3/30:

-Units a cuh

Quantity	Unit Name (Symbol)	Formula
Frequency	hertz (Hz)	s^{-1}
Force	newton (N)	$kg.m/s^2$
Energy or work	joule (J)	$N.m$
Power	watt (W)	J/s
Electric charge	coulomb (C)	$A.s$
Electric potential	volt (V)	J/C
Electric resistance	ohm (Ω)	V/A
Electric conductance	siemens (S)	A/V
Electric capacitance	farad (F)	C/V
Magnetic flux	weber (Wb)	$V.s$
Inductance	henry (H)	Wb/A

Quantity	Basic Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	Ampere	A
Thermodynamic temperature	degree Kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

Prefix	Symbol	Power
atto	a	10^{-18}
femto	f	10^{-15}
pico	p	10^{-12}
nano	n	10^{-9}
micro	µ	10^{-6}
milli	m	10^{-3}
centi	c	10^{-2}
deci	d	10^{-1}

Prefix	Symbol	Power
deka	da	10
hecto	h	10^2
kilo	k	10^3
mega	M	10^6
giga	G	10^9
tera	T	10^{12}

-Voltage and Current definition

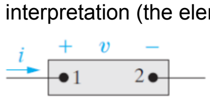
- Potential difference or voltage between two points
  - The work done in moving a unit charge from one point to another in an electric field, measured in j per C (volts)
  - Voltage is the energy per unit of charge, $v = \frac{dw}{dq}$
- Current
  - The current i through an area A is defined as the amount of charge passing through A per second, measured in C per s (amps)
  - Current is the rate of flow of charge, $i = \frac{dq}{dt}$

-Power agane

- Power associated with a circuit element is consumed by that circuit element when the value of power is positive.
- Conversely, power is generated, or produced by the element if the value consumed is negative.

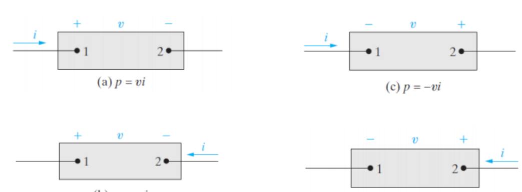
$$p = \frac{dw}{dt} = \left(\frac{dw}{dq}\right)\left(\frac{dq}{dt}\right) = vi$$

-Power sign convention and interpretation (the element in pic is not a battery)



- Interpretation

	Positive Value	Negative Value
v	voltage drop from terminal 1 to terminal 2 *or* voltage rise from terminal 2 to terminal 1	voltage rise from terminal 1 to terminal 2 *or* voltage drop from terminal 2 to terminal 1
i	positive charge flowing from terminal 1 to terminal 2 *or* negative charge flowing from terminal 2 to terminal 1	positive charge flowing from terminal 2 to terminal 1 *or* negative charge flowing from terminal 1 to terminal 2



(a) $p = vi$ (b) $p = -vi$ (c) $p = -vi$ (d) $p = vi$

- Positive power means the power is consumed by the element and negative power means the power is produced by the element.

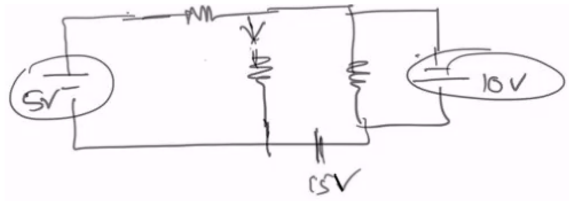
$V_1 = ??$ $V_2 = 20V$

$= 20 + (-10)$

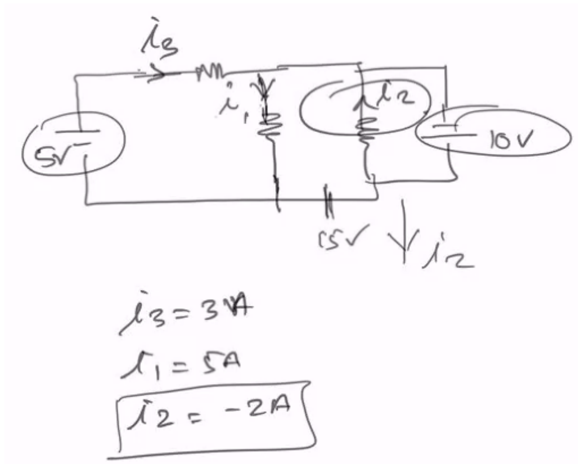
$= 10V$

-Current direction guessing process

-Say you have this wonky 3 battery circuit. What is the direction of current through each resistor?

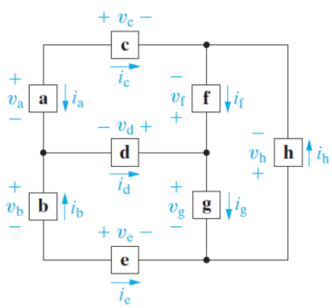


-It's hard to tell, so just guess and check. Then if the voltage is negative, the current guessed is the opposite way.



-i_2 was the opposite way.

-Example diagram



Component	$v\ (V)$	$i\ (A)$	$P(W)$
a	120	10	1200
b	120	-9	-1080
c	10	10	1000
d	10	-1	-10
e	−10	−9	90
f	−100	5	-500
g	120	4	480
h	−220	−5	1100

- b, d, f and h produce power and the other elements consume power.

Book Chapter 1: Circuit Variables

-Assumptions for our study of circuits

-Lumped-Parameter System: A circuit small enough that the effects happen basically instantaneously.

-The net charge on every component in the system is always zero.

-There is no magnetic coupling between the components in a system.

-Lumped-Parameter System

-If the dimension of the system is 1/10th (or smaller) of the dimension of the wavelength, you have a lumped-parameter system.

-US power system, the wavelength is 5,000,000 m (60 Hz @ speed of light). So Lumped-Parameter System is 500,000 m or less.

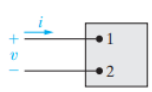
-Radio system, 0.3 m (1,000,000,000 Hz @ speed of light). So Lumped-Parameter System is 3 cm or less.

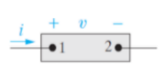
-Ideal Circuit Element

-2 terminals

-Described mathematically in terms of current and/or voltage

-cannot be subdivided into smaller elements





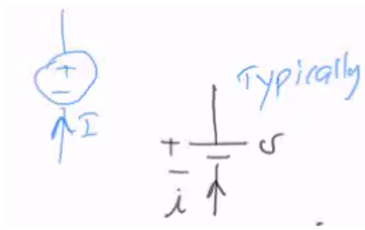
-Summary

Summary

- The International System of Units (SI) enables engineers to communicate in a meaningful way about quantitative results. Table 1.1 summarizes the base SI units; Table 1.2 presents some useful derived SI units. (See pages 8 and 9.)
- Circuit analysis is based on the variables of voltage and current. (See page 11.)
- **Voltage** is the energy per unit charge created by charge separation and has the SI unit of volt ($v = dw/dq$). (See page 12.)
- **Current** is the rate of charge flow and has the SI unit of ampere ($i = dq/dt$). (See page 12.)
- The **ideal basic circuit element** is a two-terminal component that cannot be subdivided; it can be described mathematically in terms of its terminal voltage and current. (See page 12.)
- The **passive sign convention** uses a positive sign in the expression that relates the voltage and current at the terminals of an element when the reference direction for the current through the element is in the direction of the reference voltage drop across the element. (See page 13.)
- **Power** is energy per unit of time and is equal to the product of the terminal voltage and current; it has the SI unit of watt ($p = dw/dt = vi$). (See page 15.)
- The algebraic sign of power is interpreted as follows:
  - If $p > 0$, power is being delivered to the circuit or circuit component.
  - If $p < 0$, power is being extracted from the circuit or circuit component. (See page 16.)

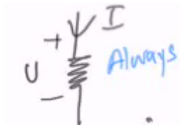
4/1:

-Source giving power (typically + to -)

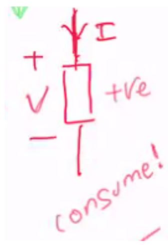


-It is possible for an irl source to consume power like a rechargeable battery.

-Resistor is always + to -



-If current is flowing from high potential to low potential (+ to -), it’s consuming power. The opposite current flow is giving power.



-Power: Work & Energy

Energy

$\Rightarrow E = P \times t$

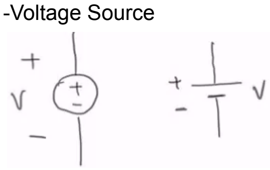
$\Rightarrow E = \int P\,dt$

Power $P = \frac{dw}{dt} \Rightarrow W = \int P\,dt$

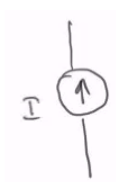
-Circuit elements

-Independent Sources

-Voltage Source

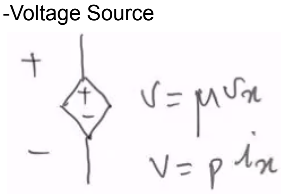


-Current Source

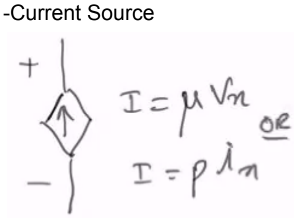


-Dependent Sources (The source depends on the voltage or current) (We only covering linear relationship.) (\mu and p are scalars for the dependent realtionship)

-Voltage Source



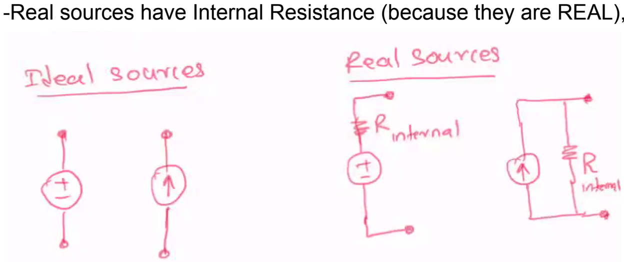
-Current Source



-Ohm’s not Law V=IR: If Voltage is constant, Current varies. If Current is constant, Voltage varies.

-Ideal Sources VS Real Sources

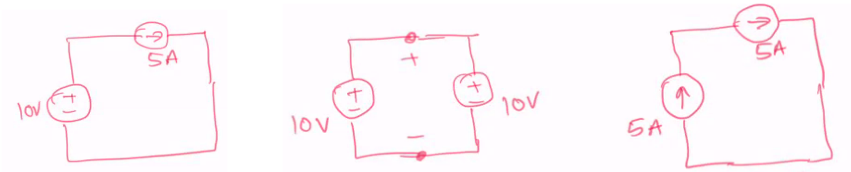
-Real sources have Internal Resistance (because they are REAL), creating some calculation error from Ideal.



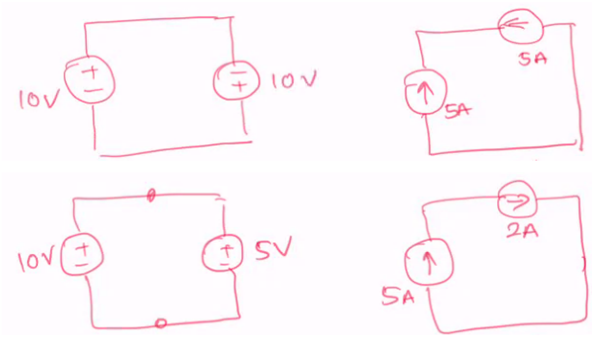
-Connecting Ideal Sources

-Within one loop, there is only ONE current. Between 1 terminal, there is only ONE voltage.

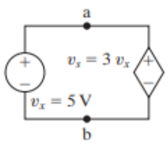
-Legal because same direction and same value.



-Illegal. Will cause Terrorist Wins irl.



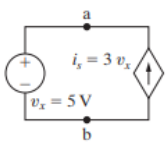
-With Dependant Source



(a)

✗ $V_s = 3V_x$
$= 3 \times 5$
$= 15V$

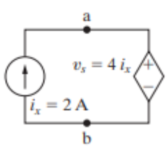
(Here, the dependant voltage element depends on the voltage)



(b)

✓ $i_s = 3V_x$
$= 3 \times 5$
$= 15A$

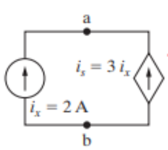
(Here, the dependant current element depends on the voltage)



(c)

✓ $V_x = 4i_x$
$= 4 \times 2$
$= 8V$

(Here, voltage depends on current)



(d)

✗ $i_x = 3i_x$
$= 3 \times 2$
$= 6A$

(Here, current depends on current)

-Ohm's Law: V=IR

-This is true when Temp is constant

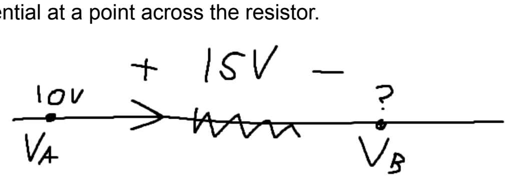
$$\boxed{R_t = R_0(1 + \alpha t)}$$

-Conductance (SI: Siemens gachibass)

$$G = \frac{1}{R}$$

conductance / S (Siemens)

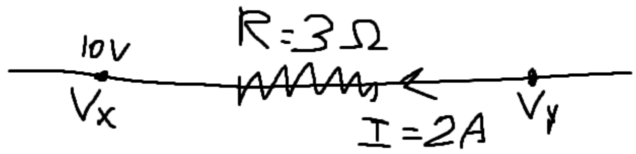
-Finding Potential at a point across the resistor.



$V_A = 10V$

$V_B = V_A - 15V = (10-15)V = -5V$

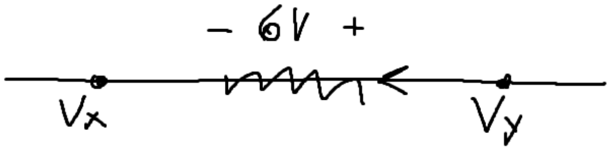
-Combine with Ohm’s Law



$V_x = 10V$

$V = IR = 2 \cdot 3 = 6V$

$\downarrow$

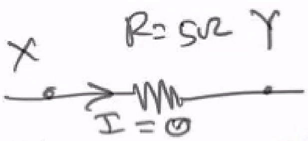


$V_y = V_x + 6V$

$= 10 + 6V$

$= 16V$

-When there’s no current



$V_x = 20V$

$V = IR$

$V = 0$

$V_Y = ?$

$V_x = V_Y$

$V_Y = 20V$

-Power & Ohm’s Law (\deltaV btw) (Power supplied/consumed)

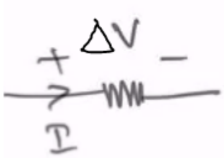
$P = VI$

$I = V/R$

$P = V \cdot \frac{V}{R} = \frac{V^2}{R}$

$\boxed{P = \frac{V^2}{R}}$

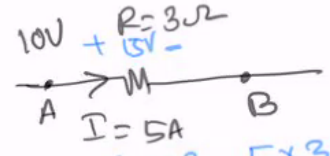
-Calculating Power dissipated/absorbed (causes lose in V) by resistor



$\boxed{P = \Delta V I = I^2 R = \frac{\Delta V^2}{R}}$

I ⇒ current going through the resistor

ΔV ⇒ voltage difference accross the resistor



$V = IR = 5 \times 3 = 15V$

$V_B = ?$

$V_B = V_A - 15$
$= 10 - 15$
$= -5V$

$P_R = VI$
$= 15 \times 5 = \underline{75W}$

$P_R = I^2 R$
$= 5^2 \times 3$
$= \underline{75W}$

$P_R = V^2/R = \frac{15^2}{3}$
$= 75W$

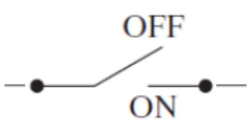
4/6:

-Conductance and Ohm's Law

$\boxed{V = IR}$

$\boxed{I = GV}$

-Switch a cuh

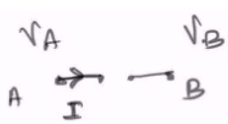


-Short Circuit

short circuit

$V_A = V_B$

-Open Circuit



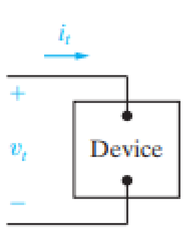
open circuit

$V_A \neq V_B$

$I = 0$

-Example 2.5

The voltage and current are measured at the terminals of the device illustrated in Fig. 2.13(a), and the values of v_t and i_t are tabulated in Fig. 2.13(b). Construct a circuit model of the device inside the box.

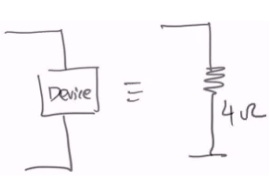


(a)

v_t (V)	i_t (A)
−40	−10
−20	−5
0	0
20	5
40	10

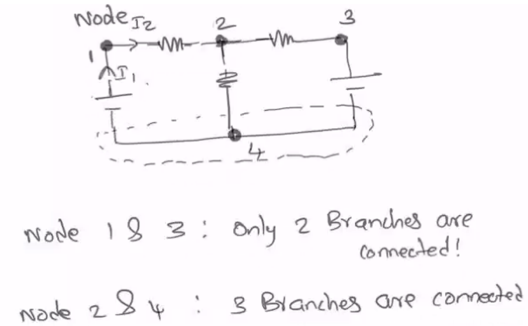
(b)

-Since linear relationship, Ohm's Law: R=4 ohms internal resistance.

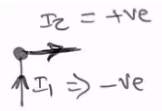


-U can model it like this, just a resistor unless the device is specified.

-Node and Branches

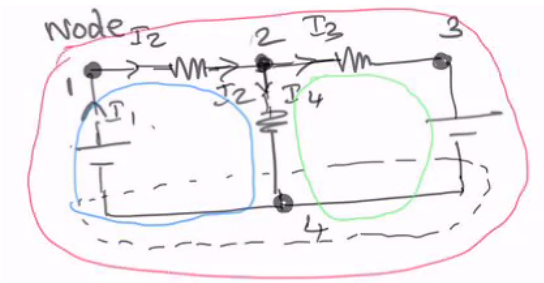


-Current entering vs leaving sign convention (idk why v_e, should be I for current?)

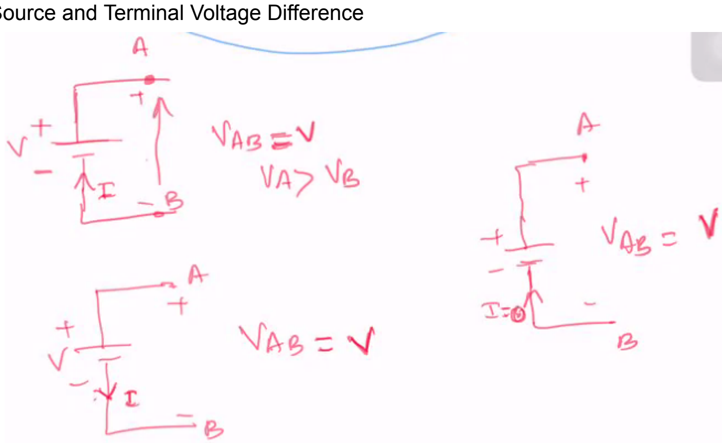


-Closed Loop (Mesh)

-There's 3 here (but 2 is good enough here since outer loop is not that important)



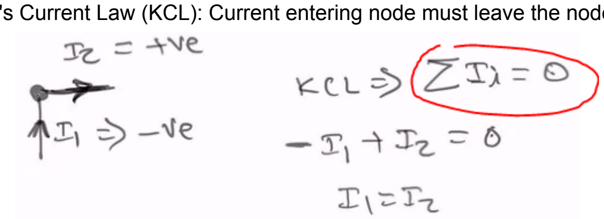
-Voltage Source and Terminal Voltage Difference



-Source always has a constant voltage (and current for Current Source).

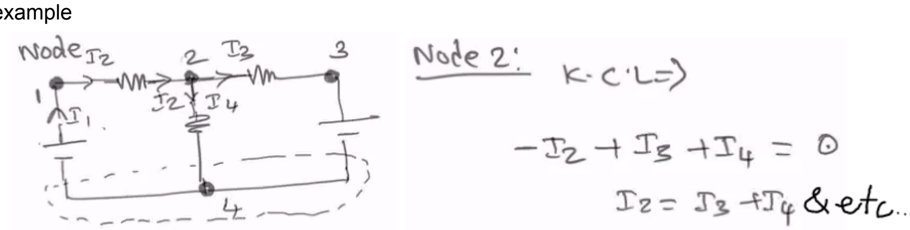
-Kirchoff's Laws

-Kirchhoff's Current Law (KCL): Current entering node must leave the node. Or, the algebraic sum of all current (so source I too) through a node = 0.

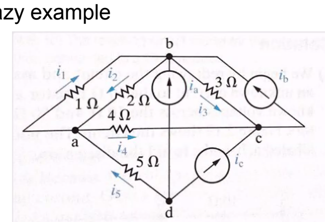


-

-Node 2 example



-More crazy example



(The middle is not a node, it's 2 independent wire. Pic is poop. Needs the little hump)

Node a ⇒

$+i_1 - i_2 + i_4 - i_5 = 0$

Node b ⇒

$-i_1 + i_2 + i_3 - i_a - i_b = 0$

Node c ⇒

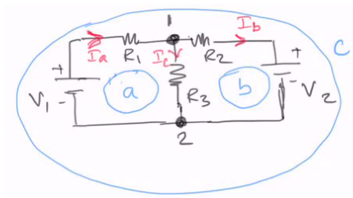
$-i_3 - i_4 + i_b - i_c = 0$

Node D ⇒

$+i_5 + i_a + i_c = 0$

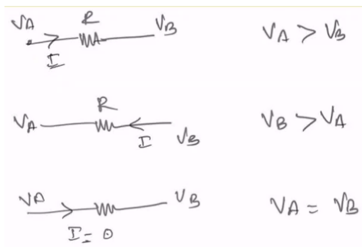
-Node D looks wrong, but this diagram is a "prediction" notation. After calculating values, they will cancel out, some current is negative, meaning the direction is actually flipped.

-Kirchhoff's Voltage Law (KVL): An algebraic sum of the voltage around a closed loop is 0.

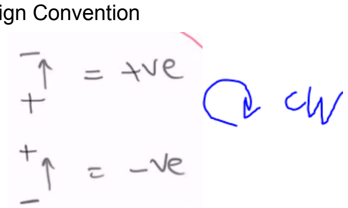


(The other two is avoided because they are junctions, 2 branches are boring)

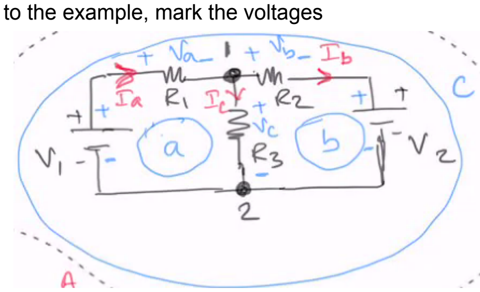
-Voltage and Current



-Book's Sign Convention



-So back to the example, mark the voltages



(This is an assumption. Until we calculate the values, it might be the wrong notated direction.)

-KCL

Node 1 :

$-I_a + I_b + I_c = 0$

Node 2:

$+I_a - I_b - I_c = 0$

-KVL (These signs are the opposite of the book's convention)

Closed path (mesh) a ⇒

$V_1 - V_a - V_c = 0$

(V_1 is considered +, V_a and V_c are opposite. Relationship is key)

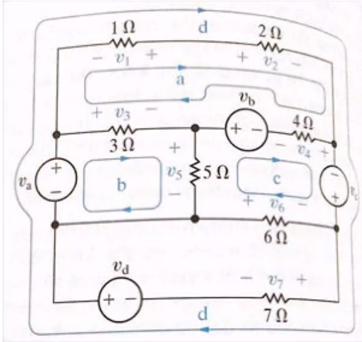
Mesh b ⇒

$+V_c - V_b - V_2 = 0$

mesh c ⇒

$+V_1 - V_a - V_b - V_2 = 0$

-Another example



-KCL

mesh a ⇒

$$-v_1 + v_2 + v_4 - v_b - v_3 = 0$$

mesh b ⇒

$$-v_a + v_3 + v_5 = 0$$

mesh c ⇒

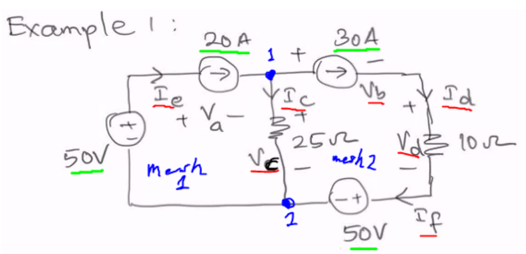
$$-v_5 + v_b - v_4 - v_c - v_6$$

mesh d ⇒

$$-v_a - v_1 + v_2 - v_c + v_7 - v_d = 0$$

-Example of both KVL and KCL:

-Prof random circuit



$$I_e = 20A$$

$$I_f = 30A$$

$$I_d = 30A$$

KCL Node 1: $-I_e + I_c + I_d = 0 \rightarrow I_c = -10A$

$-20 + I_c + 30 = 0$

$V_c = IR = (-10A)(25\Omega) = -250\ V$

$V_d = IR = (10\Omega)(30A) = 300V$

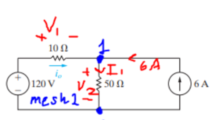
V_a = KVL mesh 1: $V_c + V_a - 50V = 0$

$-250 + V_a - 50 = 0 \longrightarrow V_a = 300\ V$

V_b = KVL mesh 2: $V_b + V_d + 50V - V_c = 0 \rightarrow V_b = -600V$

$V_b + 300 + 50 + 250 = 0$

-Example 2.8



1. Mark all currents
2. Mark all Voltages
3. Mark nodes

i_0 = KCL Node 1: $I_1 - i_0 - 6A = 0$

KVL mesh 1: $-120V + V_1 + V_2$

Ohm's $10\Omega = i_0 \cdot 10\Omega$

Ohm's $50\Omega = I_1 \cdot 50\Omega$

Lin. Algebra $\begin{cases} -\underline{120}V + 10i_0 + 50I_1 = 0 \\ -i_0 - \underline{6A} + I_1 = 0 \end{cases}$

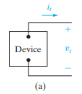
$i_0 = -3A$

$I_1 = 3A$

* Underlined are just constants. I_1 & i_0 are Variables

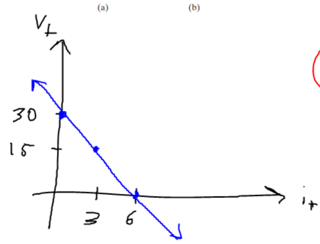
(The linear algebra part is a set of KVL equations, they're voltages.)

-Example 2.9 (modelling a circuit device)



v_1 (V)	i_1 (A)
30	0
15	3
0	6

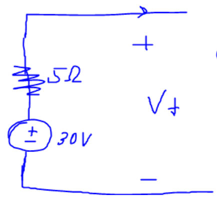
(b)



$V_+ = -5(i_+) + 30$

or

$i_+ = \frac{-1}{5}(V_+) + 6$



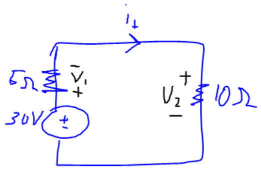
model

KCL: $-30V + V_1 + V_+ = 0$

$V_+ = 30 - V_1$

$V_+ = 30 - 5(i_+)$

-Part is finding power with the device in a circuit with 10 ohms.



$P = ?$

KVL: $-30 + V_1 + V_2 = 0$

Ohm's: $V_1 = 5i_+$

$V_2 = 10i_+$

$-30 + 5i_+ + 10i_+ = 0$

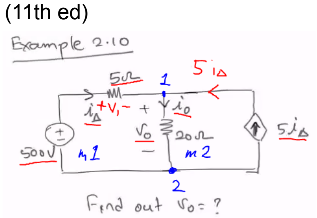
$\therefore i_+ = 2A$

$P = I^2R$

$= (2A)^2(10\Omega)$

$= 40\ W$

-Example 2.10 (11th ed)



KCL Node 1: $-i_\Delta - 5i_\Delta + i_o = 0 \rightarrow i_o = 6i_\Delta$

KVL m1: $V_o - 500V + V_1 = 0$

Ohm's: $V_1 = 5i_\Delta$

$V_o = 20i_o$

$-500 + 5i_\Delta + 20i_o = 0$

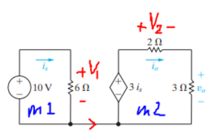
$-500 + 5i_\Delta + 20 \cdot 6i_\Delta = 0$

$i_o = 24A$

$V_o = 20i_o = 20 \cdot 24 = 480V$

-Example 2.10

$V_o = ?$



KVL m1: $-10 + V_1 = 0 \rightarrow V_1 = 10V$

ohm's: $i_s = V_1/(6\Omega) = \frac{10}{6}A$

KVL m2: $-3i_s + V_2 + V_o = 0$

$-\frac{30}{6} + V_2 + V_o$

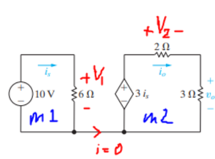
ohm's: $V_2 = i_o(2\Omega) = 2i_o$

$V_o = i_o 3\Omega = 3i_o$

$-5 + 2i_o + 3i_o = 0$

$i_o = 1A$

$\therefore$ Ohm's $V_o = 3\Omega(i_o) = 3V$



Part b.

• Power Consume VS Dissipated

• should be =

• Battery consume

• Ressistor dissipate

Dissipated:

$P_{6\Omega} = V_1^2/R = \frac{100}{6}W$

$P_{3\Omega} = I^2R = 1^2(3) = 3W$

$P_{2\Omega} = I^2R = 1^2(2) = 2W$

$P_D = \frac{100}{6} + 3 + 2 = 21.7\ W$

Consumed:

$P_{10V} = (10V)(i_s) = \frac{100}{6}W$

$P_{3i_s} = (3i_s)(i_o) = \frac{30}{6} = 5W$

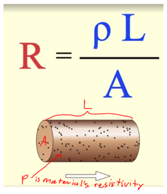
$P_C = \frac{100}{6} + 5 = 21.7W$ ✓

4.13 Resistive Circuits:

-Making a circuit, we want the least resistance possible to accomplish our goal. More resistance is wasting power.

-Material resistance

-rho = resistivity (SI: ohm*m)



-But resistance increases with temperature

$$R(T) = R_0\left(1 + \alpha\,(T - T_0)\right)$$

$T_0 \Rightarrow$ Reference Temperature (20°C)

-alpha is a "fudge factor" constant that depends on the material (SI: ohm).

-alpha for aluminium and copper. The higher it is, the more temperature raises resistivity.

$$\alpha_{Al} \Rightarrow 0.004308$$

$$\alpha_{Cu} \Rightarrow 0.004041$$

-sigma = conductance (SI: ohm^-1*m^-1= S)

$$R = \frac{\ell}{\sigma A} \qquad \sigma = \frac{1}{\rho}$$ is the conductivity of the metal.

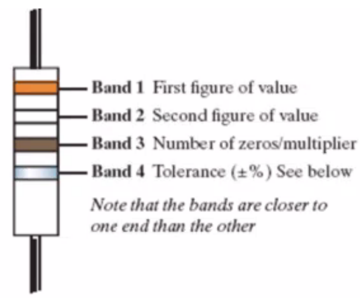
-Resistance identification

Resistor color code

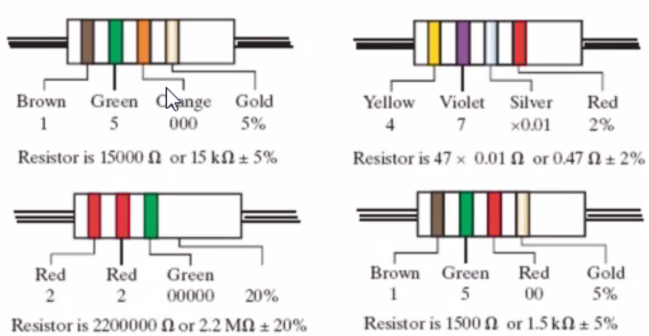
Band color	Value
Black	0
Brown	1
Red	2
Orange	3
Yellow	4
Green	5
Blue	6
Violet	7
Grey	8
White	9
Gold	0.1
Silver	0.01

Tolerance color code

Band color	±%
Brown	1
Red	2
Gold	5
Silver	10
None	20



- Examples:



-Effective Resistance (these relationship comes from doing KVL and Ohm's Law)

-Resistance in series

$$R_{tot} = R_1 + R_2 + R_3 + \ldots$$ (He uses R_eq)

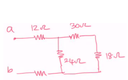
-Resistance in parallel (can use to REDUCE resistance opposed to the singular resistor)

$$\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$ or $$R_{eq} = \frac{R_1 + R_2 + \ldots}{R_1 R_2 \ldots}$$

-If all same resistors

$$R_{eq} = \frac{R}{n}$$

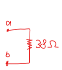
-Examples

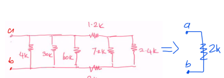


$$12 + \left(\frac{1}{30+18} + \frac{1}{24}\right)^{-1} + 10 = R_{eq}$$

$$38\,\Omega = R_{eq}$$

$\therefore$

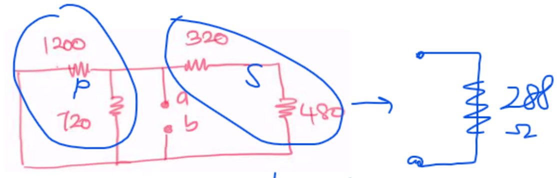




$$\left(\frac{1}{4k} + \frac{1}{30k} + \frac{1}{60k} + \frac{1}{1.2k + \left(\frac{1}{7.2k} + \frac{1}{2.4k}\right)^{-1} + 2k}\right)^{-1} = R_{eq}$$

$$2000 = R_{eq}$$

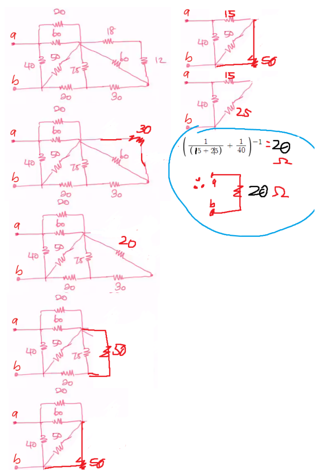
-Terminal a & b are in the middle



$$\left(\frac{1}{(320+480)} + \frac{1}{\left(\frac{1}{1200}+\frac{1}{720}\right)^{-1}}\right)^{-1} = R_{eq}$$

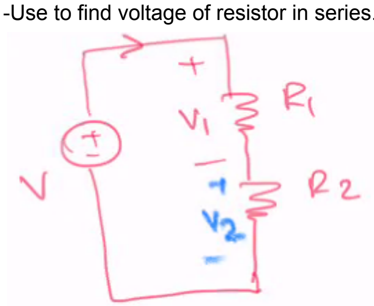
$$288\ \Omega = R_{eq}$$

-Super aids



-Voltage Divider Rule & Circuit (2 in series resistance, use KVL & Ohm's)

-Use to find voltage of resistor in series.



$$V = V_1 + V_2$$

$$V = IR_1 + IR_2$$

$$I = \frac{V}{R_1 + R_2}$$

$$V_1 = IR_1$$

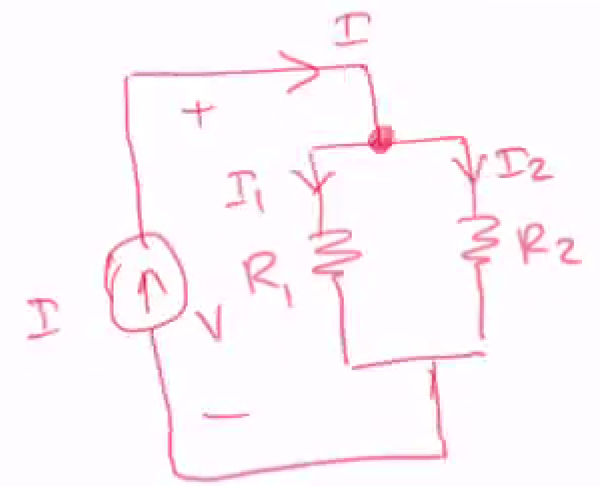
$$V_1 = \frac{V}{R_1+R_2} \cdot R_1$$

$$\boxed{V_1 = \left(\frac{R_1}{R_1+R_2}\right) V}$$

$$V_1 = \frac{V}{R_1+R_2} \times R_1$$

$$V_2 = \frac{V}{R_1+R_2} \times R_2$$

-Current Divider Rule & Circuit (2 resistor in parallel, use KCL & Ohm's)



$$K.C.L \Rightarrow I = I_1 + I_2$$

$$V = I_1 R_1 = I_2 R_2 \quad \text{—} \quad (1)$$

$$I = \frac{V}{R_1} + \frac{V}{R_2}$$

$$= \left(\frac{R_1 + R_2}{R_1 \cdot R_2}\right) V$$

$$V = I\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

(Steps)

$$(1) \Rightarrow \quad I_1 = \frac{V}{R_1}$$

$$I_1 = \frac{I\,(R_1 R_2)}{(R_1 + R_2)} \cdot \frac{1}{R_1}$$

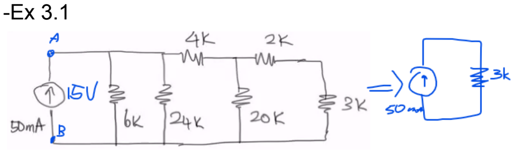
$$\boxed{I_1 = \frac{I}{R_1 + R_2} \cdot R_2} \qquad I_2 = \frac{I}{R_1 + R_2} \times R_1$$

(Multiply by the other resistor)

4.15:

-More Examples

-Ex 3.1

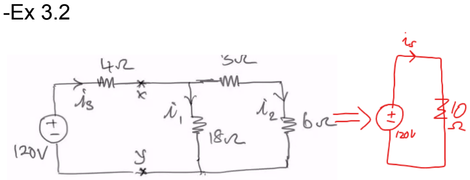


$R_{eq} = 3000\Omega$

$V = IR = 0.0050A(3000\Omega) = 15V$

$P_{source} = I^2 R = 7.5W$

-Ex 3.2



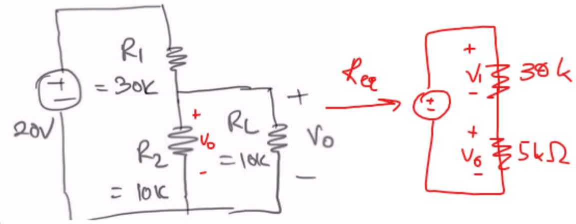
$R_{eq} = 10\Omega$

$i_s = \frac{V}{R} = \frac{120}{10} = 12A$

$i_2 = \frac{i_s(6+3)}{18+(6+3)} = 4A \qquad i_1 = \frac{i_s(18)}{18+(6+3)} = 8A \qquad i_1 + i_2 = i_s$

(U can also use KCL instead of Voltage/Current Divider)

-Ex 3.4



a) $V_o = ?$

$= \frac{20V}{35k}(5k) = 2.86\ V$

(b) $R_2 = 400\Omega$

$R_1 = 1200\Omega$

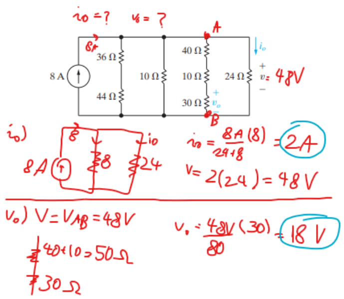
$R_L = 10k\Omega$

$R_{eq} = \frac{400 \times 10k}{400 + 10k} = 384.6\ \Omega$

$V_o = V_{R_{eq}} = \frac{20}{384.6 + 400} \times 384.6$

$= 4.85\ V$

-Ex 3.7



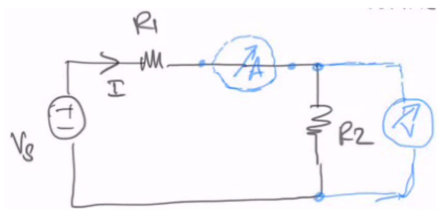
(Lesson: Don't simplify the branch that the question wants answer to)

-Using Voltmeter and Ammeter

-Ammeter connected in series

-Voltmeter connected in parallel

-Example



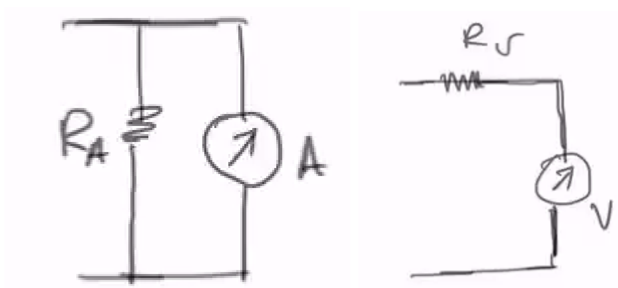
A-meter measures I, V-meter measures V across R2.

-IRL, the A-meter will not read exactly I amps since there is internal resistance in the A-meter.

-IRL, V-meter does not have infinite internal resistance. There will be current through the V-meter branch, throwing off R2 V reading.

-Also, A&V-meters have a range. "Full scale range" is the actual max input the meter can measure, and can differ from original range.

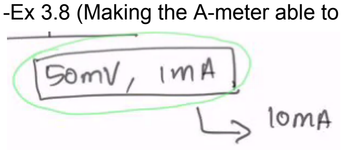
-Modeling real A-meter & V-meter



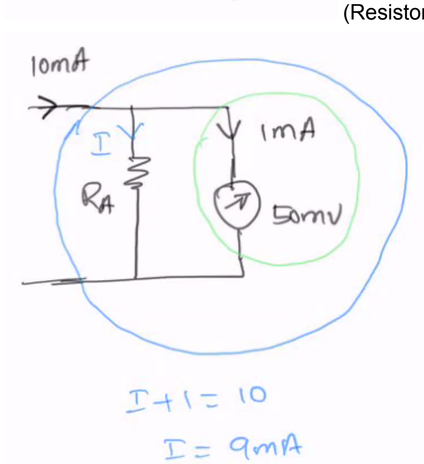
R_A & R_V is the internal resistance of each

-U can connect resistance in series for A-meter & parallel for V-meter, but it complicates calculations later on (conversion from current source to and from voltage source).

-Ex 3.8 (Making the A-meter able to accept higher current by adding resistance, changing the full scale range)



(Resistor diverts current, allow the meter to have more range. So, if meter reads 1mA w/ R_A, it's 10mA)



$R_A = ?$

$V = IR$

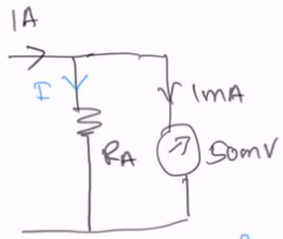
$50 \times 10^{-3} = 9 \times 10^{-3} \times R_A$

$R_A = 50/9\ \Omega$

$I + 1 = 10$

$I = 9mA$

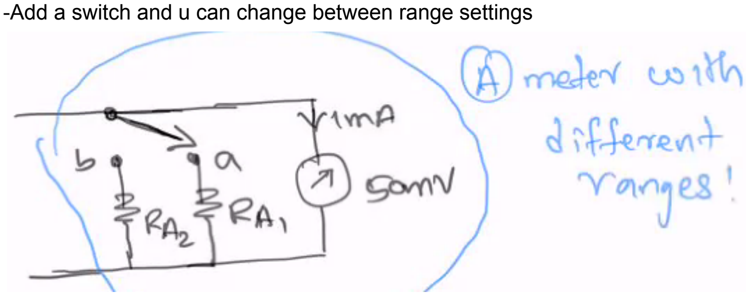
-Again but to make reading 1A.



$I = 1 - 1\times10^{-3}$
$= 999\ mA$

$V = IR$
$50\times10^{-3} = 999\times10^{-3}\ R_A$
$R_A = 50/999 = 0.050\ \Omega$
$= 50\ m\Omega$

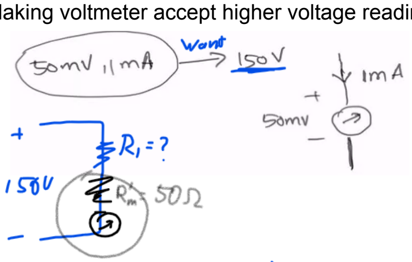
-Add a switch and u can change between range settings



-Same resistance range manipulation can be applied to V-meter.

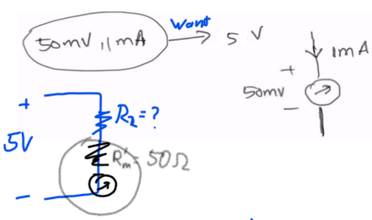
4/20:

-Ex 3.9 (Making voltmeter accept higher voltage readings, add resistance in series to lower voltage)



Orig resistance
$50mV = (1mA)(R_m)$
$50\Omega = R_m'$

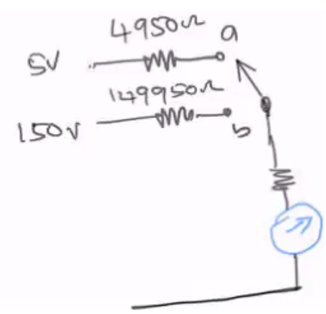
$150V = 0.001\,(R_1 + R_m')$
$0.0015 = R_1 + R_m'$
$\boxed{R_1 = 149950\ \Omega}$



Orig resistance
$50mV = (1mV)(R_m)$
$50\Omega = R_m'$

$5V = 0.001\,(R_2 + R_m')$
$0.005 = R_2 + R_m'$
$\boxed{R_2 = 4950\ \Omega} = 5k\Omega$

-Add a switch and u can switch between different voltage maximum readings

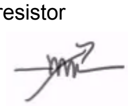


-Changing V-meter and A-meter maximum

-V-meter, add resistor in series

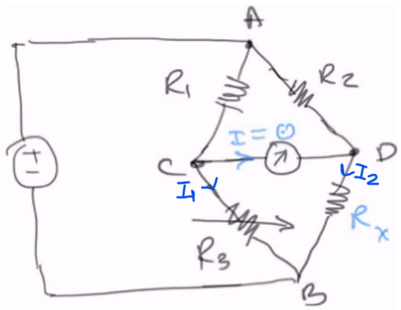
-A-meter, add resistor in parallel

-Variable resistor



-Measuring resistance (Ohm meter)

-The Wheatstone Bridge circuit/method (w/ Ammeter)



R_1 & R_2 = Known Resistors

R_3 = Variable Resistor

R_x = To be measured!

-Change R_3 to make the Ammeter read 0. That means nodes C & D have the same voltage, current doesn't travel across the A-meter bridge to favor a route.

-Using KVL to show mathematically

K.V.L; mesh ACD;

$$I_1 R_1 = I_2 R_2 \quad — \text{①}$$

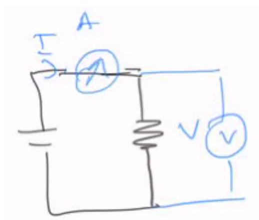
K.V.L, mesh CDB;

$$I_1 R_3 = I_2 R_x \quad — \text{②}$$

$$\frac{R_3}{R_1} = \frac{R_x}{R_2}$$

$$R_x = \left(\frac{R_2}{R_1}\right) \cdot R_3$$

-Well, why this method instead of Ohm's Law?



$V = IR$

$R = \frac{V}{I}$

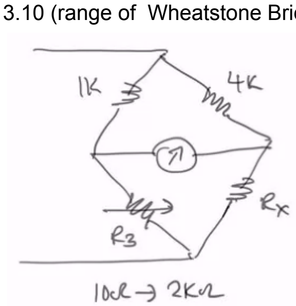
-This works, but IRL A-meter & V-meter has internal resistance, throwing off the readings more than to only A-meter only from Wheatstone.

-Well, we don't need the V-meter because we know the V-source value already!

-But IRL, the battery has internal resistance too. So without a V-meter, it's off too.

-The V-source in Wheatstone doesn't need to be accurately known too, since we don't care for it when making the A-meter read 0, and mathematically through KVL doesn't care for it too.

-Example 3.10 (range of Wheatstone Bridge)



$R_x = \frac{R_2}{R_1} R_3$

$R_x = \frac{4k}{1k} R_3$

if $R_3 = 10$
$R_x = 40\Omega$

if $R_3 = 2k\Omega$
$R_x = 8k\Omega$

$\therefore$ range : 40Ω to $8k\Omega$

-V-source to A-source & vice versa (Source Transformation)

$V_S = I_S R_{in}$

R_{in}

$V_S = I_S \cdot R_{in}$

$I_S = \frac{V_S}{R_{in}}$

$R_{in} = R_{in}$

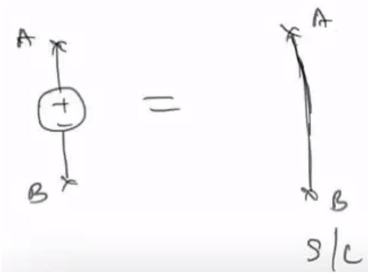
$V_S = I_S \cdot R_{in}$

$I_S = \frac{V_S}{R_{in}}$

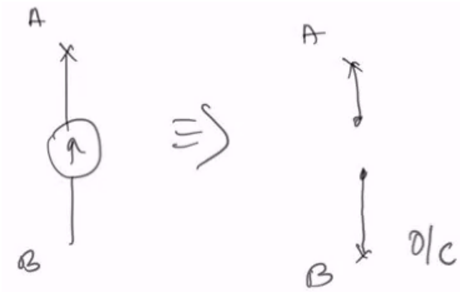
-The 2 sources are essentially the same. (When converting, the arrow points to the voltage's +)

-Deactivating Sources

-Voltage

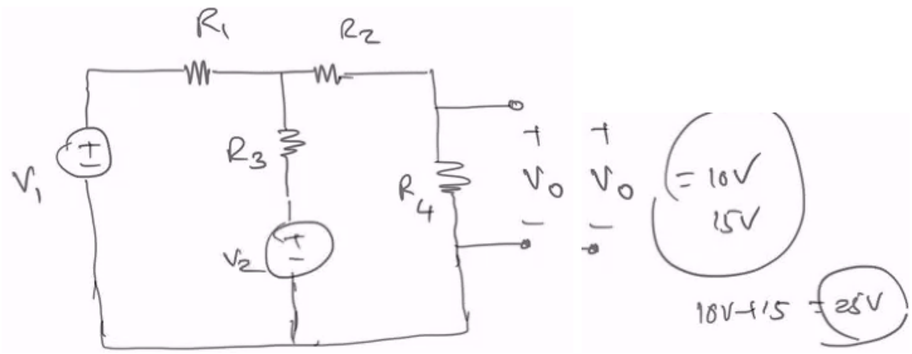


-Current



-Superposition (of sources' effect)

-Voltage

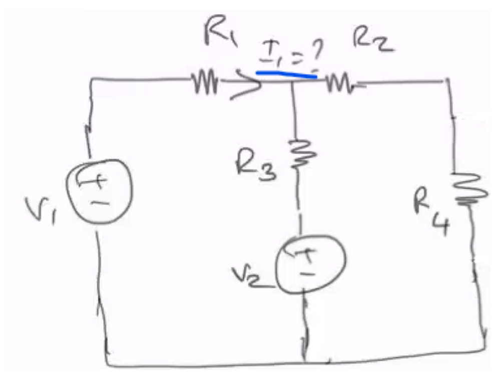


-Deactivate V1, Vo reads 15V

-Deactivate V2, Vo reads 10V

-Therefore V1 and V2 will makes Vo read 25V

-Current

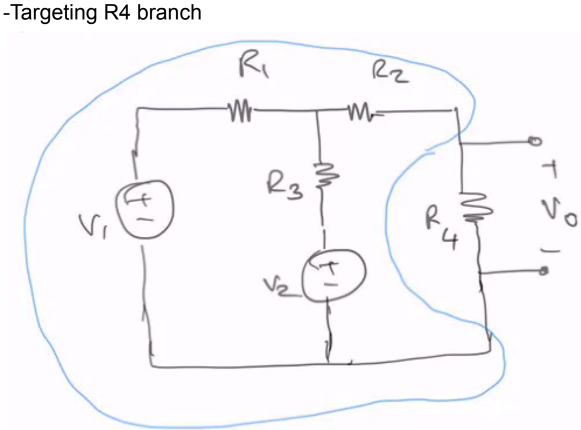


-Deactivate V1 and V2 one by one and get I1's readings for both.

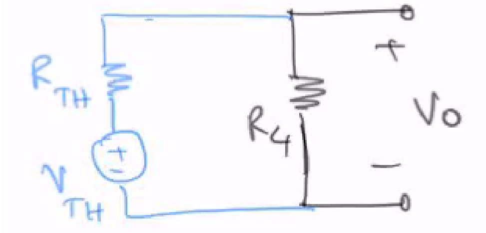
-IRL, this reduces parts.

-Thevenin & Norton Theorem (reduce everything but the targeted branch by either V-source or A-source with equivalent internal resistance)

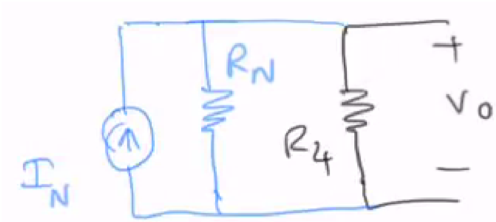
-Targeting R4 branch



-Thevenin (V-source)



-Norton (A-source)



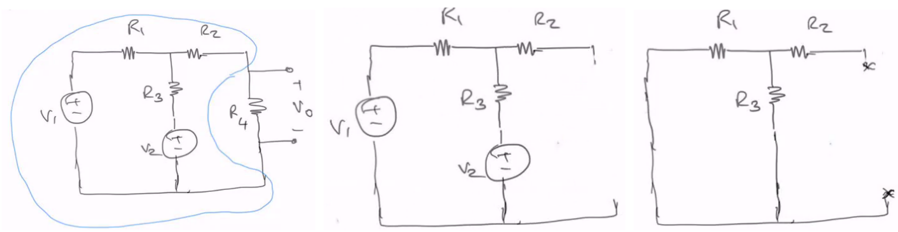
-You can convert Norton to Thevenin

-R_N = R_TH

-V_TH = I_N * R_N

-Finding the reduced circuits (Thevenin)

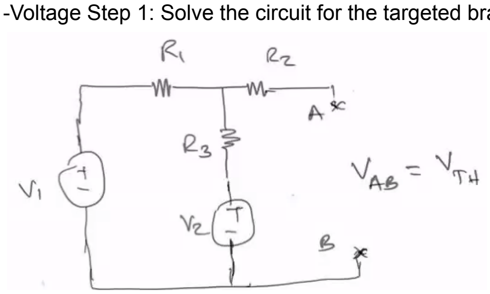
-Resistance Step 1: Deactivate Source to use Superposition



-Resistance Step 2: Find equivalent resistance (R_TH)

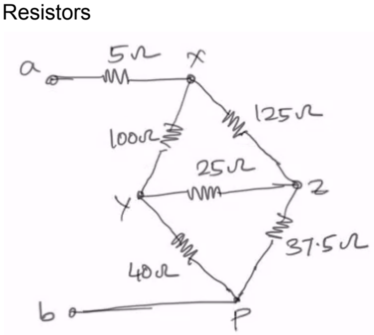
$$R_{eq} = R_1 // R_3 + R_2 \qquad R_{TH} = R_{eq} = \frac{R_1 R_3}{R_1 + R_3} + R_2$$

-Voltage Step 1: Solve the circuit for the targeted branch's terminals (V_TH)



4/22:

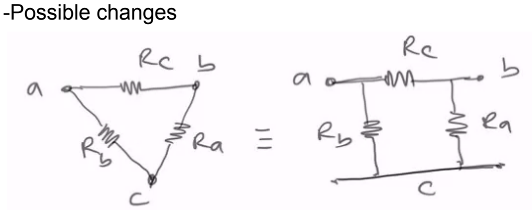
-Jebaiting Resistors



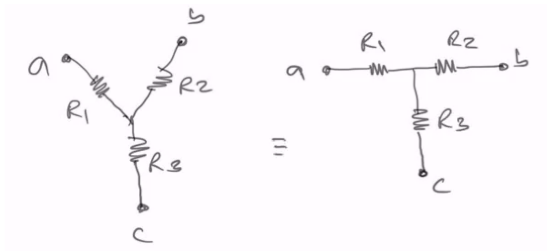
-There is no parallel or in series resistance. R_eq can't be calculated from this

-U need to change the circuit to calculate the resistance

-Possible changes

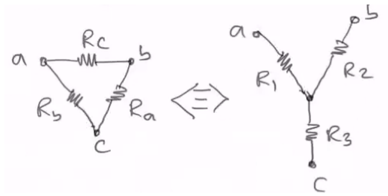


(Delta connection/configuration)



(Y / Star connection/config)

-We can convert from Delta to Y by keeping the voltages at each node the same.



$V_a = V_a$

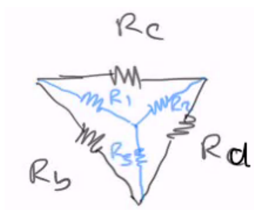
$V_b = V_b$

$V_c = V_c$

-Delta to Y

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \qquad R_2 = \frac{R_a R_c}{R_a + R_b + R_c} \qquad R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

-A way to remember



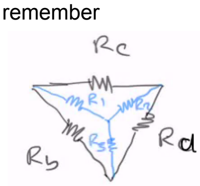
-Y to Delta

$$R_a = R_2 + R_3 + \frac{R_2 R_3}{R_1} = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_3}$$

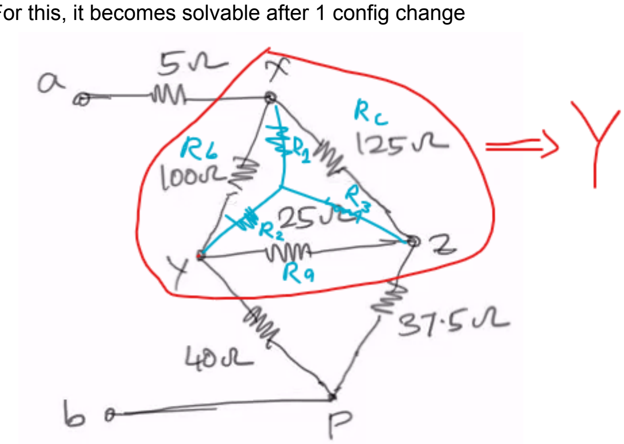
-A way to remember



The denominator is the opposite inner resistance.

-Back to example

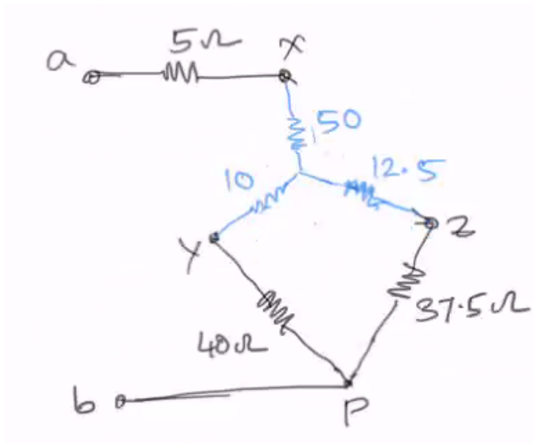
-For this, it becomes solvable after 1 config change



$$R_1 = \frac{100 \cdot 125}{100 + 125 + 25} = 50\,\Omega$$

$$R_2 = \frac{100 \cdot 25}{100 + 125 + 25} = 10\,\Omega$$

$$R_3 = \frac{25 \cdot 125}{100 + 125 + 25} = 12.5\,\Omega$$



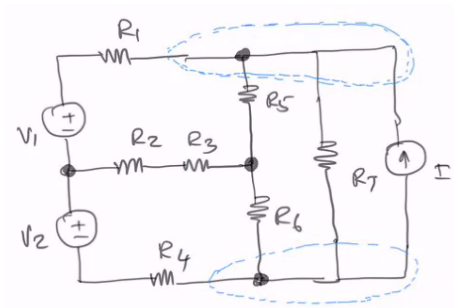
$$5+50+\left(\frac{1}{10+40}+\frac{1}{12.5+37.5}\right)^{-1}$$

$$= 80$$

-Circuit Analysis: Vocab

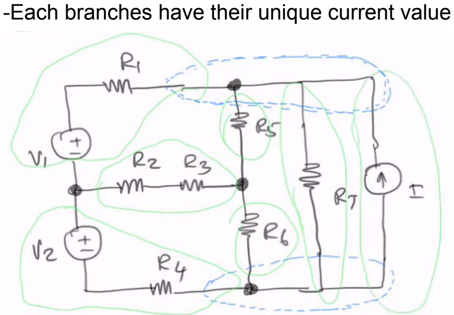
-The black dots are Essential Nodes (ALSO: Nodes don't have to be in the middle of an intersection.)

-Blue dotted are 2 examples of confusable (nonessential/junction) nodes. There's no component between the nodes to make it essential to consider.



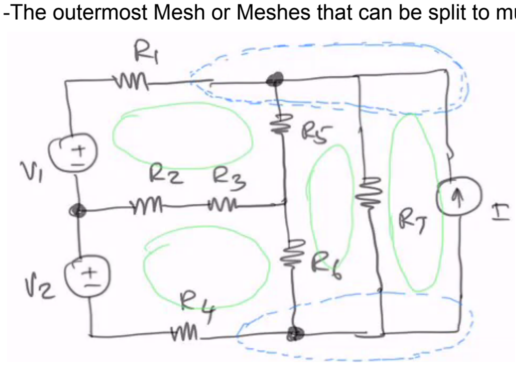
-Green are the Essential Branches

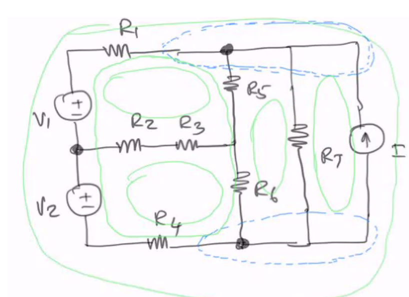
-Each branches have their unique current value running through.



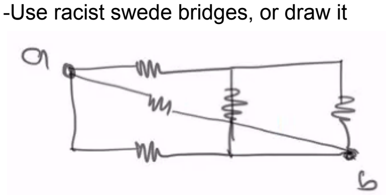
-Green loops are Meshes

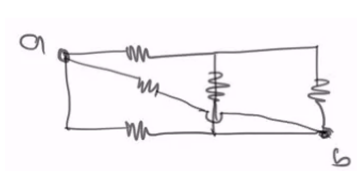
-The outermost Mesh or Meshes that can be split to multiple Meshes are called Super Mesh.

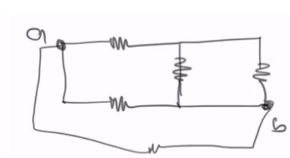




-Use racist swede bridges, or draw it



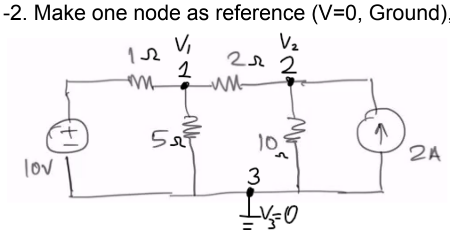




-Node Voltage Method

-1. Make all essential node

-2. Make one node as reference (V=0, Ground), other nodes are relative to that node.



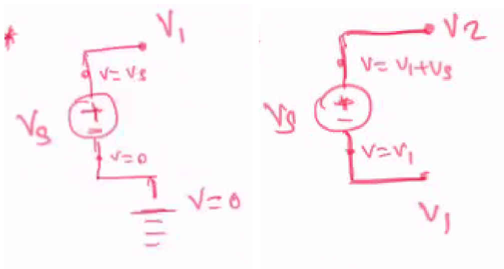
(V_1 & V_2 is with respect to V_3. It's relative!)

-That means Ohm's law is actually

$$I = \frac{V_1 - V_2}{R}$$

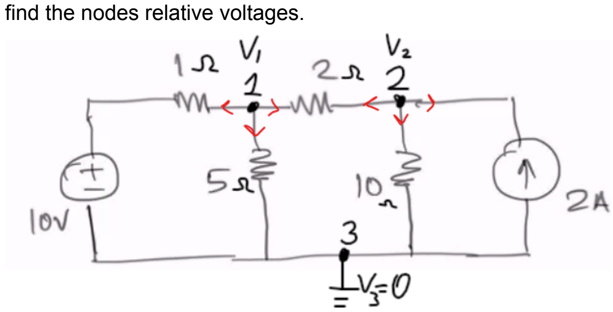
(Current flow from higher voltage to lower)

-Other voltages from node examples



$V_1 = V_S$ $\qquad$ $V_2 = V_1 + V_S$

-3. KCL and find the nodes relative voltages.



KCL (1): $\frac{V_1 - V_2}{2\Omega} + \frac{V_1 - 0}{5\Omega} + \frac{V_1 - 10V}{1\Omega} = 0$

KCL (2): $\frac{V_2 - V_1}{2\Omega} + \frac{V_2 - 0}{10\Omega} - 2A = 0$

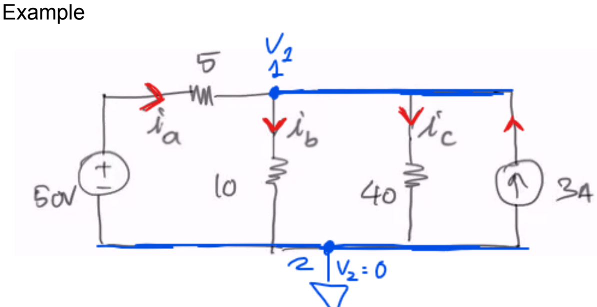
Lin Algebra

$V_1 = 9.09\,V$

$V_2 = 10.91\,V$

(Always consider the current leaving (or stay consistent with entering but nah))

-Another Example



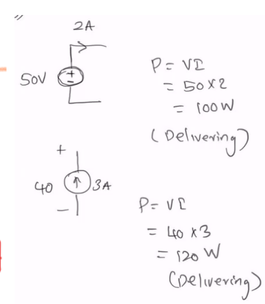
KCL(1): $\left(\frac{V_1 - 0}{10} + \frac{V_1 - 0}{40} - 3 + \frac{50 - V_1}{5}\right) A = 0$

$V_1 = 40\,V$

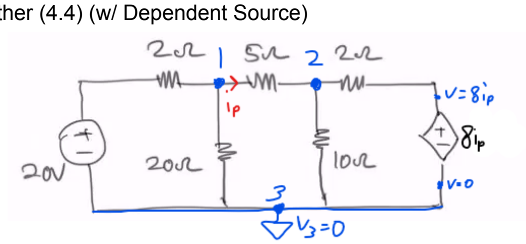
$i_b = \frac{V_1 - 0}{40} = 4A$

$i_c = \frac{V_1 - 0}{40} = 1A$

$i_a = \frac{50 - V_1}{5} = 2A$



-And Another (4.4) (w/ Dependent Source)



KCL(1): $\left(\frac{V_1 - 20}{2} + \frac{V_1 - 0}{20} + \frac{V_1 - V_2}{5}\right) A = 0$

KCL(2): $\frac{V_2 - V_1}{5} + \frac{V_2 - 0}{10} + \frac{V_2 - 8i_p}{2} = 0$

i_p: $i_p = \frac{V_1 - V_2}{5}$

Lin Algebra

$V_1 = 16\,V$

$V_2 = 10\,V$

$i_p = 1.2\,A$

$P_{5\Omega} = I^2 R$

$= 1.2^2 \times 5$

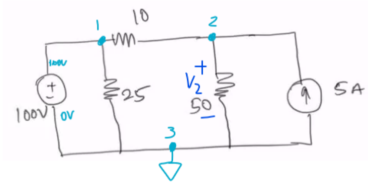
$= 7.2\,W$

(i_p direction does not come into account when doing the KCL(2). It's only when we want i_p's current value)

4/27:

-Node Voltage Special Cases

-Here, the 100V is connected in series with 2 nodes, so we know the voltage of a node immediately



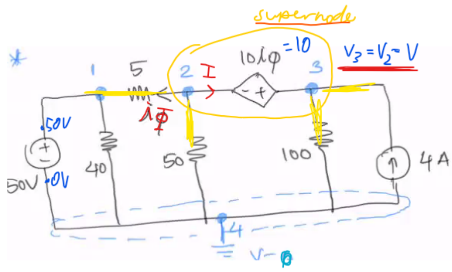
$V_1 = 100V$

$$\left.\begin{aligned} KCL(2) &= \frac{V_2 - V_1}{10} + \frac{V_2 - 0}{50} - 5A = 0 \\ KCL(1) &= \frac{V_2 - 100}{10} + \frac{V_2}{50} - 5 = 0 \end{aligned}\right\} V_2 = 125V$$

(he names 2 V2, its ugly)

-Here, the source is between two nodes like above, but one is not ground. So consider V3 and V2 as one node, Supernode!

-If there's only one source in between 2 nodes, consider it as Supernode.



$KCL(Super 2,3) = \frac{V_2 - V_1}{5} + \frac{V_2 - 0}{50} + \frac{V_3 - 0}{100} - 4 = 0$

$V_1 = 50V$ $\quad V_3 = V_2 + 10 i_\Phi$

$$\left\{\begin{aligned} KCL(2) &= \frac{V_2 - V_1}{5} + \frac{V_2 - 0}{50} + I = 0 \\ KCL(3) &= \frac{V_3 - 0}{100} - 4A - I = 0 \end{aligned}\right.$$

$$\left.\begin{aligned} &\frac{V_2 - V_1}{5} + \frac{V_2 - 0}{50} + \frac{V_3 - 0}{100} - 4 = 0 \\ &V_3 = V_2 + 10 i_\Phi \\ &I_\Phi = \frac{V_2 - V_1}{5} \end{aligned}\right\} \rightarrow V_2, V_3, I_\Phi$$

(<--Standard method here)

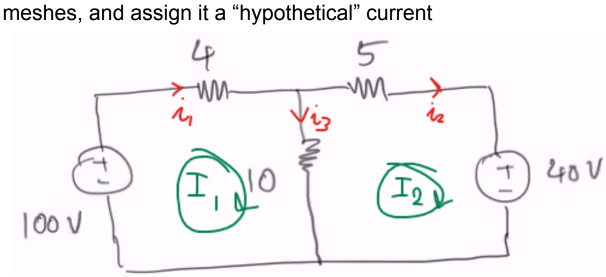
-Then the supernode is

$$V_3 = V_2 + 10 i_\phi$$
$$i_\phi = \frac{V_2 - V_1}{5}$$

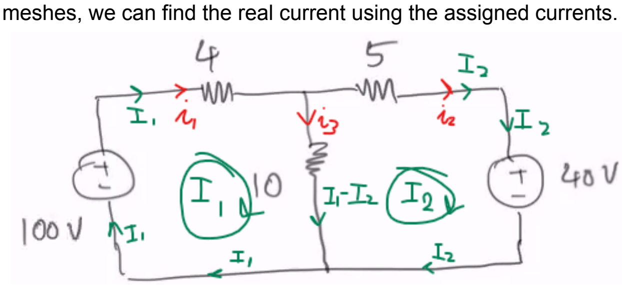
because $10 I_\phi = V_3 - V_2$ potential difference cheese

-Mesh Current Methodology

-Make ur meshes, and assign it a "hypothetical" current

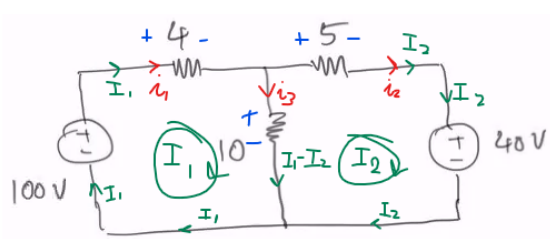


-With the meshes, we can find the real current using the assigned currents. The right and left is unique for each mesh, but the middle is affected by both.



(in the mid, I1 goes down & I2 goes up)

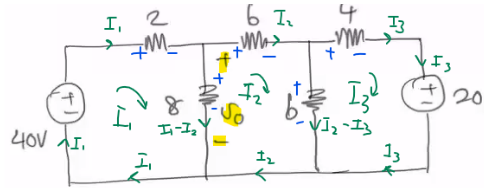
-Do KVL



$$\left.\begin{aligned} KVL(M1)&: -100 + 4I_1 + 10(I_1 - I_2) = 0 \\ KVL(M2)&: -10(I_1 - I_2) + 5I_2 + 40 = 0 \end{aligned}\right\} \text{Sys. Eq.}$$

(then solve it)

-Example 4.6



$KVL(1) = 2I_1 + 8(I_1 - I_2) - 40 = 0$

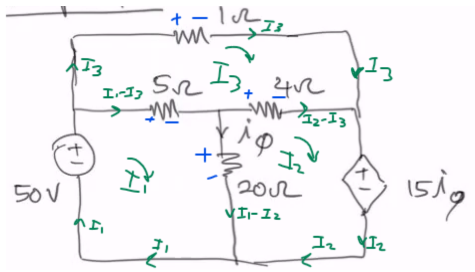
$KVL(2) = -8(I_1 - I_2) + 6I_2 + 6(I_2 - I_3) = 0$

$KVL(3) = 20 - 6(I_2 - I_3) + 4I_3 = 0$

blah $I_1 = 5.6A,\ I_2 = 2.0A,\ I_3 = -0.8A$

blah $V_0 = 28.8\,V$

-Example 4.7 (Dependent Source)



$$\left.\begin{aligned} &KVL(3) = I_3 - 4(I_2 - I_3) - 5(I_1 - I_3) = 0 \\ &KVL(1) = -50 + 5(I_1 - I_3) + 20(I_1 - I_2) = 0 \\ &KVL(2) = -20(I_1 - I_2) + 4(I_2 - I_3) + 15 i_\phi \\ &i_\phi = I_1 - I_2 \end{aligned}\right\} \text{Lin Algebra}$$

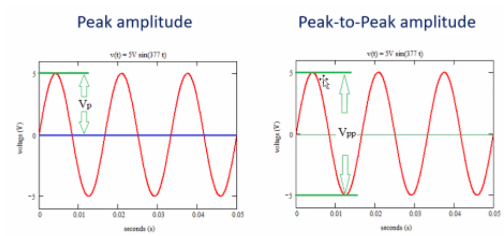
-AC circuit

-Alternating cheese, the voltage goes pos and neg, making the current switch direction.

-Modelled with a wave function usually. (Can be sin, square, triangle, sound synthesis wth)

-So Physics 3 cheese, \omega, freq, and everything.

-Peak to Peak Amplitude

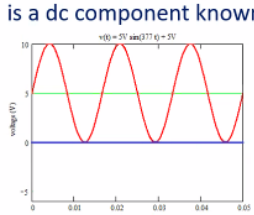


-Average

-The default sine is 0.

-If we add a DC Offset, the average increases. (Shift the wave up)

- The average value of a sinusoid signal is the integral of the sine wave over one full cycle. This is always equal to zero.
- If the average of an ac signal is not zero, then there is a dc component known as a DC offset.



$$V_{Ave} = \frac{\int_0^T v(t).dt}{T}$$

-RMS (Root Mean Square)

-This is used to calculate the avg power of AC

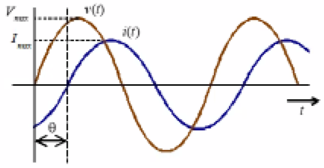
- Most equipment that measure the amplitude of a sinusoidal signal displays the results as a root mean square value. This is signified by the unit V_{ac} or V_{RMS}.
- RMS voltage and current are used to calculate the average power associated with the voltage or current signal in one cycle.

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T [v(t)]^2 dt} \qquad V_{RMS} = \frac{V_P}{\sqrt{2}} = 0.707V_P$$

$$P_{Ave} = \frac{V_{RMS}^{\ 2}}{R}$$

-Phase Angle (the difference between phase)

- Consider the following sinusoidal waveforms.

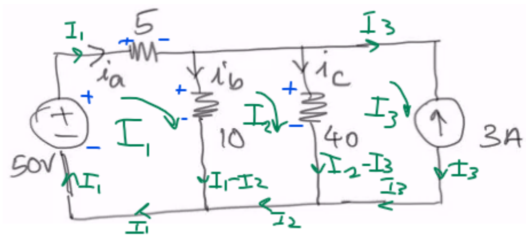


- $v(t) = V_{max}\sin(\omega t) = V_P\sin(\omega t)$
- $i(t) = I_{max}\sin(\omega t - \theta) = I_P\sin(\omega t - \theta)$
- where, ω - the angular frequency measured in rad/s;
- $\omega = 2\pi f$

4/29:

-Mesh Current Special Cases

-The source gives away a current ez



$\star\ I_3 = -3A$

$KVL(1) = -50 + 5I_1 + 10(I_1 - I_2) = 0$

$KVL(2) = -10(I_1 - I_2) + 40(I_2 - I_3) = 0$

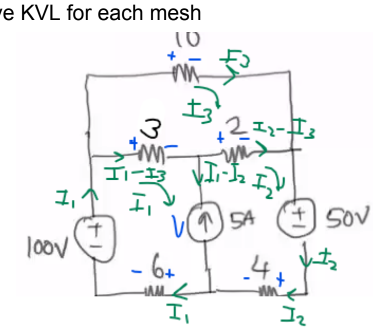
$-10(I_1 - I_2) + 40(I_2 + 3) = 0$

$\Big\}\ I_1 = 2A,\ I_2 = -2A$

$\therefore\ i_a = I_1 = 2A,\ i_b = I_1 - I_2 = 4A,\ i_c = I_2 - I_3 = 1A$ -i_3 is ez

-Using Supermesh

-Normal, u have KVL for each mesh



$I_1 - I_2 = -5A$

K.V.L for mesh 1:

$-100 + 3(I_1 - I_3) + V + 6I_1 = 0$ — ①

K.V.L for mesh 2:

$-V + 2(I_2 - I_3) + 50 + 4I_2 = 0$ —

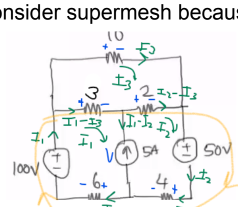
K.V.L for mesh 3:

$-3(I_1 - I_3) + 10I_3 - 2(I_2 - I_3) = 0$ —

②+③ ⇒

$\star\ -100 + 3(I_1 - I_3) + 6I_1 + 2(I_2 - I_3) + 50 + 4I_2 = 0$ (V cancels out, that's why he used it)

-But u can consider supermesh because V cancels out.



$I_1 - I_2 = -5A$

$KVL(3) = 10I_3 - 2(I_2 - I_3) - 3(I_1 - I_3)$

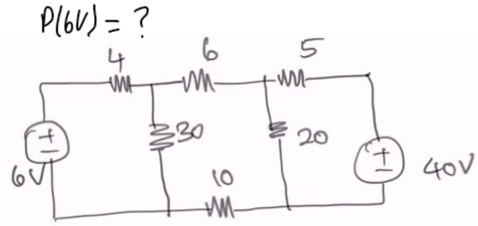
K.V.L for Supermesh:

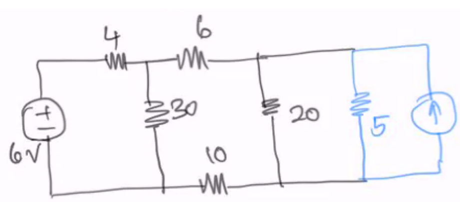
$-100 + 3(I_1 - I_3) + 2(I_2 - I_3) + 50 + 4I_2 + 6I_1 = 0$

-Source Transformation to solve problems

-Ex 4.12 (Source Transformation to switch resistance to be in series/parallel in order to R_eq) (When converting, the arrow points to the voltage's +)

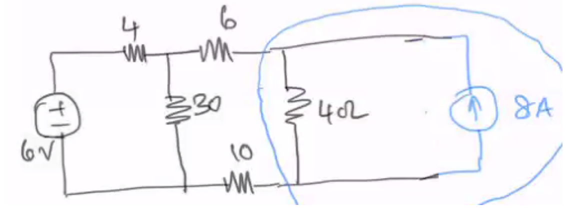
$P(6V) = ?$



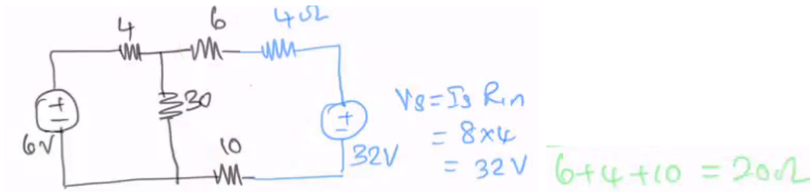


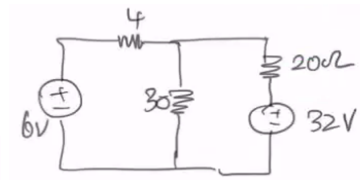
$I_S = \frac{V_S}{R_{in}} = \frac{40}{5} = 8A$

-Start simplifying

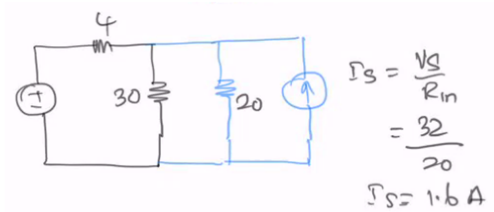


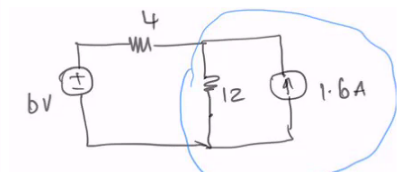
-Oh another hard one to simplify, just change the source to voltage to make the resistors in series and ez



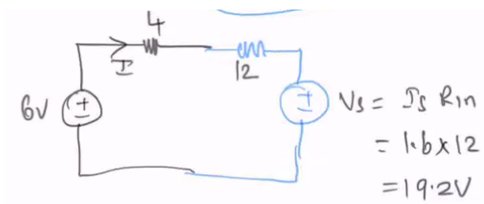


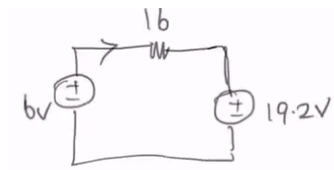
-And another road roadblock? Change it to a current source again. The resistor becomes in series.





-Again.





-Wow, super simple now, we can do the example's problem.

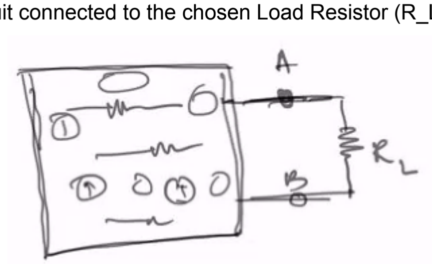
-Current through the circuit & Power of 6V (absorbing)

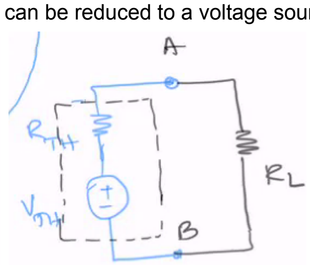
$$-6 + 16I + 19.2 = 0$$
$$16I = 13.2$$
$$I = 0.825\,A$$

$$P_{6V} = VI$$
$$= 6 \times 0.825$$
$$= 4.95\,W$$

-Thevenin's Theorem

-The circuit connected to the chosen Load Resistor (R_L) can be reduced to a voltage source.





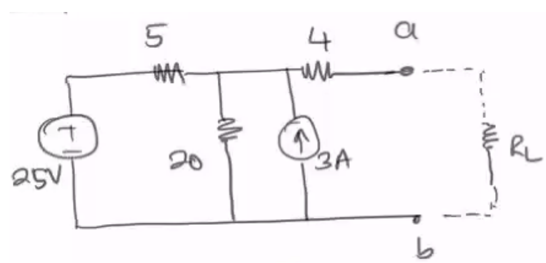
-Big steps

-1. Remove R_L

-2. Deactivate all sources (Voltage=short circuited, Current=open circuited) to find R_eq which is R_TH

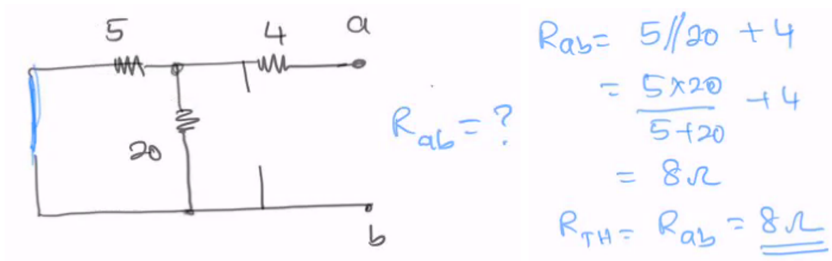
-3. Find the terminal (A&B) voltage which is V_TH

-Example 14.4

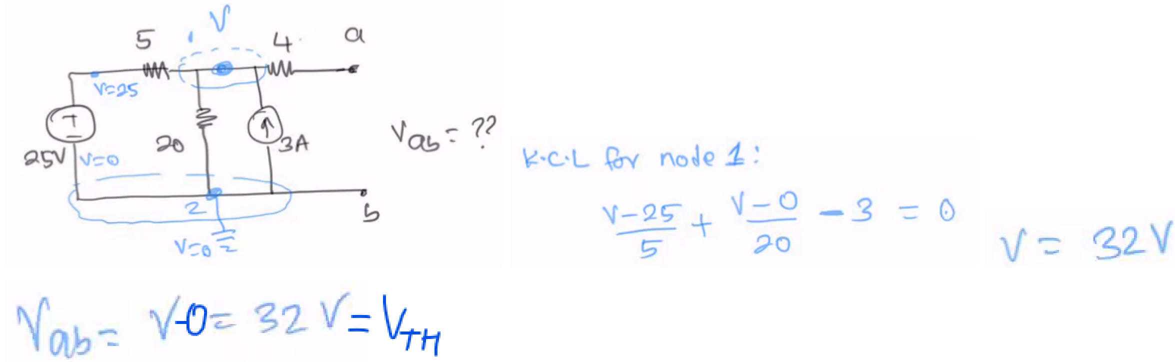


-1. Remove R_L

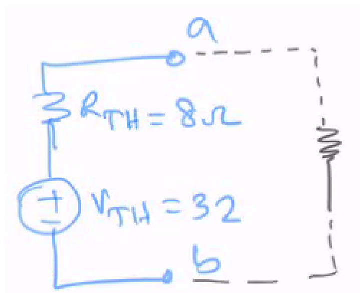
-2. Deactivate sources to find R_TH



-3. Find the terminal voltage which is V_TH

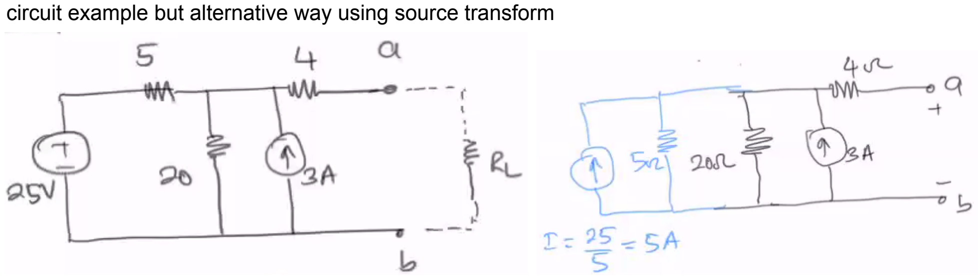


-BIg juice joy ending

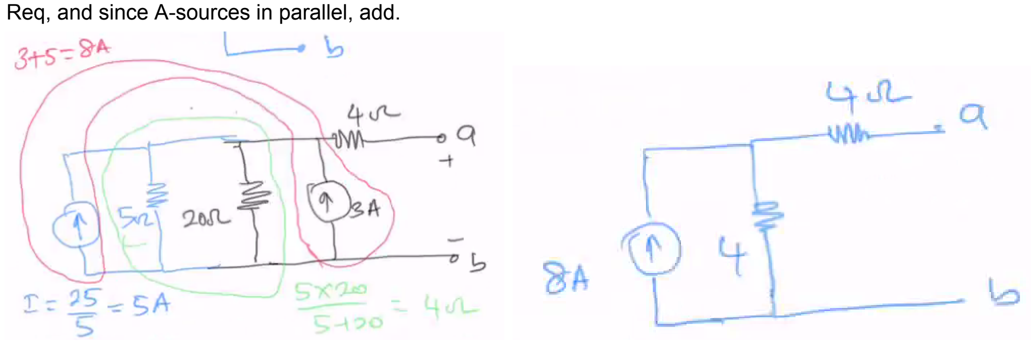


5/6:

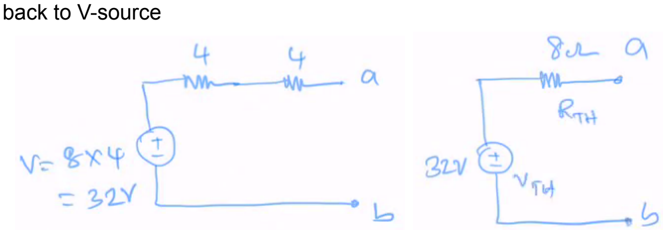
-Again TH circuit example but alternative way using source transform



-Then find Req, and since A-sources in parallel, add.

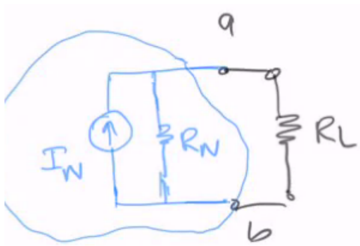


-Convert back to V-source

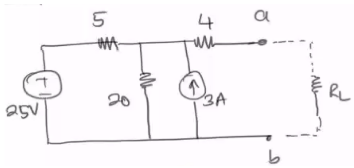


-Norton's Theorem

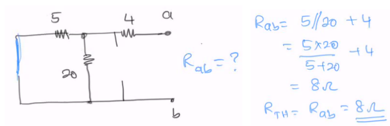
-Like Thevenin, but reduced to an A-source and parallel resistor.



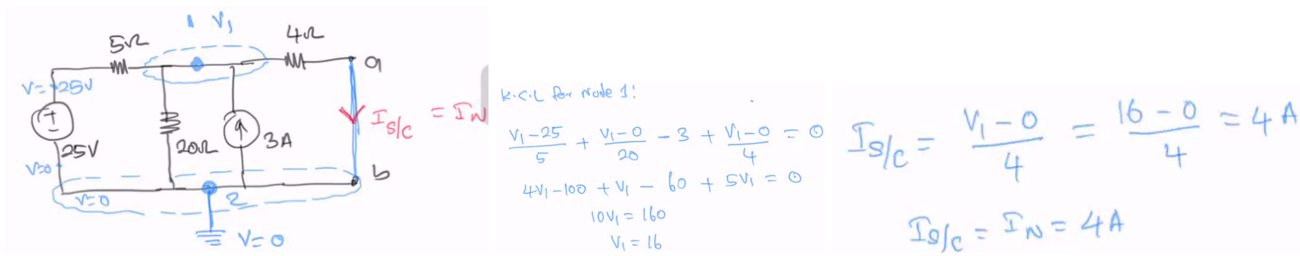
-1. Finding Rn is the same as Rth. So transform it to V-source. Then deactivate the sources and find Req.
-2. Finding In is by short circuit the terminals, find the current through targeted branch (Isc)
-Back to the TH example



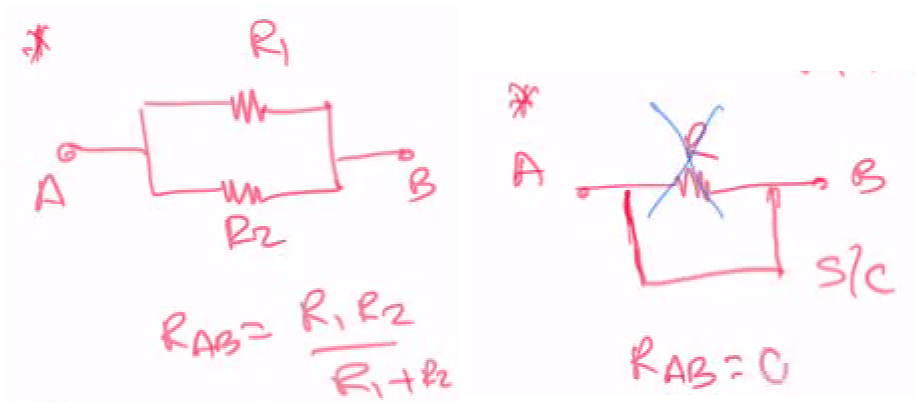
-Deactivate 25V & 3A to find Req=8ohms (Same as TH).



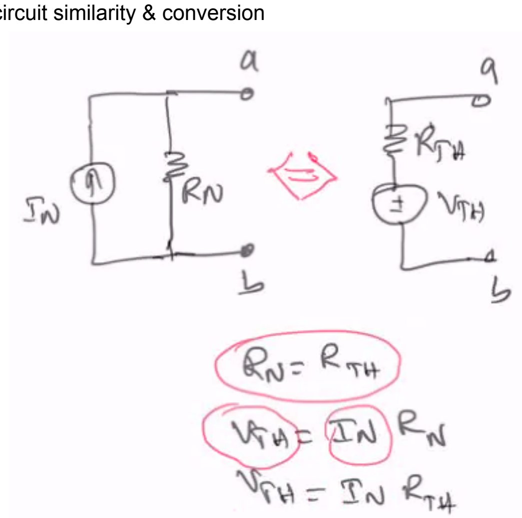
-Shortcircuit a&b, allowing you to find Isc or In.



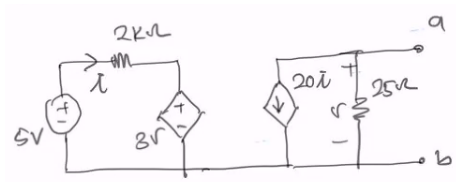
-Short circuited & resistance. If resistor connected in parallel with s/c, it's does nothing because current prefer no resistacne



-TH & N circuit similarity & conversion

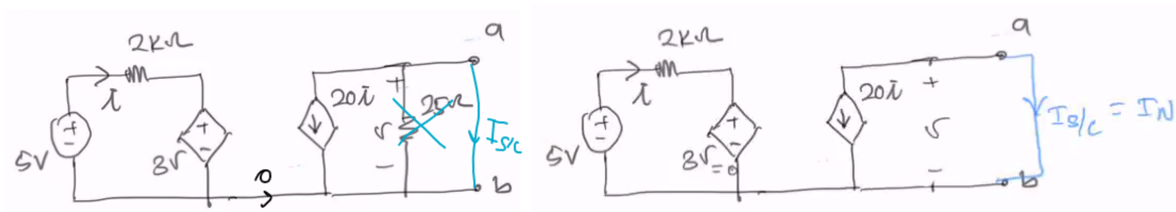


-If you cannot find Rn or Rth because of dependent sources, you need to find Vth & In (Isc) through each Th & N steps.
    -Example 4.18 (dependent source, can't deactivate it to find Req)

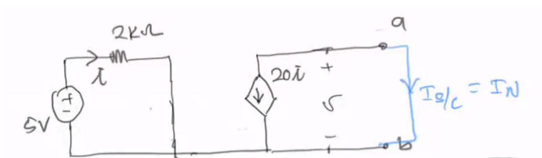


-Finding In (Isc)

-Simplifying because current prefer short circuit branch



-Now find i for dependent source & In



$V = IR$

$i = V/R = \frac{5}{2k} = 2.5mA$

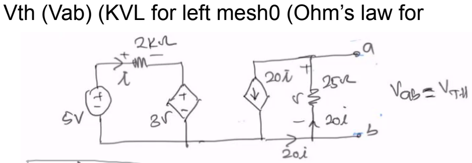
$I_{s/c} = -20i$

$I_{s/c} = -20 \times 2.5 \times 10^{-3}$

$= -50mA$

$I_{s/c} = I_N = -50mA$

-Finding Vth (Vab) (KVL for left mesh0 (Ohm's law for



K.V.L ⇒

$-5 + 2ki + 3v = 0$ — ①

$v = IR$

$v = -20i \times 25$

$v = -500i$ — ②

① ⇒

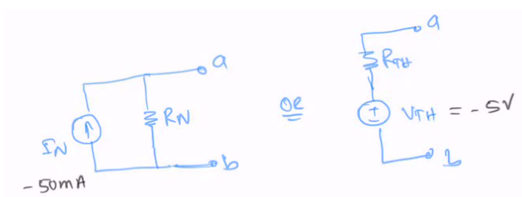
$-5 + 2\times10^3 \times \frac{v}{-500} + 3v = 0$

$-5 - 4v + 3v = 0$

$v = -5V$

-5V = Vab

-So Req/Rn/Rth & the TH & N circuit are:



$V = IR$

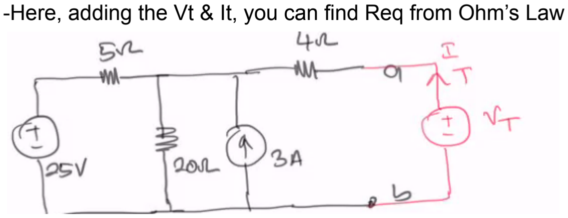
$-5 = -50\times10^{-3} R$

$R = 100\Omega$

$R_N = R_{TH} = R = 100\Omega$

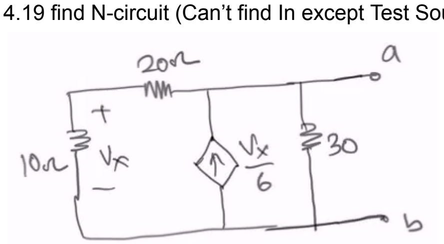
-Finding Rth/Rn another method: use a test source to find out Req. (works with dependent and independent sources)

-Here, adding the Vt & It, you can find Req from Ohm's Law after solving.

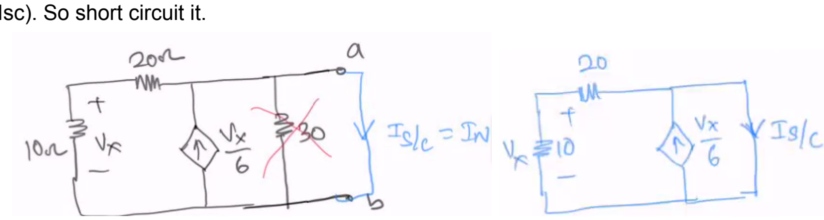


$$\frac{V_T}{I_T} = R_{TH} = R_N$$

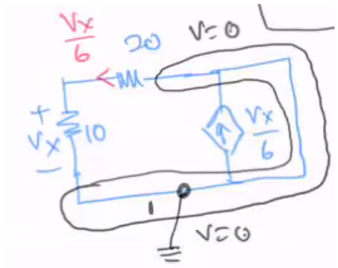
-Example 4.19 find N-circuit (Can't find In except Test Source)



-Find In (Isc). So short circuit it.



-Node Voltage (there's only one node because s/c, can't solve)

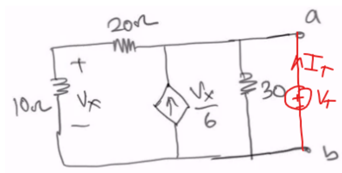


K.V.L

$$-V_x - 20\frac{V_x}{6} = 0$$

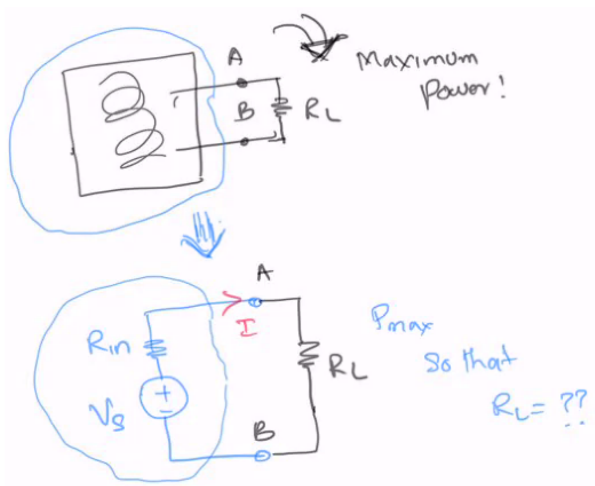
$$V_x = 0$$

-So use the test source



-Maximum Power Transfer

-We have a circuit and want to maximize the power used by a resistor (R_L)



$$I = \frac{V_S}{R_{in}+R_L}$$

(The current through R_L can be found with TH or N circuit)

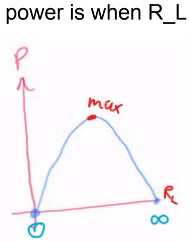
-That means the power is

Power at $R_L \Rightarrow$

$$P = I^2 R$$

$$P = \frac{V_S^2}{(R_{in}+R_L)^2} \cdot R_L$$

-Maximum power is when R_L vs P is at max. So need derivative



$$\frac{dP}{dR_L} = V_S^2 \frac{\left[(R_{in}+R_L)^2 \times 1 - R_L \cdot 2(R_{in}+R_L)\cdot 1\right]}{(R_{in}+R_L)^4}$$

$$(R_{in}+R_L)^2 - 2R_L(R_{in}+R_L) = 0$$

$$(R_{in}+R_L) - 2R_L = 0$$

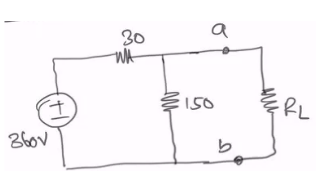
$$\boxed{R_L = R_{in}}$$

(When external resistance is the same as internal resistance, u get max power)

-That means that total resistance is R_in + R_L, and max power can be simplify to this.

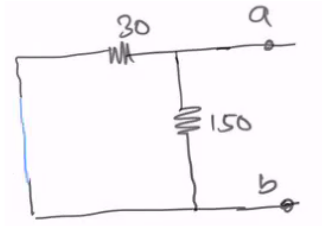
$$\boxed{P_{max} = \frac{V_S^2}{4R_{in}} = \frac{V_S^2}{4R_L}}$$

-Ex 4.21



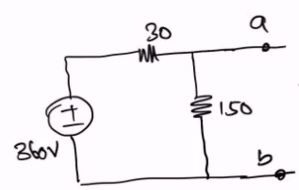
-TH theorem

-Rth (deactivate sources and o/c a&b)



$$R_{ab} = R_{TH} = \frac{30 \times 150}{30+150} = 25\Omega$$

-Vth (VDR)



$$V_{ab} = V_{TH} = \frac{360}{30+150} \times 150 = 300\text{ V}$$

-For max power, R_L is by theorem

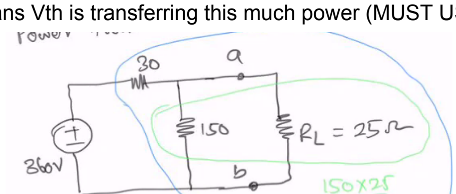
$$R_L = R_{in} = R_{TH}$$

$$R_L = 25\Omega$$

-So max power is

$$I = \frac{V_{TH}}{R_{TH}+R_L} = \frac{300}{25+25} = 6A \qquad P_{max} = I^2R = 6^2 \times 25 = 900W$$

-That means Vth is transferring this much power (MUST USE ORIGINAL CIRCUIT because current through V-source original is not same through R_L):



$$\frac{150 \times 25}{150+25} = 21.42$$

$$30 + 21.42 = 51.42$$

$$V = IR$$

$$360 = I \times 51.42$$

$$I = 7A$$

$$P = VI = 360 \times 7 = 2520\text{ W}$$

-The % of power from the source to R_L is: 900/2520 = 35.7% (efficiency)

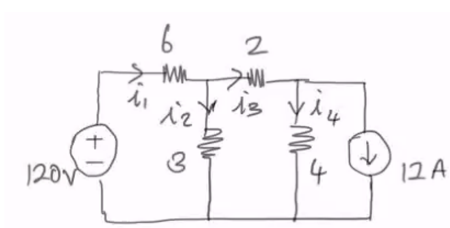
5/11:

-Superposition Theorem (Linear Systems)

-If there's more than one source, the effect of each source can be added up.

-So, consider one source at a time and deactivate other sources, then add up their effect.

-Example 4.22



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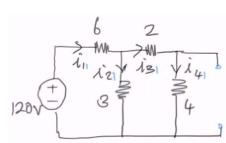

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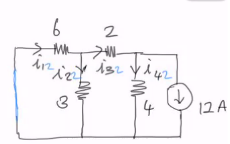
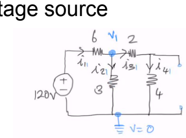


$$i_1 = i_{11} + i_{12}$$
$$i_2 = i_{21} + i_{22}$$
$$i_3 = i_{31} + i_{32}$$
$$i_4 = i_{41} + i_{42}$$

-Only voltage source



K.C.L for node 1:

$$\frac{V_1-120}{6} + \frac{V_1-0}{3} + \frac{V_1-0}{2+4} = 0$$

$$V_1 - 120 + 2V_1 + V_1 = 0$$

$$4V_1 = 120$$

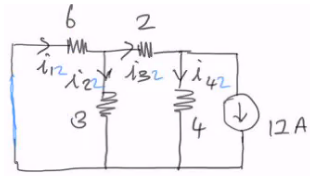
$$V_1 = 30V$$

$$i_{11} = \frac{120-V_1}{6} = \frac{120-30}{6} = 15A$$

$$i_{21} = \frac{V_1 - 0}{3} = \frac{30-0}{3} = 10A$$

$$i_{31} = i_{41} = \frac{V_1-0}{2+4} = \frac{30}{6} = 5A$$

-Only other current source



$$R_{eq} = \left(\frac{1}{\left(\frac{1}{6}+\frac{1}{3}\right)^{-1}+2} + \frac{1}{4} \right)^{-1}$$

$R_{eq} = 2$

$i_{32} = 6A$

$i_{42} = -\frac{12}{2} = -6A$

$i_{12} = \frac{6}{6+3} \times 3 = 2A$

$i_{22} = -4A$

-Superposition combine

$i_1 = i_{11} + i_{12} = 15 + 2 = \underline{17A}$

$i_2 = i_{21} + i_{22} = 10 + (-4) = \underline{6A}$

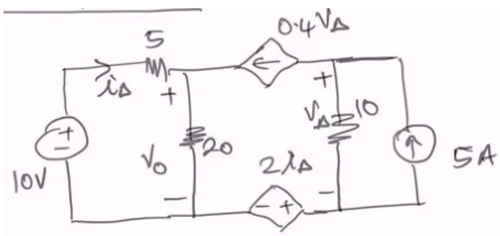
$i_3 = i_{31} + i_{32} = 5 + 6 = \underline{11A}$

$i_4 = i_{41} + i_{42} = 5 + (-6) = \underline{-1A}$

-Superposition Theorem (for dependent sources)

-Just deactivate the independent sources.

-4.23 (solving for v_o)



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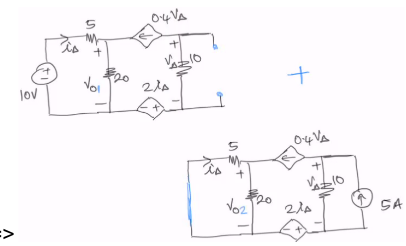
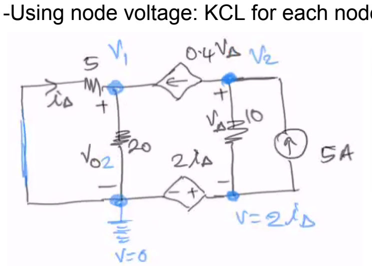


-Only A-source

-Using node voltage: KCL for each node, & 2 more equations for the 2 dependent sources' dependent variables.



KCL ⇒

for node 1 ⇒

$\frac{V_1 - 0}{5} + \frac{V_1 - 0}{20} - 0.4V_\Delta = 0$ — ①

for node 2 ⇒

$0.4V_\Delta + \frac{V_2 - 2i_\Delta}{10} - 5 = 0$ — ②

$i_\Delta = \frac{0 - V_1}{5}$ — ③

$V_\Delta = (V_2 - 2i_\Delta) \cdot 10$ — ④

$\boxed{V_{01} = V_1}$

$V_{01} = \underline{8V}$

-Only V-source

-Man skipped process. Just do nV rule or something

$\boxed{V_{02} = V_1}$

$V_{02} = \underline{16V}$

-Combine

$\boxed{V_0 = V_{01} + V_{02}}$

$= 8 + 16$

$= \underline{24V}$

-Capacitor & Inductor

- Inductors store energy, in the form of a magnetic field, and capacitors store energy in the form of an electrostatic field.
- The response of networks containing inductance and capacitance is time varying because of the time necessary for the exchange of stored energy.

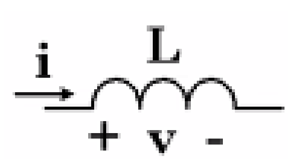
-Inductor stuff (V drop of inductor & energy stored)

- The changing flux linkage $d\emptyset/dt$ corresponds to an induced voltage V (Faraday's Law).
- The relationship between the voltage across an inductor and the current through the inductor.

$$V = L\frac{di}{dt}$$

- Note that V and i in the above equations are instantaneous values - functions of time.

- Stored energy
  - $E = \frac{1}{2}Li^2$

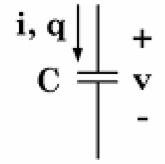


-Equivalent of in series & parallel inductance is same like resistor

-Capacitor stuff

$C = \frac{q}{V}$ $\quad i = C\frac{dv}{dt}$

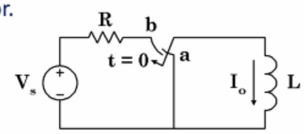
- Stored energy
  - $E = \frac{1}{2}Cv^2$



-Equivalent of in series & parallel capacitance is opposite of resistance

-RL circuit

- Make-before-break switch changes from position a to b at $t = 0$.
- For $t < 0$, I_0 circulates unchanged through inductor.

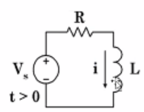


- For $t > 0$, circuit is as below.
- Initial value of inductor current, i, is i_0.

-When switch is at b, voltage&current is this: (charging)

- The KVL equation provides the differential equation.

$$V_s = Ri + L\frac{di}{dt}$$



$$i(t) = i_{SS} + ke^{-\frac{R}{L}t}$$

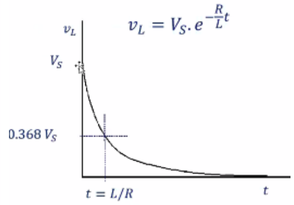
Steady State Response

Transient Response

- Solution has two parts: Steady State Response and Transient Response

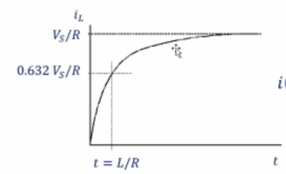
$v_L = L\frac{di}{dt}$

$v_L = V_S.e^{-\frac{R}{L}t}$



If $I_0 = 0$

Inductor current rises exponentially with a time constant L/R from its' initial value of zero to a steady-state value equal V/R.



$$i(t) = \frac{V_S}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

If $I_0 = 0$

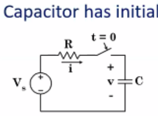
-Steady State is when di/dt is zero, doesn't change with time

-Transient is the component that depends with time.

-RC Circuit

-Charging

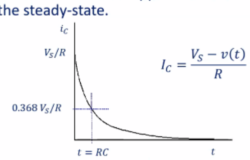
- Switch closes at $t = 0$.
- Capacitor has initial voltage, v_0.



- Current, $i = C\frac{dv}{dt}$
- By KVL & Ohm's Law: $V_S = RC\frac{dv}{dt} + v$
- Solution has two parts: Steady State Response and Transient Response

-V & I relationship with time is reversed. (The graphs are for charging)

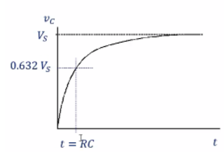
- The capacitor charging current has an initial value of (V/R) but falls exponentially with a time constant of RC until it approaches a value of zero in the steady-state.



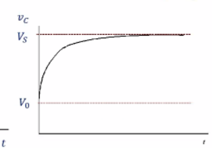
$$I_C = \frac{V_S - v(t)}{R} = \frac{V_S}{R}e^{-\frac{t}{RC}}$$

If $v(0) = 0$

- Capacitor voltage rises exponentially with a time constant RC from its' initial value of zero to a steady-state value equal to the source voltage V_S.



If $v(0) = 0$

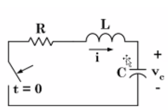


If $v(0) = V_0$

-RLC Circuit

-2nd Order DE

- KVL for t ≥ 0:
- $0 = Ri + L\frac{di}{dt} + v_C$
- But,
- $v_C = \frac{1}{C}\int_0^t i.dt + v_{C0}$
- Therefore,
- $0 = Ri + L\frac{di}{dt} + \frac{1}{C}\int_0^t i.dt + v_{C0}$
- Differentiate:
- $0 = L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i$



-Crit, Under, Over Damped (ew)

- From experience with 1st order problems: $i = Ie^{at}$
- Substitute into last KVL equation:

$$0 = La^2Ie^{at} + RaIe^{at} + \frac{1}{C}Ie^{at}$$

- Divide out, then

$$0 = La^2 + Ra + \frac{1}{C}$$

- Solve for a:

$$a = -\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$

- Let, $\alpha \equiv \frac{R}{2L}$ and $\omega_0^2 \equiv \frac{1}{LC}$

Then,

$$a_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

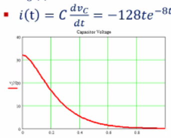
$$a_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

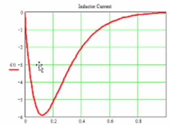
Three types of response:

- real and unequal (both negative) - Overdamped
- real and equal (negative) - Critically Damped
- complex conjugate pair - Underdamped

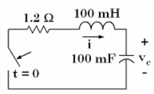
-Crit

- Solve for g_1 & g_2 using initial conditions:
- $v_C(0) = g_2 = 32$
- $i(0) = 0 = g_1 - 8g_2$
- Then, $g_1 = 256$
- $v_C(t) = 256te^{-8t} + 32e^{-8t}$
- $i(t) = C\frac{dv_C}{dt} = -128te^{-8t}$



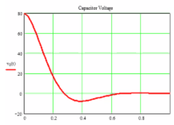


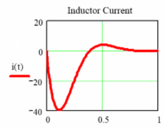
-Under



$v_C(t) = e^{-6t}(80cos(8t) + 60sin(8t))$

$i(t) = -100e^{-6t}sin(8t)$





5/13:

-Inductance Review (SI: Henry)

* Stored as magnetic field

* ϕ – flux

$\phi \propto i$

$\boxed{\phi = Li}$ — ①

* $L = \frac{\phi}{i}$

i – current

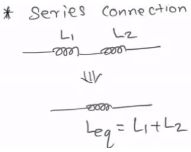
① ⇒ $\frac{d\phi}{dt} = L\frac{di}{dt}$

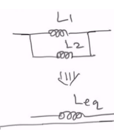
$\boxed{v = L\frac{di}{dt}}$

$P = VI$

Energy $E = \frac{1}{2}Li^2$

* Series connection:





$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}}$

-Capacitance Review (SI: Farad)

Stored as Electro static field

q - charge

$q \propto V$

$\boxed{q = CV}$ — ②

$C = \frac{q}{V}$

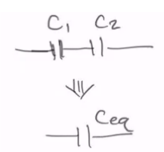
V - voltage

$\frac{dq}{dt} = C\frac{dv}{dt}$

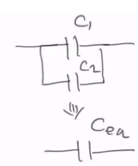
* $\boxed{i = C\frac{dv}{dt}}$

$P = VI$

$E = \frac{1}{2}Cv^2$

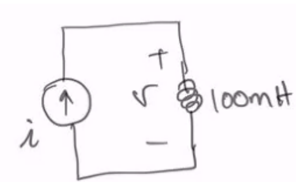


$\frac{1}{C_q} = \frac{1}{C_1} + \frac{1}{C_2}$



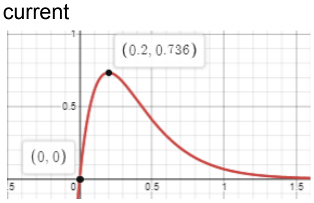
$\boxed{C_{eq} = C_1 + C_2}$

-Example 6.1 (inductor voltage: V=L(di/dt))



$i = \begin{cases} 0 & ; t < 0 \\ 10te^{-5t} & ; t > 0 \end{cases}$

-Graph of current



$i = \begin{cases} 0 & ; t < 0 \rightarrow V{:}0 \\ 10te^{-5t} & ; t > 0 \end{cases}$

$= (10te^{-5t})'$

$= (10(e^{-5t} - 5e^{-5t}t)$

$= e^{-5t} - 5e^{-5t}t$

$\rightarrow 0 = e^{-5t} - 5e^{-5t}t$

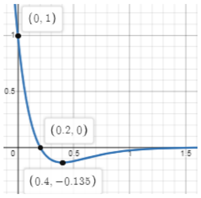
$= 5t - 1$

$t = 1/5s$ (maxima)

$\frac{di}{dt}(1/5s) = 0.7357$

(error: V(0) doesn't equal 0)

-Graph of Voltage



$V = 100\times10^{-3}\,\frac{d}{dt}\left(10te^{-5t}\right)$

$= 0.1\times10\times\left[te^{-5t}\cdot(-5) - e^{-5t}\times 1\right]$

$= e^{-5t}\left[-5t+1\right]$

$V = e^{-5t}\left[1-5t\right]\ V$

$\frac{dv}{dt} = e^{-5t}(0-5) + (1-5t)e^{-5t}(-5) = 0$

$e^{-5t}\left[-5 - 5 + 25t\right] = 0$

$25t = 10$

$t = 10/25 = 0.4\,s$

* **For Inductors**

Current cannot change instantaneously $V = L\frac{di}{dt}$

But

Voltage can change instantaneously

* **For Capacitors**

Voltage cannot change instantaneously $i = C\frac{dV}{dt}$

But

Current can change instantaneously

(This is why voltage graph can jump to (0,1))

-Opposite way

* $V = L\frac{di}{dt}$

$\int V\cdot dt = \int L\,di$

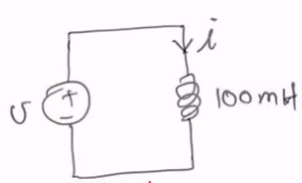
$\boxed{i = \frac{1}{L}\int v\,dt}$

$i = C\frac{dv}{dt}$

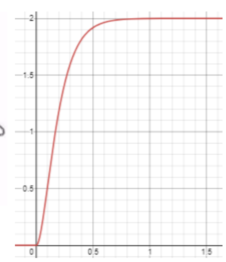
$\int i\,dt = \int C\,dv$

$\boxed{V = \frac{1}{C}\int i\,dt}$

-Example 6.2

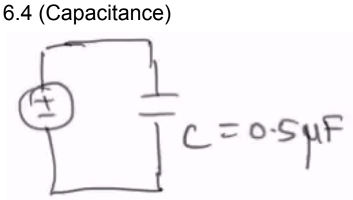


$$v = \begin{cases} v = 0 & : t < 0 \\ v = 20te^{-10t} & : t > 0 \end{cases}$$



$i = \frac{1}{L}\int_0^t 20te^{-10t}\,dt = 2(-10e^{-10t}t - e^{-10t} + 1)\ A$

-Example 6.4 (Capacitance)



$$v = \begin{cases} 0 & t \le 0 \\ 4t & 0 \le t \le 1 \\ 4e^{-(t-1)} & t \ge 1 \end{cases}$$

$0 \le t \le 1 \rightarrow \frac{dV}{dt} = 4V \rightarrow i = \frac{0.5}{10^6}4 = \underline{2\mu A}$

$t \ge 1 \rightarrow \frac{dV}{dt} = -4e^{-(t-1)}V \rightarrow i = \frac{0.5}{10^6}\frac{dV}{dt} = \underline{-2e^{-(t-1)}}\,A$

-Power

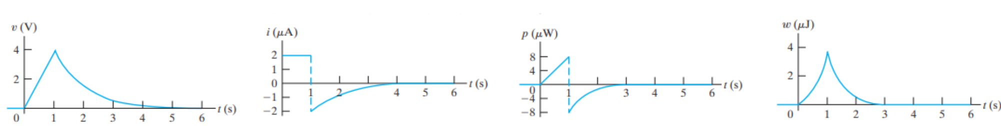
$$P = \begin{cases} 0 & ; t \le 0 \\ 8t\ \mu W & ; 0 \le t \le 1 \\ -8e^{-2(t-1)}\ \mu W & ; t \ge 1 \end{cases}$$

-Energy (U_E, electrical potential energy function)

$$E = \begin{cases} 0 & t \le 0 \\ \frac{1}{2}\times0.5\times\mu\times(4t)^2 = 4t^2\ \mu J & ; 0 \le t \le 1 \\ \frac{1}{2}\times0.5\times\mu\times\left(4e^{-(t-1)}\right)^2 = 4e^{-2(t-1)}\ \mu J & ; t \ge 1 \end{cases}$$

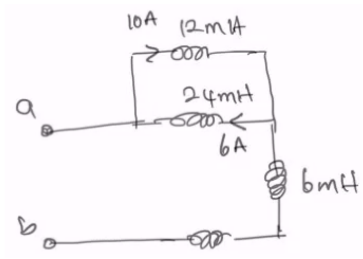
Energy $E = \frac{1}{2}CV^2$

-Graphs



5/18:

-Example 6.6

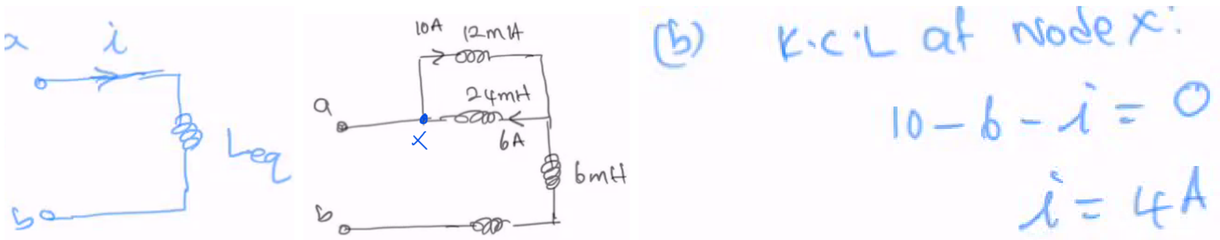


-L_eq

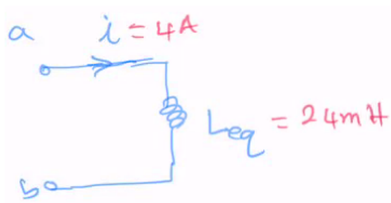
$$\left(\frac{1}{12}+\frac{1}{24}\right)^{-1}+6+10$$

$= 24$ mH

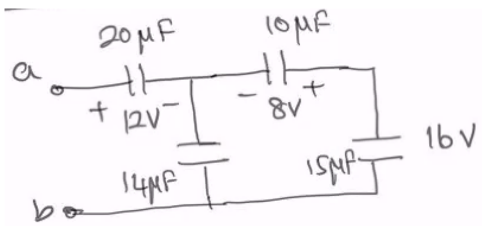
-Initial current (right in from terminal a)



-Ez



-Example 6.7

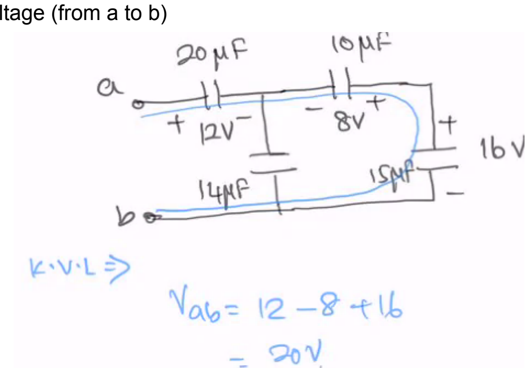


-C_eq

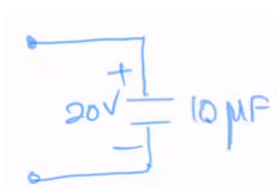
$$\left(\frac{1}{20}+\frac{1}{\left(\frac{1}{10}+\frac{1}{15}\right)^{-1}+14}\right)^{-1}$$

$= 10$ mF

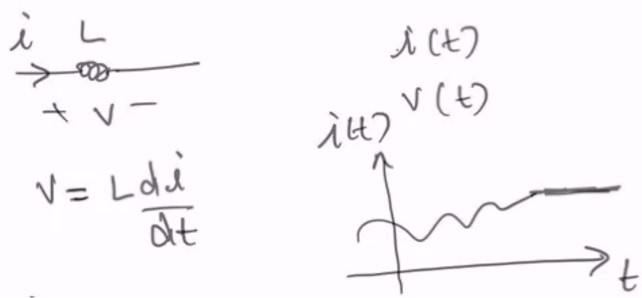
-Initial voltage (from a to b)



-Ez



-Natural Response of RL circuit (discharge)

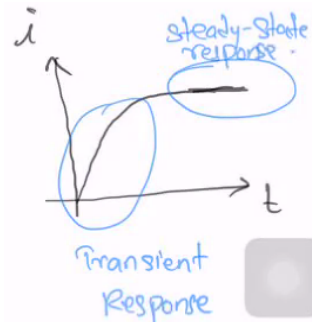


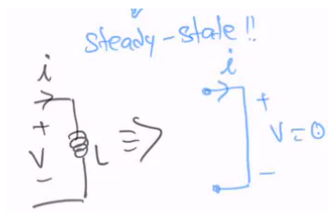
when $\frac{di}{dt} = 0$

$v = 0$

(When v=0, it's steady state)

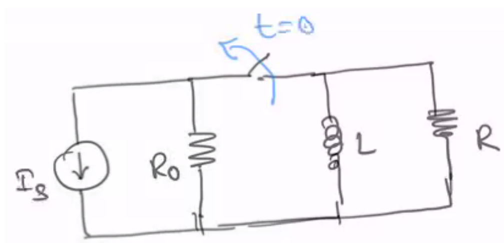
-Steady State & Transient Response





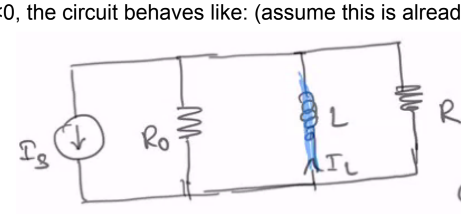
(inductor behaves as s/c when v=di/dt=0)

-Example



(R_o is internal of I_s & R is internal of L)

-When t<0, the circuit behaves like: (assume this is already like this for a long time)

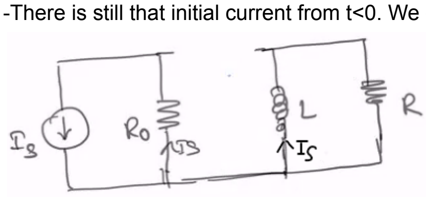


(L is s/c)

-I_L = I_S because the more current goes through less resistance.

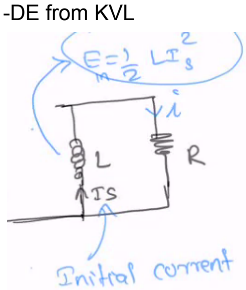
-When t=0: Switch is off

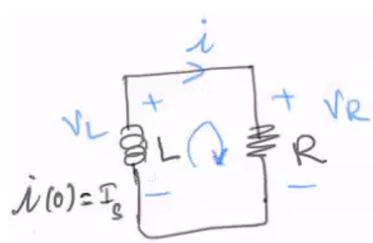
-There is still that initial current from t<0. We only care about L&R because the left side is constant.



-When t>0:

-DE from KVL





K.V.L

$$V_L + V_R = 0$$

$$L\frac{di}{dt} + iR = 0$$

(1st order DE eww)

-DE separable

$$\frac{di}{dt} = -\frac{R}{L}i$$

$$\int \frac{di}{i} = \int -\frac{R}{L}\,dt$$

$$\int_{i(0)}^{i(t)} \frac{1}{i}\,di = -\frac{R}{L}\int_0^t dt$$

$$\ln(i)\Big|_{i(t_0)}^{i(t)} = -\frac{R}{L}t\Big|_{t_0}^{t}$$

$$\ln\left(\frac{i(t)}{i(t_0)}\right) = -\frac{R}{L}(t-t_0)$$

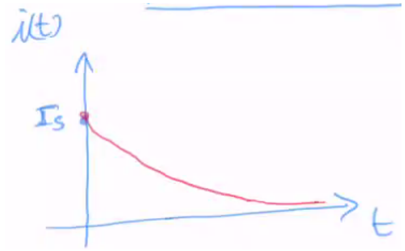
$$\frac{i(t)}{i(t_0)} = e^{-\frac{R}{L}(t-t_0)}$$

$$i(t) = i(t_0)\cdot e^{-\frac{R}{L}(t-t_0)}$$

-If t_o = o

$$\boxed{i(t) = i(0)\cdot e^{-\frac{R}{L}t}} \qquad \boxed{i(t) = I_S \cdot e^{-\frac{R}{L}t}}$$

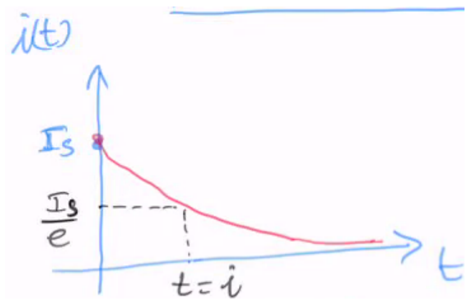
-Exponential decay & time constant



$$i(t) = I_S\, e^{-t/\tau}$$

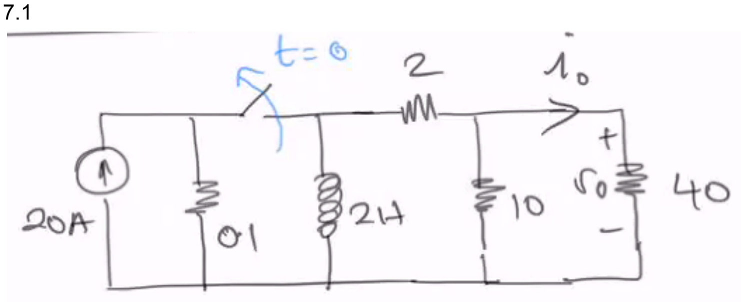
τ - Time-Constant (tau = L/R. If tau is higher, decay slower.)

-When t = tau



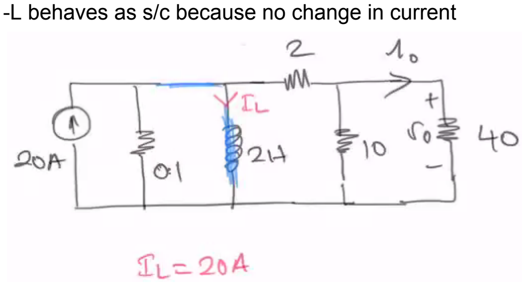
(Is/e is a 63% decrease from initial Is)

-Example 7.1



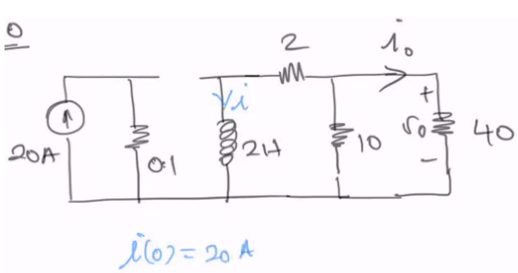
-When t<0 after a long time (so steady state)

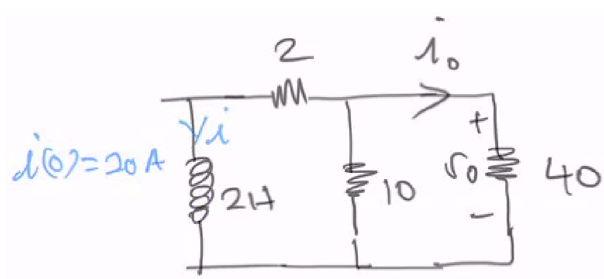
-L behaves as s/c because no change in current



-When t>0, after the switch is open.

-Exponential decay. We must graph it!



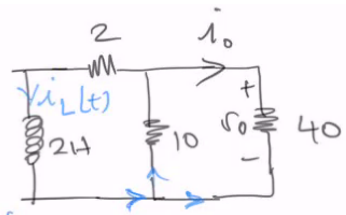


-To graph it, we need tau, so we need R_eq.

$$2 + \left(\frac{1}{10} + \frac{1}{40}\right)^{-1}$$

$= 10$ ohms so $i(t) = 20e^{-\frac{10}{2}t}$

-i_0 is the current through the 40 ohms resistor.



Current-devider ⇒

$$i_0 = -\frac{i_L(t)}{10+40} \times 10$$

$$= -0.2\, i_L(t) = -4e^{-5t}$$

(V-div & exponential decay)

-v_o(t)

Ohm's law

$$V = IR$$

$$v_0(t) = i_0(t) \cdot R$$

$$= -4e^{-5t} \cdot 40$$

$$v_0(t) = -160\, e^{-5t} \text{ V}$$

(ohm's law)

-Energy dissipated

-Initial stored energy in inductor

$$E_{initial} = \frac{1}{2} L i^2$$

$$= \frac{1}{2} \times 2 \times 20^2 = \underline{\underline{400 J}}$$

-Through 10 ohms resistor

-Current

$$i_{10} = \frac{V_0}{10} = -\frac{160\, e^{-5t}}{10}$$

$$= -16\, e^{-5t} \text{ A}$$

-Power

$$P_{10} = V_0 \cdot i_{10}$$

$$= 160\, e^{-5t} \cdot 16\, e^{-5t}$$

$$P_{10} = 2560\, e^{-10t} \text{ W}$$

-Energy (integration of power)

$$E_{10} = \int P_{10}\, t$$

$$= \int_0^{\infty} 2560\, e^{-10t} \cdot dt$$

$$E_{10} = 2560 \int_0^{\infty} e^{-10t}\, dt = 2560 \cdot \left. \frac{e^{-10t}}{-10} \right|_0^{\infty}$$

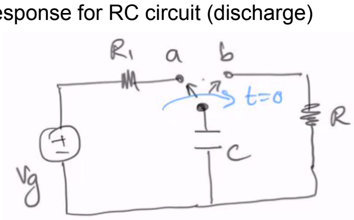
$$= \frac{2560}{10}(0-1)$$

$$= \underline{\underline{256 J}}$$

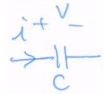
$$\% \text{ of Energy} = \frac{E_{10}}{E_{initial}} \times 100\%$$

$$= \frac{256}{400} \times 100 = \underline{\underline{64\%}}$$

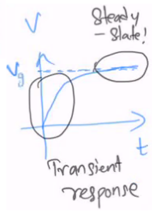
-Natural response for RC circuit (discharge)

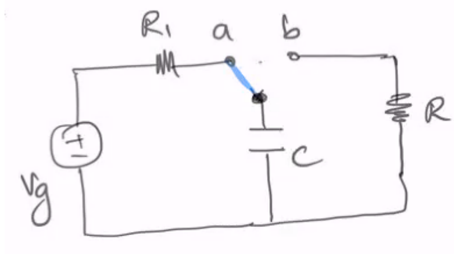


-When t<0, the capacitor is charged up and in steady state.



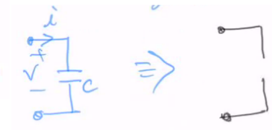
$i = C\frac{dV}{dt}$





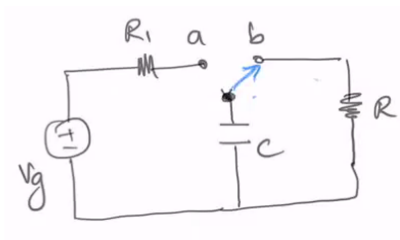
when $\frac{dV}{dt} = 0$

$i = 0$



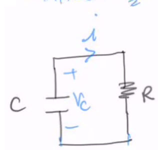
$$\boxed{V_c = V_g}$$

-t>0, capacitor discharge. Exponential decay (like Inductor but with voltage, wow Tphys 2)



$V_c(0) = V_g$

$E_{initial} = \frac{1}{2} C V_c^2$



$$i = C\frac{dV_c}{dt} = \frac{V_c}{R}$$

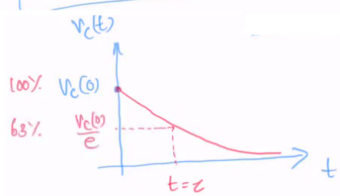
$$\int_{V_c(t_0)}^{V_c(t)} \frac{dV_c}{V_c} = -\int_{t_0}^{t} \frac{1}{CR}\, dt$$

$$V_c(t) = V_c(t_0) \cdot e^{-\frac{(t-t_0)}{RC}}$$

$t_0 = 0$

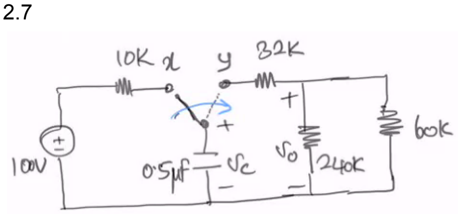
$$\boxed{V_c(t) = V_c(0) \cdot e^{-\frac{t}{RC}}} \quad \boxed{V_c(t) = V_c(0) \cdot e^{-t/\tau}}$$

$$\boxed{\tau = RC}$$



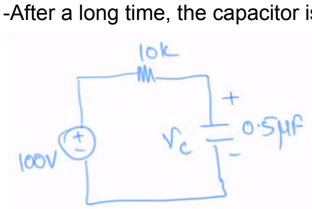
5/20:

-Example 2.7



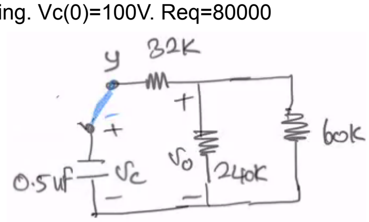
-t<0

-After a long time, the capacitor is like o/c. Vc = 100V



-t>0

-Discharging. Vc(0)=100V. Req=80000



$$\left(\frac{1}{240000} + \frac{1}{80000}\right)^{-1} + 32000$$

$= 80000$ ohms

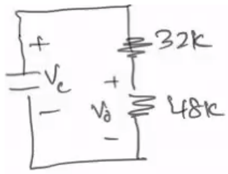
-Discharging Natural Response Vc(t)

$$v_c(t) = v_c(0)\, e^{-t/RC}$$

$$v_c(t) = 100 \cdot e^{-t/80\times10^{3}\times0.5\times10^{-6}}$$

$$v_c(t) = 100 \cdot e^{-25t} \quad V$$

-Vo(t) use VDR



$$V_o(t) = \frac{V_c(t)}{32+48} \times 48$$

$$V_o(t) = 60\, e^{-25t} \quad V$$

-Io(t) use Ohm's Law because Vo(t) is same voltage through 60K resistor

$$i_o(t) = \frac{V_o(t)}{R} = \frac{60\, e^{-25t}}{60k}$$

$$i_o(t) = e^{-25t} \ mA$$

-Power of 60K resistor

$$P = I^2 R$$

$$= i_o^2(t) \cdot 60k$$

$$= \left(e^{-25t} \times 10^{-3}\right)^2 \times 60 \times 10^{3}$$

$$= 60\, e^{-50t} \times 10^{-3}$$

$$P = 0.06\, e^{-50t} \quad W$$

-Energy

$$E = \int P(t)\, dt$$

$$E = \int_0^{\infty} 0.06\, e^{-50t}\, dt$$

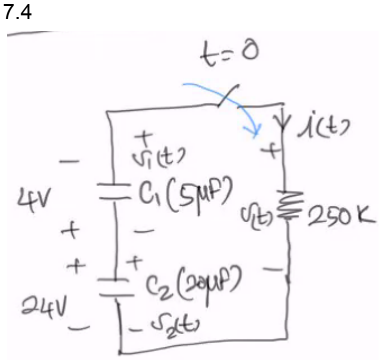
$$= 0.06 \int_0^{\infty} e^{-50t}\, dt$$

$$= 0.06 \left. \frac{e^{-50t}}{-50} \right|_0^{\infty}$$

$$= \frac{0.06}{50} \left[1 - 0 \right]$$

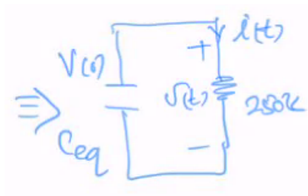
$$E = 1.2 \times 10^{-3} \ J = 1.2 \ mJ$$

-Example 7.4



(they gave initial voltage instead of V-source. Current is actually CCW)

-Reduce the capacitors so we can find the response



$\left(\frac{1}{5}+\frac{1}{20}\right)^{-1}$

$= 4$ uF

-Vc(0)

$$V_c(0) = 24 - 4 = 20V$$

(because opposite direction capacitors)

-V(t)

$$V(t) = V(0)\,e^{-t/RC} = 20\,e^{-t/250\times10^{3}\times4\times10^{-6}} = 20\,e^{-1t} = 20e^{-t}\ V$$

-I(t)

$$V = IR$$

$$i(t) = \frac{V(t)}{R} = \frac{20e^{-t}}{250k} = 80e^{-t}\times10^{-6} = 80e^{-t}\ \mu A$$

-V1(t)   (take integral of both side wrt V & I for the correct side)

$$i = C\frac{dV}{dt}$$

$$V_1(t) = (16e^{-t} - 20)$$

-V2(t)

$$\int_{24}^{V_2(t)} dV = \frac{-1}{20\times10^{-6}}\int_0^t 80e^{-t}\times10^{-6}\,dt$$

$$V_2(t) - 24 = -4\int_0^t e^{-t}\,dt$$

$$= -4\left.\frac{e^{-t}}{-1}\right|_0^t$$

$$V_2(t) - 24 = 4e^{-t} - 4$$

$$V_2(t) = 4e^{-t} + 20$$

-Initial energy in capacitors

$$E = \frac{1}{2}CV^2$$

$C_1 \Rightarrow$ $E_1 = \frac{1}{2}\times5\times10^{-6}\times4^2 = 40\ \mu J$

$C_2 \Rightarrow$ $E_2 = \frac{1}{2}\times20\times10^{-6}\times24^2 = 5760\ \mu J$

$$E_T = E_1 + E_2 = 40 + 5760 = 5800\ \mu J$$

-Energy in capacitors when t->infinna

-Not zero because there's two capacitors and they are poop

$$V_1(t) = 16e^{-t} - 20$$

$$V_1(\infty) = -20V$$

$$V_2(t) = 4e^{-t} + 20$$

$$V_2(t) = +20V$$

$$E_1(\infty) = \frac{1}{2}\times5\times10^{-6}\times20^2 = 1mJ$$

$$E_2(\infty) = \frac{1}{2}\times20\times10^{-6}\times20^2 = 4mJ$$

$$E_T = 1 + 4 = \underline{5mJ}$$

-Energy Delivered VS Dissipated

Energy delivered $= 5.8 - 5$
$= 0.8\,mJ$
$= 800\,\mu J$

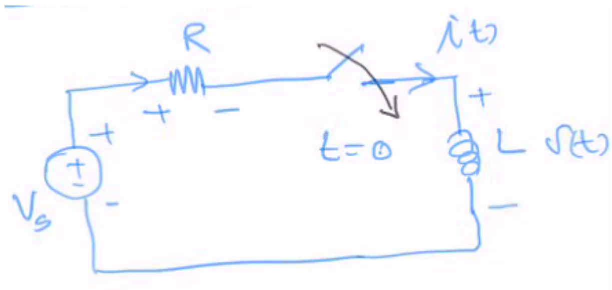
-or u can do it like

$$P = I^2 R = \left(80 e^{-t}\mu\right)^2 \times 250\,k = 1600\, e^{-2t}\ W$$

$$E = \int_0^{\infty} p\, dt = 1.6\times10^{-3}\int_0^{\infty} e^{-2t}\, dt\ \mu = 1.6\times10^{-3}\left.\frac{e^{-2t}}{-2}\right|_0^{\infty}\mu = 0.8\times10^{-3}\,\mu = 800\,\mu J$$

-Step Response of RL (charging)

-Step Response is when the source is charging the inductor (inductance doesn't have to be 0)



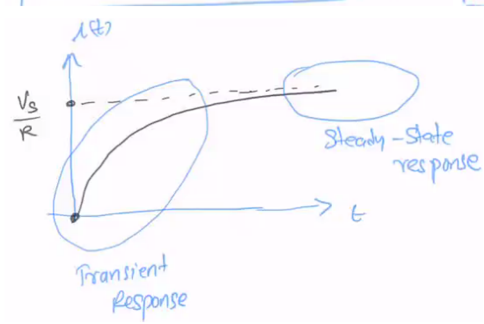
-KVL & DE -> i(t)

$$-V_s + iR + v(t) = 0$$

$$-V_s + iR + L\frac{di}{dt} = 0 \qquad L\frac{di}{dt} + Ri - V_s = 0$$

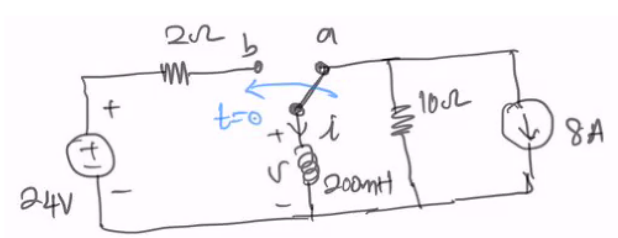
If $I_0 = 0$

$$\boxed{i(t) = \frac{V_s}{R}\left(1 - e^{-\frac{R}{L}t}\right)} \qquad \boxed{i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right)e^{-\frac{R}{L}t}}$$

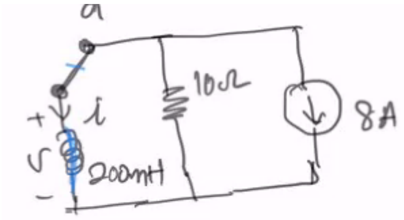


(steady state = s/c)

-Example 7.5

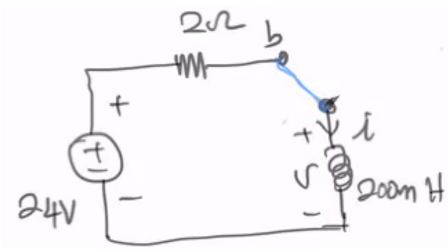


-t<0 (long time, already steady state)



$$i = -8A$$

-t>0 (step response when switched)



$$i(0) = -8A$$

-1 inductor, 1 resistor, ez i(t)

$$i(t) = \frac{V_S}{R} + \left(I_0 - \frac{V_S}{R}\right) e^{-\frac{R}{L}t}$$

$$i(t) = \frac{24}{2} + \left(-8 - \frac{24}{2}\right) e^{-\frac{2}{200 \times 10^{-3}}t}$$

$$= 12 + (-20) e^{-10t}$$

$$i(t) = 12 - 20e^{-10t} \text{ A}$$

-V(0)

$$V = L\frac{di}{dt}$$

$$V = 200 \times 10^{-3} \frac{d\left(12 - 20e^{-10t}\right)}{dt}$$

$$= 0.2\left[0 - 20e^{-10t}(-10)\right]$$

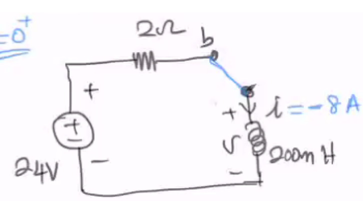
$$V(t) = 40e^{-10t}$$

$$t \to 0$$

$$V(0) = \underline{40V}$$

-KVL

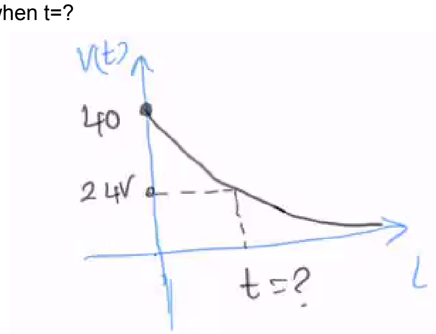
$\underline{t=0^+}$



$$-24 + 2\times -8 + V = 0$$

$$V = \underline{40V}$$

-V=24V when t=?



$$V(t) = 40e^{-10t} = 24$$

$$e^{-10t} = \frac{24}{40}$$

$$e^{10t} = 40/24$$

$$10t = \ln 40/24$$

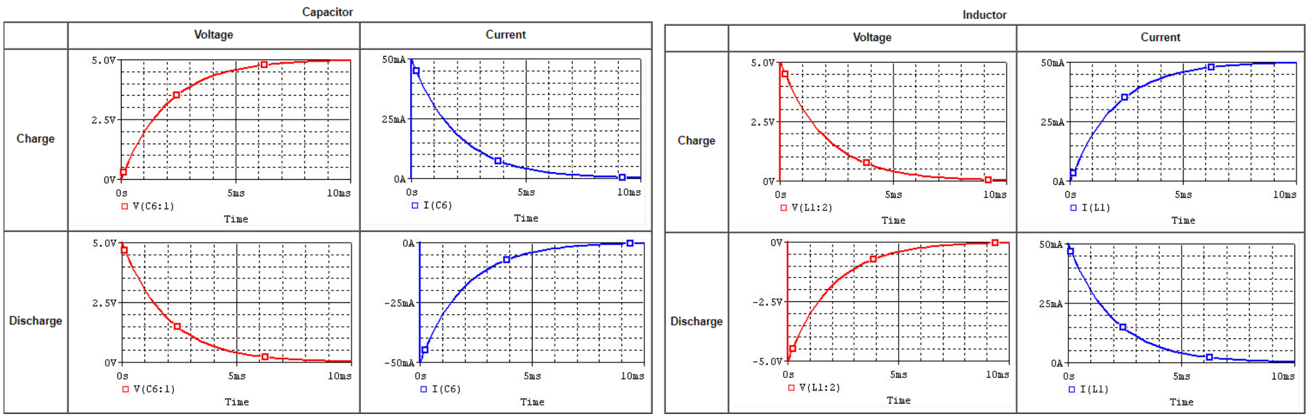
$$t = 0.0511 \text{ s}$$

$$t = \underline{51.1 \text{ ms}}$$

-Inductor & Capacitor Responses Recap

Exponential responses of capacitors and inductors

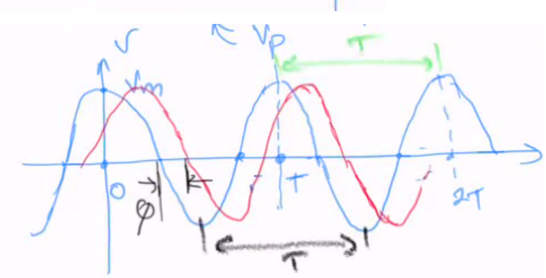
	Discharging	Charging	Time Constant
Capacitor	$v_C(t) = V_0 e^{-\frac{t}{RC}}$	$v_C(t) = V_0(1 - e^{-\frac{t}{RC}})$	RC
Inductor	$i_L(t) = I_0 e^{-\frac{R}{L}t}$	$i_L(t) = I_0(1 - e^{-\frac{R}{L}t})$	$\frac{L}{R}$



5/25:

-Sinusoidal Analysis

$v = V_m \cos(\omega t + \phi)$



$V_m \Rightarrow V_{maximum}$

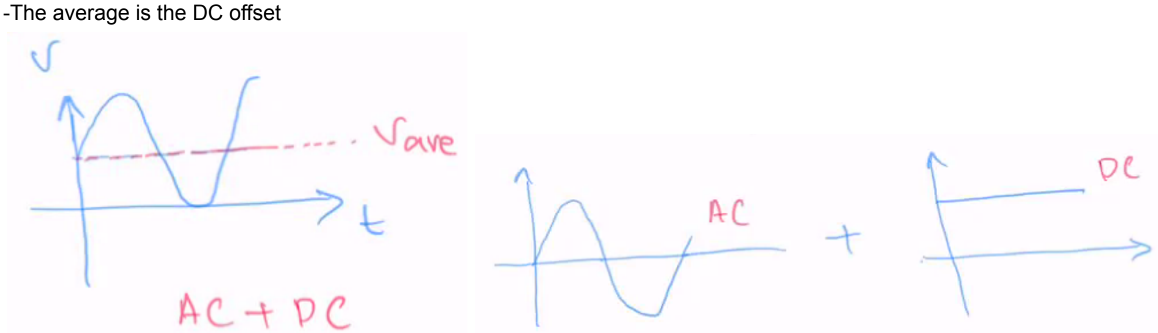
$V_p \Rightarrow V_{peak}$ (aka amplitude)

-ϕ Offset is in angle rad or degrees.
-ω(rad/s)=2πf
-frequency (Hz) in US & Canada is 60Hz
-Other parts of the world is usually 50Hz
-Period T=1/f
-Average voltage

$$\text{Average } v = 0\,V = \frac{\int_0^T v(t)\,dt}{T} = 0\,V$$

(Avg is always DC)

-DC Offset (Vertical shift)
-The average is the DC offset



-Root Mean Square Value (RMS)
-Square root of the mean value from a periodic function

$$V_{rms} = \sqrt{\frac{\int_0^T v(t)^2 \cdot dt}{T}} = \sqrt{V_{avg}} = \frac{V_m}{\sqrt{2}}$$

-Example 9.1

$I_m = 20A \quad T = 1ms \quad i(0) = 10A$

-f

$$f = \frac{1}{T} = \frac{1}{1\times10^{-3}} = 1000\ Hz$$

-ω

$$\omega = 2\pi f$$
$$= 2\pi \times 1000$$
$$= 2000\pi$$

-i(t) and ϕ

$$i(t) = I_m \cos(\omega t + \phi)$$
$$i(0) = I_m \cos(\omega \times 0 + \phi)$$
$$10 = 20 \cos\phi$$
$$\cos\phi = 0.5$$
$$\phi = \cos^{-1}(0.5) = 60^\circ$$
$$180^\circ \rightarrow \pi$$
$$60^\circ \rightarrow \pi/3 \text{ rad}$$

$$i(t) = 20 \cos(2000\pi t + 60^\circ)$$

-RMS

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$
$$= \frac{20}{\sqrt{2}} = \underline{\underline{14.14\ A}}$$

-Phasor Representation

$$e^{j\theta} = \cos\theta + j\sin\theta$$
$$e^{-j\theta} = \cos\theta - j\sin\theta$$

$$V_m \cos(\omega t + \phi) = V_m e^{j\phi}$$

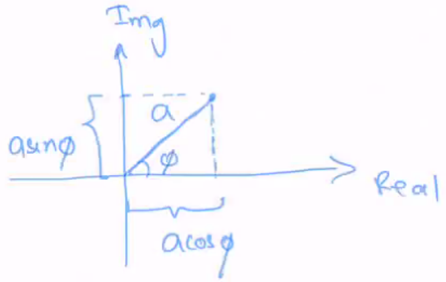
-Any periodic stuff can be represented in this complex number thingy. It's easier for calculations & you can convert back to cos.

$V_m \angle \phi$ (That's not an L, it's an angle $A\angle\theta$.)

-More on Complex Numbers: https://www.electronics-tutorials.ws/accircuits/complex-numbers.html

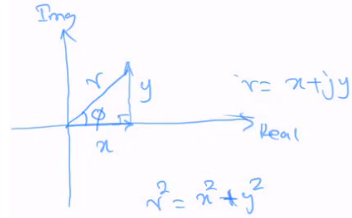
-Phasor to Complex

-Complex to Phasor



$$a\angle\phi = a\cos\phi + j\,a\sin\phi$$

Phasor complex



$$r^2 = x^2 + y^2$$
$$r = \sqrt{x^2+y^2}$$
$$\tan\phi = y/x$$
$$\phi = \tan^{-1}(y/x)$$
$$r = \sqrt{x^2+y^2}\ \angle \tan^{-1}(y/x)$$

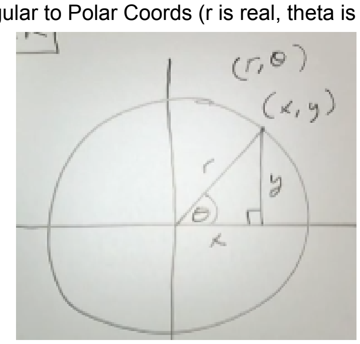
-Phasor Calculator: https://ejl.neocities.org/phasorMath.html

-Complex Number math

$$A = a\angle\theta_a$$
$$B = b\angle\theta_b$$
$$A * B = a\angle\theta_a * b\angle\theta_b$$
$$= ab\angle\theta_a+\theta_b$$
$$\frac{A}{B} = \frac{a\angle\theta_a}{b\angle\theta_b} = \frac{a}{b}\angle\theta_a-\theta_b$$

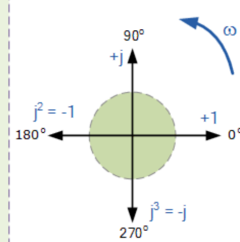
$$A \pm B = a\angle\theta_a \pm b\angle\theta_b$$
$$= a\cos\theta_a + j\,a\sin\theta_a \pm b\cos\theta_b \pm j\,b\sin\theta_b$$
$$= \underline{(a\cos\theta_a \pm b\cos\theta_b)} + j\,\underline{(a\sin\theta_a \pm b\sin\theta_b)}$$
$$= \underline{X} + j\,\underline{Y}$$
$$= \sqrt{X^2+Y^2}\ \angle\tan^{-1}\left(\frac{Y}{X}\right)$$

-Rectangular to Polar Coords (r is real, theta is angle)



$$x^2 + y^2 = r^2$$
$$\boxed{r = \sqrt{x^2+y^2}}$$
$$\tan\theta = \frac{y}{x}$$
$$\boxed{\theta = \tan^{-1}\left(\frac{y}{x}\right)}$$

90° rotation:	$j^1 = \sqrt{-1} = +j$
180° rotation:	$j^2 = \left(\sqrt{-1}\right)^2 = -1$
270° rotation:	$j^3 = \left(\sqrt{-1}\right)^3 = -j$
360° rotation:	$j^4 = \left(\sqrt{-1}\right)^4 = +1$



-Example 9.5

$$Y_1 = 20\cos(\omega t - 30^\circ) \quad Y_2 = 40\cos(\omega t + 60^\circ) \quad Y = Y_1 + Y_2 = ??$$

$$Y = Y_1 + Y_2 = 20\cos(\omega t - 30^\circ) + 40\cos(\omega t + 60^\circ)$$
$$= 44.72\cos(\omega t + 33.43^\circ)$$

-Phasor Rep Method

$$Y_1 = 20\angle -30^\circ \qquad Y = Y_1 + Y_2 = 20\angle -30^\circ + 40\angle 60^\circ$$
$$Y_2 = 40\angle 60^\circ \qquad = 44.72\angle 33.43^\circ$$

-Then simplify by converting to Phasor & Complex

$$y = y_1 + y_2 = 20\angle{-30^\circ} + 40\angle 60^\circ$$

$$= [20\cos(-30) + j\,20\sin(-30)] + [40\cos(60) + j\,40\sin(60)]$$

$$= [20\cos(-30) + 40\cos 60] + j[20\sin(-30) + 40\sin(60)]$$

$$= 37.32 + j\,24.64$$

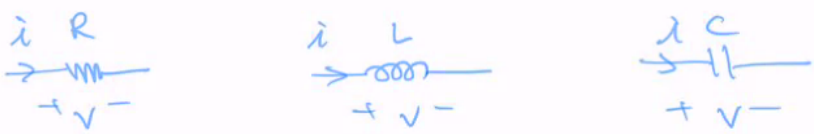
$$= \sqrt{37.32^2 + 24.64^2}\ \angle \tan^{-1}\left(\frac{24.64}{37.32}\right)$$

$$= 44.72\angle 33.43^\circ$$

-Finally, convert to time domain (cos)

$$= 44.72\cos(\omega t + 33.43^\circ)$$

-Circuit Elements in Phasor Domain



$$v = V_m\cos(\omega t + \phi)$$

$v = IR$ $\qquad v = L\frac{di}{dt}$ $\qquad i = C\frac{dv}{dt}$

$V_m = \omega L I_m$

$V_m = I_m X$

$\therefore X = \omega L$

Reactance

Impedance Z

$Z = jX$

$i = \frac{V_m}{R}\cos(\omega t + \phi)$

$i = I_m\cos(\omega t + \phi)$

$\therefore I_m = \frac{V_m}{R}$

$\boxed{V_m = R I_m}$

$i = C\frac{d(V_m\cos(\omega t + \phi))}{dt}$

$= \omega C V_m\cos(\omega t + \phi - 90)$

$i = I_m\cos(\omega t + \phi - 90)$

$\therefore I_m = \omega C V_m$

$V_m = \frac{1}{\omega C} I_m$

$X = \frac{1}{\omega_c}$

Impedance

$Z = -jX$

$V_m = I_m R$ $\qquad V_m = I_m X$ $\qquad V_m = I_m X$

$R = R$ $\qquad \boxed{X = j\omega L}$ $\qquad \boxed{X = \frac{-j}{\omega C}}$

$X = \frac{-j^2}{j\omega C}$

$\boxed{X = \frac{1}{j\omega C}}$

-Big juice here(Even better if scrolled down). Impedance (Ohms) and Reactance

$$Z = R + jX$$

Impedance = Resistance $+j$ Reactance

TABLE 9.1

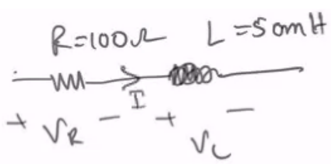
	Impedance (Z)	Reactance (X)
Resistor (R)	R	—
Inductor (L)	$j\omega L$	ωL
Capacitor (C)	$\frac{-j}{\omega C}$	$\frac{-1}{\omega C}$

-That means AC Circuit

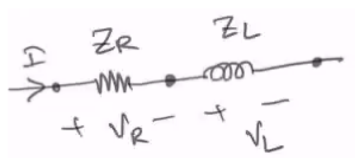
$$\boxed{V = IZ}$$

(V≠IX because it doesn't have angle info whatever idk, just use V=IZ)

-Example 9.6



$$i = 50\cos(1000t + 45^\circ)\ \text{mA}$$



-Z_R (Impedance of Resistor)

-Z_R =100 ohms ez

-Z_L (Impedance of Inductor)

$$Z_L = j\omega L$$
$$= j\,1000 \times 50 \times 10^{-3}$$
$$= j50\ \Omega$$

-Current

$$I = 50\angle 45^\circ\ \text{mA}$$
$$I = 0.5\angle 45^\circ\ \text{A}$$

-V_R

$$V_R = I_R Z_R$$
$$V_R = 0.5\angle 45^\circ * 100$$
$$= 50\angle 45^\circ\ \text{V}$$

-V_L

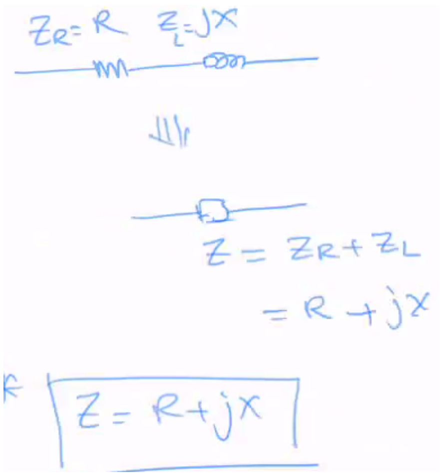
$$V_L = I_L Z_L$$
$$= 0.5\angle 45^\circ * j50$$
$$= 0.5\angle 45^\circ * 50\angle 90^\circ$$
$$= 2.5\angle 135^\circ\ \text{V}$$

-V_L + V_R

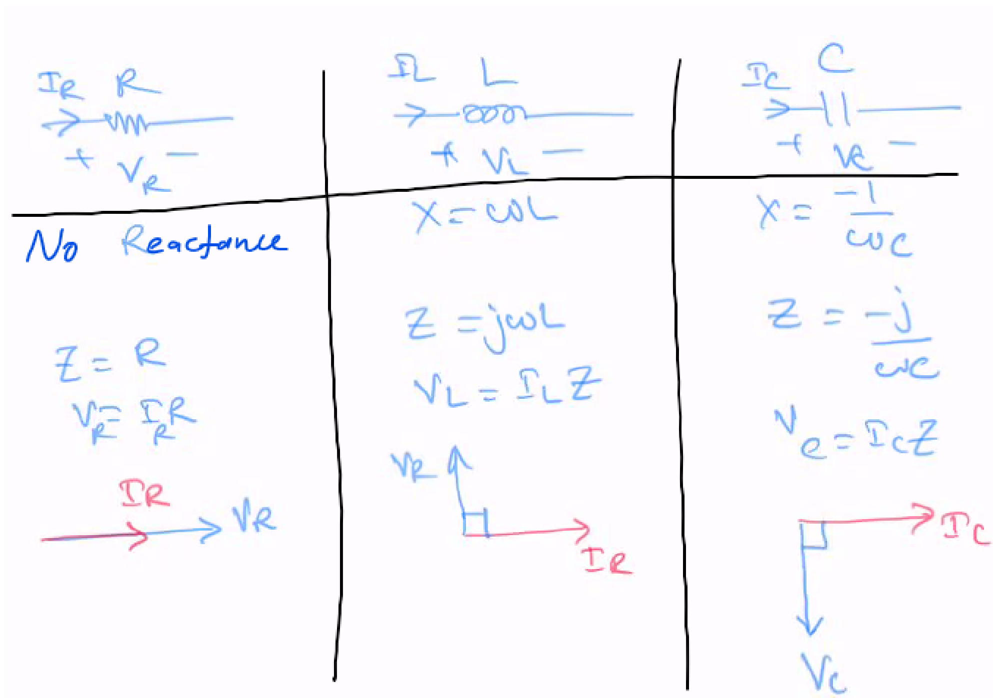
$$V = V_L + V_R$$
$$= 2.5\angle 135^\circ + 50\angle 45^\circ$$

-Impedance addition

-Since both are ohms, you can add if it's in series and simplify it.



5/27:



V_R & I_R are "In-Phase"

V_R leads I_R, I_R lags V_R

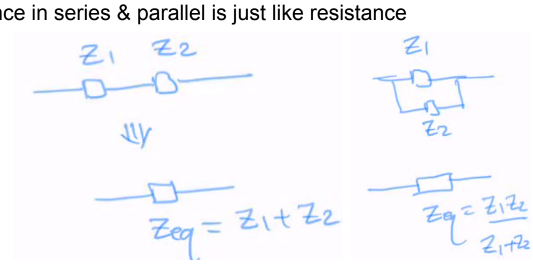
I_C leads V_C, V_C lags I_C

-Impedance is like Resistance

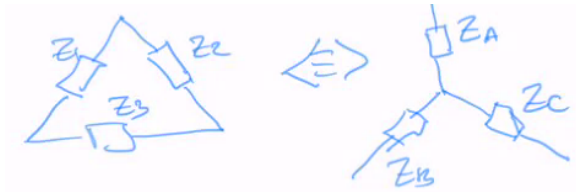
-Impedance extends the concept of resistance to alternating current (AC) circuits, and possesses both magnitude and phase, unlike resistance, which has only magnitude. Impedance is a complex number, with the same units as resistance, for which the SI unit is the ohm (Ω).

-The units of both is ohms.

-Impedance in series & parallel is just like resistance



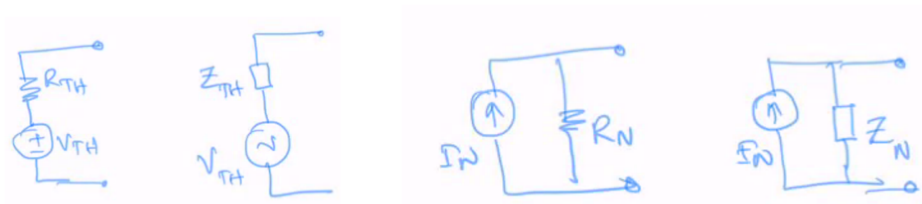
-Same thing when wanting to change delta or y shape



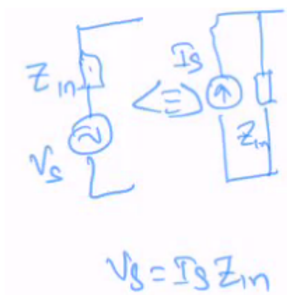
-Ohm's Law, KVL, & KCL too

Ohm's law	$V = IR$	$V = IZ$
K.C.L	$\sum I_i = 0$	$\sum I_i = 0$
K.V.L	$\sum V_i = 0$	$\sum V_i = 0$

-Thevenin & Norton for Impedance



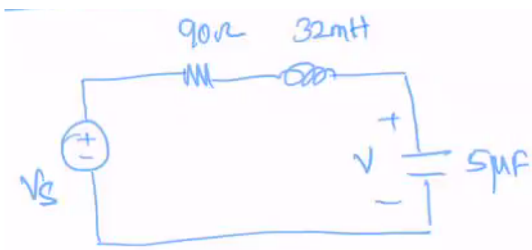
-Source transformation w/ impedance



-Node Voltage & Mesh Current

-Impedance just like resistance

-Example 9.8



$V_s = 750\cos(5000t + 30^\circ)\text{ V} \Rightarrow 750\angle 30^\circ$

$\omega = 5000 \text{ rad/s}$

-Impedance of inductor

$L = 32\text{mH} \Rightarrow X_L = \omega L = 5000 \times 32 \times 10^{-3} = 160\,\Omega$

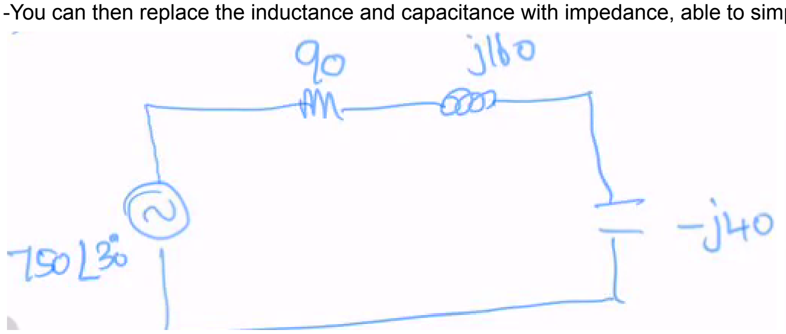
$Z_L = j160$

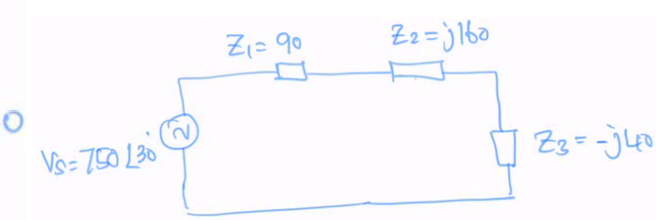
-Impedance of capactior

$C \Rightarrow 5\mu F \Rightarrow X_C = \frac{-1}{\omega C} = \frac{-1}{5000 \times 5 \times 10^{-6}} = -40\,\Omega$

$Z_C = -j40$

-You can then replace the inductance and capacitance with impedance, able to simplify future calculation.





-Voltage across capacitor by VDivRule

$$v = \frac{750\angle 30^\circ}{Z_1 + Z_2 + Z_3} \cdot Z_3 = \frac{750\angle 30^\circ}{90 + j160 - j40} \cdot (-j40) = 200 \angle -113.13^\circ \ \text{V}$$

-Convert back to time domain

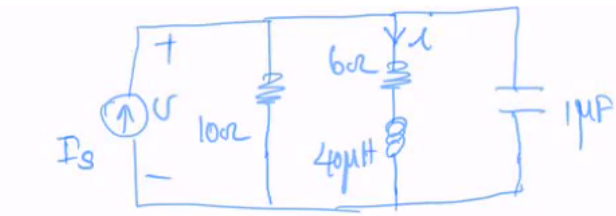
$$v(t) = 200 \cos(5000t - 113.13^\circ) \ \text{V}$$

-Assessment Problem 9.1

  -Time domain to phasor domain

  -When give time domain as sin, -90* to phi to get it to be cos

-Example 9.9



$i_s = 8\cos(200\,000\,t)$ A

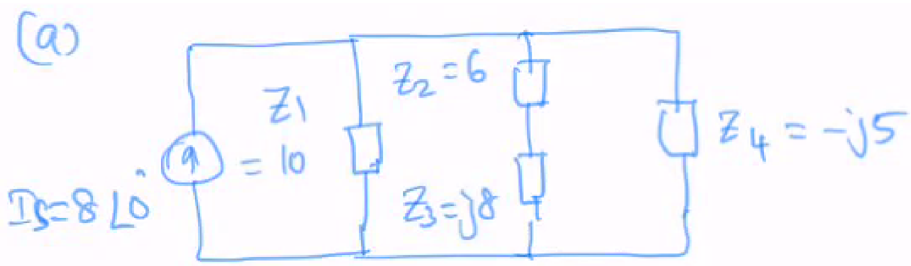
$\omega = 200\,000$ rad/s

$I_s = 8\angle 0^\circ$

  -Impedance of L & C

$$Z_L = j\omega L = j\,200\,000 \times 40 \times 10^{-6} = j8$$

$$Z_C = \frac{-j}{\omega C} = \frac{-j}{200\,000 \times 1 \times 10^{-6}} = -j5$$



  -Z_eq

    Blah blah just like resistor but with phasor

  -Current

$$V = IZ$$

$$I = \frac{V}{Z_{eq}} = \frac{8\angle 0^\circ}{Z_{eq}}$$

$$I =$$

poop

-Conductance, SUSceptance, & Admittance

	Impedance	
Resistor (R)	$Z_R = R$	$G = 1/R$ (S) (conductance)
Inductor (L)	$Z_L = j\omega L = X$	$B = \frac{1}{X}$ (S)
Capacitor (C)	$Z_C = \frac{-j}{\omega C} = X$	$B = \frac{1}{X}$ (S) (Suceptance)

$$Y = \frac{1}{Z} \quad (S)$$

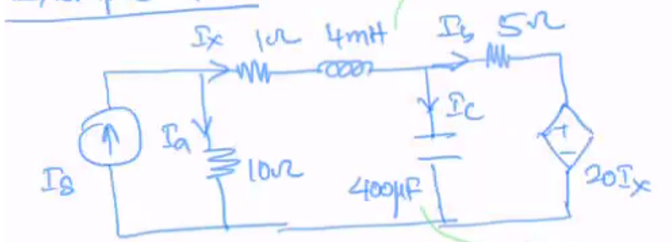
(Admittance)

-Example 9.13 (Node Voltage Analysis w/ impedance)

  -First convert to current, L, C to phasor

Example 9.13

$j\omega L = j \times 500 \times 4 \times 10^{-3} = j2$

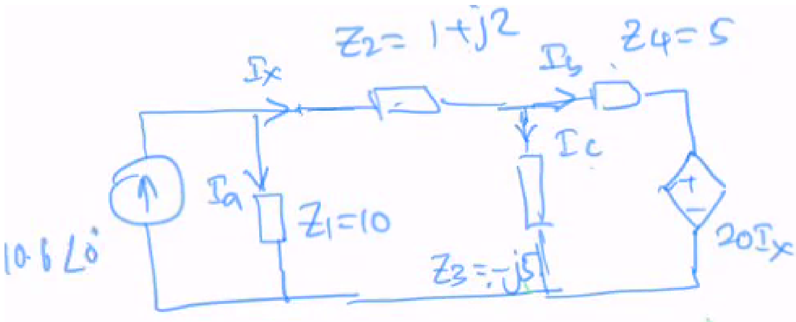


$\frac{-j}{\omega C} = \frac{-j}{500 \times 400 \times 10^{-6}} = -j5$

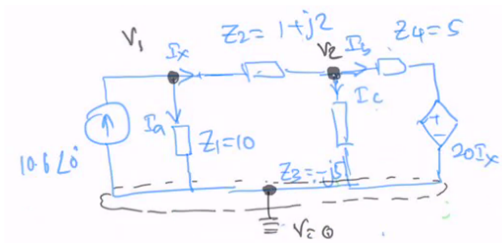
$I_s = 10.6 \cos(500t)$ A

$\omega = 500 \text{ rad s}^{-1}$

$I_s = 10.6 \angle 0^\circ$



-Nodes



-KCL

K.C.L for node 1:

$$-10.6 \angle 0^\circ + \frac{V_1 - 0}{Z_1} + \frac{V_1 - V_2}{Z_2} = 0$$

$$-10.6 \angle 0^\circ + \frac{V_1}{10} + \frac{V_1 - V_2}{1 + j2} = 0$$

K.C.L for node 2:

$$\frac{V_2 - V_1}{Z_2} + \frac{V_2 - 0}{Z_3} + \frac{V_2 - 20I_x}{Z_4} = 0$$

$$\frac{V_2 - V_1}{1 + j2} + \frac{V_2}{-j5} + \frac{V_2 - 20I_x}{5} = 0$$

$$I_x = \frac{V_1 - V_2}{Z_2} = \frac{V_1 - V_2}{1 + j2}$$

$V_1 = \checkmark$

$V_2 = \checkmark$

$I_x = \checkmark$

-Currents

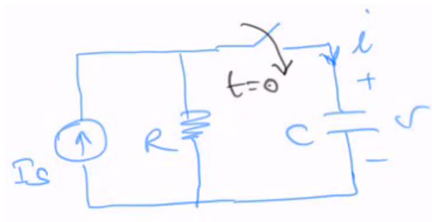
$I_a = \frac{V_1}{10} = \checkmark$

$I_b = \frac{V_2 - 20I_x}{5} = \checkmark$

$I_c = \frac{V_2}{-j5} = \checkmark$

6/1:

-Step Response of RC (charging)

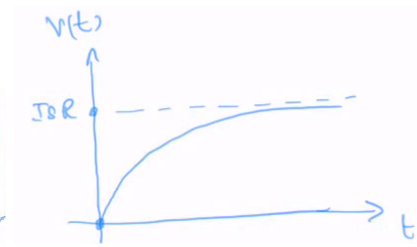


-KCL

$$C\frac{dv}{dt} + \frac{v}{R} - I_s = 0$$

-DE solved

$$v(t) = I_sR + (V_0 - I_sR)e^{-t/RC}$$



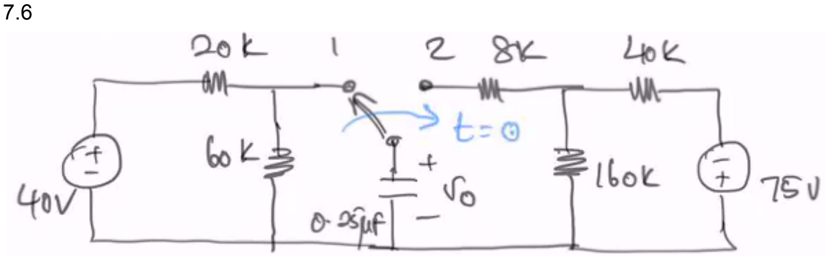
-If v=0

$$v(t) = I_sR - I_sRe^{-t/RC}$$
$$= I_sR(1 - e^{-t/RC})$$

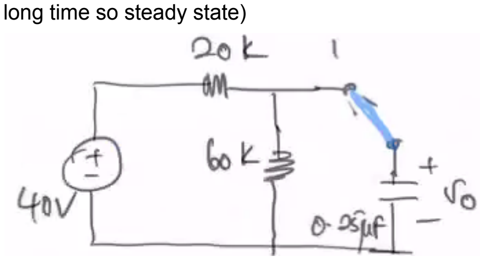
-tau

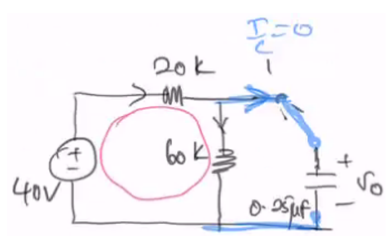
$$\tau = RC$$

-Example 7.6



-t<0 (after long time so steady state)

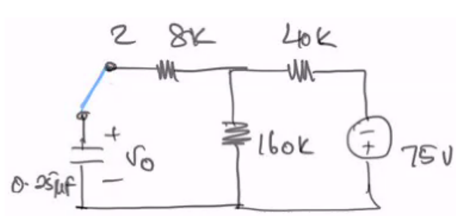




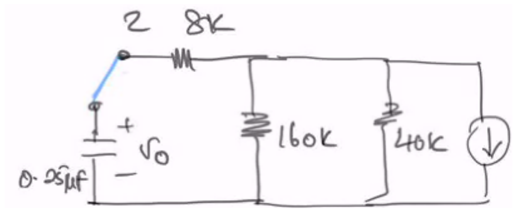
-Ic=0

$$V_{60k} = \frac{40}{20+60} \times 60$$
$$= 30V$$
$$V_0 = 30V$$

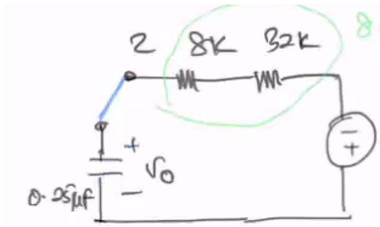
-t>0



-Req by source conversion



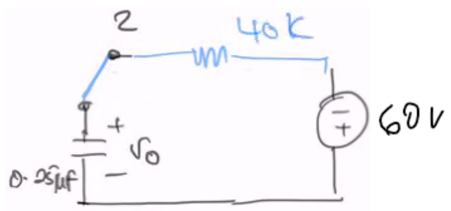
$$\frac{75}{40} = 1.875\ A$$

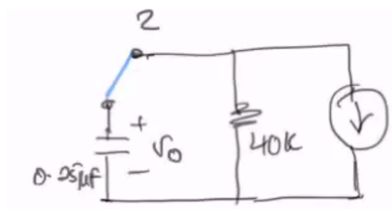


$8k + 32k = 40k$

$$1.875 \times 32k = 60\ V$$

Req = 40k





$$\frac{60}{40} = 1.5\ mA$$

-So the voltage step response (charging) is

$$v(t) = -I_s R + (V_0 - I_s R)\, e^{-t/RC}$$

$$= -1.5\times10^{-3} \times 40\times10^{3} + (30 + 1.5\times10^{-3} \times 40\times10^{3})\, e^{-\frac{t}{40k \times 0.25\times\mu}}$$

$$= -60 + (30 + 60)\, e^{-100t}$$

$$= -60 + 90 e^{-100t}$$

$$\underline{v(t) = -60 + 90 e^{-100t}\ V}$$

-The current step response is

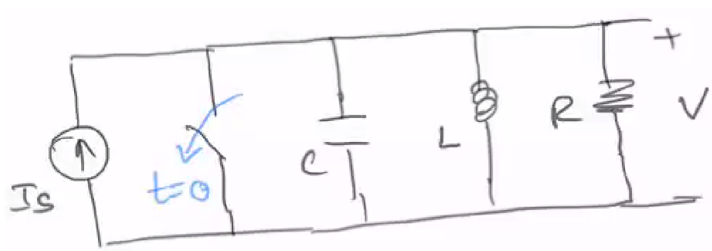
$$i = C \frac{dv}{dt}$$

$$i(t) = 0.25\times\mu \frac{d}{dt}\left[-60 + 90 e^{-100t}\right]$$

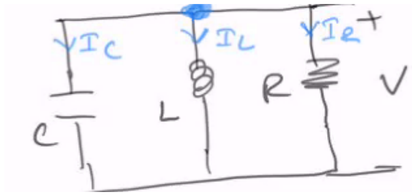
$$= 0.25\times10^{-6} \left[0 + 90 e^{-100t}(-100)\right]$$

$$\underline{= -2.25 e^{-100t}\ mA}$$

-Parallel RLC Circuit Natural Response



-KCL



$v_C(0) = V_{C0}$

$i_L(0) = I_{L0}$

$i_C = C\dfrac{dv_C}{dt}$

$v_L = L\dfrac{di_L}{dt}$

K.C.L =>

$I_C + I_L + I_R = 0$

$$C\frac{dv_C}{dt} + \frac{1}{L}\int_0^t v_L\,dt + \frac{v_R}{R} = 0$$

$$v_C = v_L = v_R = V$$

$$C\frac{dv}{dt} + \frac{1}{L}\int_0^t v\,dt + \frac{v}{R} = 0 \quad \text{------>} \quad C\frac{d^2v}{dt^2} + \frac{v}{L} + \frac{1}{R}\frac{dv}{dt} = 0$$

-Ew DE using CP

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{v}{LC} = 0$$

Assume the solution

$$v = Ae^{st}$$

$$As^2e^{st} + \frac{As}{RC}e^{st} + \frac{Ae^{st}}{LC} = 0$$

$$s^2 + \frac{s}{RC} + \frac{1}{LC} = 0$$

$$\boxed{\begin{array}{c} ax^2+bx+c=0 \\ x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} \end{array}}$$

If

$$\alpha = \frac{1}{2RC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$s_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$s_2 = -\frac{1}{2RC} - \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

-Overdamped, Underdamped, Crit Damped Responses

- Overdamped Response

  $\omega_0^2 < \alpha^2$

  s_1 & s_2 – Real numbers!

  $v = A_1e^{s_1t} + A_2e^{s_2t}$

- Underdamped Response

  $\omega_0^2 > \alpha^2$

  s_1 & s_2 – Complex numbers!

  $v = A_1e^{s_1t} + A_2e^{s_2t}$ → cosine stuff

- Critically damped

  $\omega_0^2 = \alpha^2$

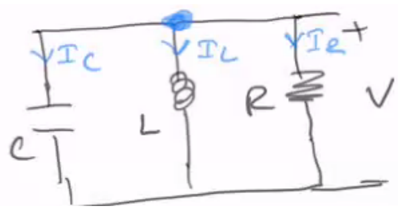
  $s_1 = s_2$ = Real numbers!

  $v = A_1e^{st} + A_2te^{st}$

-Underdamped cosine stuff

$$v = A_1 e^{-\alpha t} \cos\beta t + A_2 e^{-\alpha t} \sin\beta t$$

-Example 8.1



$R = 200\,\Omega$
$L = 50\,mH$
$C = 0.2\,\mu F$

-Solve CP

$$s^2 + \frac{s}{RC} + \frac{1}{LC} = 0$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 200 \times 0.2 \times 10^{-6}} = 12500$$

$$\omega_0^2 = \frac{1}{LC} = \frac{1}{50 \times 10^{-3} \times 0.2 \times 10^{-6}} = 10^8$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$
$$= -12500 + \sqrt{12500^2 - 10^8}$$
$$= -5000$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$
$$= -20000$$

- $\omega_0^2 < \alpha^2 \longrightarrow$ so overdamped!
- s_1 & s_2 are Real & different
  $\hookrightarrow$ so overdamped!

-If resistance is 312.5Ohms

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 312.5 \times 0.2 \times 10^{-6}}$$
$$= 8000$$

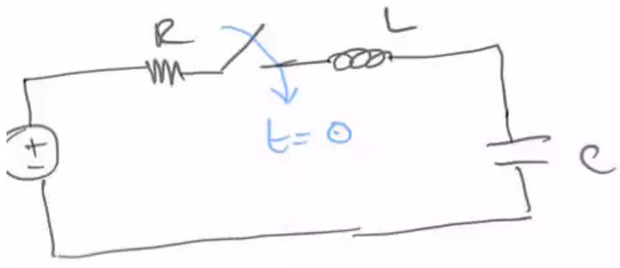
$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$
$$= -8000 + \sqrt{8000^2 - 10^8} \quad \sqrt{-6000^2}$$
$$= -8000 + j6000$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$
$$= -8000 - j6000$$

- $\alpha^2 < \omega_0^2$
- s_1 & s_2 are complex numbers!

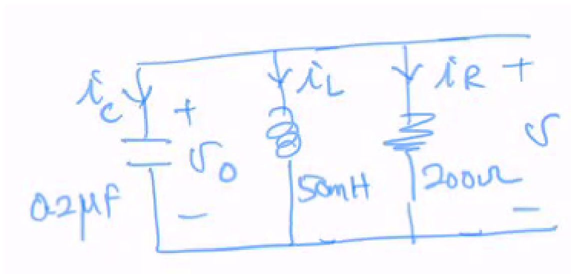
  So, it is underdamped

-Series



Use KVL

-Example 8.2 Parallel Natural Response Full Walkthrough



$$v(0^+) = 12V$$
$$i_L(0^+) = 30mA$$

-Find ur alpha & omega, then s1 & s2 for CP

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 200 \times 0.2 \times 10^{-6}} = 12500 \text{ rad/s}$$

$$\omega_0^2 = \frac{1}{LC} = \frac{1}{50 \times 10^{-3} \times 0.2 \times 10^{-6}} = 10^8 \text{ rad}^2/s^2$$

$$S_1 = -12500 + \sqrt{12500^2 - 10^8} = -5000 \text{ rad/s}$$

$$S_2 = -12500 - \sqrt{12500^2 - 10^8} = -20000 \text{ rad/s}$$

-s1 & s2 are real and different numbers, so Overdamped Solution with IVP

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$
$$v(t) = A_1 e^{-5000t} + A_2 e^{-20000t}$$

-IVP by system of solution

-Vo

$$v(0) = A_1 + A_2 = 12$$

$$v_L(t) = A_1 e^{-5000t} + A_2 e^{-20000t}$$

-Current across inductor

$$i_L(t) = C\frac{dV_L(t)}{dt}$$

$$i_L(t) = C\frac{d\left(A_1 e^{-5000t} + A_2 e^{-20000t}\right)}{dt} = CA_1 e^{-5000t}(-5000) + CA_2 e^{-20000t}(-20000)$$

$$i_L(0) = -5000\,CA_1 - 20000\,CA_2 = 30mA$$
$$-5000 \times 0.2 \times 10^{-6} A_1 - 20000 \times 0.2 \times 10^{-6} A_2 = 30 \times 10^{-3}$$
$$-A_1 - 4A_2 = 30 \quad \text{——} \quad ②$$

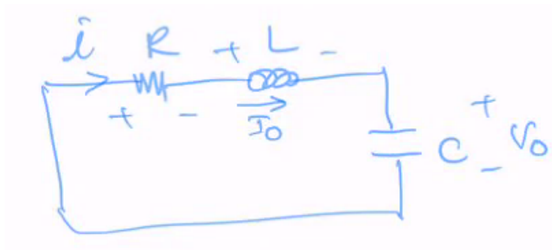
-Both equations gives

$$A_1 = -14$$
$$A_2 = 26$$

-Therefore plug back in

$$v(t) = (-14e^{-5000t} + 26e^{-20,000t})\text{ V}, \quad t \geq 0.$$

-Series RLC Circuit Natural Response



-KVL

$$K.V.L \Rightarrow \quad Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0$$

-DE

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0 \qquad i(t) = Ae^{st}$$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \qquad \alpha = -\frac{R}{2L}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$s_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$s_2 = -\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

-All Responses RLC

	Parallel	Series
Natural (Discharge)	$s^2 + \frac{s}{RC} + \frac{1}{LC} = 0$ $s_{12} = \frac{-1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$ $\alpha = \frac{1}{2RC}$ $\omega = \frac{1}{\sqrt{LC}}$ $s_{12} = -\alpha \pm \sqrt{a^2 - \omega^2}$	$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$ $s_{12} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$ $\alpha = \frac{R}{2L}$ $\omega = \frac{1}{\sqrt{LC}}$ $s_{12} = -\alpha \pm \sqrt{a^2 - \omega^2}$
Step (Charge)		

TABLE 8.3 In Determining the Natural Response of a Second-Order Circuit, We First Determine Whether it is Over-, Under-, or Critically Damped, and Then We Solve the Appropriate Equations

Damping	Natural Response Equations	Coefficient Equations
Overdamped	$x(t) = A_1e^{s_1t} + A_2e^{s_2t}$	$x(0) = A_1 + A_2$; $dx/dt(0) = A_1s_1 + A_2s_2$
Underdamped	$x(t) = (B_1 \cos \omega_d t + B_2 \sin \omega_d t)e^{-\alpha t}$	$x(0) = B_1$; $dx/dt(0) = -\alpha B_1 + \omega_d B_2$, where $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$
Critically damped	$x(t) = (D_1t + D_2)e^{-\alpha t}$	$x(0) = D_2$, $dx/dt(0) = D_1 - \alpha D_2$

TABLE 8.4 In Determining the Step Response of a Second-Order Circuit, We Apply the Appropriate Equations Depending on the Damping

Damping	Step Response Equations[a]	Coefficient Equations
Overdamped	$x(t) = X_f + A_1' e^{s_1 t} + A_2' e^{s_2 t}$	$x(0) = X_f + A_1' + A_2'$; $dx/dt(0) = A_1' s_1 + A_2' s_2$
Underdamped	$x(t) = X_f + (B_1' \cos \omega_d t + B_2' \sin \omega_d t)e^{-\alpha t}$	$x(0) = X_f + B_1'$; $dx/dt(0) = -\alpha B_1' + \omega_d B_2'$
Critically damped	$x(t) = X_f + D_1' te^{-\alpha t} + D_2' e^{-\alpha t}$	$x(0) = X_f + D_2'$; $dx/dt(0) = D_1' - \alpha D_2'$

[a] where X_f is the final value of $x(t)$.

-Xf aka the steady state value