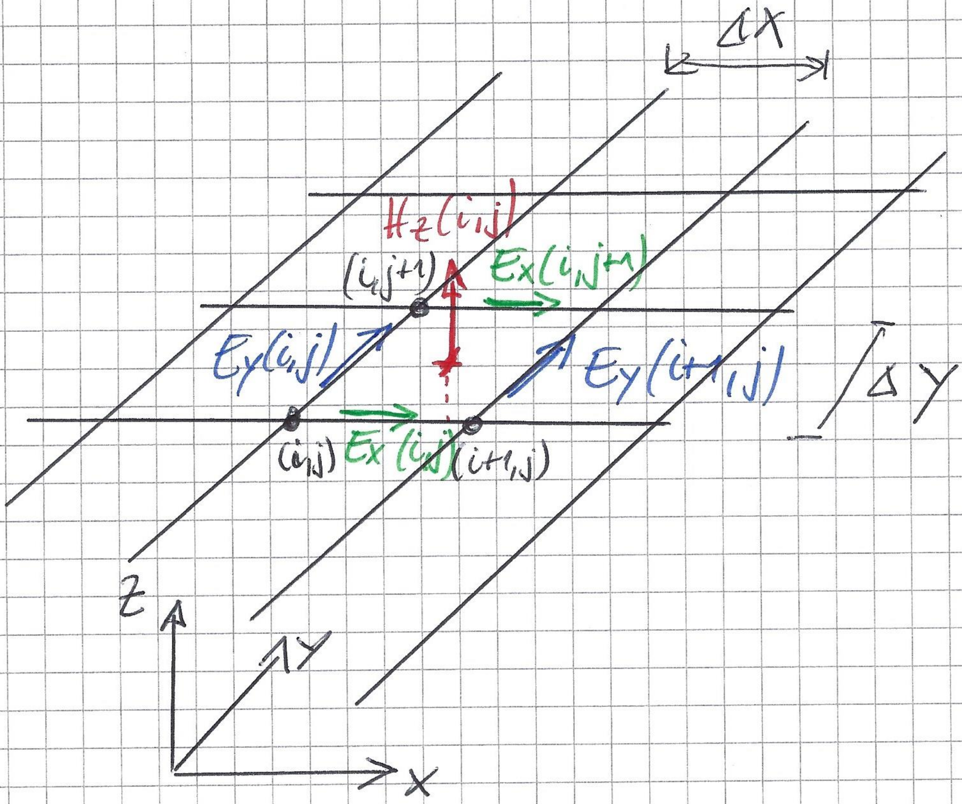
TE-Case: E_x, E_y, H_z



$$E(x = i \cdot \Delta x, y = j \cdot \Delta y) =: E(i,j)$$

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \iint \mu \vec{H}\, d\vec{A}$$

$$\Rightarrow \cancel{-\Delta E_y(i,j) + E_x(i,j}$$

$$-E_y(i,j) \cdot \Delta y + E_x(i,j) \cdot \Delta x + E_y(i+1,j)\, \Delta y$$

$$-E_x(i,j+1)\, \Delta x = -\frac{d}{dt}\left(\mu \cdot H_z(i,j) \cdot \Delta x \cdot \Delta y\right)$$

$$\Rightarrow \frac{d}{dt} H_z(i,j) = -\frac{1}{\mu} \cdot \left(\frac{E_x(i,j) - E_x(i,j+1)}{\Delta y} + \frac{E_y(i+1,j) - E_y(i,j)}{\Delta x} \right)$$

$$\Delta x = \Delta y = \Delta s$$

$$\Rightarrow \frac{d}{dt} H_z(i,j) = -\frac{1}{\mu \Delta s} \left(E_x(i,j) - E_x(i,j+1) + E_y(i+1,j - E_y(i,j) \right)$$

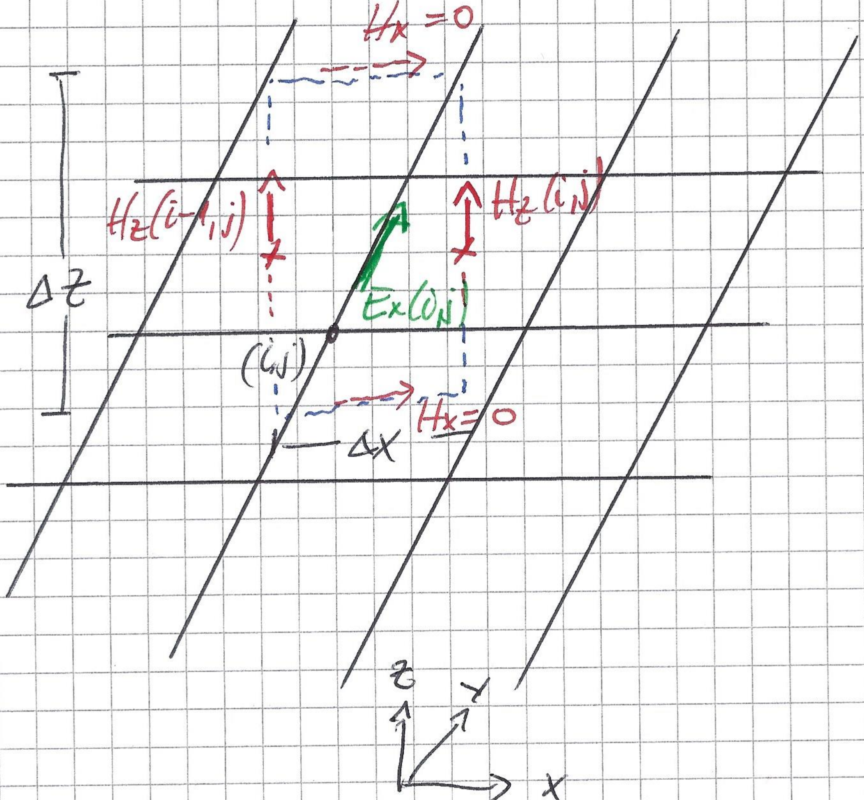
$$\frac{d}{dz} H_z(i,j) = \frac{H_z^{new}(i,j) - H_z^{prev}(i,j)}{\Delta t}$$

(same as 1D)

$$\Rightarrow \boxed{H_z^{new}(i,j) = H_z^{prev}(i,j) - \frac{\Delta t}{\mu \Delta s} \left(E_x(i,j) - E_x(i,j+1) + E_y(i+1,j - E_y(i,j) \right)}$$

$\Rightarrow$ Update equation for magnetic field component.

Similar for $\vec{E}$ - Field:



$$\oint \vec{H} d\vec{s} = \iint \varepsilon \cdot \frac{d\vec{E}}{dt} d\vec{A}$$

1) $H_x = 0$ (we only consider TM case with $H_z \neq 0$)

$$\Rightarrow H_z(i-1,j) \cdot \Delta z + 0 - H_z(i,j) \cdot \Delta z - 0$$

$$= \varepsilon \cdot \frac{d}{dt} E_x(i,j) \cdot \Delta x \cdot \Delta z$$

$\Delta x = \Delta S$ (dividing by Δz)

$$\Rightarrow \frac{dE_x(i,j)}{dt} = \frac{1}{\varepsilon \Delta s}\left(H_z(i-1,j) - H_z(i,j)\right)$$

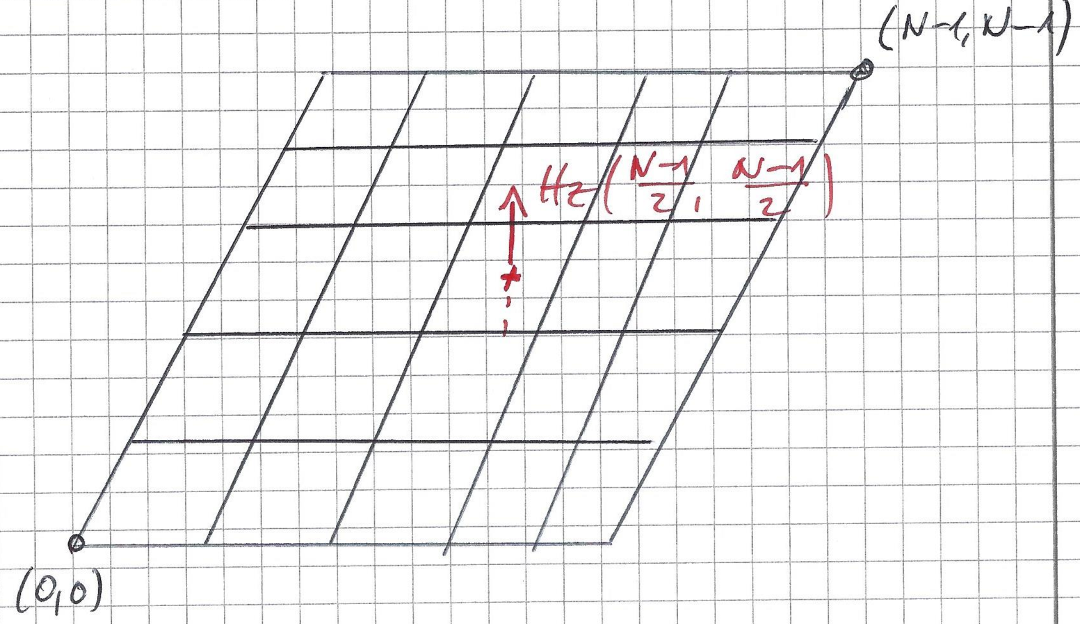
$$\frac{dE_x(i,j)}{dt} = \frac{E_x^{new}(i,j) - E_x^{prev}(i,j)}{\Delta t}$$

$$\Rightarrow \boxed{E_x^{new}(i,j) = E_x^{prev}(i,j) + \frac{\Delta t}{\varepsilon \Delta s}\left(H_z(i-1,j) - H_z(i,j)\right)}$$

Updating equation for Ex.

The updating equation for Ey can be derived in the same way. Please try it yourself.

Excitation:



I suggest to put the excitation to the H_z field component in the middle.

If we choose N to be an odd number, we get a symmetric excitation.