NUMERISCHE METHODEN

ZUR EMV SIMULATION

Peter Thoma

OUTLINE

1. **Describing Electromagnetic Phenomena**

2. **EMC – Relevant Types of Coupling**

3. **Numerical Methods for Electromagnetics**
   - The Finite Element Method
   - The Finite Integration Technique
   - Obtaining Broadband Results
   - Modeling Dispersive Materials
   - Multiscale Cable Modeling
   - High Performance Computing Options

4. **Numerical Simulation for EMC**

DESCRIBING ELECTROMAGNETIC PHENOMENA

MAXWELL'S EQUATIONS

Describing the behavior of **all kinds** of electromagnetic fields at macroscopic level:

$$\vec{\nabla} \times \vec{E}(\vec{r},t) = -\frac{\partial \vec{B}(\vec{r},t)}{\partial t}$$

$$\vec{\nabla} \times \vec{H}(\vec{r},t) = \vec{J}(\vec{r},t) + \frac{\partial \vec{D}(\vec{r},t)}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B}(\vec{r},t) = 0$$

$$\vec{\nabla} \cdot \vec{D}(\vec{r},t) = \rho(\vec{r})$$

Material relations:

$$\vec{D}(\vec{r},t) = \epsilon(\vec{r}) \cdot \vec{E}(\vec{r},t)$$

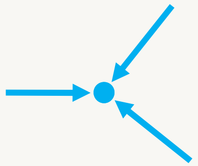
$$\vec{B}(\vec{r},t) = \mu(\vec{r}) \cdot \vec{H}(\vec{r},t)$$

$$\vec{J}(\vec{r},t) = \kappa(\vec{r}) \cdot \vec{E}(\vec{r},t) + \vec{J_0}(\vec{r},t)$$

OHMS LAW AND KIRCHHOFF'S CIRCUIT LAWS

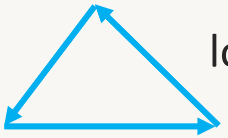
$$U = R \cdot I$$

1. $\sum_i I_i = 0$



node

2. $\sum_i U_i = 0$



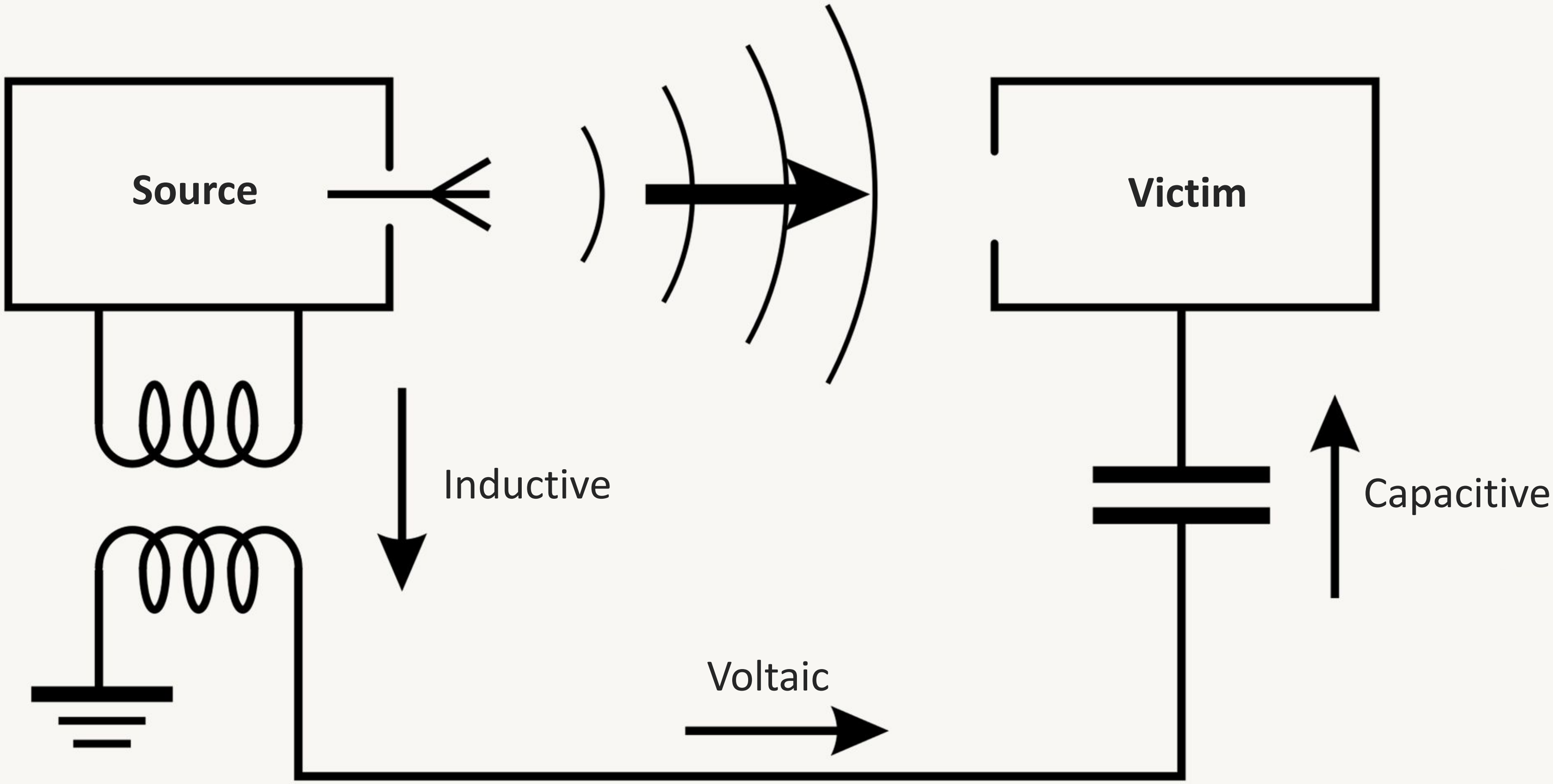
loop

$$U(t) = \int_L \vec{E}(\vec{r},t) \cdot d\vec{s}$$

$$I(t) = \iint_A \vec{J}(\vec{r},t) \cdot d\vec{A}$$

Kirchhoff's laws are derived from Maxwell's equations for $\frac{d}{dt} = 0$

EMC - RELEVANT TYPES OF COUPLING



- Interactions related to **voltages and currents** (voltaic, inductive, capacitive) can be described by **circuit theory** (derived from Maxwell's equations)
- Interaction related to **radiation** requires solving **Maxwell's equations**
- **Maxwell's equations** generally describe all coupling phenomena

Source: S. Reible, https://commons.wikimedia.org/wiki/File:EMI_coupling_modes.svg

MAXWELL'S EQUATIONS FOR NON-STATIONARY PROBLEMS

$$\vec{\nabla} \times \vec{E}(\vec{r},t) = -\frac{\partial \vec{B}(\vec{r},t)}{\partial t}$$

$$\vec{\nabla} \times \vec{H}(\vec{r},t) = \vec{J}(\vec{r},t) + \frac{\partial \vec{D}(\vec{r},t)}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B}(\vec{r},t) = 0$$

$$\vec{\nabla} \cdot \vec{D}(\vec{r},t) = \rho(\vec{r})$$

$$\vec{D}(\vec{r},t) = \epsilon(\vec{r}) \cdot \vec{E}(\vec{r},t)$$

$$\vec{B}(\vec{r},t) = \mu(\vec{r}) \cdot \vec{H}(\vec{r},t)$$

Homogeneous medium: $\epsilon(\vec{r}) = \epsilon_0, \quad \mu(\vec{r}) = \mu_0$

$$\boxed{\begin{gathered}\Delta\vec{E}(\vec{r},t) - \frac{1}{c^2}\frac{\partial^2 \vec{E}(\vec{r},t)}{\partial t^2} = \mu_0 \frac{\partial \vec{J}(\vec{r},t)}{\partial t} \\ c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}\end{gathered}}$$

Wave Equation

(Second order partial differential equation)

SOLVING THE WAVE EQUATION

Numerical solution of the wave equation: $$\Delta\vec{E}(\vec{r},t) - \frac{1}{c^2}\frac{\partial^2\vec{E}(\vec{r},t)}{\partial t^2} = \mu_0 \frac{\partial\vec{J}(\vec{r},t)}{\partial t}$$

1. **Full Wave Methods (Rigorous Solutions of the Wave Equation)**

   Consider all aspects of electromagnetic coupling: voltaic, inductive, capacitive, and radiation

   - Finite Element Method (FEM)
   - Finite Integration Technique (FIT)
   - Transmission Line Matrix Method (TLM)
   - Finite Difference Time Domain Method (FDTD)
   - Integral Equation based methods, e.g. Method of Moments (MOM)

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2. **Approximative Approaches, e.g. Asymptotic Methods**

   Consider some aspects of electromagnetic coupling, e.g. radiation

   - Geometrical Optics (GO)
   - Shooting and Bouncing Rays (SBR)
   - Physical Optics (PO)
   - Uniform Theory of Diffraction (UTD)

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Numerical solution of the wave equation: $\Delta\vec{E}(\vec{r},t) - \frac{1}{c^2}\frac{\partial^2\vec{E}(\vec{r},t)}{\partial t^2} = \mu_0 \frac{\partial\vec{J}(\vec{r},t)}{\partial t}$

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The Finite Element Method

1 **Wave Equation** in Frequency Domain $\Delta\vec{E}(\vec{r},\omega) + \frac{\omega^2}{c^2}\vec{E}(\vec{r},\omega) = \mu_0\, i\omega\, \vec{J}(\vec{r},\omega)$

2 Derive **Weak Form** of the Wave Equation

$$\int_\Omega \nabla\vec{E}(\vec{r},\omega)\cdot\nabla\vec{v}(\vec{r},\omega)\, d\Omega - \int_\Omega \frac{\omega^2}{c^2}\vec{E}(\vec{r},\omega)\cdot\vec{v}(\vec{r},\omega)\, d\Omega = \int_\Omega \mu_0\, i\omega\, \vec{J}(\vec{r},\omega)\cdot\, \vec{v}(\vec{r},\omega)\, d\Omega$$

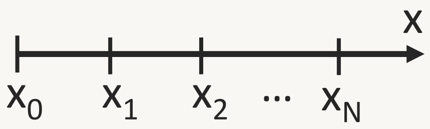
3 Approximate Solution on **mesh** elements via **Basis Functions** and select **Test Functions**

$$\vec{E}(\vec{r},\omega) \approx \sum_{i=0}^{N} e_i\, \vec{\varphi}_i(\vec{r},\omega) \qquad \vec{v}(\vec{r},\omega) \in \{\vec{\varphi}_i(\vec{r},\omega)\}$$

4 **Assemble** Equation System and **Solve** $\left(A - \frac{\omega^2}{c}M\right)e = j$

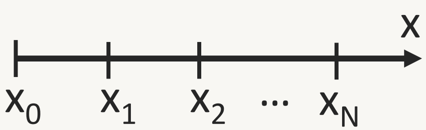
THE FINITE ELEMENT METHOD

1D Discretization

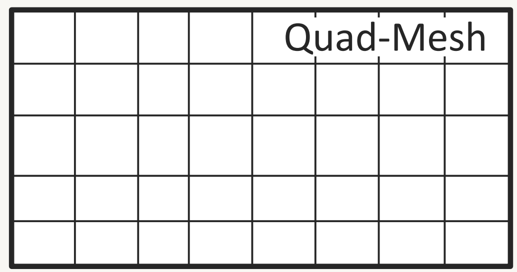


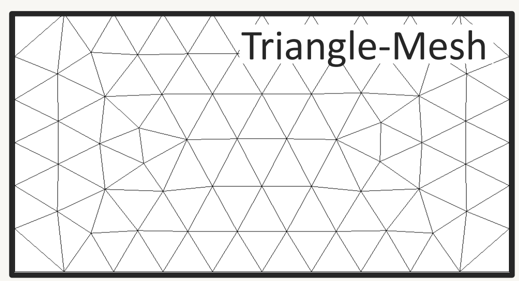
THE FINITE ELEMENT METHOD

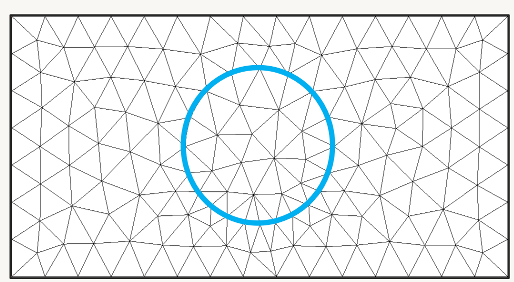
1D Discretization



2D Discretizations







THE FINITE ELEMENT METHOD

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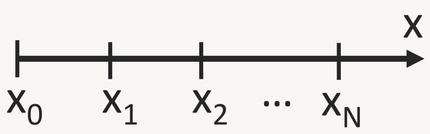
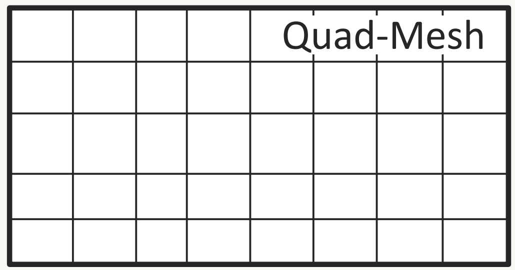
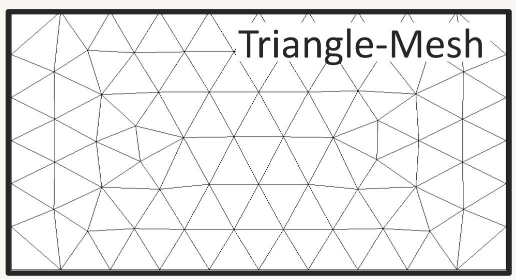
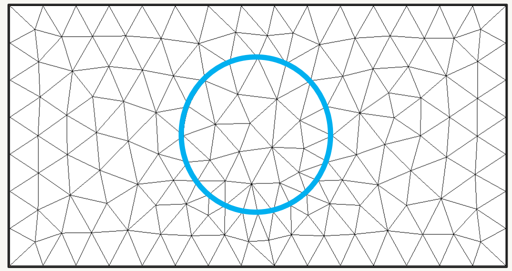


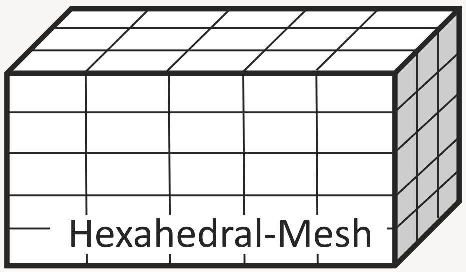
2D Discretizations

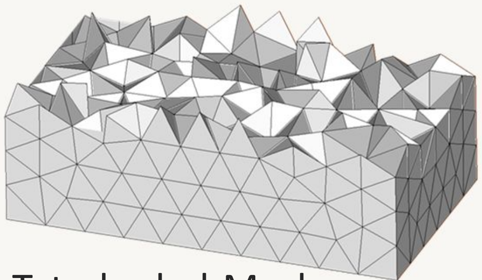




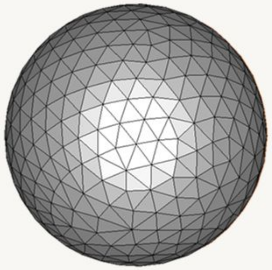


3D Discretizations



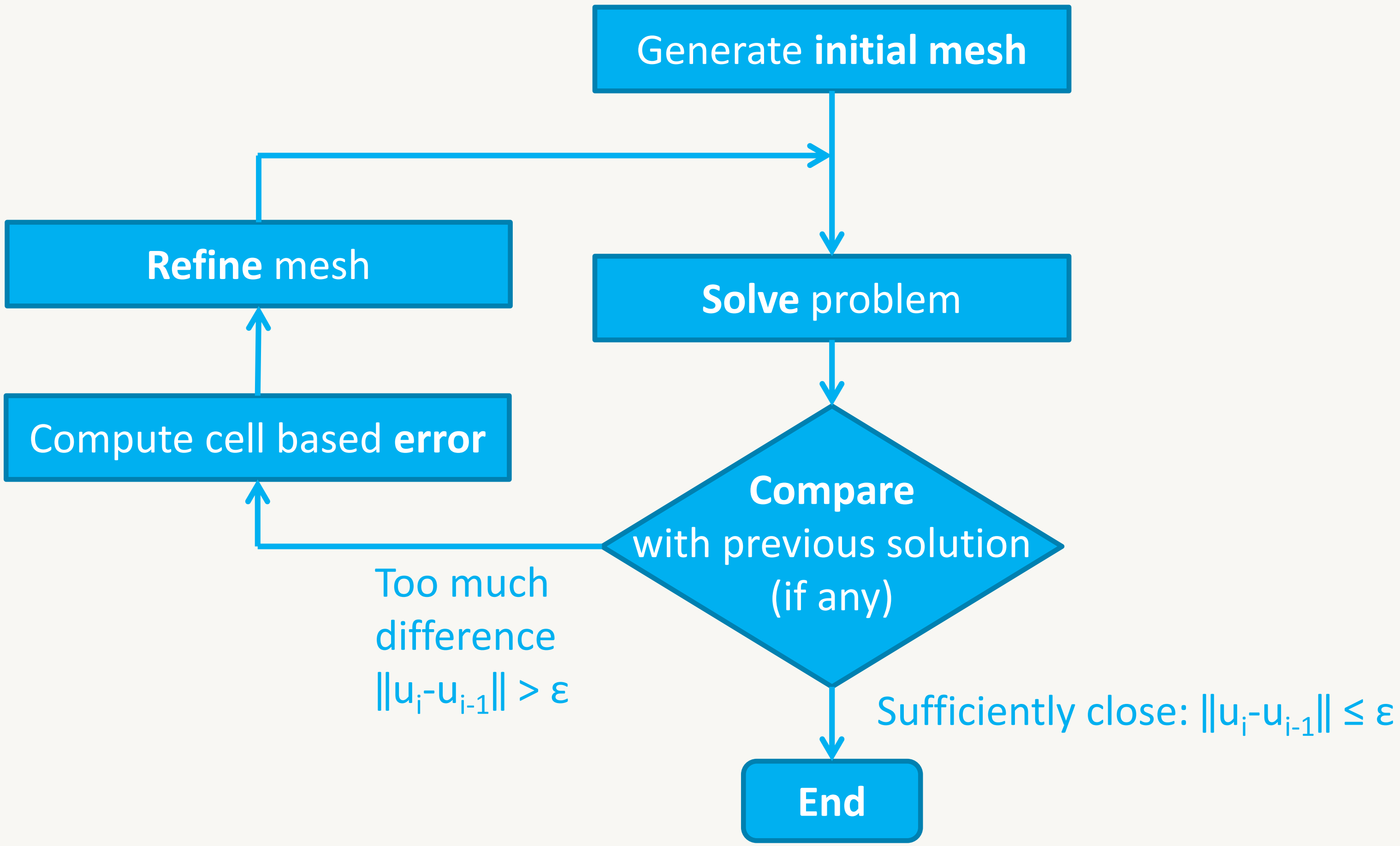


Tetrahedral-Mesh



THE FINITE ELEMENT METHOD

AUTOMATIC MESH ADAPTATION



THE FINITE ELEMENT METHOD

SOLUTION STEPS

1. Derive the **weak form** of the partial differential equation
2. **Discretize** (mesh) the computational domain
3. Select **basis functions** and apply them to the formulation
4. Compute (**assemble**) the matrix entries of the **mass matrix** and the **stiffness matrix**
5. Introduce the **boundary conditions** and set up the **excitation vector**
6. **Solve** the FEM system of equations
7. **Estimate** the cell-based error
8. **Refine** the mesh where needed
9. **Re-run** the scheme until **convergence** is obtained

THE FINITE INTEGRATION TECHNIQUE

Differential Form

$$\vec{\nabla} \times \vec{E}(\vec{r},t) = -\frac{\partial \vec{B}(\vec{r},t)}{\partial t}$$

$$\vec{\nabla} \times \vec{H}(\vec{r},t) = \vec{J}(\vec{r},t) + \frac{\partial \vec{D}(\vec{r},t)}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B}(\vec{r},t) = 0$$

$$\vec{\nabla} \cdot \vec{D}(\vec{r},t) = \rho(\vec{r})$$

$\Leftrightarrow$

Integral Form

$$\oint_{\partial A} \vec{E}(\vec{r},t) \cdot d\vec{s} = -\frac{\partial}{\partial t} \iint_A \vec{B}(\vec{r},t) \cdot d\vec{A}$$

$$\oint_{\partial A} \vec{H}(\vec{r},t) \cdot d\vec{s} = \iint_A \left(\vec{J}(\vec{r},t) + \frac{\partial \vec{D}(\vec{r},t)}{\partial t}\right) \cdot d\vec{A}$$

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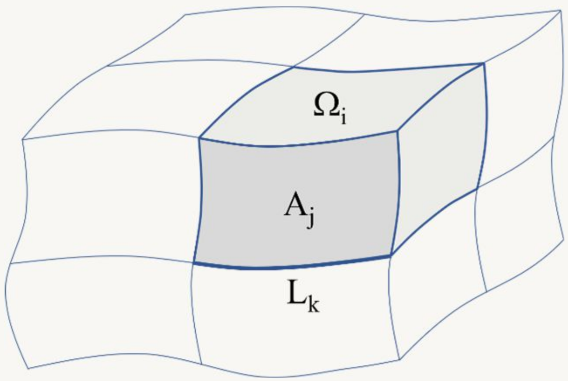
Finite Integration Technique

THE FINITE INTEGRATION TECHNIQUE

1. DECOMPOSITION OF COMPUTATION DOMAIN Ω INTO ARBITRARILY SHAPED SUBDOMAINS Ω_i:



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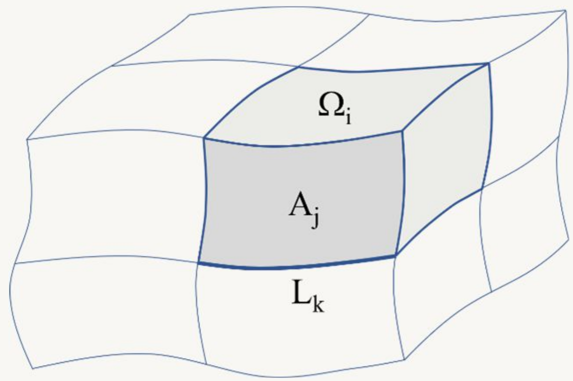

2. DEFINE INTEGRAL QUANTITIES OF ELECTROMAGNETIC FIELDS AS UNKNOWNS (VECTOR COMPONENTS):

$$e_k = \int_{L_k} \vec{E}(\vec{r},t) \cdot d\vec{s}\,, \qquad b_j = \iint_{A_j} \vec{B}(\vec{r},t) \cdot d\vec{A}$$

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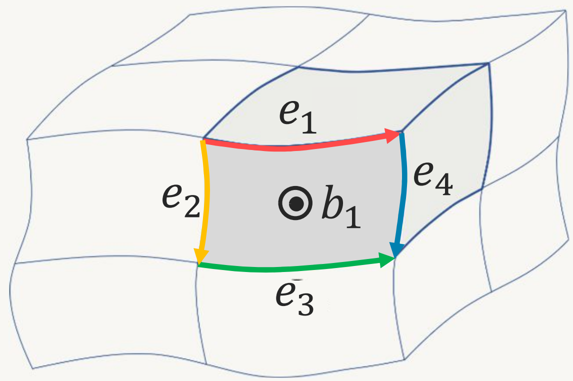

$$-e_1 + e_2 + e_3 - e_4 = -\frac{d}{dt} b_1$$

2. DEFINE INTEGRAL QUANTITIES OF ELECTROMAGNETIC FIELDS AS UNKNOWNS (VECTOR COMPONENTS):

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3. APPLY MAXWELL'S EQUATIONS IN INTEGRAL FORM TO SUBDOMAINS (A_j, Ω_i):

$$\oint_{\partial A_j} \vec{E}(\vec{r},t) \cdot d\vec{s} = -\frac{\partial}{\partial t} \iint_{A_j} \vec{B}(\vec{r},t) \cdot d\vec{A}$$

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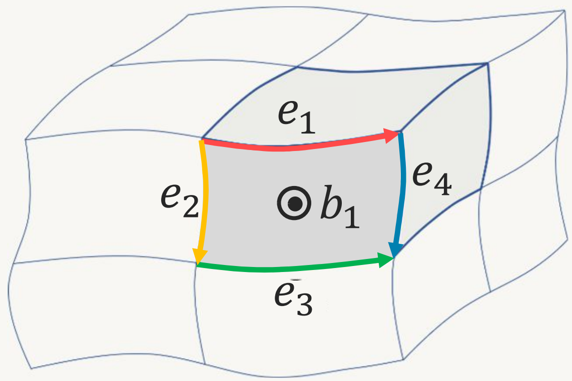

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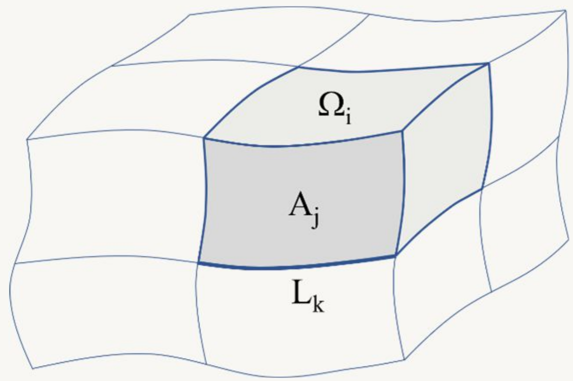
$$\oint_{\partial A_j} \vec{E}(\vec{r},t) \cdot d\vec{s} = -\frac{\partial}{\partial t} \iint_{A_j} \vec{B}(\vec{r},t) \cdot d\vec{A} \Rightarrow \sum_{k \in \partial A_j} c_k\, e_k = -\frac{d}{dt}\, b_j \Rightarrow \boxed{\boldsymbol{C}\,\boldsymbol{e} = -\frac{d}{dt}\,\boldsymbol{b}}$$

THE FINITE INTEGRATION TECHNIQUE

1. DECOMPOSITION OF COMPUTATION DOMAIN Ω INTO ARBITRARILY SHAPED SUBDOMAINS Ω_i:



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$$\oiint_{\partial \Omega_i} \vec{B}(\vec{r},t) \cdot d\vec{A} = 0 \Rightarrow \sum_{k \in \partial \Omega_i} s_k\, b_k = 0 \Rightarrow \boxed{\boldsymbol{S}\,\boldsymbol{b} = \boldsymbol{0}}$$

THE FINITE INTEGRATION TECHNIQUE

INTEGRAL FORM OF MAXWELL'S EQUATIONS

$$\oint_{\partial A} \vec{E}(\vec{r},t) \cdot d\vec{s} = -\frac{\partial}{\partial t} \iint_A \vec{B}(\vec{r},t) \cdot d\vec{A}$$

$$\oint_{\partial A} \vec{H}(\vec{r},t) \cdot d\vec{s} = \iint_A \left(\vec{J}(\vec{r},t) + \frac{\partial \vec{D}(\vec{r},t)}{\partial t} \right) \cdot d\vec{A}$$

$$\oiint_{\partial V} \vec{B}(\vec{r},t) \cdot d\vec{A} = 0$$

$$\oiint_{\partial V} \vec{D}(\vec{r},t) \cdot d\vec{A} = \iiint_V \rho(\vec{r})\, d\vec{r}$$

MAXWELL'S GRID EQUATIONS

$$\boldsymbol{C}\, \boldsymbol{e} = -\frac{d}{dt}\, \boldsymbol{b}$$

$$\widetilde{\boldsymbol{C}}\, \boldsymbol{h} = \boldsymbol{j} + \frac{d}{dt} \boldsymbol{d}$$

$$\boldsymbol{S}\, \boldsymbol{b} = \boldsymbol{0}$$

$$\tilde{\boldsymbol{S}}\, \boldsymbol{d} = \boldsymbol{q}$$

$$\vec{D}(\vec{r},t) = \epsilon(\vec{r}) \cdot \vec{E}(\vec{r},t)$$

$$\vec{B}(\vec{r},t) = \mu(\vec{r}) \cdot \vec{H}(\vec{r},t)$$

$$\vec{J}(\vec{r},t) = \kappa(\vec{r}) \cdot \vec{E}(\vec{r},t) + \vec{J_0}(\vec{r},t)$$

$$\boldsymbol{d} = \boldsymbol{D}_\epsilon\, \boldsymbol{e}$$

$$\boldsymbol{b} = \boldsymbol{D}_\mu\, \boldsymbol{h}$$

$$\boldsymbol{j} = \boldsymbol{D}_\kappa\, \boldsymbol{e} + \boldsymbol{j}_0$$

THE FINITE INTEGRATION TECHNIQUE

EXPLICIT TRANSIENT SCHEME

$$\boldsymbol{C}\,\boldsymbol{e} = -\frac{d}{dt}\,\boldsymbol{b} \quad \overset{\boldsymbol{b} = \boldsymbol{D}_\mu\,\boldsymbol{h}}{\Rightarrow} \quad \boxed{\boldsymbol{C}\,\boldsymbol{e} = -\boldsymbol{D}_\mu \frac{d\boldsymbol{h}}{dt}}$$

$$\widetilde{\boldsymbol{C}}\,\boldsymbol{h} = \frac{d}{dt}\boldsymbol{d} \quad \overset{\boldsymbol{d} = \boldsymbol{D}_\epsilon\,\boldsymbol{e}}{\Rightarrow} \quad \boxed{\widetilde{\boldsymbol{C}}\,\boldsymbol{h} = \boldsymbol{D}_\epsilon \frac{d\boldsymbol{e}}{dt}}$$

THE FINITE INTEGRATION TECHNIQUE

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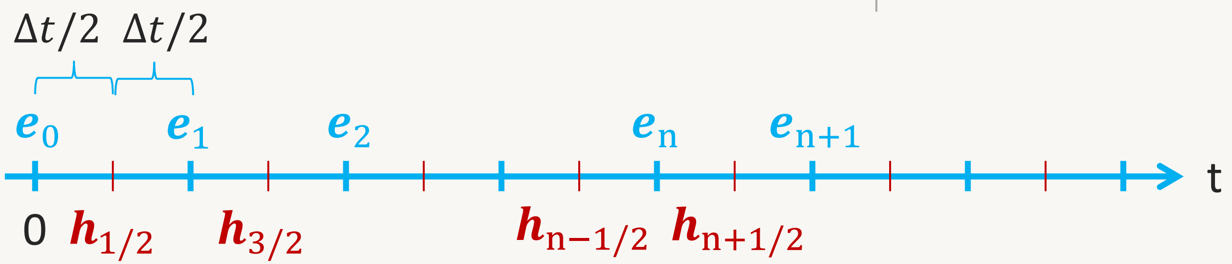


$$\boldsymbol{e}_{\mathrm{n}} = e(t = n\,\Delta t)$$

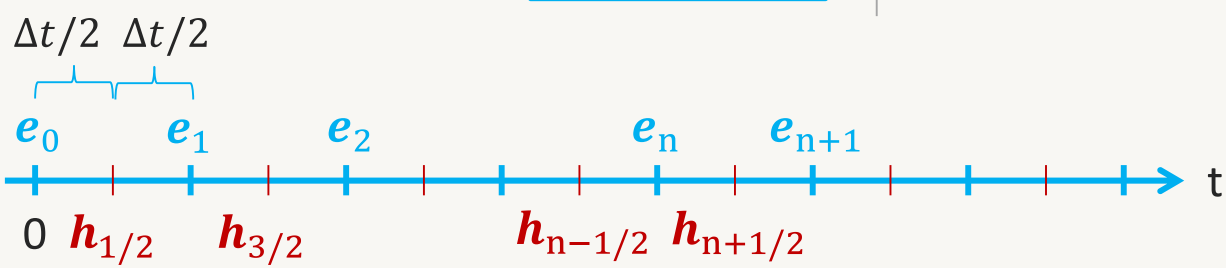
$$\boldsymbol{h}_{\mathrm{m}} = h(t = m\,\Delta t)$$

THE FINITE INTEGRATION TECHNIQUE

EXPLICIT TRANSIENT SCHEME

$$\boldsymbol{C}\,\boldsymbol{e} = -\frac{d}{dt}\boldsymbol{b} \quad \overset{\boldsymbol{b} = \boldsymbol{D}_\mu \boldsymbol{h}}{\Rightarrow} \quad \boxed{\boldsymbol{C}\,\boldsymbol{e} = -\boldsymbol{D}_\mu \frac{d\boldsymbol{h}}{dt}}$$

$$\widetilde{\boldsymbol{C}}\,\boldsymbol{h} = \frac{d}{dt}\boldsymbol{d} \quad \overset{\boldsymbol{d} = \boldsymbol{D}_\epsilon \boldsymbol{e}}{\Rightarrow} \quad \boxed{\widetilde{\boldsymbol{C}}\,\boldsymbol{h} = \boldsymbol{D}_\epsilon \frac{d\boldsymbol{e}}{dt}}$$



$$\boldsymbol{e}_{\mathrm{n}} = e(t = n\,\Delta t)$$

$$\boldsymbol{h}_{\mathrm{m}} = h(t = m\,\Delta t)$$

LEAPFROG TIME INTEGRATION

$$\boldsymbol{C}\,\boldsymbol{e}_n = -\boldsymbol{D}_\mu \left.\frac{d\boldsymbol{h}}{dt}\right|_{t=n\Delta t} \approx -\boldsymbol{D}_\mu \frac{\boldsymbol{h}_{n+1/2} - \boldsymbol{h}_{n-1/2}}{\Delta t} \Rightarrow \boldsymbol{h}_{n+1/2} = \boldsymbol{h}_{n-1/2} - \Delta t\,\mathbf{D}_\mu^{-1}\,\mathbf{C}\,\boldsymbol{e}_n$$

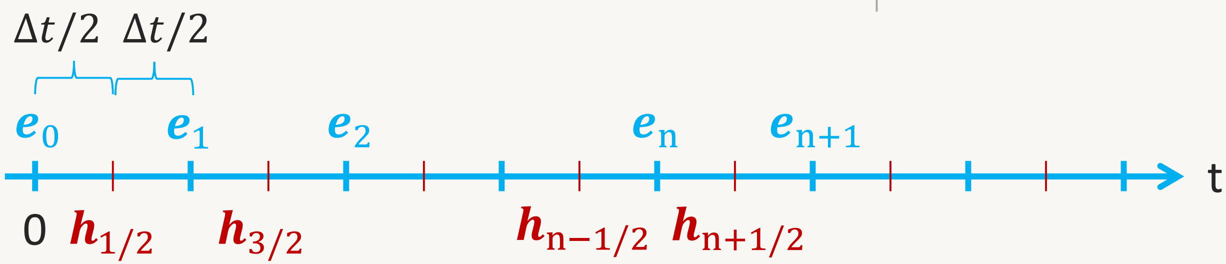
$$\widetilde{\boldsymbol{C}}\,\boldsymbol{h}_{n+1/2} = \boldsymbol{D}_\epsilon \left.\frac{d\boldsymbol{e}}{dt}\right|_{t=(n+1/2)\Delta t} \approx \boldsymbol{D}_\epsilon \frac{\boldsymbol{e}_{n+1} - \boldsymbol{e}_n}{\Delta t} \Rightarrow \boldsymbol{e}_{n+1} = \boldsymbol{e}_n + \Delta t\,\mathbf{D}_\epsilon^{-1}\,\widetilde{\boldsymbol{C}}\,\boldsymbol{h}_{n+1/2}$$

THE FINITE INTEGRATION TECHNIQUE

EXPLICIT TRANSIENT SCHEME

$$\boldsymbol{C}\,\boldsymbol{e} = -\frac{d}{dt}\,\boldsymbol{b} \quad \overset{\boldsymbol{b} = \boldsymbol{D}_\mu \boldsymbol{h}}{\Rightarrow} \quad \boxed{\boldsymbol{C}\,\boldsymbol{e} = -\boldsymbol{D}_\mu \frac{d\boldsymbol{h}}{dt}}$$

$$\widetilde{\boldsymbol{C}}\,\boldsymbol{h} = \frac{d}{dt}\boldsymbol{d} \quad \overset{\boldsymbol{d} = \boldsymbol{D}_\epsilon \boldsymbol{e}}{\Rightarrow} \quad \boxed{\widetilde{\boldsymbol{C}}\,\boldsymbol{h} = \boldsymbol{D}_\epsilon \frac{d\boldsymbol{e}}{dt}}$$



$$\boldsymbol{e}_\mathrm{n} = e(t = n\,\Delta t)$$

$$\boldsymbol{h}_\mathrm{m} = h(t = m\,\Delta t)$$

LEAPFROG TIME INTEGRATION

$$\boldsymbol{C}\,\boldsymbol{e}_n = -\boldsymbol{D}_\mu \left.\frac{d\boldsymbol{h}}{dt}\right|_{t=n\Delta t} \approx -\boldsymbol{D}_\mu \frac{\boldsymbol{h}_{n+1/2} - \boldsymbol{h}_{n-1/2}}{\Delta t} \Rightarrow \boldsymbol{h}_{n+1/2} = \boldsymbol{h}_{n-1/2} - \Delta t\,\mathbf{D}_\mu^{-1}\,\mathbf{C}\,\boldsymbol{e}_n$$

$$\widetilde{\boldsymbol{C}}\,\boldsymbol{h}_{n+1/2} = \boldsymbol{D}_\epsilon \left.\frac{d\boldsymbol{e}}{dt}\right|_{t=(n+1/2)\Delta t} \approx \boldsymbol{D}_\epsilon \frac{\boldsymbol{e}_{n+1} - \boldsymbol{e}_n}{\Delta t} \Rightarrow \boldsymbol{e}_{n+1} = \boldsymbol{e}_n + \Delta t\,\mathbf{D}_\epsilon^{-1}\,\widetilde{\boldsymbol{C}}\,\boldsymbol{h}_{n+1/2}$$

STABILITY

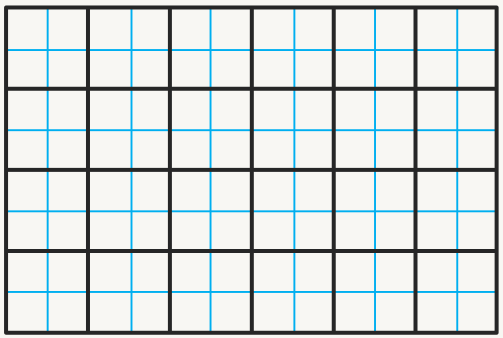
$$\boxed{\Delta t \le \frac{\Delta s}{K}}$$

Courant-Friedrichs-Levy stability criterion
(Δs is the mesh step width, K a constant)

THE FINITE INTEGRATION TECHNIQUE

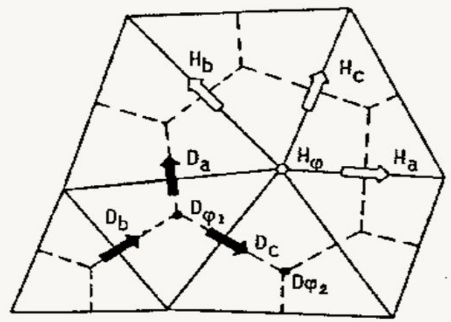
POTENTIAL MESH TYPES

Cartesian

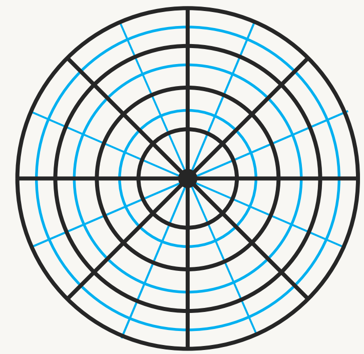


mesh, dual mesh

Triangular

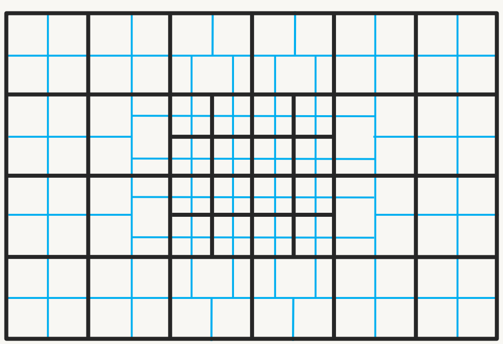


Cylindrical

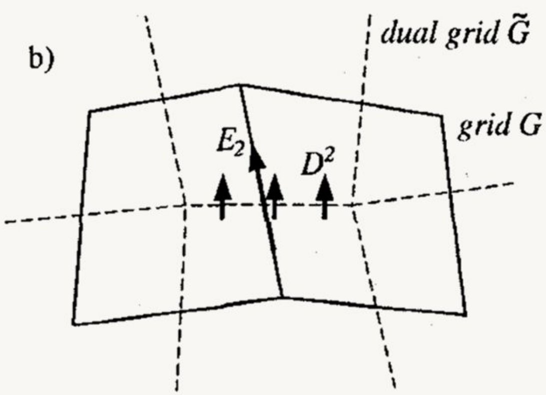


mesh, dual mesh

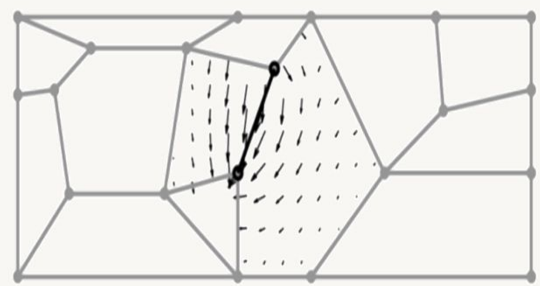
Subgrids



Non-orthogonal



Polygonal



THE FINITE INTEGRATION TECHNIQUE

WHY CARTESIAN MESHES?

Often complex (and faulty) geometric models for practical applications

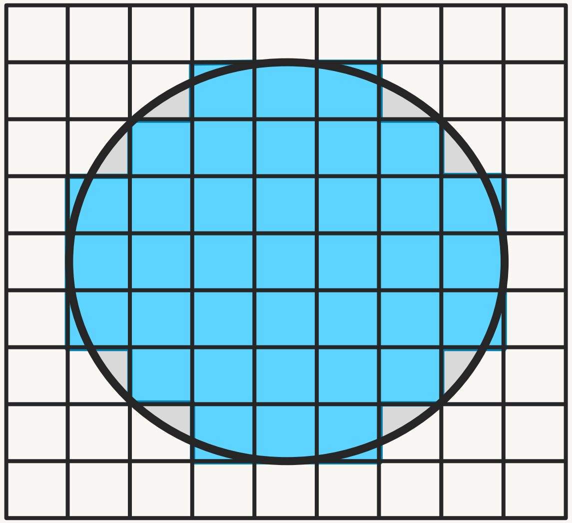
Source: aip-automotive.de

Due to **robust mesh generation** and **low memory requirements** for storage, **cartesian meshes** are still the method of choice for many **high frequency applications**.

THE FINITE INTEGRATION TECHNIQUE

REPRESENTATION OF GEOMETRIC BOUNDARIES IN CARTESIAN MESHES

Stairstep Approximation



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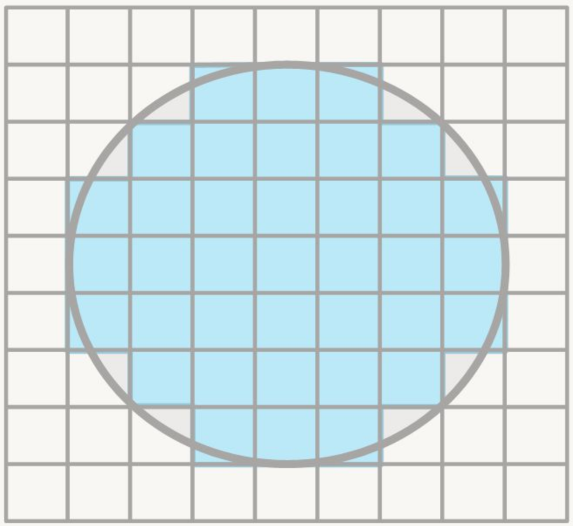
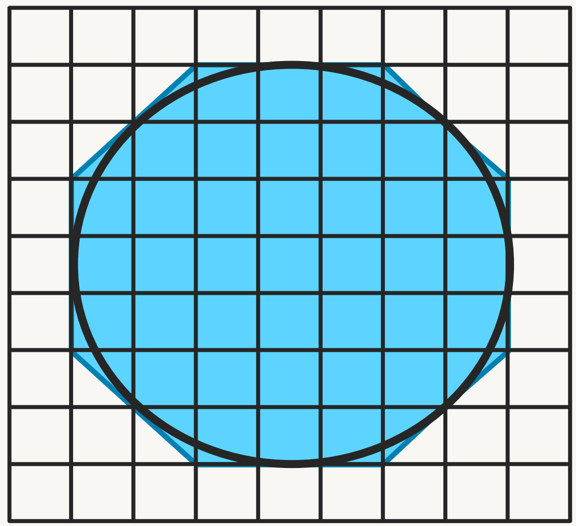


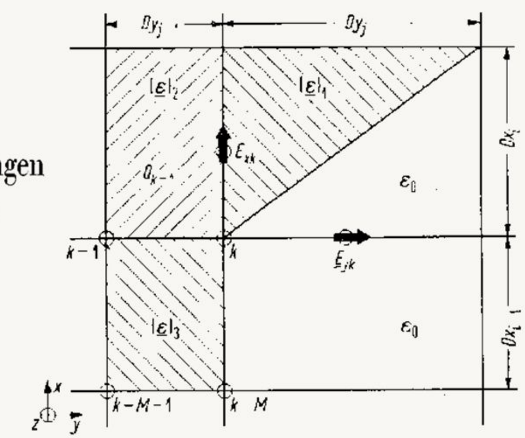
Prismatic Sub-fillings



1977

Eine Methode zur Lösung der Maxwellschen Gleichungen für sechskomponentige Felder auf diskreter Basis

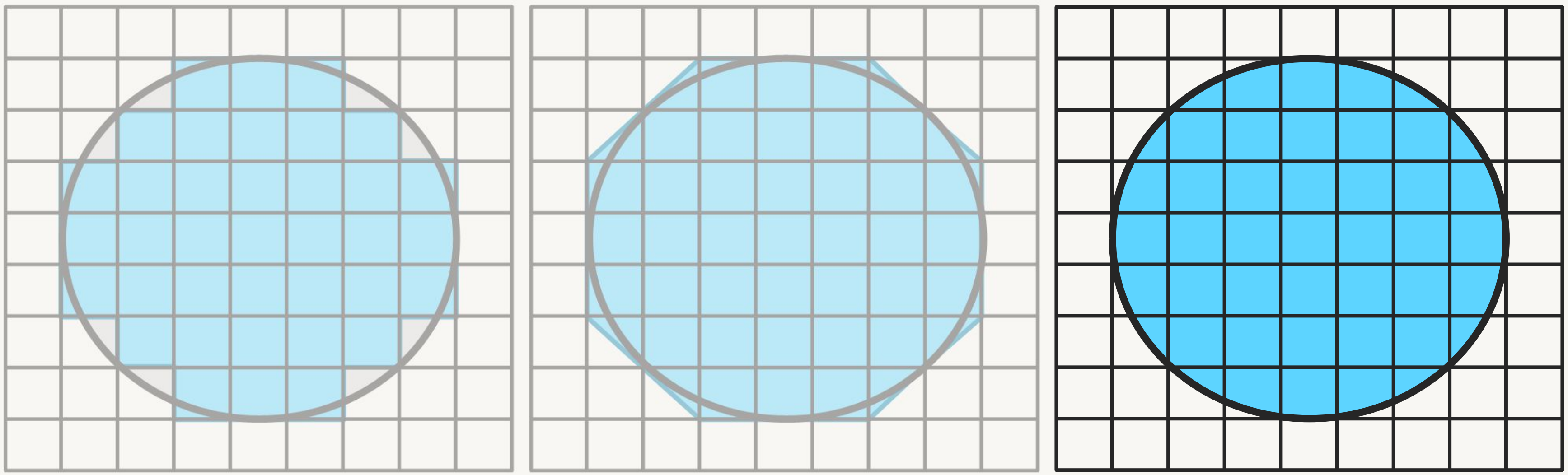
von Thomas Weiland*

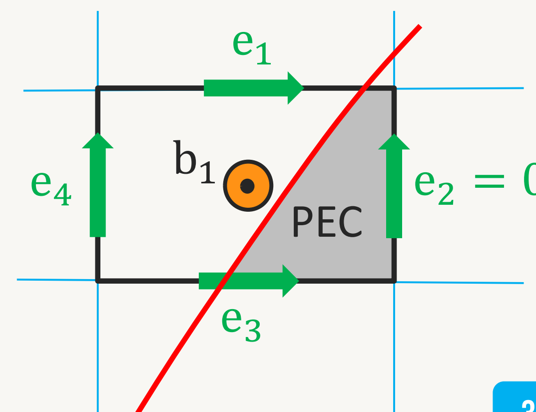


THE FINITE INTEGRATION TECHNIQUE

REPRESENTATION OF GEOMETRIC BOUNDARIES IN CARTESIAN MESHES

Stairstep Approximation | Prismatic Sub-fillings | **Generalized Sub-fillings (PBA)**





THE FINITE INTEGRATION TECHNIQUE

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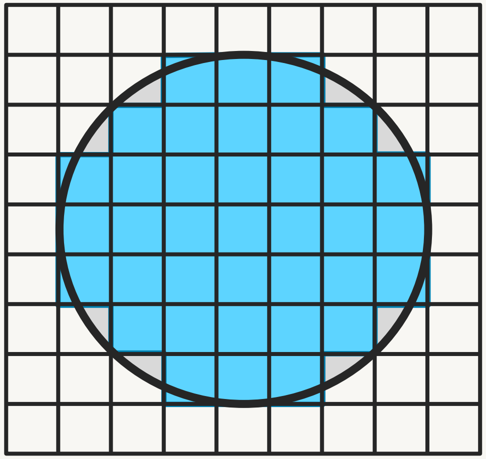
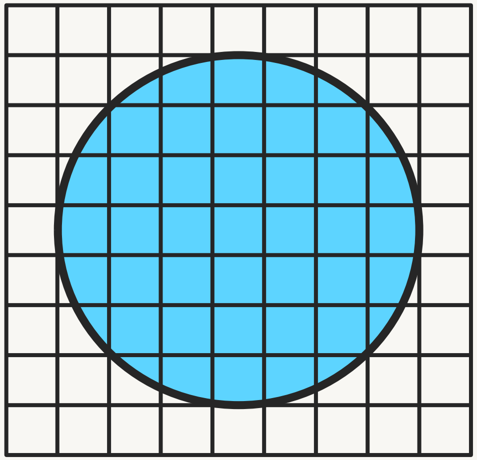
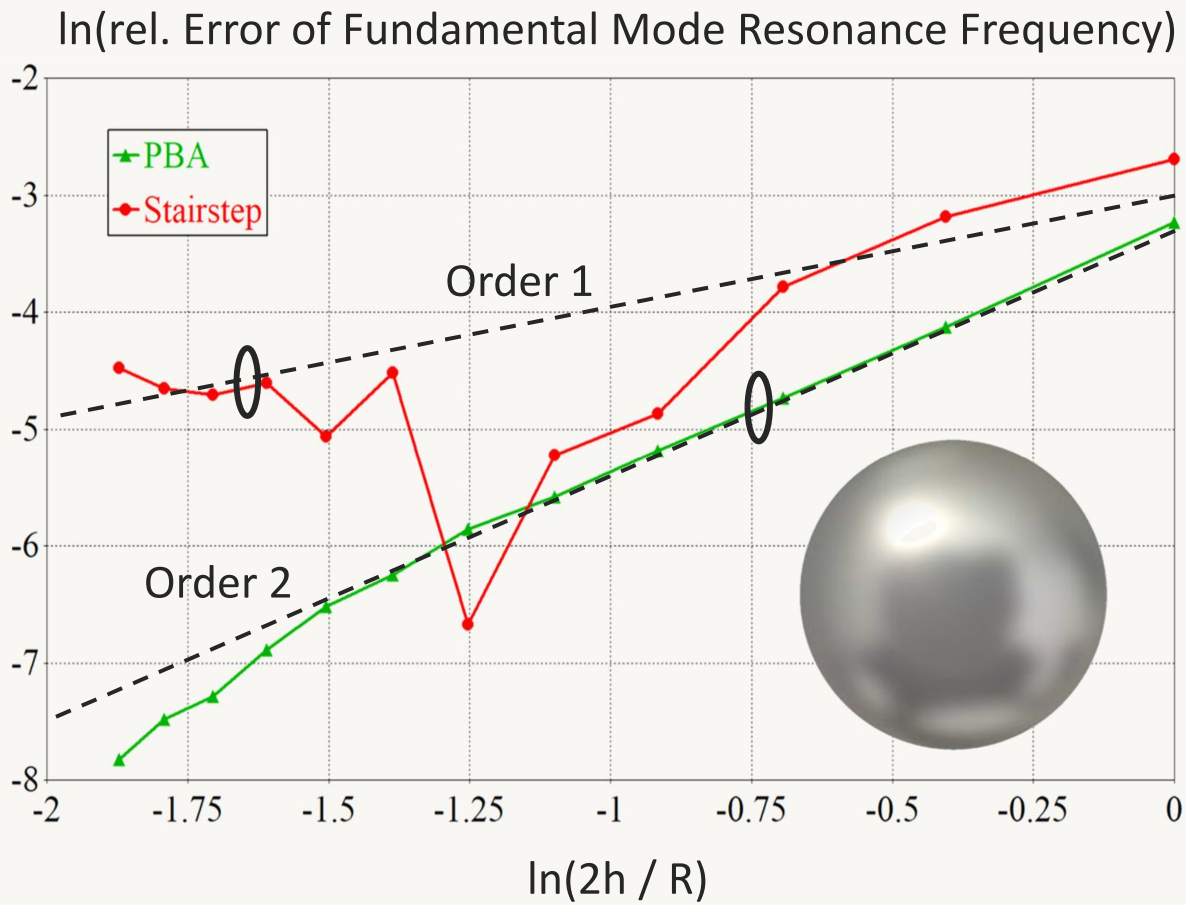
Stairstep Approximation



Generalized Sub-fillings (PBA)





OBTAINING **BROADBAND RESULTS**

TD

- Transient schemes obtain time signal responses for the entire excitation spectrum from **one simulation**
- Apply **Fast Fourier Transform** to obtain broadband frequency response

OBTAINING BROADBAND RESULTS

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FD

Frequency Domain schemes solve an **equation system** for each frequency, e.g.:

$$\left(A - \frac{\omega^2}{c} M\right) e = j$$

Many frequency samples (solutions) are required to obtain broadband results (especially in case of many resonances)

Reduced Order Modeling can help to get broadband responses faster:

Large FEM system → Small reduced system → Frequency sweep on small system

3-4 times effort of single FEM solution

Very fast sweep (small system)

MODELING DISPERSIVE MATERIALS

REQUIREMENTS FOR ACCURATE MATERIAL MODELING IN TRANSIENT SIMULATIONS

- Transient simulations obtain a broadband system response from a single computation
- Therefore, material models need to be accurate for a broad range of frequencies
- Dielectric and permeable material properties are typically frequency dependent

⇨ **Specific modeling required to handle frequency dependency in transient method**

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COMMONLY USED MODELS FOR MODELING MATERIALS

1. **Debeye Model** (most dielectric materials at microwave frequencies)

$$\epsilon(\omega) = \epsilon_\infty + \sum_{k=1}^{N} \frac{\Delta\epsilon_k}{1 + i\omega\tau_{ki}}$$

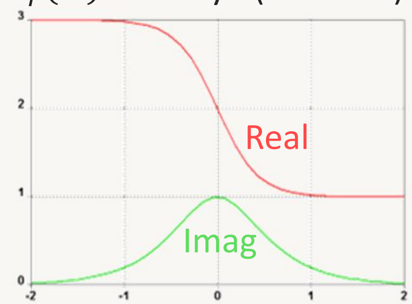
2. **Lorentz Model** (modeling resonance, often needed for higher frequencies)

$$\epsilon(\omega) = \epsilon_\infty + \sum_{k=1}^{N} \frac{\Delta\epsilon_k \omega_k^2}{\omega_k^2 + 2i\omega\delta_k - \omega^2}$$

3. **Drude Model** (often used for optical frequencies or cold plasma)

$$\epsilon(\omega) = \epsilon_\infty - \sum_{k=1}^{N} \frac{\omega_k^2}{\omega^2 - i\omega\gamma_k}$$

$\epsilon_r(\omega)$ - Debeye (1st order)



$\log(\omega\tau_1)$

$\epsilon_r(\omega)$ - Lorentz (1st order)

ω/ω_1

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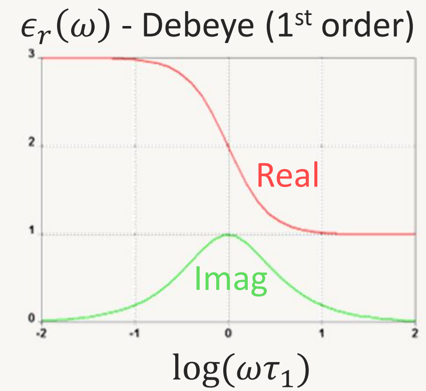

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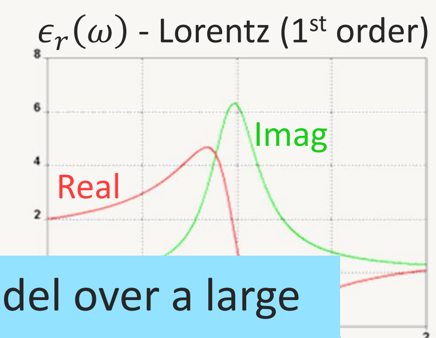


3. **Drude Model** (often used for optical frequencies or cold plasma)

Real materials can often not be modeled accurately by a single model over a large frequency range ⇨ combinations of models, e.g. Drude-Lorentz

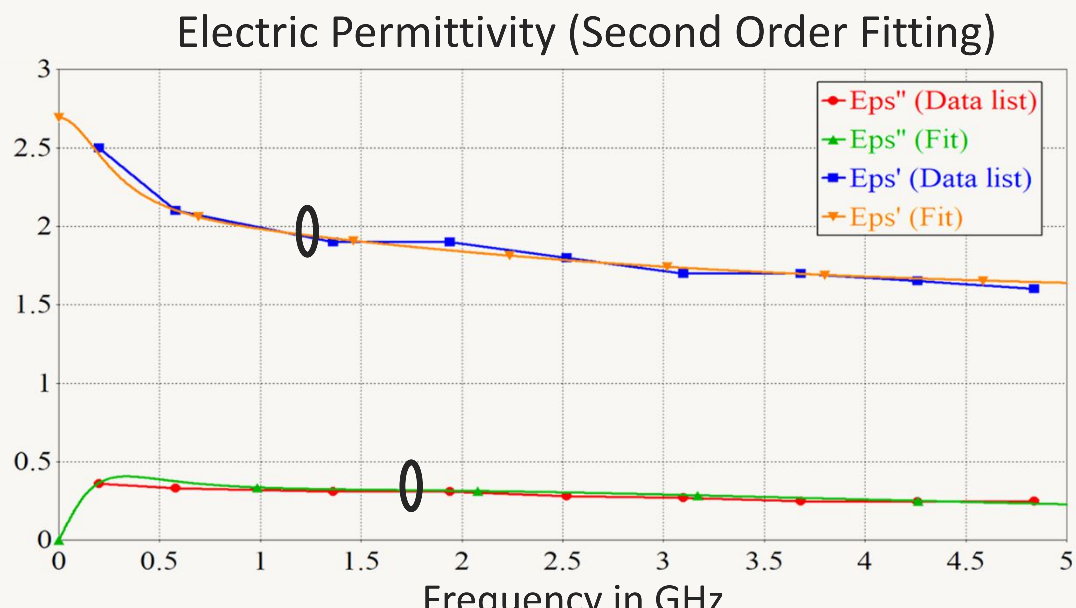
MODELING DISPERSIVE MATERIALS

GENERALIZED HIGHER ORDER MATERIAL MODEL

Instead of using the physically motivated Debeye, Drude or Lorentz models, a generalized **mathematical model** can be used:

$$\epsilon(\omega) = \epsilon_\infty + \sum_{n=1}^{N} \frac{\beta_{0,n}}{\alpha_{0,n} + i\omega} + \sum_{n=1}^{M} \frac{\gamma_{0,n} + i\omega\gamma_{1,n}}{\delta_{0,n} + i\omega\delta_{1,n} - \omega^2} = \frac{a_m\omega^m + a_{m-1}\omega^{m-1} + \cdots + a_1\omega + a_0}{b_n\omega^n + b_{n-1}\omega^{n-1} + \cdots + b_1\omega + b_0}$$

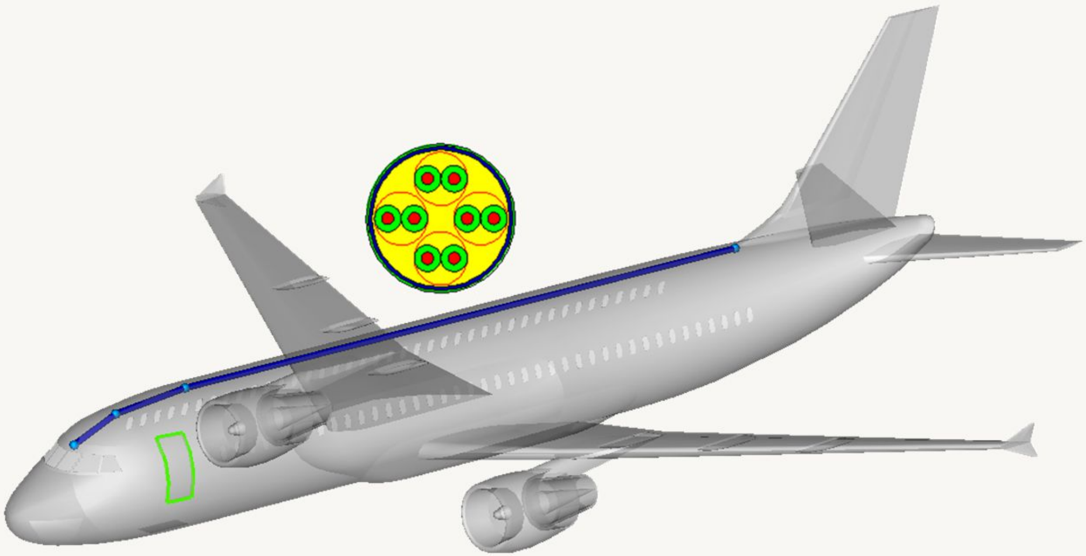
Parameters are obtained by **fitting** to the given material dispersion curves. Fitting needs to ensure **causality** and **stability** of the model.

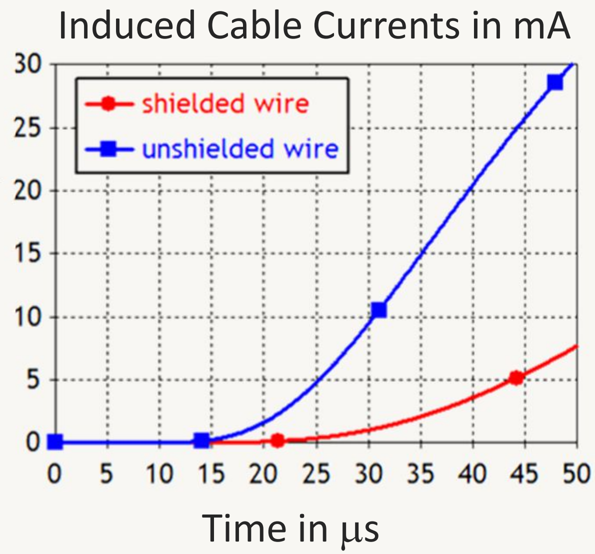


MULTISCALE CABLE MODELING

Enhancements of the basic scheme (e.g. PBA, TST) and maintaining the **broadband capabilities** (e.g. material properties) yield **performance improvements** over basic scheme by orders of magnitude.

Other example (multiscale problem): **Cable in airplane** (mm vs. tens of m)

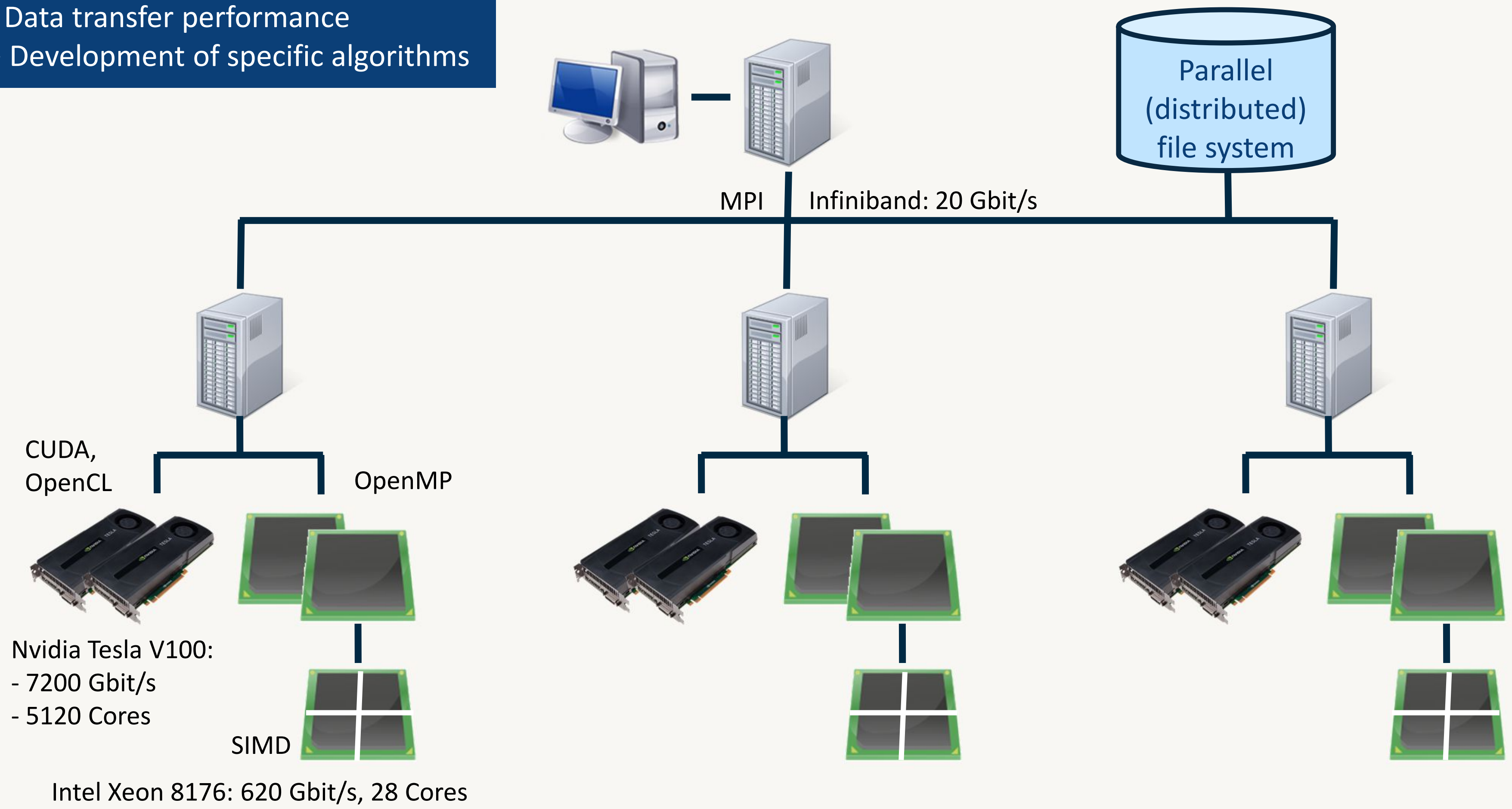




- Cable is modeled by a **multimodal transmission line** model
- Fields around the cable are simulated by **FIT or TLM** method
- Common mode **bi-directional field-cable coupling** through tangential magnetic fields and currents for every time step

HIGH PERFORMANCE COMPUTING OPTIONS

- Distribution of data / load balancing
- Data transfer performance
⇨ Development of specific algorithms



GP-GPUs provide good performance / cost ratio for EMC type simulations.

NUMERICAL SIMULATION FOR EMC

- Often **complex geometries** and **broadband responses** with many resonances
- **FIT** provides **robust meshing** capabilities, **FEM might need simplifications**
- **Time Domain** methods are well suited to obtain **broadband responses**
- **Explicit Time Domain** methods become less efficient at **lower frequencies** due to stability criterion ⇨ **Frequency Domain** methods (usually FEM)
- **Integral Equation** based techniques are less able to deal with complex geometries
- **Asymptotic** techniques shall be considered as „**add on**“

SUMMARY – 3 KEY TAKE-AWAYS

1. **EMC** involves **multiple coupling mechanisms** (inductive, capacitive, radiation) which can be treated by a variety of different numerical schemes

2. **Full wave** schemes provide the **highest level of accuracy** since they consider all possible couplings

3. The choice of the **best suited numerical** scheme depends on the **frequency range**
   - **Low frequencies** (LF): parasitic extraction provides seamless connection to circuit simulation
   - **High frequencies** (HF): full wave methods should be preferred
   - **Finite Integration Technique** (FIT) and **Finite Element method** are optimal choices for most HF problems while the FIT meshing is more robust for complex geometries

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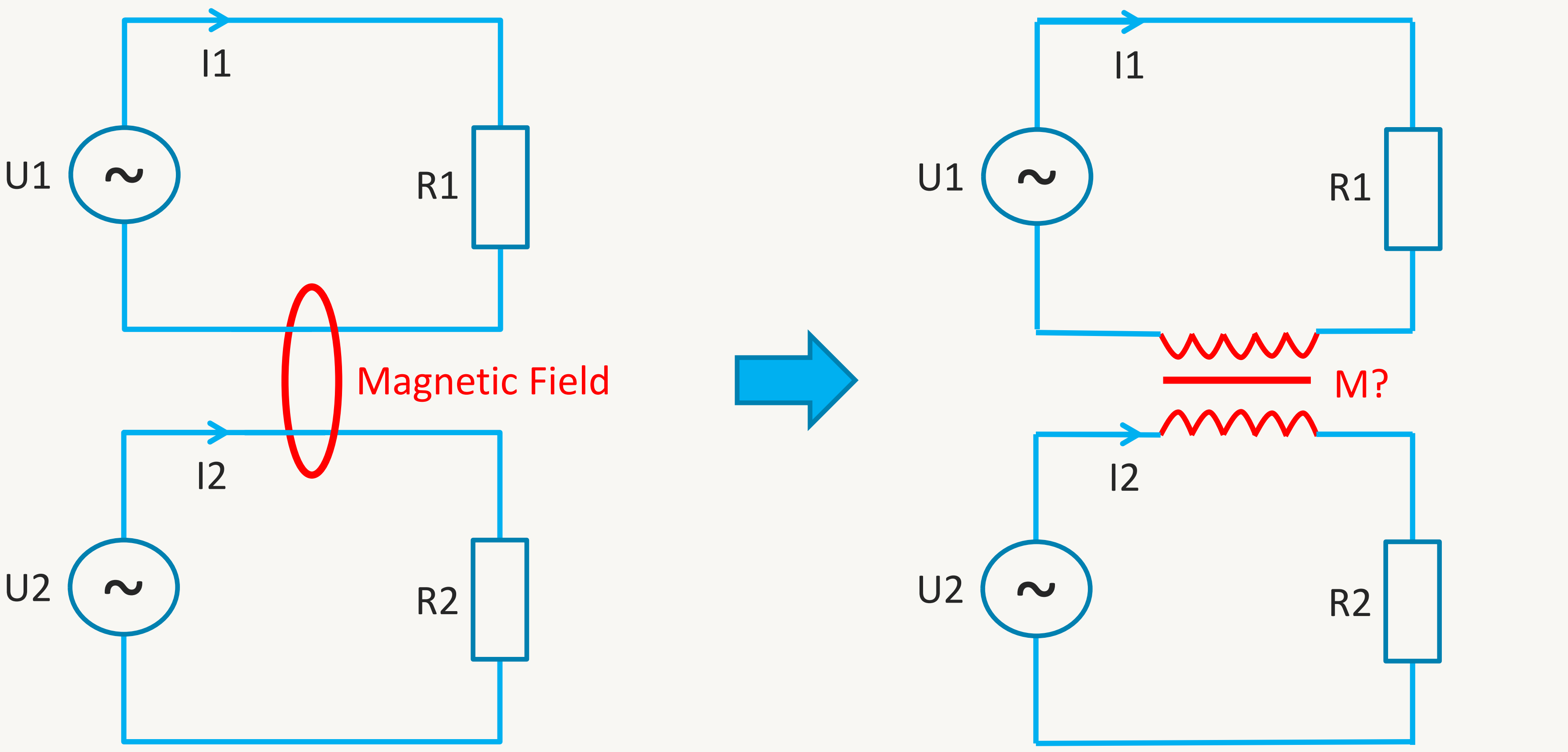
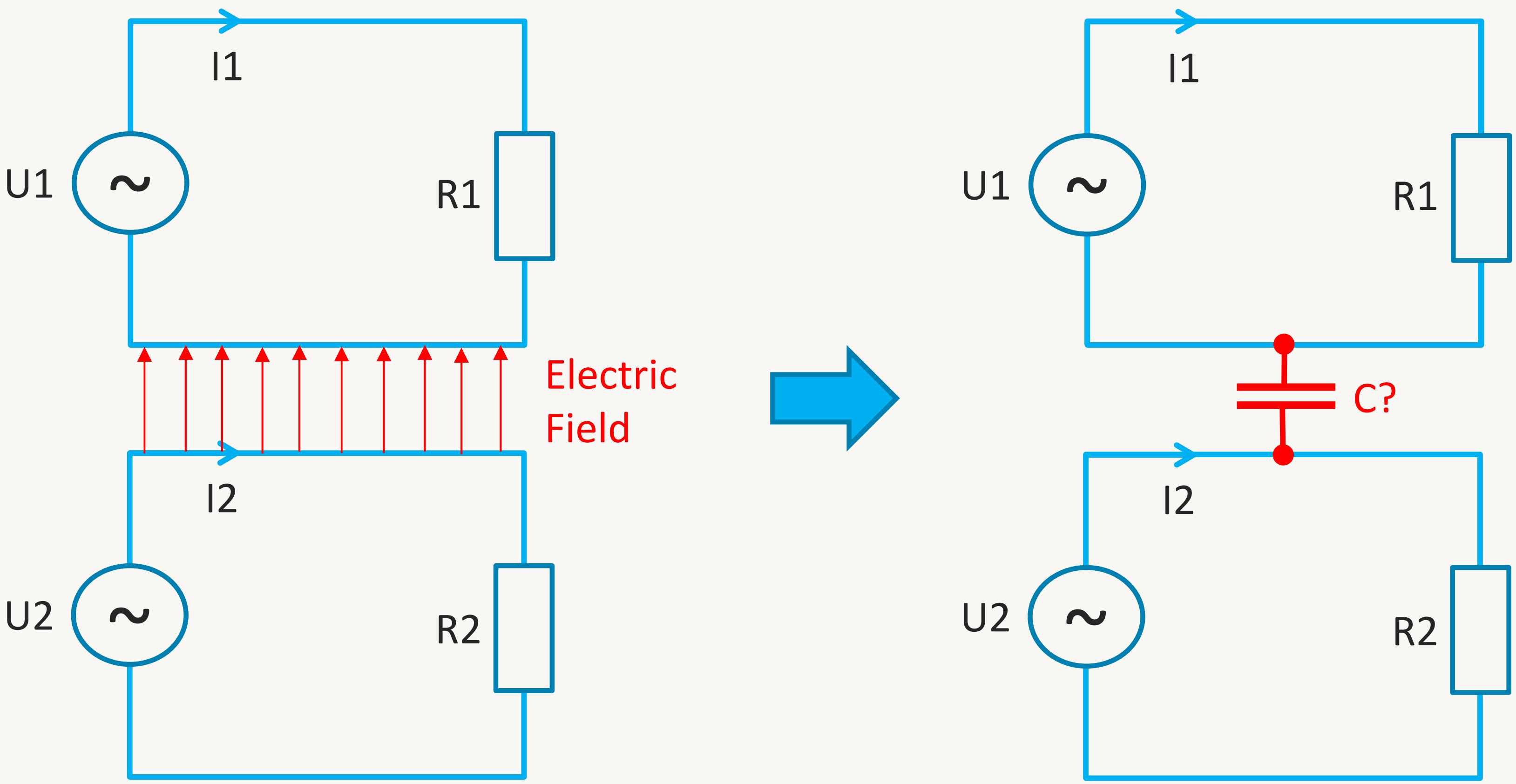
THANK YOU!

INDUCTIVE COUPLING



Self- and mutual coupling inductances between all branches of the network need to be determined and can then be considered in the circuit simulation.

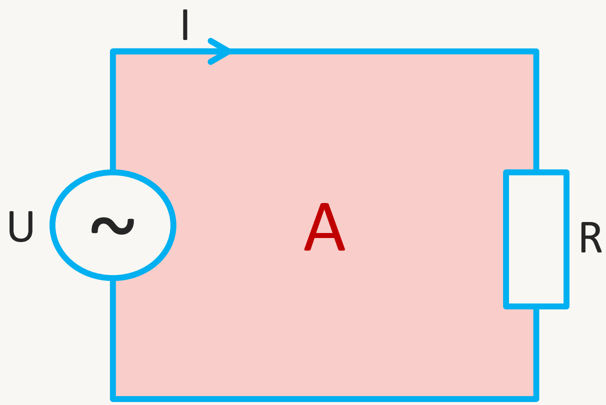
CAPACITIVE COUPLING



Coupling capacitances between all branches of the network need to be determined and can then be considered in the circuit simulation.

CONCEPT OF PARTIAL INDUCTANCES

The inductance L is a quantity of a **current loop**:

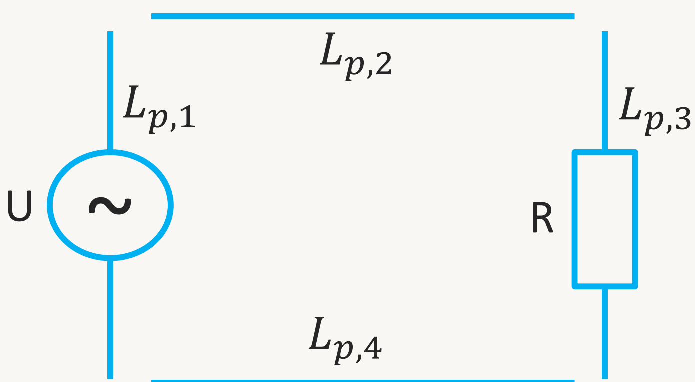


$$L = \frac{\iint_A \vec{B}(\vec{r}, t) \cdot d\vec{A}}{I}$$

The inductance is directly influenced by the loop area.

Note: Individual current segments do not have an inductance (no closed loop).

We can introduce the mathematical concept of **partial inductances** L_p. The inductance of a loop is given by **adding up** the partial inductances of its segments:



$$L = L_{p,1} + L_{p,2} + L_{p,3} + L_{p,4}$$

The same concept holds for **partial capacitances**, **partial mutual inductances**, and **partial coupling capacitances**, so called **parasitics**.

NUMERICAL METHODS TO DETERMINE PARASITICS

The **numerical solution** needs to determine the partial mutual inductances and coupling capacitances for individual branches of the network.

1. **Partial Element Equivalent Circuit Method (PEEC) / Method of Moments**
   - Based on Integral Equation solution of Maxwell's equations (usually stationary, neglecting displacement current)
   - Compute partial inductances and capacitances
     ⇨ well suited for combination with circuit simulation
   - Most efficient for stationary problems at low frequencies (but can be extended towards higher frequencies, e.g. „retarded PEEC“)

2. **Extraction from a full solution of Maxwell‘s equations**
   - More accurate and more efficient than PEEC at higher frequencies
   - Naturally compute physical inductances and capacitances, but not the contributions of individual branches (partial values)
   - Recent developments support the calculation of partial values as well

INTEGRAL EQUATION BASED METHODS

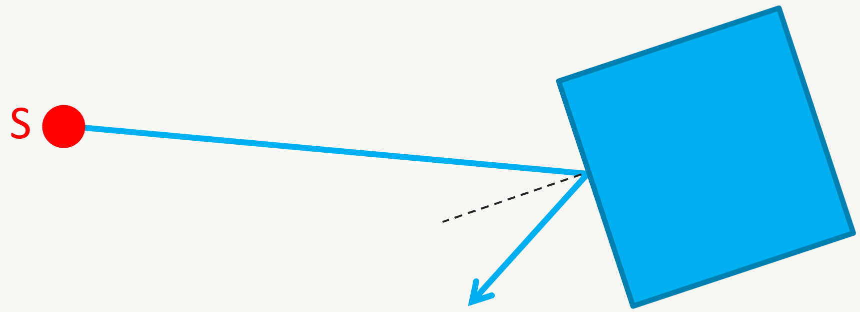
- Based on the **integral form** of Maxwell's equations (usually) in Frequency Domain
- All **material boundaries** (objects) are **discretized** and unknown electric and magnetic **currents** are introduced at the boundaries
- Field is given by a superposition of solutions of Maxwell's equations for homogeneous problems (e.g. the **Greens functions**) driven by the surface currents:

$$\vec{E}(\vec{r},\omega) = \vec{E}^{inc}(\vec{r},\omega) + \iiint_V \bar{\bar{G}}_{EJ}(\vec{r},\vec{r}',\omega)\,\vec{J}(\vec{r}',\omega)\,d\vec{r}' + \iiint_V \bar{\bar{G}}_{EM}(\vec{r},\vec{r}',\omega)\,\vec{M}(\vec{r}',\omega)\,d\vec{r}'$$

- Currents on the surface are described in terms of **basis functions**
- Fields need to satisfy the **boundary conditions** (e.g. tangential electric field vanishes at perfect electric conductors, continuity of field components across a material interfaces, etc.)
- Applying a **Galerkin** approach yields an **equation system** which needs to be solved for each frequency point
- The equation system is **dense** ⇨ the solution is **slow for large problems** (high frequencies, complex geometries with many boundaries)
- Solution can be accelerated by using **"fast" techniques**, e.g. FMM or MLFMM

ASYMPTOTIC METHODS

- **Geometrical optics** (GO) assumes point sources and models wave propagation by rays launched from the sources



- Rays are traced through the computation domain including **reflections** at objects
- Rays carry information about **field strength and phase**
- Field strength at an observer is given by the **superposition** of all rays hitting it
- Due to the approximations, the method is **only valid at very high frequencies**

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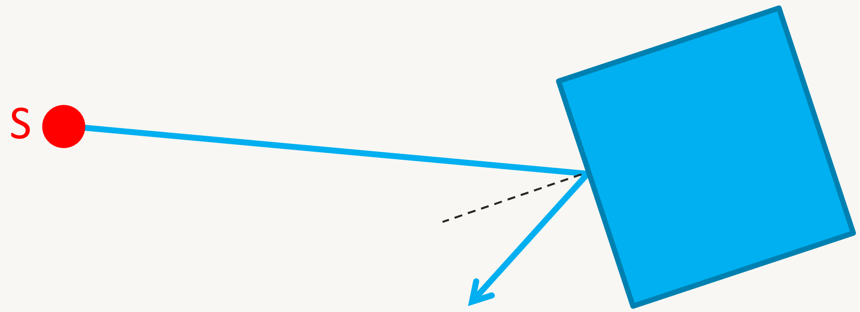

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- Rays carry information about **field strength and phase**
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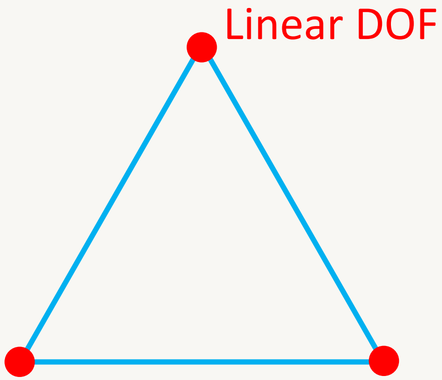
- **Physical optics** (PO) provides more accurate modeling including interference and diffusion at edges - but is still **not a rigorous full wave method**
- In Physical optics the **rays illuminate** the object surfaces resulting in surface currents. The fields are derived from the currents by using **Greens functions**.
- Objects are typically modeled by **surface triangles** (either flat or curved)

THE FINITE ELEMENT METHOD

HIGHER ORDER METHODS



THE FINITE ELEMENT METHOD

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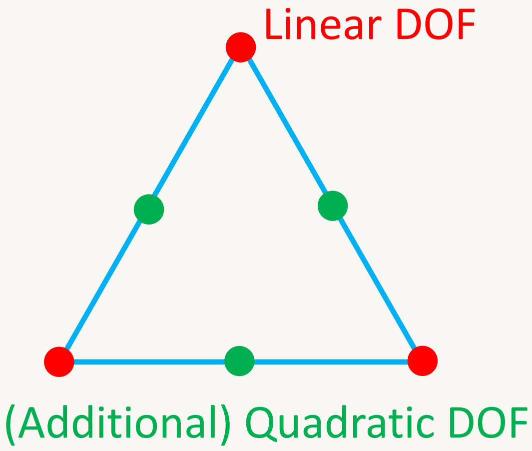

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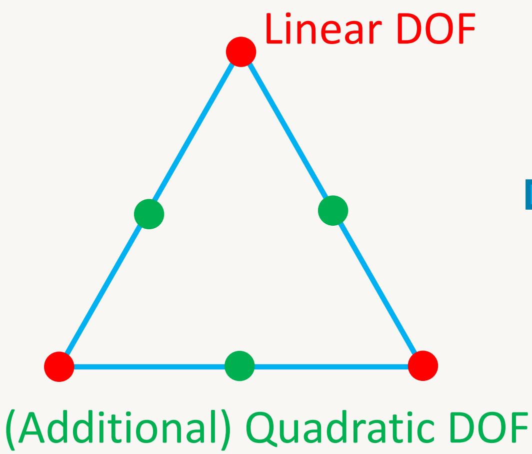
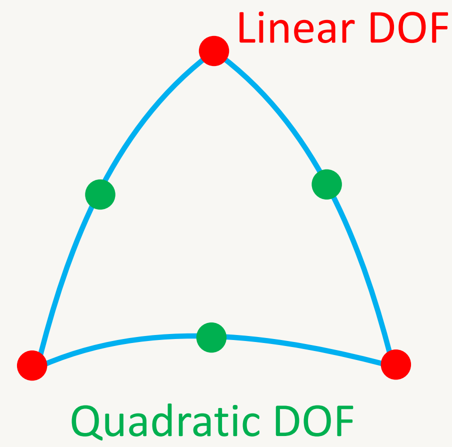


Higher order geometric description (curved elements)



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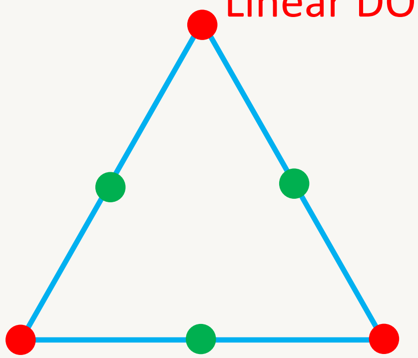

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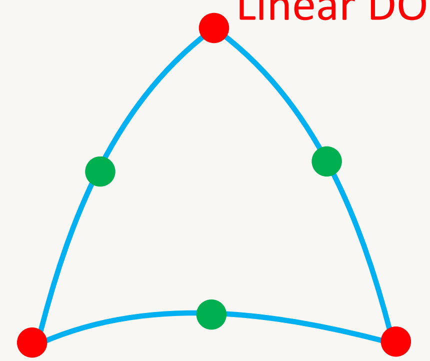
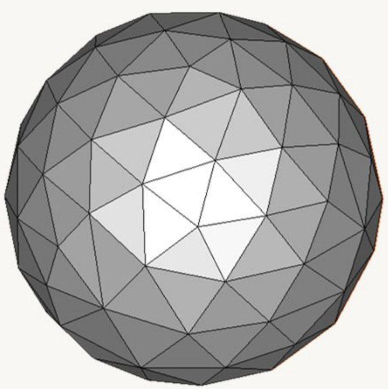


Linear Elements



Quadratic Elements