

# SIMULATION METHODS

**FACULTY 2, M.SC. PROGRAM HIGH INTEGRITY SYSTEMS** 

Winter-Semester 2020/2021, Prof. Dr.-Ing. Peter Thoma

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4 FINITE DIFFERENCE METHOD

ORDINARY DIFFERENTIAL EQUATIONS

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### **SUMMARY WHERE WE ARE...**

$$\frac{\partial^2 u(x,t)}{\partial x^2} - k \frac{\partial^2 u(x,t)}{\partial t^2} = 0$$

**Wave Equation** 





#### **Initial value problem**

- Time Domain simulation
- Start from an initial state and simulate how the system behaves as function of time (iterative)
- No further excitation during the simulation

#### **Eigenvalue problem**

- Frequency Domain simulation
- Determine natural resonances of a system (solve eigenvalue problem)
- No excitation at all
- Eigenvectors (eigenmodes) describe the shapes of the resonances
- Eigenvalues describe the resonance frequencies



**DRIVEN PROBLEMS** 



### FINITE DIFFERENCE METHOD AGENDA

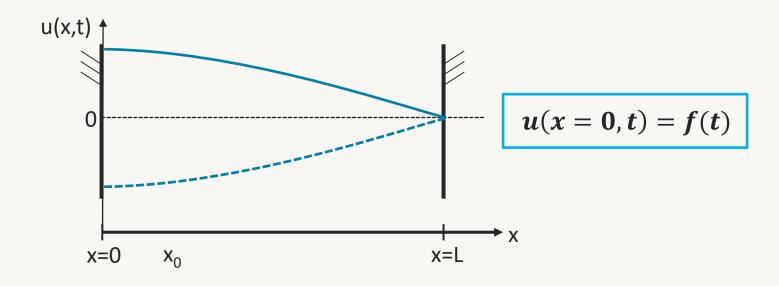
- 1. Wave equation with excitation
- 2. Excitation functions
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### **SOLUTIONS OF THE WAVE EQUATION**

$$\frac{\partial^2 u(x,t)}{\partial x^2} - k \frac{\partial^2 u(x,t)}{\partial t^2} = 0$$

Now we will consider a driven problem where we will have an excitation at the left end of the string.



## **BOUNDARY CONDITIONS FOR DIFF. EQUATIONS**

#### TYPES OF BOUNDARY CONDITIONS FOR DIFFERENTIAL EQUATIONS

#### 1. Dirichlet (first-type) boundary condition

Describe the value of the solution at the domain boundary, e.g.

$$u(x = 0, t) = f(t)$$

#### 2. Neumann (second-type) boundary condition

Describe the derivative of the solution normal to the domain boundary, e.g.

$$\left. \frac{\partial u(x,t)}{\partial x} \right|_{x=0} = f(t)$$

#### 3. Robin (third-type) boundary condition

Combination of Dirichlet and Neumann boundary conditions, e.g.:

$$\alpha \left. \frac{\partial u(x,t)}{\partial x} \right|_{x=0} + \beta u(x=0,t) = f(t)$$

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### TRANSIENT SOLUTION OF THE DRIVEN PROBLEM

$$\frac{\partial^2 u(x,t)}{\partial x^2} - k \frac{\partial^2 u(x,t)}{\partial t^2} = 0$$

Previously we derived the following transient iteration scheme:

$$u_{i,j} = cu_{i+1,j-1} + cu_{i-1,j-1} + 2(1-c)u_{i,j-1} - u_{i,j-2} \quad \text{with} \quad c = \frac{\Delta t^2}{k\Delta x^2}$$

Now we assume that the string is **straight** and **at rest** at the **beginning**:

$$\frac{u(x,t=0)=0}{\left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0}} \Rightarrow u_{i,0}=0$$
Initial conditions

The end of the string is still fixed, but we apply an excitation at the beginning:

$$u(0,t)=f(t) \Rightarrow u_{0,j}=f_j$$
  $u(L,t)=0 \Rightarrow u_{N,j}=0$  Boundary conditions (Dirichlet type)



### TRANSIENT SOLUTION OF THE DRIVEN PROBLEM

$$c = \frac{\Delta t^2}{k \Delta x^2}$$

Based on this information, we obtain the following **iteration scheme** in the same way as before:

$$i=1,\dots,N-1$$
 :  $u_{i,0}=0$  Initialization 
$$u_{0,1}=f_1$$
 First step 
$$i=1,\dots,N-1$$
 :  $u_{i,1}=cu_{i+1,0}+cu_{i-1,0}+2(1-c)u_{i,0}-u_{i,0}$  
$$u_{N,1}=0$$

$$j=2,\ldots:\ u_{0,j}=f_{j}$$
 Iteration 
$$i=1,\ldots,N-1, j=2,\ldots:\ u_{i,j}=cu_{i+1,j-1}+cu_{i-1,j-1}+2(1-c)u_{i,j-1}-u_{i,j-2}$$
 
$$j=2,\ldots:\ u_{N,j}=0$$



### TRANSIENT SOLUTION OF THE DRIVEN PROBLEM

$$c = \frac{\Delta t^2}{k \Delta x^2}$$

Based on this information, we obtain the following **iteration scheme** in the same way as before:

# FINITE DIFFERENCE METHOD AGENDA

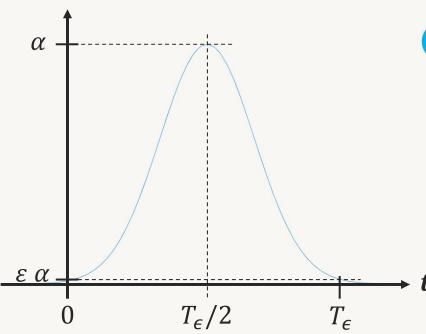
- **1**
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#### **EXCITATION FUNCTIONS**

The excitation function should be zero at the beginning of the simulation. An often-used smooth excitation function is a shifted gaussian pulse:

$$f(t) = \alpha e^{-\beta (t-\gamma)^2}$$



1 
$$f(T_{\varepsilon}/2) = \alpha \Rightarrow \gamma = T_{\varepsilon}/2$$

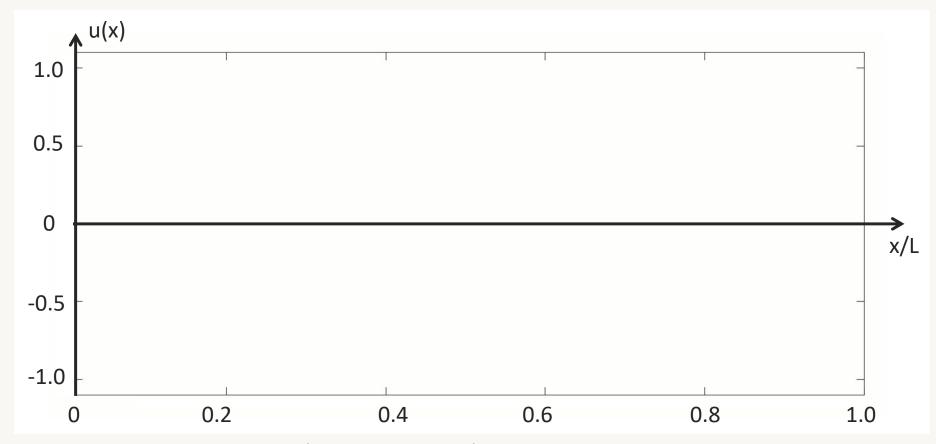
2 
$$f(T_{\varepsilon}) = \alpha e^{-\beta (T_{\varepsilon}/2)^2} = \varepsilon \alpha$$
  

$$\Rightarrow \beta = -\left(\frac{2}{T_{\varepsilon}}\right)^2 \ln \varepsilon$$

$$f(t) = \alpha e^{-\beta (t - T_{\varepsilon}/2)^{2}}$$
$$\beta = -\left(\frac{2}{T_{\varepsilon}}\right)^{2} \ln \varepsilon$$

Typical value for sufficient smoothness:  $\varepsilon = 0.001$ 

Run the transient scheme with **Gaussian excitation** at the left end of the string:



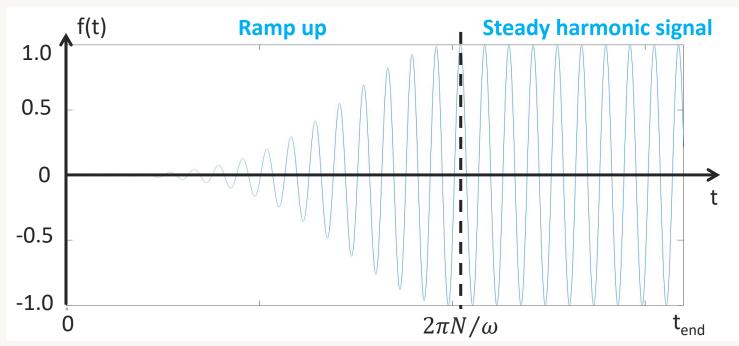
Video: Guitar\_string\_with\_Gaussian\_Excitation



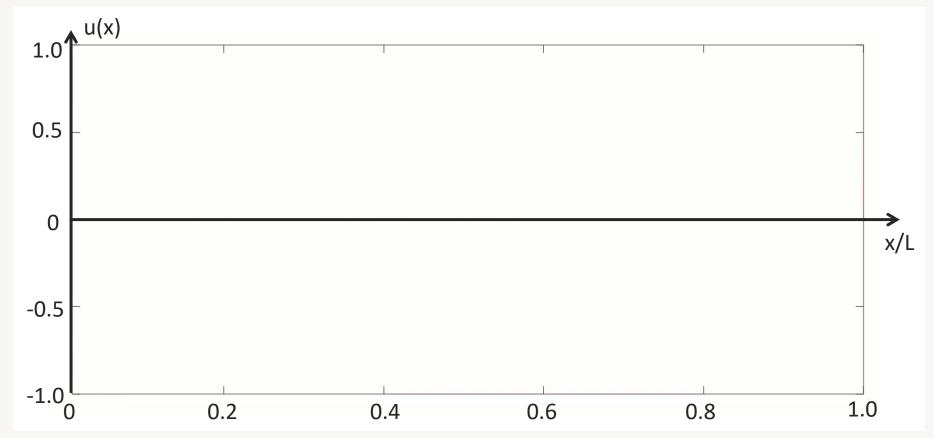
As an alternative to a pulse shaped excitation, a ramped up harmonic excitation can be used, too. A ramped up sinusoidal excitation function can be defined as follows:

$$f(t) = \begin{cases} \frac{\ln 0.001}{e^{\left(N\frac{2\pi}{\omega}\right)^2}} (t - N\frac{2\pi}{\omega})^2 \\ e^{\left(N\frac{2\pi}{\omega}\right)^2} & sin(\omega t), & t < 2\pi N/\omega \\ sin(\omega t) & , & t \ge 2\pi N/\omega \end{cases}$$

Ramp up for N periods

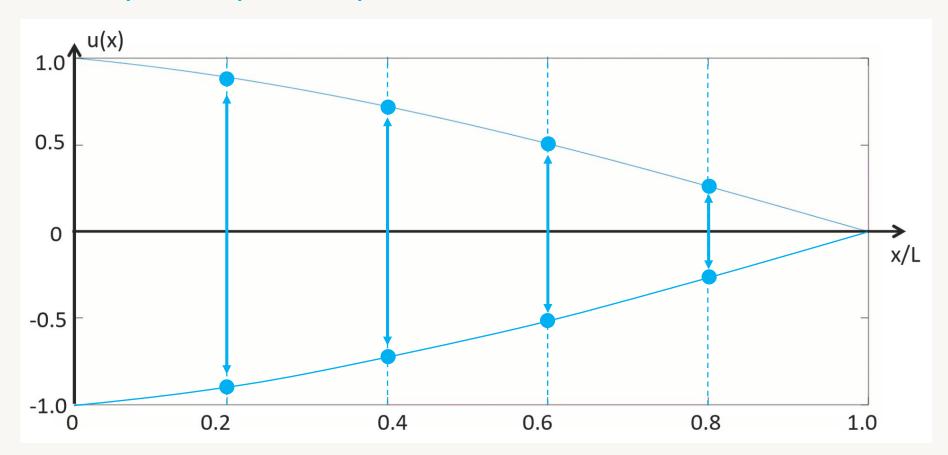


Now run the transient scheme with a **slowly ramped up sinusoidal** excitation at the left end of the string:



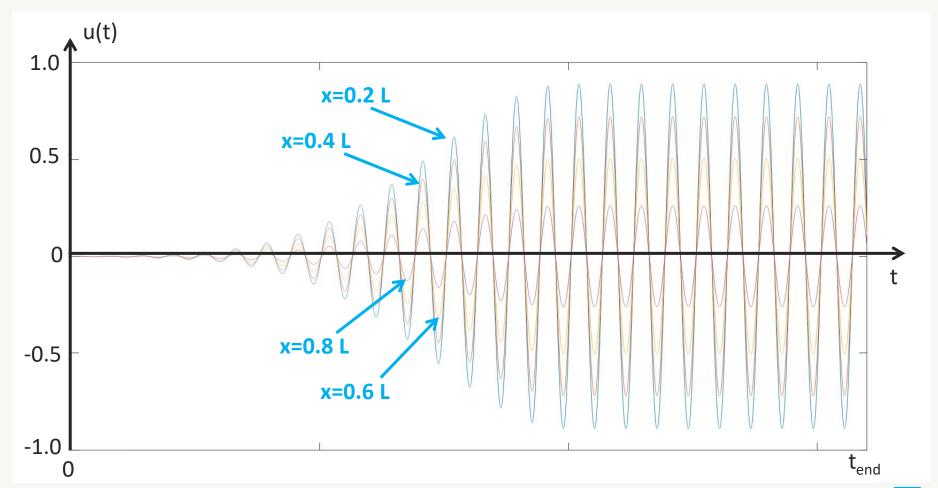
Video: Guitar\_string\_with\_harmonic\_excitation

At each position along the string, the string performs a harmonic oscillation with a position dependent amplitude:

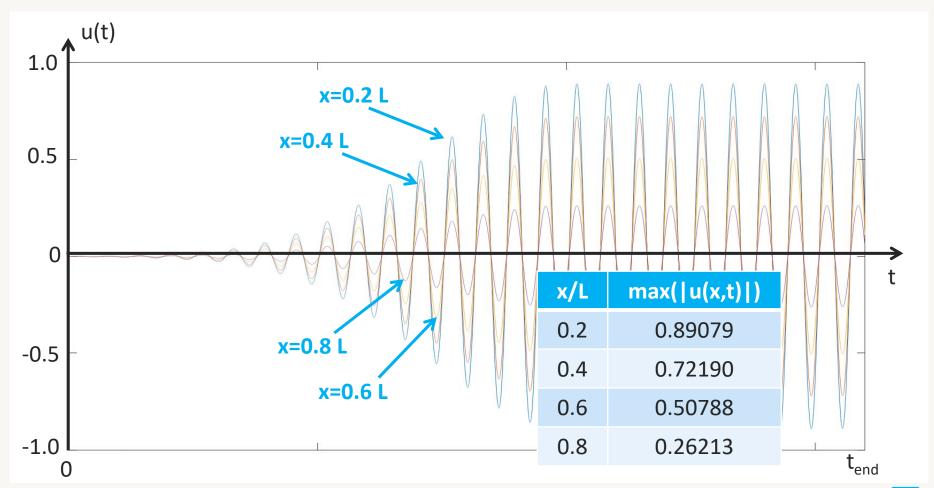


Thus, after a while the string performs a continuous harmonic steady-state oscillation

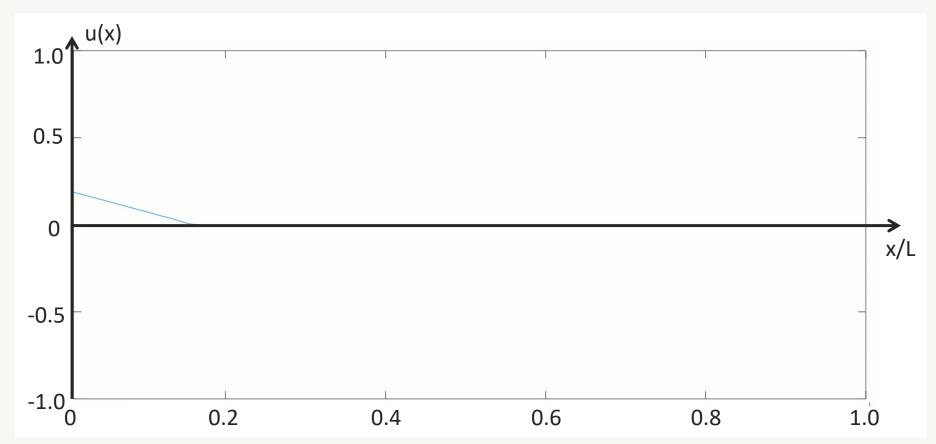
At each position along the string, the string performs a harmonic oscillation with a position dependent amplitude:



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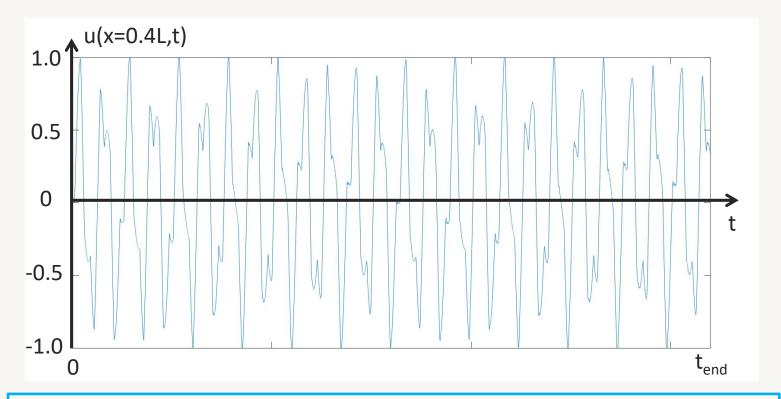


What happens if we switch on the sinus without ramping up the amplitude?



Video: Guitar\_string\_with\_step\_harmonic\_excitation

The step function type excitation creates a lot of high frequency content stimulating many higher order modes which disturb the harmonic solution:



Note: Smooth excitation functions need to be used if harmonic steady-state solutions shall be obtained from transient simulations directly

## FINITE DIFFERENCE METHOD AGENDA

- ✓ 1. Wave equation with excitation
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#### FREQUENCY DOMAIN SOLUTION OF THE DRIVEN PROBLEM

Time harmonic **steady-state** solutions can be obtained from the **Frequency Domain** formulation:

$$\frac{\partial^2 \underline{u}(x)}{\partial x^2} = -k \ \omega^2 \ \underline{u}(x)$$

Applying a spatial discretization (central difference) again, we obtain:

$$\frac{\underline{u}_{i+1} - 2\underline{u}_i + \underline{u}_{i-1}}{\Delta x^2} = -k \ \omega^2 \underline{u}_i$$

For the special case i = 1 we get:

$$\frac{\underline{u}_2 - 2\underline{u}_1 + \underline{u}_0}{\Lambda x^2} = -k \ \omega^2 \underline{u}_1$$

Our time harmonic excitation function defines the value of  $\underline{u}_0 = f$ 

$$\frac{\underline{u}_2 - 2\underline{u}_1}{\Delta x^2} = -k \ \omega^2 \underline{u}_1 - \frac{\underline{u}_0}{\Delta x^2}$$



#### FREQUENCY DOMAIN SOLUTION OF THE DRIVEN PROBLEM

In matrix form, we can then write (the first row contains the excitation):

$$\begin{pmatrix} \ddots & & & \\ & \ddots & & \\ & 1 & -2 & 1 \\ & & \ddots & \\ & & & \ddots \end{pmatrix} \begin{pmatrix} \underline{u}_1 \\ \underline{u}_2 \\ \vdots \\ \underline{u}_{N-1} \end{pmatrix} = -k \ \omega^2 \Delta x^2 \begin{pmatrix} \underline{u}_1 \\ \underline{u}_2 \\ \vdots \\ \underline{u}_{N-1} \end{pmatrix} + \begin{pmatrix} -\underline{f} \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

Or by adding the term  $k \omega^2 \Delta x^2 \underline{u}$ :

$$\begin{pmatrix} \ddots & & & & \\ & \ddots & & & \\ & 1 & k \omega^2 \Delta x^2 - 2 & 1 \\ & & \ddots & \\ & & & & \ddots \end{pmatrix} \begin{pmatrix} \underline{u}_1 \\ \underline{u}_2 \\ \vdots \\ \underline{u}_{N-1} \end{pmatrix} = \begin{pmatrix} -\underline{f} \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$M$$

$$\underline{u}$$

$$b$$

#### FREQUENCY DOMAIN SOLUTION OF THE DRIVEN PROBLEM

The resulting **equation system** has the form of:

$$M u = b$$

where b is called the excitation vector or often simply right-hand side.

The system can be solved by calculating the matrix inverse

$$\underline{u} = M^{-1} b$$

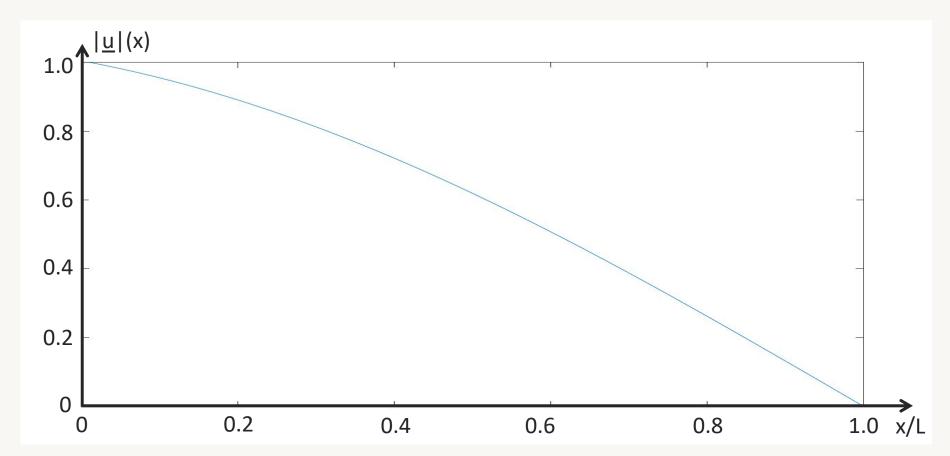
or by applying any algorithm to solve this linear system of equations:

- 1. **Direct methods**, e.g.
  - 1. Gaussian elimination
  - 2. Matrix factorization
- 2. **Iterative methods**, e.g.
  - 1. Preconditioned Krylow subspace methods (e.g. CG)
  - 2. Multigrid methods



### FREQUENCY DOMAIN SOLUTION OF THE DRIVEN PROBLEM

Solving the equation system in MATLAB using linsolve yields:





### MATLAB: USEFUL TIPS AND TRICKS

#### SOLVING MATRIX EQUATIONS M u = b

```
u = inv(M)*b % or better: u = M\b

u = linsolve(M, b)% direct solution using factorization

% iterative solvers for sparse matrices
S = sparse(M) % Convert a full matrix into sparse form

u = pcg(S, b) % preconditioned conjugate gradients method u = bicg(S,b) % BiConjugate gradients method u = bicgstab(S,b) % BiConjugate gradients stabilized method u = qmr(S,b) % Quasi-Minimal Residual (QMR) method u = tfqmr(S,b) % Transpose-Free QMR method
```

There is a large collection of other solvers and additional libraries available for solving linear equation systems. Optimal solver selection depends on matrix properties and influences runtime and memory requirements

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### **RESULTS IN TIME AND FREQUENCY DOMAIN**

**Comparing** the **Time Domain** and **Frequency Domain** results for the harmonic steady-state behavior, we get:

x/L	Transient max( u(x,t) )	Frequency Domain   <u>u</u> (x)	Deviation  TD from FD
0.2	0.89079	0.89376	0.33%
0.4	0.72190	0.72787	0.82%
0.6	0.50788	0.51672	1.74%
0.8	0.26213	0.27345	4.14%

#### Reasons for the differences:

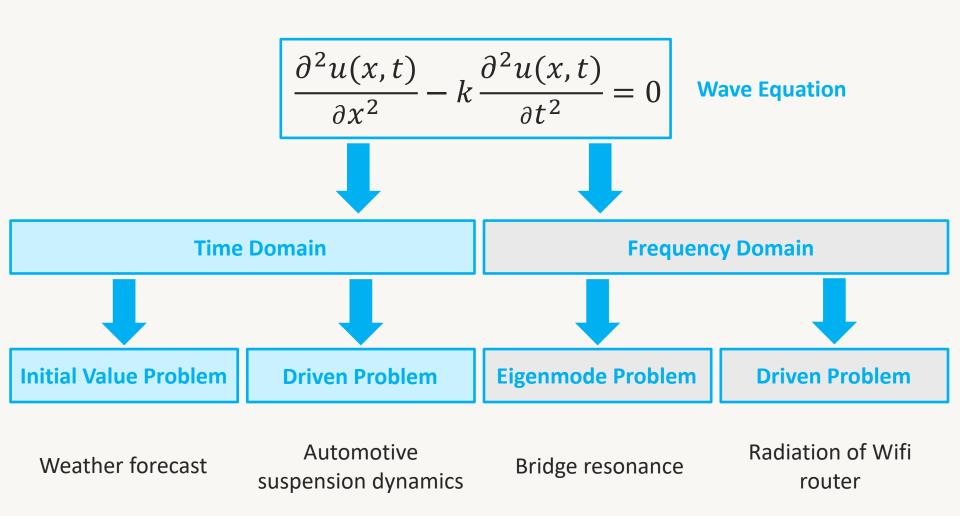
- 1. Steady-state solution not yet reached in transient simulation
- 2. Excitation of higher order eigenmodes in transient simulation (smoothness of excitation)
- 3. Discretization error of the time derivatives (numerical dispersion)
- 4. Numerical error when solving the linear equation system

**Conclusion:** It is not straight-forward to compute harmonic steady-state solutions from transient simulations, but possible.

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# **SOLUTIONS OF THE WAVE EQUATION**



## **SOLUTIONS OF THE WAVE EQUATION**

### **SUMMARY OF FORMULATIONS**

#### 1. Transient

- Discretization of spatial and temporal derivatives
- Ideally suited for transient phenomena
- Explicit scheme, matrix times vector multiplications only
- Can be less efficient for calculating harmonic steady-state solutions

#### 2. Frequency Domain

- Transformation into frequency domain and discretization of spatial derivatives
- Ideally suited for computing harmonic steady-state solutions
- Requires solutions of equation system, either direct or iterative
- Can be less efficient for calculating transient solutions due to large number of required frequency samples

#### 3. Eigenmode

- Special variant of the frequency domain formulation without excitation
- Ideally suited for computing the eigenmodes (resonances) of a system

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#### MATERIAL PROPERTIES

So far, we have solved the wave equation for **constant** k:

$$\frac{\partial^2 u(x,t)}{\partial x^2} - k \frac{\partial^2 u(x,t)}{\partial t^2} = 0$$

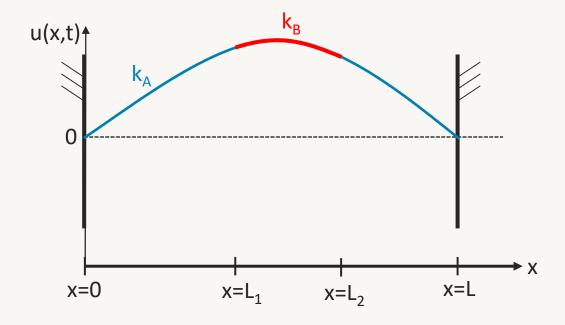
The parameter k describes the **speed** of the spatial **wave propagation** and is determined by the **properties of the materials** in which the wave propagates

In our example, it is described by the stiffness of the string

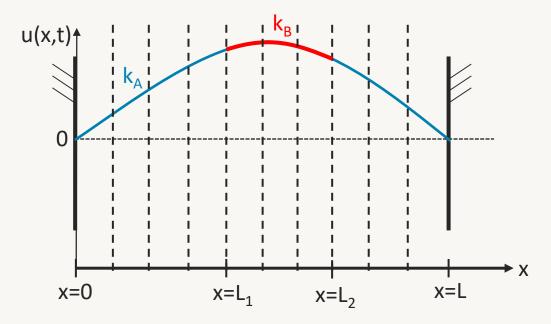
What happens if the string contains a segment with a different stiffness?



### **MATERIAL PROPERTIES**



#### **MATERIAL PROPERTIES**



- 1. Perform a spatial discretization of the string as before
- 2. Apply a specific value of k for each segment (instead of using a constant k):

$$\frac{\underline{u}_{i+1} - 2\underline{u}_i + \underline{u}_{i-1}}{\Delta x^2} = -\mathbf{k}_i \,\omega^2 \underline{u}_i$$

# •

### **VARYING MATERIAL PROPERTIES**

#### **EXAMPLE: TRANSIENT SOLUTION OF THE DRIVEN PROBLEM**

Based on this approach, we obtain the following iteration scheme:

$$\mathbf{c_i} = \frac{\Delta t^2}{k_i \Delta x^2}$$

$$i=1,\ldots,N-1$$
 :  $u_{i,0}=0$  Initialization

$$u_{1,1} = f_1$$

First step

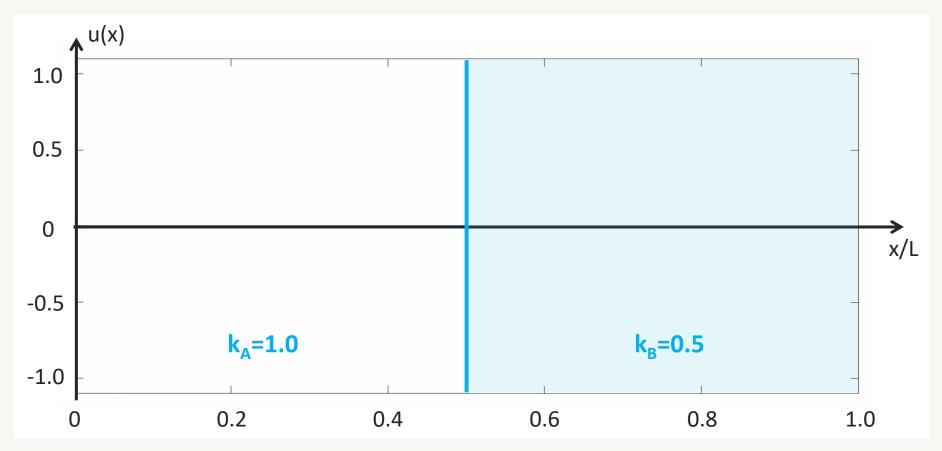
$$i = 2, ..., N - 1$$
 :  $u_{i,1} = c_i (u_{i+1,0} + u_{i-1,0} - 2u_{i,0}) + u_{i,0}$ 

$$j = 2, \dots : u_{1,j} = f_j$$

**Iteration** 

$$i=2,\ldots,N-1, j=2,\ldots:\ u_{i,j}=c_i\left(u_{i+1,j-1}+u_{i-1,j-1}-2u_{i,j-1}\right)+2u_{i,j-1}-u_{i,j-2}$$

Run the transient scheme with **Gaussian** excitation at the left end of the string and **two different values for the string stiffness**:



Video: Guitar\_string\_with\_two\_materials

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**CST Studio Suite**® is a high-performance **3D Electromagnetic and Thermal analysis** software package for designing, analyzing and optimizing components and systems. Next week, we will start using this commercial software package to run some simulations

A academic version is available in the computer rooms at the university

However, due to the current restrictions it is **highly recommended** (but optional) that you install a student edition of the software on your own computer

Since the confirmation of your registration will take a **few days**, please go ahead and download and install the software **as soon as possible** 

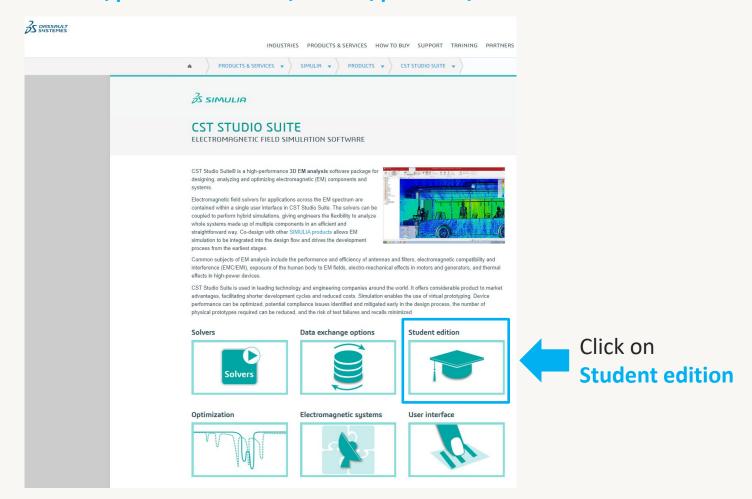
The Student Edition can be downloaded free of charge from <a href="https://www.3ds.com/products-services/simulia/products/cst-studio-suite">https://www.3ds.com/products-services/simulia/products/cst-studio-suite</a>

The following slides will explain how to install the student edition on your own computer



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The CST Studio Suite® Student Edition introduces you to the world of electromagnetic simulation, making Maxwell's equations easier to understand than ever. With this free edition, you have access to our powerful visualization engine and some of the most advanced solvers of CST Studio Suite - with some restrictions related to the Student Edition.

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#### Student Edition







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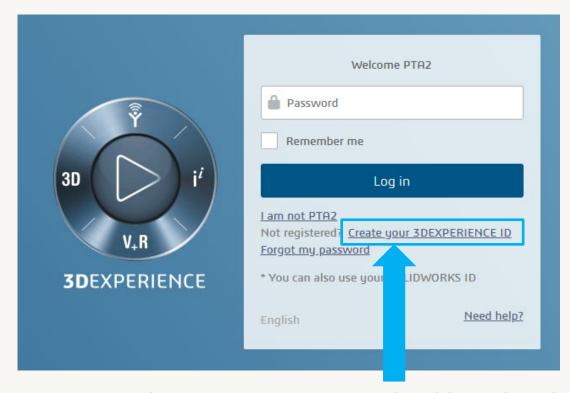


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CST Studio Suite Student Edition is ideal for use as part of coursework as well as for anyone wishing to become more proficient with CST Studio Suite.

Even if you are not affiliated with an academic institution, you can immediately download the CST Studio Suite Student Edition for free and get access to the tutorials at the SIMULIA Learning Commuity (SLC)



You need to create an account to be able to download the software

The video contains introductions about **how to create an account** (starting at 0:15): <a href="https://youtu.be/Z8cMPTVeWsl">https://youtu.be/Z8cMPTVeWsl</a>



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# **THANK YOU!**

See you in the exercise!