one) along each of the three coordinate axes $x$, $y$ and $z$ to be $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ respectively (figure 2). The following three vectors in the three coordinate directions can now be defined.

$$\vec{\mathbf{V}}_x = V_x\hat{\mathbf{i}}, \quad \vec{\mathbf{V}}_y = V_y\hat{\mathbf{j}}, \quad \vec{\mathbf{V}}_z = V_z\hat{\mathbf{k}}.$$

Using the triangle rule for vector addition twice, this gives,

$$\vec{\mathbf{V}} = \vec{\mathbf{V}}_x + \vec{\mathbf{V}}_y + \vec{\mathbf{V}}_z = V_x\hat{\mathbf{i}} + V_y\hat{\mathbf{j}} + V_z\hat{\mathbf{k}}.$$

This is known as the **unit vector notation** of a vector.

-Essentially, instead of writing the full vector, you multiply the magnitude of that vector component (vector3.x,vector3.y,vector3.z) with a unit vector corresponding to each axis.

-(hats) i = (1,0,0), j= (0,1,0), k = (0,0,1)

-Fundamental Interaction

-Grav, Weak, Electromagnetic, Strong (From weakest to strongest)

- An interaction is **fundamental** if it cannot be explained in terms of other interactions.
- All known interactions can be traced to the four fundamental interactions listed below.

**Table 7.1** Fundamental Interactions

| Type | Required attribute | Relative strength | Range | Gauge particle | Propagation speed |
|---|---|---|---|---|---|
| gravitational | mass | 1 | $\infty$ | graviton? | $c$? |
| weak | weak charge | $10^{25}$ | $10^{-18}$ m | vector bosons | varies |
| electromagnetic | electrical charge | $10^{36}$ | $\infty$ | photon | $c$ |
| strong | color charge | $10^{38}$ | $10^{-15}$ m | gluon | $c$ |

The relative strength is a measure of the magnitude of the effects of these interactions on two protons separated by about $10^{-15}$ m. The question marks in the last two columns indicate that the information provided has not yet been verified experimentally. The symbol $c$ represents the speed at which lights travels.

- Strong nuclear interactions act only between quarks: to form protons and neutrons>>>> binds protons and neutrons to the nuclei .
- Electromagnetic interactions: act between electrically charges particles.
- The weak nuclear interactions act between all quarks and leptons.
- The gravitational interactions act between all particles.

-Macroscopic Interactions

-Long Range Interactions

-Grav, Weak Electromagnetic, Strong

-Contact Interactions

-Friction, Tension, Spring,

-Inertial reference frame & Newton's 1st Law

-where vel is constant or 0.

**Newton's 1st Law of Motion**

**Newton's first law of motion**, first formulated by Galileo Galilei, is what we called the *law of inertia* in Chapter 6:

*In an inertial reference frame, any isolated object that is at rest remains at rest, and any isolated object that is in motion keeps moving at a constant velocity.*

-The ball will go forever with vel = v, unless some force slows/speeds it down/up.

-Static vs Kinetic Friction (http://hyperphysics.phy-astr.gsu.edu/hbase/frict.html) (Coefficient: https://www.youtube.com/watch?v=ErsbmryQIX8)

**Static Friction and Kinetic Friction**

- Friction, like the normal force, is exerted by a surface.
- *Kinetic friction,* denoted by $\vec{f}_k$, acts as an object slides across a surface. Kinetic friction is a force that always "opposes the motion."
- *Static friction,* denoted by $\vec{f}_s$, is the force that keeps an object "stuck" on a surface and prevents its motion relative to the surface. Static friction points in the direction necessary to *prevent* motion.

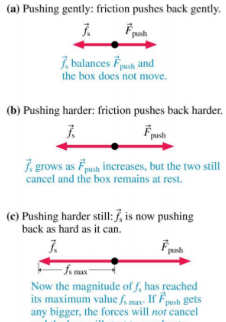
$\mu_s > \mu_k$

**Let's talk about Static Friction a little bit more:**

- The harder one pushes, the harder the friction force from the floor pushes back.
- If one pushes hard enough, the box will slip and start to move.
- The static friction force has a maximum possible magnitude:

$$f_{s\,\max} = \mu_s N$$

where $\mu_s$ is called the **coefficient of static friction.**




$\boxed{f_s \leq \mu_s \cdot N}$

$\boxed{f_{s_{max}} = \mu_s \cdot N}$ bound =>

When static fric exceeds max, turns into kine fric (which is constant)

-Uncertainty combining

## Addition and Subtraction

If $R$ is some combination of sums and differences, that is,

$$R = aA \pm bB \pm c$$

then

$$u_R = \sqrt{(au_A)^2 + (bu_B)^2}$$

## Multiplication and Division

If $R = cA^aB^b$, then

$$u_R = R\sqrt{\left(\frac{au_A}{A}\right)^2 + \left(\frac{bu_B}{B}\right)^2}$$

-Example of finding Static Friction from Free Body and Sin and Cos

**Problem 3:**

A 25.0 N book is held at rest against a rough vertical surface by force $F_{push}$ of magnitude 18.0 N, directed 60.0° above the horizontal.

a) Draw a free body diagram for the book.

b) What is the magnitude and direction of the friction force acting on the book?

$W = 25.0\,N$ $\quad \theta = 60.0°$

$F_p = 18.0\,N$

$\vec{f}_s = ?$

$\sum \vec{F}_x = 0$

$-F_N + F_p \cdot \cos 60° = 0$

$\sum \vec{F}_y = 0$

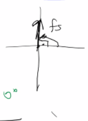
$\boxed{f_s - W + F_p \sin 60° = 0}$

$\boxed{f_s - 25.0N + 18.0N \cdot \sin 60° = 0}$

$\boxed{f_s = 9.41\,N}$

| Forces | x | y |
|---|---|---|
| $\vec{f}_s$ | 0 | $f_s$ |
| $\vec{F}_N$ | $-F_N$ | 0 |
| $\vec{W}$ | 0 | $-W$ |
| $\vec{F}_p$ | $F_p \cdot \cos 60°$ | $F_p \cdot \sin 60°$ |

$\vec{f}_s = (9.41N, 90°)$




-Only y-axis was useful.

-Impulse (BETTER VERSION DOWN BELLOW)

-Equals \deltaMomentum = F_net * \deltaTime

-Therefore F_net = \deltaMomentum / \deltaTime

-2D collision

-Just split it into x and y components 4Head

### 10.8: Collisions and momentum in two dimensions

- As we saw in Chapter 5, momentum conservation states that the momentum of an isolated system of colliding objects does not change, or $\Delta\vec{p} = 0$.
- Momentum is a vector, so in two dimensions momentum change must be expressed in terms of the components.
- Conservation of momentum in two dimensions is given by

$$\Delta p_x = \Delta p_{1x} + \Delta p_{2x} = m_1(v_{1x,f} - v_{1x,i}) + m_2(v_{2x,f} - v_{2x,i}) = 0$$
$$\Delta p_y = \Delta p_{1y} + \Delta p_{2y} = m_1(v_{1y,f} - v_{1y,i}) + m_2(v_{2y,f} - v_{2y,i}) = 0.$$

$\Delta\vec{P}_x = 0$

$\Delta\vec{P}_y = 0$



-Ex

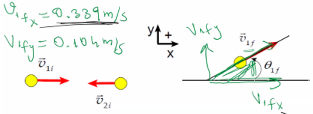
A puck of mass $m_1$ = 0.151 kg slides along a frictionless horizontal surface with a velocity of $\vec{v}_{1i}$ = 0.450 m/s directed horizontally to the right. This puck makes a glancing collision with a second puck of mass $m_2$ = 0.250 kg sliding with a velocity $\vec{v}_{2i}$ = 0.135 m/s directed horizontally to the left.

After the collision, $m_1$ is observed to be moving with a speed $v_{1f}$ = 0.355 m/s at an angle of $\theta_{1f}$ = 17.1 degrees counterclockwise from its original direction of motion (when viewed from above).

What is the velocity of puck $m_2$ after the collision (both magnitude and direction)?

$v_{1fx} = 0.339\,m/s$

$v_{1fy} = 0.104\,m/s$




$\underbrace{m_1 v_{1i} - m_2 v_{2i}}_{} = \underbrace{m_1 v_{1fx} + \vec{P}_{2fx}}_{}$

$\Delta\vec{P}_x = 0$

$\boxed{\vec{P}_{ix} = \vec{P}_{fx}}$

$\Delta\vec{P}_y = 0$

$\vec{P}_{iy} = \vec{P}_{fy}$

$0.151kg \times 0.450\frac{m}{s} - 0.250kg \times 0.135\frac{m}{s} =$

$0.151kg \times \left(0.339\frac{m}{s}\right) + \vec{P}_{2fx}$

$$\boxed{\vec{P}_{2fx} = -0.017\,kg\frac{m}{s}}$$

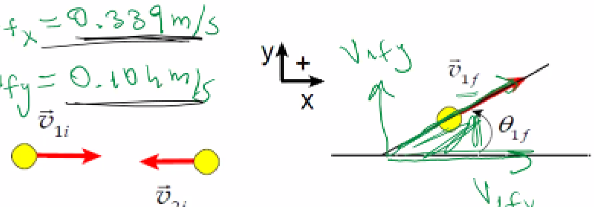
$0.017\frac{kgm}{s} = 0.250kg \times v_{2fx}$

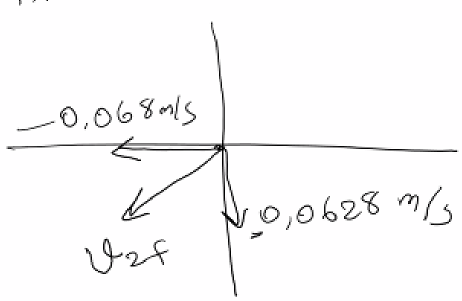
$v_{2fx} = -0.068\,m/s$

$v_{2fx} = -0.068\,m/s$

$v_{1fx} = 0.339\,m/s$

$v_{1fy} = 0.104\,m/s$




$0 = 0.151kg \times 0.104\frac{m}{s} + 0.250kg \cdot \vec{v}_{2fy}$

$\vec{v}_{2fy} = -0.0628\,m/s$.

-Then arctan to find angle

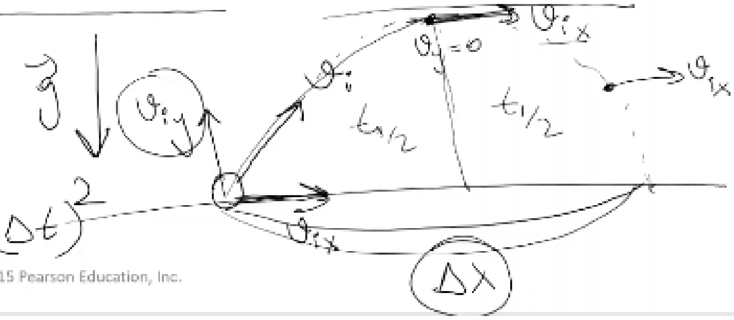
-Projectile Motion

-An object launched but not self-propelled is in PROJECTILE MOTION Kkona bullet.

$g = 9.81\ m/s^2$

$$\boxed{\Delta \vec{x} = \vec{v}_{ix} \cdot \Delta t}$$

$$\Delta \vec{y} = \vec{v}_{iy} \cdot \Delta t - \frac{1}{2} g (\Delta t)^2$$





-Where only vel_y change, and vel_x is same init and final

-Where ½ of \deltaX is the peak.

-Kinematic equations and notes restate (also tan(\theta) relationship)

- In Earth's reference frame, the ball's initial velocity is

$$\vec{v}_i = v_{x,i}\hat{i} + v_{y,i}\hat{j}$$

$\vec{v} = (v_x, v_y)$

- The ball's launch angle relative to the x axis is

$$\tan\theta = \frac{v_{y,i}}{v_{x,i}}$$

$\tan\theta = \frac{v_{yi}}{v_{xi}}$

- Using $a_x = 0$ and $a_y = -g$ in Equations 3.4 and 3.8, we get

$$\boxed{v_{x,f} = v_{x,i} \quad \text{(constant velocity)}}$$
$$\boxed{v_{y,f} = v_{y,i} - g\Delta t \quad \text{(constant acceleration)}}$$
$$\boxed{x_f = x_i + v_{x,i}\Delta t \quad \text{(constant velocity)}}$$
$$\boxed{y_f = y_i + v_{y,i}\Delta t - \tfrac{1}{2}g(\Delta t)^2 \quad \text{(constant acceleration)}}$$

$\Delta \vec{x} = \vec{v}_{x_i} \cdot \Delta t$

-Dra. Work (SI: Joules)

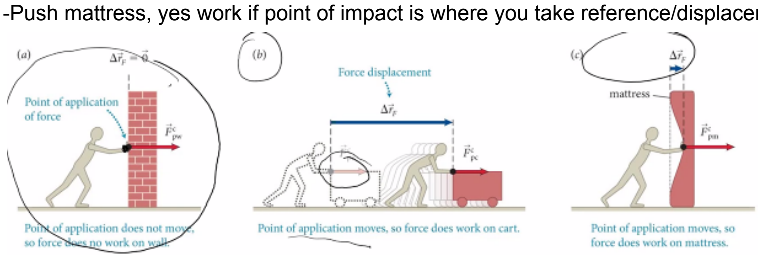
-Work amounts to a mechanical transfer of energy, either from a system to environment or vice versa.

-To do work, there has to be displacement..

-Push wall, no work.

-Push cart, yes work. (Pic: +work as it speeds up cart.)

-Push mattress, yes work if point of impact is where you take reference/displacement point.



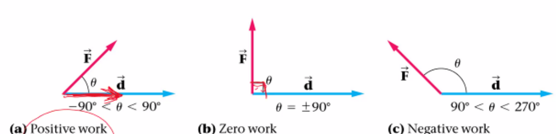

-+work, where K energy increases.

--work, where K energy decreases.

-Work at an angle

-b) No work because perpendicular

- The work done may be positive, zero, or negative, depending on the angle between the force and the displacement:

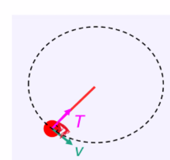



$\cos 90° = 0$

$W = F \cdot d \cdot \cos\theta$

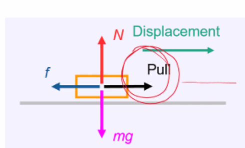
A ball tied to a string is being whirled around in a circle. What can you say about the work done by tension?

a) Tension does no work at all
b) Tension does negative work
c) Tension does positive work




A box is being pulled across a rough floor at a constant speed. What can you say about the work done by friction?

a) Friction does no work at all
b) Friction does negative work
c) Friction does positive work



-Total Work

-Get ur net force.

**Total work done by the net force:**

- When one or more constant forces cause a particle or a rigid object to undergo a displacement $\Delta x$ in one dimension, the work done by the force or forces on the particle or object is given by the **work equation**:

$$W = \left(\sum F_x\right)\Delta x_F$$

- In one dimension, the work done by a set of constant nondissipative forces on a system of particles or on a deformable object is

$$W = \sum_n (F_{\text{ext } nx}\Delta x_{Fn})$$

$$W_{Total} = F_{x\,net} \cdot \Delta X \cdot \cos\theta$$

-Use F * displacement * angle

-Work is Area Under Force Curve

**Variable forces**

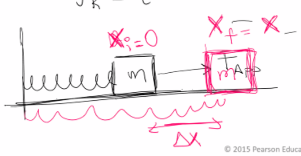
$$\boxed{W = \vec{F} \cdot \vec{r}} = \boxed{|\vec{F}| \cdot |\vec{r}|} \cdot \cos\theta$$

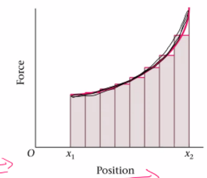
- The work done by a variable nondissipative force on a particle or object is

$$\boxed{W = \int_{x_i}^{x_f} F_x(x)\,dx}$$

**Derive work-energy theorem....**

$$f_k = \mu_k \cdot F_N$$




$$\vec{F}_{sp} = -k\,\vec{\Delta X}$$

$$\boxed{F_{App} = k\,\Delta X}$$

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-Work Energy Theorem (W = \deltaK) - Math Behind it

**Variable forces**

$$\boxed{W_T = \Delta K}$$

Work-Energy.

- The work done by a variable nondissipative force on a particle or object is

$$W = \int_{x_i}^{x_f} F_x(x)\,dx$$

**Derive work-energy theorem....**

$$\frac{dv_x}{dt} = \frac{dv_x}{dx} \cdot \frac{dx}{dt}$$

$$W = \int_{v_i}^{v_f} m \cdot v_x \cdot dv_x$$

$$= \left. m \cdot \frac{v_x^2}{2} \right|_{v_i}^{v_f} = \boxed{\frac{1}{2} m v_{f_x}^2 - \frac{1}{2} m v_{i_x}^2}$$

$\Delta K$

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Let's review work and energy equations together:

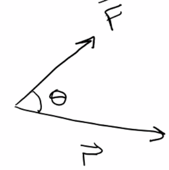
$$W = \vec{F} \cdot \vec{r}$$

Force — scalar product — displacement

$$\vec{r} = \underbrace{(x, y, z)}$$

$$W = \underbrace{F \cdot r \cdot \cos(\theta)}$$

$$J = N \cdot m$$

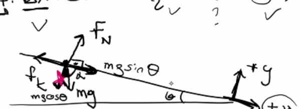



-Ex: (Solve for F_net and then v_f)

A 50.0 kg skier is skiing down hill with a slope of 30.0°. Assume that the magnitude of kinetic frictional force is 60.0 N. Her initial speed is 3.00 m/s. Ignore the air resistance and determine her speed at a point 50.0 m downhill.

$$W_T = \Delta K$$

$$F_{net} \cdot \Delta X \cdot \cos\theta = \frac{1}{2} m (v_f^2 - v_i^2)$$




$$\alpha + \theta = 90°$$

$F_N \Rightarrow$ no work

$f_k$ — work

$$\vec{F}_{net_x} = mg\sin\theta - f_k$$

$$\vec{F}_{net_x} = 50\,kg \times 9.81 \times \sin 30° - 60\,N$$

$$\vec{F}_{net_x} = 185\ N$$

A 50.0 kg skier is skiing down hill with a slope of 30.0°. Assume that the magnitude of kinetic frictional force is 60.0 N. Her initial speed is 3.00 m/s. Ignore the air resistance and determine her speed at a point 50.0 m downhill.

$$W_T = \Delta K$$

$$F_{net} \cdot \Delta X \cdot \cos\theta = \frac{1}{2} m (v_f^2 - v_i^2)$$

$$185\ N \cdot 50\ m \cdot \cos(0) = \frac{1}{2} 50\,kg \left(v_f^2 - 9\frac{m^2}{s^2}\right)$$

$$\frac{2 \times 9250\ Nm}{50\ kg} = \frac{\cancel{2}\ \frac{1}{\cancel{2}}\ 50\,kg\left(v_f^2 - 9\frac{m^2}{s^2}\right)}{50\ kg}$$

$$370\ \frac{m^2}{s^2} = v_f^2 - 9\frac{m^2}{s^2}$$

$$\vec{F}_{net_x} = mg\sin\theta - f_k$$

$$\vec{F}_{net_x} = 50\,kg \times 9.81 \times \sin 30° - 60\,N$$

$$\vec{F}_{net_x} = 185\ N$$

$$\boxed{v_f = 19.5\ m/s}$$

-Power (SI unit: W or J/s)

- **Power** is the *rate* at which energy is either converted from one form to another or transferred from one object to another.

- The SI unit of power is the **watt** (W), where 1 W = 1 J/s.

The **instantaneous power** is

- $\boxed{P = \frac{dE}{dt}}$ (Very general)

-If there is constant Work: $\boxed{P = \frac{W}{t}}$ ---> $P = F_{\text{ext}\,x}\,v_x$ (F_ext is all external forces)

-Ex:

A sports car accelerates from zero to 30 mph in 1.5 s. How long does it take for it to accelerate to 60 mph, if the power delivered by the engine is independent of velocity and neglecting friction?

a) 2.0 s
b) 3.0 s
c) 4.5 s
d) 6.0 s
e) 9.0 s
f) 12.0 s

$m \Rightarrow v_i = 0, \; v_f = 30\text{mph}$ ] $1.5s$

if

$m \Rightarrow v_i = 0, \; v_f = 60\text{ mph}$ ] $t = ?$

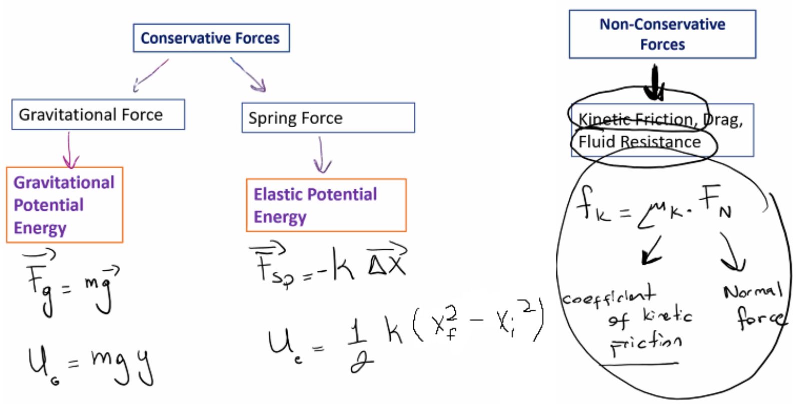
$$\frac{3^2}{1.5s} = \frac{6^2}{t_2}$$

$$9t^2 = 36 \times 1.5s$$

$$t^2 = \frac{36}{9} \times 1.5 s$$



-Conservative Force and NonConservative Force relating to Potential Energy (U)
https://courses.lumenlearning.com/boundless-physics/chapter/work-done-by-a-variable-force/#:~:text=The%20work%20done%20by%20a%20constant%20force%20of%20magnitude%20F,done%20by%20a%20constant%20force.

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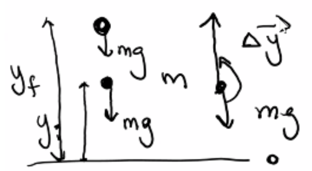
-Getting Conservative Forces(from grav):




$$W_G = mg\cdot\Delta y\cdot\cos(180°)$$

$$\boxed{W_G = -mg\Delta y}$$

$$U_G = m\cdot g\cdot y$$


$$W_G = mg\cdot\Delta y\cdot\cos(180°)$$

$$W_G = -mg\Delta y$$

$$\boxed{\Delta U_G = mg\Delta y}$$

Therefore: $\boxed{W_c = -\Delta U}$ (The big idea: Path Independent)

-Mechanical Energy (E) is conserved!

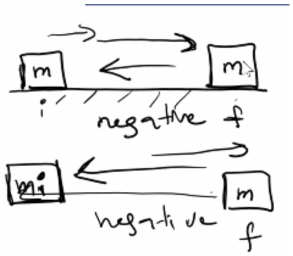
$$\Delta K + \Delta U = 0$$

$$E_f - E_i = 0$$

$$E_i = E_f$$

-Spring Force has a similar walkthrough.

-Getting NonConservative Forces(from friction):




-Friction always goes opposite of displacement. (The big idea: Path Dependent, so can't \deltaW = \delta-U)

-Where Mechanical Energy (E) is not conserved!

$$\Delta K + \Delta U \neq 0$$

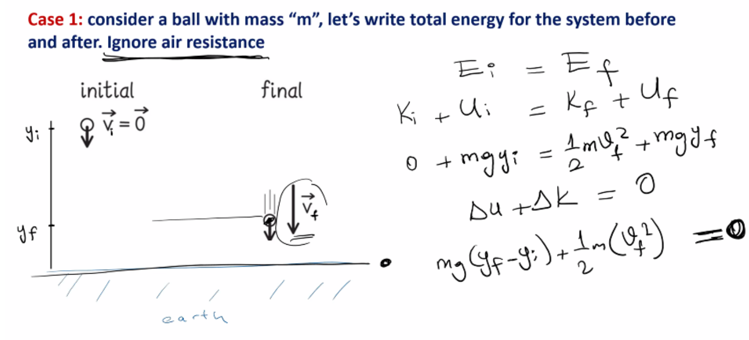
$$W_T = \Delta K$$

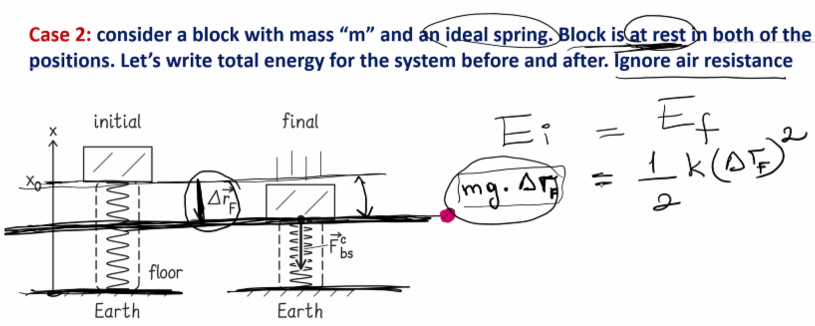
$$W_c + W_{NC} = \Delta K$$

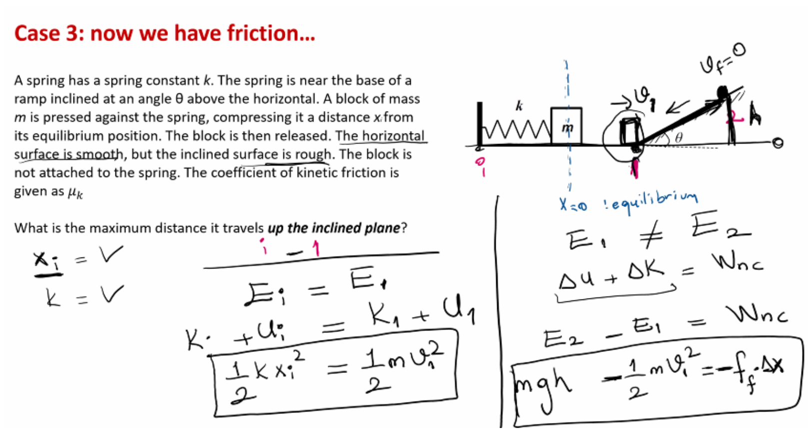
$$-\Delta U + W_{NC} = \Delta K$$

$$\boxed{\Delta K + \Delta U = W_{NC}}$$

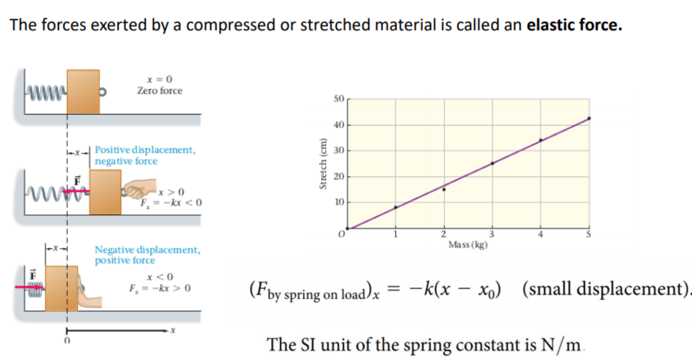
-Ex:



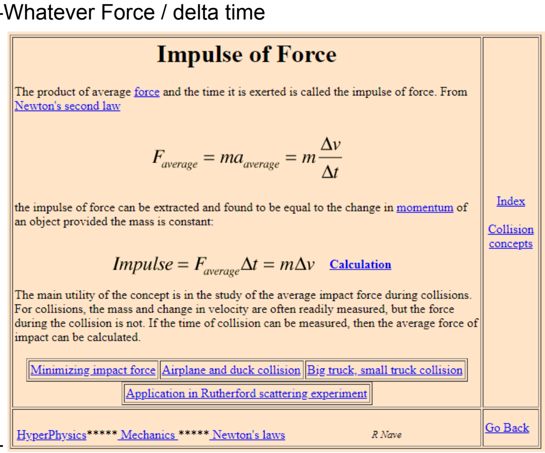




-Hooke's Law




-Average Force: http://hyperphysics.phy-astr.gsu.edu/hbase/impulse.html#:~:text=The%20most%20straightforward%20way%20to.use%20the%20impulse%20of%20force

-Whatever Force / delta time

- 

## Impulse of Force

The product of average force and the time it is exerted is called the impulse of force. From Newton's second law

$$F_{average} = ma_{average} = m\frac{\Delta v}{\Delta t}$$

the impulse of force can be extracted and found to be equal to the change in momentum of an object provided the mass is constant:

$$Impulse = F_{average}\Delta t = m\Delta v$$ Calculation

The main utility of the concept is in the study of the average impact force during collisions. For collisions, the mass and change in velocity are often readily measured, but the force during the collision is not. If the time of collision can be measured, then the average force of impact can be calculated.

Minimizing impact force | Airplane and duck collision | Big truck, small truck collision

Application in Rutherford scattering experiment

Index

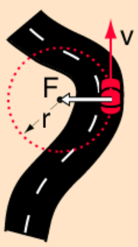
Collision concepts

HyperPhysics***** Mechanics ***** Newton's laws R Nave

Go Back

-Impulse is F_avg * \deltaTime = m \delta

-Centripetal Force (Circle pi wow) http://hyperphysics.phy-astr.gsu.edu/hbase/cf.html



# Centripetal Force

Any motion in a curved path represents accelerated motion, and requires a force directed toward the center of curvature of the path. This force is called the centripetal force which means "center seeking" force. The force has the magnitude

$$F_{centripetal} = m\frac{v^2}{r}$$

Swinging a mass on a string requires string tension, and the mass will travel off in a tangential straight line if the string breaks.

The centripetal acceleration can be derived for the case of circular motion since the curved path at any point can be extended to a circle.

$$F_{centripetal} = m\frac{v^2}{r}$$

$\frac{v^2}{r}$ is the centripetal acceleration



Note that the centripetal force is proportional to the square of the velocity, implying that a doubling of speed will require **four times** the centripetal force to keep the motion in a circle. If the centripetal force must be provided by friction alone on a curve, an increase in speed could lead to an unexpected skid if friction is insufficient.

**Calculation**

Centripetal force on banked highway curve

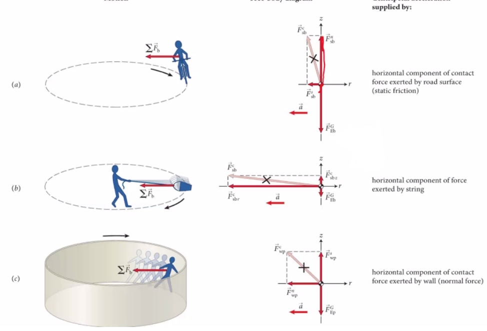
-Centripetal Accel is just Centripetal Force without mass.

-Modelling Downward Spiral (tied to string)


-

-Types of Circular Motion (Free Body)

- In each case the vector sum of the forces exerted on the object points toward the center of the trajectory.

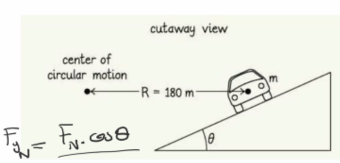



-Centripetal Force Nascar Example

## Practice Problem 1- set it up and let students solve it

**Guided Problem 11.4 It's in the bank**

Highway curves are often banked to reduce a vehicle's reliance on friction when negotiating the turn: On a banked curve, there is a centripetal component of normal force acting on the vehicle. When the angle and speed are such that friction plays no role in a vehicle's motion in the curve, the nature of the road surface is immaterial and thus the posted speed limit applies in both wet and dry weather. Suppose the posted speed limit is 100 km/h on a curve of radius 180 m. At what angle $\theta$ to the horizontal should the curve be banked so that reliance on friction is not necessary?




$F_{yN} = F_N \cdot \cos\theta$

$F_{xN} = F_N \cdot \sin\theta$

$\frac{\cancel{F_N} \cdot \cos\theta}{\cancel{F_N} \cdot \sin\theta} = \frac{\cancel{m}g}{\cancel{m}v^2/R}$

$\frac{\cos\theta}{\sin\theta} = \frac{g}{v^2/R}$


$\frac{\sin\theta}{\cos\theta} = \frac{v^2/R}{g}$

$\boxed{\tan\theta = \frac{v^2}{Rg}}$

-\theta = 23.7*

-Angular Displacement

## Angular displacement:

uniform circular motion $\Rightarrow \omega$ constant

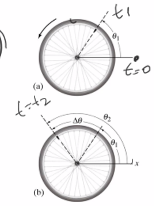
$$\Delta\theta = \boxed{\theta_2 - \theta_1}$$

The average angular velocity is defined as the total angular displacement divided by time:

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t} = \text{angular velocity}$$

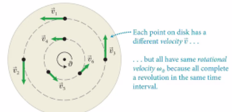
The instantaneous angular velocity:

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} \quad \frac{d\theta(t)}{dt}$$




Every point on a rotating body has an angular velocity $\omega$ and a linear velocity $v$.

They are related: $v = \omega r$




-She wants \theta in radians. -\omega units: rad/s

-Angular Acceleration

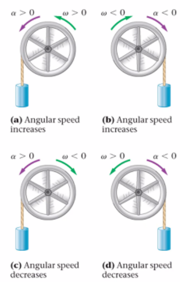
**The angular acceleration** is the rate at which the angular velocity changes with time:

$$\bar{\alpha} = \frac{\omega_2 - \omega_1}{\Delta t} = \frac{\Delta\omega}{\Delta t}$$

**The instantaneous acceleration**:

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}.$$

$$\omega = \omega_0 + \alpha t$$




-Where \omega is changing. -\alpha units: rad/s^2

-CCW = +

-CW = -

-Angular Example

## Example Problem 2

A computer disk starts from rest at time $ti$ = 0 and accelerates uniformly for 0.500 s to an angular speed of 2000 rpm (rev/min), then coasts at steady angular velocity for another 0.500 s.

$\omega_i = 0$ $\quad \omega_1 = 2000 \frac{rev}{min}$ $\quad \omega_2 = 2000 \frac{rev}{min}$

$\Delta t_1 = 0.500s$ $\quad \Delta t_2 = 0.5s$

a) What is the disk's angular acceleration during the first 0.500 s interval? Express the result in rad/s2.

$\omega_f = \omega_i + \alpha t$ $\quad \alpha_1 = 418.6 \text{ rad/s}^2$

$209.3 \frac{rad}{s} = 0 + \alpha \cdot (0.5s)$ $\quad \alpha_1 = 419 \text{ rad/s}^2$

d) What is the disk's angular velocity at $t$ = 1.00 s?

209 rad/s

b) Through how many radians has it turned during the first 0.500 s interval?

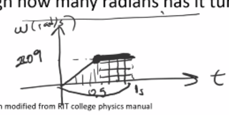
$\Delta\theta_1 = \omega_i t + \frac{1}{2}\alpha_1 t^2 = \frac{1}{2} \cdot 419 \text{ rad/s}^2 \cdot (0.5s)^2 = \boxed{52.3 \text{ rad}}$

c) Through how many radians has it turned after the first 1.00 s?

$\Delta\theta_2 = \omega_i \cdot \Delta t_2$

$\Delta\theta_2 = 209 \frac{rad}{s} \times (0.5s) = 104.5 \text{ rad}$

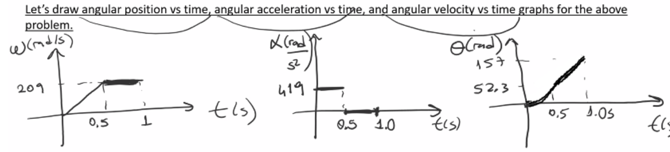
$\Delta\theta = \Delta\theta_1 + \Delta\theta_2 = 52.3 + 104.5 = 157 \text{ rad}$




This question has been modified from RIT college physics manual

A computer disk starts from rest at time $ti$ = 0 and accelerates uniformly for 0.500 s to an angular speed of 2000 rpm (rev/min), then coasts at steady angular velocity for another 0.500 s.

Let's draw angular position vs time, angular acceleration vs time, and angular velocity vs time graphs for the above problem.

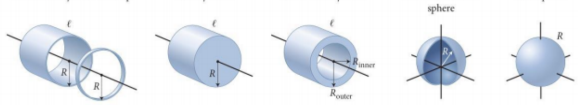



-Rotational Inertia / Moment of Inertia

-I = \Sum(m*r^2)

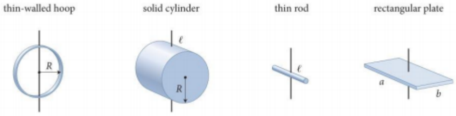
**Table 11.3** Rotational inertia of uniform objects of inertia $M$ about axes through their center of mass

**Rotation axes oriented so that object could roll on surface:** For these axes, rotational inertia has the form $cMR^2$, where $c = I/MR^2$ is called the *shape factor*. The farther the object's material from the rotation axis, the larger the shape factor and hence the rotational inertia.



| | thin-walled cylinder or hoop | solid cylinder | hollow-core cylinder | thin-walled hollow sphere | solid sphere |
|---|---|---|---|---|---|
| Shape | | | | | |
| Rotational inertia | $MR^2$ | $\frac{1}{2}MR^2$ | $\frac{1}{2}M(R^2_{outer} + R^2_{inner})$ | $\frac{2}{3}MR^2$ | $\frac{2}{5}MR^2$ |
| Shape factor $c = I/MR^2$ | 1 | $\frac{1}{2}$ | $\frac{1}{2}\left[1 + \left(\frac{R_{inner}}{R_{outer}}\right)^2\right]$ | $\frac{2}{3}$ | $\frac{2}{5}$ |

**Other axis orientations**



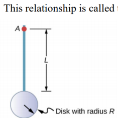
| | thin-walled hoop | solid cylinder | thin rod | rectangular plate |
|---|---|---|---|---|
| Shape | | | | |
| Rotational inertia | $\frac{1}{2}MR^2$ | $\frac{1}{4}MR^2 + \frac{1}{12}M\ell^2$ | $\frac{1}{12}M\ell^2$ | $\frac{1}{12}M(a^2 + b^2)$ |



- Sometimes you need to know the moment of inertia about an axis through an unusual position
- You can find it if you know the rotational inertia about a *parallel axis* through the center of mass:

$$I = I_{cm} + md^2$$

- This relationship is called the **parallel-axis theorem**.




-Parallel Axis theorem

## -Period, Frequency, and Angular Velocity

The period (T) is the time to complete one revolution ( $2\pi$rad).

The frequency is the number of complete revolutions per second:

$$T = \frac{1}{f}.$$

Frequencies are measured in hertz.

$$1 \text{ Hz} = 1 \text{ s}^{-1}$$

Angular velocity : $\omega = 2\pi f = \dfrac{2\pi}{T}$

-

## -Tangential Velocity

$$v_t = \omega r$$

$v_t$ = tangential velocity

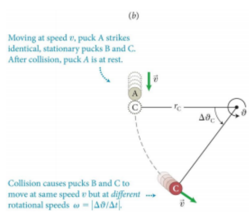
$\omega$ = angular velocity

$r$ = wheel radius

## -Rotational Kinetic Energy and Rotational Inertia

- Let us consider the following experiment: A stationary puck C fastened to a string of length $r$ is struck by an identical puck moving at speed $\upsilon$. Treating the puck C as a particle,
  - its kinetic energy can be written as $K = \frac{1}{2}m\upsilon^2 = \frac{1}{2}m(r\omega)^2 = \frac{1}{2}(mr^2)\omega^2$
  - Defining the term in the parenthesis as the **rotational inertia** $I$ of the particle about the axis of rotation, $I \equiv mr^2$, we get

    where $K_{rot}$ is the **rotational kinetic energy**. $K_{rot} = \frac{1}{2}I\omega^2$
  - The SI units of $I$ are kg · m$^2$.




## -Translational VS Rotational Motion

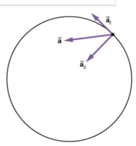
**Review : relate translational motion and rotational motion in circular motion**

| Rotational | Translational | Relationship ($r$ = radius) |
|---|---|---|
| $\theta$ | $s$ | $\theta = \frac{s}{r}$ |
| $\omega$ | $v_t$ | $\omega = \frac{v_t}{r}$ |
| $\alpha$ | $a_t$ | $\alpha = \frac{a_t}{r}$ |
| | $a_c$ | $a_c = \frac{v_t^2}{r}$ |

**Tangential Versus Centripetal Acceleration**

$a_t = r\alpha$ due to changing angular speed

$a_{cp} = r\omega^2$ due to changing direction of motior



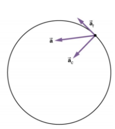

(Tangential Acceleration here 2 ways)

## -Rotational Kinematic Equations

**Review Rotational Kinematics**

| | Linear | Rotational |
|---|---|---|
| Position | $x$ | $\theta$ |
| Velocity | $v = \frac{dx}{dt}$ | $\omega = \frac{d\theta}{dt}$ |
| Acceleration | $a = \frac{dv}{dt}$ | $\alpha = \frac{d\omega}{dt}$ |

| Rotational | Translational |
|---|---|
| $\theta_f = \theta_0 + \bar{\omega}t$ | $x = x_0 + \bar{v}t$ |
| $\omega_f = \omega_0 + \alpha t$ | $v_f = v_0 + at$ |
| $\theta_f = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ | $x_f = x_0 + v_0 t + \frac{1}{2}at^2$ |
| $\omega_f^2 = \omega_0^2 + 2\alpha(\Delta\theta)$ | $v_f^2 = v_0^2 + 2a(\Delta x)$ |



-Torque (SI: N*m) http://hyperphysics.phy-astr.gsu.edu/hbase/torq.html

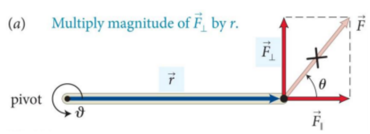
-Torque is the product of the magnitude of the force and its lever arm distance.

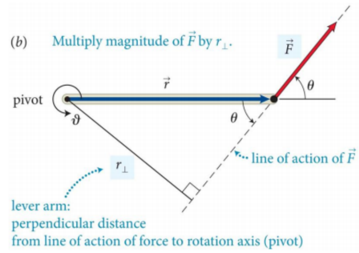
-A torque is an influence which tends to change the rotational motion of an object.

- Torque is the product of the magnitude of the force and its lever arm distance.

Two ways to determine torque:

torque $= rF_\perp = r(F \sin\theta)$ | torque $= r_\perp F = (r \sin\vartheta)F$



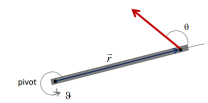




The mathematical expression for torque is

torque $= r(F \sin\theta)$

The effectiveness of a force to rotate an object about an axis depends on:

a) the magnitude of the applied force ($F$).
b) the distance from the pivot to the point force is applied ($r$).
c) the angle at which the force is applied ($\theta$).




## Rotational Motion and Newton's 2nd Law

- A force $\vec{F}$ is exerted on a particle constrained to move in a circle.

Continue in class :

$\sum \vec{\tau} = 0$ ] mechanical equilibrium

$\sum \vec{F} = 0$

$\vec{\tau}$ ccw +

cw −

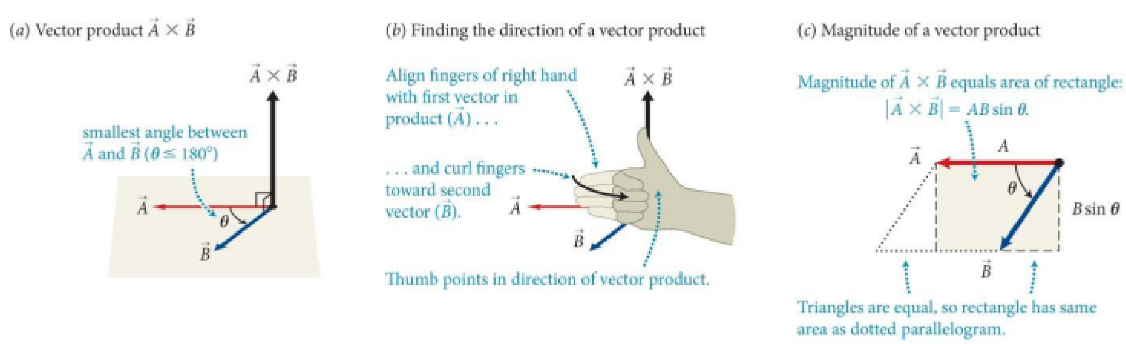
-Ex: https://www.youtube.com/watch?v=JOmGfR87Mh4 Ladder against wall

-Torque is a Vector

-Found by cross product r x F. calc3 poop. vector3.normal

## Torque is a vector product of position vector and force vector.

- This figure shows geometry of the vector product between two vectors $\vec{A}$ and $\vec{B}$. The vector product is a mathematical operation that combines two vectors to obtain a third vector.



-Torque and Newton's Second Law (2nd Law)

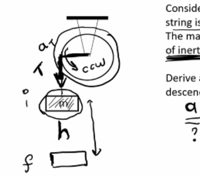
iss : translational motion $\qquad \vec{F}_{net} = m \cdot \vec{a}$

rotational motion $\qquad \vec{\tau}_{net} = I \cdot \vec{\alpha}$

\SumT = all torque added together as well

-Ex:

**Practice 3:**



Consider a fixed pulley of radius *R* that is free to rotate about a frictionless axle. A light string is wrapped around the pulley. The other end of the string is attached to a mass *m*. The mass is released from rest and descends with a constant acceleration. The moment of inertia of the pulley with respect to its axis of rotation is *I*. $I, R, m, v_i = 0$

Derive an expression for the magnitude of the acceleration *a* of the falling mass as it descends.

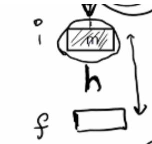
$a = \alpha R$

Translational: $\sum \vec{F} = m\vec{a}$, $\quad mg - T = ma$

rotational: $\sum \vec{\tau} = I\alpha$, $\quad \boxed{T \cdot R = I\alpha}$

$mg - T = ma$

$TR = I\alpha$



$\alpha = \frac{a}{R}$

$\boxed{TR = I\frac{a}{R}}$

$T = I \cdot \frac{a}{R^2}$

$mg - \frac{Ia}{R^2} = ma$

$mg = ma + \frac{Ia}{R^2}$

$mg = a\left(m + \frac{I}{R^2}\right)$

$$a = \frac{mg}{\left(m + \frac{I}{R^2}\right)}$$

-Ex:

**Practice 4**

A propeller is accelerated from rest to an angular velocity of 1000 rev/min over a period of 6.0 seconds by a constant torque of 2.0×10³Nm.

What is the moment of inertia of the propeller?

$\omega_i = 0$

$\omega_f = 1000 \frac{rev}{min} \times \frac{1 min}{60 s} \times \frac{2\pi rad}{rev}$

$\omega_f = 105 \ rad/s$

$\vec{\tau} = I \vec{\alpha}$ (rotational Inertia; ✓, ?, solve)

$\tau_{net} = I \alpha$

$2.0 \times 10^3 \ Nm = I \cdot \frac{17.4 rad}{s^2}$

$I = \frac{2.0 \times 10^3}{17.4} kgm^2$

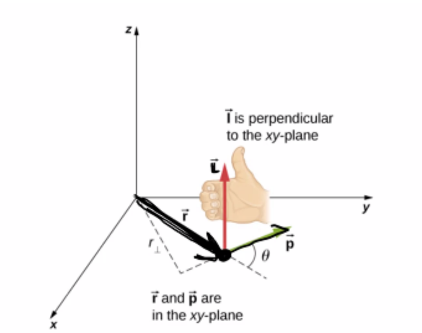
$\alpha = 17.4 \frac{rad}{s^2}$

$I = 115 \ kgm^2$

$I = 120 \ kgm^2$

-Angular Momentum (L)

- -p=mv Linear
- -L = I*\omega Circular





$\vec{L} = \vec{r} \times \vec{p}$

$L = I\omega$

$$\frac{d\vec{L}}{dt} = \sum \vec{\tau}.$$

If the sum of the torques caused by the external forces on a extended object is zero (isolated system), then angular momentum is conserved:

$\Delta L = 0$ thus $L_i = L_f$

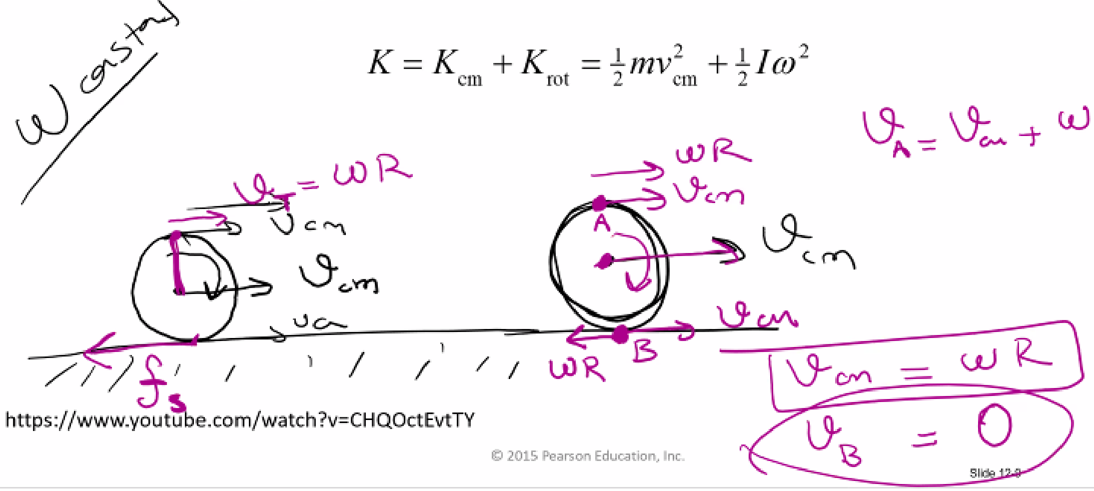
$I_i \omega_i = I_f \cdot \omega_f$

-Right Hand Rule

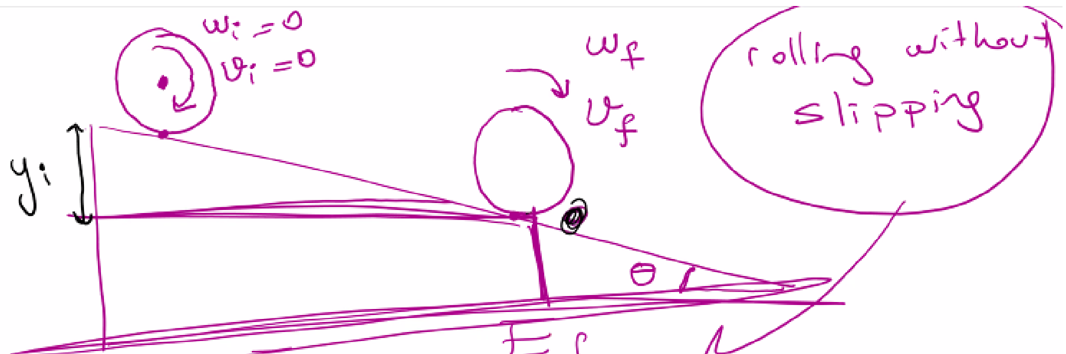
-Rolling Motion w/o Slipping

- Now, if the object is in both translational and rotational motion, then its kinetic energy is given by

$$K = K_{cm} + K_{rot} = \tfrac{1}{2}mv_{cm}^2 + \tfrac{1}{2}I\omega^2$$




-v_cm is center of mass velocity
-Distance travel is arc length of \deltaTheta

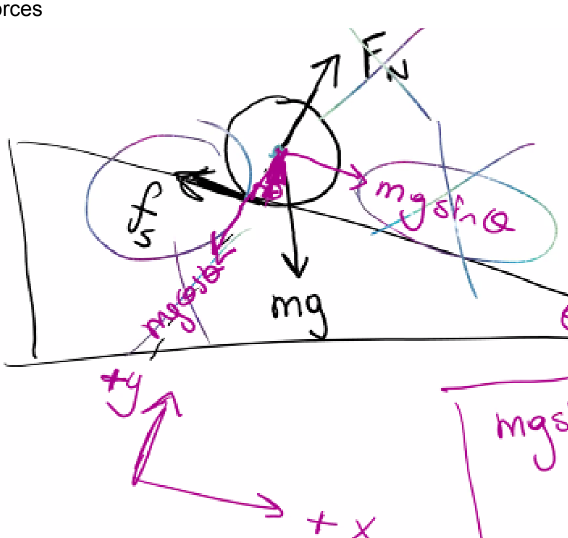



$$E_i = E_f$$

$$U_i + \cancel{K_i} = \cancel{U_f} + K_f$$

$$\boxed{mgy_i = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega^2}$$

-With Forces




① $\sum \vec{F} = m\vec{a}_{cm}$

② $\sum \vec{\tau} = I\alpha$

$a_{cm} = \alpha R$

$$\boxed{\begin{aligned} mg\sin\theta - f_s &= ma \\ f_s \cdot R &= I \cdot \alpha \end{aligned}}$$

-Newton's Law of Universal Gravity

# Chapter 13: Newton's Law of Universal Gravitation

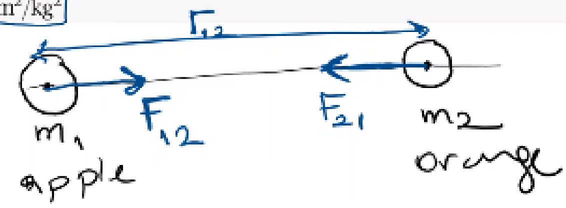
weakest dominant

The Law of Universal Gravitation states that every point mass attracts every other point mass in the universe by a force pointing in a straight line between the centers-of-mass of both points, and this force is proportional to the masses of the objects and inversely proportional to their separation.

$$\boxed{F = G\frac{m_1 m_2}{r^2}} \qquad \left[ G \cdot \frac{m_1 \cdot m_2}{r^2} \right] \quad \text{N} \qquad 12\text{-}1$$

In this expression, $r$ is the distance between the masses, and $G$ is a constant referred to as the **universal gravitational constant.** Its value is

$$\boxed{G = 6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2} \qquad 12\text{-}2$$




$$|\vec{F}_{12}| = |\vec{F}_{21}|$$

-m is the masses of both objects
-G is Universal Gravitational Constant (6.67 * 10^-11)
-r is distance between the two objects

## What about the gravitational force on objects at the surface of the Earth?

- What about the gravitational force on objects at the surface of the Earth? The center of the Earth is one Earth radius away, so this is the distance we use:

$$F = G\frac{mM_E}{R_E^2} = m\left(\frac{GM_E}{R_E^2}\right)$$

- Therefore,

m = inertia

$$m\left(\frac{GM_E}{R_E^2}\right) = mg$$