FRANKFURT UNIVERSITY OF APPLIED SCIENCES

STATE OF THE ART SIMULATION

OF ELECTROMAGNETIC FIELDS IN TIME DOMAIN USING THE FINITE INTEGRATION TECHNIQUE

Peter Thoma

OUTLINE

1. Introduction to the Finite Integration Technique

– Maxwell's grid equations
– Transient formulation and stability
– Discretizations

2. Advanced topics

– Material modeling
– Waveguide ports
– Efficient simulations / HPC
– Hybrid solver framework

3. Industrial application example

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1. Introduction to the Finite Integration Technique

- Maxwell’s grid equations
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2. Advanced topics

- Material modeling
- Waveguide ports
- Efficient simulations / HPC
- Hybrid solver framework
- Optimization

3. Industrial application example

This presentation summarizes the outstanding work of many people.
Appropriate references are given in the conference paper.

THE FINITE DIFFERENCE TIME DOMAIN METHOD

ORIGINAL FDTD EQUATIONS

Numerical Solution of Initial Boundary Value Problems Involving Maxwell's Equations in Isotropic Media

KANE S. YEE

$$\frac{\partial B}{\partial t} + \nabla \times E = 0,$$

$$-\frac{\partial B_x}{\partial t} = \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z},$$

$$\boxed{\begin{aligned}\frac{B_x^{n+1/2}(i, j+\frac{1}{2}, k+\frac{1}{2}) - B_x^{n-1/2}(i, j+\frac{1}{2}, k+\frac{1}{2})}{\Delta t} &= \frac{E_y^n(i, j+\frac{1}{2}, k+1) - E_y^n(i, j+\frac{1}{2}, k)}{\Delta z} \\ &\quad - \frac{E_z^n(i, j+1, k+\frac{1}{2}) - E_z^n(i, j, k+\frac{1}{2})}{\Delta y}.\end{aligned}}$$

TE waves:

$$\begin{aligned} H_z^{n+1/2}(i+\tfrac{1}{2}, j+\tfrac{1}{2}) = H_z^{n-1/2}(i+\tfrac{1}{2}, j+\tfrac{1}{2}) \\ - \frac{1}{Z}\frac{\Delta\tau}{\Delta x}[E_y^n(i+1, j+\tfrac{1}{2}) - E_y^n(i, j+\tfrac{1}{2})] \\ + \frac{1}{Z}\frac{\Delta\tau}{\Delta y} E_x^n(i+\tfrac{1}{2}, j+1) - E_x^n(i+\tfrac{1}{2}, j)] \quad (13a) \end{aligned}$$

$$\begin{aligned} E_x^{n+1}(i+\tfrac{1}{2}, j) = E_x^n(i+\tfrac{1}{2}, j) \\ + Z\frac{\Delta\tau}{\Delta y}[H_z^{n+1/2}(i+\tfrac{1}{2}, j+\tfrac{1}{2}) - H_z^{n+1/2}(i+\tfrac{1}{2}, j-\tfrac{1}{2})] \quad (13b) \end{aligned}$$

$$\begin{aligned} E_y^{n+1}(i, j+\tfrac{1}{2}) = -Z\frac{\Delta\tau}{\Delta x}[H_z^{n+1/2}(i+\tfrac{1}{2}, j+\tfrac{1}{2}) \\ - H_z^{n+1/2}(i-\tfrac{1}{2}, j+\tfrac{1}{2})]. \quad (13c) \end{aligned}$$

TM waves:

$$\begin{aligned} E_z^{n+1}(i, j) = E_z^n(i, j) \\ + Z\frac{\Delta\tau}{\Delta x}[H_y^{n+1/2}(i+\tfrac{1}{2}, j) - H_y^{n+1/2}(i-\tfrac{1}{2}, j)] \\ - Z\frac{\Delta\tau}{\Delta y}[H_x^{n+1/2}(i, j+\tfrac{1}{2}) - H_x^{n+1/2}(i, j-\tfrac{1}{2})] \quad (14a) \end{aligned}$$

$$\begin{aligned} H_x^{n+1/2}(i, j+\tfrac{1}{2}) = H_x^{n-1/2}(i, j+\tfrac{1}{2}) \\ - \frac{1}{Z}\frac{\Delta\tau}{\Delta y}[E_z^n(i, j+1) - E_z^n(i, j)] \quad (14b) \end{aligned}$$

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$$-\frac{\partial B_x}{\partial t} = \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z},$$

$$\frac{B_x^{n+1/2}(i, j+\frac{1}{2}, k+\frac{1}{2}) - B_x^{n-1/2}(i, j+\frac{1}{2}, k+\frac{1}{2})}{\Delta t} = \frac{E_y^n(i, j+\frac{1}{2}, k+1) - E_y^n(i, j+\frac{1}{2}, k)}{\Delta z} - \frac{E_z^n(i, j+1, k+\frac{1}{2}) - E_z^n(i, j, k+\frac{1}{2})}{\Delta y}.$$

Multiply by $\Delta y\ \Delta z$

$$\frac{\Delta y\,\Delta z}{\Delta t}\left(B_x^{n+\frac{1}{2}}\left(i,j+\frac{1}{2},k+\frac{1}{2}\right) - B_x^{n-\frac{1}{2}}\left(i,j+\frac{1}{2},k+\frac{1}{2}\right)\right)$$
$$= \Delta y\left(E_y^n\left(i,j+\frac{1}{2},k+1\right) - E_y^n\left(i,j+\frac{1}{2},k\right)\right)$$
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$$\frac{\Delta y\,\Delta z}{\Delta t}\left(B_x^{n+\frac{1}{2}}\left(i, j+\frac{1}{2}, k+\frac{1}{2}\right) - B_x^{n-\frac{1}{2}}\left(i, j+\frac{1}{2}, k+\frac{1}{2}\right)\right)$$
$$= \Delta y\left(E_y^n\left(i, j+\frac{1}{2}, k+1\right) - E_y^n\left(i, j+\frac{1}{2}, k\right)\right)$$
$$- \Delta z\left(E_z^n\left(i, j+1, k+\frac{1}{2}\right) - E_z^n\left(i, j, k+\frac{1}{2}\right)\right)$$

Integral quantities

$$b_x^{n+\frac{1}{2}}\left(i, j+\frac{1}{2}, k+\frac{1}{2}\right) - b_x^{n-\frac{1}{2}}\left(i, j+\frac{1}{2}, k+\frac{1}{2}\right)$$
$$= \Delta t\left(e_y^n\left(i, j+\frac{1}{2}, k+1\right) - e_y^n\left(i, j+\frac{1}{2}, k\right)\right)$$
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$$\frac{\Delta y\,\Delta z}{\Delta t}\left(B_x^{n+\frac{1}{2}}\left(i,j+\frac{1}{2},k+\frac{1}{2}\right) - B_x^{n-\frac{1}{2}}\left(i,j+\frac{1}{2},k+\frac{1}{2}\right)\right)$$
$$= \Delta y\left(E_y^n\left(i,j+\frac{1}{2},k+1\right) - E_y^n\left(i,j+\frac{1}{2},k\right)\right)$$
$$- \Delta z\left(E_z^n\left(i,j+1,k+\frac{1}{2}\right) - E_z^n\left(i,j,k+\frac{1}{2}\right)\right)$$

$$b_x^{n+\frac{1}{2}}\left(i,j+\frac{1}{2},k+\frac{1}{2}\right) - b_x^{n-\frac{1}{2}}\left(i,j+\frac{1}{2},k+\frac{1}{2}\right)$$
$$= \Delta t\left(e_y^n\left(i,j+\frac{1}{2},k+1\right) - e_y^n\left(i,j+\frac{1}{2},k\right)\right)$$
$$- \Delta t\left(e_z^n\left(i,j+1,k+\frac{1}{2}\right) - e_z^n\left(i,j,k+\frac{1}{2}\right)\right)$$

Matrix form

$$\boldsymbol{b}_{n+1/2} = \boldsymbol{b}_{n-1/2} - \Delta t\,\boldsymbol{C}\,\boldsymbol{e}_{\mathrm{n}}$$

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FINITE INTEGRATION TECHNIQUE IN TIME DOMAIN

Transient FIT formulation

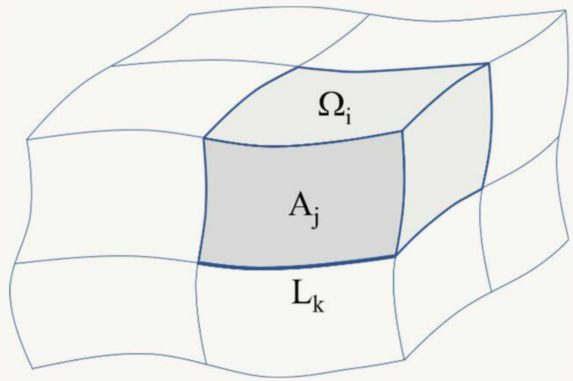
$$\boldsymbol{b}_{n+1/2} = \boldsymbol{b}_{n-1/2} - \Delta t\, \boldsymbol{C}\, \boldsymbol{e}_{\mathrm{n}}$$

$$b_x^{n+\frac{1}{2}}\left(i, j+\frac{1}{2}, k+\frac{1}{2}\right) \quad \ldots \quad {}_x^{n-\frac{1}{2}}\left(i, j+\frac{1}{2}, k+\frac{1}{2}\right)$$
$$= \Delta t \left(e_y^n\left(i, j+\frac{1}{2}, k+ \ldots\right) - e_y^n\left(i, j+\frac{1}{2}, k\right)\right)$$
$$- \Delta t \left(e_z^n\left(i, j+1, k+ \ldots\right) - e_z^n\left(i, j, k+\frac{1}{2}\right)\right)$$

$$\frac{\Delta y\, \Delta z}{\Delta t}\left(B_x^{n+\frac{1}{2}}\left(i, j+\frac{1}{2}, k+ \ldots\right) - B_x^{n-\frac{1}{2}}\left(i, j+\frac{1}{2}, k+\frac{1}{2}\right)\right)$$
$$\ldots - E_y^n\left(i, j+\frac{1}{2}, k\right)\Big)\Big)$$
$$- \Delta z\left(E_z^n\left(i, j+1, k+\frac{1}{2}\right) - E_z^n\left(i, j, k+\frac{1}{2}\right)\right)$$

THE FINITE INTEGRATION TECHNIQUE

1. DECOMPOSITION OF COMPUTATION DOMAIN Ω INTO ARBITRARILY SHAPED SUBDOMAINS Ω_i:



2. DEFINE INTEGRAL QUANTITIES OF ELECTROMAGNETIC FIELDS AS UNKNOWNS (VECTOR COMPONENTS):

$$e_k = \int_{L_k} \vec{E}(\vec{r},t) \cdot d\vec{s}\,, \qquad b_j = \iint_{A_j} \vec{B}(\vec{r},t) \cdot d\vec{A}$$

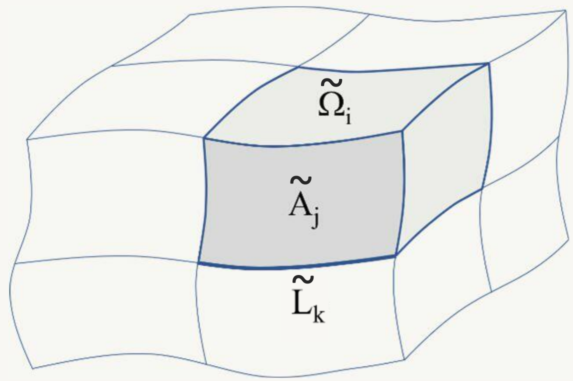
3. APPLY MAXWELL'S EQUATIONS IN INTEGRAL FORM TO SUBDOMAINS (A_j, Ω_i):

$$\oint_{\partial A_j} \vec{E}(\vec{r},t) \cdot d\vec{s} = -\frac{\partial}{\partial t} \iint_{A_j} \vec{B}(\vec{r},t) \cdot d\vec{A} \Rightarrow \sum_{k \in \partial A_j} c_k\, e_k = -\frac{\partial}{\partial t} b_j \Rightarrow \boxed{\boldsymbol{C}\,\boldsymbol{e} = -\frac{\partial}{\partial t}\boldsymbol{b}}$$

$$\oiint_{\partial \Omega_i} \vec{B}(\vec{r},t) \cdot d\vec{A} = 0 \Rightarrow \sum_{k \in \partial \Omega_i} s_k\, b_k = 0 \Rightarrow \boxed{\boldsymbol{S}\,\boldsymbol{b} = \boldsymbol{0}}$$

THE FINITE INTEGRATION TECHNIQUE

4. SECOND (DUAL) DECOMPOSITION OF COMPUTATION DOMAIN Ω INTO ARBITRARILY SHAPED SUBDOMAINS $\widetilde{\Omega}_i$:



5. DEFINE INTEGRAL QUANTITIES OF ELECTROMAGNETIC FIELDS AS UNKNOWNS (VECTOR COMPONENTS):

$$d_j = \iint_{\tilde{A}_j} \vec{D}(\vec{r},t) \cdot d\vec{A}, \quad h_k = \int_{\tilde{L}_k} \vec{H}(\vec{r},t) \cdot d\vec{s}, \quad j_j = \iint_{\tilde{A}_j} \vec{J}(\vec{r},t) \cdot d\vec{A}, \quad q_i = \iiint_{\tilde{\Omega}_i} \rho(\vec{r})\, d\vec{r}$$

6. APPLY MAXWELL'S EQUATIONS IN INTEGRAL FORM TO SUBDOMAINS (A_j, Ω_i):

$$\oint_{\partial\tilde{A}_j} \vec{H}(\vec{r},t) \cdot d\vec{s} = \iint_{\tilde{A}_j} \left(\vec{J}(\vec{r},t) + \frac{\partial \vec{D}(\vec{r},t)}{\partial t} \right) \cdot d\vec{A} \Rightarrow \sum_{k\,\in\,\partial\tilde{A}_j} \tilde{c}_k\, h_k = j_j + \frac{\partial}{\partial t}\, d_j \Rightarrow \boxed{\tilde{\boldsymbol{C}}\, \boldsymbol{h} = \boldsymbol{j} + \frac{\partial}{\partial t} \boldsymbol{d}}$$

$$\oiint_{\partial\tilde{\Omega}_i} \vec{D}(\vec{r},t) \cdot d\vec{A} = \iiint_{\tilde{\Omega}_i} \rho(\vec{r})\, d\vec{r} \Rightarrow \sum_{k\,\in\,\partial\tilde{\Omega}_i} \tilde{s}_k\, d_k = q_i \Rightarrow \boxed{\tilde{\boldsymbol{S}}\, \boldsymbol{d} = \boldsymbol{q}}$$

THE FINITE INTEGRATION TECHNIQUE

INTEGRAL FORM OF MAXWELL'S EQUATIONS

$$\oint_{\partial A} \vec{E}(\vec{r},t) \cdot d\vec{s} = -\frac{\partial}{\partial t} \iint_{A} \vec{B}(\vec{r},t) \cdot d\vec{A}$$

$$\oint_{\partial A} \vec{H}(\vec{r},t) \cdot d\vec{s} = \iint_{A} \left(\vec{J}(\vec{r},t) + \frac{\partial \vec{D}(\vec{r},t)}{\partial t} \right) \cdot d\vec{A}$$

$$\oiint_{\partial V} \vec{B}(\vec{r},t) \cdot d\vec{A} = 0$$

$$\oiint_{\partial V} \vec{D}(\vec{r},t) \cdot d\vec{A} = \iiint_{V} \rho(\vec{r})\, d\vec{r}$$

$$source\ curl\ \vec{A} = 0 \ (\text{or } \nabla \cdot (\nabla \times \vec{A}) = 0\ )$$

MAXWELL'S GRID EQUATIONS

$$\boldsymbol{C}\,\boldsymbol{e} = -\frac{\partial}{\partial t}\,\boldsymbol{b}$$

$$\widetilde{\boldsymbol{C}}\,\boldsymbol{h} = \boldsymbol{j} + \frac{\partial}{\partial t}\boldsymbol{d}$$

$$\boldsymbol{S}\,\boldsymbol{b} = \boldsymbol{0}$$

$$\tilde{\boldsymbol{S}}\,\boldsymbol{d} = \boldsymbol{q}$$

$$\boldsymbol{S}\,\boldsymbol{C} = \boldsymbol{0}, \qquad \tilde{\boldsymbol{S}}\,\widetilde{\boldsymbol{C}} = \boldsymbol{0}$$

Exact representation of integral quantities

THE FINITE INTEGRATION TECHNIQUE

MATERIAL PROPERTIES

$$\vec{D}(\vec{r},t) = \epsilon(\vec{r}) \cdot \vec{E}(\vec{r},t)$$

$$\vec{B}(\vec{r},t) = \mu(\vec{r}) \cdot \vec{H}(\vec{r},t)$$

$$\vec{J}(\vec{r},t) = \kappa(\vec{r}) \cdot \vec{E}(\vec{r},t) + \overrightarrow{J_0}(\vec{r},t)$$

e, b are defined on primary decomposition Ω_i

d, h, j are defined on dual decomposition $\widetilde{\Omega}_i$

⇨ **Connection of Ω_i and $\widetilde{\Omega}_i$ needed to derive discrete material relations**

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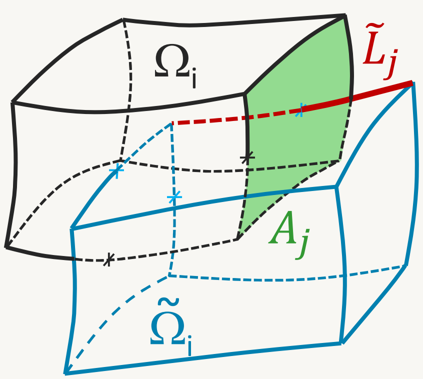
RELATION BETWEEN PRIMARY DECOMPOSITION AND DUAL DECOMPOSITION



1. Dual edge $\tilde{L}_j$ intersects with face A_j (and no other face)

THE FINITE INTEGRATION TECHNIQUE

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$$\vec{D}(\vec{r},t) = \epsilon(\vec{r}) \cdot \vec{E}(\vec{r},t)$$

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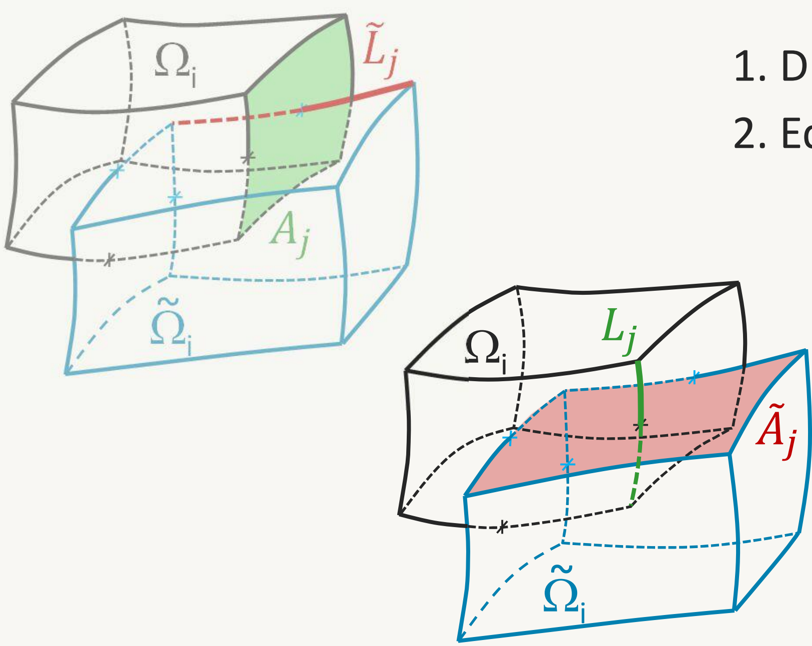

1. Dual edge $\tilde{L}_j$ intersects with face A_j (and no other face)
2. Edge L_j intersects with dual face $\tilde{A}_j$ (and no other face)

THE FINITE INTEGRATION TECHNIQUE

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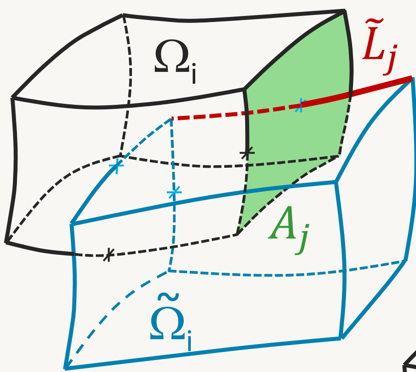

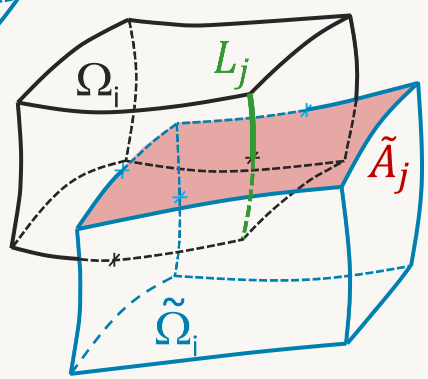

1. Dual edge $\tilde{L}_j$ intersects with face A_j (and no other face)
2. Edge L_j intersects with dual face $\tilde{A}_j$ (and no other face)

$$\boxed{\boldsymbol{C} = \widetilde{\boldsymbol{C}}^T}$$

$$\boxed{\boldsymbol{d} = \boldsymbol{D}_\epsilon\, \boldsymbol{e},\ \ \boldsymbol{b} = \boldsymbol{D}_\mu\, \boldsymbol{h},\ \ \boldsymbol{j} = \boldsymbol{D}_k\, \boldsymbol{e} + \boldsymbol{j}_0}$$

Material relations

THE FINITE INTEGRATION TECHNIQUE

INTEGRAL FORM OF MAXWELL'S EQUATIONS

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$$source\ curl\ \vec{A} = 0 \text{ (or } \nabla \cdot (\nabla \times \vec{A}) = 0 \text{)}$$

Duality of Ω_i and $\tilde{\Omega}_i$

MAXWELL’S GRID EQUATIONS

$$\boldsymbol{C}\, \boldsymbol{e} = -\frac{\partial}{\partial t}\, \boldsymbol{b}$$

$$\widetilde{\boldsymbol{C}}\, \boldsymbol{h} = \boldsymbol{j} + \frac{\partial}{\partial t} \boldsymbol{d}$$

$$\boldsymbol{S}\, \boldsymbol{b} = \boldsymbol{0}$$

$$\tilde{\boldsymbol{S}}\, \boldsymbol{d} = \boldsymbol{q}$$

$$\boldsymbol{d} = \boldsymbol{D}_\epsilon\, \boldsymbol{e}$$

$$\boldsymbol{b} = \boldsymbol{D}_\mu\, \boldsymbol{h}$$

$$\boldsymbol{j} = \boldsymbol{D}_\kappa\, \boldsymbol{e} + \boldsymbol{j}_0$$

$$\boldsymbol{S}\, \boldsymbol{C} = \boldsymbol{0}, \qquad \tilde{\boldsymbol{S}}\, \widetilde{\boldsymbol{C}} = \boldsymbol{0}$$

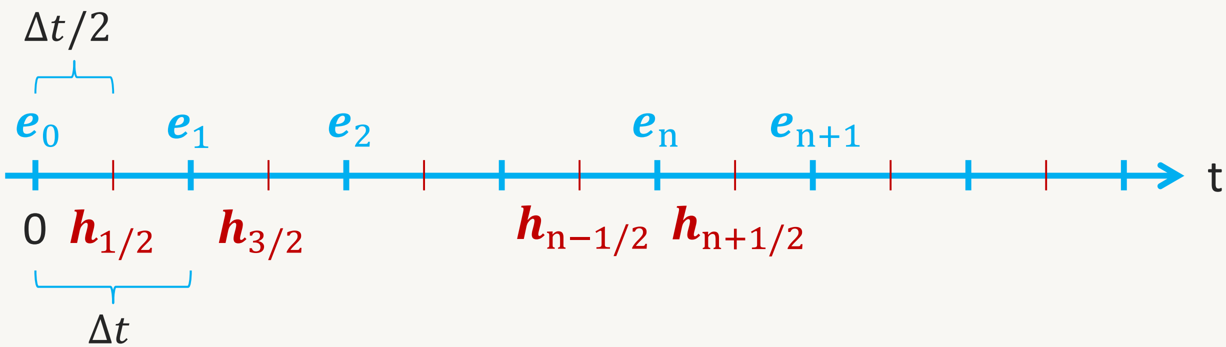
$$\boldsymbol{C} = \widetilde{\boldsymbol{C}}^T$$

THE FINITE INTEGRATION TECHNIQUE

TRANSIENT FORMULATION (WAVE EQUATION) – LOSSLESS CASE, NO CURRENT PATHS $\Rightarrow \boldsymbol{j} = 0$

$$\boldsymbol{C}\,\boldsymbol{e} = -\frac{\partial}{\partial t}\,\boldsymbol{b} \quad \overset{\boldsymbol{b} = \boldsymbol{D}_\mu\,\boldsymbol{h}}{\Rightarrow} \quad \boldsymbol{C}\,\boldsymbol{e} = -\boldsymbol{D}_\mu \frac{\partial \boldsymbol{h}}{\partial t}$$

$$\widetilde{\boldsymbol{C}}\,\boldsymbol{h} = \frac{\partial}{\partial t}\boldsymbol{d} \quad \overset{\boldsymbol{d} = \boldsymbol{D}_\epsilon\,\boldsymbol{e}}{\Rightarrow} \quad \widetilde{\boldsymbol{C}}\,\boldsymbol{h} = \boldsymbol{D}_\epsilon \frac{\partial \boldsymbol{e}}{\partial t}$$



$$\boldsymbol{e}_\mathrm{n} = e(t = n\,\Delta t)$$

$$\boldsymbol{h}_\mathrm{m} = h(t = m\,\Delta t)$$

LEAPFROG TIME INTEGRATION

$$\boldsymbol{C}\,\boldsymbol{e}_n = -\boldsymbol{D}_\mu \left.\frac{\partial \boldsymbol{h}}{\partial t}\right|_{t=n\Delta t} \approx -\boldsymbol{D}_\mu \frac{\boldsymbol{h}_{n+1/2} - \boldsymbol{h}_{n-1/2}}{\Delta t} \Rightarrow \boldsymbol{h}_{n+1/2} = \boldsymbol{h}_{n-1/2} - \Delta t\,\mathbf{D}_\mu^{-1}\,\mathbf{C}\,\boldsymbol{e}_n$$

$$\widetilde{\boldsymbol{C}}\,\boldsymbol{h}_{n+1/2} = \boldsymbol{D}_\epsilon \left.\frac{\partial \boldsymbol{e}}{\partial t}\right|_{t=(n+1/2)\Delta t} \approx \boldsymbol{D}_\epsilon \frac{\boldsymbol{e}_{n+1} - \boldsymbol{e}_n}{\Delta t} \Rightarrow \boldsymbol{e}_{n+1} = \boldsymbol{e}_n + \Delta t\,\mathbf{D}_\epsilon^{-1}\,\widetilde{\boldsymbol{C}}\,\boldsymbol{h}_{n+1/2}$$

THE FINITE INTEGRATION TECHNIQUE

STABILITY

$$\left.\begin{aligned} \boldsymbol{h}_{n+1/2} &= \boldsymbol{h}_{n-1/2} - \Delta t\, \mathbf{D}_\mu^{-1}\, \mathbf{C}\, \boldsymbol{e}_n \\ \boldsymbol{e}_{n+1} &= \boldsymbol{e}_n + \Delta t\, \mathbf{D}_\epsilon^{-1}\, \widetilde{\boldsymbol{C}}\, \boldsymbol{h}_{n+1/2} \end{aligned}\right\} \boldsymbol{e}_{n+1} + \left(\Delta t^2 \boldsymbol{D}_\epsilon^{-1} \widetilde{\boldsymbol{C}} \boldsymbol{D}_\mu^{-1} \boldsymbol{C} - 2\boldsymbol{I}\right)\boldsymbol{e}_n + \boldsymbol{e}_{n-1} = \mathbf{0}$$

Wave equation

$$\boldsymbol{C}\,\boldsymbol{e} = -\frac{\partial}{\partial t}\,\boldsymbol{b} \qquad \widetilde{\boldsymbol{C}}\,\boldsymbol{h} = \frac{\partial}{\partial t}\boldsymbol{d}$$

Frequency Domain

$$\left.\begin{aligned} \boldsymbol{C}\,\boldsymbol{e} &= -i\omega\,\boldsymbol{b} = -i\omega\,\boldsymbol{D}_\mu\,\boldsymbol{h} \\ \widetilde{\boldsymbol{C}}\,\boldsymbol{h} &= i\omega\,\boldsymbol{d} = i\omega\,\boldsymbol{D}_\epsilon\,\boldsymbol{e} \end{aligned}\right\} \widetilde{\boldsymbol{C}}\,\boldsymbol{D}_\mu^{-1}\,\boldsymbol{C}\,\boldsymbol{e} = \omega^2 \boldsymbol{D}_\epsilon\,\boldsymbol{e}$$

Eigenvalue problem

$\boldsymbol{C} = \widetilde{\boldsymbol{C}}^T$ and $\boldsymbol{D}_\epsilon, \boldsymbol{D}_\mu$ being **positive definite Hermitian** matrices $\Rightarrow \omega \in \mathbb{R}, \omega \geq 0.$

Any solution is a sum of **eigen-solutions** e^m ⇨ consider each eigen-solution separately:

$$\Rightarrow \quad e_{n+1}\boldsymbol{e}^m + \underbrace{\left(\Delta t^2 \boldsymbol{D}_\epsilon^{-1} \widetilde{\boldsymbol{C}} \boldsymbol{D}_\mu^{-1} \boldsymbol{C} - 2\boldsymbol{I}\right)}_{\Delta t^2 \omega_m^2} e_n \boldsymbol{e}^m + \boldsymbol{e}_{n-1}\boldsymbol{e}^m = \mathbf{0}$$

Schur-Cohn-Jury stability test:

$$\Delta t \leq \frac{2}{\omega_m}$$

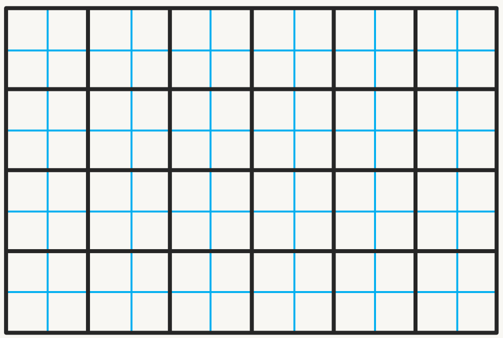
System needs to be stable for all eigen-solutions $\Rightarrow \boxed{\Delta t \leq \frac{2}{\omega_{max}}}$

THE FINITE INTEGRATION TECHNIQUE

MESH TYPES

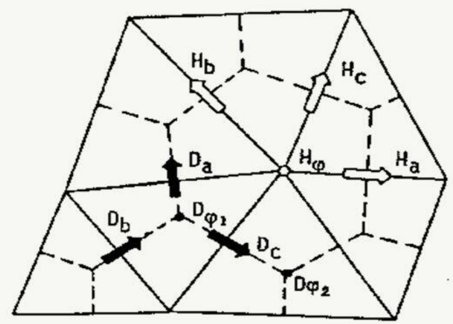
So far, no specialization for a particular mesh type: **mesh** and a **dual mesh** with $C = \widetilde{C}^T$

Cartesian

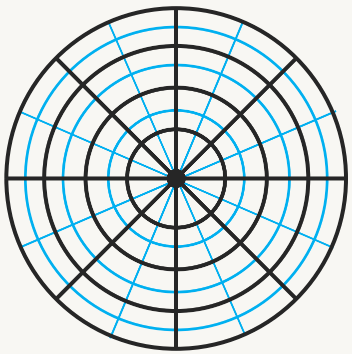


mesh, dual mesh

Triangular

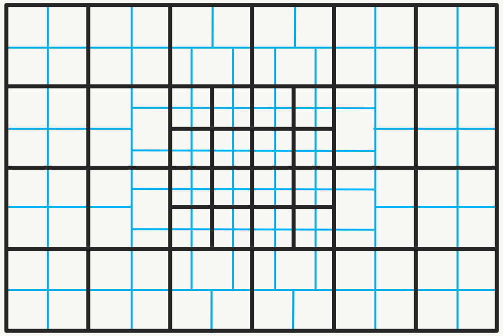


Cylindrical

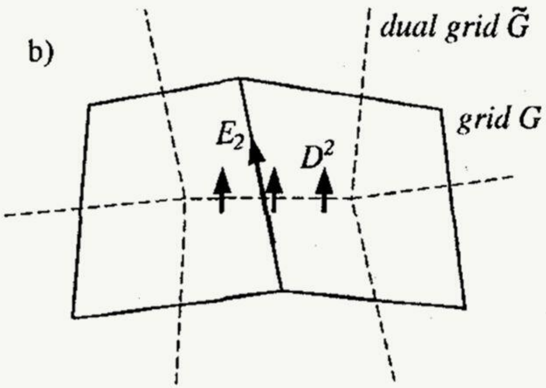


mesh, dual mesh

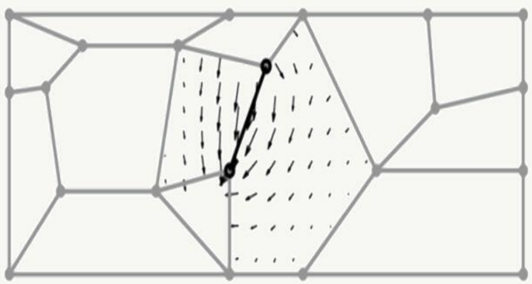
Subgrids



Non-orthogonal



Polygonal



THE FINITE INTEGRATION TECHNIQUE

WHY CARTESIAN MESHES?

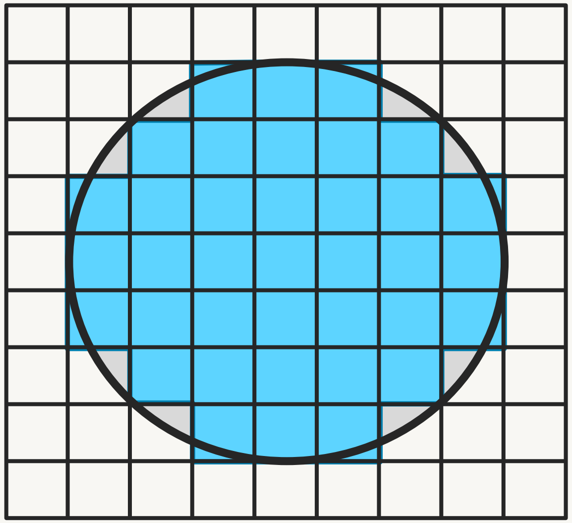
Often complex (and faulty) geometric models for practical applications

Due to **robust mesh generation** and **low memory requirements** for storage, **cartesian meshes** are still the method of choice for many **high frequency applications**.

THE FINITE INTEGRATION TECHNIQUE

REPRESENTATION OF GEOMETRIC BOUNDARIES IN CARTESIAN MESHES

Stairstep Approximation



THE FINITE INTEGRATION TECHNIQUE

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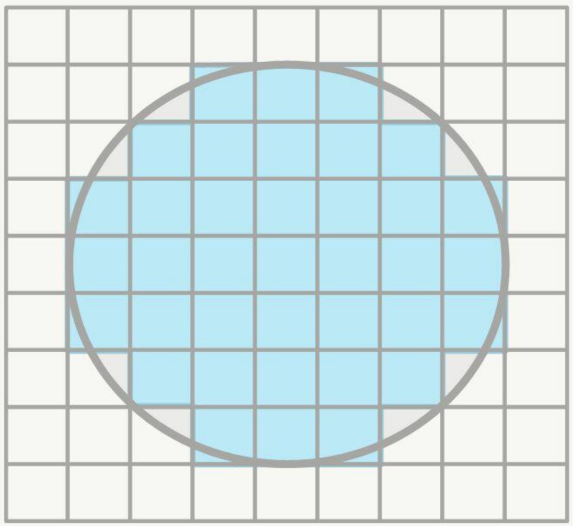
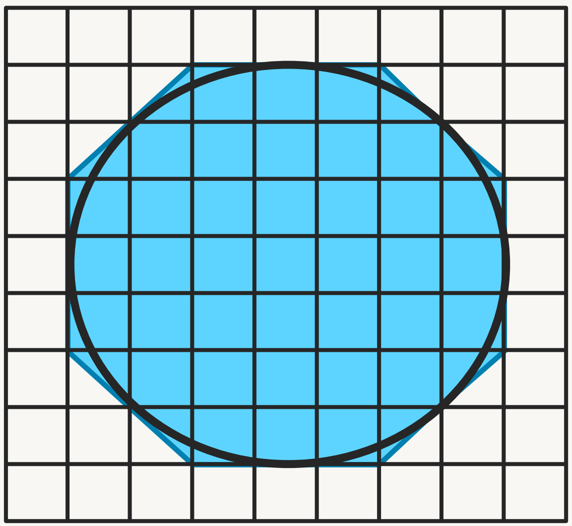


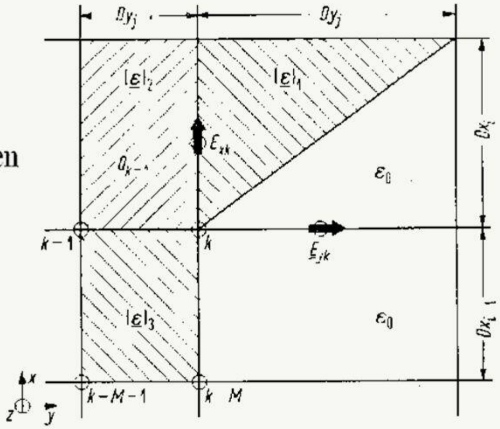
Prismatic Sub-fillings



1977

Eine Methode zur Lösung der Maxwellschen Gleichungen für sechskomponentige Felder auf diskreter Basis

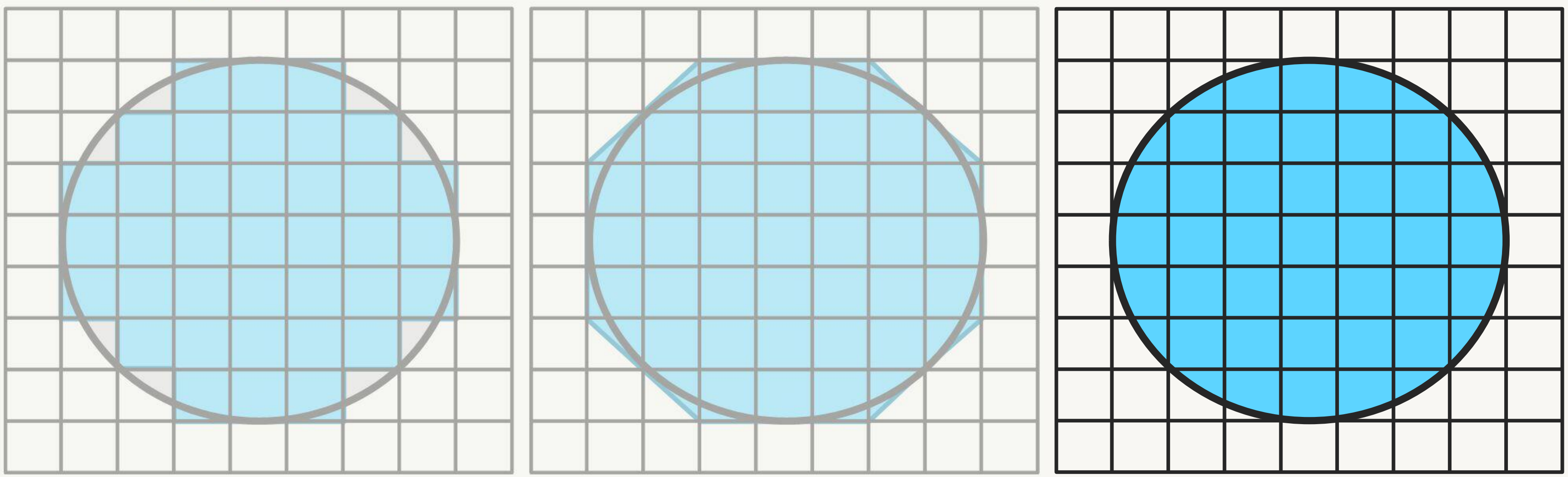
von Thomas Weiland*

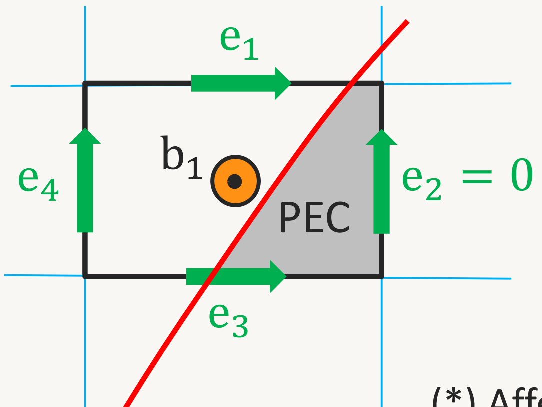


THE FINITE INTEGRATION TECHNIQUE

REPRESENTATION OF GEOMETRIC BOUNDARIES IN CARTESIAN MESHES

Stairstep Approximation | Prismatic Sub-fillings | **Generalized Sub-fillings (PBA)**





Material relations:

$$d_3 = \epsilon \frac{\tilde{A}_3}{\boxed{L_3}} e_3$$

$$b_1 = \mu \frac{\boxed{A_1}}{\tilde{L}_1} h_1$$

Reduce length and area to non-PEC region (*)

Stability requirements maintained, but reduction affects stable time step

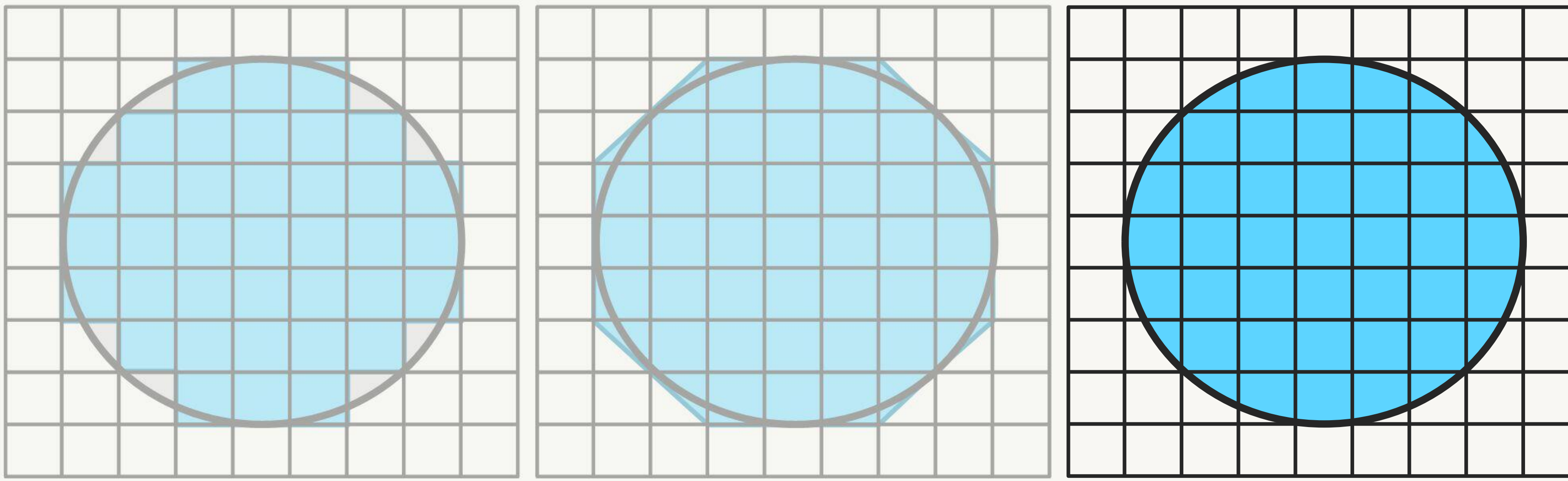
⇨ **local time stepping** for individual cells needed

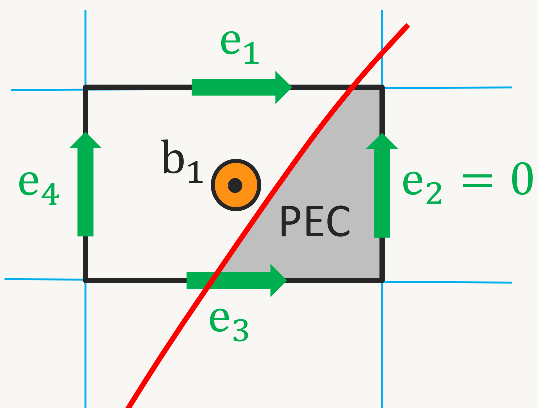
(*) Affects $\boldsymbol{D}_\epsilon, \boldsymbol{D}_\mu$ only, both remain positive definite Hermitian matrices

THE FINITE INTEGRATION TECHNIQUE

REPRESENTATION OF GEOMETRIC BOUNDARIES IN CARTESIAN MESHES

Stairstep Approximation | Prismatic Sub-fillings | Generalized Sub-fillings (PBA)





THE FINITE INTEGRATION TECHNIQUE

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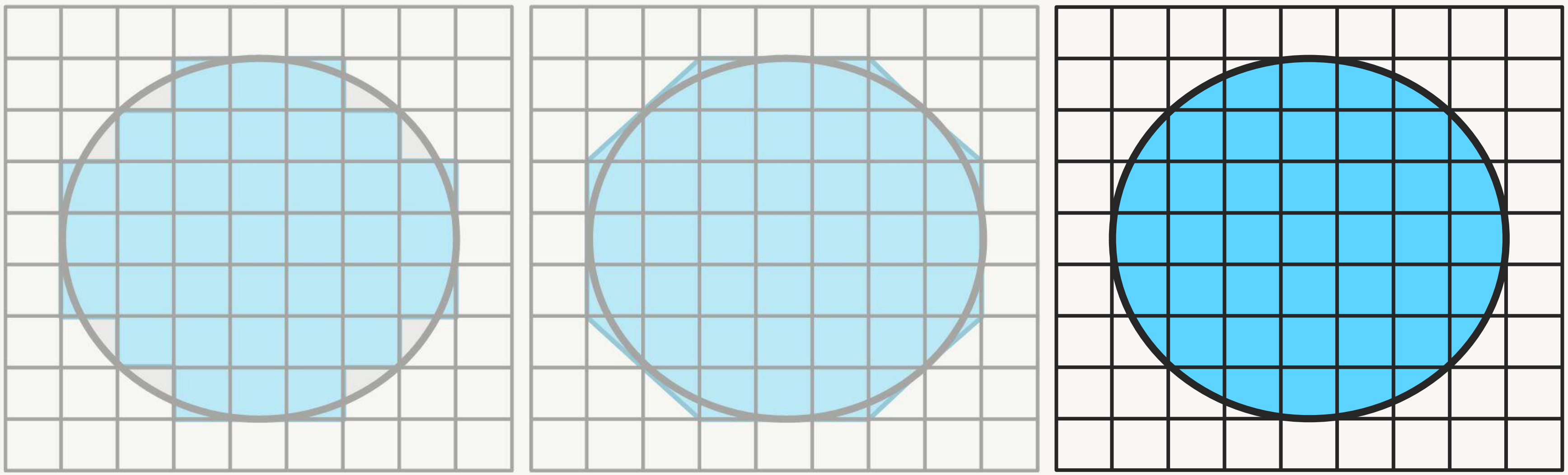
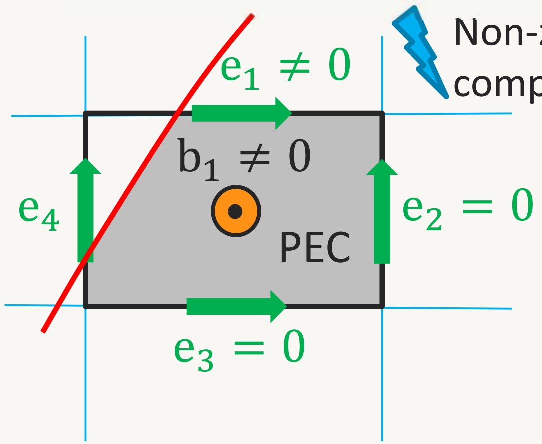
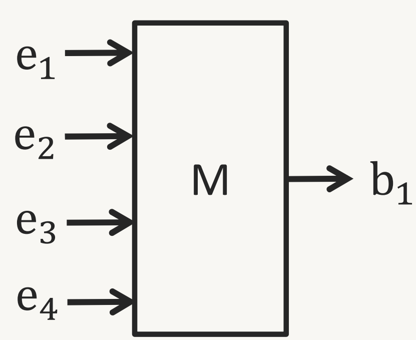




Non-zero field components in PEC

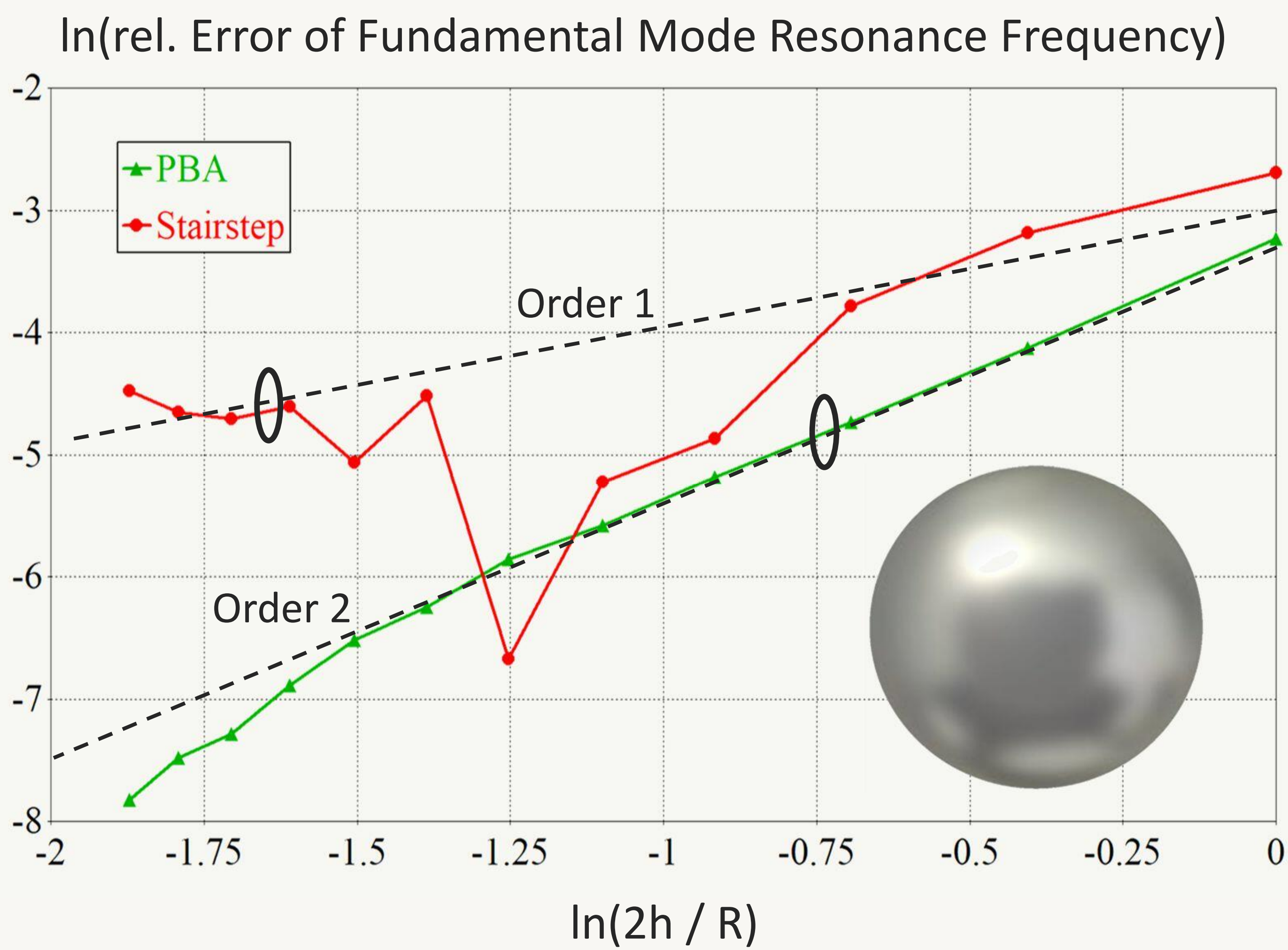
“System view”



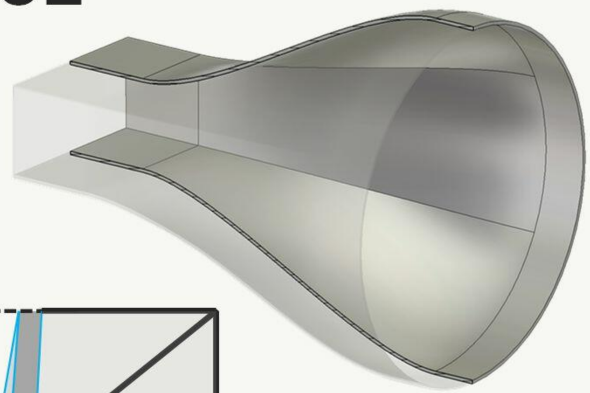
The transfer function M is a smooth function of the fill factor

THE FINITE INTEGRATION TECHNIQUE

THE “PERFECT BOUNDARY APPROXIMATION” – 2ND ORDER CONVERGENCE FOR PEC BOUNDARIES

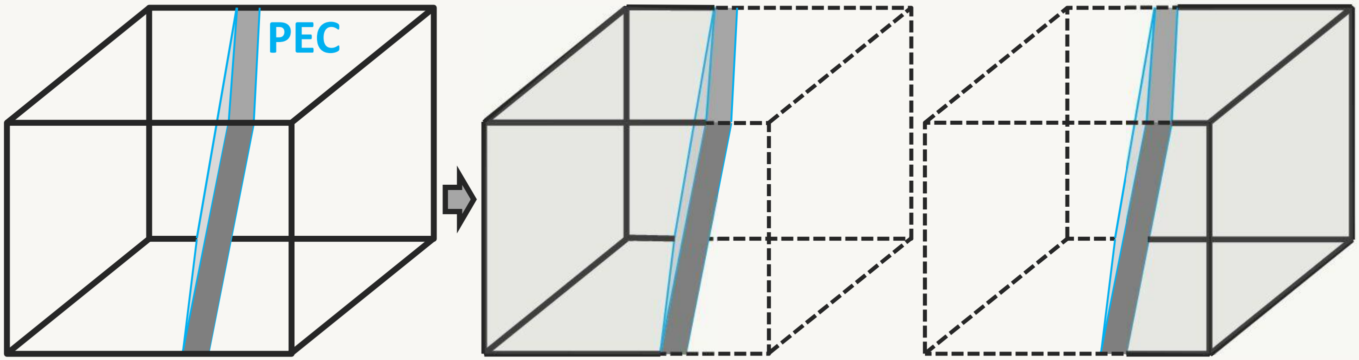


THE FINITE INTEGRATION TECHNIQUE



CUTTING MESH CELLS BY PERFECTLY CONDUCTING SHEETS

Many structures contain curved thin metallic sheets
$\Rightarrow$ mesh cells are cut into **two independent** domains



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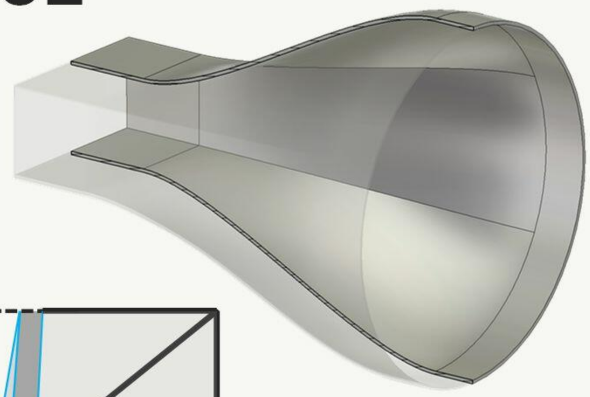

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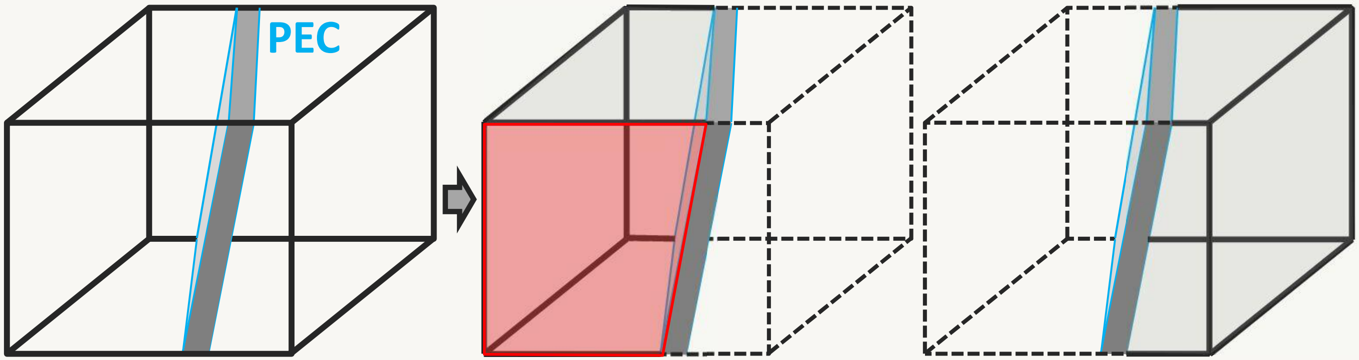
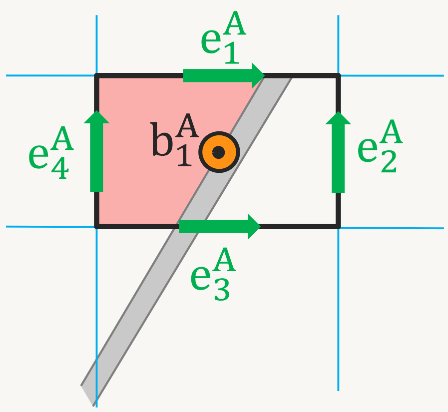


$\Rightarrow$ Introduce **two** separate **sets of degrees of freedom** for the cut cells



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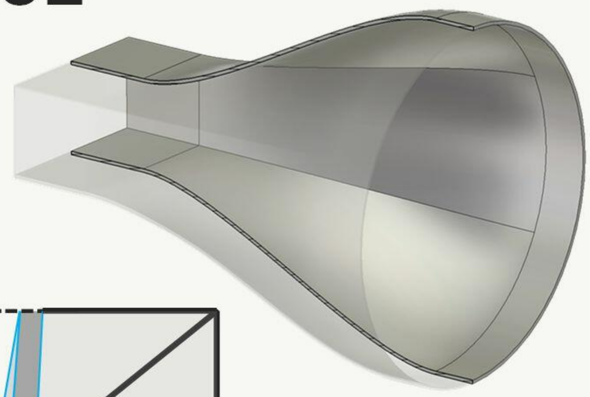

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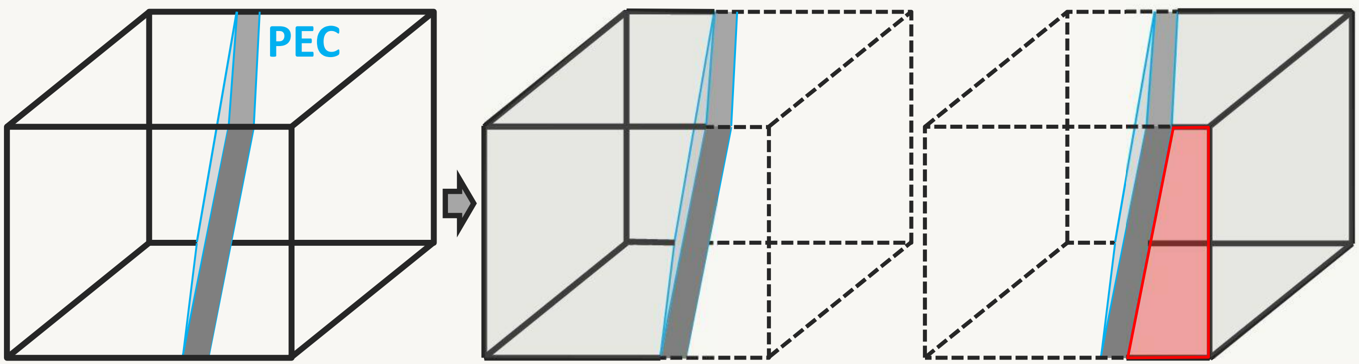

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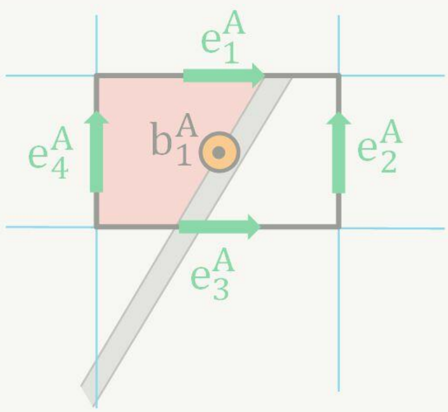

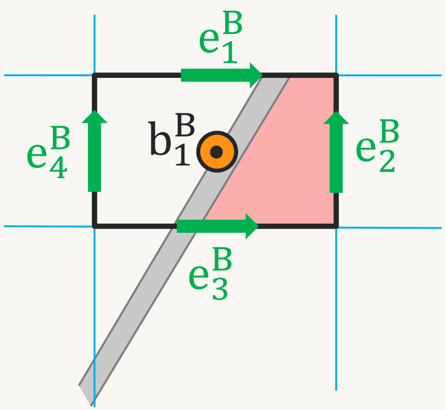

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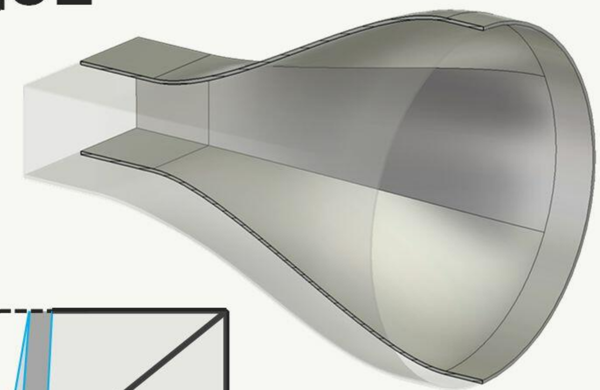

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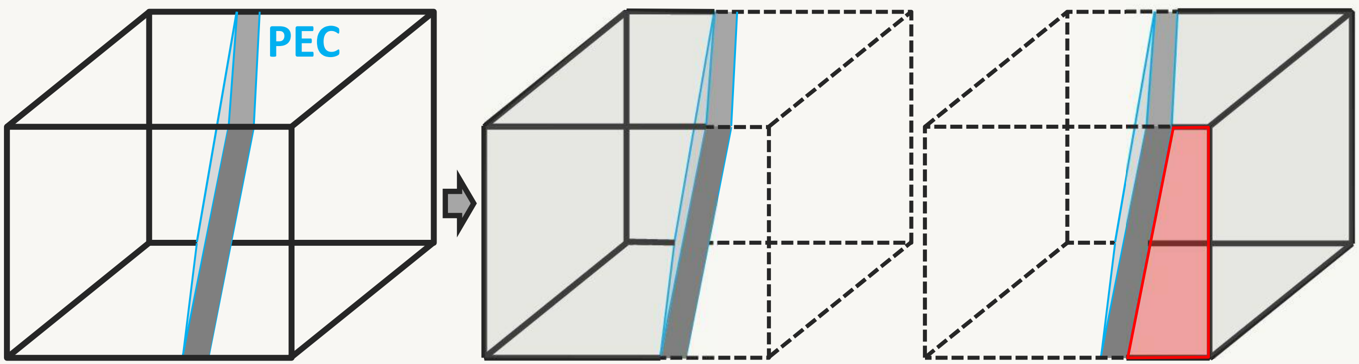

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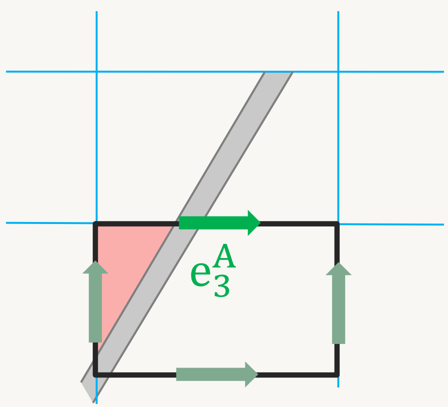
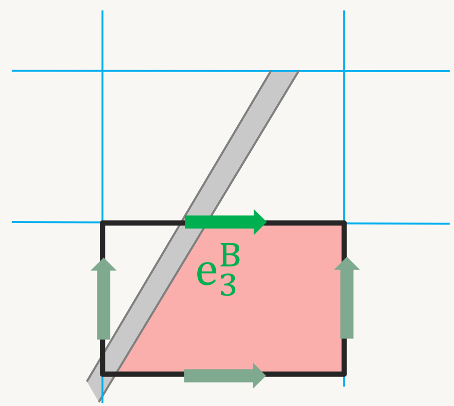
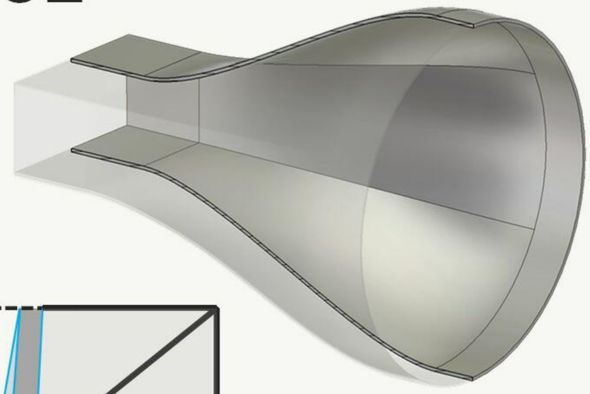




…and use the spawned DOFs with the corresponding $\boldsymbol{C}$ or $\boldsymbol{S}$ operators.

THE FINITE INTEGRATION TECHNIQUE

CUTTING MESH CELLS BY PERFECTLY CONDUCTING SHEETS



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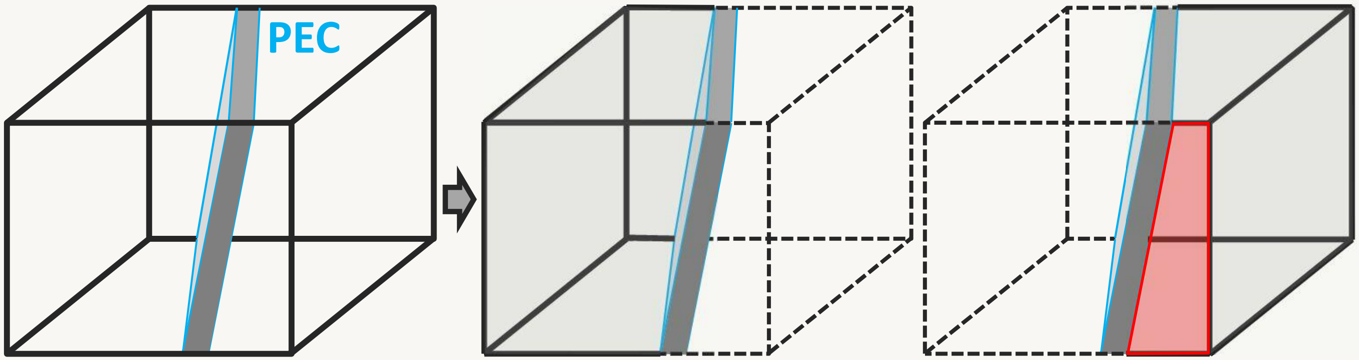

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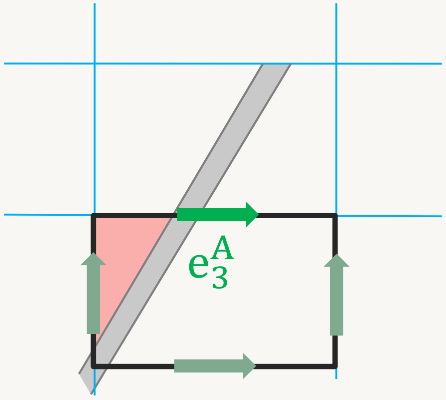

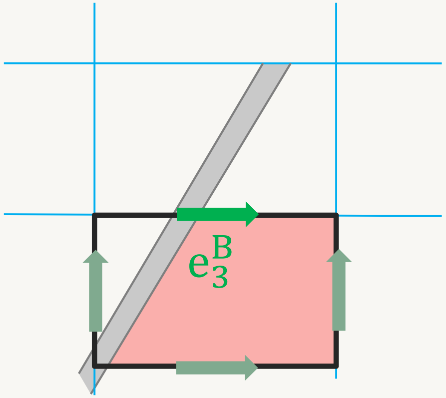

This extension maintains:

$$\boldsymbol{C} = \widetilde{\boldsymbol{C}}^T$$
$$\boldsymbol{S}\,\boldsymbol{C} = \boldsymbol{0}$$
$$\widetilde{\boldsymbol{S}}\,\widetilde{\boldsymbol{C}} = \boldsymbol{0}$$

…and use the spawned DOFs with the corresponding $\boldsymbol{C}$ or $\boldsymbol{S}$ operators.

MODELING DISPERSIVE MATERIALS

REQUIREMENTS FOR ACCURATE MATERIAL MODELING IN TRANSIENT SIMULATIONS

- Transient simulations obtain a broadband system response from a single computation
- Therefore, material models need to be accurate for a broad range of frequencies
- Dielectric and permeable material properties are typically frequency dependent

⇨ **Specific modeling required to handle frequency dependency in transient method**

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COMMONLY USED MODELS FOR MODELING MATERIALS

1. **Debeye Model** (most dielectric materials at microwave frequencies)

$$\epsilon(\omega) = \epsilon_\infty + \sum_{k=1}^{N} \frac{\Delta\epsilon_k}{1 + i\omega\tau_{ki}}$$

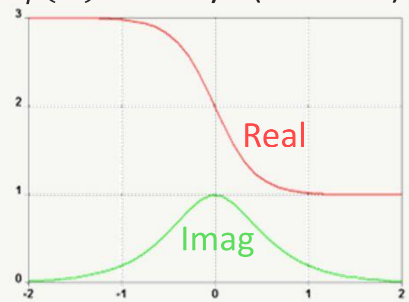
2. **Lorentz Model** (modeling resonance, often needed for higher frequencies)

$$\epsilon(\omega) = \epsilon_\infty + \sum_{k=1}^{N} \frac{\Delta\epsilon_k \omega_k^2}{\omega_k^2 + 2i\omega\delta_k - \omega^2}$$

3. **Drude Model** (often used for optical frequencies or cold plasma)

$$\epsilon(\omega) = \epsilon_\infty - \sum_{k=1}^{N} \frac{\omega_k^2}{\omega^2 - i\omega\gamma_k}$$

$\epsilon_r(\omega)$ - Debeye (1st order)



$\log(\omega\tau_1)$

$\epsilon_r(\omega)$ - Lorentz (1st order)

ω/ω_1

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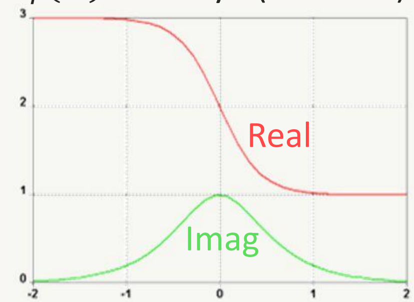

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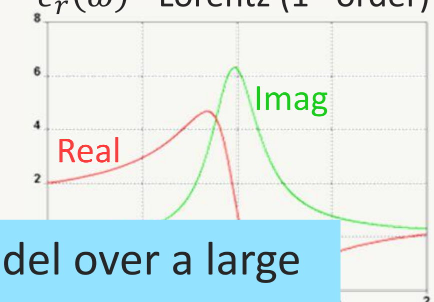

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Real materials can often not be modeled accurately by a single model over a large frequency range ⇨ combinations of models, e.g. Drude-Lorentz

MODELING DISPERSIVE MATERIALS

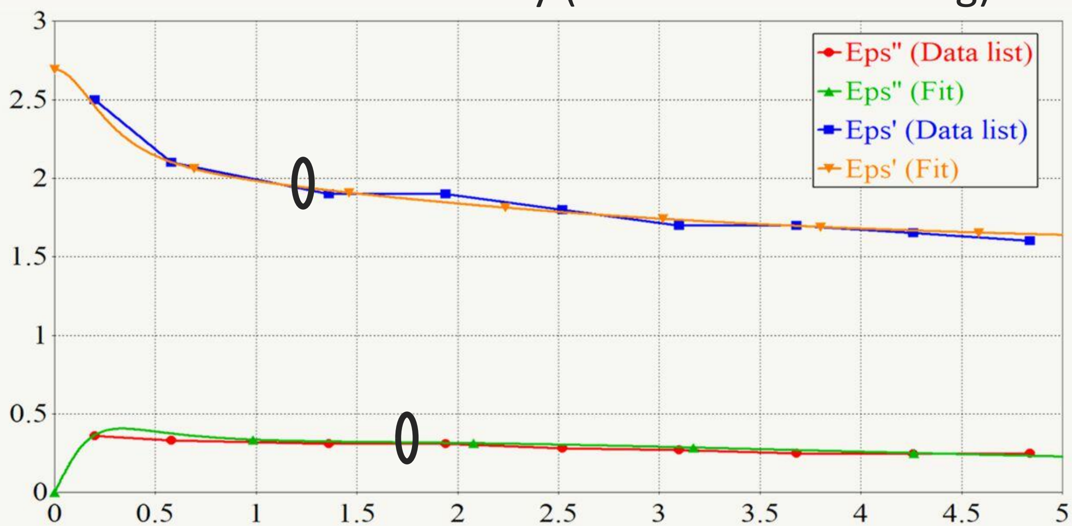
GENERALIZED HIGHER ORDER MATERIAL MODEL

Instead of using the physically motivated Debeye, Drude or Lorentz models, a generalized **mathematical model** can be used:

$$\epsilon(\omega) = \epsilon_\infty + \sum_{n=1}^{N} \frac{\beta_{0,n}}{\alpha_{0,n} + i\omega} + \sum_{n=1}^{M} \frac{\gamma_{0,n} + i\omega\gamma_{1,n}}{\delta_{0,n} + i\omega\delta_{1,n} - \omega^2}$$

Parameters are obtained by **fitting** to the given material dispersion curves. Fitting needs to ensure **causality** and **stability** of the model.

Electric Permittivity (Second Order Fitting)

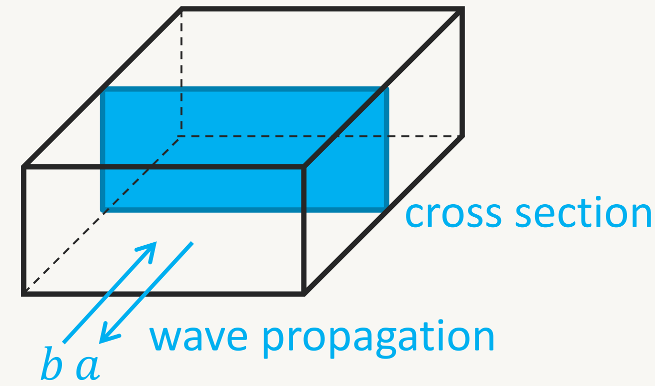


Frequency in GHz

WAVEGUIDE PORTS

FUNDAMENTAL CONCEPT – 2 PLANE EXTRACTION

The **tangential electric** field in a **cross section** of a homogeneous wave guide $\boldsymbol{e}_t(\mathrm{t})$ can be written as:



amplitude of mode propagating into direction

$$\boldsymbol{e}_t(t) = \sum_{i=0}^{N-1}\left(\overset{\text{a}}{\downarrow}\!\!a_i(t)\,\boldsymbol{e}_t^i + \overset{\text{b}}{\downarrow}\!\!b_i(t)\,\boldsymbol{e}_t^i\right) + \Delta\boldsymbol{e}_t(t)$$

transversal electric field of mode i

transversal electric field not covered by N modes

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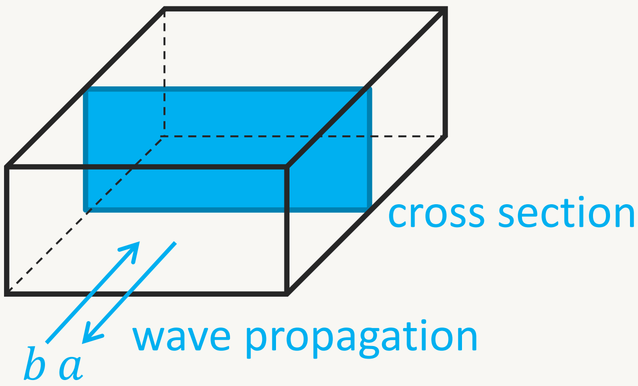

amplitude of mode propagating into direction

a

b

$$\boldsymbol{e}_t(t) = \sum_{i=0}^{N-1} \left(a_i(t)\, \boldsymbol{e}_t^i + b_i(t)\, \boldsymbol{e}_t^i \right) + \Delta \boldsymbol{e}_t(t)$$

transversal electric field of mode i

transversal electric field not covered by N modes

Homogeneous wave guides: cross section mode pattern is frequency independent

$\Rightarrow \boldsymbol{e}_t^i$ is time independent. Modes are orthogonal: $< \boldsymbol{e}_t^i, \boldsymbol{e}_t^j > = \delta_{i,j}$

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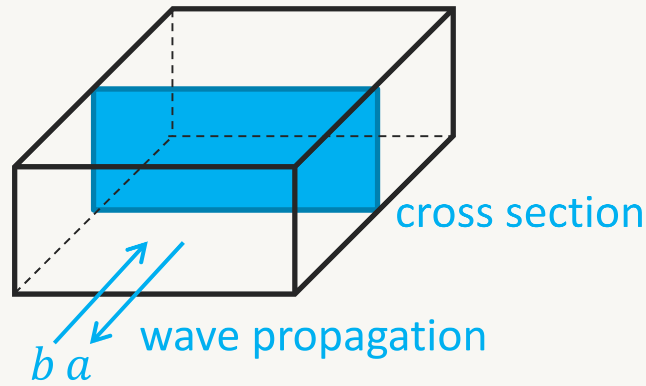

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a b

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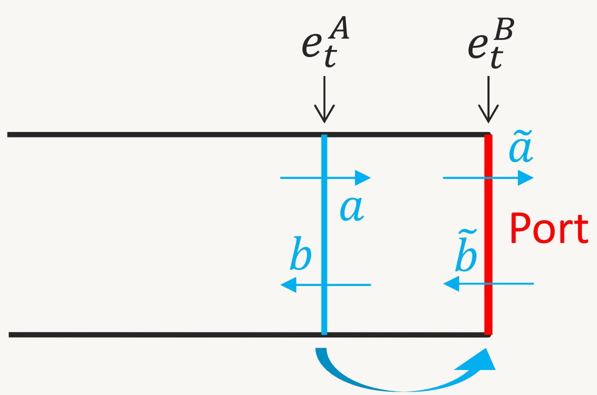
Homogeneous wave guides: cross section mode pattern is frequency independent

$\Rightarrow \boldsymbol{e}_t^i$ is time independent. Modes are orthogonal: $< \boldsymbol{e}_t^i, \boldsymbol{e}_t^j > = \delta_{i,j}$



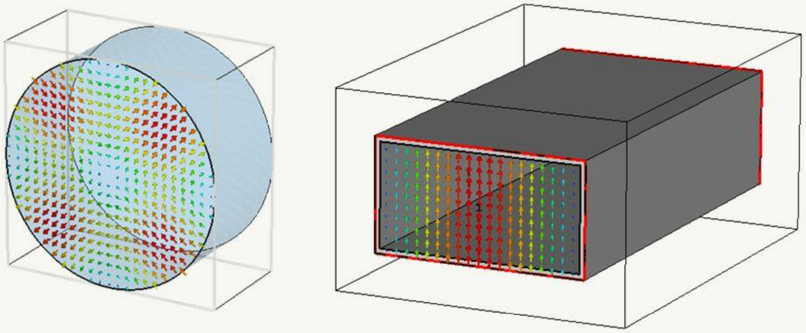
Propagation operator P (transmission line, ROM, etc.)

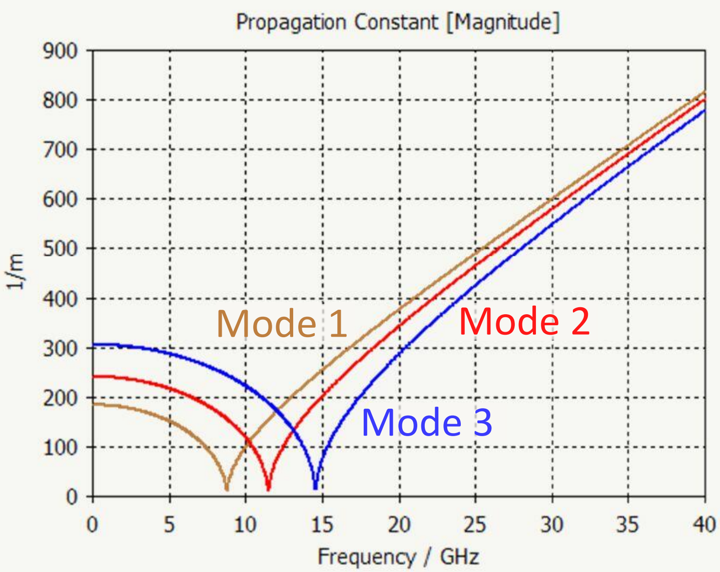
Application as wave guide boundary condition:

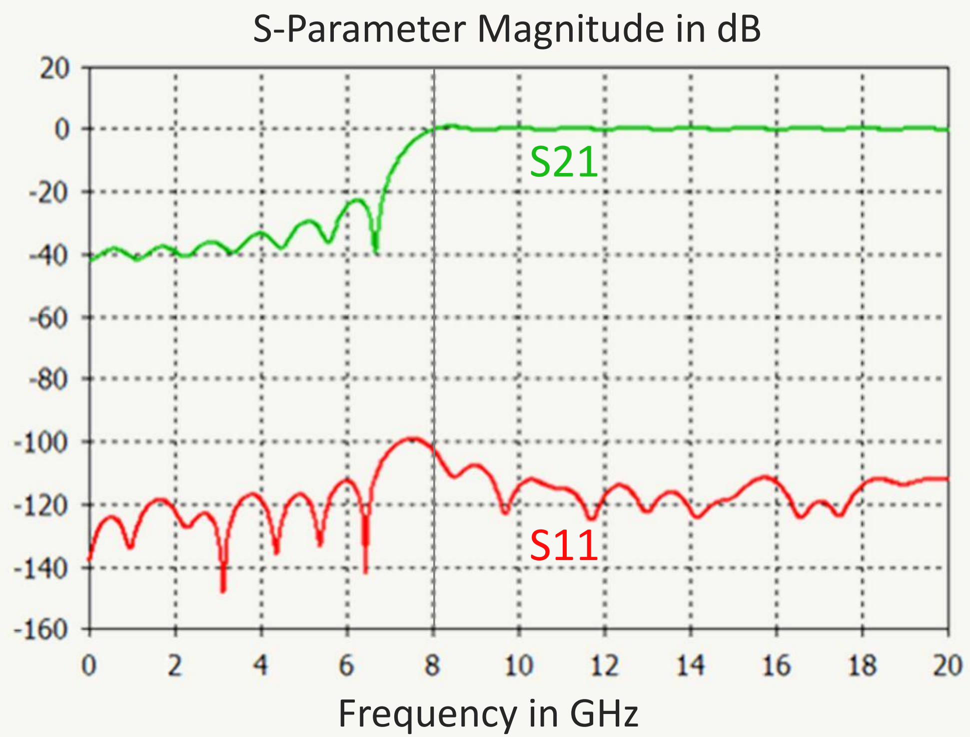
1. Known excitation $\tilde{b}_i \Rightarrow b_i = P(\tilde{b}_i)$
2. Model decomposition: $a_i + b_i = < e_t^A, e_t^i >$
3. With known $b_i \Rightarrow a_i \;\Rightarrow\; \tilde{a}_i = P(a_i)$
4. Obtain $e_t^B = \sum_i (\tilde{a}_i + \tilde{b}_i)\, e_t^i$

WAVEGUIDE PORTS

BROADBAND PORT OPERATOR FOR HOMOGENEOUS PORTS

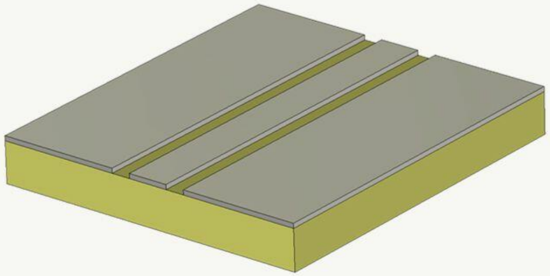




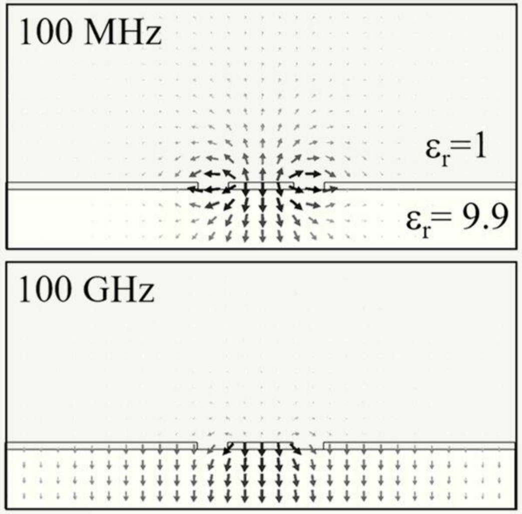


Transversal mode field is frequency independent
⇨ transmission-line based approach

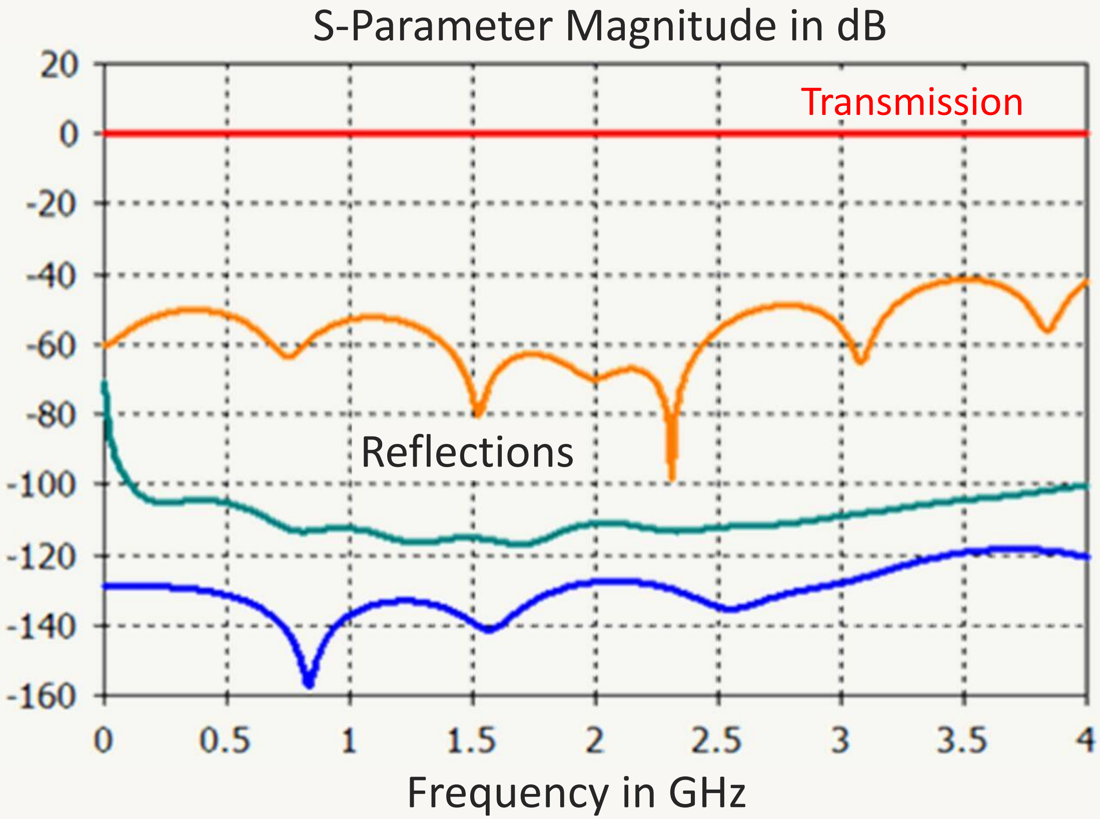
WAVEGUIDE PORTS



BROADBAND OPERATOR FOR INHOMOGENEOUS PORTS



Inhomogeneous material distribution in port cross section
$\Rightarrow$ transversal mode field is **frequency dependent**: $e_t^i(\omega)$
$\Rightarrow$ modes need to be calculated at **multiple frequencies**



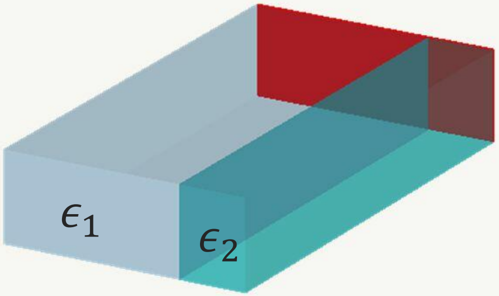
Mode calculated at center frequency (same a for homogeneous ports)

Mode calculated at center frequency, but de-embedding based on multiple frequency samples

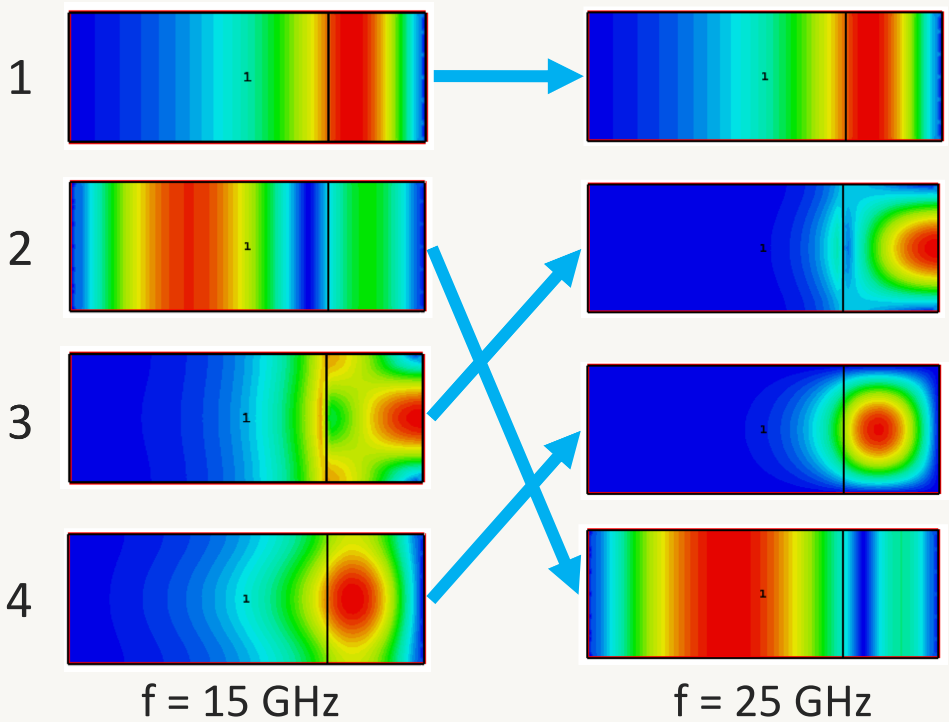
Propagation operator based on reduced order model

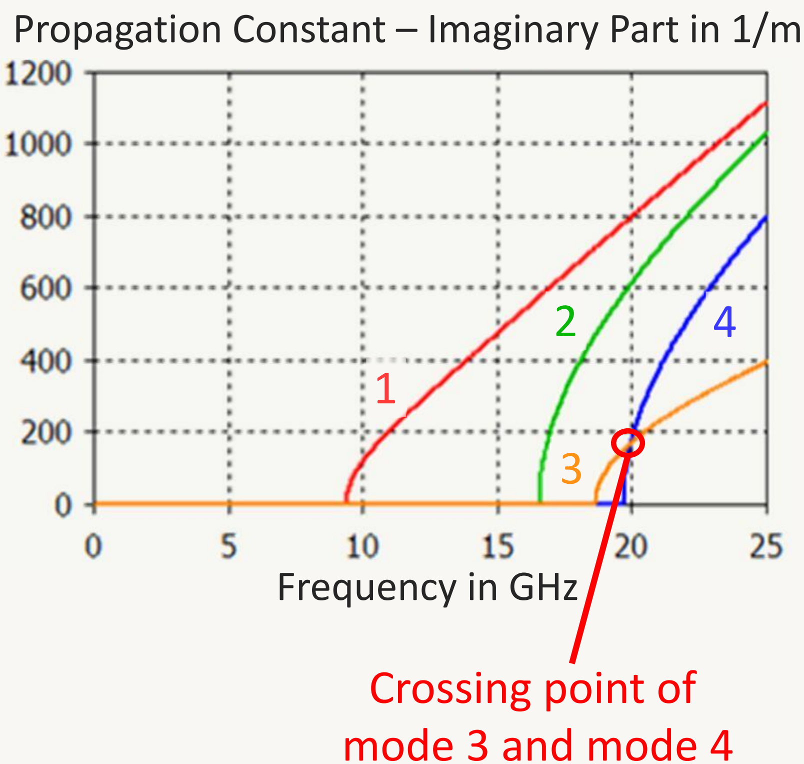
WAVEGUIDE PORTS

MODE TRACKING IN INHOMOGENEOUS WAVE GUIDES



Order of the modes change depending on frequency:





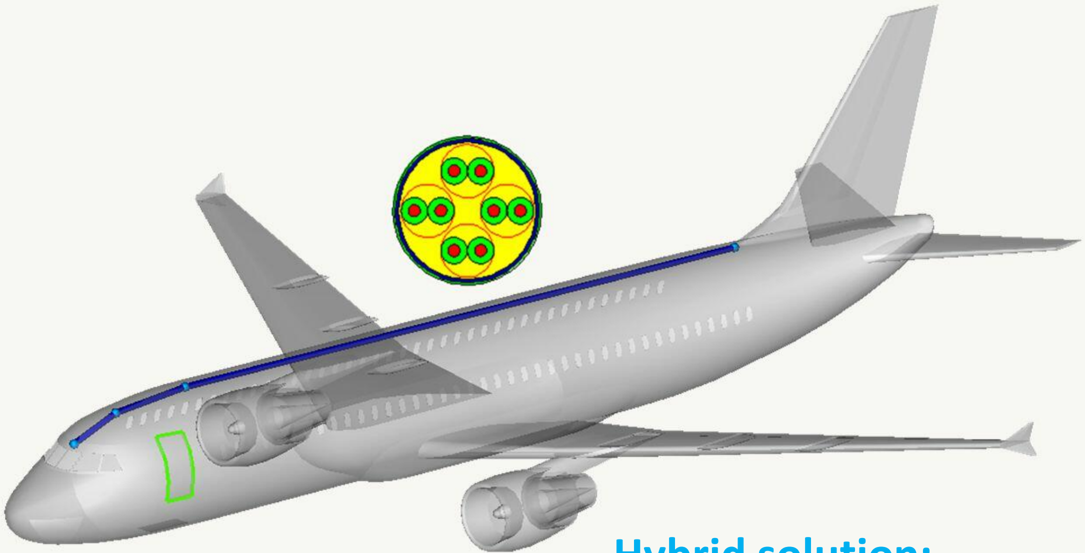
More complex situation in case of **complex modes**
$\Rightarrow$ branches in curves of propagation constant

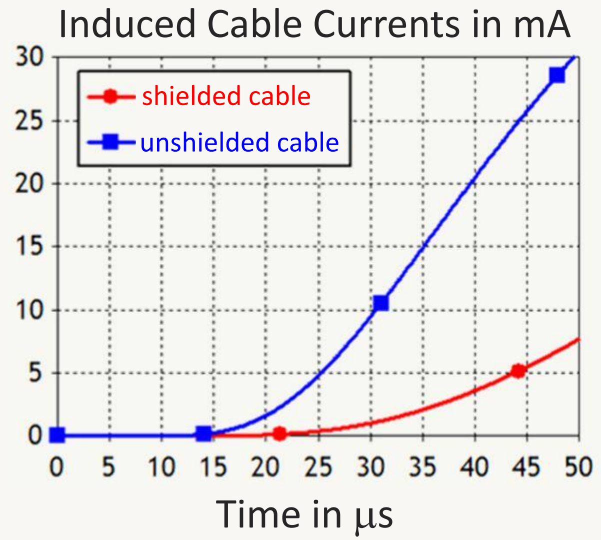
OPTIMIZING SIMULATION PERFORMANCE

SIMULATION EFFICIENCY = COMBINATION OF METHOD EFFICIENCY AND HPC

Enhancements of the basic scheme (e.g. PBA, TST) and maintaining the **broadband capabilities** (e.g. waveguide ports, material properties) yield **performance improvements** over basic scheme by orders of magnitude.

Other example (multiscale problem): **Cable in airplane** (mm vs. tens of m)



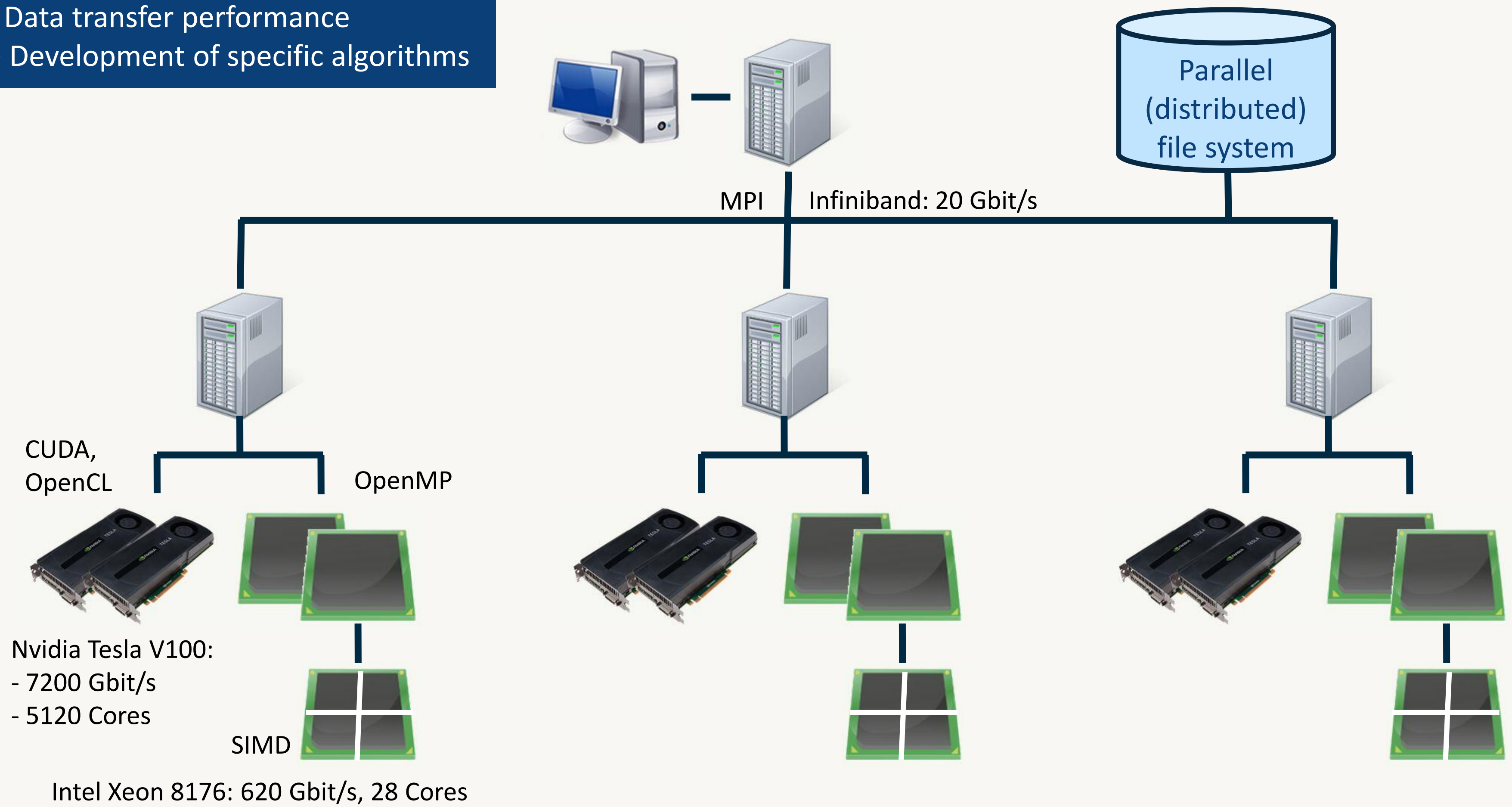


Hybrid solution:

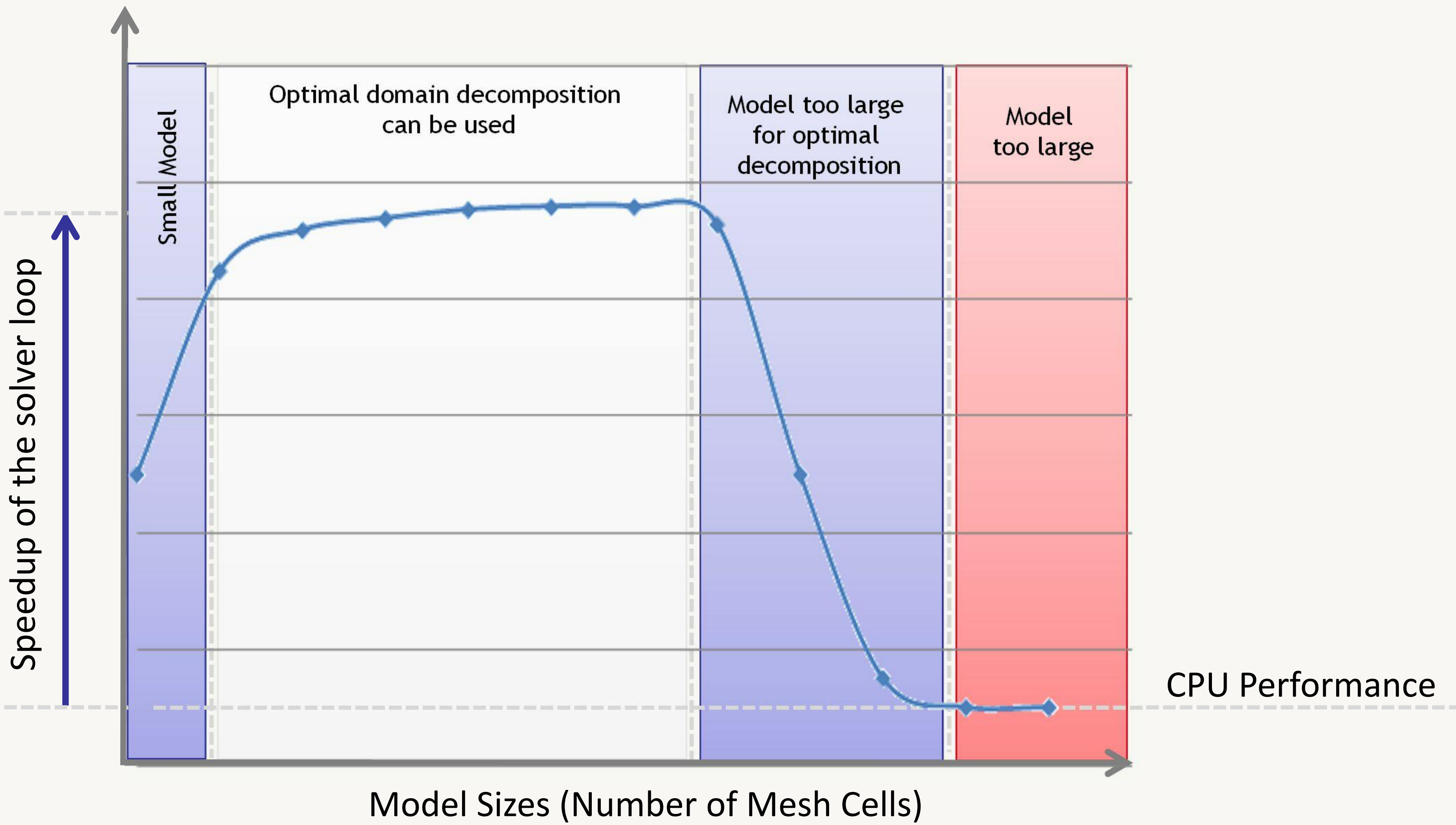
- FIT transient EM simulation of fields in airplane
- Simulation of cable by modal transmission line model
- Bi-directional coupling via tangential fields along cable

HIGH PERFORMANCE COMPUTING OPTIONS

– Distribution of data / load balancing
– Data transfer performance
⇨ Development of specific algorithms

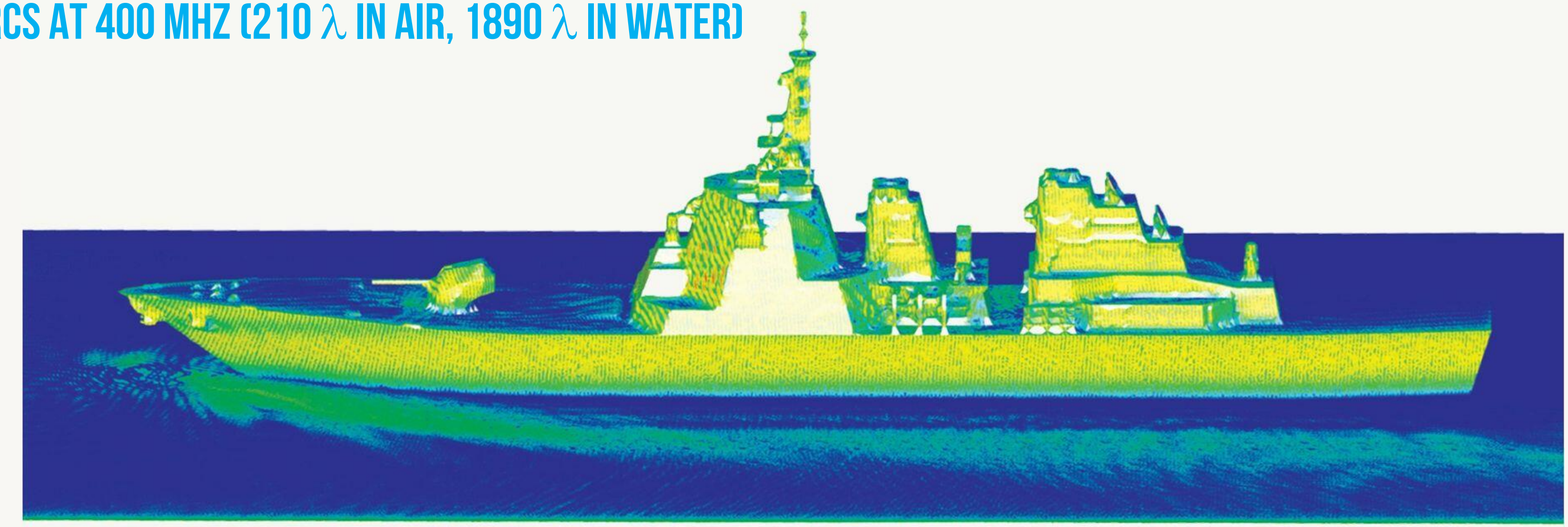


HIGH PERFORMANCE COMPUTING GP-GPU

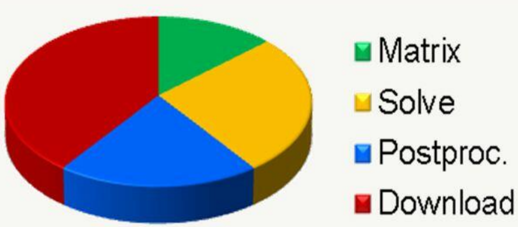


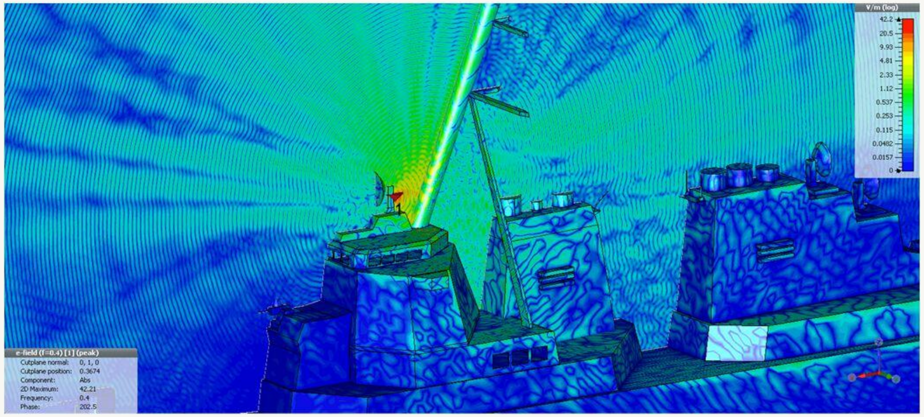
HIGH PERFORMANCE COMPUTING EXAMPLE

RCS AT 400 MHZ (210 λ IN AIR, 1890 λ IN WATER)



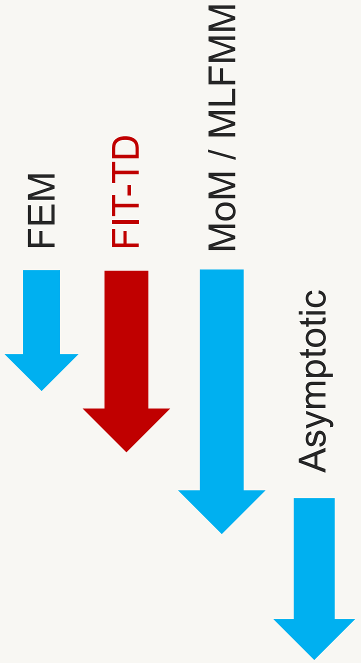
- 1.5 billion mesh cells
- HPC cluster: 1152 cores on 96 nodes
- Total runtime: 30h
- 40% percent of time for downloading results (1.1 TB)





CHOOSING OPTIMAL METHOD EXAMPLE

Frequency	El. Length	El Vol.	Application
1 MHz	0.5λ	0.125 λ^3	Lightning
10 MHz	5λ	125 λ^3	HF communications
100 MHz	50λ	125e3λ^3	EMP
1 GHz	500λ	125e6 λ^3	L-Band Communications
10 GHz	5000λ	125e9 λ^3	Radar



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FEM

FIT-TD: Antenna system

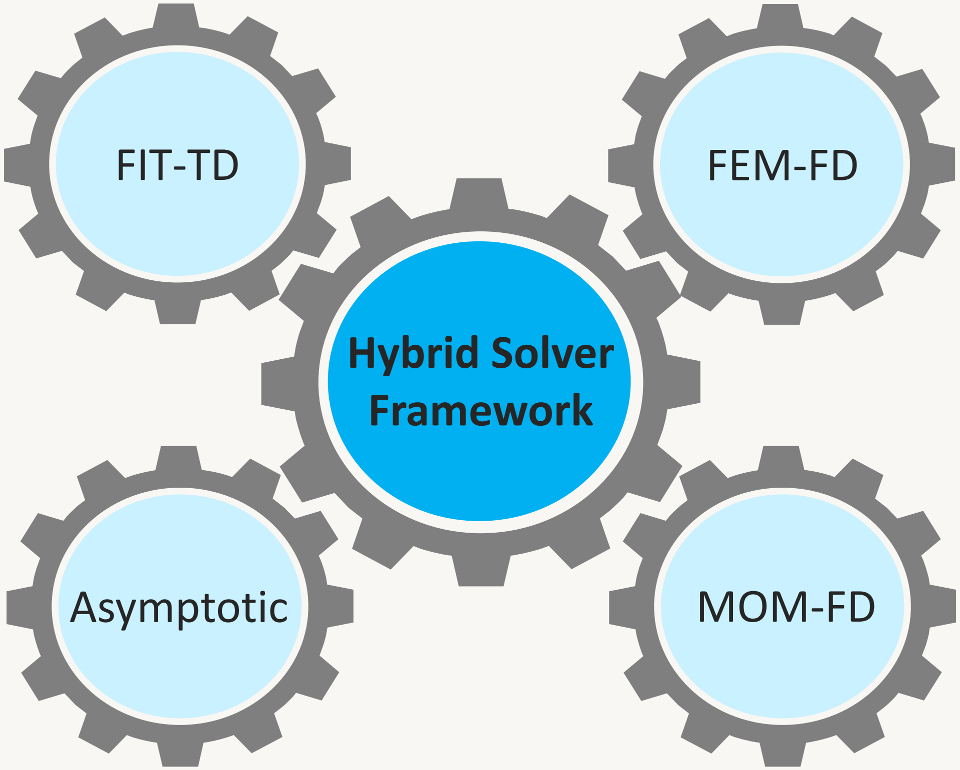
MoM / MLFMM

Asymptotic: Installed performance

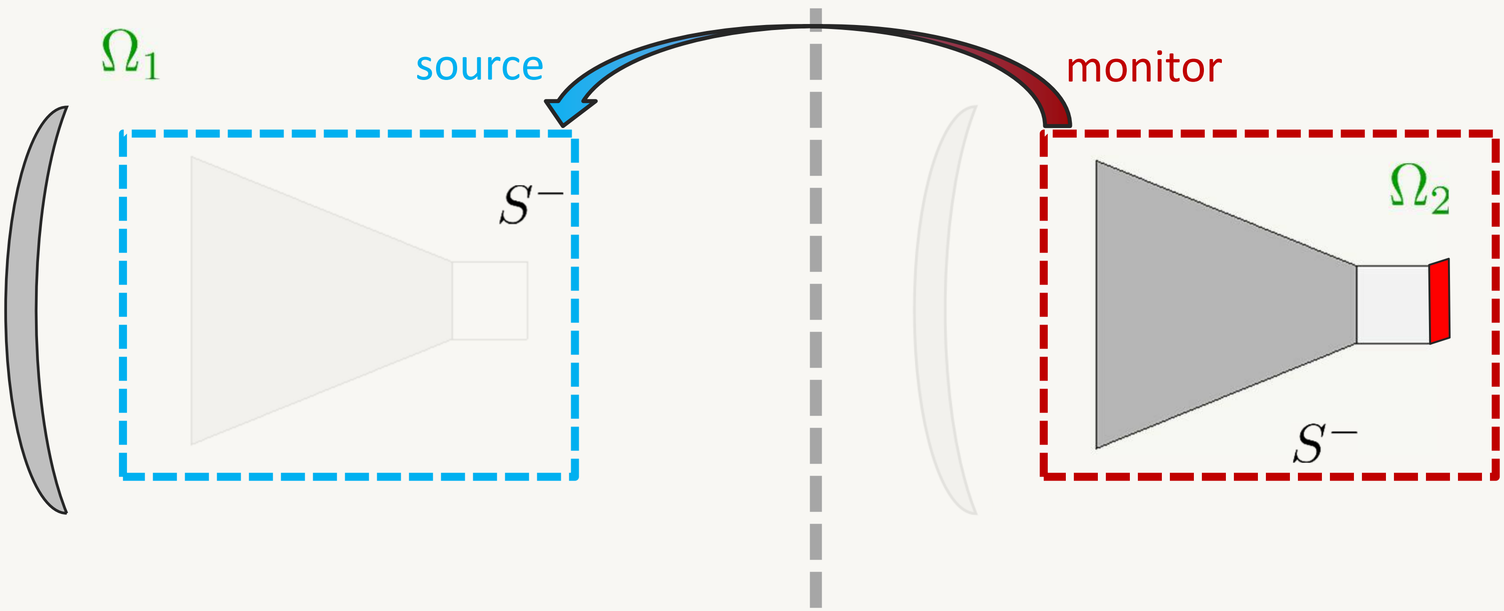
Often a combination of methods is required for best efficiency

HYBRID SOLVER FRAMEWORK

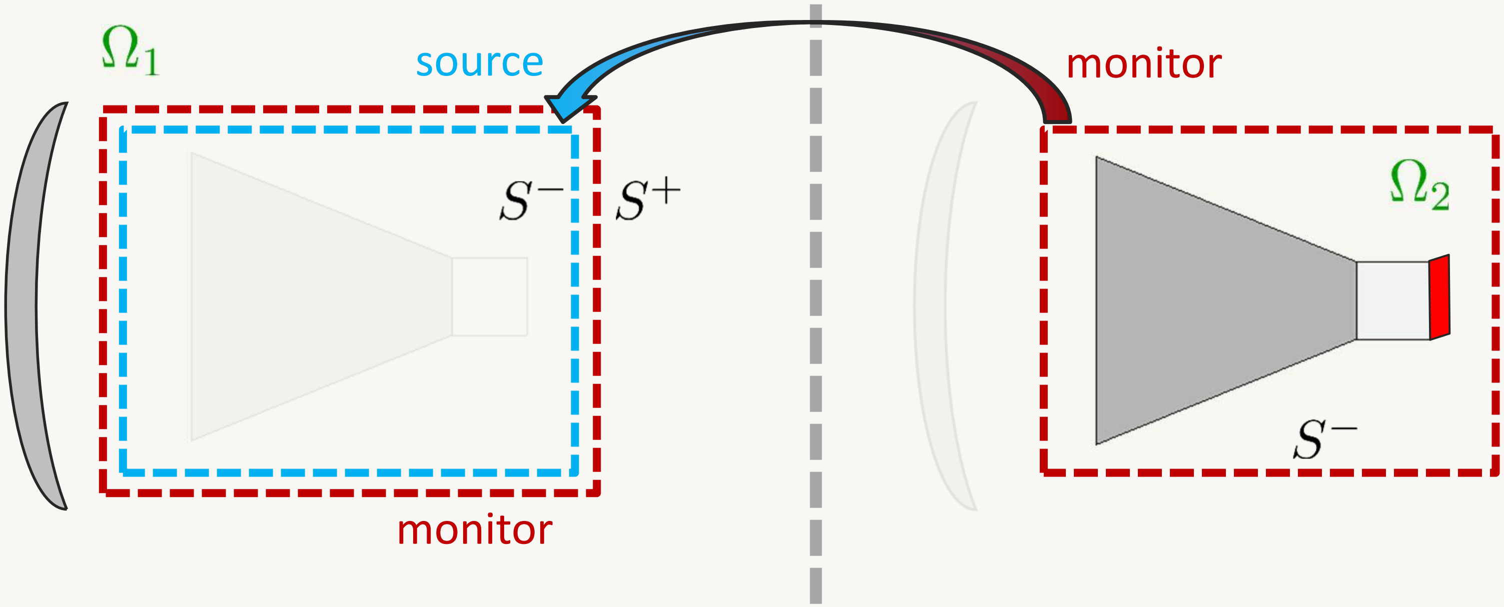
Hybrid solver framework allows **flexible unidirectional** and **bi-directional coupling** of various **simulation methods** based on **field sources**



HYBRID SOLVER ITERATION SCHEME



HYBRID SOLVER ITERATION SCHEME



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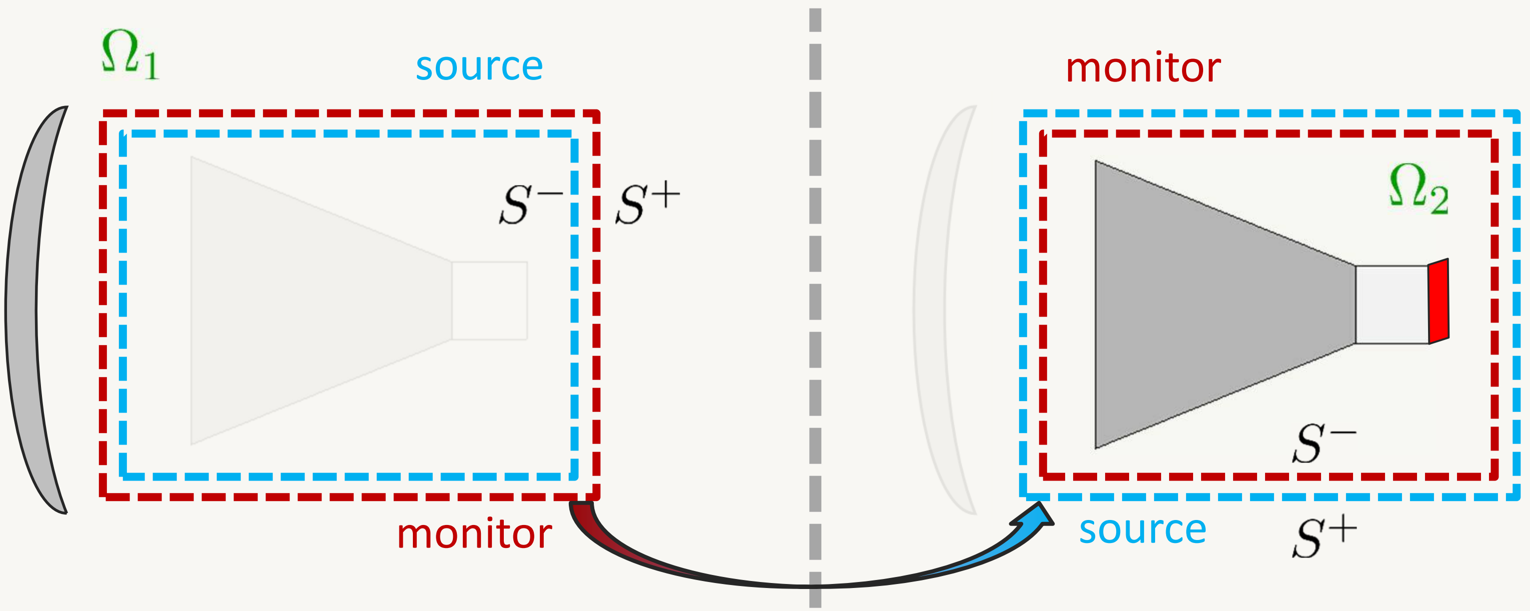
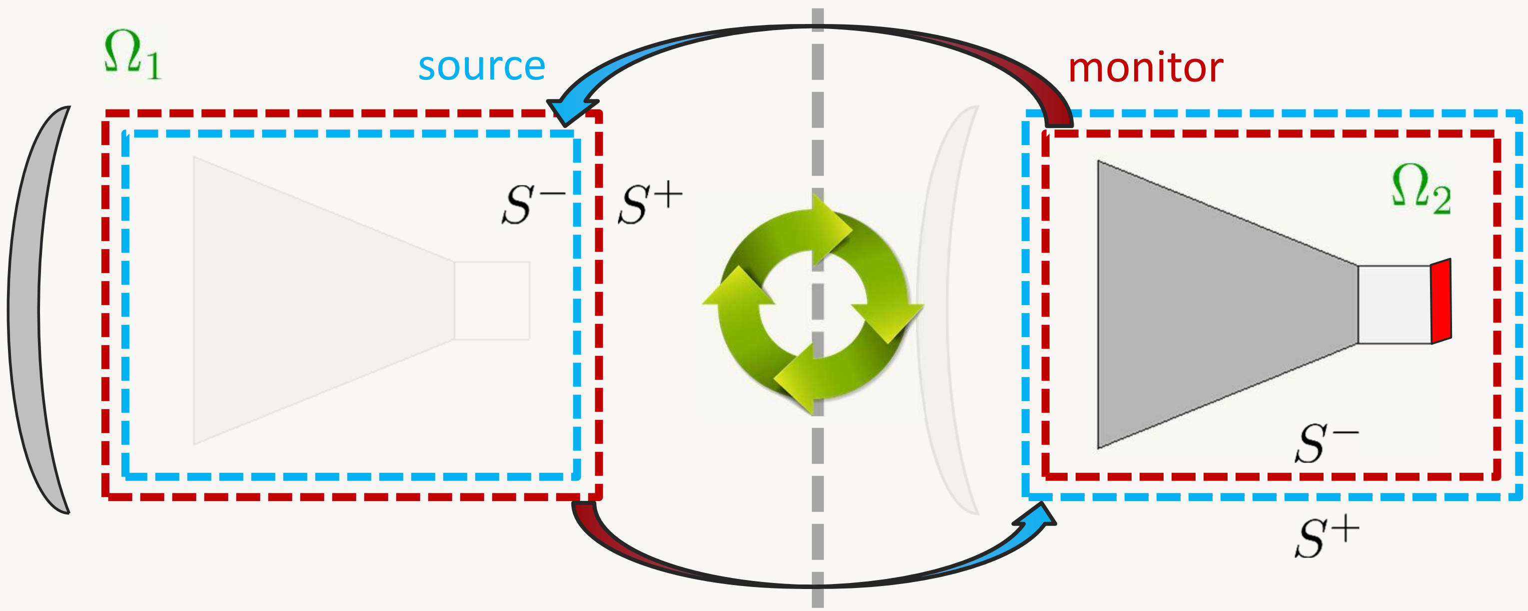


HYBRID SOLVER ITERATION SCHEME

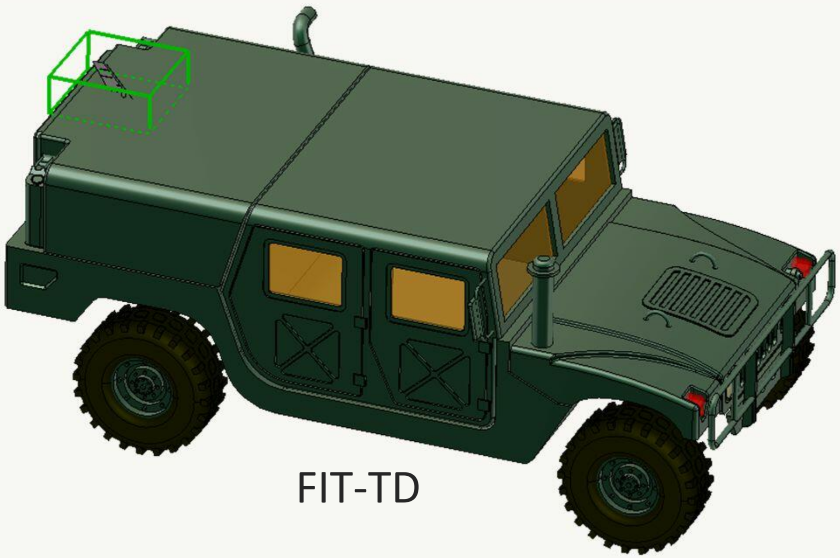


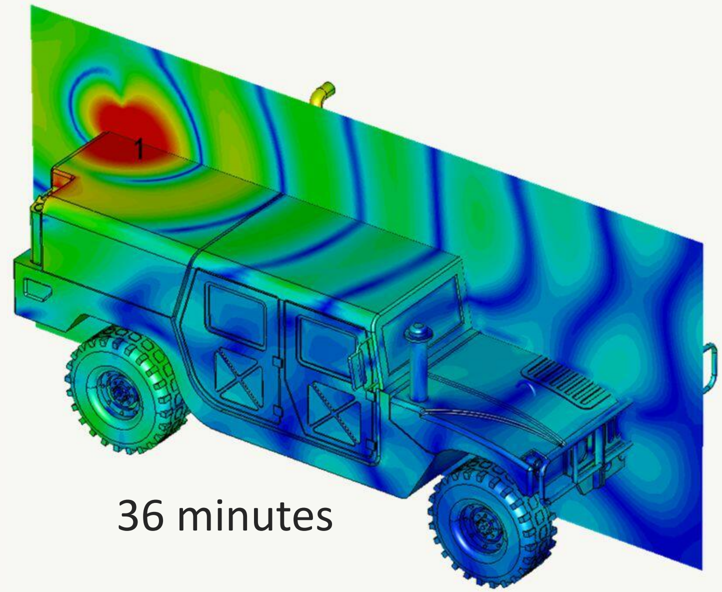
Resulting system:

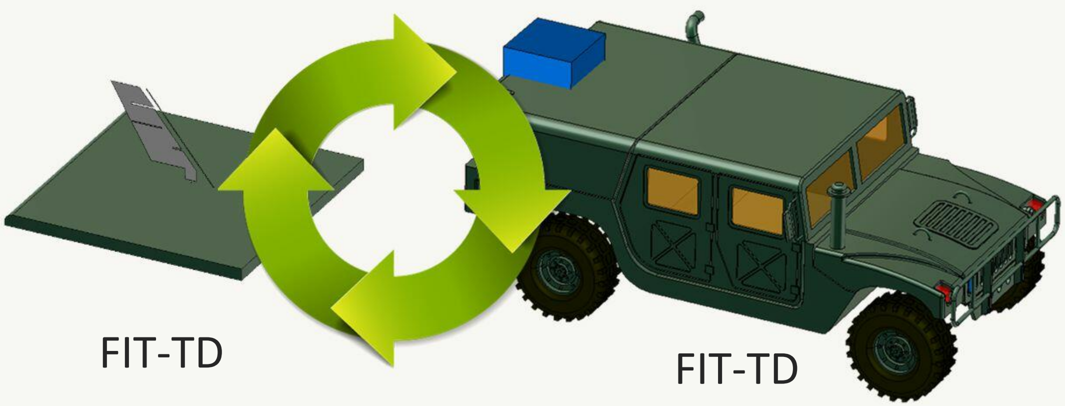
$$\begin{bmatrix} \boldsymbol{I} & \boldsymbol{R}_1 \boldsymbol{A}_1^{-1} \boldsymbol{C}_{12} \\ \boldsymbol{R}_2 \boldsymbol{A}_2^{-1} \boldsymbol{C}_{21} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} x_1^{S^+} \\ x_2^{S^-} \end{bmatrix} = \begin{bmatrix} b_1^{S^+} \\ b_2^{S^-} \end{bmatrix}$$

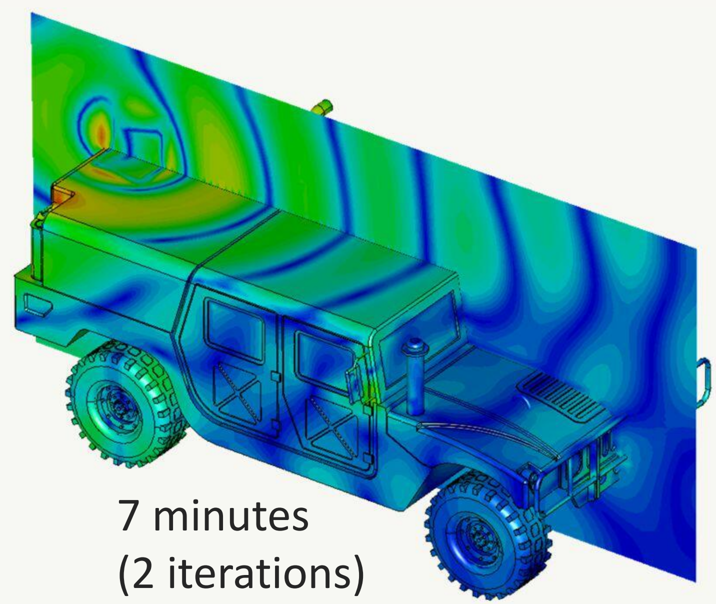
Applying a **pre-conditioned solver** to the system reduces number of iterations.

HYBRID SOLVER EXAMPLE





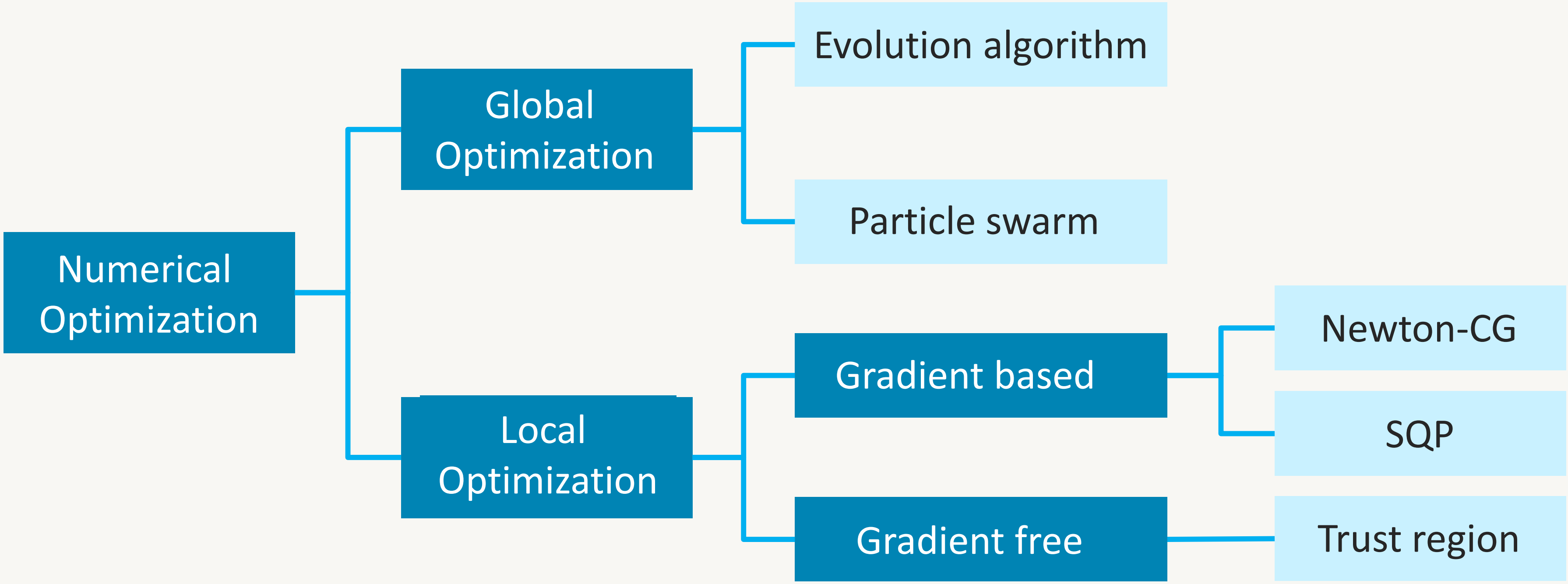




OPTIMIZATION STRATEGIES

FROM VERIFICATION TO DESIGN

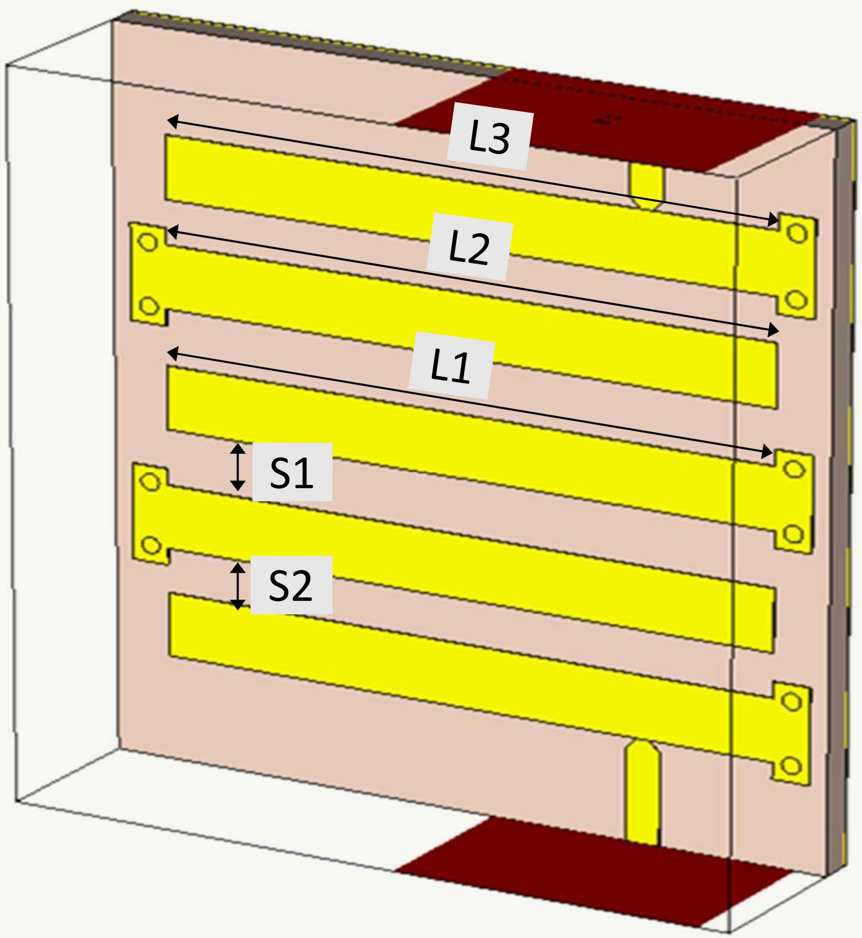
Many applications require the design to be **optimized** rather than just **verified**.



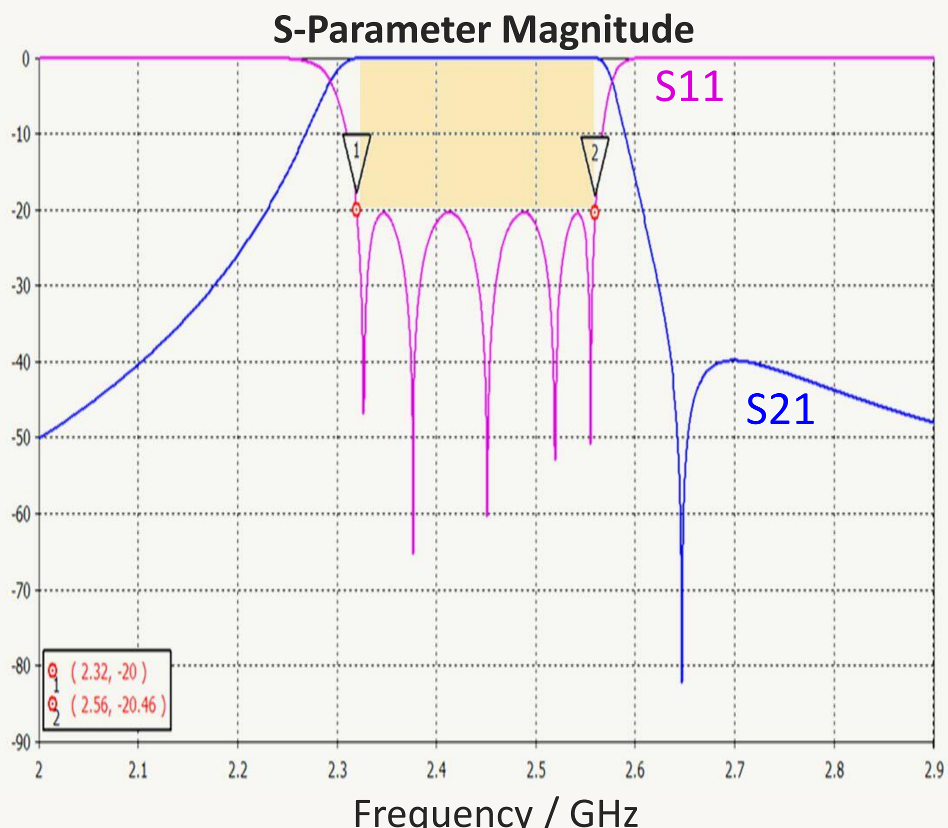
In real world applications, often a **combination of such methods** is applied (e.g. starting with global optimization and then refining the solution by local methods).

COMPARISON OF OPTIMIZATION STRATEGIES

SIMPLE FILTER EXAMPLE WITH 5 PARAMETERS



Result of Trust-Region based optimization
(350 computations required)

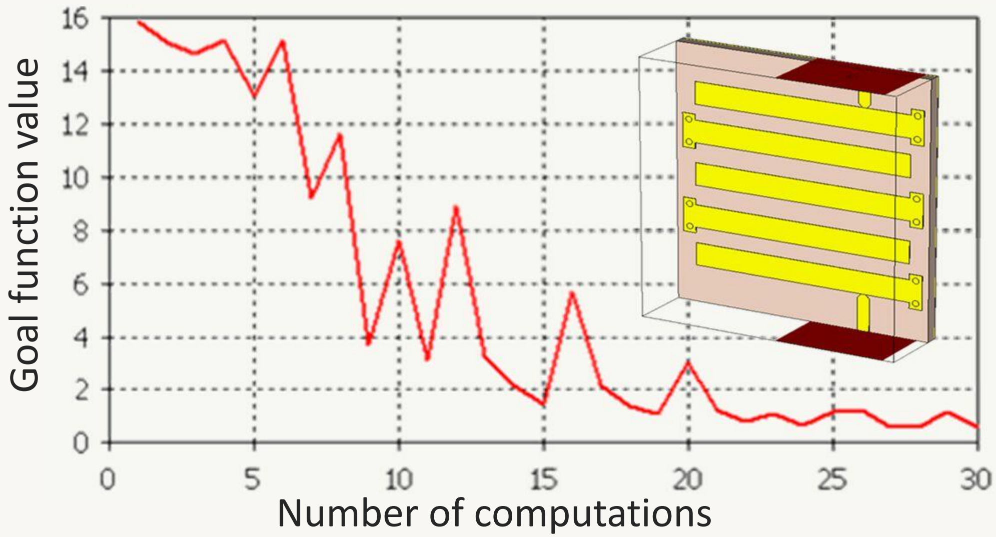


ADJOINT SENSITIVITY COMPUTATION

- Optimization often requires **gradients of the solution**
- Numerical calculation of gradients is **expensive** and **unstable**
- Here: **Sensitivity** of S-parameter vs. parameter change

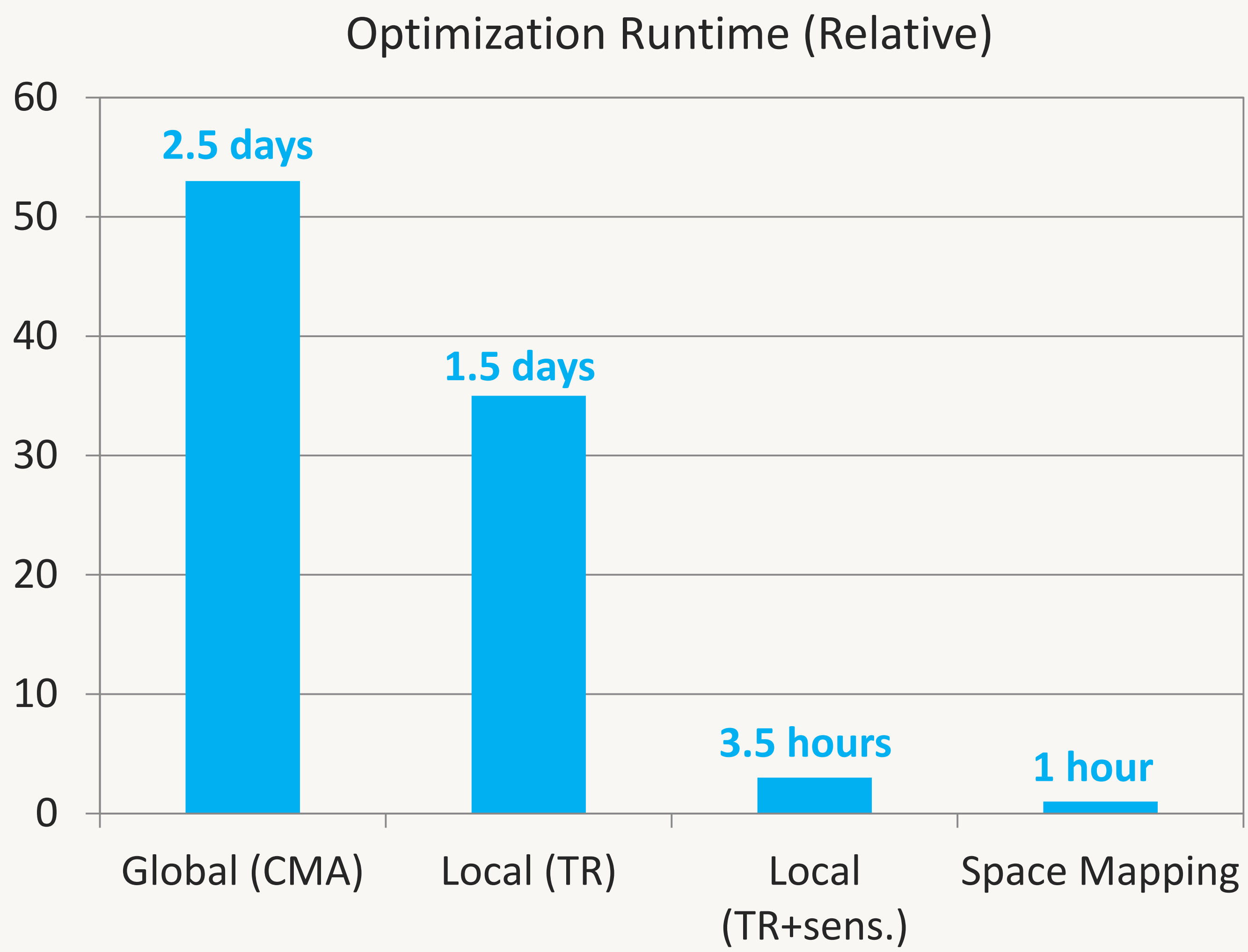
$$S(\omega,p) = \frac{1}{-j\omega\mu_0} E^T(\omega,p) K^T(\omega,p) E(\omega,p) \Rightarrow j\omega\mu_0 \boxed{\frac{\partial S}{\partial p}} = E^T \boxed{\frac{\partial K}{\partial p}} E$$

- **Matrix differentiation** required only, no further solution required
- Very efficient computation of sensitivities



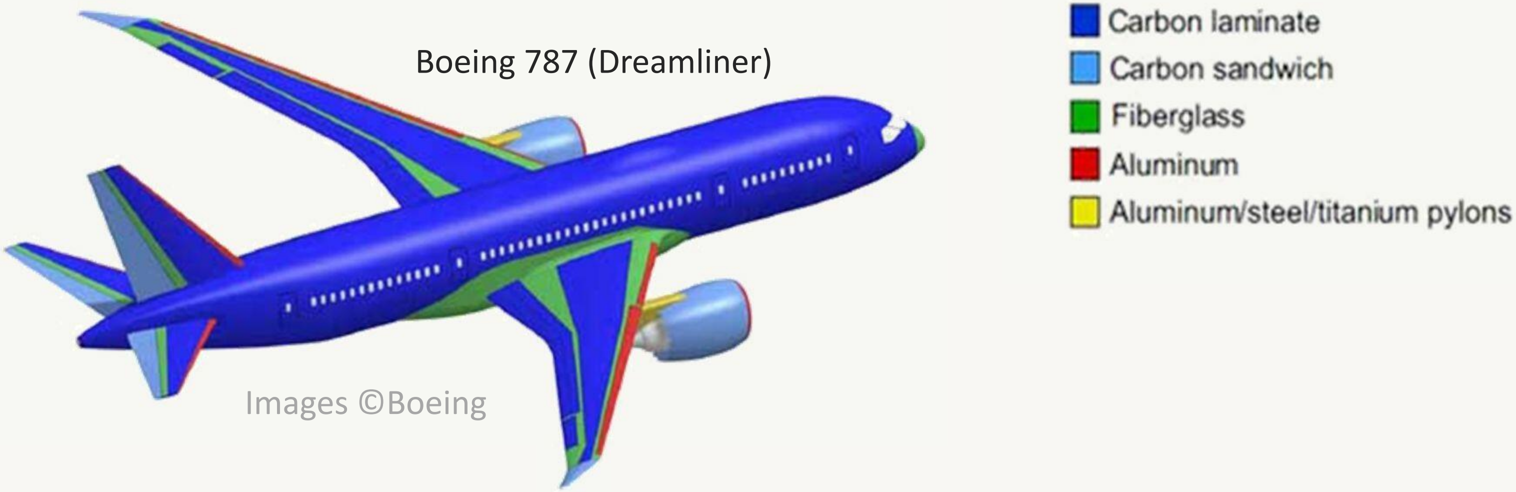
Using sensitivity information reduces number of required computations: 350 ⇨ 30

COMPARISON OF OPTIMIZATION STRATEGIES



VIRTUAL PRODUCT DEVELOPMENT
IN THE AEROSPACE INDUSTRY

AIRFRAME VULNERABILITY LIGHTNING PROTECTION



Boeing 787 (Dreamliner)

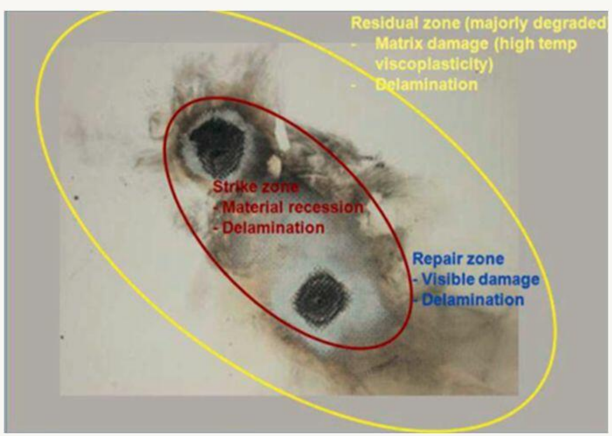
Images ©Boeing

AIRFRAME VULNERABILITY LIGHTNING PROTECTION

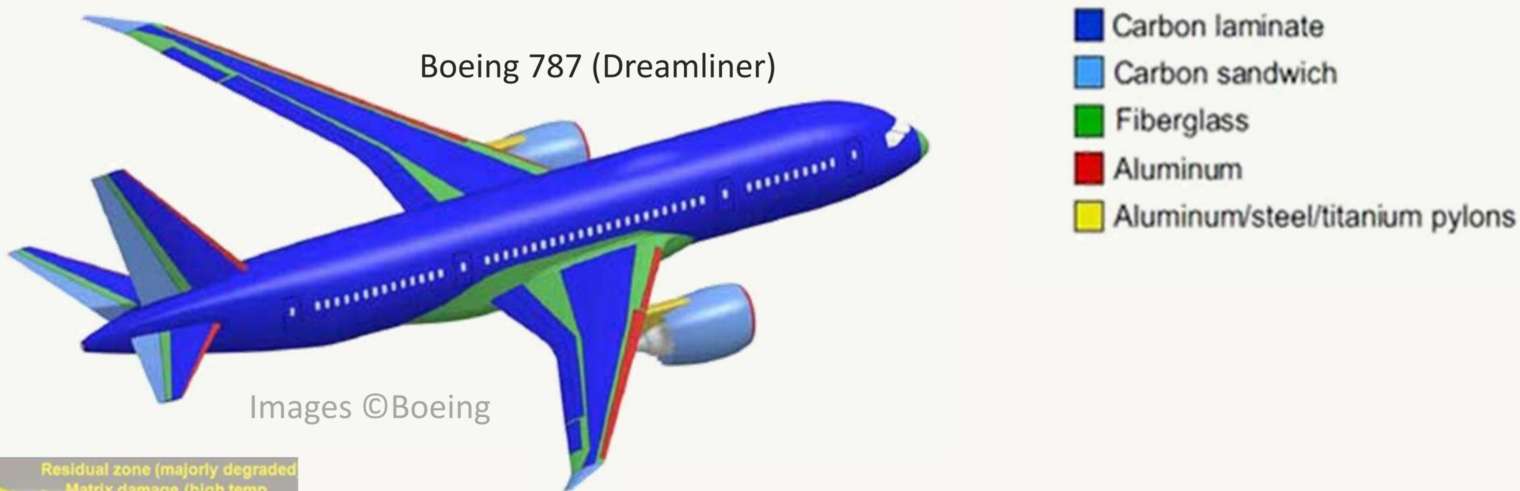
Boeing 787 (Dreamliner)

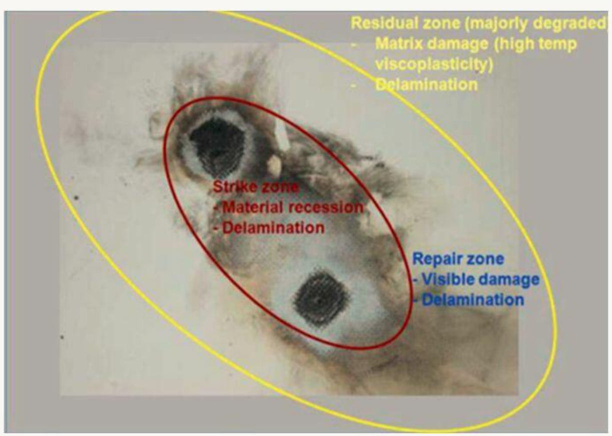
Images ©Boeing

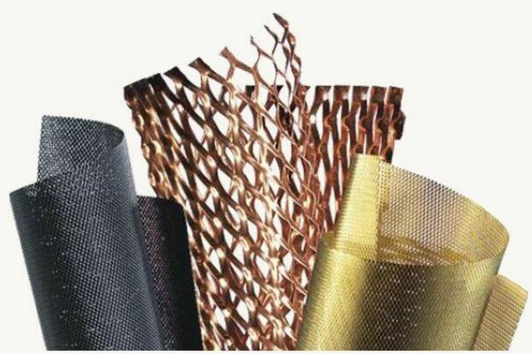
- Carbon laminate
- Carbon sandwich
- Fiberglass
- Aluminum
- Aluminum/steel/titanium pylons



AIRFRAME VULNERABILITY LIGHTNING PROTECTION







AIRFRAME VULNERABILITY LIGHTNING PROTECTION

Boeing 787 (Dreamliner)

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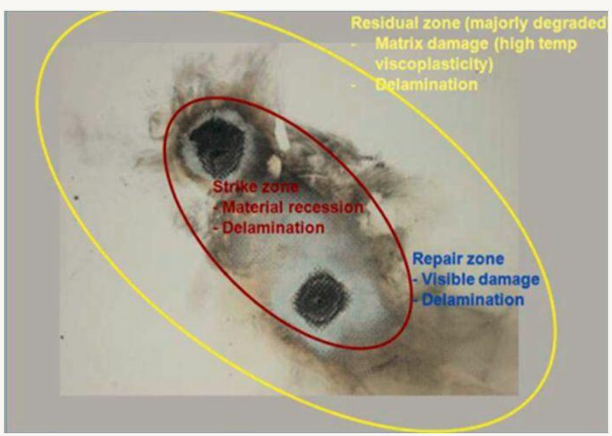

AIRFRAME VULNERABILITY LIGHTNING PROTECTION

Boeing 787 (Dreamliner)

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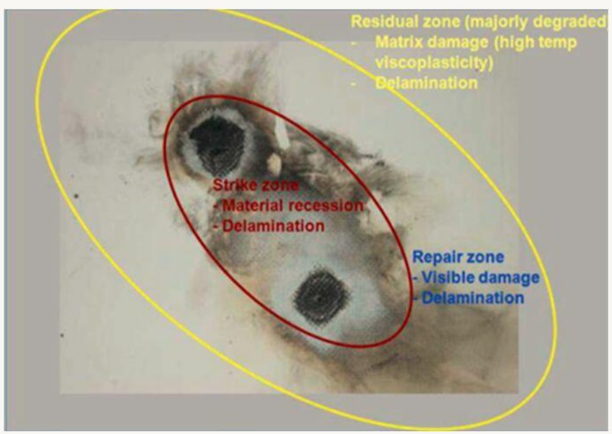
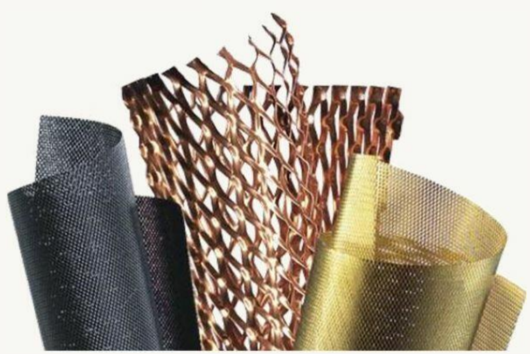
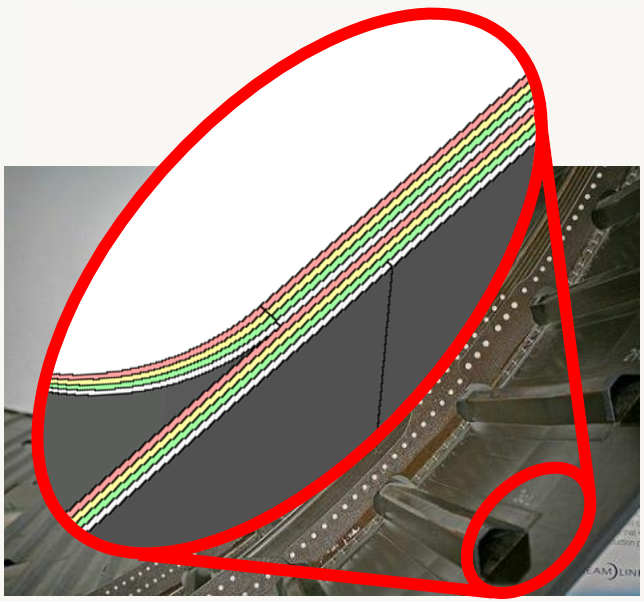
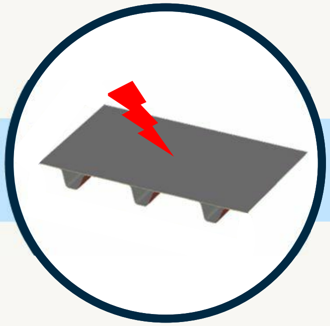


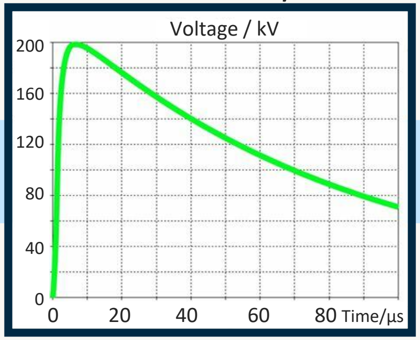




LIGHTNING STRIKE STRUCTURAL INTEGRITY



D6-16050-5/B



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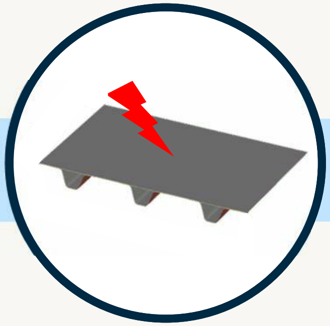
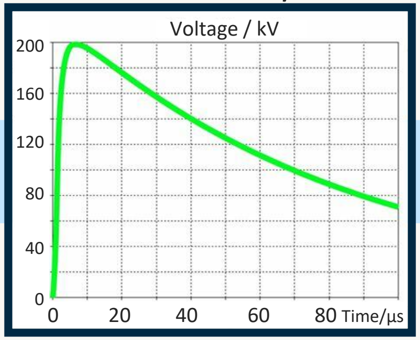
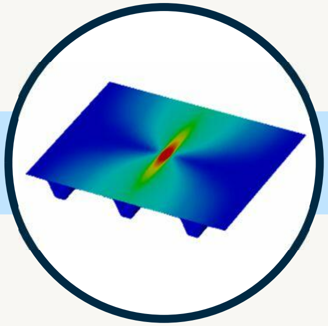
LIGHTNING STRIKE STRUCTURAL INTEGRITY



D6-16050-5/B



Electromagnetics



Electric power loss density

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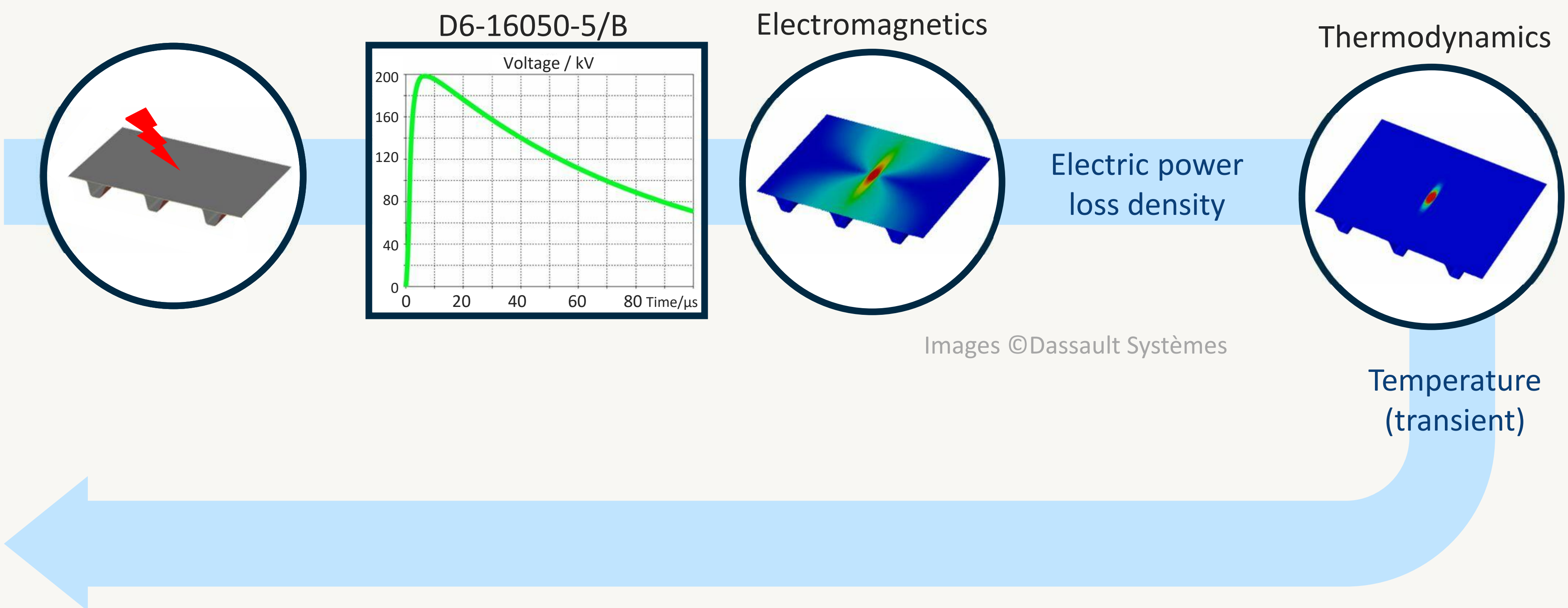
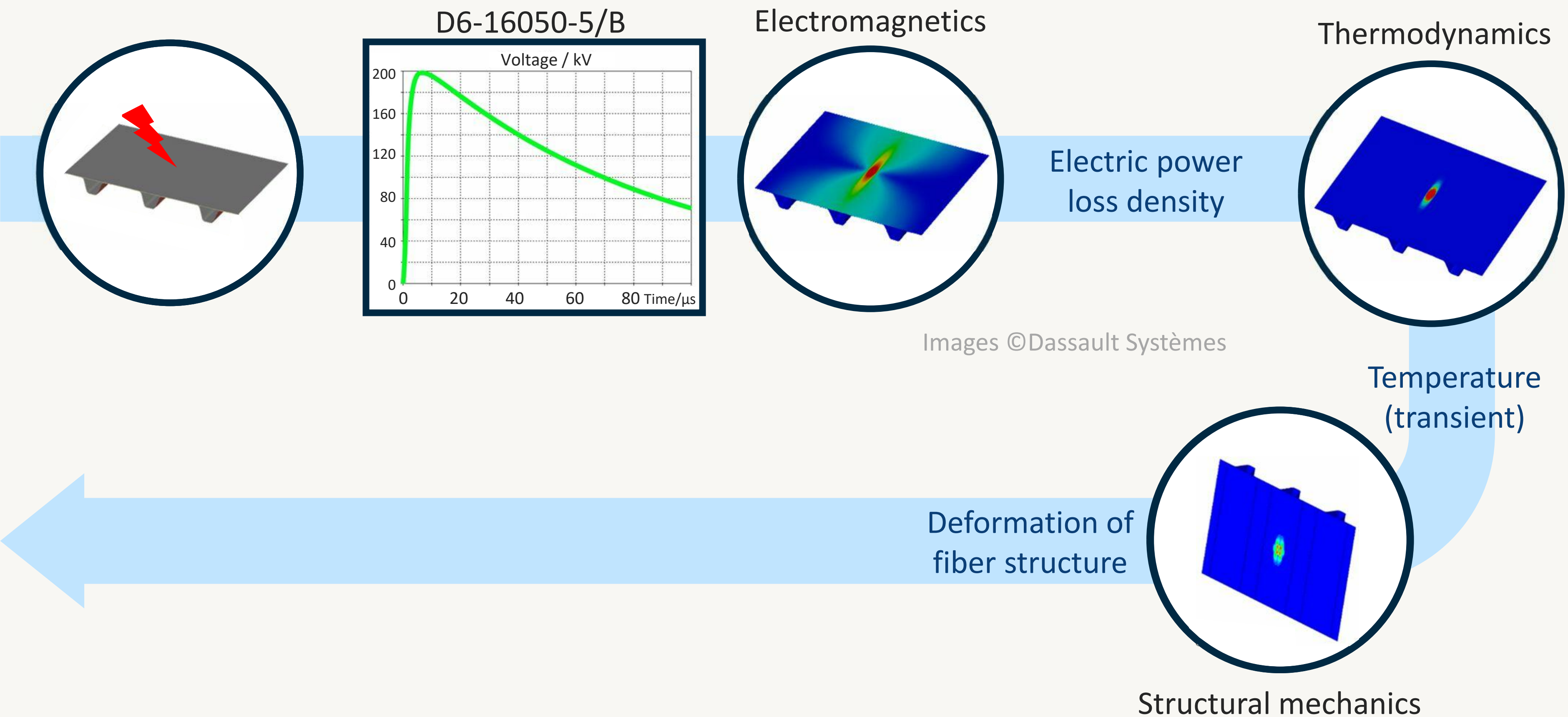
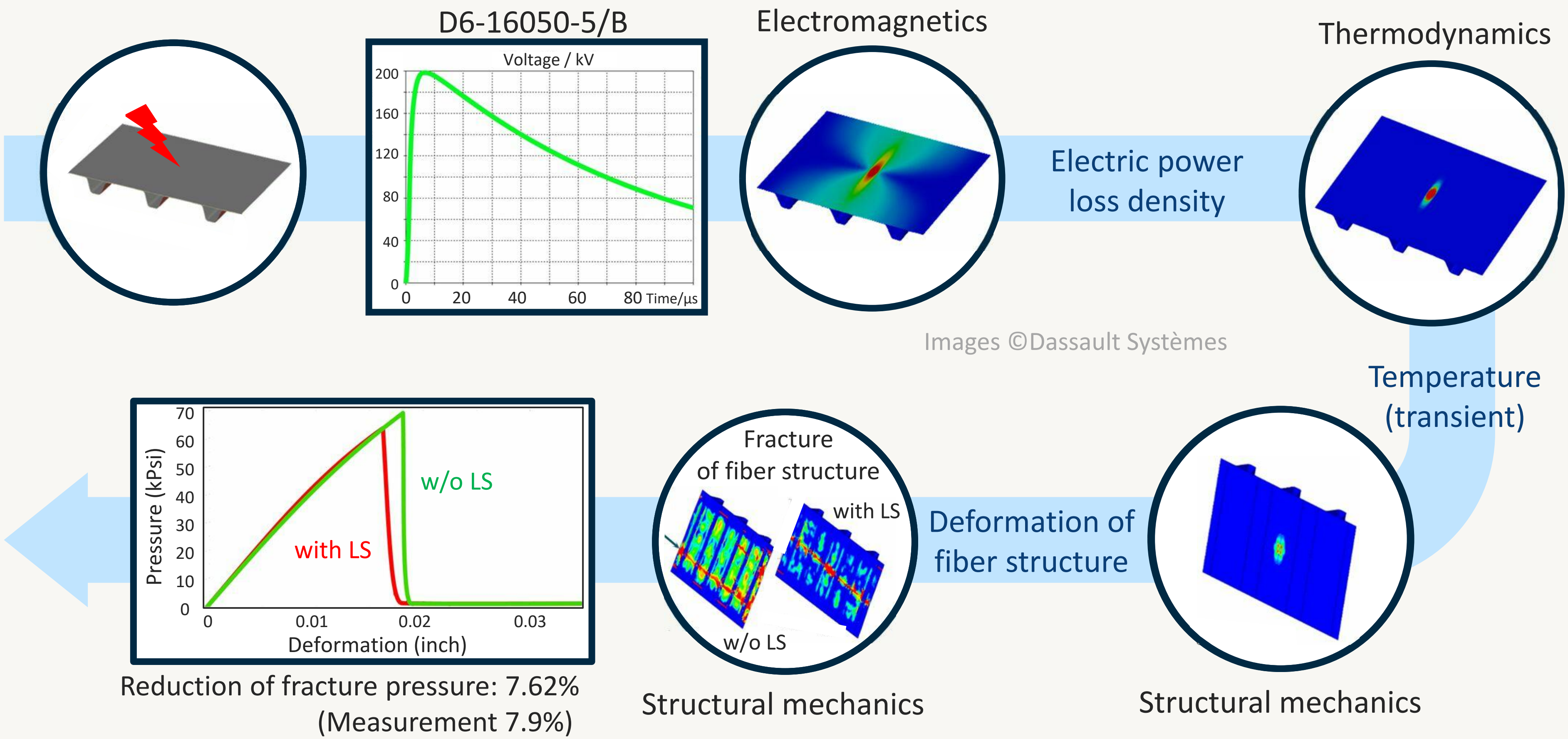
LIGHTNING STRIKE STRUCTURAL INTEGRITY



LIGHTNING STRIKE STRUCTURAL INTEGRITY



LIGHTNING STRIKE STRUCTURAL INTEGRITY



SUMMARY – 3 KEY TAKE-AWAYS

1. The **Finite Integration Technique** is a general and consistent framework for discretizing Maxwell's equations which allows developing stable and consistent extensions of the basic scheme

2. There is **no single "best method"** for all types of applications, so a portfolio of different methods is needed. **Hybridization** can enhance efficiency even further.

3. Typical **industrial applications** involve **multiple physical domains** and multiple scales. **Consistent integration** of various simulation disciplines will become more and more important in future.

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THANK YOU!