

TextBook:

-Matter has mass as a property, and now we are learning about **charge** as another property.

-How can it be observed?

-Is charge a property of all objects like mass is?

-Can we quantify the amount of force that charged objects exert on each other?

-What happens if there are multiple charged objects in the same area?

Chapter 22

-Electric Interaction: Things like comb pulling up pieces of paper to it, cat getting styrofoam packaging piece stuck to hair.



-Attract & Repel are 2 interactions

-Electric Force: Objects that participate in electrical interactions exerts an electric force on each other. This is a field force, meaning the objects are not physically touching.

-Electric Charge (aka Charge): Attribute responsible for Electric Interaction

-Not permanent (can be discharged)

-Rubbing the charged tape will discharge it by distributing the charge to your body.

-Only two types of charges (+ & -).

-Negatively charged is arbitrarily given as the charge acquired by a comb passing through hair a couple of times. (comb = -, hair = +)

-The +/- sign means the object contains more +/- charge, not that it only has one type

-*Like Charges repel*

-*Opposite Charges attract* (since all matter wants to be neutral, so them being opposite can cancel their charge)

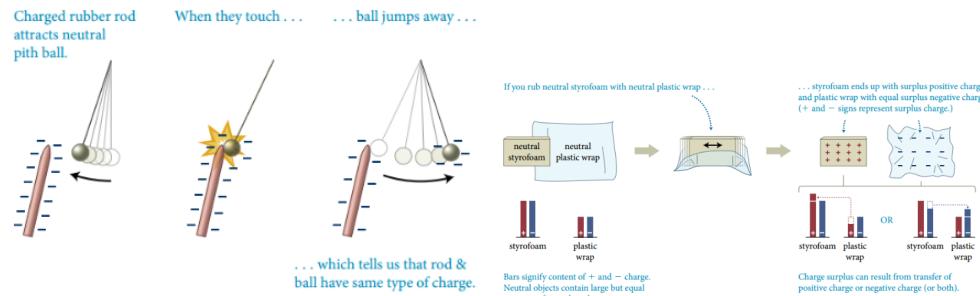
-They never appear independently

-Tape Ex has an equal amount of opposite charges when taken apart.

-Combining the tape will make both of them neutral again.

-Therefore, neutral matter has an equal amount of + & - charge.

-Charge can be transferred from one object to another by bringing the 2 into contact. (- rod attracts neutral ball, touch and both objects are - and repel)



-Charge Carrier: Any microscopic object that carries a charge. I.E. electron & ion.

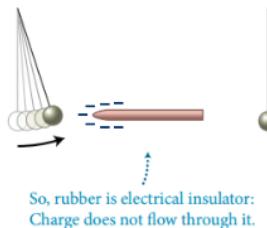
-Neutral Object: Objects that have no charge.

-No interaction with other neutral objects

-but interacts with charged objects (attraction)

-Electrical Insulator: Materials through which charge carriers cannot flow easily. Any charge transferred to an insulator remains near the spot at which it was deposited.

Charged end of rubber rod attracts pith ball uncharged end doesn't.



-Glass, rubber, wood are Ex

-Transferring large amounts of charge because of it being clumped together by insulation is bad for your computer components. Wear bracelet like the Verge guy

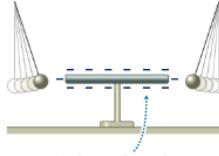
*****Insulator to Conductor is a range. Not like binary genders.

-Electrical Conductor: Opposite of insulator, the charge is spread throughout the object. Materials where charge carriers can flow through are this.

-The flow of charge is called conduction.

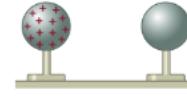
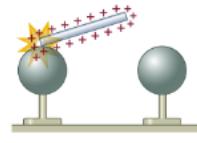
If we transfer charge to one end of metal rod . . .

. . . both ends of charged rod attract pith balls equally . . .

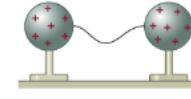


Use charged rod . . .

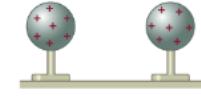
. . . to charge one metal sphere.



Connect spheres with wire.



Charge distributes equally.



-Electroscope example: https://www.youtube.com/watch?v=HupFY_24o-4

-Example material is metal (solid and works at room temp). Water is good too because of its impurity.

-Earth (except for outer layer of soil) is good too. *Grounding* is where a charged, conducting object is connected to Earth.

-Human bodies are conductors too because of water. Charge that flows to you is discharged to Earth. When it does not, it builds up & you become palm tree

Figure 22.15 Charge spreads over the human body, so a large charge will cause your hairs to repel one another and stand on end.



-Sub Atomic Particles

-Electron is Neg (electron's charge can't be removed)

-Nucleus is Neutral

-Proton is Pos

-All electrical charge comes in whole-number multiples of the electrical charge on the electron

- e (elementary charge), this is designated for the magnitude of charge on the electron, the smallest observed amount. $e=1.60E-9$

-Since atoms are neutral, a proton magnitude electrical charge is e too.

-electron charge = $-e$

-proton charge = $+e$

-When an atom has an unequal amount of electrons to protons, it is called an Ion.

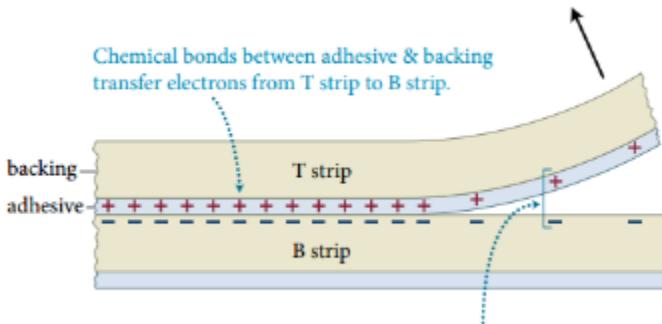
-Ion has electrical charge, depending on if there's more proton(+) or electron(-).

-NaCl example

Ions in solids are always immobile, but ions in liquids can move freely. For instance, in table salt, a compound made of pairs of sodium (Na^+) and chloride (Cl^-) ions, the charged ions hardly move at all, meaning that solid table salt is an electrical insulator. Dissolve table salt in water, however, and the solution contains large quantities of positively charged sodium ions and negatively charged chloride ions. Because these ions can move freely, the solution is an electrical conductor.

-Tape example

Figure 22.17 How strips of tape can acquire opposite charges when pulled apart.



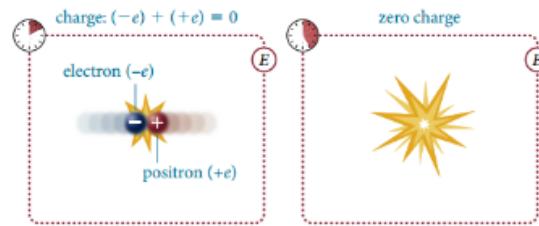
When strips pull apart, some electrons remain on B strip, leaving surplus positive charge on T strip.

-Any two dissimilar materials become charged when brought into contact with each other. When they are separated rapidly, small amounts of opposite charge may be left behind on each material.

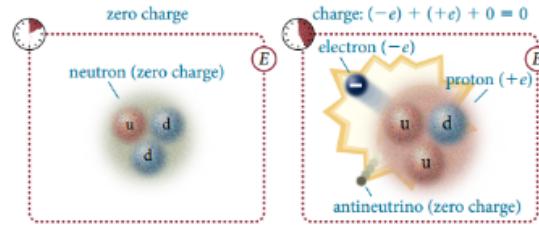
-Conservation of Charge: Electrical charge can be created or destroyed only in identical positive-negative pairs such that the charge of a closed system always remains constant.

Figure 22.18 Because charge is conserved, the charge of a closed system does not change even when particles are created or destroyed.

(a) Electron-positron annihilation



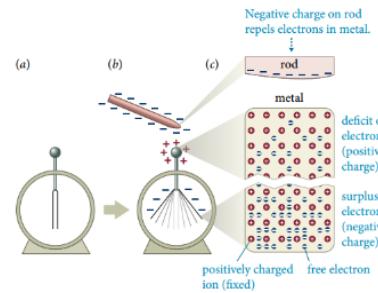
(b) Decay of a free neutron



-Charge Polarization: Any separation of charge carriers in an object is called Charge Polarization (aka Polarization). Alt explanation is a single object that has an uneven distribution of charge.

-A neutral electroscope is polarized by a charged rod <https://www.youtube.com/watch?v=B1WmizUghMo>

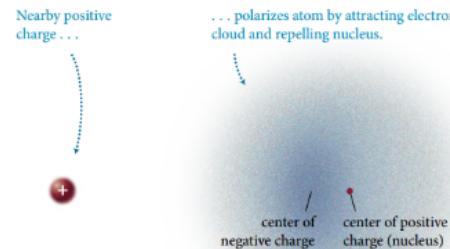
Figure 22.19 In (a) and (b), a charged rod induces polarization in an electroscope. (c) A schematic atom-level view.



- The top metal part's electrons are being repelled to the leaves. The leaves then are more negative than the metal, the leaves repel because like charge
- The electroscope is still neutral!!! Just repositioned electrons (in this case) to the leaves

-This happens in atomic level

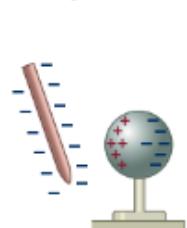
Figure 22.20 Polarization of a neutral atom.



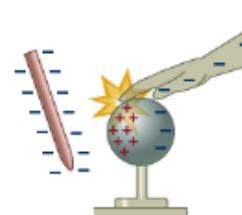
-Another example, charging neutral conducting objects

Figure 22.22 Polarization can be exploited to charge neutral conducting objects.

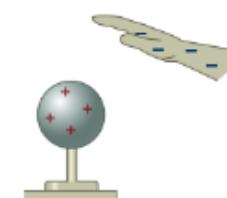
① Charged rod induces polarization in metal sphere.



② When you touch sphere, negative charge gets farther from rod by spreading onto you.



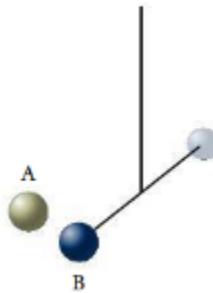
③ When you let go, you retain surplus of one type of charge and sphere retains surplus of opposite type of charge.



-Coulomb's Law (Electrostatic Force):

-Man did experiment with balls and found the Electric Force is proportional to the charge on each sphere.

Figure 22.25 Schematic diagram of Coulomb's apparatus for measuring the electric force between two charged spheres.



-The Law: the magnitude of the electric force (N) exerted by 2 charged particles separated by a distance r_{12} (m) & carrying charges q_1 & q_2 (C) is

$$F_{12}^E = k \frac{|q_1| |q_2|}{r_{12}^2}$$

$r_{12} = |\vec{r}_2 - \vec{r}_1|$ is the distance between the particles (CAREFUL THIS IS SQUARED in equation)

- k is the constant of proportionality (like G and whatnot) $k = 9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$.

-Kinda similar to Newton's law of gravity.

-Coulomb (C): SI unit for charge

-the quantity of electrical charge transported in 1 sec by a current of 1 ampere.

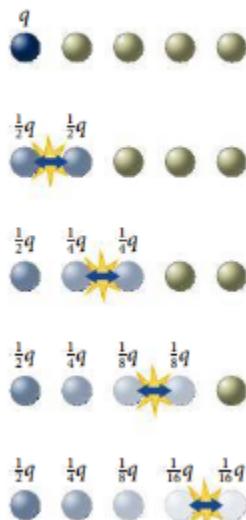
-1 Coulomb is the magnitude of the charge on about 6.24×10^{18} electrons

-**Finding amount of electrons from charge:** $q = ne$ (q is charge, n is # of electrons, e is the constant)

$$e = 1/6.24 \times 10^{18} \text{ C} = 1.60 \times 10^{-19} \text{ C}$$
 Elementary Charge

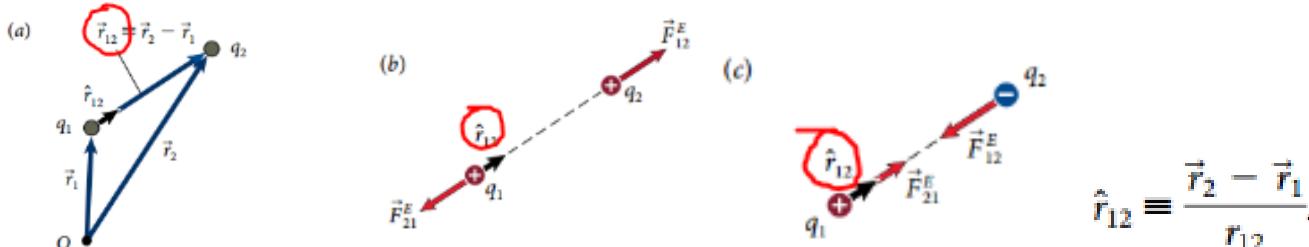
-Charged Sphere contacting other unchanged sphere neighbor

Figure 22.26 By successively allowing a charged sphere to touch an initially uncharged neighbor, we can distribute an amount of charge in ever-lessening amounts over a number of spheres.



-Since electric force is central:

Figure 22.27 (a) Position vectors for two charged particles. (b) Repulsive forces exerted on each other by two particles carrying like charges. (c) Attractive forces exerted on each other by two particles carrying opposite charges.



-Coulomb's Law but on an assembly of objects (Superposition)

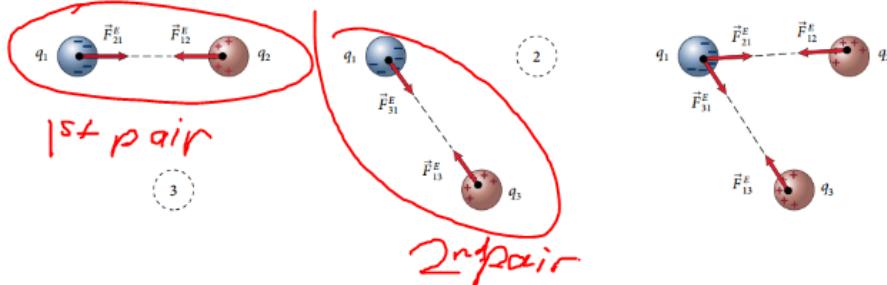
-Just Sum it 4head

$$\sum \vec{F}_1^E = \vec{F}_{21}^E + \vec{F}_{31}^E + \vec{F}_{41}^E + \dots,$$

$$\sum \vec{F}_1^E = k \frac{q_2 q_1}{r_{21}^2} \hat{r}_{21} + k \frac{q_3 q_1}{r_{31}^2} \hat{r}_{31} + k \frac{q_4 q_1}{r_{41}^2} \hat{r}_{41} + \dots.$$

Figure 22.30

(a) Sphere 1 interacts with just sphere 2 (b) Sphere 1 interacts with just sphere 3 (c) Sphere 1 interacts with both spheres



Lab1

-Diff materials rubbed with even the same thing can gain different charge. This is not accurate because it was the aluminum conductivity situation

-Seems to be 3 types of things in the world, only 2 are able to cause repulsive force

-Charged, Uncharged, Neutral

-Charged splits to - & +. Excess of - or +.

-Uncharged means it has no charge

-Neutral means + & - cancels out. So has charge still

-Uncharged and Neutral are usually synonyms though. But physicists claim they discover truly uncharged particles, where there is no charge, like where photons don't have mass.

-A Convention

-The acrylic rod becomes positively charged (+) when rubbed with wool.

-The Teflon rod becomes negatively charged (-) when rubbed with wool.

-Why are there not 3 types of charge?

-Not found with observation

-What types of objects are there?

-Neutral, + & - cancels out.

-Charged

-Excess of -

-Excess of +

-Neutral that is polarized. Uneven distribution

-Charging Objects

-Charging by Contact

-Rubbing of 2 neutral object

-Touching a charged object to a neutral one

-Charging by Induction (<https://youtu.be/-JsVZwc1dOo?t=122>)

-Polarize it by bringing a charged object, then provide a path for a charge to enter or leave. Remove that path, then the charged object that causes the polarization.

-Or have a "separable" conductor

Comparison to gravity

- Notice that the form of Coulomb's law is the same as the form of the universal law of gravitation:

$$\vec{F}_G = G \frac{m_1 m_2}{r^2} \hat{r}$$

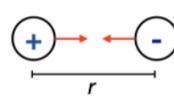
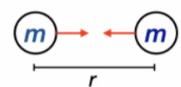
- There are a couple important differences

- Gravity is always attractive; there seems to be only one type of mass. The electric interaction can be attractive or repulsive because there are two types of charge.

- Gravity is much, much, much weaker than the electric interaction. This is clear from the constants of proportionality:

$$k = 9.0 \times 10^9 \text{ N m}^2 / \text{C}^2 \text{ (also written as } \frac{1}{4\pi\epsilon_0})$$

$$G = 6.67 \times 10^{-11} \text{ N m}^2 / \text{kg}^2$$



Mini-lecture: Coulomb's law

- The interaction between charged particles can be quantified and depends on the amount of excess charge each particle has and the distance between the particles:

$$\vec{F}_C = k \frac{|q_1 q_2|}{r^2} \hat{r}$$

- k is an experimentally-determined constant of proportionality.

- If charge is measured in coulombs (C) and the distance is measured in meters (m), k turns out to have a value of $9.0 \times 10^9 \text{ N m}^2 / \text{C}^2$.

- The direction of the force depends on the charges involved: opposite charges attract; the same charge repels.

Chapter 23 Time to use fields to describe the effect of electric force. Stationary objects for now.

-The Field Model:

-Force of electric is not instantaneous, distance matters.

-Interaction Field (aka Field):

-in a field model, an interacting object fills the space around itself with a field.

-The field is always there, even when no other objects are interacting with it.

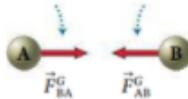
-The field model still shares Newton's Third Law, that being how each of the object's force is symmetrical.

Figure 23.2 The field model for interaction at a distance.

(a) Model of direct interaction at a distance

(b) Field model of interaction at a distance

We model A and B as exerting forces directly on each other.



field of A



force due to field of A at location of B

\vec{F}_{AB}^G

AND

force due to field of B at location of A

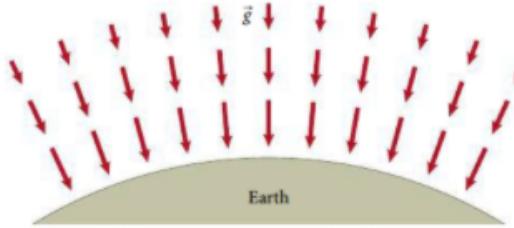
\vec{F}_{BA}^G

field of B



Fields of A and B shown separately for clarity; both are present at same time.

Figure 23.5 Vector field diagram for the gravitational field in a region near Earth.



At any given location in the space surrounding a source object S, the magnitude of the gravitational field created by S is the magnitude of the gravitational force exerted on an object B placed at that location divided by the mass of B.

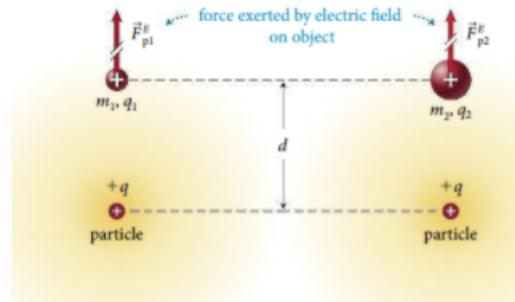
-Test Particle: an idealized particle whose mass is small enough that its presence does not perturb the object whose grav field we are measuring.

-Electric Field Diagrams: Measured in Newton/Coulomb (Grav field is Newton/Kilogram)

Figure 23.6 Electric force exerted on two objects of different inertia m and charge q by the electric fields created by two identical charged particles.

(a)

(b)



At any given location in the space surrounding a source object S, the electric field created by S is the electric force exerted on a charged test particle placed at that location divided by the charge of the test particle: $\vec{E}_S = \vec{F}_{St}^E / q_t$.

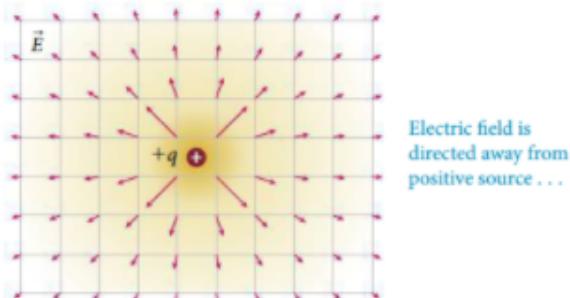
-Unlike Grav field which is always attraction, electric fields can describe repulsion.

-Gravitational fields are measured in units of N/kg while electric fields are measured in N/C. Think about this! What it means is that the field (in each case) is telling you how many newtons of force an object would feel at that location on a per mass or per charge basis. So if you know the value of the field at a particular location, and the mass or charge of an object, you can predict the force that object will feel at that location.

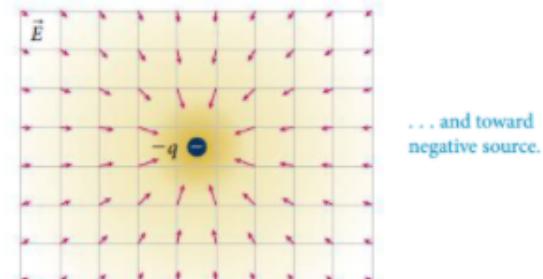
The direction of the electric field at a given location is the same as the direction of the electric force exerted on a positively charged object at that location.

Figure 23.7 Vector field diagrams for positively and negatively charged particles. The lengths of the vectors show that the electric field magnitude decreases with increasing distance from the source.

(a) Electric field of positively charged particle

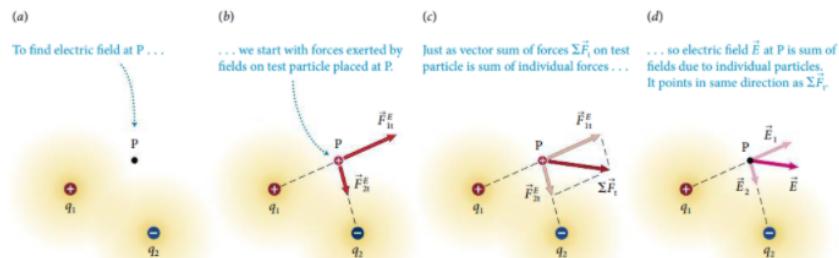


(b) Electric field of negatively charged particle



-Superposition of Electric Fields

Figure 23.9 The electric field due to multiple charged objects (here, a pair of charged particles) is the vector sum of the fields created by the individual objects.

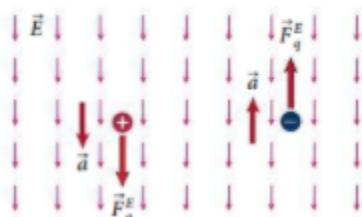


The combined electric field created by a collection of charged objects is equal to the vector sum of the electric fields created by the individual objects.

-Electric Fields and Forces

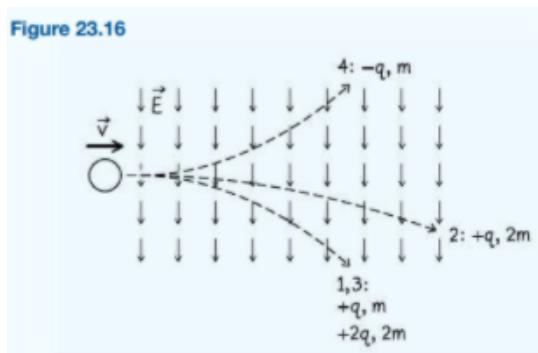
- Uniform Electric Field is where every point, the magnitude and direction of the Electric potential is the same
- NonUniform is not like Uniform bruv

Figure 23.15 Forces exerted by a uniform electric field on a positively and a negatively charged particle.



A charged particle placed in a uniform electric field undergoes constant acceleration.

-Shooting particle into uniform field



A positively charged particle placed in a nonuniform electric field has an acceleration in the same direction as the electric field; a negatively charged particle placed in such a field has an acceleration in the opposite direction.

-Shooting Particle in field that doesn't get deflected: (Grav and Electric Force CANCELLATION)

23.19

\vec{F}_C

\vec{F}_g

$$\vec{F}_C = q\vec{E} = F_g \hat{j}$$

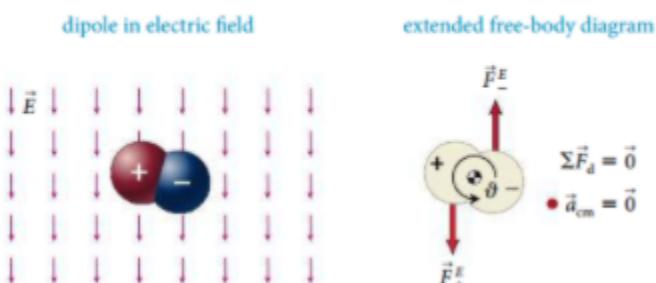
$$\vec{E} = \frac{F_g \hat{j}}{q} = \frac{mg \hat{j}}{q} = \frac{(30 \times 10^{-6} \text{ kg})(9.8 \frac{\text{m}}{\text{s}^2}) \hat{j}}{3.5 \times 10^{-6} \text{ C}}$$

$$= 84 \frac{\text{N}}{\text{C}} \hat{j}$$

-Electric Dipole: Equal amount of positive and negative charge separated by a small distance.

- Ex: Water molecule which is a permanent dipole
- Dipole in Uniform Field

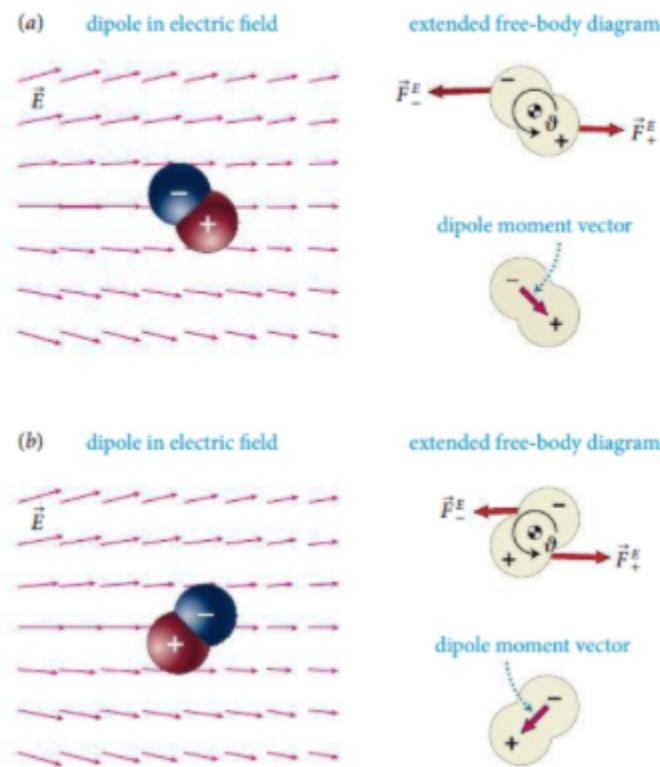
Figure 23.17 Extended free-body diagram for a permanent dipole placed in a uniform electric field.



-Dipole can spin because of the electric force applying torque.

-Dipole Moment: The orientation of an electric dipole can be characterized by a vector

Figure 23.18 Extended free-body diagrams for permanent dipoles in nonuniform electric fields. The electric field shown is due to a positively charged particle to the left of the figure.



A permanent electric dipole placed in an electric field is subject to a torque that tends to align the dipole moment with the direction of the electric field. If the field is uniform, the dipole has zero acceleration; if the electric field is nonuniform, the dipole has a nonzero acceleration.

-Calculating Electric Field Value

Coulomb's law could be written as follows:

$$\vec{F}_C = q \frac{kQ}{r^2} \hat{r} = q \vec{E}$$

-Q is just q but dif for clarity

Figure 23.22 To determine the electric field at P generated by a charged source particle, we place a test particle at P.

$$\vec{r}_{st} \equiv \vec{r}_t - \vec{r}_s$$

$$\vec{E}_s(P)$$

$$\vec{E} = \frac{kQ}{r^2} \hat{r}$$

-E = the force (N/m) at a distance r away from the charge

$$\vec{F}_C = q\vec{E}$$

-F_C = the force (N) on another charge (q) at the location r away (r is in E)

What is the magnitude and direction of the electric field 4.52 cm to the left of a 8.15 mC negative charge?

$$E = \frac{kQ}{d^2}$$

$$E = \frac{9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}}{(0.0452 \text{ m})^2} (8.15 \times 10^{-3} \text{ C})$$

$$E = 3.59 \times 10^{10} \text{ N/C, to the right}$$

https://www.youtube.com/watch?v=lJqqA_ZzF8c

-Infinitesimal source particles

Quantitative Tools The electric field due to a set of infinitesimal source particles of charge dq_s is

$$\vec{E} = k \int \frac{dq_s}{r_{sp}^2} \hat{r}_{sp} \quad (23.15)$$

-Electric Field Custom Formulas

A uniformly charged rod of length ℓ carrying charge q oriented along the y axis and centered at the origin produces an electric field

$$E_x = \frac{kq}{x\sqrt{\ell^2/4 + x^2}}$$

along the x axis.

A uniformly charged ring of charge q and radius R oriented with the plane of the ring perpendicular to the z axis and the ring center at the origin produces an electric field

$$E_z = k \frac{qz}{(z^2 + R^2)^{3/2}}$$

along the z axis.

A uniformly charged disk of surface charge density σ and radius R oriented with the disk face perpendicular to the z axis and the disk center at the origin produces an electric field

$$E_z = 2k\pi\sigma \left[1 - \frac{z}{(z^2 + R^2)^{1/2}} \right]$$

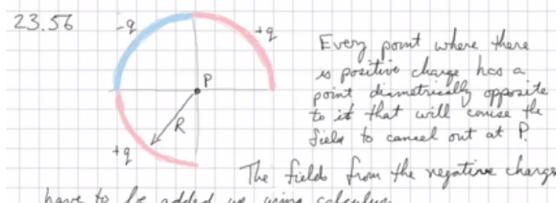
along the z axis.

Outside a solid sphere of radius R carrying a uniformly distributed charge q , the electric field produced by the sphere is

$$E_{\text{sphere}} = k \frac{q}{r^2},$$

where $r > R$ is the distance from the sphere's center. This field is the same as the field that would be produced if all the charge were concentrated at the sphere's center.

-Ex Arc:



$$\vec{E} = \int_{\frac{\pi}{2}}^{\pi} \frac{kq dq}{R^2} \hat{r}, \text{ where } dq = \frac{\sigma}{\ell} R d\theta \text{ but } \ell = \frac{2\pi R}{4} \\ \text{so we have } \vec{E} = \int_{\frac{\pi}{2}}^{\pi} \frac{kq R d\theta}{R^2} \left(\frac{2\pi}{2\pi R} \right) \hat{r} = \frac{2kq}{\pi R^2} \int_{\frac{\pi}{2}}^{\pi} \frac{d\theta}{\hat{r}}$$

But \hat{r} changes over the integral. To add up vectors, we need to use \hat{x} and \hat{y} . To express \hat{r} in terms of \hat{x} and \hat{y} , use $\sin\theta$ and $\cos\theta$: $\hat{r} = \cos\theta \hat{x} + \sin\theta \hat{y}$

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Now we have to do two separate integrals; one for the \hat{x} term and one for the \hat{y} term.

$$\vec{E} = \frac{2kq}{\pi R^2} \left[\begin{array}{l} \int_{\frac{\pi}{2}}^{\pi} \cos\theta d\theta \\ \int_{\frac{\pi}{2}}^{\pi} \sin\theta d\theta \end{array} \right] = \frac{2kq}{\pi R^2} \left[\begin{array}{l} \sin\theta \Big|_{\frac{\pi}{2}}^{\pi} \\ -\cos\theta \Big|_{\frac{\pi}{2}}^{\pi} \end{array} \right]$$

$$= \frac{2kq}{\pi R^2} \left[\begin{array}{l} 0 - 1 \\ -(-1 - 0) \end{array} \right] = \frac{2kq}{\pi R^2} \left[\begin{array}{l} -1 \\ 1 \end{array} \right] = \frac{2\sqrt{2} kq}{\pi R^2} @ 135^\circ$$

-Ex: Percent error of Bisector Rod (Line Charge) vs Point Charge

23.60

The percent error from treating this as a point charge will be

$$\frac{E_p - E_d}{E_d} \quad \text{where } E_d = \frac{kq}{x\sqrt{\frac{l^2}{4} + x^2}}$$

$$\text{and } E_p = \frac{kq}{x^2}$$

$$\frac{1}{E_d} (E_p - E_d) = \frac{1}{E_d} E_p - 1 = \frac{x\sqrt{\frac{l^2}{4} + x^2}}{kq} - 1$$

We want this percent error to be less than $\rho = 0.05$

so we solve for x in

$$\rho = \sqrt{\frac{l^2}{4x^2} + 1} - 1 \Rightarrow (\rho + 1)^2 = \frac{l^2}{4x^2} + 1 \Rightarrow x = \sqrt{\frac{l^2}{4(\rho + 1)^2} - 1}$$

With $\rho = 0.05$ and $l = 0.250\text{ m}$, we get

$$x = 0.39\text{ m}$$

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-Ex: Multiple Ring Charge

23.62

$$\begin{aligned} r_1 &= 0.050\text{ m} & q_1 &= 1.0\mu\text{C} \\ r_2 &= 0.070\text{ m} & q_2 &= -2.0\mu\text{C} \\ r_3 &= 0.090\text{ m} & q_3 &= 1.0\mu\text{C} \\ P &= (0, 0.100\text{ m}, 0) \end{aligned}$$

$$\begin{aligned} E_z &= \frac{kq_1 z}{(z^2 + r_1^2)^{3/2}} + \frac{kq_2 z}{(z^2 + r_2^2)^{3/2}} + \frac{kq_3 z}{(z^2 + r_3^2)^{3/2}} \\ &= \frac{900 \frac{\text{N m}^3}{\text{C}^2}}{0.0014 \text{m}^3} - \frac{120 \frac{\text{N m}^3}{\text{C}^2}}{0.0018 \text{m}^3} + \frac{900 \frac{\text{N m}^3}{\text{C}^2}}{0.0024 \text{m}^3} \\ &= 23905 \frac{\text{N}}{\text{C}} \quad \text{or } 24000 \frac{\text{N}}{\text{C}} \end{aligned}$$

To counter that, you would need something to make $-23905 \frac{\text{N}}{\text{C}}$

$$E = \frac{kq}{r^2} \quad q = \frac{Er^2}{K} = \frac{-23905 \frac{\text{N}}{\text{C}} (0.200\text{ m})^2}{9.0 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}} = -1.1 \times 10^{-7} \text{ C}$$

-Only looking at Z direction because ring cancels itself. Red is find q to make $E=0$

-Calculating Electric Dipole Moment

An **electric dipole** consists of equal quantities q_p of positive and negative charge separated by a small distance.



-Both Charge have equal q_p because dipole. -From - to + is dir

-Torque of Dipole in Uniform Electric Field

The torque $\sum \vec{\tau}$ produced on a dipole by a uniform electric field is

$$\sum \vec{\tau} = \vec{p} \times \vec{E}$$

$$|\vec{\tau}| = |\vec{p} \times \vec{E}| = pE \sin \theta$$

-Ex:

23.70

$$\begin{aligned} \vec{E} &= 8500 \frac{\text{N}}{\text{C}} @ 42^\circ \\ &6.186 \times 10^{-30} \text{ C m} \hat{z} \end{aligned}$$

$$\begin{aligned} \vec{\tau} &= \vec{p} \times \vec{E} = p E \sin \theta \hat{y} \quad (\text{by right-hand rule}) \\ &= (6.186 \times 10^{-30} \text{ C m})(8500 \frac{\text{N}}{\text{C}}) \sin 48^\circ \hat{y} \\ &= 3.9 \times 10^{-26} \text{ N m} \hat{y} \end{aligned}$$

-Induced Dipole Moment

The **induced dipole moment** \vec{p}_{ind} of a neutral atom placed in an external electric field that is not too large is

$$\vec{p}_{\text{ind}} = \alpha \vec{E},$$

-Electric Field Near Dipole

The text gives two formulas that are useful approximations for the electric field near (but not too near) dipoles. If the coordinate system is chose to align the y axis with the dipole moment with the x axis as the perpendicular bisector of the dipole, then the electric field along the y axis can be approximated

$$\vec{E} \approx \frac{2kp}{y^3}$$

-And the electric field along the x axis can be approximated as... Notice that this field points downward (in the -y direction) along the x axis.

$$\vec{E} \approx \frac{-kp}{|x|^3}$$

-Charge Density

$$\lambda = \frac{q}{\ell}$$

$\sigma \equiv \frac{q}{A}$ **Concepts** Charge density can be linear (λ , C/m), over a surface area (σ , C/m²), or $\rho \equiv \frac{q}{V}$ throughout a volume (ρ , C/m³).

-Ex:

For a line of charge, we can think about the charge per unit length, which is often given the symbol λ . An infinitesimal portion of the rod with length dy will thus have a charge $dq = \frac{q}{\ell} dy = \lambda dy$.

The electric field at point B due to some portion of the rod at position y will be

$$dE_y = \frac{k dq}{r^2} = \frac{k \lambda dy}{(b-y)^2}, \text{ where } b \text{ is the } y \text{ component of point B which is also the distance from point B to the center of the rod.}$$

We need to add up all these bits of electric field contributed by each of the points along the length of the rod from $-\frac{\ell}{2}$ to $\frac{\ell}{2}$:

$$E_y = \int dE_y = k \lambda \int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} \frac{dy}{(b-y)^2} = k \lambda \left[\frac{-1}{b-y} \right]_{-\frac{\ell}{2}}^{\frac{\ell}{2}} = k \lambda \left(\frac{-1}{b - \frac{\ell}{2}} + \frac{1}{b + \frac{\ell}{2}} \right)$$

This formula will work for any point B that is along the central axis of the rod. If we know the length of the rod and the charge on the rod, we can find the electric field at a distance b from the center of the rod.

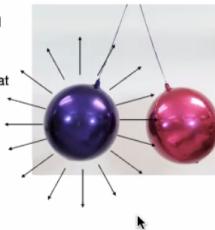
(You can find another version of this example in Worked Problem 23.5 on page 417 of the Practice text. Even though our formulas look different, can you show that they are the same?)

Lab2:

-Fields are useful to explain "action at a distance" by saying force/grav:

How can we get around "action at a distance"

- Imagine that the first object "exudes" a field that the second object experiences at its own location.
- This field can be quantified as the force per unit charge that would be felt at that location.
- We need a convention to settle the direction of the field... we have chosen to have it point in the direction that a positive charge at that location would be pushed.
- $\vec{E} = \frac{\vec{F}}{Q}$
- Since the force on point charges is known, we can write a formula for the field:
- $\vec{E} = \frac{kq}{r^2} \hat{r}$
- (Notice that the direction works... no need for absolute value symbols! But it is radial—outward in all directions.)



-Superposition = vector addition.

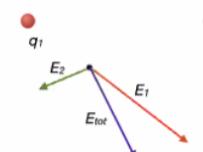
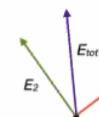
-Halving charge is halving the Force. (in $F = E/q$)

Superposition

- Electric fields add up... as vectors
- At any point, you can find the field due to a point charge if you know the location of the charge
- If there is more than one charge in the region, the total electric field at any point is the vector sum of each of the individual fields due to each charge:

$$\vec{E}_{\text{tot}} = \vec{E}_1 + \vec{E}_2 + \dots$$

- So you still have to practice doing vector addition!



-Superposition = vector addition.

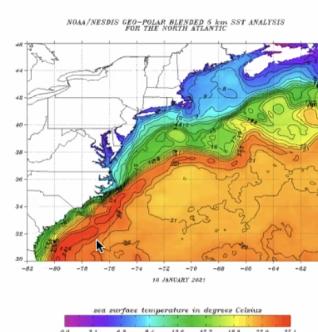
-Halving charge is halving the Force. (in $F = E/q$)

Field

Scalar Fields

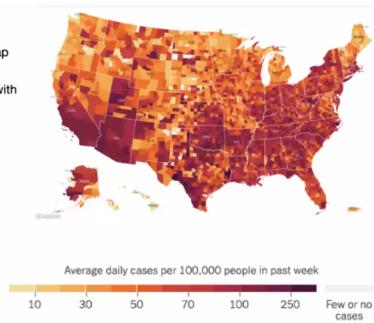
Visualizing fields

- Let's start with scalar fields.
- A scalar field associates a number with any point in space.
- For example, we could look at surface temperatures on the ocean.
- Color can be used to visualize different ranges of numbers (different scalar values).



Visualizing fields

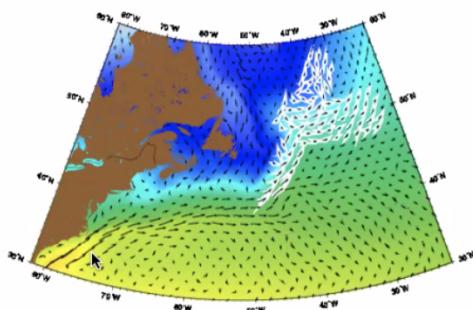
- As another example, consider a map we've seen a lot over the past year.
- What number does this associate with each location?



-Vector Fields

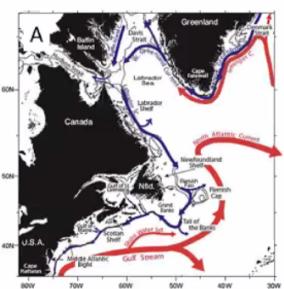
Vector fields

- Vector fields have both a value and a direction, so they are harder to map.
- For example, returning to the ocean, think about mapping surface currents.
- This map shows both... the colors represent temperatures, and the arrows represent the strength and direction of currents.



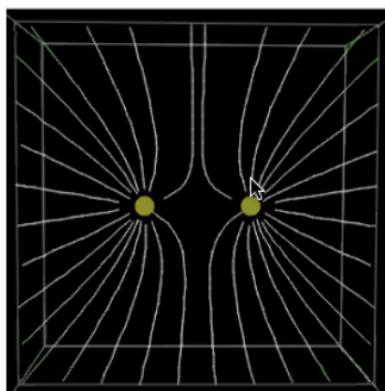
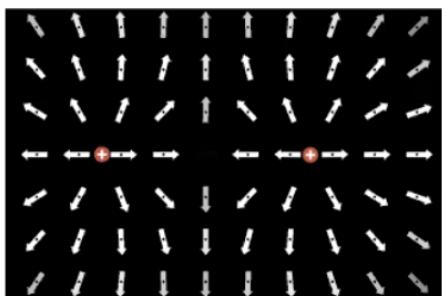
Vector fields

- But we often don't try to put separate arrows at each location
(After all, we can't actually put them at every location, and they overlap each other in areas where the field is strong.)
- Do these maps give you as much information about the vector fields of ocean currents as the one with all the little arrows?



Electric field lines

- We can make the electric field visible with arrows, but it isn't easy.
- Field lines are easier to see, but you need to add arrowheads along the lines.
- By convention, the field lines go out of positive charge and into negative charge.
- You can scale your map in terms of "lines per unit of charge."



-Field Lines NEVER CROSSES.

-Electric Field Line Rules

The electric field \vec{E} that is created by a distribution of static (non-moving) charge has the following properties

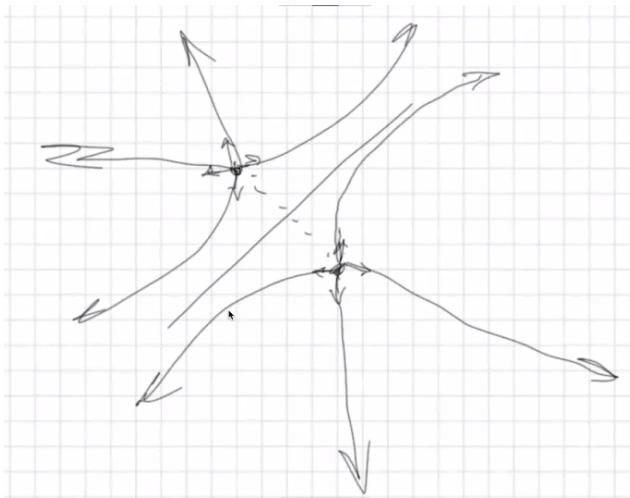
- It obeys the **superposition principle**: The total electric field due to many charged objects is the vector sum of the fields generated by each object acting alone.
- The electric field inside a conductor is zero.
- The electric field immediately outside a conducting surface is perpendicular to that surface (or else it is zero in magnitude).
- The electric field has the same symmetry as the charge distribution that creates it.

An electric field can be conveniently represented by a collection of **electric field lines**. A drawing of electric field lines should obey the following rules:

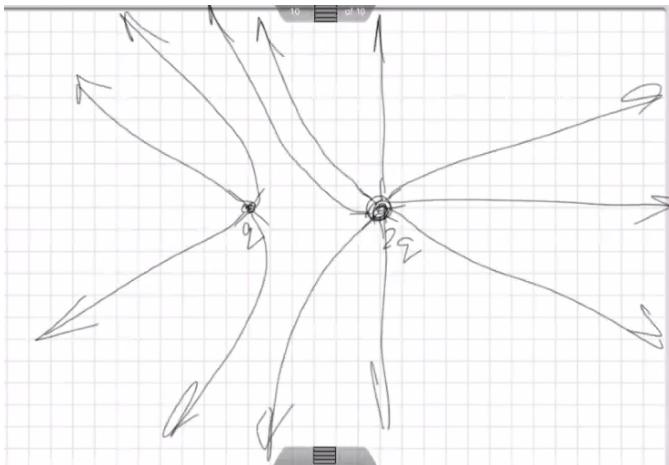
1. Electric field lines **start at positive charges (or infinity)** and **end at negative charges (or infinity)**.
2. The **number** of field lines originating or ending on a point charge is proportional to the **magnitude of the charge**, and the field lines are **uniformly distributed** around the charge when very close to the charge.
3. At **any point** on an electric field line the **electric field itself is tangent** to the field line at that point.

A drawing of electric field lines will immediately reveal (1) the relative magnitude of different charges (proportional to the number of lines that begin or end on each), (2) the sign of different charges (since lines go into negative charges and come out of positive charges), (3) the relative magnitude of the electric field at any point (because the magnitude of the field is proportional to how closely spaced the field lines are), and (4) the symmetry of the charge distribution (since they match the symmetry of the underlying field).

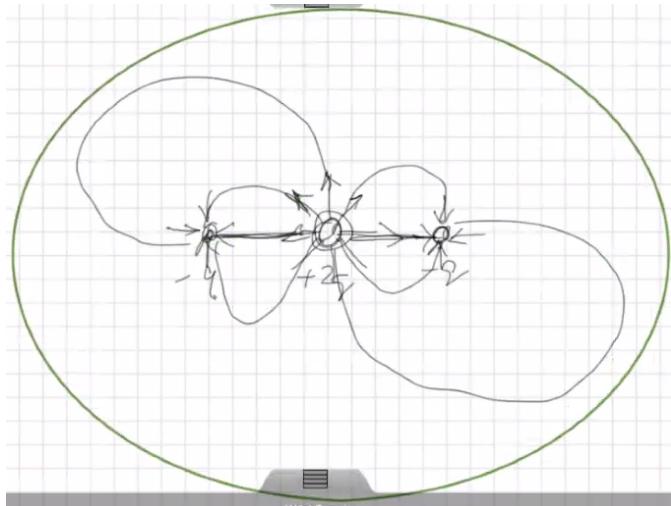
-q & q (Drawing with



-q & 2q



--q, 2q, -q



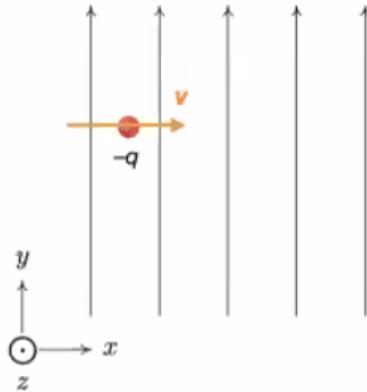
-Fields and Projectile Motion

Using electric field maps

- Imagine that we don't know anything about the charges near a region, but we do know the strength and direction of the electric field. What can that tell us?
- For example, consider an electric field that is uniformly upward.
- If we place a positive charge in this region, what happens?
- If we place a negative charge in this region, what happens?
- What if a negative charge has a velocity to the right?
- The key is to remember that the field is a "could be force" and you can determine how much force if you know the field and the charge that is in it:

$$\vec{F} = Q\vec{E}$$

- From there, you can use Newton's second law and all the things we learned in 121 about motion.



Chapter 24 Gauss's law

-Electric Field Lines

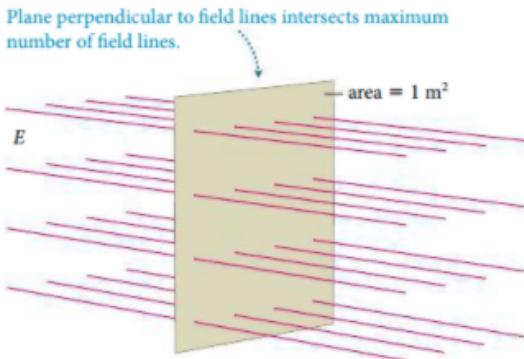
The number of field lines that emanate from a positively charged object or terminate on a negatively charged object is proportional to the charge carried by the object.

-Field Line Density

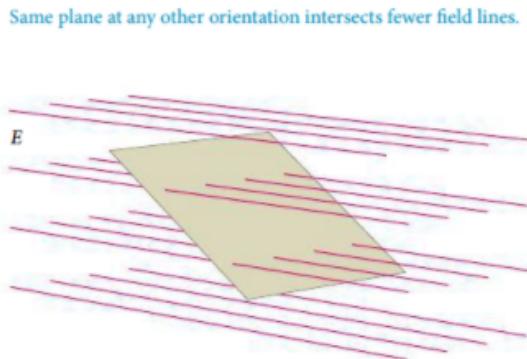
The field line density at a given position is the number of field lines per unit area that cross a surface perpendicular to the field lines at that position.

Figure 24.6 The number of field lines that cross a given surface depends on the orientation of the surface relative to the field lines.

(a)



(b)



-The number of field lines that crosses the surface is the most when the surface is perpendicular to the field lines.

-But wth, Field Line Density is arbitrary. Why bother?

-The number of field lines emanating from or terminating on charged objects is proportional to the magnitude of the charge carried by these obje

At every position in a field line diagram, the magnitude of the electric field is proportional to the field line density at that position.

-Property of Field Lines Rekapp

Properties of electric field lines

When working with electric field lines, keep the following points in mind:

1. Field lines emanate from positively charged objects and terminate on negatively charged objects.
2. At every position, the direction of the electric field is given by the direction of the tangent to the electric field line through that position.

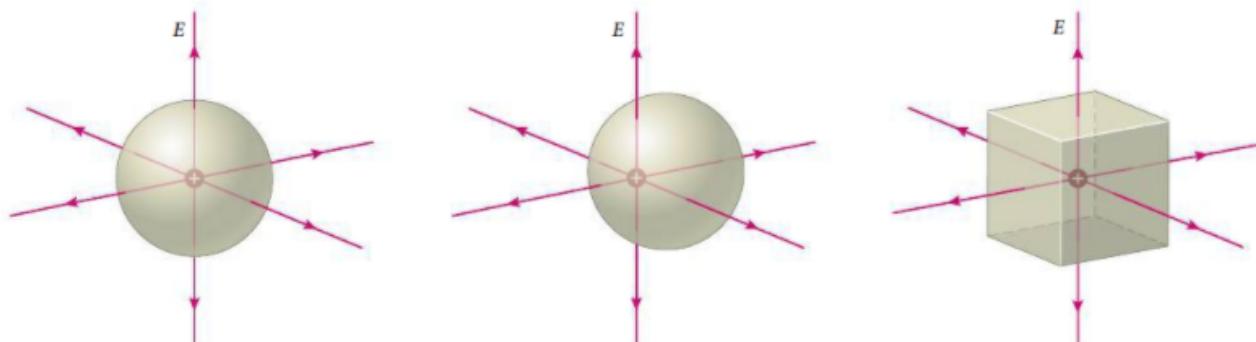
3. Field lines never intersect or touch.

4. The number of field lines emanating from or terminating on a charged object is proportional to the magnitude of the charge on the object.
5. At every position, the magnitude of the electric field is proportional to the field line density.

-Closed Surfaces (& Enclosed Charge)

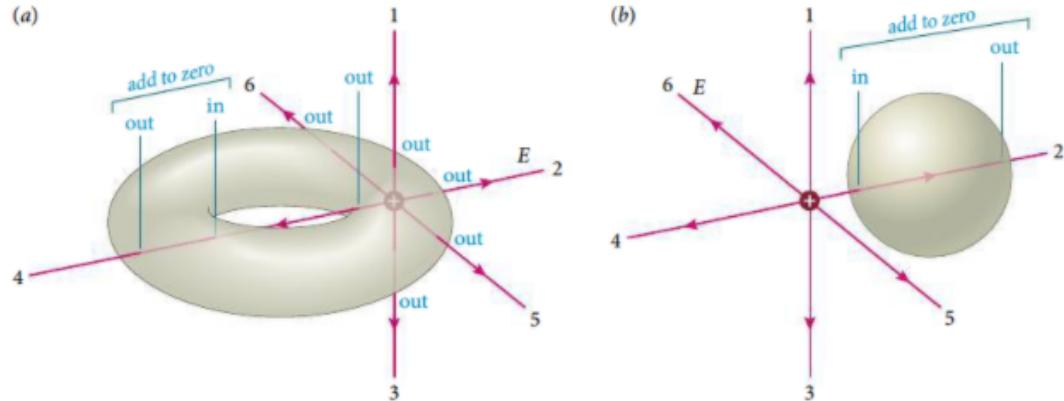
-If any charged thing(s) is placed inside a hollow space, whatever field lines that would lead to infinity in the case of open space, will pierce the object instead.

Figure 24.8 Any surface that encloses a positively charged particle is pierced by all the field lines that emanate from that particle, regardless of the shape of the surface and the position of the particle within the surface.



-If the line leave and renters closed surface

Figure 24.9 The number of field lines exiting a closed surface minus the number entering it is always equal to the number of field lines generated inside the surface.



-Field Line Flux

-Outward = +1

-Inward = -1

For any closed surface, the field line flux is the number of outward field lines crossing the surface minus the number of inward field lines crossing the surface.

$$\text{Flux} = \text{Outward} - \text{Inward}$$

The field line flux through a closed surface is equal to the charge enclosed by the surface multiplied by the number of field lines per unit charge.

$$\text{Flux through closed surface} = \# \text{ Enclosed Charges} * \text{Field Lines per Charge}$$

The field line flux through a closed surface due to charged objects outside the volume enclosed by that surface is always zero.

$$\text{Flux from Charge Outside} = \text{Outward} - \text{Inward} = 0$$

-Electric Flux Calculation (Φ_E)

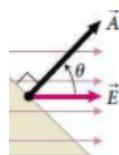
-Uniform Field & Through Single Segment: The magnitude of the electric flux through a surface with area A in a uniform electric field of magnitude E is defined as

$$\Phi_E = EA \cos \theta \quad (\text{uniform electric field}), \quad \text{where } \theta \text{ is the angle between the electric field and the normal to the surface.}$$

-Or DOT PRODUCT

$$= \vec{E} \cdot \vec{A} \quad (\text{uniform electric field}),$$

-Since



where θ is the angle between \vec{E} and \vec{A} . Electric Flux is a scalar (if not doing dot product) & SI unit is N * m^2 / C

-Uniform Field & All Segments

-Sum it.

$$\Phi_E = \sum \vec{E}_i \cdot \delta \vec{A}_i$$

-If you have a bendy surface with infinite segments, Integral it.

$$\Phi_E = \lim_{\delta A_i \rightarrow 0} \sum \vec{E}_i \cdot \delta \vec{A}_i = \int \vec{E} \cdot d\vec{A}.$$

-Add a circle on integral sign to denote it is of an Entire Closed Surface and dA points outwards.

$$\Phi_E = \oint \vec{E} \cdot d\vec{A},$$

-Deriving Gauss's Law

$$\Phi_E = 4\pi kq = \frac{q}{\epsilon_0},$$

-q can be replaced with Charge Density!

-where the Electrical Constant

$$\epsilon_0 = \frac{1}{4\pi k} = 8.85418782 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2).$$

-Special Case which states that the electric flux through the Closed Surface of an arbitrary volume is

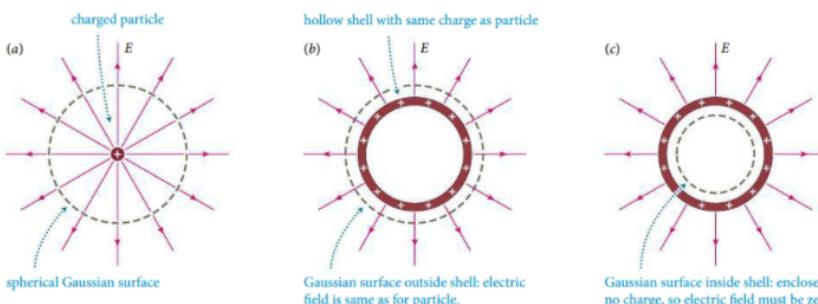
$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$$

q_{enc} is the Enclosed Charge, the sum of all charge on an object or portion of object enclosed

-Symmetry and Gaussian Surface

-Exploit symmetry = win because Gauss Law sucks elsewhere and is approx technique.

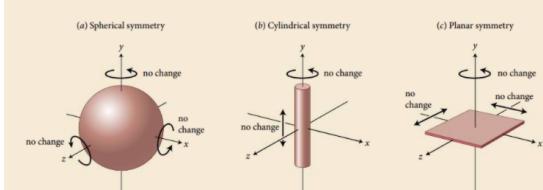
Figure 24.12 Using spherical Gaussian surfaces to examine the electric fields of a charged particle and a uniformly charged spherical shell. The electric fields, Gaussian surfaces, and charged shell are spherical and are shown here in cross section.



The electric field outside a uniformly charged spherical shell is the same as the electric field due to a particle that carries an equal charge located at the center of the shell.

-24.12b

Figure 24.13 Three symmetries important for applications of Gauss's law.



-3 good Gaussian Surface

Figure 24.14 The electric field of an infinite uniformly charged wire exhibits cylindrical symmetry. We can examine this field by surrounding the wire with a concentric cylindrical Gaussian surface.

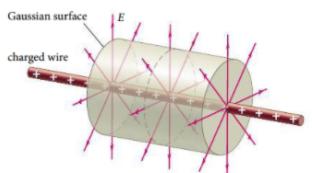
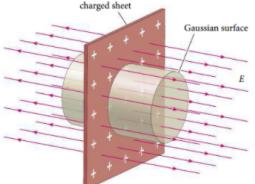


Figure 24.15 The electric field of a uniformly charged sheet exhibits planar symmetry. We examine this field by drawing a cylindrical Gaussian surface that straddles the sheet.



24.5 Charged conducting objects

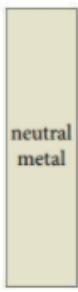
Let us now apply the relationship between field line flux and enclosed charge to charged conducting objects. As we saw in Chapter 22, conducting materials permit the free flow of pepelaugh

-Charge Conducting Objects

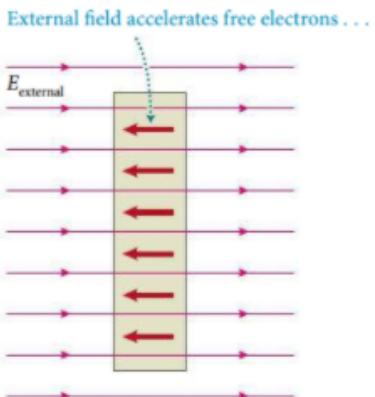
-Electrostatic Equilibrium: the condition in which the distribution of charge in a system doesn't change. (Time to reach is about 10^{12} s)

Figure 24.16 Why the electric field inside the bulk of a conducting object is zero when the object is in electrostatic equilibrium.

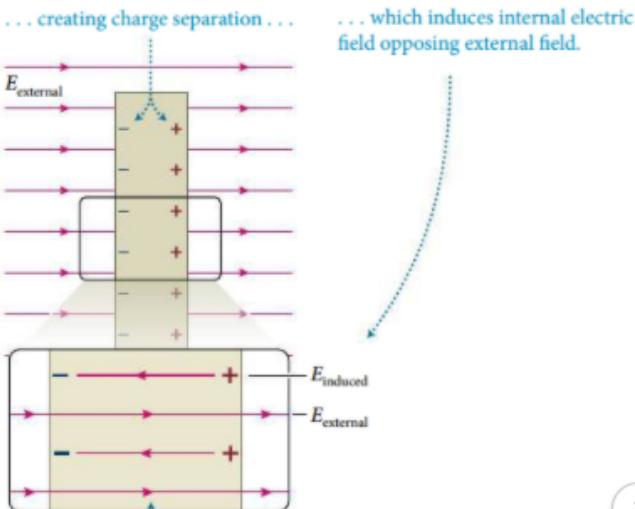
(a) No electric field



(b) Electric field just switched on



(c) Electrostatic equilibrium established

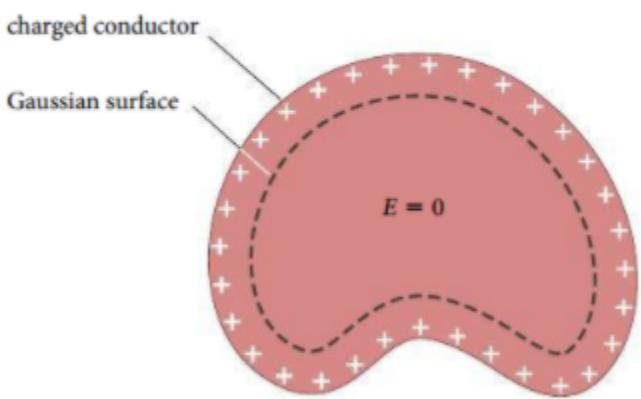


Once field within metal is zero, $\vec{E}_{\text{inside}} = \vec{E}_{\text{external}} + \vec{E}_{\text{induced}} = \vec{0}$, equilibrium is reached.

The electric field inside a conducting object that is in electrostatic equilibrium is zero.

-Stuff that uses power (moving charged particle) like your computer and fridge has an electric field though.

Figure 24.17 Because the electric field inside a conducting object in electrostatic equilibrium is zero, we conclude that there cannot be any surplus charge inside the object.



Any surplus charge placed on an isolated conducting object arranges itself at the surface of the object. No surplus charge remains in the body of the conducting object once it has reached electrostatic equilibrium.

-Applying Gauss's Law

-Using Gauss Law and Symmetry, we can skip integration of area & Electric Field because both will stay constant through a well placed Gaussian Surface (ie Sphere in a Sphere)

-Usually we have to do Integration (23.7), but instead, we can simplify it using Gauss's Law (24.8)

Procedure: Calculating the electric field of continuous charge distributions by integration

To calculate the electric field of a continuous charge distribution, you need to evaluate the integral in Eq. 23.15. The following steps will help you evaluate the integral.

1. Begin by making a sketch of the charge distribution. Mentally divide the distribution into small segments. Indicate one such segment that carries a charge dq , in your drawing.
2. Choose a coordinate system that allows you to express the position of the segment in terms of a minimum number of coordinates (x, y, z, r , or θ). These coordinates are the integration variables. For example, use a radial coordinate system for a charge distribution with radial symmetry. Unless the problem specifies otherwise, let the origin be at the center of the object.
3. Draw a vector showing the electric field caused by the segment at the point of interest. Examine how the components of this vector change as you vary the position of the segment along the charge distribution. Some components may cancel, which greatly simplifies the

calculation. If you can determine the direction of the resulting electric field, you may need to calculate only one component. Otherwise express r_{ip} in terms of your integration variable(s) and evaluate the integrals for each component of the field separately.

4. Determine whether the charge distribution is one-dimensional (a straight or curved wire), two-dimensional (a flat or curved surface), or three-dimensional (any bulk object). Express dq , in terms of the corresponding charge density of the object and the integration variable(s).
5. Express the factor $1/r_{ip}^2$, where r_{ip} is the distance between dq and the point of interest, in terms of the integration variable(s).

At this point you can substitute your expressions for dq , and $1/r_{ip}^2$ into Eq. 23.15 and carry out the integral (or component integrals), using what you determined about the direction of the electric field (or substituting your expression for r_{ip}).

Procedure: Calculating the electric field using Gauss's Law

Gauss's law allows you to calculate the electric field for charge distributions that exhibit spherical, cylindrical, or planar symmetry without having to carry out any integrations.

1. Identify the symmetry of the charge distribution. This symmetry determines the general pattern of the electric field and the type of Gaussian surface you should use (see Figure 24.27).
2. Sketch the charge distribution and the electric field by drawing a number of field lines, remembering that the field lines start on positively charged objects and end on negatively charged ones. A two-dimensional drawing should suffice.
3. Draw a Gaussian surface such that the electric field is either parallel or perpendicular (and constant) to each face of the surface. If the charge distribution divides

space into distinct regions, draw a Gaussian surface in each region where you wish to calculate the electric field.

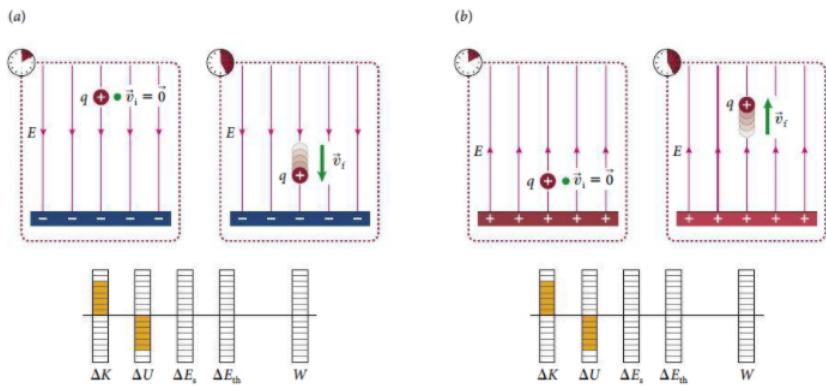
4. For each Gaussian surface determine the charge q_{enc} enclosed by the surface.
5. For each Gaussian surface calculate the electric flux Φ_E through the surface. Express the electric flux in terms of the unknown electric field E .
6. Use Gauss's law (Eq. 24.8) to relate q_{enc} and Φ_E and solve for E .

You can use the same general approach to determine the charge carried by a charge distribution given the electric field of a charge distribution exhibiting one of the three symmetries in Figure 24.27. Follow the same procedure, but in steps 4–6, express q_{enc} in terms of the unknown charge q and solve for q .

Chapter 25: Electric Potential

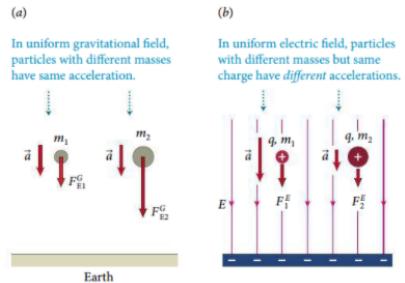
-So 121, we worked with Potential Energy (U with SI: Joules) before (ie gravity using height to judge.)

Figure 25.1 Energy diagrams for closed systems in which a positively charged particle is released from rest near a large stationary object that carries (a) a negative or (b) positive charge.



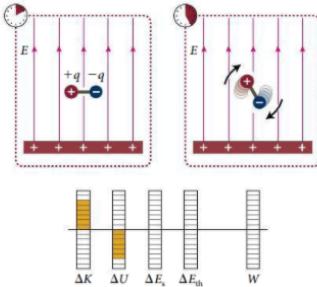
-Grav VS Electric

Figure 25.2 The free motion of charged particles in an electric field is different from the free fall of objects in a gravitational field.



-Electric Dipole

Figure 25.3 Energy diagram for a system in which a dipole is released from rest near a positively charged stationary object.



-Electric Potential is "Electric Potential Energy per Charge"

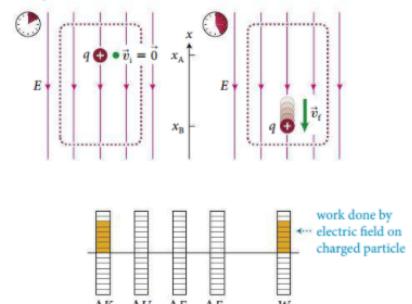
-Volts = Joules/Coulombs

$U=qV$ (Electric Potential Energy = Charge * Electric Potential)

-Electrostatic Work (Work done by Electrostatic Field)

-We'll be considering a charge in a uniform field. That thingy making the uniform field is not in the system, so the system is not closed.

Figure 25.4 Energy diagram for a positively charged particle in the uniform electric field of a stationary, negatively charged object, that is not part of the system.



-Work from 121

$$W = \vec{F} \cdot \Delta \vec{x} = \int \vec{F} \cdot d\vec{r}$$

-work as the dot product between an applied force and the displacement caused by that force

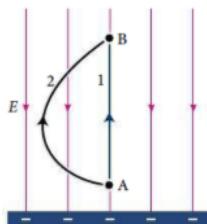
-Electrostatic Work is independent from path.

The electrostatic work done on a charged particle as it moves from one point to another is independent of the path taken by the particle and depends on only the positions of the endpoints of the path.

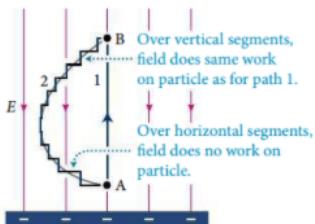
-Doesn't care about what happens to get from A->B

Figure 25.6 The electrostatic work done on a charged particle as the particle moves from point A to point B is independent of the path taken; it depends only on the positions of the endpoints of the path.

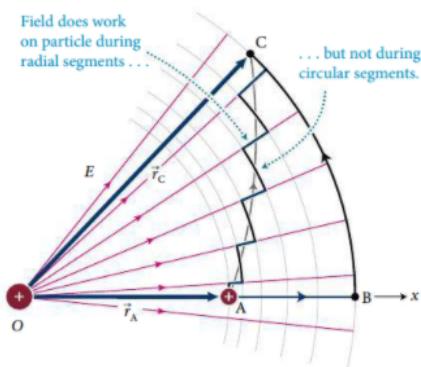
(a) Two paths by which particle can move from A to B



(b) Path 2 approximated by vertical & horizontal segments



(c) Same argument applied to nonuniform electric field



-Electric Field is not affecting particle when moving horizontal to it.

-If the work from A->B is 1 with 1q charge, the work from A->B with 2q charge is Doubled

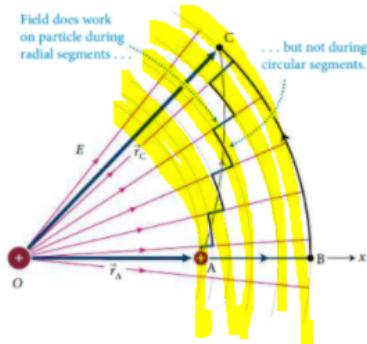
-Electric Potential Difference

The potential difference between point A and point B in an electrostatic field is equal to the negative of the electrostatic work per unit charge done on a charged particle as it moves from A to B.

-Scalar. Joules/Coulomb. If Work is +, PotenDif is -.

-Equipotentials

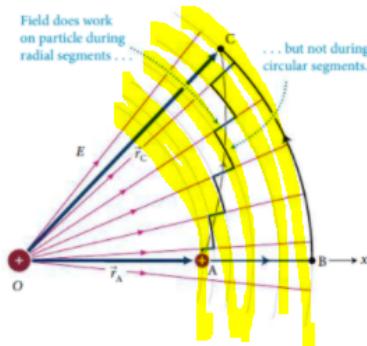
-Equipotential Lines, where Work is 0 & Electro Potential doesn't change.



An equipotential line is a line along which the value of the electrostatic potential does not change. The electrostatic work done on a charged particle as it moves along an equipotential line is zero.

-Like contour lines on map, work of grav is 0 if along it

-Equipotential Surfaces, like Lines, but 3D.

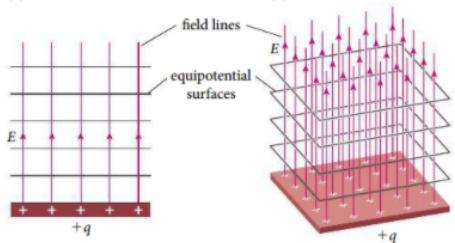


-If 3D, each Line is actually a sphere. Δx on the sphere surface, no work.

-Some arrangement of charges

Figure 25.8 Equipotential surfaces in a uniform electric field in (a) two dimensions; and (b) three dimensions.

(a)



(b)

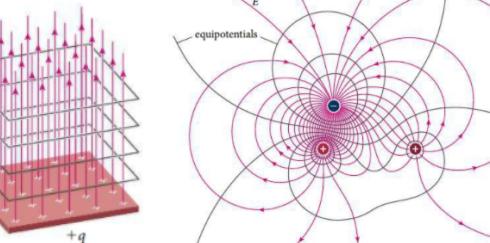
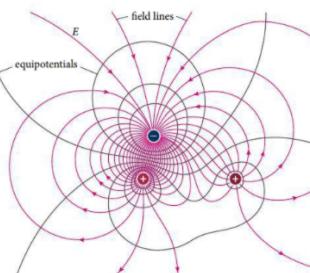
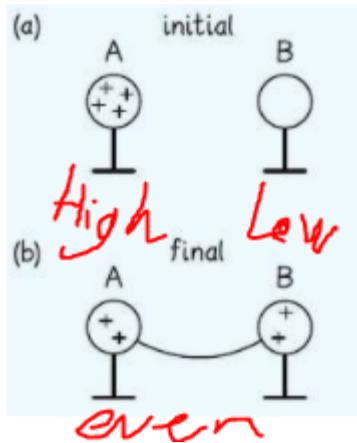


Figure 25.9 Field lines and equipotentials for three stationary charged particles.



The equipotential surfaces of a stationary charge distribution are everywhere perpendicular to the corresponding electric field lines.



-Even means 0

An electrostatic field is directed from points of higher potential to points of lower potential.

In an electrostatic field, positively charged particles tend to move toward regions of lower potential, whereas negatively charged particles tend to move toward regions of higher potential.



If the potential difference between A and B is positive, is the field directed from A to B or from B to A?

from B to A

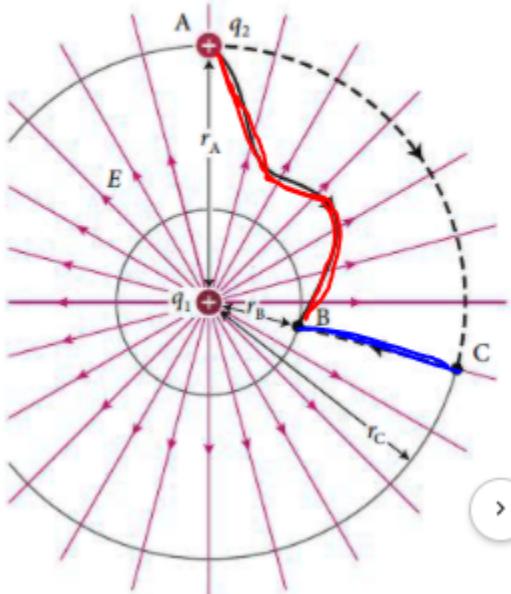
-Here, B has higher Potential Difference

-Calculating work and energy in electrostatics

$$W_{12}(A \rightarrow B) = \int_A^B \vec{F}_{12}^E \cdot d\vec{\ell},$$

-Where F^E is Force of Electric Field
-d ℓ is infinitesimal segment of path

Figure 25.15 The electrostatic work done by particle 1 on particle 2 as the latter is moved from A to B is the same for the meandering solid path and for the dashed path ACB.



-Work through red & blue path are same! So use dr instead of d ℓ for this case (use radius instead).

$$W_{12} = \frac{q_1 q_2}{4\pi\epsilon_0} \left[\frac{1}{r_{12,i}} - \frac{1}{r_{12,f}} \right],$$

-Where r_{12i} and r_{12f} is the start and end

-Electric Potential Energy (SI: Joules)

Thus, the **electric potential energy** for two particles carrying charges q_1 and q_2 and separated by distance r_{12} is

$$U^E = \frac{q_1 q_2}{4\pi\epsilon_0} \frac{1}{r_{12}} \quad (U^E \text{ zero at infinite separation}).$$

(25.8) An electrostatic field is directed from points of higher potential to points of lower potential.

-And Positive Charge is HIGH

-U of multiple charges

\Sum of all the U

-Work from one charge

$$W_{12} = -\frac{q_1 q_2}{4\pi\epsilon_0} \frac{1}{r_{12}}.$$

-Work from multiple charges

\Sum of the W

-Potential Difference (Volts: J/C)

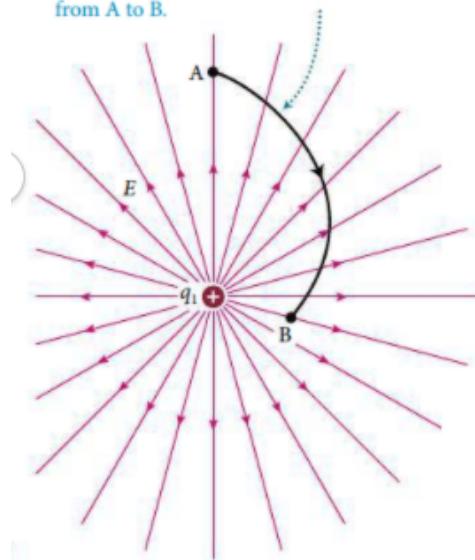
The negative of the electrostatic work per unit charge done on a particle that carries a positive charge q from one point to another is defined as the **potential difference** between those points:



$$V_{AB} \equiv V_B - V_A \equiv \frac{-W_q(A \rightarrow B)}{q}. \quad (25.15)$$

Figure 25.17 Once we know the electrostatic work done by a particle carrying a charge q_1 on a particle carrying a charge q_2 as the latter is moved from A to B in Figure 25.15, we can determine the potential difference between A and B in the electric field of particle 1.

Potential difference between A and B,
 $V_{AB} = V_B - V_A$, is negative of electrostatic work per unit charge done along a path from A to B.



Once we know the potential difference V_{AB} between A and B, we can obtain the electrostatic work done on *any* object carrying a charge q as it is moved along any path from A to B:

$$W_q(A \rightarrow B) = -qV_{AB}. \quad (25.17)$$

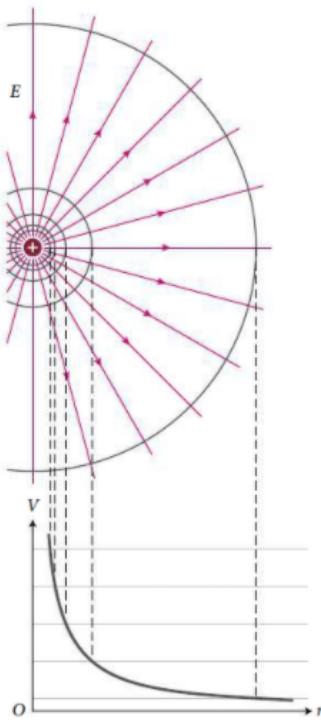
The potential difference V_{AB} between points A and B due to an electrostatic field \vec{E} is

$$V_{AB} \equiv \frac{-W_q(A \rightarrow B)}{q} = - \int_A^B \vec{E} \cdot d\vec{\ell}. \quad (25.25)$$

For any electrostatic field,

$$\oint \vec{E} \cdot d\vec{\ell} = 0. \quad (25.32)$$

Figure 25.18 Equipotentials, field lines, and graph of potential for a charged particle.



$$V(r) = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r} \quad (\text{potential zero at infinity}), \quad V_p = \frac{1}{4\pi\epsilon_0} \sum_n \frac{q_n}{r_{np}}$$

-For a group of charges

$$V = \frac{kq}{r} \quad (\text{Same thing})$$

-Work, Potential Difference, Kinetic Energy, and Force

$$W_{field} = -Vq = \Delta K_{field} = F_{field} * d \quad \text{Lab 4 4.6 is using this.}$$

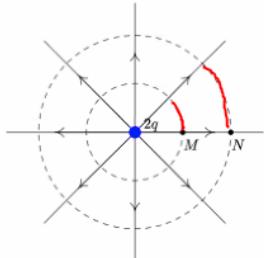
Lab 3: Gauss's Law

-Figure 5, Case B > Case A magnitude

-Area Vector is up or down. Don't have convention yet.

- $dA = A \hat{r}$

-Field Line Density



M has more line density, so stronger field

-3.8 Net is 0

-4.2 Charge Net, since Gauss law said it's proportional.

-Gauss Law only concern inside

-4.3 The Area makes up for the weak Field

$$\Phi = E_1 A_1 + E_2 A_2 + \dots = \frac{\sigma}{\epsilon_0} = \frac{\sigma A_1}{\epsilon_0}$$

$$A_1 = A_2$$

$$E_1 + E_2 = \frac{\sigma}{\epsilon_0} \quad E = \frac{\sigma}{2\epsilon_0}$$

A_3

-4.4

Lab 4: Electric Potential

-Work

-Force acting through a distance

-Dot Product of Force and Displacement

-Constant Speed = NO WORK

- $W_{net} = \Delta KE$

-2 Same Charge Particles

-To push one to move at a constant speed to the other (the other is stuck), then we have to push not at a constant force.

-Electrostatic Work

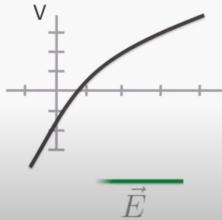
-Work of Electric Field (Uniform or Point Charges)

Chapter 25.6 & 25.7

-Electric Field and Electric Potential Relationship

$$\vec{E} = -\nabla V$$

$$V = - \int_a^b \vec{E} \cdot d\vec{s}$$



$-ds$ is the change in distance.

-Electric Breakdown

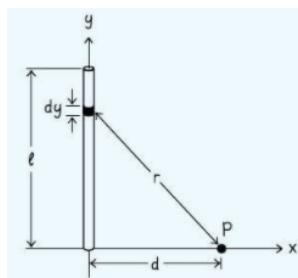
The process by which an insulating material in a strong electric field becomes electrically conductive.

-Electrostatic potentials of continuous charge distribution (V)

$$dV_s = \frac{1}{4\pi\epsilon_0} \frac{dq_s}{r_{sp}},$$

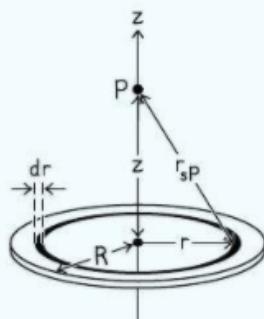
$$V_p = \int dV_s = \frac{1}{4\pi\epsilon_0} \int \frac{dq_s}{r_{sp}},$$

-Rod



$$= \frac{1}{4\pi\epsilon_0} \frac{q}{\ell} \ln \left(\frac{\ell + \sqrt{\ell^2 + d^2}}{d} \right).$$

-Disk



$$V_p = \frac{2\pi\sigma}{4\pi\epsilon_0} \int_0^R \frac{r dr}{\sqrt{z^2 + r^2}} = \frac{\sigma}{2\epsilon_0} (\sqrt{z^2 + R^2} - |z|).$$

-Obtaining the electric field from the potential. (V allows us to simplify direct integration of E)

$$W_q(P \rightarrow P') = -qV_{PP'} = -q(V_{P'} - V_P) = -q dV$$

and

$$\begin{aligned} W_q(P \rightarrow P') &= \vec{F}_q^E \cdot d\vec{s} = (q\vec{E}) \cdot d\vec{s} \\ &= q(\vec{E} \cdot d\vec{s}) = qE \cos\theta ds, \end{aligned}$$

$$-q dV = qE_s ds$$

$$E_s = -\frac{dV}{ds}.$$

So

$$E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z}. \quad \vec{E} = -\frac{\partial V}{\partial x} \hat{i} - \frac{\partial V}{\partial y} \hat{j} - \frac{\partial V}{\partial z} \hat{k}.$$

Chapter 26

-Charge Separation

- The system's electric potential energy depends on the configuration of the positive and negative charge carriers in the system.
 - The potential difference between points on the rod and the fur is a measure of the electrostatic work done on a particle carrying a unit of charge (not part of the system) while moving between those points.

- Positive work must be done on a system to cause charge separation of the positive and negative charge carriers in the system. This work increases the system's electric potential energy.

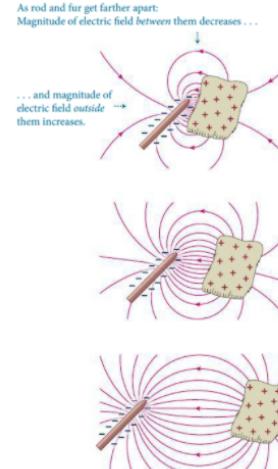
-That means the Positive Work changes orientation of charges in the system.

The amount of stored electric potential energy depends on the amount of charge that is separated and the distance that separates the charge carriers. More charge or a greater separation means more electric potential energy is stored. These arguments apply to *all* devices that separate charge.

-Charge-separating / charging devices have some mechanism to move charge carriers against electric fields. Requires work to be done on the system.

-like pulling apart attracted things

Figure 26.2 Change in the electric field pattern as the distance between the rod and fur increases.



-Royal Capacitors

-Aperturatus to store electric potential energy, consisting of two conductors

Figure 26.4 A parallel-plate capacitor

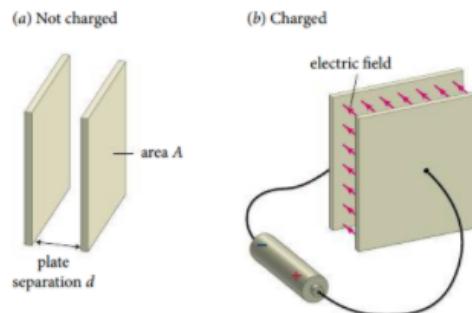
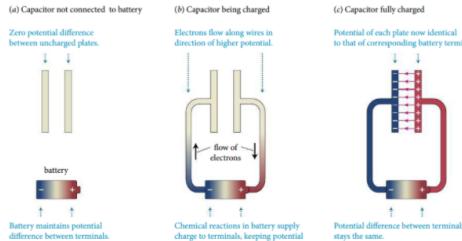


Figure 26.5 Charging a capacitor.



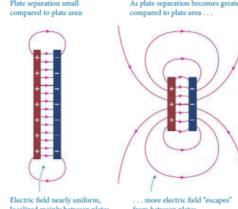
When a charge-separating device such as a battery is connected between the plates of a capacitor, it 'charges' the capacitor, creating an excess charge on each plate. The amount of the charge is given by the formula

ula Capacitance

-Battery has the same charge: looped

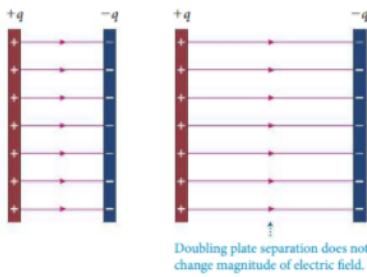
-Size. The smaller the plate, the more leakage. When spacing compare to area of plates are small, treat just like infinna sheet.

Figure 26.6 Effect of plate separation in relation to plate area on the field of a parallel-plate capacitor.



-Spacing

Figure 26.8 Doubling the plate separation of a parallel-plate capacitor.



For a given potential difference between the plates of a parallel-plate capacitor, the amount of charge stored on its plates increases with increasing plate area and decreases with increasing plate separation.

-Electrical Breakdown

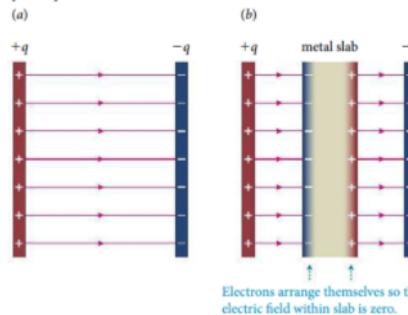
-We can't increase the amount of charge stored by the plates from making infinitely closing the separation.

-If the potential difference is fixed, E will increase. If the magnitude is about 3×10^6 V/m, air will ionize and breakdown happens.

-We can raise the breakdown threshold by putting a nonconducting material in between plates.

-But first, if we put conductive in between.

Figure 26.9 Inserting a conducting slab between the plates of a parallel-plate capacitor.



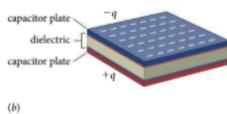
Electrons arrange themselves so that electric field within slab is zero.

-E inside conductive = 0. This reduces V from each plate.

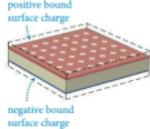
-Nonconductive is a Dielectric. Place in the middle, and it allows plates to carry more charge (capacitance up) with the same E in between.

Figure 26.13 The polarization induced on a dielectric in a parallel-plate capacitor is equivalent to two thin sheets carrying opposite charge.

(a) Dielectric sandwiched between capacitor plates



(b)

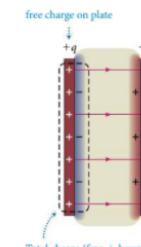


Polarization induced on dielectric is equivalent to pair of charged sheets.

Figure 26.14 The presence of a polarized dielectric reduces the strength of the electric field between the plates of a capacitor.

(a)

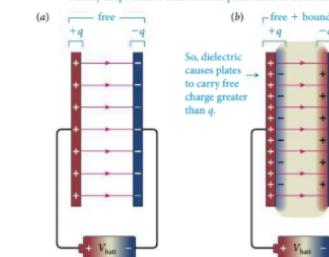
(b)



Total charge (free + bound) is less than q.

Figure 26.15 The presence of a polarized dielectric increases the charge on the plates of a capacitor connected to a battery.

Battery keeps electric field between plates same in both cases:



-Not Phasmophobia EMF

Emf % (V) The emf of a charge-separating device is the work per unit charge done by nonelectrostatic interactions in separating positive and negative charge carriers inside the device:

$$\mathcal{E} = \frac{W_{\text{nonelectrostatic}}}{q}. \quad (26.7)$$

The emf of a charge-separating device is the work per unit charge done by nonelectrostatic interactions in separating positive and negative charge carriers inside the device.

The energy stored in a capacitor is supplied to it by the charging device—such as a generator, a battery, or a solar cell. Inside this device, nonelectrostatic interactions cause a separation of charge by doing work on charged particles. The work per unit charge done by the nonelectrostatic interactions on the charge carriers inside the device is called the emf and is denoted by \mathcal{E} :

If no energy is dissipated inside the charging device, all of the energy can be transferred to charge carriers outside the device. This transfer takes place through electric interactions. In Figure 26.25, for example, electric forces remove electrons from one object and push them onto the other, charging the capacitor. In the absence of any energy dissipation, the nonelectrostatic work done on charge carriers inside the device is equal to the electrostatic work done on charge carriers outside it. Because the electrostatic work per unit charge is the potential difference between the negative and positive terminals of the charging device, we have, for an ideal charging device,

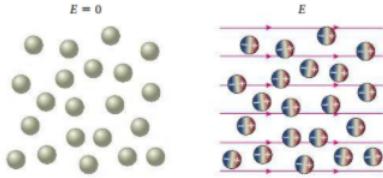
$$V_{\text{device}} = \mathcal{E} \quad (\text{ideal device}). \quad (26.8)$$

-Otherwise, if V_{device} sucks and does like 75%: $V_{\text{device}} = (0.75)\text{EMF}$

-Polarization of Nonpolar molecules

Figure 26.11 Polarization of nonpolar and polar molecules in an electric field.

(a) Polarization of nonpolar molecules



(b) Polarization of polar molecules

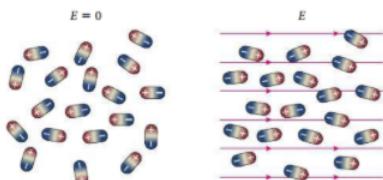
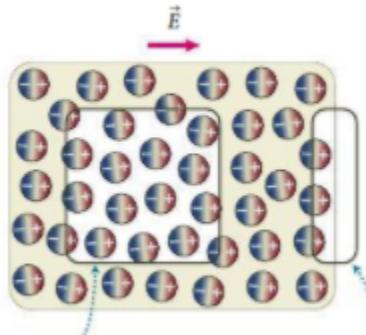
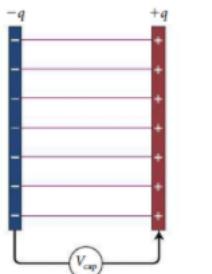
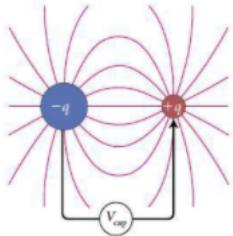
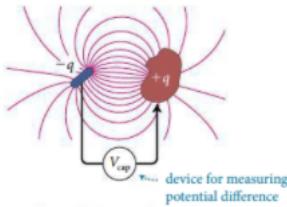


Figure 26.12 The reason a polarized dielectric exhibits a macroscopic polarization.



-Capacitance (SI: Farad = Coulomb/Volt)

Figure 26.19 The electric fields and potential differences of three different capacitors.



$$C = \frac{q}{V_{cap}} \quad q = CV_{cap}$$

-q is the charge on each object. V_cap is the magnitude of the potential difference between the conducting plates.

$$C = \frac{A}{\epsilon_0 d}$$

-Plate Capacitor

-d distance between plates surface & A is plates' area.

-Electric Field from parallel plate capacitor

$$E = \frac{Q}{\epsilon_0 A} = \frac{CV}{\epsilon_0 A} = \frac{V}{d}$$

(Gauss's Law was used here bruv)

-Potential Energy from Capacitor (got from parallel plates type beat)

$$U_C = \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} QV$$

-U_c is U^E btw

-Potential Energy from Capacitance more relations

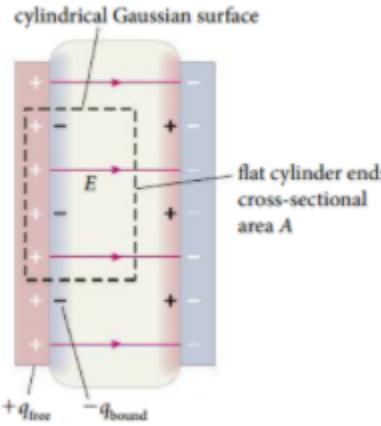
$$U^E = \frac{1}{2} CV_{cap}^2 = \frac{1}{2} \left(\frac{\epsilon_0 A}{d} \right) (Ed)^2 = \frac{1}{2} \epsilon_0 E^2 (Ad).$$

-(Ad) is the volume of space between capacitor's surfaces.

-Potential Energy Density

$$u_E \equiv \frac{U^E}{\text{volume}} = \frac{1}{2} \epsilon_0 E^2.$$

Figure 26.31 A cylindrical Gaussian surface used to calculate the electric field inside a dielectric-filled parallel-plate capacitor.



Because the dielectric constant affects the value of the electric field, **Gauss's law in matter** is usually written in the form

$$\oint \kappa \vec{E} \cdot d\vec{A} = \frac{q_{\text{free, enc}}}{\epsilon_0}. \quad (26.25)$$

Dielectric constant κ (unitless) The factor by which the potential across an isolated capacitor is reduced by the insertion of a dielectric:

$$\kappa = \frac{V_0}{V_d}, \quad (26.9)$$

-\kappa =

- Technically, we should adjust all of our laws about electric fields and force by replacing ϵ_0 with $\epsilon = \kappa\epsilon_0$

-\kappa = 1 when we in a vacuum (USE \EPSILON WHEN USING DIELECTRIC OR DIE)

-Dielectrics and Capacitance

- $C = \epsilon_0 \frac{A}{d}$ becomes $C = \epsilon \frac{A}{d}$ or $C = \kappa\epsilon_0 \frac{A}{d}$
- So the capacitance of the capacitor increases when there is a dielectric.
- This means it can store more charge which also means that it stores more energy at a given voltage:
- This can also be seen in the formula for the energy per unit volume stored in the electric field:
- $u_E = \frac{1}{2}\epsilon_0 E^2$ becomes $u_E = \frac{1}{2}\epsilon E^2$ or $u_E = \frac{1}{2}\kappa\epsilon_0 E^2$

out a dielectric. Therefore, the capacitance changes by the factor κ when a dielectric is inserted:

$$C_d = \kappa C_0. \quad (26.11)$$

-Dielectric Materials Table

Table 26.1 Dielectric properties

Material	Dielectric constant κ	Breakdown threshold E_{max} (V/m)
Air (1 atm)	1.00059	3.0×10^6
Paper	1.5–3	4.0×10^7
Mylar (polyester)	3.3	4.3×10^8
Quartz	4.3	8×10^6
Mica	5	2×10^8
Oil	2.2–2.7	
Porcelain	6–8	
Water (distilled, 20 °C)	80.2	$6.5–7 \times 10^7$
Titania ceramic	126	8×10^6
Strontium titanate	322	
Barium titanate	1200	8×10^7

-\Epsilon Naught

This gives a value of free space permittivity

$$\epsilon_0 = 8.854187817 \times 10^{-12} \text{ F/m} = 8.85 \times 10^{-12} \text{ F/m}$$

Farad / Meter or C/Vm or C/Vm

Capacitance $C = \epsilon_0 \frac{A}{d}$

(2)

$$\frac{2A}{d} > \frac{A}{d}$$

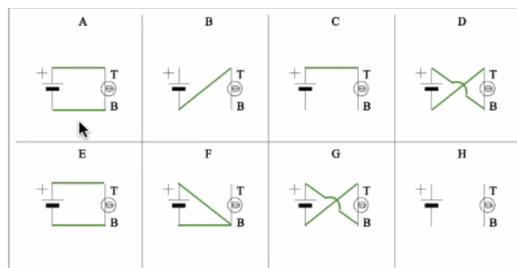
Potential diff $V = \frac{Q}{C}$ same Q , so if C .

Electric field $E = \frac{V}{d} = \frac{Q}{dC} = \frac{Q}{\epsilon_0 A} = \frac{Q}{\epsilon_0 A}$

Energy $U_C = \frac{1}{2} CV^2 = \frac{1}{2} QV$ same

Energy density $u_E = \frac{1}{2} \epsilon_0 E^2$ so same order as E : $u_i = u_E$

Lab 5



Chapter 31

-Current

- The amount of charge that passes through an area in a given amount of time.
- Charge per unit time

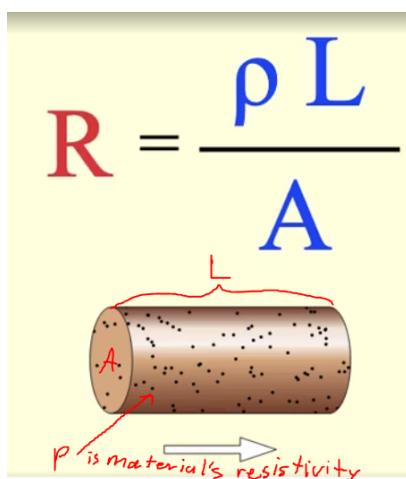
-Conductor and Insulators

- Conductor: Where charge is able to flow easily.
- Insulator: Where it requires a large potential difference to get charge to flow through.
- This is a continuum/gradient, not binary, like gender fluid people.

-Vocab

- Voltage / Potential Difference is the push... but more like a pressure than a force. It causes the flow as the charges try to reach equilibrium.
 - Measured across elements or between points in the circuit (compare one pos to another pos).
- Current is the flow
 - Measured at a point.
- Resistance is the resistance to the flow, how hard it is to get through. Phet Dog had 1000000 \(\Omega\).
 - No resistance is "ideal". So even wire, there is resistance. Longer wires have more resistance.
- Since current is the flow through a section per unit time, it also makes sense that resistance is less if the cross-sectional area of the wire is greater.
- Resistance of an object as a function of the properties of that object.

$$R = \rho \frac{\ell}{A} \quad \text{or} \quad R = \frac{\ell}{\sigma A} \quad \text{where } \rho \text{ is the resistivity and } \sigma = \frac{1}{\rho} \text{ is the conductivity of the metal.}$$



-Measurement

- Voltage is measured in volts (V)
- Current is measured in amperes (A = C/s)
- Resistance is measured in ohms (\(\Omega\))

-Ohm's Law (The simplest case)

$$V = IR \quad \text{or} \quad I = \frac{V}{R}$$

Note: This linear model is only true for an 'Ohmic' device, and only for a reasonable range of voltages in such a device.

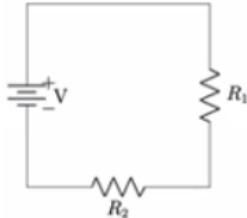
-I is current

-V is voltage

-R is resistance

-What if we have more than one resistor?

-Just add them 4head. This pattern is true if the resistors are in series.



From the perspective of the current, if it has to go through both resistors, it's the same as if it was just one, long resistor:

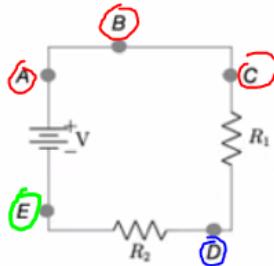
$$R_{\text{tot}} = R_1 + R_2$$

is the same as

$$R_{\text{tot}} = R_1 + R_2 + R_3 + \dots$$



-Potential Difference example



-A,B,C have the same potential.

-E has the inverse of A,B,C's potential.

-The potential difference between A, B, or C to E is that of the battery's voltage.

-D has different potential because of resistance

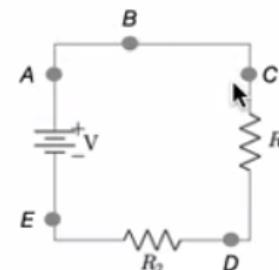
If the current through the resistor is known, the voltage drop across it is given by Ohm's law: $V = IR$.

The potential difference between C and D plus the potential difference between D and E would equal the voltage of the battery.

$$V_{CD} + V_{DE} = \text{Battery's Voltage}$$

Example with numbers

- Let's put a 9 V battery in this circuit and use two resistors: $R_1 = 12 \Omega$ and $R_2 = 24 \Omega$
- For a series circuit, we know there is only one current throughout the loop, and we can find that current from Ohm's law if we know the total resistance of the circuit.
- Since these resistors are in series, they just add up to a total resistance of $R_{\text{tot}} = R_1 + R_2 = 36 \Omega$ and $I = \frac{V}{R} = \frac{9 \text{ V}}{36 \Omega} = 0.25 \text{ A}$
- We can find the voltage drop across each resistor because we know the current through each resistor is 0.25 A.
- So the potential difference between points C and D must be $V = IR = (0.25 \text{ A})(12 \Omega) = 3 \text{ V}$.
- And the potential difference between points D and E must be $V = IR = (0.25 \text{ A})(24 \Omega) = 6 \text{ V}$.

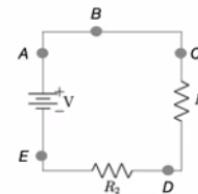


-Resistors dissipate energy, and the pattern is simpler: The energy used each second is equal to the current times the potential difference across the resistor.

$$P = IV \quad (\text{Watt} = \text{Joules/Second}) \quad (\text{Also } P = I^2 R)$$

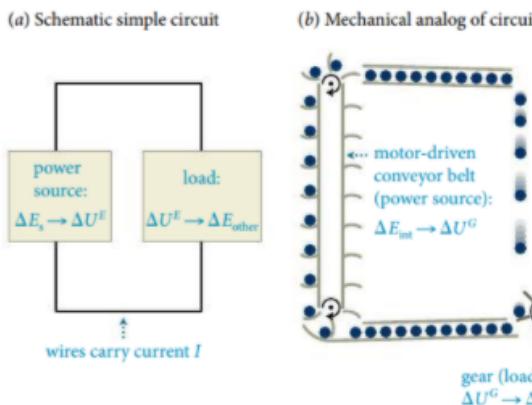
Example

- For example, returning to our 9 V battery connected to two resistors: $R_1 = 12 \Omega$ and $R_2 = 24 \Omega$, what is the power dissipated in each resistor and the total power supplied by the battery?
- We found that the current everywhere in this circuit was $I = 0.25 \text{ A}$
- We found the voltage drop across R_1 was $V_1 = 3 \text{ V}$; therefore the power dissipated in R_1 is $P = IV = (0.25 \text{ A})(3 \text{ V}) = 0.75 \text{ W}$ or 0.75 joules per second.
- The voltage drop across R_2 was $V_2 = 6 \text{ V}$; therefore the power dissipated in R_2 is $P = IV = (0.25 \text{ A})(6 \text{ V}) = 1.5 \text{ W}$.
- So the circuit uses a total of 2.25 joules of energy per second, and the battery has to supply that. This is also equal to the current times the voltage of the battery: $P = IV = (0.25 \text{ A})(9 \text{ V}) = 2.25 \text{ W}$.
- A real battery will need to use more energy than this each second because it has some internal resistance to overcome.



-Load & Energy Conversion in Circuit

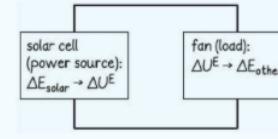
Figure 31.2 The energy conversions in a single-loop circuit and in a mechanical analog of the circuit.



In a single-loop DC circuit, electric potential energy acquired by the carriers in the power source is converted to another form of energy in the load.

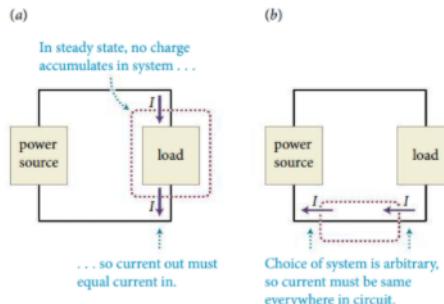
SOLUTION The solar cell is the power source, and the fan is the load. **Figure 31.3** shows my diagram for this circuit. The solar cell converts solar energy to electric potential energy, and the fan converts electric potential energy to another form of energy by setting the air in motion. I add these conversions to my diagram. ✓

Figure 31.3



-Steady State

Figure 31.6 Because charge is conserved, in steady state it doesn't accumulate in the load or in any other part of the circuit. Hence, in steady state the current into any part of the circuit must be the same as the current out of that part.



In a steady state, the current is the same at all locations along a single-loop electric circuit.

Because we can ignore the energy dissipation that takes place in the wires:

Every point on any given wire is essentially at the same potential.

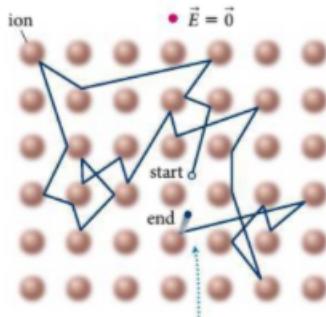
The potential difference across circuit elements connected in series is equal to the sum of the individual potential differences across each circuit element.

The resistance of any element in an electric circuit is a measure of the potential difference across that element for a given current in it.

The derived SI unit of resistance is the **ohm** ($1 \Omega = 1 \text{ V/A}$). Resistance is always positive, so in Eq. 31.10 the direction in which V and I are measured must be such that they both have the same algebraic sign. The resistance of most circuit elements is typically in the range of 10Ω to $100,000 \Omega$.

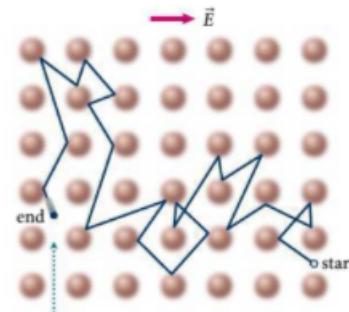
Figure 31.30 The effect of an applied electric field on the motion of a free electron through a lattice of ions.

(a) Motion in absence of an electric field



Electron's displacement is zero over long time interval.

(b) Motion with applied electric field



Electron undergoes displacement in direction opposite to electric field.

$$\vec{v}_f = \vec{v}_i + \vec{a}\Delta t = \vec{v}_i - \frac{e\vec{E}}{m_e} \Delta t, \quad (31.1)$$

where \vec{v}_i is the electron's initial velocity on that path (its velocity just after the first collision), Δt is the time interval the electron spends on that path (in other words, the time interval between collisions), e is the elementary charge (Eq. 22.3), \vec{E} is the applied electric field, and m_e is the mass of the electron. The magnitude of \vec{v}_i is roughly 10^5 m/s , as noted above, and the direction is determined by the first of the two collisions. Because of the high electron speed, the time interval Δt between collisions is extremely short—on the order of 10^{-14} s .

To calculate the average velocity of all the electrons, we take the average of Eq. 31.1 for all of the electrons:

$$(\vec{v}_t)_{\text{av}} = (\vec{v}_i)_{\text{av}} - \frac{e\vec{E}}{m_e} (\Delta t)_{\text{av}}. \quad (31.2)$$

(I have assumed here that the electric field is either constant over time or takes a time interval much longer than Δt to change significantly.)

Even though the magnitude of \vec{v}_i is quite large, its average value for all electrons is zero because the collisions produce a random distribution of the directions of the initial velocities. The resulting average velocity, called the **drift velocity** \vec{v}_d of the electrons, is thus

$$\vec{v}_d = -\frac{e\vec{E}}{m_e} \tau, \quad (31.3)$$

where $\tau = (\Delta t)_{\text{av}}$ is the average time interval between collisions. (The value of τ depends on the number density, size, and charge of the lattice ions, and on

-Single Loop Circuits

-Dont know what EMF is yet.

Let's next consider the energy transformations that occur in the circuit shown in Figure 31.34. Nonelectrostatic work done on the negative charge carriers as they travel from point a to point b through the source raises the electric potential energy of the charge carriers. In the load, this electric potential energy is converted to other forms of energy, such as thermal energy, mechanical energy, radiation energy:

$$\Delta E_{\text{other}} = W_{\text{nonelectrostatic}}(a \rightarrow b). \quad (31.16)$$

Considering just the load by itself, we also know that the amount of energy converted to other forms of energy must be equal to the work done by the electrostatic field on the charge carriers as they travel from point b to point a through the load:

$$\Delta E_{\text{other}} = W_{\text{electrostatic}}(b \rightarrow a), \quad (31.17)$$

so $W_{\text{nonelectrostatic}}(a \rightarrow b) = W_{\text{electrostatic}}(b \rightarrow a).$ (31.18)

Using Eqs. 26.7 and 25.15 this becomes

$$q\mathcal{E} = -qV_{ba} \quad (31.19)$$

or $\mathcal{E} + V_{ba} = 0.$ (31.20)

For circuits that contain many elements, Eq. 31.20 can be generalized by replacing each term by a sum. In that case the algebraic sum of the emfs and the potential differences around the loop is zero:

$$\sum \mathcal{E} + \sum V = 0 \quad (\text{steady state, around loop}). \quad (31.21)$$

$$I = \frac{\mathcal{E}}{R}.$$

Equation 31.21 is called the **loop rule**.

-Power in Electric Circuits

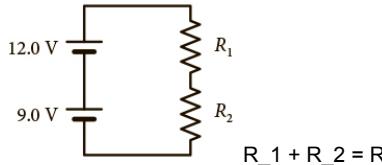
$$P = \frac{\Delta E}{\Delta t} = \frac{-qV_{ab}}{\Delta t} = -IV_{ab}$$

$$P = I^2R.$$

$$P = \frac{V_{ab}^2}{R}.$$

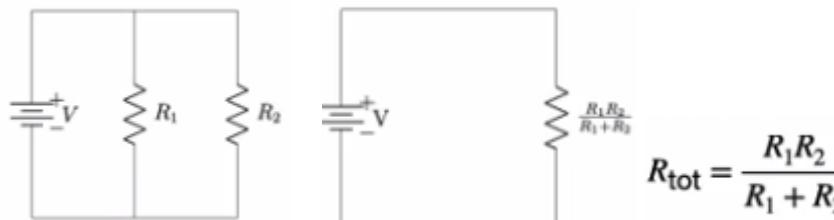
-Multiple Batteries in a Single Loop Circuit

-When they are in series like this (opposite terminals): Add them



-Multiloop Circuits: Parallel Resistors

-The next simplest. The current has 2 paths to take from one end of the battery to the other.



This is for 2 resistors

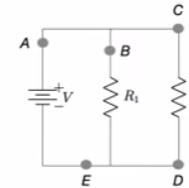
-This time, you don't add up the resistors.

It turns out that an easy way to write the equivalent resistance for resistors in parallel is

$$\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

-Potential Difference of Parallel Resistors Circuit

- Recalling what we learned about the potential at points along the straight sections of a circuit diagram, we can see that points A, B, and C all have the same potential.
- Similarly, D and E both have the same potential as the negative side of the battery, which is usually taken as zero.
- So the potential difference across each resistor is the full voltage of the battery.



-Current of Parallel Resistors Circuit

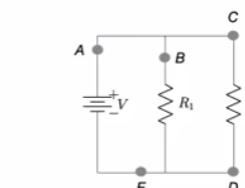
- Since there is more than one path through the circuit, there are different currents in different parts of the circuit.
- All of the current must pass through points A and E, but different currents can pass through points B and C.
- The same current that passes through C must pass through D.
- Since we know the voltage across each resistor, we can find the current through each resistor by using Ohm's law:

$$I_1 = \frac{V}{R_1} \quad \text{and} \quad I_2 = \frac{V}{R_2}$$

- The total current through points A and E will be the sum of these: $I_{tot} = I_1 + I_2$

-And with numbers

- Let's return to our 9 V battery with two resistors: $R_1 = 12 \Omega$ and $R_2 = 24 \Omega$, but this time we will connect the resistors in parallel.
- The current in the first resistor is $I_1 = \frac{V}{R_1} = \frac{9 \text{ V}}{12 \Omega} = 0.75 \text{ A}$.
- The current in the second resistor is $I_2 = \frac{V}{R_2} = \frac{9 \text{ V}}{24 \Omega} = 0.375 \text{ A}$.
- So the total current provided by the battery is 1.125 A.
- Notice that this is much more than the 0.25 A we found when these two resistors were connected in series to a 9 V battery.
- This is because the total resistance of this circuit is only $R_{tot} = \frac{R_1 R_2}{R_1 + R_2} = 8 \Omega$, whereas the total resistance in series was 36 Ω .



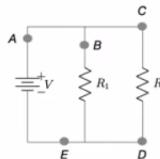
-Resistance is less because flow splits and picks less a resistant path.

-More parallel circuits, less resistance (if same ω for each).

-Power of Parallel Resistors Circuit

-Same P=IV

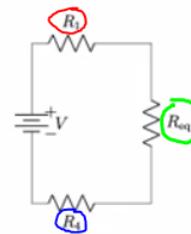
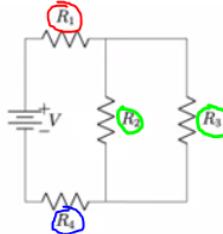
- Power is calculated the same way as before:
 $P = IV$
- Once you know the current through and voltage across any element, you can calculate the power.
- For our 9 V battery connected to two resistors in parallel with $R_1 = 12 \Omega$ and $R_2 = 24 \Omega$, we found a total current of 1.125 A, so the power provided by the battery must be
 $P = IV = (1.125 \text{ A})(9.0 \text{ V}) = 10.125 \text{ W}$
- The power dissipated in each resistor can also be calculated:
 $P_1 = I_1 V_1 = (0.75 \text{ A})(9.0 \text{ V}) = 6.75 \text{ W}$
 $P_2 = I_2 V_2 = (0.375 \text{ A})(9.0 \text{ V}) = 3.375 \text{ W}$
- As expected, these add up to the total power supplied by the battery.



-Complex Circuits

-Turn to split it up and simplify. Single and Parallel.

- Let's try this! While we could use any values, let's start by setting all four of these resistors to 12Ω . We'll use our 9 V battery again.
- What is the current through, voltage across, and power dissipated by each resistor?
- Here, the best first step is to combine R_1 and R_2 ; they are in parallel with each other, so they can be replaced with an equivalent resistance of 6Ω .
- This leaves us with three resistors in series; they have a total resistance of 30Ω .
- So the total current provided by the battery is $I = \frac{V}{R} = \frac{9 \text{ V}}{30 \Omega} = 0.30 \text{ A}$.
- The power supplied by the battery is therefore 2.7 W .
- All of the current goes through R_1 so the voltage drop across R_1 will be
 $V_1 = (0.30 \text{ A})(12 \Omega) = 3.6 \text{ V}$
- The same calculation applies to the voltage across R_4 , and both resistors dissipate 1.08 W .
- Now the tricky part.... Do you see that the voltage across the parallel resistors must be 1.8 V ?
- This means that the current through each must be 0.15 A , which makes sense because half the current should go through each (since each path has the same resistance).
- So these resistors dissipate 0.27 W each.



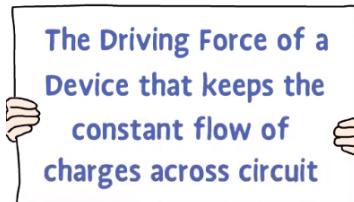
https://www.youtube.com/watch?v=kjW4H3fKi8o&feature=emb_title

-EMF (Electromotive Force) Scanner VS Voltage

(<https://circuitglobe.com/difference-between-emf-and-voltage.html#:~:text=The%20EMF%20is%20the%20measure.from%20one%20point%20to%20another.&text=The%20EMF%20is%20generated%20by.the%20electric%20and%20magnetic%20field.>)

Basis for Comparison	EMF	Voltage
Definition	The amount of energy supply by the source to each coulomb of charge.	Energy use by unit charge to move from one point to another
Formula	$E = I(R + r)$	$V = IR$
Symbol	ϵ	V
Measure	Measure between the end point of the source, when no current flows through it.	Measure between any two points.
Source	Dynamo, electrochemical cell, transformer, solar cell, photodiodes etc.	Electric and magnetic field

potential difference or voltage makes electrons to flow, while electromotive force maintains the potential difference.



E.M.F
ELECTRO MOTIVE FORCE



-Similar to driving force pushing car up the hill, giving the car potential energy to coast down.

Magnetic fields

-Magnetism

- The surprise came in 1820. Hans Christian Oersted accidentally noticed that magnetic compasses moved when they were near a wire and current was turned on through the wire.
- Investigations found that any charged particle moving through a magnetic field will feel a force.

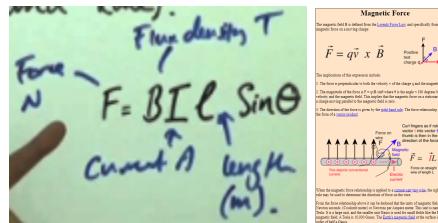
-Magnetic Force (SI: Tesla = Newton/(Ampere*Meter))

-Only when there are two poles. (Unlike electric force where point charge exerts force)

$$\vec{F} = q \vec{v} \times \vec{B}$$

- q is charge
- \vec{v} is velocity
- \vec{B} is Magnetic field

-Or other form $F = BIL$ (Lorentz force) (<http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/magfor.html>)



- The direction of the force is perpendicular to both the direction of the field and the direction of motion of the charge!

-Right Hand Rule since Cross Product

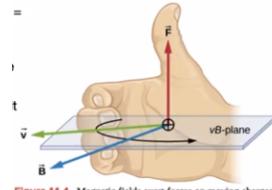
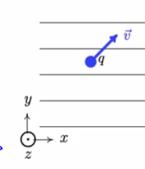


Figure 11.4 Magnetic fields exert forces on moving charges. The direction of the magnetic force on a moving charge is perpendicular to the plane formed by \vec{v} and \vec{B} and follows the right-hand rule-1 (RHR-1) as shown. The magnitude of the force is proportional to q , v , B , and the sine of the angle between \vec{v} and \vec{B} .

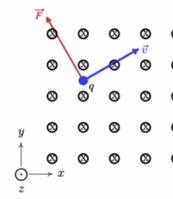
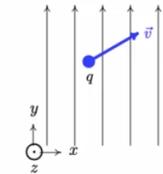
- What is the direction of the force on this positive charge moving through a region of magnetic field that is uniform and directed to the right?
- What is the direction of the force on this particle? \rightarrow
- What if the charge was negative instead of positive? \leftarrow
- If the direction of motion was the same but the field pointed to the left, what direction would the force on the particle be? \rightarrow

$$\vec{F} = q \vec{v} \times \vec{B}$$



$$\vec{F} = q \vec{v} \times \vec{B}$$

- What is the direction of the force on this positive charge moving through a region of magnetic field that is uniform and directed upward?
- What if the charge was negative instead of positive? \uparrow
- What if the field were reversed? \downarrow

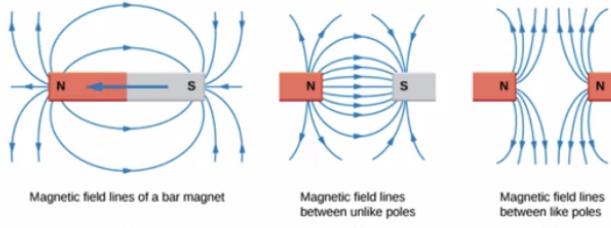


-Magnetic Field Lines

-Always a loop

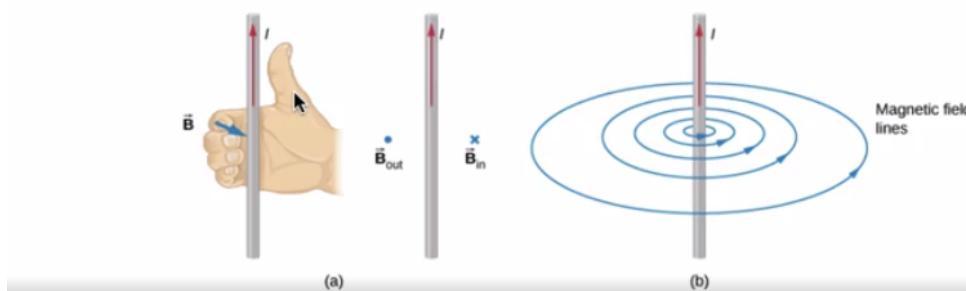
-Field lines goes out of North, goes into South, and loops to North through the magnet (seen in a).

- While the previous exercises used uniform fields, the field lines around a magnet can be more complex.
- They are similar to electric field lines except that they end up going in loops that go out of 'north' and into 'south' on the outside but circle back from south to north inside the magnet.



-Magnetic Field of a Wire with Current

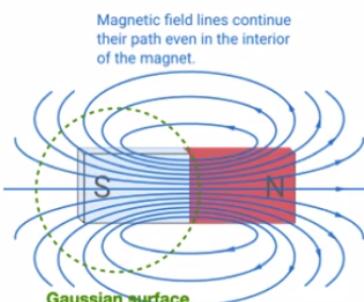
- But those magnetic fields were around permanent magnets. What about Oersted's discovery... magnetic fields around a current-carrying wire?
- By using a magnetic compass, you can clearly find that the magnetic field around the wire is circular, with the wire at the center.
- And yet another right-hand rule applies: point your right thumb in the direction of the current, and your fingers will curl around the wire in the direction of the magnetic field.



-Magnetic Flux

- In the same way that we had electric flux, we can define magnetic flux as the amount of magnetic field that cuts through a surface: $\Phi_B = \vec{B} \cdot d\vec{A}$
- You may jump to the conclusion that we might have a magnetic equivalent to Gauss's law, and we do!
- But there are no magnetic monopoles, so you can never enclose anything other than magnetic dipoles.
- The magnetic field lines always loop back and close on themselves.
- This means that the magnetic flux through a closed surface will always be zero!

$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0$$

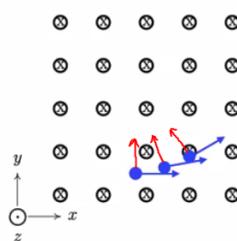


- This result — $\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0$ — makes Gauss's law for magnetism less useful than it was for electric fields, but the concept of magnetic flux will turn out to be very important in chapter 29.
- It also tells us something important about the world: It is the expression of the fact that there are no magnetic monopoles... no single sources of a magnetic 'north' or 'south'.
- This should leave you wondering... Where do magnetic fields come from?
(We'll get to that in chapter 28.) *cliff hanger*

-Charged Particle moving through Magnetic Field

-Path of travel is a circle

- Let's return to the magnetic force formula and investigate further what happens when a charged particle moves through a magnetic field.
- We start with a uniform magnetic field pointed into the page (or screen) in the $-\hat{z}$ direction.
- What happens when a positively charged particle has a velocity in the positive \hat{x} direction (to the right)?
- It will be deflected upward, but then what?
- Once it starts going a little upward, the force on it changes direction slightly... it is upward and a little to the left.
- In fact, the force on the particle is always perpendicular to the velocity of the particle. (The cross product in $q\vec{v} \times \vec{B}$ guarantees that.)



- So the path of the particle will be circular!
- We can use our formula for circular motion caused by a center-directed force:

$$F_c = \frac{mv^2}{r} \text{ with } F_c = |q\vec{v} \times \vec{B}| = |qvB| \text{ (since } \vec{v} \perp \vec{B}).$$

- If $\frac{mv^2}{r} = |qvB|$, we can solve for the radius of the circle:

$$r = \frac{mv}{|qB|} \quad (\text{equation 27.23})$$

- Interestingly, this circular motion is periodic, and we can find the time to complete one cycle (the period):

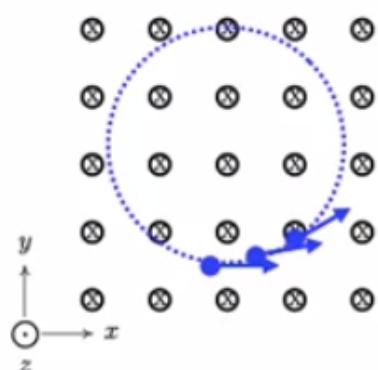
$$T = \frac{2\pi r}{v} = \frac{2\pi mv}{v|qB|} = \frac{2\pi m}{|qB|} \quad (\text{equation 27.24})$$

and we can find the frequency (number of cycles per unit of time):

$$f = \frac{1}{T} = \frac{|qB|}{2\pi m}$$

from which we can find the angular frequency (number of radians per unit of time):

$$\omega = 2\pi f = \frac{|qB|}{m} \quad (\text{equation 27.25})$$

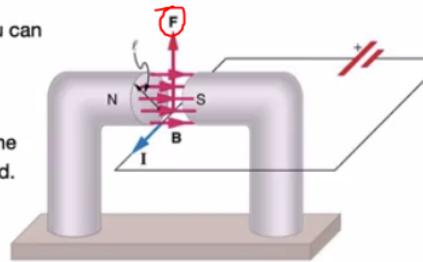


-Magnetic Forces on Current-Carrying Wires

- If a current-carrying wire is located in a region where there is a magnetic field, it is as if there are positive charge carriers moving through the field in the direction of the current.
- This means the wire will feel a magnetic force perpendicular to the direction of the current.
- I still encourage you to think in terms of $q \vec{v} \times \vec{B}$, but you can rewrite the force in terms of the current as

$$\vec{F} = I\vec{l} \times \vec{B}$$

where \vec{l} is in the direction of positive current flow, I is the current, and ℓ is the length of the wire exposed to the field.



-27.1 Magnetism

-Magnetic materials are attracted by magnets.

-The magnets induce a magnetic polarization below.

Figure 27.4 Both the north and south poles of a magnet attract an unmagnetized iron object.

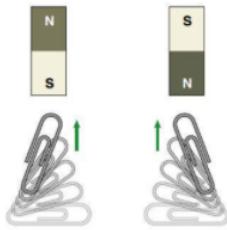
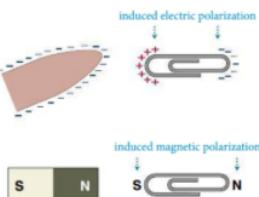


Figure 27.5 Comparing induced electric and magnetic polarization. Magnetic polarization can be induced only in an object made from magnetic material.



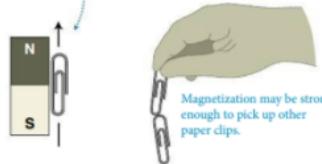
Shungite Magnet. Human biofield has electromagnetic nature. Electric and **magnetic** fields of home appliances suppress biofield and influence pathogenic to organism. **Shungite** plates creates a field round itself, neutralised geopathogenic radiations and restoring power of the person.



-Real. If you orientate right, electrons deflected!

Figure 27.6 If you stroke a paper clip several times in the same direction with a magnet, the clip retains some magnetic polarization.

Paper clip can be magnetized by stroking on magnet several times in one direction.



-Some retain the magnetic polarization..

-Elementary Magnets, when you can't split (like 27.7 pic) it down anymore (represented by the pokeball looking circles)

Figure 27.7 When a magnet is cut in two, each piece retains both an N and S pole.

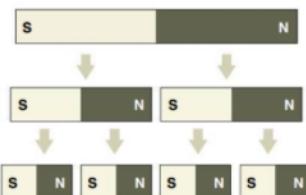
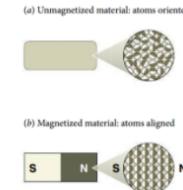


Figure 27.8 The concept of elementary magnets explains why cutting a magnet in two reveals new N and S poles.

Figure 27.9 (a) Unmagnetized and (b) magnetized pieces of magnetic material.



-27.2 Magnetic Fields

Figure 27.11 Effect of a bar magnet on a compass needle.

(a) Forces exerted by magnet on poles of compass needle cause a torque.



(b) North pole of compass needle points toward south pole of magnet



Figure 27.12 A magnetic field line can be traced by moving a compass small distances in the direction in which its needle points.

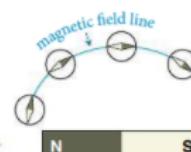
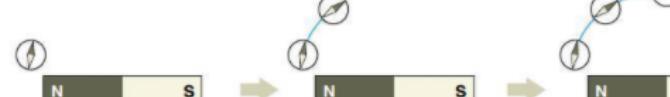
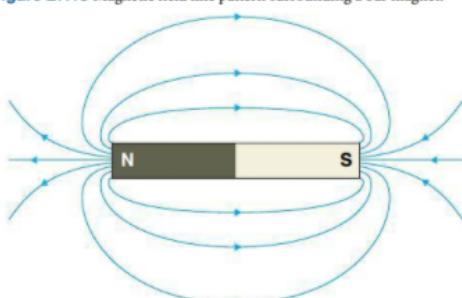
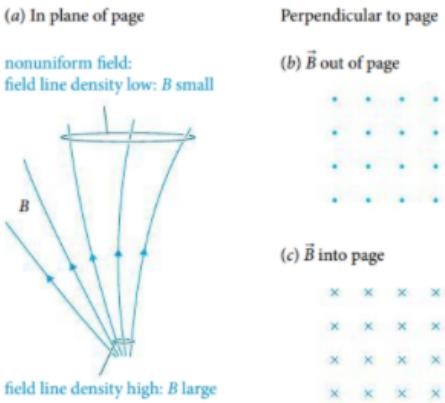


Figure 27.13 Magnetic field line pattern surrounding a bar magnet.



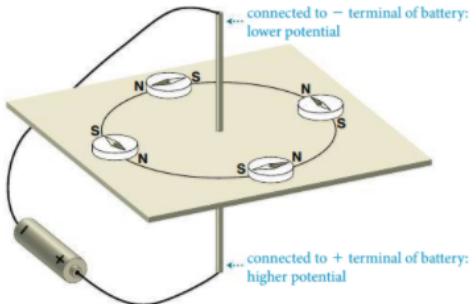
-Field Line Density still applies for the magnitude of the field.

Figure 27.15 Conventions for representing a magnetic field.



-27.3 Charge Flow and Magnetism

Figure 27.16 A flow of charge carriers through a conducting rod causes a circular alignment of compass needles.



-Like current through a wire

Figure 27.17 A flow of positive charge carriers in one direction is equivalent to a flow of negative charge carriers in the other direction.

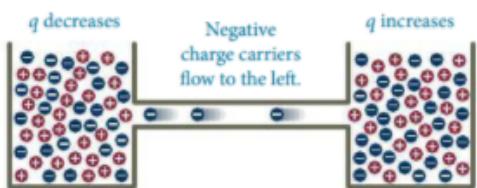
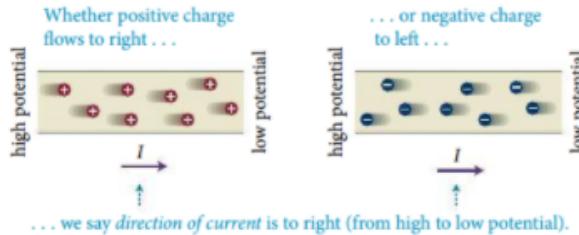


Figure 27.18 By definition, current has the direction in which positive charge carriers would flow, even if it is actually carried by negative charge carriers moving in the opposite direction.



-Wire and Bar Magnet

Figure 27.20 The magnetic force exerted by a bar magnet on a current-carrying wire depends on the magnet's orientation.

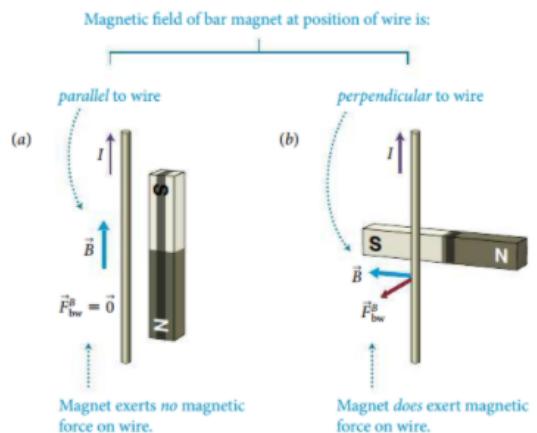


Figure S27.11

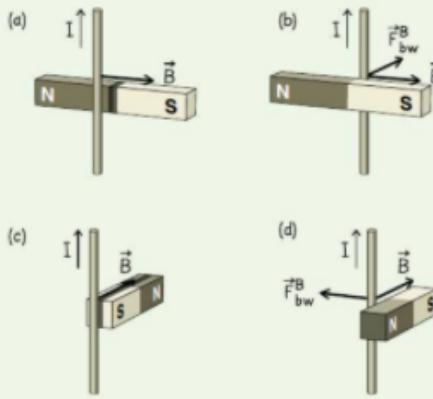
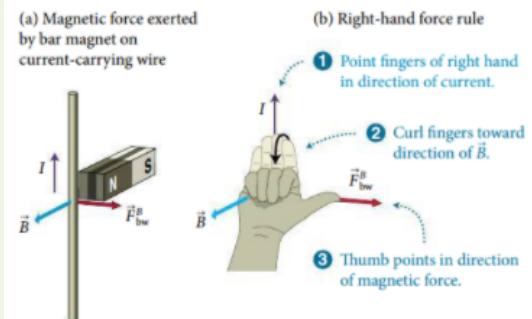


Figure 27.22 When a magnet exerts a force on a current-carrying wire, the right-hand force rule relates the direction of the force to that of the current and the magnet's magnetic field.

(a) Magnetic force exerted by bar magnet on current-carrying wire



-Right Hand Rules Bruv

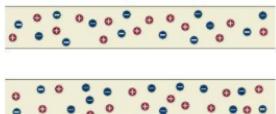
Table 27.1 Right-hand rules in magnetism

Right-hand rule	thumb points along	fingers curl
current rule	current	along B -field
force rule	magnetic force	from current to B -field

-Magnetism and Relativity

Figure 27.24 Schematic view of the interaction between two current-carrying rods.

(a) Metal rods don't interact when they carry no current



(b) When carrying a current, they do interact

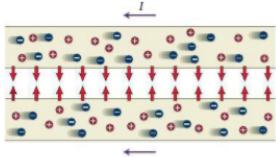
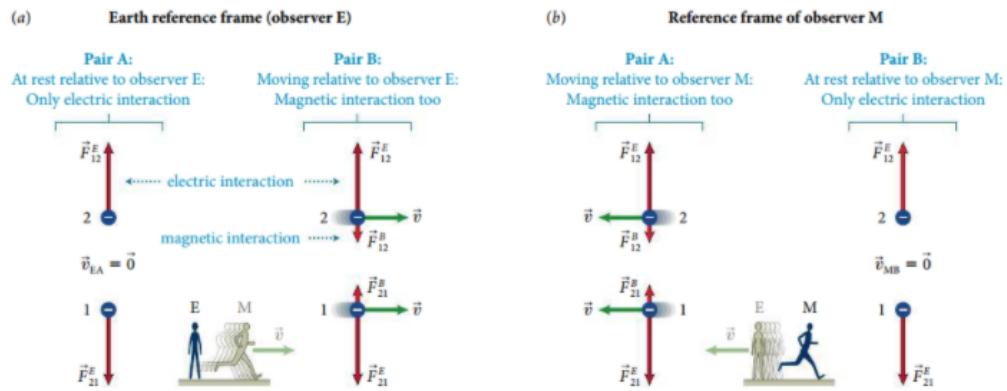


Figure 27.25 A relativistic view of the interaction between two charged particles.



-When at rest, only electrical forces. It's when they move that there are magnetic forces.

-Since motion is relative:

The observed interaction between charge carriers depends on their motion relative to the observer: The interaction can be purely electric, purely magnetic, or a combination of the two.

-Because you would think both are happening constantly.

-Ex:

If two charged particles M and S are at rest relative to each other, there is no magnetic force between them. Suppose instead that particle M is moving relative to particle S while S is at rest in the Earth reference frame.

Is a magnetic force exerted on S?

- yes
 no

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Correct

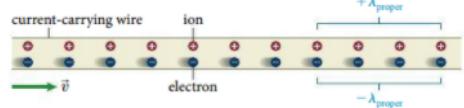
Not in the Earth reference frame. In the Earth reference frame, particle M is moving, so it produces a magnetic field. But because particle S is at rest, there is no magnetic force exerted on S. The same is true in the reference frame moving along with particle M, because in this reference frame particle M is at rest and so does not produce a magnetic field. There are reference frames in which both particles are moving, and in these a magnetic force is exerted on S.

-Special Relativity and Charge Density of Current in Wire

Figure 27.27 Observers E (at rest in the Earth reference frame) and M (moving in the Earth reference frame) observe a current-carrying wire.

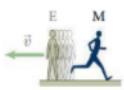
(a) Earth reference frame (observer E)

To observer E, ions and electrons in wire have same charge density, so wire is electrically neutral and has no electric field: $\vec{E} = \vec{0}$.



(b) Reference frame of observer M

To observer M, charge density of ions in wire is greater than λ_{proper} , and that of electrons is smaller than λ_{proper} , so wire is positively charged, and $\vec{E} \neq \vec{0}$.

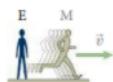


To observer M, charged wire exerts electric force
on particle.

$$\vec{F}_{wp}^E$$

(c) Earth reference frame

To observer E, wire cannot exert electric force on particle because it is electrically neutral.



27.5 Current and Magnetism

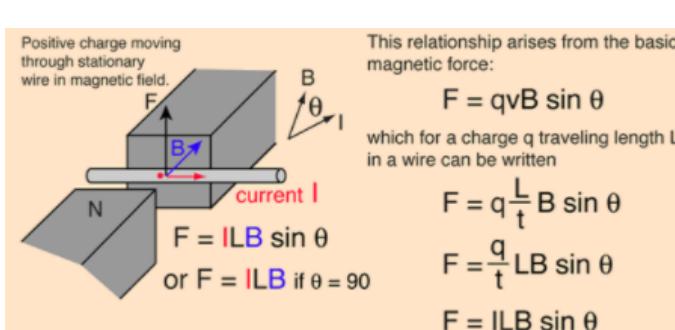
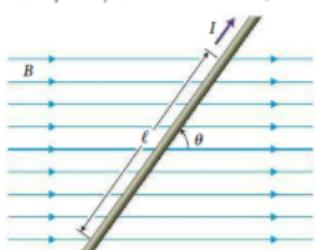
-Magnitude of Uniform Magnetic Field with Straight Wire Perpendicular

$$B \equiv \frac{F_w^B, \text{ max}}{|I| \ell} \quad (\text{straight wire perpendicular to uniform magnetic field}).$$

-Magnitude of Uniform Magnetic Field with Straight Wire

$$F_w^B = |I| \ell B \sin \theta \quad (0 < \theta < 180^\circ). \quad \text{OR} \quad \vec{F}_w^B = I \vec{\ell} \times \vec{B} \quad (\text{straight wire in uniform magnetic field}).$$

Figure 27.31 A current-carrying wire in an external magnetic field (that is, a magnetic field created by an object other than the wire).



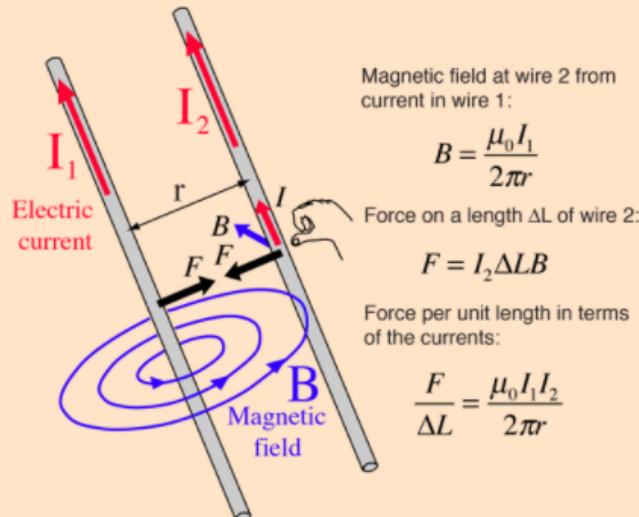
-Some Magnetic Fields Magnitudes

Table 27.2 Magnetic fields

Object	B (T)
Earth's surface	5×10^{-5}
small bar magnet	0.01
neodymium magnet	0.2
laboratory magnet	10
neutron star surface	10^8

-Magnetic Force Between Wires (Permeability / Magnetic Constant here)

Magnetic Force Between Wires



The [magnetic field](#) of an infinitely long straight wire can be obtained by applying [Ampere's law](#). The expression for the magnetic field is

$$B = \frac{\mu_0 I}{2\pi r} \quad \text{Show}$$

Once the magnetic field has been calculated, the [magnetic force expression](#) can be used to calculate the force. The direction is obtained from the [right hand rule](#). Note that two wires carrying current in the same direction attract each other, and they repel if the currents are opposite in direction. The calculation below applies only to long straight wires, but is at least useful for estimating forces in the ordinary circumstances of short wires. Once you have calculated the force on wire 2, of course the force on wire 1 must be exactly the same magnitude and in the opposite direction according to [Newton's third law](#).

-Magnetic Field of One Wire

The strength of the magnetic field depends on the current I in the wire and r , the distance from the wire.



$$B = \frac{\mu_0 I}{2\pi r} \quad , \quad \mu_0 = 4\pi \times 10^{-7} \left(\frac{Tms}{C} \right)$$

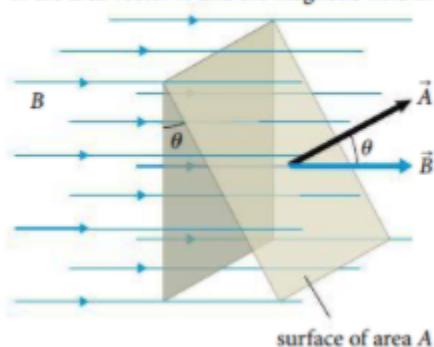
-27.6 Magnetic Flux

-SI: Weber ($Wb = T \cdot m^2$)

-Uniform Magnetic Field

$$\Phi_B \equiv \vec{B} \cdot \vec{A} = \vec{B} \cdot \vec{A} \quad (\text{uniform magnetic field}),$$

Figure 27.35 The magnetic flux through a surface of area A is given by the scalar product of the area vector \vec{A} and the magnetic field \vec{B} .



-Magnetic Flux (which = 0 because field always loop)

$$\Phi_B \equiv \int \vec{B} \cdot d\vec{A},$$

-27.7 Moving Particles in Electric and Magnetic Field

-Magnetic Force

$$I \equiv \frac{Q}{\Delta t} = \frac{nA(v\Delta t)q}{\Delta t} = nAqv.$$

What is the direction of the magnetic field at any point that lies midway between the wires and in the plane defined by them?

• The magnitude of the resultant magnetic field is zero.

Permeability constant

$$\mu_0$$

$$4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$$

Figure 27.38 If the charge carriers in a straight current-carrying wire move at speed v , they advance a distance $\ell = v\Delta t$ in a time interval Δt . All the charge carriers in the shaded volume pass through the cross-sectional area A in that time interval.

$$F_w^B = |\underline{nA\ell}qv|B \sin \theta = nA\ell|q|vB \sin \theta,$$

$$\vec{F}_p^B = |q|vB \sin \theta$$

$$\vec{F}_p^B = q\vec{v} \times \vec{B}$$

-Electromagnetic Force

$$\vec{F}_{pEB} = q\vec{E} + q\vec{v} \times \vec{B} = q(\vec{E} + \vec{v} \times \vec{B})$$

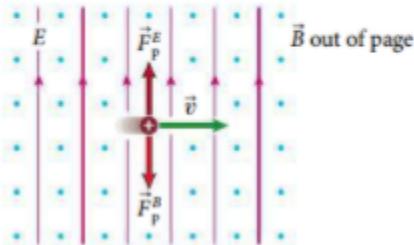
-When Electric and Magnetic Force Cancellation (Do superposition for both E and B)

Figure 27.41 Charged particles whose speed

satisfies Eq. 27.26 move in a straight line

through magnetic and electric fields that are oriented perpendicular to each other.

$$v = \frac{E}{B} \quad (\text{electric and magnetic force cancel}).$$



-27.8 Magnetism and Electricity Unified

-urumum

-Magnetic Field from an Arc of Current

The magnetic field dB from a representative piece of the wire is:

$$dB = \frac{\mu_0 I dl \times \hat{r}}{4\pi r^2}$$

$$r = R, \text{ and } dl \times \hat{r} = dl \hat{k}$$

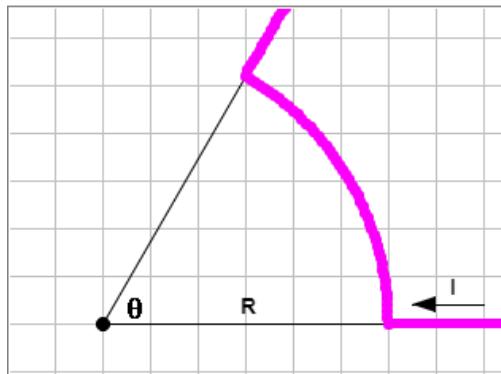
Our integral for the net field is easy:

$$B = \frac{\mu_0 I}{4\pi R^2} \int dl \hat{k}$$

$$B = \frac{\mu_0 I s}{4\pi R^2} \hat{k}$$

where s is the arc length, equal to Rθ.

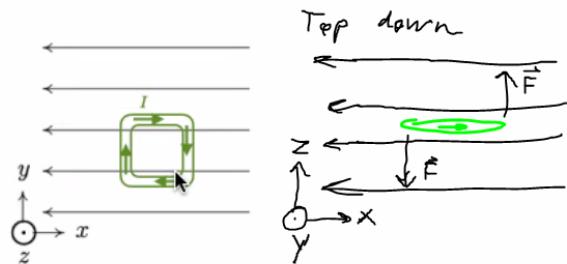
$$\text{Therefore } B = \frac{\mu_0 I \theta}{4\pi R} \hat{k}$$



Lab 7

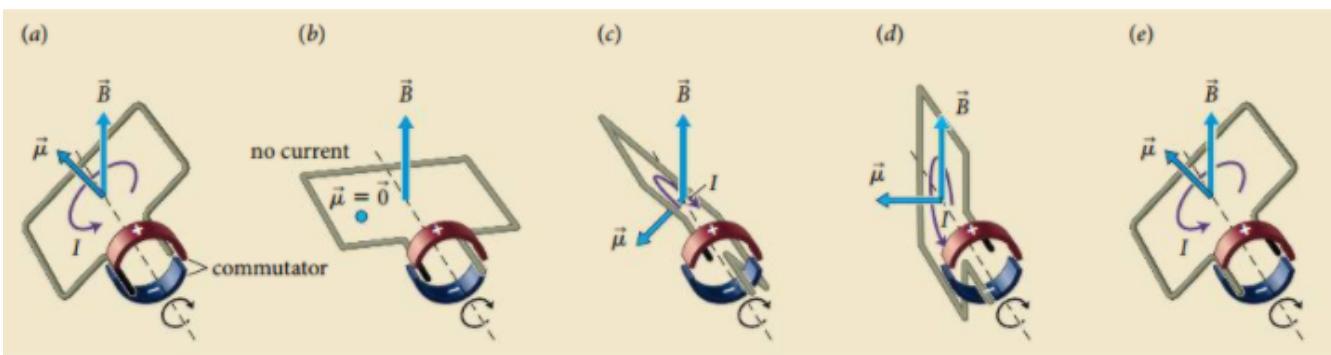
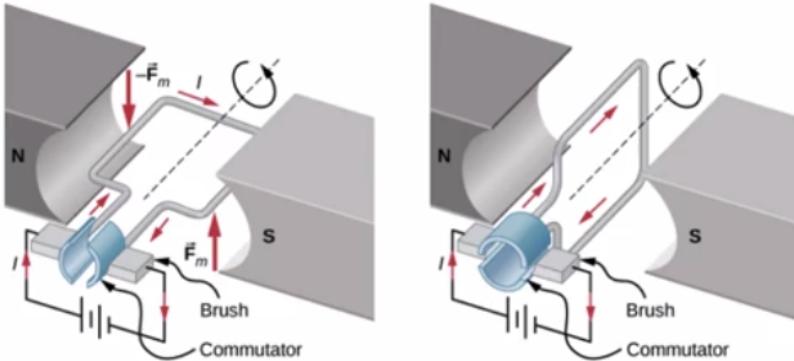
-Torque on Current Loop

You can either think in terms of $\vec{F}_B = q\vec{v} \times \vec{B}$ (right hand rule for cross product) or in terms of the magnetic moment produced by the current loop (right hand rule for current).



-DC / Electric Motor works by cutting the current depending on the rotation of the loop in order to only allow torque to keep the loop spinning. Or, reverse the current each half of the loop.

- If the current loop feels a torque, can we use that torque to make it spin?
- Yes... but the problem is keeping it spinning.
- When the magnetic moment of the loop is aligned with the external field, the torque is zero.
- Inertia will cause it to continue to spin past the equilibrium point, but then the torque reverses, slowing the loop down and causing it to rotate back the other way.
- So how can we fix that to make it keep spinning?
- There are two choices...
 - Flip the direction of the field
 - Reverse the current
- This has to be repeated every 180° that the loop rotates.



Chapter 28

	Electric field	Magnetic field
Gauss's law	$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$ <small>(Charges are sources of electric field.)</small>	<small>(Magnetic field lines loop back onto themselves.)</small> $\oint \vec{B} \cdot d\vec{A} = 0$
Ampere's law	$\oint \vec{E} \cdot d\vec{l} = 0$ <small>(Potential difference of a closed loop path is zero.)</small>	$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$ <small>(Currents are sources of magnetic field.)</small>

-There are really three main ideas to grasp here.

- The source of all magnetic fields appears to be moving electric charge.
- Since currents in wires are moving electric charges, there are magnetic fields around wires with currents, and we can use Ampere's law to derive the formulas for some of those fields.
- Current loops look, magnetically, like magnetic dipoles (or magnets) and can be used to make electromagnets and electric motors.

Ampere's Law for Magnetic Field

The [magnetic field](#) in space around an [electric current](#) is proportional to the electric current which serves as its source, just as the [electric field](#) in space is proportional to the [charge](#) which serves as its source. Ampere's Law states that for any closed loop path, the sum of the length elements times the magnetic field in the direction of the length element is equal to the [permeability](#) times the electric current enclosed in the loop.

$$\sum B_{||} \Delta l = \mu_0 I$$

In the electric case, the relation of field to source is quantified in [Gauss's Law](#) which is a very powerful tool for calculating electric fields.

-reduces to that because symmetry of field

-28.1 Source of Electric Field

-Summary Picture of seen before. Magnetic interaction are moving charges

Figure 28.1 Summary of magnetic interactions. Notice that stationary charged particles do not engage in magnetic interactions.

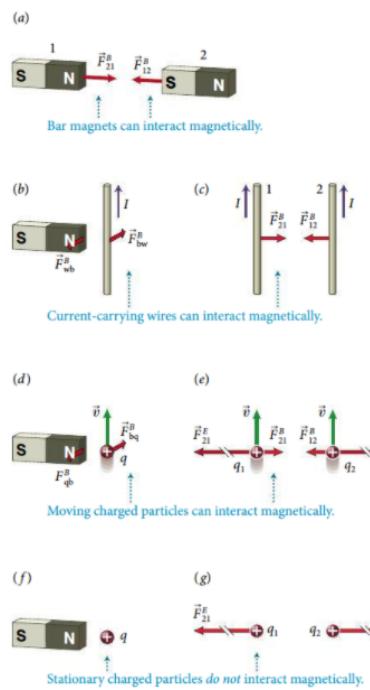
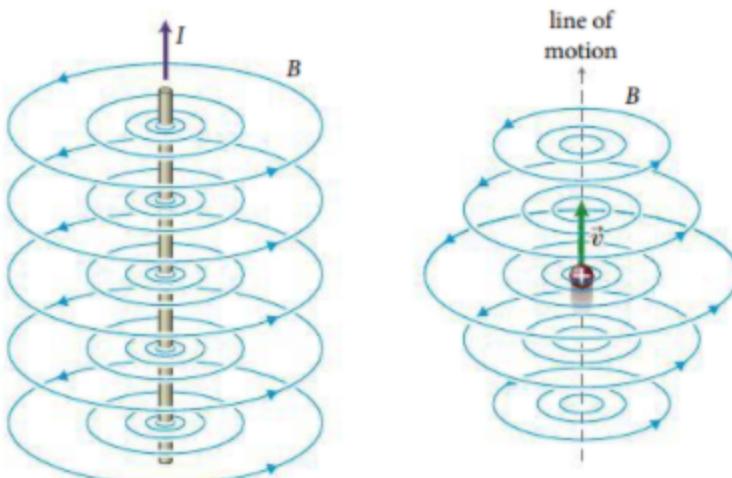


Figure 28.2 Comparing the magnetic fields of a current-carrying wire and a moving charged particle.

(a) Magnetic field of a wire carrying a constant current

(b) Magnetic field of a moving charged particle



-28.2 Current Loops and Spin Magnetism

-This section, refer to the Lab 7 with the Small Magnet spinning between the 2 Big Magnets.

-B at various positions around loop

Figure 28.5 Mapping the magnetic field of a current loop. The magnetic field contributions from (a) segment 1 and (b) segment 2 at A (c) add up to a vertical field. Magnetic fields at (d) point C at the center of the ring, (e) point D below the ring, and (f) point G to the right of the ring. Note that in all cases the magnetic field of each segment is perpendicular to the line connecting that segment to the point at which we are determining the field.

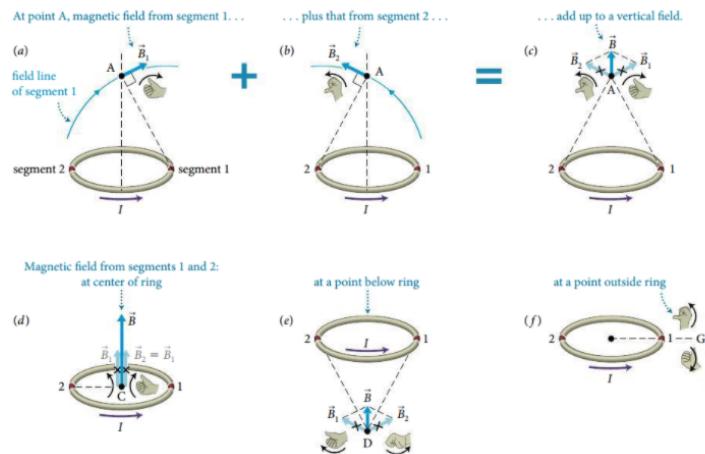
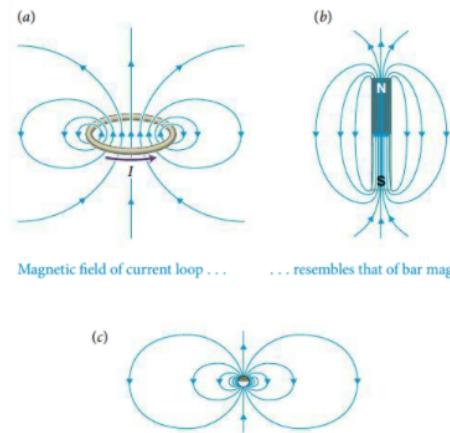


Figure 28.6 The magnetic field of a current loop (*a*) resembles that of a dipole (*b*, *c*).



A spinning charged particle has a magnetic field identical to that of an infinitesimally small magnetic dipole.

-28.3 Magnetic Dipole Moment and Torque

-This section, refer to the Lab 7 with the Small Magnet spinning between the 2 Big Magnets.

-Magnetic Dipole Moment (vector μ)

-Direction is parallel to the magnetic field

Figure 28.8 The magnetic dipole moment points in the direction of the magnetic field through the center of the dipole.

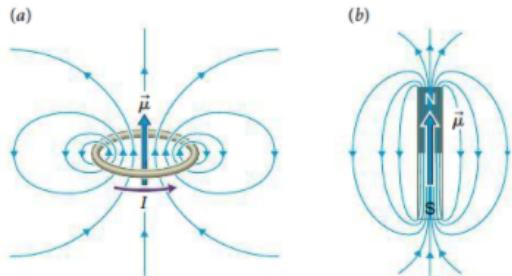


Figure 28.10 Magnetic forces exerted on a square current loop that is oriented so that its magnetic dipole moment is perpendicular to an external magnetic field.

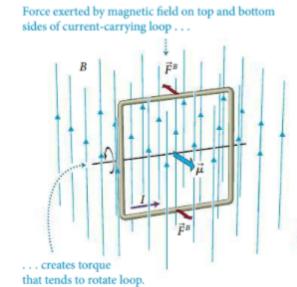
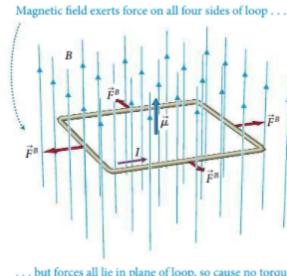


Figure 28.11 Magnetic forces exerted on a square current loop that is oriented so that its magnetic dipole moment is parallel to an external magnetic field.



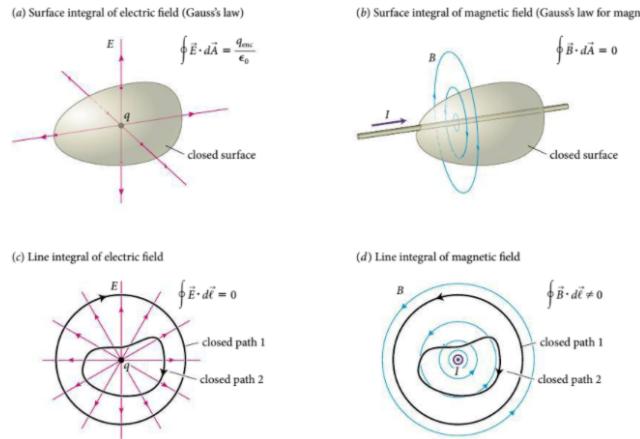
A current loop placed in a magnetic field tends to rotate such that the magnetic dipole moment of the loop becomes aligned with the magnetic field.

-Wants to get to Figure 26.11, equilibrium

-28.4 Amperian Paths

-Gauss's Law comparison and the line integral

Figure 28.15 Surface and line integrals of electric and magnetic fields.



-From the lines integral:

The value of the line integral of the magnetic field along a closed path encircling a current-carrying wire is independent of the shape of the path.

The line integral of the magnetic field along a closed path that does not encircle any current-carrying wire is zero.

Figure 28.16 (a) Two closed circular paths concentric with a wire that carries a current directed out of the page. (b) A noncircular path encircling the current-carrying wire. The two arcs each represent one-eighth of a circle.

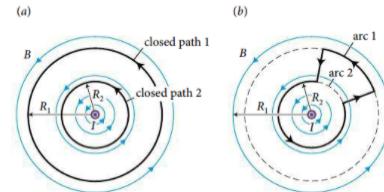


Figure 28.17 (a) A noncircular closed path encircling a current-carrying wire. (b) We can approximate the path by using small arcs and radial segments.

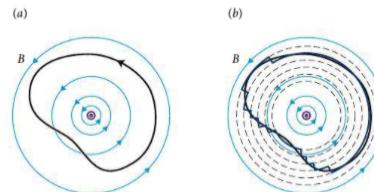
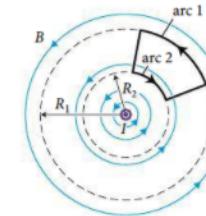


Figure 28.18 A noncircular closed path not encircling a current-carrying wire.



-The two statements above leads us to Ampere's Law:

The line integral of the magnetic field along a closed path is proportional to the current encircled by the path.

-The line path is called an Amperian path.

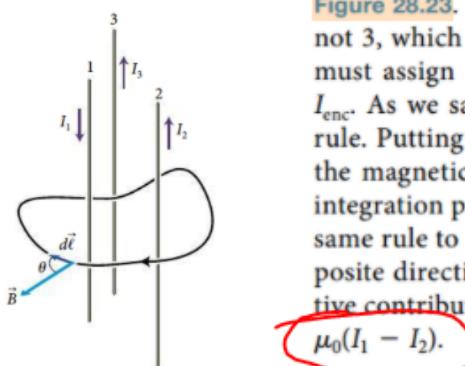
-Even when you tilt the Amperian path (no longer perpendicular to wire), it stills doesn't change outcome if the wire is enclosed still.

-If the magnetic field is in the same direction (like CW and the Amperian path is also CW), then it contributes positively to the line integral.

-28.5 Ampere's Law

-Ex:

Figure 28.23 A closed path encircling two of three straight current-carrying wires.



Let us illustrate how Eq. 28.1 is used by applying it to the closed path in

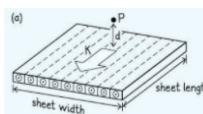
Figure 28.23. From the figure we see that the path encircles wires 1 and 2 but not 3, which lies behind the path. To calculate the right side of Eq. 28.1, we must assign an algebraic sign to the contributions of currents I_1 and I_2 to I_{enc} . As we saw in Section 28.4, we can do so using the right-hand current rule. Putting the thumb of our right hand in the direction of I_1 , we see that the magnetic field of I_1 curls in the same direction as the direction of the integration path (the Amperian path) indicated in the diagram. Applying the same rule to I_2 tells us that the magnetic field of this current curls in the opposite direction. Thus I_1 yields a positive contribution and I_2 yields a negative contribution to the line integral. The right side of Eq. 28.1 thus becomes

$$\mu_0(I_1 - I_2).$$

-Deriving Magnetic Field Magnitude of Current Wire with Ampere's law



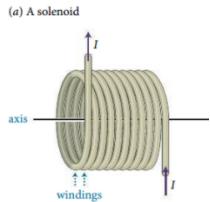
-Deriving Magnetic Field of a large current-carrying sheet with Ampere's Law



$$B = \frac{I}{2} \mu_0 K$$

-28.6 Solenoids and Toroids

-Solenoids



$$B\ell = \mu_0 n\ell I$$

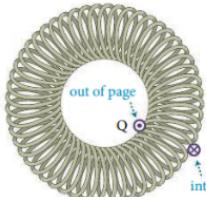
$$B = \mu_0 nI \quad (\text{infinitely long solenoid}).$$

-n = windings per length & nl = windings encircled by Amp path.

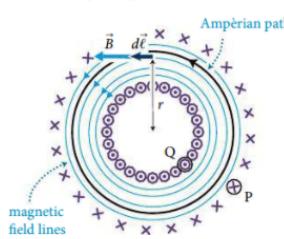
-Toroid

Figure 28.30 A toroid is a solenoid bent into a ring.

(a) Toroid



(b) Cross section showing magnetic field and a choice of Ampérian path



$$B = \frac{\mu_0 NI}{2R}$$

B = magnetic field intensity

μ_0 = permeability of free space

N = number of turns

I = current intensity

R = radius

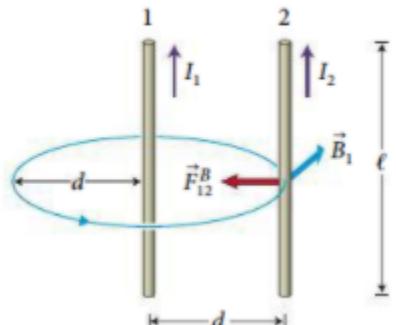
-N also = amount of windings

-28.7 Magnetic fields due to currents

-Like Gauss's Law, Ampere's Law is good for symmetrical circuits. So look in this chapter for deriving magnetic field formulas with calculus

-Magnetic Field of Parallel Wires

Figure 28.34 Calculating the magnetic force exerted by one current-carrying wire on another.



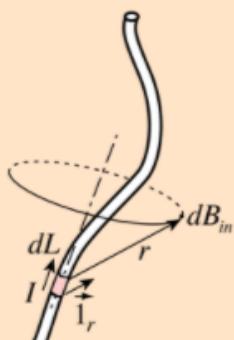
$$B_1 = \frac{\mu_0 I_1}{2\pi d} \quad F_{12}^B = \frac{\mu_0 \ell I_1 I_2}{2\pi d} \quad \frac{\vec{F}}{\ell} = \frac{\mu_0 i I}{2\pi r} \hat{r}$$

-Biot-Savart Law & Field on Axis of Current Loop Application

<http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/Biosav.html>

Biot-Savart Law

The Biot-Savart Law relates [magnetic fields](#) to the [currents](#) which are their sources. In a similar manner, [Coulomb's law](#) relates [electric fields](#) to the point charges which are their sources. Finding the magnetic field resulting from a current distribution involves the [vector product](#), and is inherently a calculus problem when the distance from the current to the field point is continuously changing.



Magnetic field of a current element

$$d\vec{B} = \frac{\mu_0 I d\vec{L} \times \hat{1}_r}{4\pi r^2}$$

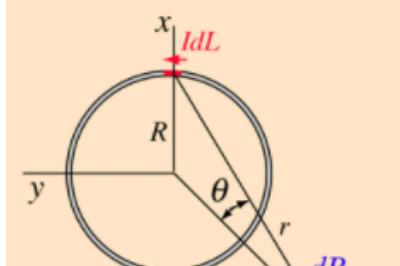
where

$d\vec{L}$ = infinitesimal length of conductor carrying electric current I

$\hat{1}_r$ = unit vector to specify the direction of the vector distance r from the current to the field point.

Field on Axis of Current Loop

The application of the [Biot-Savart law](#) on the centerline of a [current loop](#) involves integrating the z-component.



The field element labeled dB_x rotates around as you progress around the loop, and by symmetry gives a net zero field for the loop. The field at this point is in the z-direction, along the centerline of the loop.

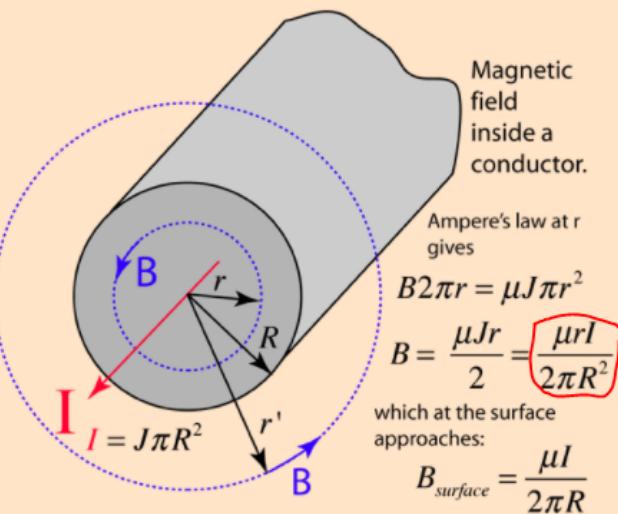
$$dB_z = \frac{\mu_0 I dL}{4\pi} \frac{R}{(z^2 + R^2)^{3/2}}$$

The symmetry is such that all the terms in this element are constant except the distance element dL, which when integrated just gives the circumference of the circle. The magnetic field is then

$$B_z = \frac{\mu_0}{4\pi} \frac{2\pi R^2 I}{(z^2 + R^2)^{3/2}}$$

Magnetic Field Inside a Conductor

The [magnetic field](#) inside a conductor with uniform current density $J = I/\pi R^2$ can be found with [Ampere's Law](#).



Outside the surface,

$$B 2\pi r' = \mu_0 I \quad \text{or} \quad B = \frac{\mu_0 I}{2\pi r'}$$

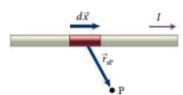
Inside the conductor the magnetic field B increases linearly with r . Outside the conductor the magnetic field becomes that of a [straight conductor](#) and decreases with radius. Note that the expressions for inside and outside would approach the same value at the surface if the [magnetic permeability](#) were the same.

-28.8 Magnetic Field of A Moving Charge Particle

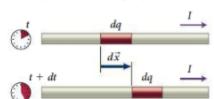
-Charged Particle move all the time in wires. If we single it out, we can get formulas.

Figure 28.38 We use the Biot-Savart law to obtain an expression for the magnetic field caused by charged particles moving at constant velocity.

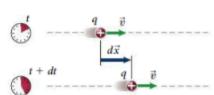
(a) Small segment of current-carrying wire causes magnetic field at point P



(b) Displacement of charge dq in time interval dt



(c) Displacement of charged particle in time interval dt



$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}_{pp}}{r_{pp}^2} \quad (\text{single particle}), \quad (28.21)$$

where r_{pp} is the distance between the particle and P, and \hat{r}_{pp} is the unit vector pointing from the particle to P.

-Now, add the electric interaction to the magnetic field interaction: Force by q_1 on q_2 .

$$\vec{F}_{EB} = \frac{k q_1 q_2}{r_{12}^2} \left(\hat{r}_{12} + \frac{\vec{v}_2 \times (\vec{v}_1 \times \hat{r}_{12})}{c^2} \right) \quad c = 3.0 * 10^8 \text{ m/s (Speed of Light)}$$

-This violates Newton's Third Law

Magnetic Induction

-A rod moving in a magnetic field. It becomes polarized.

Handwritten derivation of the magnetic field formula for a moving charge:

$$\vec{B} = ?$$

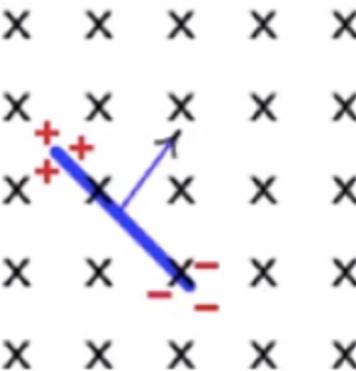
$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q \vec{v} \times \vec{r}}{r^3}$$

$$|\vec{B}| = \frac{\mu_0}{4\pi} \frac{q v \sin \theta}{r^2}$$

Definitions:

- μ_0 : permeability of free-space
- q : coulombs
- v : m/s
- r : m
- B : Teslas

- We have seen that a charge moving through a magnetic field experiences a force.
- What happens when a conducting material moves through a magnetic field?
- Conducting materials contain positive and negative charges, even if there are no currents.
- What direction will the positive charges be pushed?
- What direction will the negative charges be pushed?
- Question:** So what happens?
- Answer:** A potential difference between the two ends of the rod develops and remains there while the conductor moves through the magnetic field.
- It's as if the moving wire has an electric field along its length.



-Hall Effect

-With a wide conductor, is the + charge moving or the - charge moving?

-Experiments show that most of the time, it's the - charged particles that move.

- This leads to another interesting situation. What if you have a wide conductor with current flowing through it sitting in a magnetic field?
- If current is flowing left to right, that could be from positive particles moving right or from negative particles moving to the left (or both). Which is it?

What will happen in each case?

Use the right hand rule to figure it out.

- For most conductors, the bottom edge turns out to have a higher potential than the top edge.
- This is evidence that the majority of the current is caused by negative charges moving left... so it is the negative charge that is moving to the left.
- This is one of the few experimental results where the results depend on which charge carrier is the one that moves.
- This is called the Hall effect and give us evidence that it is the electrons which move through conductors.

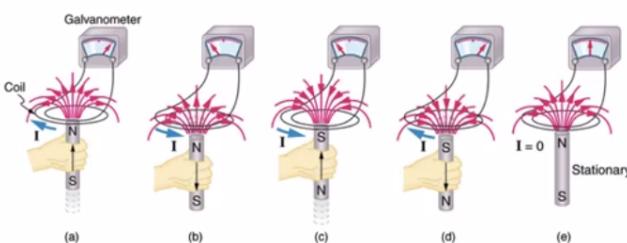


-Faraday's Law

- We can extend this idea to a loop of wire moving through a non-uniform magnetic field... and it creates a current in the wire!
- One way of quantifying this is to return to the idea of magnetic flux:
- If the magnetic flux through the loop changes, a current is induced in the loop.
- The time rate of change of the magnetic flux is a voltage, which is sometimes called an "emf"

$$\mathcal{E}_{\text{ind}} = - \frac{d\Phi_B}{dt}$$

- Challenge: Show that the units work out to volts!



-Lenz's Law

- What is the significance of the negative sign?

$$\mathcal{E}_{\text{ind}} = (-) \frac{d\Phi_B}{dt}$$

- The induced current opposes the change in the flux.
- This is known as Lenz's law. (The full idea is contained in Faraday's law, but Lenz's law provides a way to think about it.)

There are many ways the flux can change:

- If the field isn't uniform, moving a loop around will change the flux.
- Changing the size of the loop will change the flux.
- Changing the strength and/or direction of the field will change the flux.
- Changing the orientation of the loop will change the flux (for example, by spinning it).

-Faraday/Lenz Example



What is the direction of the current induced in the coil in Figure 1 if the current in the wire is...

- (a) increasing
- (b) steady
- (c) decreasing

-Wire is creating magnetic field at Coil going into the screen

-a) Wire is creating more "into the screen" magnetic field. Coil will respond with CCW current to have a magnetic field going "out of the screen"

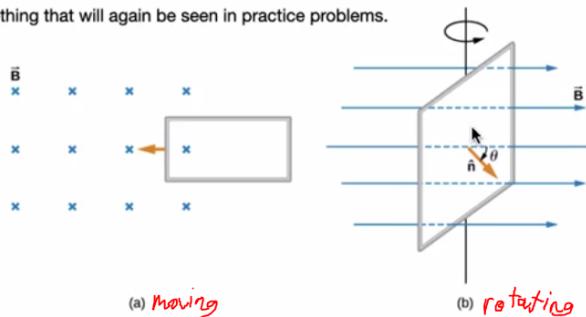
-b) No change, nothing happens

-c) Wire is reducing "into the screen" magnetic field. Coil will have CW current to create magnetic field going "into the screen".

-Changing Magnetic Flux

If you have a magnetic field, there are many ways to change the flux through a conducting loop:

- You'll see examples like (a) in the practice problems and text examples.
- If the loop in (b) rotates at a steady rate, how can we quantify the flux?
- It is sinusoidal, something that will again be seen in practice problems.

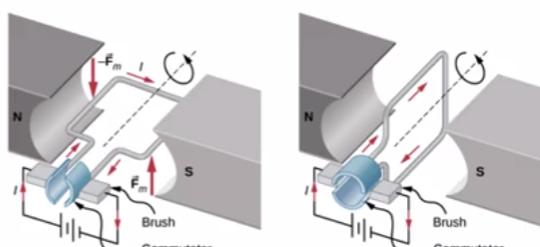
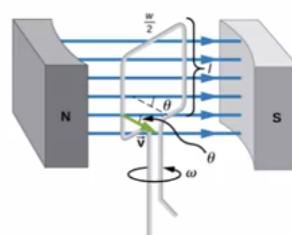


-Generator

-By rotating a loop, current appears.

A generator

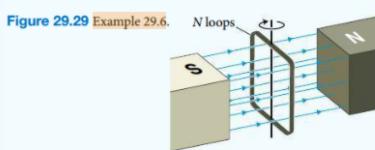
- This leads to a design for a generator.
- But this is the same design as the electric motor!
- This is, indeed, how almost all of the electricity in the world is generated. All you need is something to make the loop spin:
 - Waterwheel or turbine in a dam
 - Wind turbine
 - Steam turbine
where the steam comes from burning fossil fuels or other fuels or the heat of nuclear reactions or geothermal heat or ...



-Generator Peak EMF

Example 29.6 Generator

In an electric generator a solenoid that contains N windings each of area A is rotated at constant rotational speed ω in a uniform magnetic field of magnitude B (Figure 29.29). What is the emf induced in the solenoid?

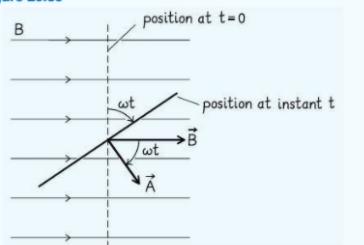


① GETTING STARTED As the solenoid rotates, the magnetic flux through it changes, and this changing flux causes an emf in the solenoid.

② DEVISE PLAN Equation 29.8 tells me that the emf induced in the solenoid equals the time rate of change of the magnetic flux through it. Therefore I must first determine how the magnetic flux varies as a function of time and then differentiate whatever expression I get to obtain the emf. I can determine the magnetic

flux through the solenoid by multiplying the magnetic flux through a single winding, $\Phi_B = \vec{B} \cdot \vec{A}$ (Eq. 27.9), by the number of windings N . To determine the magnetic flux through a single winding, I sketch a top view of a winding in the magnetic field, indicating the directions of the magnetic field \vec{B} and the area vector \vec{A} (Figure 29.30). I let the plane of the winding be perpendicular to the direction of the magnetic field at $t = 0$.

Figure 29.30



3 EXECUTE PLAN As the solenoid rotates, the scalar product $\vec{B} \cdot \vec{A}$ changes with time. At instant t shown in my sketch, the angle between \vec{A} and \vec{B} is ωt , and so the magnetic flux through a single winding is $\Phi_B = \vec{B} \cdot \vec{A} = BA \cos \omega t$. Through the N windings of the solenoid the magnetic flux is $\Phi_B = NBA \cos \omega t$. Substituting this value into Eq. 29.8, I get

$$\mathcal{E}_{\text{ind}} = -\frac{d}{dt}(NBA \cos \omega t) = \omega NBA \sin \omega t. \checkmark$$

4 EVALUATE RESULT My result shows that the emf oscillates sinusoidally. It is zero when $\omega t = n\pi$ ($n = 0, 1, 2, \dots$) and maximum when $\omega t = n\pi + \frac{\pi}{2}$ ($n = 0, 1, 2, \dots$). That result makes sense because the rate of change of the magnetic flux through the solenoid is zero when the area vector of the windings is parallel to the magnetic field ($\omega t = n\pi$) and maximum when the area vector is perpendicular to the magnetic field ($\omega t = n\pi + \frac{\pi}{2}$).

-Capacitance VS Inductors

The key symbols and formulas for capacitors and inductors are listed below. You should see that there are many parallels, yet there are subtle differences.

- capacitance: C measured in farads (F)
- inductance: L measured in henrys (H)

Relation between current and voltage (or emf):

- capacitors: since $Q = CV$, $I = \frac{dQ}{dt} = C \frac{dV}{dt}$
- inductors: $V = \mathcal{E} = -L \frac{dI}{dt}$

Energy stored:

- in capacitor: $U_C = \frac{1}{2}CV^2$
- in inductor: $U_B = \frac{1}{2}LI^2$

Energy density:

- in electric field: $u_E = \frac{1}{2}\epsilon_0 E^2$
- in magnetic field: $u_B = \frac{1}{2} \frac{B^2}{\mu_0}$

Time constant:

- in RC circuit: $\tau = RC$
- in RL circuit: $\tau = \frac{L}{R}$

Transient behavior of circuit using the time constant above:

- increasing voltage: $V = V_{\max}(1 - e^{-\frac{t}{\tau}})$
- decreasing voltage: $V = V_0 e^{-\frac{t}{\tau}}$
- increasing current: $I = I_{\max}(1 - e^{-\frac{t}{\tau}})$
- decreasing current: $I = I_0 e^{-\frac{t}{\tau}}$

When calculating emf as $\mathcal{E}_{\text{ind}} = -L \frac{dI}{dt} = -1.8 \text{ V}$, the minus sign indicates that the induced emf opposes the increasing flow of charge carriers.

-Inductance (SI Unit: Henry = Ohm / s = (V * s) / A)

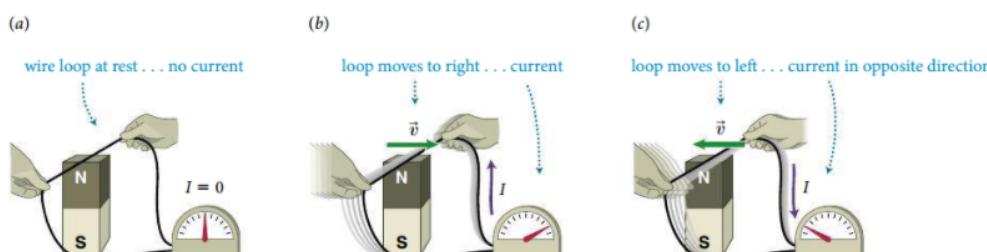
-inductance is the tendency of an electrical conductor to oppose a change in the electric current flowing through it.

$$H = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2 \cdot \text{A}^2} = \frac{\text{N} \cdot \text{m}}{\text{A}^2} = \frac{\text{kg} \cdot \text{m}^2}{\text{C}^2} = \frac{\text{J}}{\text{A}^2} = \frac{\text{T} \cdot \text{m}^2}{\text{A}} = \frac{\text{Wb}}{\text{A}} = \frac{\text{V} \cdot \text{s}}{\text{A}} = \frac{\text{s}^2}{\text{F}} = \frac{\Omega}{\text{Hz}} = \Omega \cdot \text{s}$$

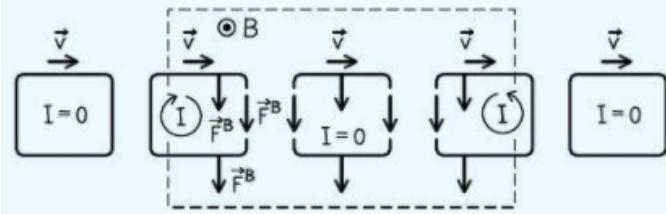
-29.1 Moving Conductors in Magnetic Fields

-Examples

Figure 29.3 Experimental observation of induced current.



-If magnet moves & string is still, there is current still.



-Literally the nintendo labo

-29.2 Faraday's Law

A changing magnetic flux through a conducting loop induces a current in the loop.

-29.3 Electric fields accompany changing magnetic fields

-The uniform magnetic field moving doesn't change the magnitude at the conductor. But how the heck is it polarized?

Figure 29.10 A stationary conducting rod in a moving magnetic field develops a charge separation, but no magnetic force is exerted on the charge carriers because their speed in the rod is zero, which means $\vec{v} \times \vec{B} = \vec{0}$.

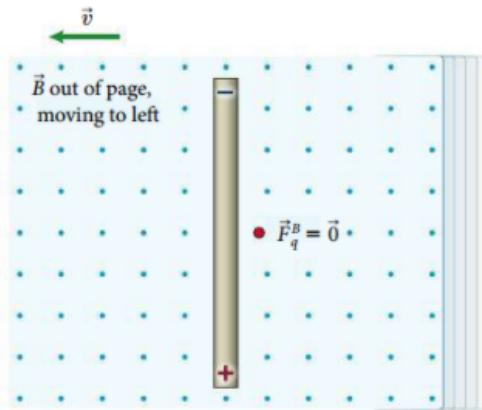
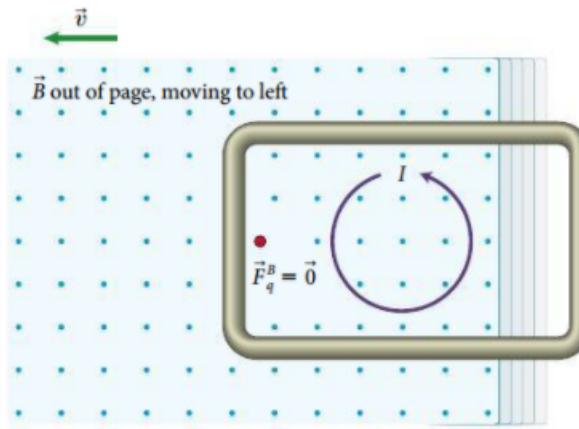


Figure 29.11 A current is induced in a stationary conducting loop as a magnetic field moves past it when not all of the loop is in the field, even though no magnetic force is exerted on the charge carriers.



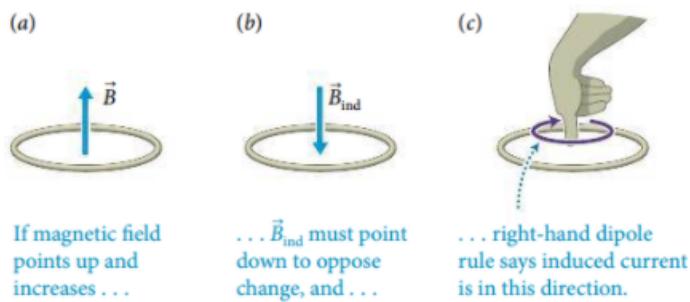
A changing magnetic field is accompanied by an electric field.

-This is more reasoning on how Electric and Magnetic Force are SAME

-29.4 Lenz's Law

The direction of an induced current through a conducting loop is always such that the magnetic flux produced by the induced current opposes the change in magnetic flux through the loop.

Figure 29.14 Applying Lenz's law.



-Magnetic Potential Energy

An induced current is always in such a direction as to oppose the motion or change that caused it.

-Because conservation of energy

Figure 29.15 Energy diagram for a magnet-loop system when the magnet moves toward the conducting loop, inducing a current through the loop. Work must be done on the system to push the magnet at constant speed against the opposing induced magnetic field of the loop. This work causes the potential energy of the magnet-loop system to increase.

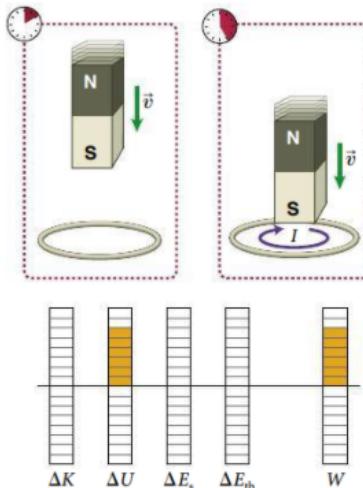
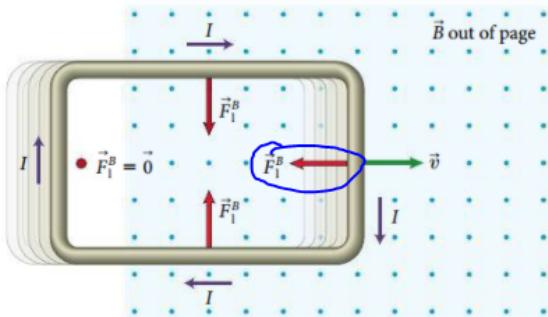
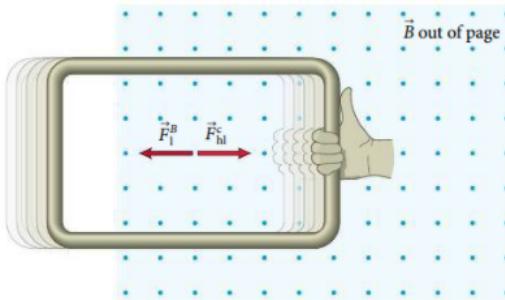


Figure 29.16 Direction of magnetic forces exerted by a magnetic field on each side of a rectangular conducting loop because of the current induced in the loop. As the loop moves into the field, the vector sum of the magnetic forces exerted on the loop resists its motion into the field.



Because of this opposing force, an agent moving the magnet at constant speed toward the loop must do work on the magnet. Where does this energy go? As work is done on the system, the current through the loop—and therefore the induced magnetic field—increases. Just as we can associate electric potential energy with an electric field, we can associate **magnetic potential energy** with a magnetic field. In the absence of dissipation, the work done on

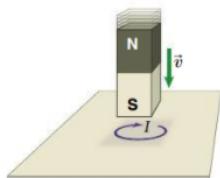
Figure 29.17 Work must be done on a conducting loop to move it into a magnetic field because magnetic forces exerted on the loop resist its motion when there is an induced current in the loop (shown in more detail in Figure 29.16).



-When loop is fully flux, no net force from current. When loop is exiting, net force is the other way compared to entering.

-Eddy Currents

Figure 29.19 The circular current loops induced in a conducting sheet by the motion of a nearby magnet are called eddy currents.

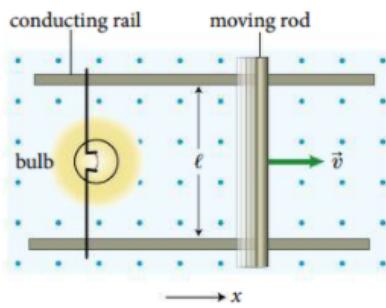


-Example of using Lenz/Faraday Law mathematically <https://www.youtube.com/watch?v=JSWOIxanzPU> (Find derivative of Flux wrt Time through B & A)

2 of 3

$$\begin{aligned} \mathcal{E}_{\text{ind}} &= - \frac{d\Phi_B}{dt} \\ \Phi_B &= \int \vec{B} \cdot d\vec{A} \\ &= \int B dA \\ &= BA \\ &= \left(5 \frac{T}{s^2} t^2\right) (0.1 m)^2 \\ \Phi_B &= 0.5 T \cdot m^2 \cdot t^2 \end{aligned}$$

Figure 29.25 The emf that develops in a moving rod can be used to drive a current when that rod is part of a closed conducting loop. The rod is connected to the rest of the loop by sliding electrical contacts.



$$W_r = B\ell v |q| \quad |\mathcal{E}_{\text{ind}}| \equiv \frac{W_r}{|q|} = B\ell v. \quad I_{\text{ind}} = \frac{\mathcal{E}_{\text{ind}}}{R},$$

$$\frac{\Delta\Phi_B}{\Delta t} = \frac{B\Delta A}{\Delta t} = \frac{B\ell\Delta x}{\Delta t} = B\ell \frac{\Delta x}{\Delta t}, \quad (29.6)$$

so Eq. 29.3 can be written as

$$|\mathcal{E}_{\text{ind}}| = B\ell v = B\ell \left| \frac{\Delta x}{\Delta t} \right| = \left| \frac{\Delta\Phi_B}{\Delta t} \right|. \quad (29.7)$$

As we let the time interval Δt approach zero, this yields

$$\mathcal{E}_{\text{ind}} = -\frac{d\Phi_B}{dt}, \quad (29.8) \quad \left| \frac{\Delta\Phi_B}{\Delta t} \right| = \left| \frac{\Delta BA}{\Delta t} \right|$$

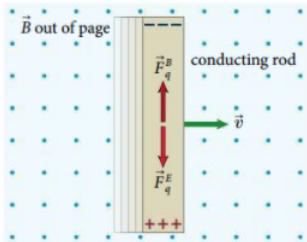
$$\mathcal{E} = -L \frac{dI}{dt}$$

-Induced emf of loop entering/exiting uniform magnetic field

$$\mathcal{E} = \frac{BdA}{dt} \text{ meaning emf} = nBhv$$

-Electric Field and Potential Diff inside the rod moving in a uniform magnetic field

Figure 29.28 When a conducting rod moves in a magnetic field, the magnetic force exerted on the negative charge carriers in the rod cause a charge separation. This separation continues until the force exerted by the electric field resulting from the charge separation exactly counters the magnetic force exerted on the charge carriers.



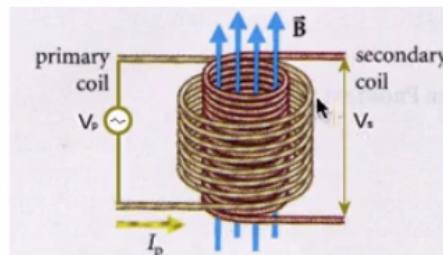
$$E = vB \quad V_{tb} = vB\ell.$$

Other Applications of Inductance

-Mutual Inductance

- Changing magnetic fields due to the current in the primary coil induce a current in the secondary coil.
- The strength of the magnetic fields produced is proportional to the number of coils in the primary coil.
- The induced voltage in the secondary coil is proportional to the number of coils in the secondary coil.
- Therefore, we can show that the primary and secondary potential differences have the same ratio as the number of coils:

$$\frac{V_p}{V_s} = \frac{N_p}{N_s}$$



-Transformer

- In practice, transformers are built with iron cores through the coils to concentrate the magnetic fields.

Example: How would you design a transformer to convert household voltage in the U.S. (120 V) to standard voltage in Europe (220 V)?

Answer: This would be a "step up" transformer with more coils on the secondary (output). Since the ratio of the voltages is 120 to 220, a simple way would be to have 120 coils on the primary and 220 on the secondary. But 1200 coils on the primary and 2200 coils on the secondary would also work.

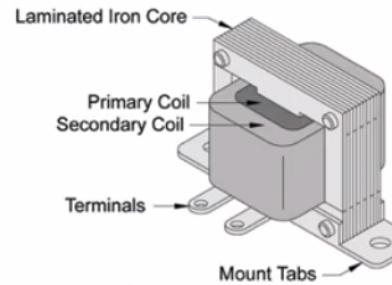


Figure 3
"E" Core Transformer

- You may be asking yourself. "Where does a step-up transformer get the increased voltage?"
- Conservation of energy still holds, and since power is current times voltage, the increased voltage comes from decreased current.
- If the transformer we just constructed to change U.S. voltages to European voltages was used to power a European device that required 0.5 A of current, how much current would flow from my home in the U.S. when I plugged it in via the transformer?
- If no power is wasted (transformed to heat), there would be the same power in the primary coil as the secondary:

$$I_P V_P = I_S V_S \quad \text{We know all of these except } I_P:$$

$$I_P = I_S \frac{V_S}{V_P} = (0.5 \text{ A}) \frac{220 \text{ V}}{120 \text{ V}} = 0.92 \text{ A}$$

$$\frac{N_S}{N_P} = \frac{V_S}{V_P} = \frac{I_P}{I_S}$$

$$\rho_{in} = \rho_{out}$$

$$V_P I_P = V_S I_S$$

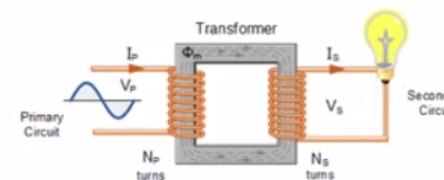
Power is same in and out

- In general, if you have a "step up" transformer, you can turn it around and use the other input as the primary to create a "step down" transformer.
- If the transformer we just constructed to change U.S. voltages to European voltages was used in a new setting and we put an AC voltage of 100 V into side of the transformer with 2200 coils, what would the voltage be on the side with 1200 coils?

You should find that the output voltage is only 55 V.

- If the current supplied on the 100 V input side was 0.5 A, what maximum current is available at the output?

You should find that the available current is greater on the output since the voltage is less: Max current = 0.92 A.



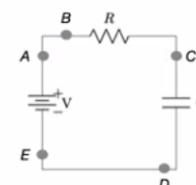
-IRL, >0.92A because transformer heats up.



-Reversible

-Capacitors Again

- Capacitors are shown as parallel plates with a gap between them. This may look like a break in the circuit (and it is), but current still flows as charge collects on the plates of the capacitor.
- We visualize positive current as flowing from the positive end of the battery to the negative side, but it is the same as thinking that negative charge flows from the negative side to the positive end of the battery.
- Positive charge will accumulate on the "high" side of the capacitor while negative charge is pulled onto the low side.
- This increases the electric field within the capacitor and causes the potential difference between the plates of the capacitor to increase.
- This, in turn, decreases the current through the loop.
- Eventually, the potential across the capacitor equals that across the battery and the current stops.
- At this point, the potential at points A, B, and C is the same.



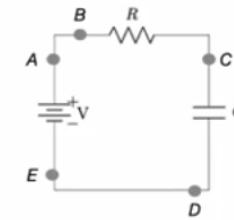
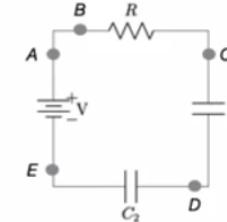
Series Capacitance

- If there are two capacitors in series, the process is the same; but it's a little more interesting to think through the resulting pattern.
- We'd like to be able to think in terms of a single equivalent capacitance the same way we did when we had two resistors.
- The total voltage drop across the two capacitors has to equal the total voltage of the battery.
- Using the definition of capacitance:

$$C_{\text{tot}} = \frac{Q_{\text{tot}}}{V_{\text{tot}}} = \frac{Q_{\text{tot}}}{V_1 + V_2}$$

- The next step is to recognize that each capacitor must have the same charge at all times. This is because of the connection between their plates (through point D). So $Q_{\text{tot}} = Q = Q_1 = Q_2$
- To separate the two terms we can take the reciprocal:

$$\frac{1}{C_{\text{tot}}} = \frac{V_1 + V_2}{Q} = \frac{V_1}{Q} + \frac{V_2}{Q} = \frac{V_1}{Q_1} + \frac{V_2}{Q_2} = \frac{1}{C_1} + \frac{1}{C_2}$$



Parallel Capacitance

- Interestingly, you can use similar logic to show that capacitors in parallel just add up, so capacitors follow the opposite rules as resistors.
- In series:

$$R_{\text{tot}} = R_1 + R_2 + R_3 + \dots$$

$$\frac{1}{C_{\text{tot}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$$

- In parallel:

$$\frac{1}{R_{\text{tot}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

$$C_{\text{tot}} = C_1 + C_2 + C_3 + \dots$$

-Reversed to resistance calculations

Time Constant

- The way we have described the process in an RC circuit, the initial current decreases over time and eventually goes to zero.
- This time-dependence in the circuit is new... and useful.
- Can we quantify it?
- We can! But it is easier to work with a capacitor that has already been charged and is connected in series to a resistor. There is no battery in this circuit, but the charge stored in the capacitor has a path that allows it to discharge.
- Again, the initial current will decrease to zero over time.
- Start with the definition of current as the flow of charge. The charge on the capacitor is changing over time, and the rate of change is equal to the current: $\frac{dQ}{dt} = I$ (Check the units to make sure it makes sense.)
- But we also know that the potential difference across the resistor is related to the current by Ohm's law: $I = \frac{V_R}{R}$.
- Finally, we know that the charge on the capacitor is related to its voltage by $V_C = \frac{Q}{C}$ and that the total voltage drop for the loop must be zero: $V_R + V_C = 0$

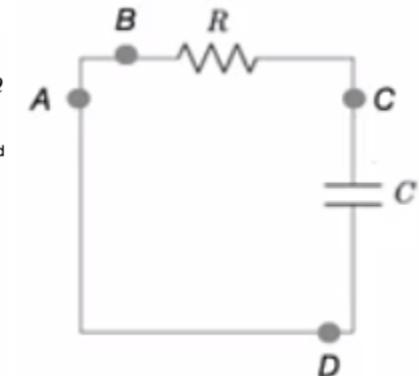
- Put all of these together:

$$\frac{dQ}{dt} = I = \frac{V_R}{R} \quad V_C = \frac{Q}{C} = -V_R \quad \text{so} \quad \frac{dQ}{dt} = -\frac{1}{RC}Q$$

- This is a differential equation!
- A common way to solve differential equations is to make an educated guess and check it.
- Here we have something that looks like it is equal to its own derivative. We know a function that does that... the exponential function.
- So we guess that $Q = Ae^{-t/\tau}$ and take the derivative of it:

$$\frac{d}{dt}Q = \frac{d}{dt}Ae^{-t/\tau} = -\frac{1}{\tau}Ae^{-t/\tau} = -\frac{1}{\tau}Q$$

- We can see that this fits the form of the equation above if $\tau = RC$
- And if we check the units, we find that this is, indeed, a time.



- The function $Q = Ae^{-t/\tau}$ is an exponential decay function with a starting value of A. It makes sense in this case to identify this with the starting charge of the capacitor. So we have

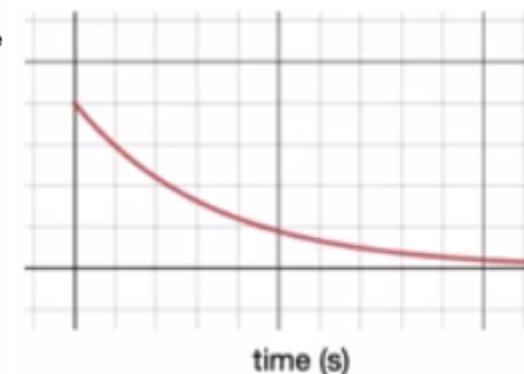
$$Q = Q_0 e^{-t/\tau}$$

- Since the charge of the capacitor is what is driving the current, it is easy to show that the current also starts from some maximum value and decays exponentially with the same time constant:

$$I = I_0 e^{-t/\tau}$$

- And the potential difference across the capacitor also decays over time from its initial voltage:

$$V = V_0 e^{-t/\tau}$$



Charging a Capacitor

- Although the integration to show it is more complex, you can also find the function for the charge on a capacitor when it is charging:

$$Q = Q_f (1 - e^{-t/\tau})$$

- Even in this situation, the current starts from some maximum value and decays exponentially with the same time constant:

$$I = I_0 e^{-t/\tau}$$

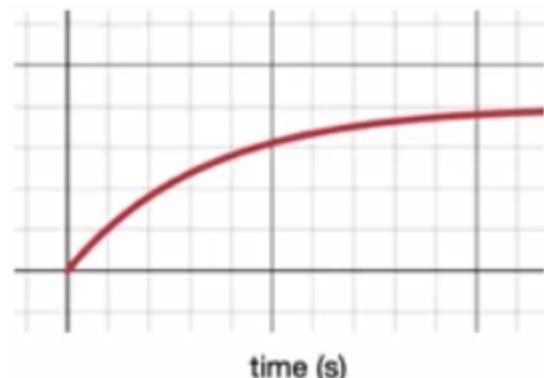
- And therefore, the potential difference across the resistor also decays over time from its initial voltage:

$$V_R = V_0 e^{-t/\tau}$$

- But the potential difference across the capacitor increases as it approaches the same potential difference as the battery:

$$V_C = V_B (1 - e^{-t/\tau})$$

- Again, for all of these $\tau = RC$

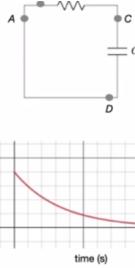


Time dependence

- Circuits with a capacitor and some resistance (which all circuits have) are called RC circuits.
- RC circuits have many applications in electronics in both DC and AC circuits (direct current and alternating current).

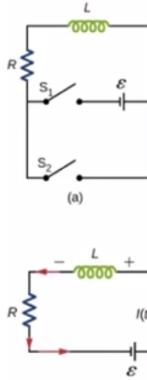
-Example:

- Let's assume that we have a 30 mF capacitor that has been fully charged using a 9 V battery, and then it is connected in series to a 1000 Ω resistor. What will the current in the circuit be 2.0 seconds after the circuit is connected?
- The time constant in this circuit is $\tau = RC = (1000 \Omega)(0.0030 F) = 3.0$ s
- The initial current can be found by thinking about the 9.0 V difference across the resistor initially created by the capacitor. This creates an initial current of $I_0 = \frac{V}{R} = \frac{9\text{ V}}{1000 \Omega} = 9.0 \text{ mA}$
- At time $t = 2.0$ s, we have $I = I_0 e^{-t/\tau} = 9.0 \text{ mA } e^{-2/3} = 4.6 \text{ mA}$
- You could also find the voltage across the resistor or the capacitor at any time. And you could calculate the charge on the capacitor at any time.



-Inductors in a circuit

- Inductors are shown as coils in circuit diagrams because they are essentially how they are constructed.
- There is always some resistance in the circuit, but it is shown separately as a resistor, the inductor is assumed not to have resistance.
- You have to be very careful when connecting and especially disconnecting circuits with inductors because the inductor will continue to drive current through the circuit after it is disconnected.
- So let's put some switches in the circuit as shown here.
- First let's close switch 1 but leave switch 2 open. What happens?



-If we close switch 1:

- The current attempts to flow through the inductor, but the self-inductance causes it to be delayed... it takes time for the current to build up to the maximum current of $I_{\max} = \frac{\mathcal{E}}{R}$. In fact the current builds up according a function just like what we saw with the capacitor:

$$I = I_{\max} (1 - e^{-t/\tau})$$

but this time the time constant is determined by the inductance and the resistance:

$$\tau = \frac{L}{R}$$

Current vs Time



-Inductor opposes change, so current increases over some time.

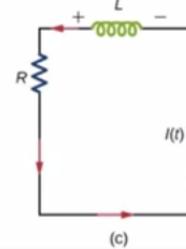
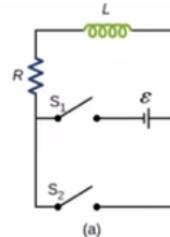
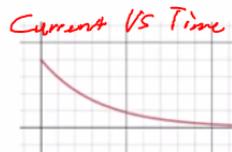
-Now, close switch 2 and open switch 1:

Inductors

- Now let's close switch 2 and open switch 1 (in that order, so that we don't get a spark jumping across switch 1)
- What happens?
- Even though there is no battery or emf in the circuit, the current will not immediately stop because the inductor resists changes in current. So the current will gradually die out in an exponential decay pattern:

$$I = I_{\max} e^{-t/\tau}$$

$$\text{again with a time constant } \tau = \frac{L}{R}$$



-Inductor opposes change, current decreases w/ time

-Inductor Storing Energy

- Clearly the inductor has stored some energy when it has current going through it, because it is able to keep that current going after the power source is disconnected.
- How much energy is there?
- The simple formula for energy stored in an inductor is

$$U_L = \frac{1}{2} L I^2$$

- Although we can also think of the energy as being stored in the magnetic field that is created in the core of the inductor.
- This energy stored in the field can be quantified most easily as an energy density as we did for electric fields, and the formula is very similar in form:

$$u_B = \frac{1}{2} \frac{B^2}{\mu_0}$$

-Inductor Instantaneous Power

Energy in an Inductor

When a electric current is flowing in an inductor, there is energy stored in the magnetic field. Considering a pure inductor L, the instantaneous power which must be supplied to initiate the current in the inductor is

$$P = iv = Li \frac{di}{dt}$$

so the energy input to build to a final current i is given by the integral

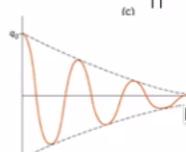
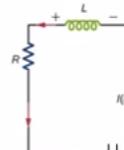
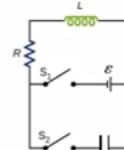
$$\text{Energy stored} = \int_0^i P dt = \int_0^i L i' di' = \frac{1}{2} L I^2$$

-Inductor and Capacitor in a Circuit

-Creates an oscillation of current.

What if you use capacitors and inductors?

- If we start by closing switch 1 to "charge up" the inductor, and then open close switch 2 and open switch 1, what happens if there is a capacitor next to switch 2?
- The current will continue to flow and charge up the capacitor. At some point the current will die out, but the stored charge in the capacitor will now act like a battery, causing current to flow in the opposite direction as before.
- This current will initially be resisted by the inductor, but it will eventually "charge up" the inductor.
- By the time the capacitor runs out of charge, the inductor will be pushing current clockwise, and it won't allow that current to just stop, even though the "battery" of the capacitor has been depleted.
- Current will continue to flow, thus charging up the capacitor again, but with the opposite polarity as before.
- When the current dies out, the capacitor will be charged and will act like a battery, pushing current counterclockwise through the circuit.
- This pattern just keeps repeating... it is an oscillating circuit!



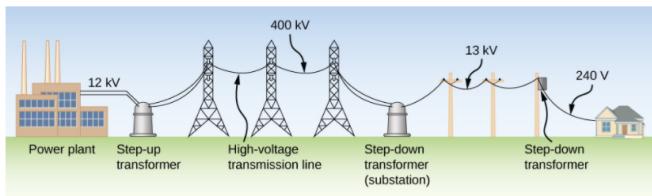


Figure 15.20 The rms voltage from a power plant eventually needs to be stepped down from 12 kV to 240 V so that it can be safely introduced into a home. A high-voltage transmission line allows a low current to be transmitted via a substation over long distances.

-A good transformer can have losses as low as 1% of the transmitted power, so this is not a bad assumption

University Physics Vol 2 On Inductors

-Capacitance & Inductor Reactance

$$\frac{V_0}{I_0} = \frac{1}{\omega C} = X_C.$$

15.3

The quantity X_C is analogous to resistance in a dc circuit in the sense that both quantities are a ratio of a voltage to a current. As a result, they have the same unit, the ohm. Keep in mind, however, that a capacitor stores and discharges electric energy, whereas a resistor dissipates it. The quantity X_C is known as the **capacitive reactance** of the capacitor, or the opposition of a capacitor to a change in current. It depends inversely on the frequency of the ac source—high frequency leads to low capacitive **reactance**.

$$\frac{V_0}{I_0} = \omega L = X_L.$$

$$X = 1/(2(\pi)fC)$$

The quantity X_L is known as the **inductive reactance** of the inductor, or the opposition of an **inductor** to a change in current; its unit is also the ohm. Note that X_L varies directly as the frequency of the ac source—high frequency causes high inductive **reactance**.

$$X_L = 2\pi f L$$

-Many equations

Molar extinction by Fe^{2+}	$\epsilon = \frac{\Delta A}{\Delta C} = \frac{\Delta A}{\Delta [Fe^{2+}]}$
Molar extinction by Cr^{3+}	$\epsilon = \frac{\Delta A}{\Delta C} = \frac{\Delta A}{\Delta [Cr^{3+}]}$
Self-extinction of a magnetite	$\epsilon = -\frac{\Delta A}{\Delta [Fe^{2+}]}$
Self-extinction of a reduced magnetite	$\epsilon_{\text{mag}} = -\frac{\Delta A}{\Delta [Fe^{2+}]}$
Self-extinction of a leached magnetite	$\epsilon_{\text{mag}} = -\frac{\Delta A}{\Delta [Fe^{2+}]}$
Energy stored in a magnetite	$E = \frac{1}{2} \epsilon A^2$
Current as a function of time for a BMR sample	$I = I_0 e^{-\epsilon A^2 / 2}$
Time constant of the ESR signal	$\tau = \frac{1}{\epsilon A^2 / 2} = \frac{2}{\epsilon A^2}$
Change in energy per unit time	$dE/dt = -\epsilon A^2 / 2$
Angular frequency in Cr^{3+} crystals	$\omega = \sqrt{\frac{g\mu_B}{M}}$
Current measured in Cr crystals	$I = I_0 e^{-\epsilon A^2 / 2}$
Change in a fraction of the PEC signal	$\delta I = -I_0 \epsilon A^2 / 2$

10.1 Maxwell's Equations

-Maxwell's Equations

-Basically, Gauss's and Ampere's Law but with induction, changing magnetic fields, changing electric fields

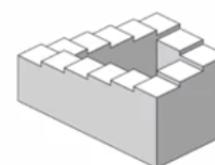
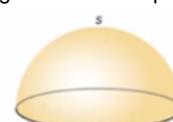
Electric field	Magnetic field
Gauss's law $\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$ <p>(Charges are sources of electric field.)</p>	$\oint \vec{B} \cdot d\vec{A} = 0$ <p>(Magnetic field lines loop back onto themselves.)</p>
Ampere's law $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$ <p>(The only way to produce a potential difference over a closed loop path is with changing magnetic flux.)</p>	$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{int}} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$ <p>(Currents (including 'displacement</p>

-Ampere's Law adjustments

-Changing magnetic flux from induction (How can potential difference appearing in a closed loop?)

- In the absence of magnetic fields, we found that $\oint \vec{E} \cdot d\vec{l} = 0$
 - And even with magnetic fields, this can be true... if the magnetic flux doesn't change.
 - But if it does change, we get an emf ($\mathcal{E}_{\text{ind}} = -\frac{d\Phi_B}{dt}$) which means there must be some electric field in the wire, pushing the charges.
 - Somehow, as we go around the loop, there is a potential difference... it's like an Escher staircase (also called Penrose stairs).
 - The potential difference comes from the changing magnetic flux **through the surface bounded by** the loop.

$$\oint \vec{E} \cdot d\vec{l} = - \frac{d\Phi_B}{dt}$$



When you walk around the loop, there is a voltage drop across the resistor, but as you continue on your path, somehow you end up back at the higher potential on the other side of the resistor and drop across it again.

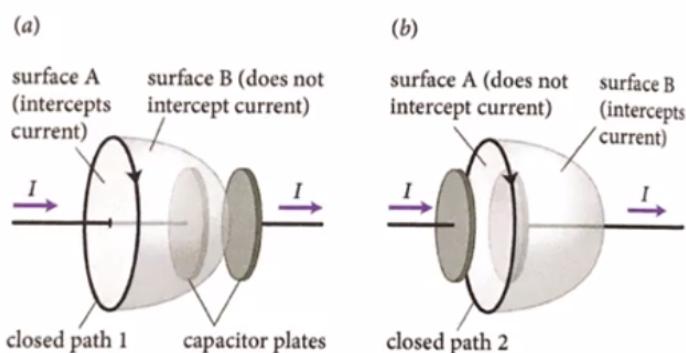
- We also have changing electric fields that cause magnetic fields.

- How can we tell how they work?...

Ampere's law for magnetism:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$$

- But something is wrong...



- The solution is to look at the electric fields in the capacitor:

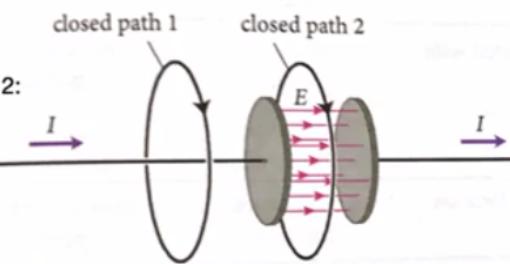
- These two paths should have equivalent path integrals.

- We can construct a 'displacement current' for path 2:

$$I_{\text{disp}} = \epsilon_0 \frac{d\Phi_E}{dt}$$

- And add that to Ampere's law for magnetism:

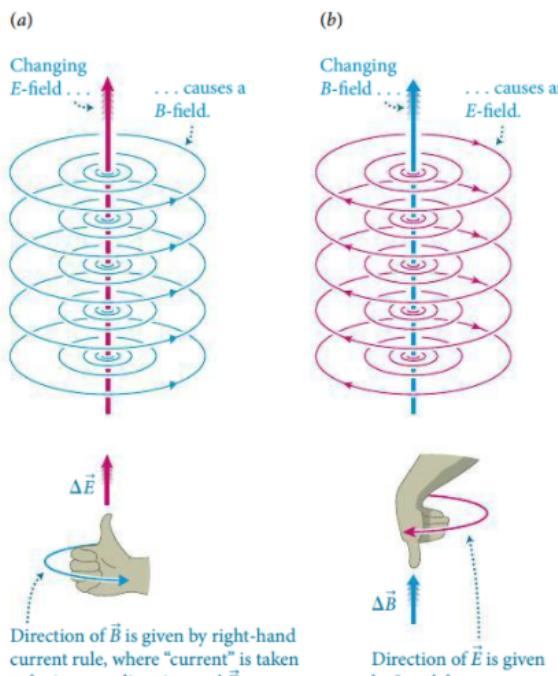
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(I_{\text{int}} + \epsilon_0 \frac{d\Phi_E}{dt} \right)$$



30.1 Magnetic fields accompany changing electric fields

-Changing magnetic field is accompanied by a magnetic field

Figure 30.5 Parallels between (a) the electric field that accompanies a changing magnetic field and (b) the magnetic field that accompanies a changing electric field.



Direction of \vec{B} is given by right-hand rule, where "current" is taken to be in same direction as $\Delta\vec{E}$.

Direction of \vec{E} is given by Lenz's law.

-Electric fields are produced by charged particles either at rest or in motion, but magnetic fields are produced only by charged particles in motion.

Table 30.1 Properties of electric and magnetic fields

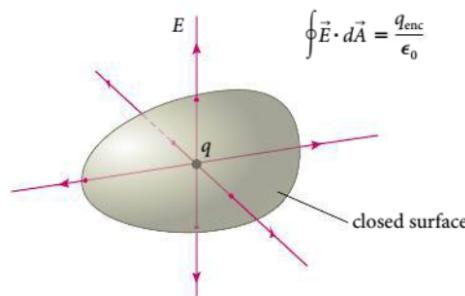
	Electric field	Magnetic field
associated with	charged particle	moving charged particle
	changing magnetic field	changing electric field
exerts force on	any charged particle	moving charged particle

Table 30.2 Electric and magnetic field lines

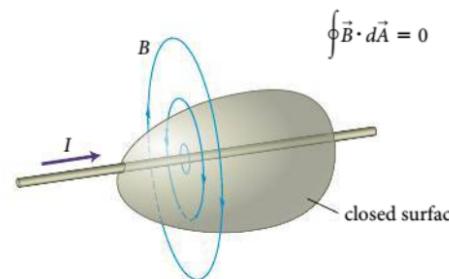
	Electric field	Magnetic field
lines emanate from or terminate on	charged particle	-
	-	moving charged particle
loops encircle		
	changing magnetic field	changing electric field

Figure 30.30 Graphical representation of the physics behind Maxwell's equations, together with their mathematical expressions. (a) Electric field surrounding a charged particle and a Gaussian surface enclosing that particle. Gaussian surfaces can be used to relate the electric field to the enclosed charge. (b) Magnetic field surrounding a current-carrying wire and a closed surface intersected by the wire; the integral of the magnetic field over a closed surface is always zero. (c) Electrostatic field and two closed paths through that field; the path integral of the electric field around either path must be zero. (d) Steady magnetic field surrounding a current and two closed paths through that field; the path integral of the magnetic field is proportional to the encircled current. (e) Changing magnetic field and two closed paths through it; an electric field accompanies the changing magnetic field; the path integral of this electric field around either path is nonzero. (f) Changing electric field and two closed paths through it; a magnetic field accompanies the changing electric field; the path integral of this magnetic field around either path is nonzero.

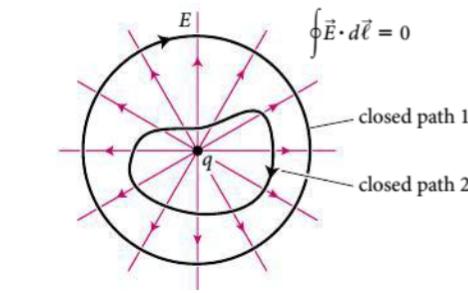
(a) Surface integral of electric field (Gauss's law)



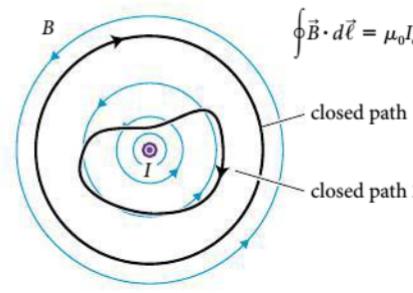
(b) Surface integral of magnetic field (Gauss's law for magnetism)



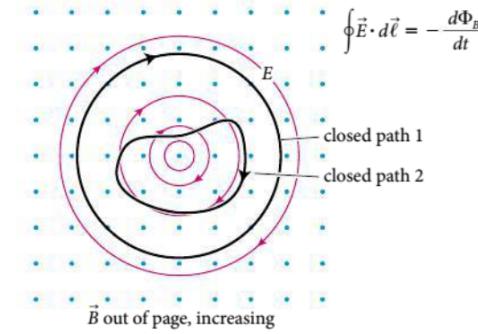
(c) Line integral of constant electric field



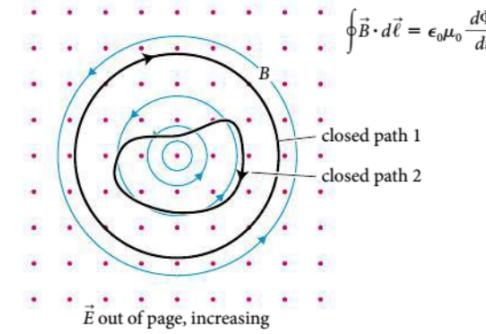
(d) Line integral of constant magnetic field (Ampère's law)



(e) Line integral of changing electric field (Faraday's law)



(f) Line integral of changing magnetic field (Maxwell's displacement current)



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{int}} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

Special Relativity

-Lead up

-Maxwell guy be like, "Is light wave?"

-If so, what is its medium which light travels through?

-This dude thought there was Luminiferous Aether mod, a hypothesized medium. It was not installed.

-Then 5Head Einstein published a 5 minute paper introducing a theory of time, distance, mass, and energy that was consistent with electromagnetism, but omitted the force of gravity.

-Ah of course: "The speed of light is the same constant relative to everything." Simple really.

-Einstein suggested that our conception of speed might be wrong (or at least limited). He decided to just believe that the speed of light will be measured to be the same by all observers regardless of the motion of those observers.

-Foundation for Special Relativity

-All non-accelerating observers will measure the speed of light to be $c = 3.0 \times 10^8$ m/s.

-Einstein's Arguments

- If something is a law of physics, then it is the same for all (non-accelerating) observers.
- Maxwell's equations, in any form, are laws of physics.
- Therefore, Maxwell's equations, in any form, are the same for all (non-accelerating) observers.
- If something is a form of Maxwell's equations, then it is the same for all (non-accelerating) observers.
- An electromagnetic wave equation where the wave speed c is constant is one form of Maxwell's equations.
- Therefore, an electromagnetic wave equation where the wave speed c is constant is the same for all (non-accelerating) observers.

Special Relativity Consequence 1: Measuring Space and Time

-Speed of Light is a conversion factor

c serves as a conversion factor (a constant of proportionality)
between space (meters) and time (seconds) FOR ALL OBSERVERS.

$$c = 300\,000\,000 \text{ m/s}$$

So 1 meter is the same as 3.33 nanoseconds of light travel.

$$1.00 \text{ m} = 3.33 \times 10^{-9} \text{ s} = 3.33 \text{ ns}$$

Or 3 meters is the same as 10 nanoseconds of light travel.

$$3.00 \text{ m} = 10.0 \times 10^{-9} \text{ s} = 10 \text{ ns}$$

-Measuring Speed

If we can measure time in units of space, we can come up with strange but simple answers to questions like this:

- How far does light travel in 12 meters of time?
It travels 12 meters of space in 12 meters of time!
- So what is light's speed?

Speed is distance divided by time, so...

$$\text{speed} = \frac{12 \text{ meters (of distance)}}{12 \text{ meters (of time)}} = 1$$

-One convention for symbolizing speed expressed as a unitless fraction of the speed of light is to use the Greek letter beta. So we would say that a particle moving at 80% the speed of light had a velocity of $\beta=0.8$.

$$\beta = \frac{v}{c}$$

Special Relativity Consequence 2: Time Dilation

-The measured time between two events is not the same for all observers.

-Proper Time

The measured time between two events is shortest in the frame where the two events occur at the same place.

This shortest time is called the "proper time."

-Measuring Time

-Let's have an example where there's a light shooting time measurement device in a rocket shot across a laboratory. The time for the light to travel is shorter in the rocket than seen in the lab observations.

-Events observed in the rocket

- Events will have a time and a location.
- We can designate this as a four-vector: (t, x, y, z)

Let's look at two events E and R .

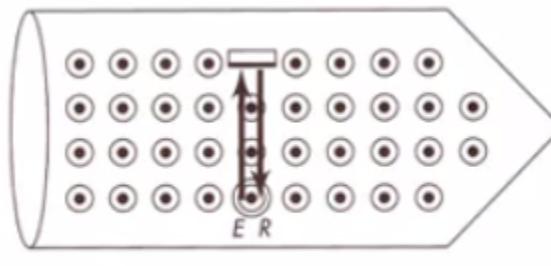
- E is the emission of a light pulse from the base of a light clock
- R is the reception of the light pulse when it returns to the base of the light clock.
- If the light clock is 3 m tall, what are the coordinates of E and R ?
- $E = (0.00 \text{ m}, 0.00 \text{ m}, 0.00 \text{ m}, 0.00 \text{ m})$
- $R = (6.00 \text{ m}, 0.00 \text{ m}, 0.00 \text{ m}, 0.00 \text{ m})$
- $R = (20.0 \text{ ns}, 0.00 \text{ m}, 0.00 \text{ m}, 0.00 \text{ m})$

-Event observed in the laboratory

If...

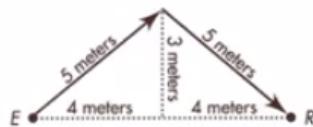
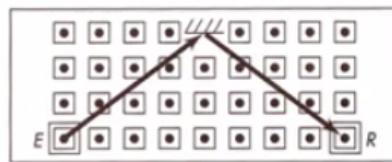
... the wave speed for all electromagnetic waves (including light) is the same for all (non-accelerating) observers,

Then what???



ROCKET PLOT

- The light clock in the previous part was on a rocket that was speeding through a laboratory where other observers also measured the events E and R .
 - The video of the rocket speeding through the lab is used to produce the trace of the path of the light shown in the plot to the right.
 - What are the coordinates of E and R as measured in this frame?
 - $E = (0.00 \text{ m}, 0.00 \text{ m}, 0.00 \text{ m}, 0.00 \text{ m})$
 - $R = (10.0 \text{ m}, 8.00 \text{ m}, 0.00 \text{ m}, 0.00 \text{ m})$
 - $R = (33.3 \text{ ns}, 8.00 \text{ m}, 0.00 \text{ m}, 0.00 \text{ m})$



-Space Time Interval (I)

Observers in the rocket measure

- $E = (0.00 \text{ m}, 0.00 \text{ m}, 0.00 \text{ m}, 0.00 \text{ m})$
 - $R = (6.00 \text{ m}, 0.00 \text{ m}, 0.00 \text{ m}, 0.00 \text{ m})$

$$I^2 = t^2 - x^2 = (6 \text{ m})^2 - (0 \text{ m})^2 = 36 \text{ m}^2$$

This "spacetime interval" can be used to compare measurements by one set of observers to measurements by another set.

It should come out the same for all (non-accelerating) observers.

Observers in the laboratory measure

- $E = (0.00 \text{ m}, 0.00 \text{ m}, 0.00 \text{ m}, 0.00 \text{ m})$
 - $R = (10.0 \text{ m}, 8.00 \text{ m}, 0.00 \text{ m}, 0.00 \text{ m})$

$$I^2 = t^2 - x^2 = (10 \text{ m})^2 - (8 \text{ m})^2 = 36 \text{ m}^2$$

1. Convert all measurements to the same units (using the speed of light as the conversion)

2. Calculate the spacetime interval:

$$J^2 \equiv t^2 - x^2$$

-What was the speed of the rocket relative to the lab? 80% of the speed of light

Observers in the laboratory measure

- $E = (0.00 \text{ m}, 0.00 \text{ m}, 0.00 \text{ m}, 0.00 \text{ m})$
 - $R = (\textcolor{red}{10.0 \text{ m}}, \textcolor{red}{8.00 \text{ m}}, 0.00 \text{ m}, 0.00 \text{ m})$
 - $R = (\textcolor{red}{33.3 \text{ ns}}, \textcolor{red}{8.00 \text{ m}}, 0.00 \text{ m}, 0.00 \text{ m})$

It covers 8 meters of space in 10 meters of time, so it must have speed $\beta = 0.8$.

-So if we assume that the speed of light is constant for all (non-accelerating) observers, we must conclude that different observers can measure different times between the same two events.

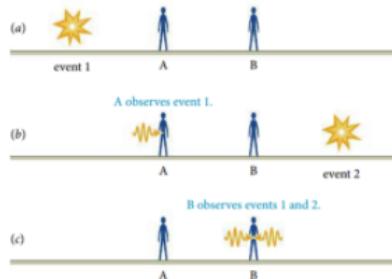
And it turns out that the **shortest time** will be measured by the observer for whom the events take place at the same place, if such an observer is possible. This shortest time between two events is sometimes called the "proper time."

-Proper Time (shortest measured time) & Proper Length (longest measured length) Equation

14.1 Time Measurements

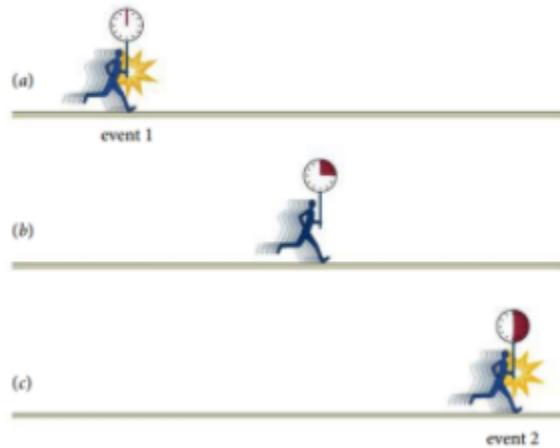
-Which event happened first?

Figure 14.1 Two observers with synchronized clocks observe two events.



-Proper Time Interval

Figure 14.2 One way to measure the time interval between two events is to travel from one event to the other, using a single clock to record the instant at which each occurs.



The time interval between any two events that occur at the same position in a particular reference frame is called the proper time interval between the two events.

-Measuring Proper Time Interval

Figure 14.5 Synchronized clocks can be used to measure the time interval between events that occur in different locations. (a) Each observer records the instant at which a nearby event occurs. (b) To synchronize their clocks, the two observers record the instant at which a single event occurs.

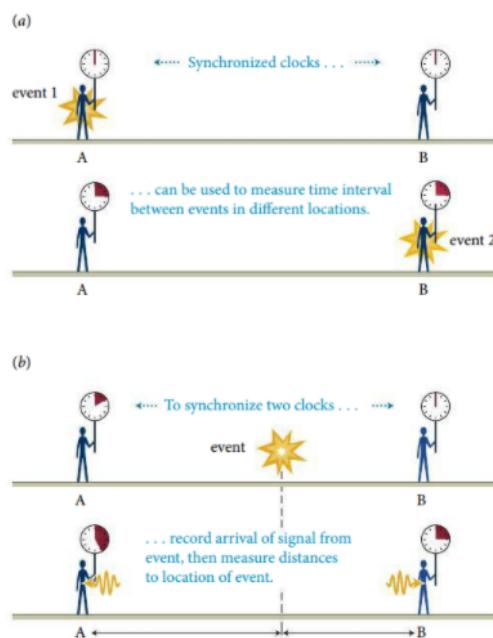
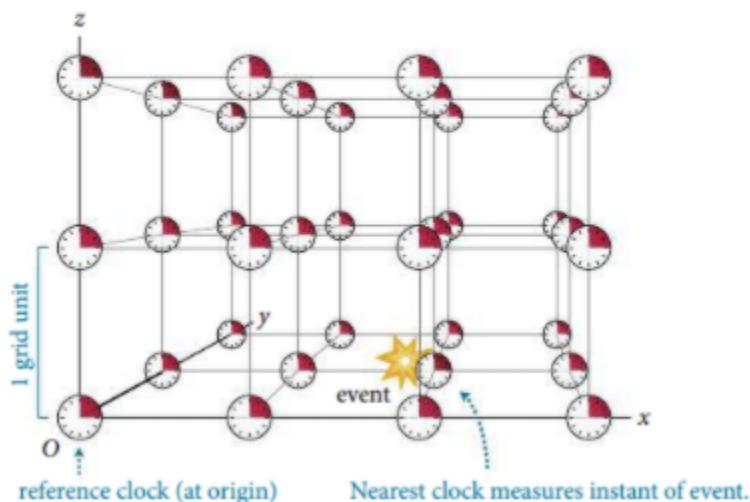


Figure 14.6 The scaffold of time: a reference frame made up of a grid of synchronized clocks. An event can occur anywhere in the grid, and the instant at which the event occurs is the instant shown on the clock nearest the event.



14.5 Time Dilation

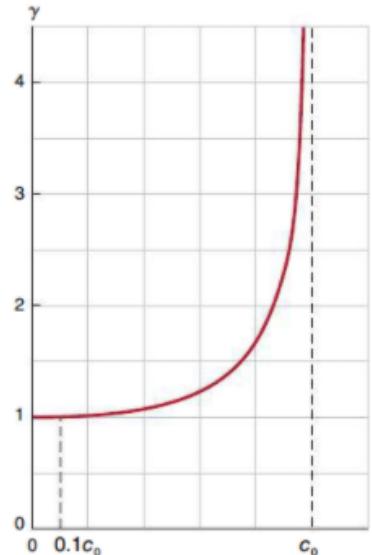
-Lorentz Factor

$$\gamma \equiv \frac{1}{\sqrt{1 - v^2/c_0^2}} \quad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

Table 14.3 Gamma factor

v/c_0	γ
0	1
0.1	1.005
0.5	1.15
0.9	2.3
0.98	5.0
0.99	7.1
0.999	22.4
0.9999	70.71
0.99999	223.61
0.999999	707.107

Figure 14.25 As the speed v at which any object travels approaches c_0 , the Lorentz factor γ approaches infinity. Up to a speed of about $0.1c_0$, γ is very close to 1.



-Space Time Interval Equations

-Proper Time

$$(c_0 \Delta t_{\text{proper}})^2 = (c_0 \Delta t)^2 - (\Delta x)^2.$$

-Proper Length (s)

$$s^2 = (c_0 \Delta t)^2 - (\Delta x)^2$$

Special Relativity Consequence 3: Length Contraction

-Length in Special Relativity

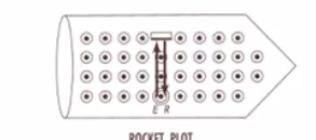
Measured lengths (the distance between events) are not the same for all observers.

The greatest length will be measured in the frame in which the 'object' is motionless.

This greatest length is called the "proper length."

-Back to the Rocket & Lab Plot

-This time, we are observing from another rocket that is going the opposite way of the first rocket, making the light travel more in that reference.
-It's still 36 m² for time interval



Observers in the rocket measure

- $E = (0.00 \text{ m}, 0.00 \text{ m}, 0.00 \text{ m}, 0.00 \text{ m})$
- $R = (6.00 \text{ m}, 0.00 \text{ m}, 0.00 \text{ m}, 0.00 \text{ m})$

$$I^2 = t^2 - x^2 = (6 \text{ m})^2 - (0 \text{ m})^2 = 36 \text{ m}^2$$

Observers in the laboratory measure

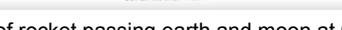
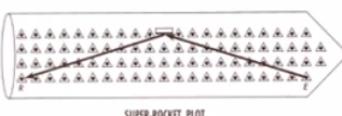
- $E = (0.00 \text{ m}, 0.00 \text{ m}, 0.00 \text{ m}, 0.00 \text{ m})$
- $R = (10.0 \text{ m}, 8.00 \text{ m}, 0.00 \text{ m}, 0.00 \text{ m})$

$$I^2 = t^2 - x^2 = (10 \text{ m})^2 - (8 \text{ m})^2 = 36 \text{ m}^2$$

Observers in a "super rocket" measure

- $E = (0.00 \text{ m}, 0.00 \text{ m}, 0.00 \text{ m}, 0.00 \text{ m})$
- $R = (20.88 \text{ m}, -20.0 \text{ m}, 0.00 \text{ m}, 0.00 \text{ m})$

$$I^2 = t^2 - x^2 = (20.88 \text{ m})^2 - (-20 \text{ m})^2 = 36 \text{ m}^2$$



-Example of rocket passing earth and moon at 0.8 speed of light

-Earth frame

Let's continue to investigate the rocket that has a speed of $\beta = 0.8$. We've set up stations on Earth and Moon to track the progress of the rocket and note that it first passes near Earth (Event 1) and then passes near Moon (Event 2).

In the Earth frame, it is known that light travels from Moon to Earth (or Earth to Moon) in 1.28 seconds. Thus we can measure the distance between Moon and Earth as 1.28 s. What will observers on Earth measure as the time between Event 1 and Event 2?

This question is all in one frame of reference, so we don't need to use relativity. We just use the fact that time should be distance divided by velocity:

$$t = \frac{d}{\beta} = \frac{1.28 \text{ s}}{0.8} = 1.6 \text{ s}$$

If we choose Event 1 as our origin for space and time, the coordinates of Event 2 in the Earth frame are (1.6 s, 1.28 s).
(time, distance)

-From the rocket frame though, the origin is 0 since we in the rocket is origin. How long is the time between each event?

What are the coordinates of these events in the rocket frame?

The first key observation is that both events take place at the same place in the rocket frame.

If you were in the rocket, Event 1 happens right outside your window as Earth zooms by. A little later, Moon zooms by that same window.

If Event 1 is used as the origin in the rocket frame, we have this much:

Rocket frame

Event 1: (0 , 0)

Event 2: (? , 0)

-We must connect the two frames. Space Time Interval!

Earth frame

Rocket frame

Event 1: (0 , 0) Event 1: (0 , 0)

Event 2: (1.6 s, 1.28 s) Event 2: (? , 0)

Is there anything we can use to connect the measurements in these frames?

That is a relativity question, and it can be answered by using the spacetime interval.

$I^2 = t^2 - x^2$ has to be the same in both frames.

Earth frame: $I^2 = (1.6 \text{ s})^2 - (1.28 \text{ s})^2 = 0.92 \text{ s}^2$

Rocket frame: $I^2 = t^2 - (0)^2 = 0.92 \text{ s}^2$, so $t = 0.96 \text{ s}$.

-Length contraction from example. (Moving faster means shorter length)

Earth frame

Rocket frame

Event 1: (0 , 0) Event 1: (0 , 0)

Event 2: (1.6 s, 1.28 s) Event 2: (0.96 s, 0)

So what do observers in the rocket think about the distance between Earth and Moon?

Remember, Earth came zooming by, and a little later the Moon zoomed by. Both seemed to be going in the same direction at the same speed: $\beta = 0.8$

This question is all about measurements in the same frame of reference, so it is not a relativity question. It can be answered using $d = t\beta = (0.96 \text{ s})(0.8) = 0.768 \text{ s}$, which is the same as 230,400 km.

In the Earth frame, the distance between Earth and Moon is 1.28 s or 384,000 km.

The observers in the rocket measure Earth and Moon to be closer together.

Earth frame

Rocket frame

Event 1: (0 , 0) Event 1: (0 , 0)

Event 2: (1.6 s, 1.28 s) Event 2: (0.96 s, 0)

In the rocket frame, the distance between Earth and Moon is 0.768 s or 230,400 km.

In the Earth frame, the distance between Earth and Moon is 1.28 s or 384,000 km.

The largest distance is measured in the frame where the "object" is at rest.

-Proper time and proper length of this example

-Rocket measures the proper time, since the origin doesn't change/stays zero.

-Earth measures the proper length, since Earth and the moon is stationary.

-Lorentz Factor and this example

Earth frame

Event 1: (0 , 0)

Event 2: (1.6 s, 1.28 s)

Earth to Moon: 1.28 s or 384,000 km

Rocket frame

Event 1: (0 , 0)

Event 2: (0.96 s, 0)

Earth to Moon: 0.768 s or 230,400 km.

Because both events occur at the same place in the rocket frame, that frame measures the shortest, proper time between the events: 0.96 s.

Because we have taken Earth and Moon to be stationary in the Earth frame, the length measured in that frame is the longest, proper length: 1.28 s or 384,000 km

Notice that you can also use a Lorentz transformation to get these same results:

$$\text{If } \beta = 0.8, \text{ then } \gamma = \frac{1}{\sqrt{1 - \beta^2}} = \frac{1}{\sqrt{1 - 0.8^2}} = 1.67$$

The proper time should be "stretched" out in the Earth frame: $\Delta t_E = \gamma \Delta t_p = (1.67)(0.96 \text{ s}) = 1.6 \text{ s.}$

The proper length should be "contracted" in the rocket frame: $\ell_r = \frac{\ell_p}{\gamma} = \frac{1.28 \text{ s}}{1.67} = 0.77 \text{ s}$

-These example can be worked from Lorentz Factor or Space Time Interval

14.6 Length Contraction

-Examples Problems

-Biggest takeaway here, DRAW TABLES

Example 14.8 Electron dash

An electron moving at $0.80c_0$ relative to the Earth reference frame travels the 100-m length of a building. What is the length of the building according to an observer moving along with the electron?

① GETTING STARTED This problem involves two reference frames: the reference frame of the building (which is the Earth reference frame) and the reference frame of the electron. Although the problem doesn't mention this explicitly, the 100-m length of the building is likely to be the length measured in Earth's reference frame, and so it is the building's proper length. I summarize this information in a table:

Electron reference frame	Earth reference frame
v_{electron}	0
ℓ_{building}	?

② DEVISE PLAN Equation 14.28 relates the building length measured by the observer in the electron's reference frame to the proper length. I know the building's proper length (100 m) and the speed of the observer relative to the reference frame in which the proper length is measured ($0.80c_0$). I can calculate γ from Eq. 14.6 and then use Eq. 14.28 to obtain the length of the building in the electron's reference frame.

③ EXECUTE PLAN $\gamma = 1/\sqrt{1 - v^2/c_0^2} = 1/\sqrt{1 - (0.80)^2} = 1.67$, and so $\ell_v = \ell_{\text{building}} = (100 \text{ m})/1.67 = 60 \text{ m.}$ ✓

④ EVALUATE RESULT From our length contraction discussion in Section 14.3, I know that an observer measuring the length of an object that is moving relative to her measures a length shorter than the proper length. In this example, the observer is in the electron's reference frame, and the building being measured is moving relative to that reference frame. Therefore I expect the measured length to be shorter than the proper length.

Example 14.9 Cosmic ray longevity

A proton strikes an air molecule in Earth's atmosphere, creating a muon 6.0 km above the ground. The muon travels toward Earth at nearly c_0 and decays just before reaching the ground. An observer traveling along with the muon determines that it lives for 2.2×10^{-6} s. According to this observer, what is the distance between the ground and the muon at the instant the muon is created?

① GETTING STARTED The problem mentions two reference frames: the Earth reference frame and the reference frame of the muon. The altitude is measured in the Earth reference frame, whereas the muon's lifetime is measured in the reference frame of the muon. The muon's lifetime is the time interval between its creation at an altitude of 6.0 km (event 1) and its decay at the ground (event 2). The speed of the muon in the Earth reference frame is (nearly) c_0 . I summarize this information in tabular form:

	Earth reference frame	Muon reference frame
Δt		2.2×10^{-6} s (proper time interval)
h_{atm}	6.0 km (proper length)	?
v_{muon}	c_0	

② DEVISE PLAN The 6.0-km distance traveled by the muon in the Earth reference frame is the proper length of the atmosphere. To calculate the (contracted) distance traveled by the muon in the muon reference frame, I can use Eq. 14.28, but I first need to determine γ for the motion of the muon. I can determine the time interval between events 1 and 2 in the Earth

reference frame by dividing the distance traveled in the Earth reference frame (6.0 km) by the muon speed, which I take to be c_0 . This time interval is the lifetime of the muon in the Earth reference frame. Because I know the muon lifetime in the reference frame of the muon (2.2×10^{-6} s), which is the proper time interval, I can use Eq. 14.13 to determine γ .

③ EXECUTE PLAN The distance traveled in the Earth reference frame is 6.0×10^3 m, and so in this reference frame the time interval between events 1 and 2 is $\Delta t_v = (6.0 \times 10^3 \text{ m}) / (3.00 \times 10^8 \text{ m/s}) = 2.0 \times 10^{-5}$ s. In the muon reference frame, the muon is at rest and exists for the proper time interval $\Delta t_{\text{proper}} = 2.2 \times 10^{-6}$ s. I can obtain γ from Eq. 14.13:

$$\gamma = \frac{\Delta t_v}{\Delta t_{\text{proper}}} = \frac{2.0 \times 10^{-5} \text{ s}}{2.2 \times 10^{-6} \text{ s}} = 9.1.$$

Using this value for γ , I determine the distance ℓ_v traveled by the muon in its reference frame from Eq. 14.28:

$$\ell_v = \frac{\ell_{\text{proper}}}{\gamma} = \frac{6.0 \times 10^3 \text{ m}}{9.1} = 6.6 \times 10^2 \text{ m. } \checkmark$$

④ EVALUATE RESULT The muon lifetime in the Earth reference frame is longer than the muon's proper lifetime, as it should be. Because the muon lives longer in the Earth reference frame, it can travel a greater distance (6.0 km instead of 660 m). I can also check my result by considering the situation from the muon's reference frame. In that reference frame, the atmosphere flies by at nearly the speed of light for a time interval corresponding to the muon's proper lifetime, and so $\ell_v = c_0 \Delta t_{\text{proper}} = (3.00 \times 10^8 \text{ m/s})(2.2 \times 10^{-6} \text{ s}) = 6.6 \times 10^2 \text{ m}$, which is the result I got.

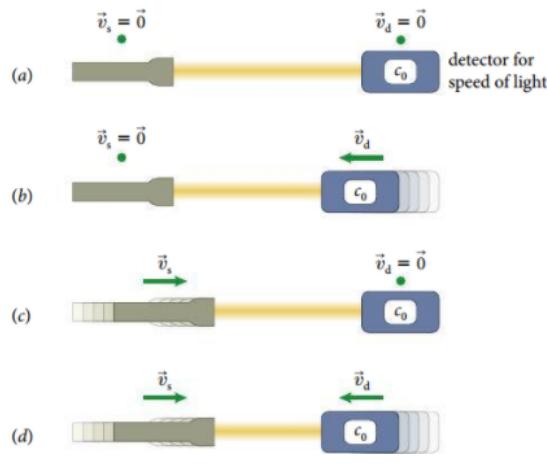
Logical consequence #4: Simultaneity

14.2 Simultaneous is a relative term

-The speed of light is invariant

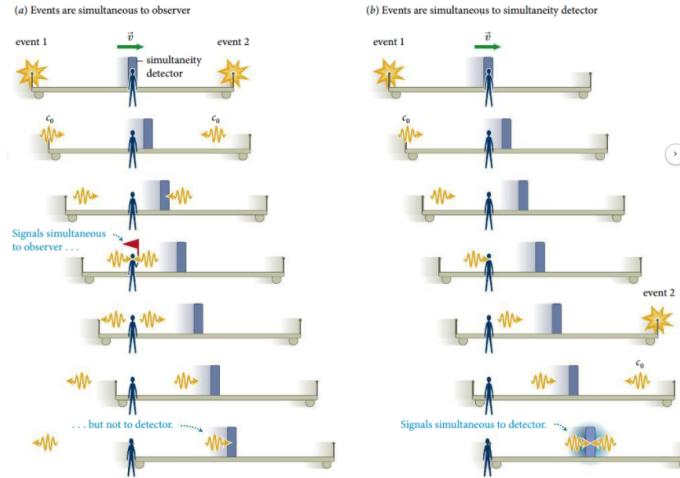
The measured value of the speed of light propagating in vacuum is c_0 , independent of the motion of the source relative to the observer making the measurement.

Figure 14.10 The measured value of the speed of light in vacuum does not depend on the speeds of the source and the measuring device.



-Moving Detector chases/runs away from light signal

Figure 14.11 If the sources and the simultaneity detector move together at high speed relative to an observer, then (a) events that are simultaneous to the observer will not be so to the detector. (b) To make the signals strike the simultaneity detector at the same instant, the source behind the detector must emit its signal before the source in front of the detector.



Special Relativity: Velocity Addition

-Velocity Addition, it just doesn't simply add up because special relativity.

$$-c = c + c$$

-Well, we suck and will only be looking at cases where velocity are added along the same axis

-Note that the velocity addition formula is for finding the speed of an object relative to the "ground" when its speed in some moving frame is known.

$$v_{Ebx} = \frac{v_{tbx} + v_{Etx}}{1 + \frac{v_{tbx} v_{Etx}}{c^2}}$$

if unitless $\beta = \frac{\beta' + \beta_{\text{rel}}}{1 + \beta' \beta_{\text{rel}}}$

$$\beta_{AB} = \frac{\beta_A - \beta_B}{1 - \beta_A \beta_B}$$

-If the speed of the object is known relative to the "ground," its speed in the moving frame can be found by rearranging the equation.

Velocity addition: From the Lorentz transformation equations, we find that v_{Aox} , the x component of the velocity of an object o measured in reference frame A , is related to v_{Box} , the x component of its velocity measured in reference frame B , by

$$v_{Box} = \frac{v_{Aox} - v_{ABx}}{1 - \frac{v_{ABx}}{c_0^2} v_{Aox}}. \quad (14.33)$$

$$\text{if unitless } \beta = \frac{\beta_{\text{rel}} - \beta'}{1 - \beta' \beta_{\text{rel}}} \quad \beta_B = \frac{\beta_A - \beta_{AB}}{1 - \beta_A \beta_{AB}}$$

-Look bruv, it works with speed of light

$$v_{Elx} = \frac{v_{ttx} + v_{Etx}}{1 + \frac{v_{ttx} v_{Etx}}{c^2}} = \frac{c + c}{1 + \frac{c \cdot c}{c^2}} = \frac{2c}{2} = c$$

Special Relativity: Momentum and Energy

-A particle's inertia depends on its speed and is related to its mass (rest mass), which is invariant.

-Moving closer to c , the inertia increases to almost infinity. Else, Lorentz factor is ~ 1 at low speed, the inertia is same as mass.

-This makes sense with "energy and momentum are conserved".

-Since mass & inertia is different with Special Relativity, we should revisit kinetic energy, momentum.

-Since inertia increases with speed, it's harder to increase a particle's speed at, say, $0.8c$ than it is at $0.2c$.

Concepts The internal energy E_{int} of an object (energy not associated with the object's motion) is an invariant and is proportional to the object's mass m (the property that determines its gravitational interaction). Mass is also an invariant.

Unlike mass, inertia (a quantitative measure of an object's tendency to resist any change in its velocity) is not an invariant: What an observer measures for an object's inertia depends on the motion of the observer relative to the object.

Quantitative Tools For an object of mass m , its inertia m_v is

$$m_v = \gamma m,$$

its momentum \vec{p} is

$$\vec{p} = \gamma m \vec{v},$$

and its kinetic energy K is

$$K = (\gamma - 1)m c_0^2.$$

The energy E of an object of mass m and inertia m_v is

$$E = m_v c_0^2,$$

and its internal energy E_{int} is

$$E_{\text{int}} = m c_0^2.$$

Special Relativity: Summary

- The speed of light can be used as a conversion factor between units for measuring space and units for measuring time.
- The measured time between two events is not the same for all observers. (proper time = shortest time where observer is fastest in situation)
- Lengths and the measured distance between two events are not the same for all observers. (proper length = longest distance where observer is at rest in situation)
- Events that are simultaneous for one observer are not for another (and the time order of events can sometimes be different for different observers).
- Relative velocities don't simply add up.
- A particle's inertia depends on its speed and is related to its mass, which is invariant.

Percent Error

$$\boxed{\% \text{ error} = \frac{\text{approximation} - \text{real}}{\text{real}} \cdot 100\%}$$