

Part 2

Methods and problems in residual stress determination by diffraction

3 Calculation of residual stress from measured strain

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3.1 General overview

Residual stress evaluation is an important stage in the improvement of the performance of materials, the control of the deformation of components and the understanding of industrial processes. In general, residual stress has various origins: mechanical (machining, shot peening), thermal (heat treatment, laser treatment), thermomechanical (forging, welding) or thermochemical (carburizing, nitriding).

According to the standard definition [1], the residual stresses are the auto-balancing stresses that exist in a material that is submitted either to no forces or external stresses and that is in a constant temperature condition.

In general, these residual stresses are caused by an inhomogeneity in deformation, from some source of local incompatibility which is generated from one or more of three fundamental physical origins: plastic flow, volume change, and thermal dilatation. Incomplete relaxation of the elastic deformation associated with these phenomena leads to residual stress.

In the solid there are additionally local incompatibilities caused by crystal defects, dislocations, grain boundaries, second phase particles, etc. The exact origin of stress is therefore going to depend on the scale of observation.

A classification of the residual stresses into three orders or types, related to the scale on which one considers the material, was proposed by Macherauch and Kloss [2].

- Residual stresses of the first order, or type I residual stresses, are homogeneous over a very large number of crystal domains of the material. Such stresses are also termed macrostresses (σ_R^I). The internal forces related to this stress are balanced on all planes. The moments related to these forces are equal to zero around all axes.
- Residual stresses of the second order, or type II residual stress, are homogeneous within small crystal domains of the material (a single grain or phase). The internal forces related to these stresses are in balance between the different grains or phases.
- Residual stresses of the third order, or type III residual stress, are homogeneous on the smallest crystal domains of the material (over a few interatomic distances). The internal forces coupled to these stresses are in balance in very small domains (such as around dislocations or point defects). Type II and III residual stresses are collectively termed microstresses (σ_R^{II} and σ_R^{III}).

In the case of real materials, the actual residual stress state at a point comes from the superposition of stresses of type I, II and III stresses [3–5, 7], as is illustrated in [Figure 3.1](#).

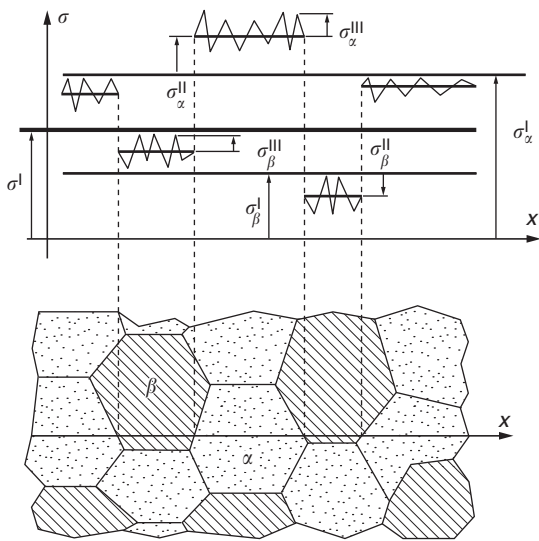


Figure 3.1 Three orders of stresses in two-phase materials.

3.2 Mechanical definition of stress and strain

In engineering science, the mechanical stress state at a point in a material and in a reference system is represented by a second-order tensor σ_{ij} :

$$\sigma_{ij} = \frac{\partial F_i}{\partial S_j}$$

where F_i represents the component of force in a direction x_i that acts on the element of volume characterized by surface dS_j , whose normal is in direction x_i .

In its complete notation σ_{ij} can be written in the referential system (1, 2, 3):

$$\sigma_{ij} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix}$$

The diagonal elements represent normal stresses on planes normal to the direction of the chosen axes, the other elements being shear stresses.

The stress tensor is symmetrical ($\sigma_{ij} = \sigma_{ji}$) and can be diagonalized, which means that for any point we can choose a system of particular axes called *principal axes* of the tensor so that only elements of the diagonal are not equal to zero: that is, an axis system where there are only normal stresses along the axes, and no shear stresses between them. These components are the principal components of the stress tensor.

The stress tensor can be transposed to other reference axes using the law of transformation:

$$\sigma'_{ij} = a_{ik}a_{jl}\sigma_{kl}$$

The stress in direction ij of the initial set of axes is equal to the stress in direction kl of the new set of axes after the transformation $a_{ik}a_{jl}$.

While developing the equation in the direction of the plane normal, the stress is given by the ellipsoid of stresses:

$$\sigma'_{11} = a_{11}^2\sigma_{11} + a_{12}^2\sigma_{22} + a_{13}^2\sigma_{33} + 2a_{12}a_{11}\sigma_{12} + 2a_{13}a_{11}\sigma_{13} + 2a_{13}a_{12}\sigma_{23}$$

The stress tensor can also be expressed in the matrix notation of Voigt

$$\sigma = \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ 2\sigma_{12} \\ 2\sigma_{13} \\ 2\sigma_{23} \end{bmatrix}$$

The advantage of this notation is that it is sometimes convenient to express the elastic tensor as a matrix.

In practice, it is not stress that is obtained by a diffraction measurement: it is imprecise to speak of stress measurement. It is necessary to transform strain in stress.

In a solid, applied external forces deform the structure. It is then necessary to define a strain field characteristic of the resulting displacements. In any reference sample the state of strain of a small element of volume is defined by the second-order tensor ε_{ij} . For small strains the expression is written:

$$\varepsilon_{ij} = \frac{1}{2} \left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right]$$

in which $u(1, 2, 3)$ is the vector of displacements.

In its complete notation and for the standard reference system, it can be written as:

$$\varepsilon_{ij} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix}$$

The diagonal elements represent strains along the direction of the chosen axes and the other elements are the shear strains between the axes.

For tensor ε_{ij} , the same rules as the stress tensor are applied: it is symmetrical,

$$\varepsilon_{ij} = \varepsilon_{ji}$$

it obeys the law of transformation,

$$\varepsilon'_{ij} = a_{ik}a_{jl}\varepsilon_{kl}$$

and, as for the stress tensor, matrix notation can be used for the strain tensor.

Table 3.1 Stress and strain notations

	Tensor	Matrix	Engineering notation
Normal stress	σ_{11}	σ_1	$\sigma_{xx}\sigma_x$
	σ_{22}	σ_2	$\sigma_{yy}\sigma_y$
	σ_{33}	σ_3	$\sigma_{zz}\sigma_z$
Shear stress	σ_{23}	σ_4	$\sigma_{yz}\tau_{yz}$
	σ_{13}	σ_5	$\sigma_{xz}\tau_{xz}$
	σ_{12}	σ_6	$\sigma_{xy}\tau_{xy}$
Longitudinal strain	ε_{11}	ε_1	$\varepsilon_{xx}\varepsilon_x$
	ε_{22}	ε_2	$\varepsilon_{yy}\varepsilon_y$
	ε_{33}	ε_3	$\varepsilon_{zz}\varepsilon_z$
Shear strain	ε_{23}	$(1/2)\varepsilon_4$	$\varepsilon_{yz}(1/2)\gamma_{yz}$
	ε_{13}	$(1/2)\varepsilon_5$	$\varepsilon_{xz}(1/2)\gamma_{xz}$
	ε_{12}	$(1/2)\varepsilon_6$	$\varepsilon_{xy}(1/2)\gamma_{xy}$

In engineering science, stress and/or strain analysis is done generally by choosing a Cartesian reference system (o, x, y, z). The relations between the different notations are given in Table 3.1. However, these definitions are not sufficient to describe the origins of the stress field in the material. It is necessary to separate clearly the macrostress and macrostrain from microstress and microstrain.

3.3 Macroscopic and microscopic relations

When a material is submitted to a applied load, it develops a field of microstress σ and microstrain ε . By definition, the macrostress $\bar{\sigma}$ is equal to the average value of the field of microstress σ in the volume V

$$\bar{\sigma} = \frac{1}{V} \int_V \sigma \, dV$$

In the same way, the macrostrain $\bar{\varepsilon}$ is written as:

$$\bar{\varepsilon} = \frac{1}{V} \int_V \varepsilon \, dV$$

If the sample is homogeneous and elastic, there is a linear correlation, on a macroscopic scale between the two terms $\bar{\varepsilon}$ and $\bar{\sigma}$:

$$\bar{\sigma} = \overline{C} \bar{\varepsilon}$$

where \overline{C} represents the macroscopic elastic constant matrix.

In the same way, on the microscopic scale:

$$\sigma = C \varepsilon$$

where C is the microscopic elastic coefficients matrix.

In most materials, the polycrystalline values of \overline{C} cannot generally be used interchangeably with the average single crystal C because there are very strong interactions between single crystals (grains).

3.3.1 Case of a monocrystal

The microscopic relation between the strain and the stress tensor is given by the generalized Hooke's law:

$$\varepsilon_{ij} = S_{ijkl}\sigma_{kl}$$

or by expressing the stress according to the strain:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl}$$

C_{ijkl} and S_{ijkl} are coefficients of stiffness and compliance of the material, respectively, and are fourth-order tensors.

For any transformation a_{ij} , tensors of compliance and stiffness can be expressed as:

$$S_{ijkl} = a_{im}a_{jn}a_{ko}a_{lp}S_{mnop}$$

$$C_{ijkl} = a_{im}a_{jn}a_{ko}a_{lp}C_{mnop}$$

These formulas represent nine equations each of nine terms. That gives 81 constants in the tensors of compliance and stiffness.

However, relations of symmetry of the stress and strain tensor decreases the number of constants to 36.

We can also demonstrate that:

$$C_{ijkl} = C_{klij}$$

$$S_{ijkl} = S_{klij}$$

This decreases the number of independent constants to 21.

In general, tensors of stiffness and compliance are expressed with the help of the matrix notation of Voigt.

$$S_{ijkl} = S_{mn} \quad \text{for } m \text{ and } n = 1, 2 \text{ or } 3$$

$$S_{ijkl} = \frac{1}{2}S_{mn} \quad \text{for } m \text{ or } n = 4, 5 \text{ or } 6$$

$$S_{ijkl} = \frac{1}{4}S_{mn} \quad \text{for } m \text{ or } n = 7, 8 \text{ or } 9$$

In this case, the generalized Hooke's law is simplified as:

$$\sigma_i = C_{ij}\varepsilon_j$$

$$\varepsilon_i = S_{ij}\sigma_j$$

The number of independent constants of the matrix of compliance or stiffness can be reduced according to these symmetries.

For a cubic monocrystal it is, for example, necessary to know only three independent constants to deduce its elastic behaviour.

Matrices of compliance and stiffness are written in the following way:

$$S_{ij} = \begin{bmatrix} S_{11} & S_{12} & S_{12} & 0 & 0 & 0 \\ S_{12} & S_{11} & S_{12} & 0 & 0 & 0 \\ S_{12} & S_{12} & S_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{44} \end{bmatrix}$$

$$C_{ij} = \begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44} \end{bmatrix}$$

If we consider the cubic monocrystal system as isotropic, the elastic constants must be the same in all directions.

So, the independent constants are S_{11} and S_{12} , with $S_{44} = 2(S_{11} - S_{12})$. In the same way, C_{11} and C_{12} with $C_{44} = (C_{11} - C_{12})/2$; by introducing Lamé constants of $\lambda = C_{12}$ and $\mu = C_{44}$, the matrix can be expressed thus:

$$C_{ij} = \begin{bmatrix} (\lambda + 2\mu) & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & (\lambda + 2\mu) & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & (\lambda + 2\mu) & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix}$$

Hooke's law is simplified by the following expression:

$$\varepsilon_{ij} = \frac{1 + \nu}{E} \sigma_{ij} - \delta_{ij} \frac{\nu}{E} \sigma_{kk}$$

$$\sigma_{ij} = \frac{E}{1 + \nu} \varepsilon_{ij} - \frac{\nu E}{(1 + \nu)(1 - 2\nu)} \delta_{ij} \varepsilon_{kk}$$

with δ being the Kronecker delta-function, E the Young's modulus and ν the Poisson's ratio of the material or of the cubic crystal variety considered:

$$E = \frac{1}{S_{11}} \quad \text{and} \quad \nu = -\frac{S_{12}}{S_{11}}$$

For a cubic monocrystal, the microscopic relations can generally be used. It is only necessary to consider the anisotropy of the single crystal.

3.3.2 Elastic constants of the single crystal (cubic system)

A single crystal is an anisotropic system and its elastic properties depend on the direction which is considered. To determine the Young's modulus a uniaxial stress σ_{app} , is applied to the cubic crystal in a direction $[hkl]$.

In this direction, the ε_{11} strain is given by:

$$\varepsilon_{11} = \frac{1}{(E)_{hkl}} \sigma_{\text{appl}}$$

Since the material is anisotropic, the equation can be written in the generalized form:

$$\varepsilon_{11} = S'_{1111} \sigma_{11}$$

It is supposed that the direction $[hkl]$ can also be expressed in terms of the compliance constants S , defined in the reference system of the crystal, with the use of the transformation law:

$$\frac{1}{(E)_{hkl}} = S'_{1111} = a_{1m}a_{1n}a_{1o}a_{1p}S_{mnop}$$

in which a_{ij} are the direction cosines of the direction $[hkl]$.

By developing the equation:

$$\begin{aligned} \frac{1}{(E)_{hkl}} = S_{11} & \left(a_{11}^4 + a_{12}^4 + a_{13}^4 \right) + S_{12} \left(a_{11}^2 a_{12}^2 + a_{11}^2 a_{13}^2 + a_{11}^2 a_{23}^2 \right) \\ & + S_{44} \left(a_{11}^2 a_{12}^2 + a_{11}^2 a_{13}^2 + a_{11}^2 a_{23}^2 \right) \end{aligned}$$

Since:

$$a_{ij}a_{jk} = 1 \quad \text{if } i = j$$

$$a_{ik}a_{jk} = 0 \quad \text{if } i \neq j$$

The equation becomes

$$\frac{1}{(E)_{hkl}} = S'_{1111} = S_{11} - 2 \left\{ (S_{11} - S_{12}) - \frac{S_{44}}{2} \right\} \left(a_{11}^2 a_{12}^2 + a_{11}^2 a_{13}^2 + a_{11}^2 a_{23}^2 \right)$$

and according to the Miller indices [3]:

$$\frac{1}{(E)_{hkl}} = S_{1111} = S_{11} - 2A_{hkl}S$$

or

$$S = (S_{11} - S_{12}) - \frac{S_{44}}{2} \quad \text{and} \quad A_{hkl} = \frac{h^2 k^2 + h^2 l^2 + l^2 k^2}{(h^2 + k^2 + l^2)^2}$$

From the equation, we get the Young's modulus:

$$E_{hkl} = \frac{1}{S_{11} - 2A_{hkl}S}$$

The Poisson's ratio can be obtained by considering the direction perpendicular to the loading direction:

$$\varepsilon_{\perp} = \frac{1}{(E_{\perp})_{hkl}} \sigma_{\text{appl}}$$

With the help of the transformation law:

$$S'_{2111} = a_{2i}a_{2j}a_{1k}a_{1l}S_{ijkl} = \frac{1}{(E_{\perp})_{hkl}} = S_{12} + 2A_{hkl}$$

and therefore:

$$(E_{\perp})_{hkl} = \frac{1}{S_{12} + 2A_{hkl}}$$

So, the Poisson's ratio for the plane (hkl) is:

$$(v)_{hkl} = -\frac{(E_{\parallel})_{hkl}}{(E_{\perp})_{hkl}} = \frac{S_{12} + 2A_{hkl}}{S_{11} - 2S_{hkl}}$$

3.3.3 Case of one phase in an isotropic polycrystalline material

For a cubic and isotropic polycrystalline material, Hooke's law is simplified by the following expression where the micro-macro transformation takes place by knowledge of $E(hkl)$ and $\nu(hkl)$:

$$\bar{\sigma}_{ij} = \frac{E(hkl)}{1 + \nu(hkl)} \varepsilon_{ij} + \frac{\nu(hkl)E(hkl)}{1 + \nu(hkl)(1 - 2\nu(hkl))} \delta \varepsilon_{kk}$$

δ being the Kronecker delta-function.

3.3.4 Elastic constants of a polycrystalline material (cubic system)

The behaviour of a polycrystal plane (hkl) in a non-textured sample is generally considered to be isotropic. However, the material is composed of aggregated monocrystals whose properties are anisotropic. The elastic behaviour of the polycrystal depends not only on the elastic constants of the single crystal, but also on the interaction between grains. A rigorous theoretical calculation of the elastic constants of the polycrystal requires a complete theoretical solution of the influence of the elastic anisotropy of every crystallite and the interaction between crystallites on the elastic response of polycrystalline material. Since such a precise solution has not yet been obtained, it is necessary to use coupling models for the interaction between crystallites. Historically several models have been proposed, such as the models of Voigt, Reuss and Kroner [8–13].

3.3.4.1 Model of Voigt

In the approximation of Voigt the field of distortions (strain) is uniform in the polycrystal, and therefore each grain experiences the same strain. There is a continuity strain on grain

boundaries. In this hypothesis the elastic constants of the material are calculated from five stiffness constants of the single crystal and are independent of the crystallographic direction.

For example, for a non-textured cubic material:

$$E_{hkl} = \frac{2S_{44} + 6(S_{11} - S_{12})}{2S_{11}(S_{11} + S_{12} + 2S_{44}) - 2S_{12}(2S_{12} + S_{44})}$$

$$\nu_{hkl} = -\frac{S_{11}(2S_{11} + 2S_{12} - S_{44}) + S_{12}(3S_{44} - 4S_{12})}{2S_{11}(S_{11} + S_{12} + 2S_{44}) - 2S_{12}(2S_{12} + S_{44})}$$

3.3.4.2 Model of Reuss

In the approximation of Reuss, the stress field in the material is considered to be uniform, and therefore each crystallite develops a strain that is proportional to the anisotropic modulus in a particular direction. In this hypothesis, the elastic constants therefore depend on the crystallographic orientation $[hkl]$.

For example, for a non-textured cubic material:

$$E_{hkl} = \frac{1}{S_{11} - A_{hkl}\{2(S_{11} - S_{12}) - S_{44}\}}$$

$$\nu_{hkl} = -\frac{S_{12} + A_{hkl}\{(S_{11} - S_{12}) - S_{44}/2\}}{S_{11} - A_{hkl}\{2(S_{11} - S_{12}) - S_{44}\}}$$

where A_{hkl} is the orientation factor of the plane (hkl) .

3.3.4.3 The self-consistent model of Kröner

In this extended Eshelby model, the method of Kröner concerns the calculation of the global elastic properties of a polycrystalline material, from the elastic constants of the single crystal. This model is based on the treatment of one anisotropic crystallite, spherical or ellipsoidal, that is embedded in an infinite, elastic and isotropic matrix.

For every crystallite, the distortion is directly proportional to the average stress that exists in the polycrystal:

$$\varepsilon_{ij} = (S_{ijkl} + t_{ijkl}(\Omega))\bar{\sigma}_{kl}$$

where S_{ijkl} is the compliance tensor of the single crystal of volume Ω , and t_{ijkl} is a fourth-order tensor that describes the interaction between crystallites.

After integration of the equation over all orientations, one gets the macroscopic average values:

$$\bar{\varepsilon}_{ij} = (S_{ijkl})_B \bar{\sigma}_{kl}$$

$(S_{ijkl})_B$ being the compliance tensor of the material. So

$$\int_D t_{ijkl} dD = 0$$

From calculations using the Kröner model, for a spherical, anisotropic crystal and cubic symmetry, the equation becomes:

$$G^3 + \alpha G^2 + \beta G + \gamma = 0$$

with

$$\alpha = \frac{9A_1 + 4A_2}{8}$$

$$\beta = -\frac{(3A_1 + 12A_3)A_2}{8}$$

$$\gamma = -\frac{3A_1A_2A_3}{8}$$

$$A_1 = \frac{1}{3(S_{11} + S_{12})} \quad A_2 = \frac{1}{S_{44}} \quad A_3 = \frac{1}{2(S_{11} - S_{12})}$$

The solution of G represents the macroscopic Coulomb modulus and enables us to calculate E and ν :

$$E_{hkl} = \frac{\omega}{1 - 2\omega(1 - 5A_{hkl})t_{44}}$$

$$\nu_{hkl} = -\frac{2G\{1 + \omega(1 - 5A_{hkl})t_{44}\} - \omega}{2G\{1 - 2\omega(1 - 5A_{hkl})t_{44}\}}$$

where the tensor is composed as:

$$t_{44} = \frac{(G - A_2)(3A_1 + 6G)}{2G\{8G^2 + G(9A_1 + 12A_2) + 6A_1A_2\}} \quad \text{and} \quad \omega = \frac{9A_1G}{3A_1 + G}$$

Kröner's model gives the most realistic value of the elastic constants of a polycrystal, whereas the models of Voigt and Reuss give lower and upper bounds of these values, respectively. A simpler procedure, introduced by Hill and Neerfeld, consists of taking the arithmetic average of values of Reuss and Voigt, and which yields a result which will be closer to the value of Kröner.

3.3.5 Case of a two-phase polycrystalline material

In a two-phase polycrystalline material, the grains belonging to different phases have different physical and elastic properties and hence, the second order stresses averaged over one phase are not equal to zero. It is possible to approximate this problem. The mean value of stress calculated over one phase is called the phase stress ($\bar{\sigma}^{\text{ph}}$) and it can be subdivided into the following terms with the formalism proposed by Clyne and Withers [14]:

$$\bar{\sigma}_{ij}^{\text{ph}} = \bar{\sigma}_{ij}^{\text{I}} + \sigma_{ij}^{\text{phE}} + \sigma_{ij}^{\text{phTh}} + \sigma_{ij}^{\text{phPl}}$$

where the superscript I is used for the macro (first order) stress, and the mismatch elastic thermal and plastic stresses are denoted by E, Th and Pl, respectively [6, 14, 15].

The mismatch stresses are defined as the average value of the second-order stresses over all grains belonging to one phase; for both phases of the material the second-order stresses must sum to zero over a large volume of material and we obtain:

$$\overline{\sigma_{ij}^I} = f \overline{\sigma_{ij}^{\text{ph1}}} + (1 - f) \overline{\sigma_{ij}^{\text{ph2}}}$$

The overall macrostress is simply the summation of two macrostress terms characteristic of each phase in the material. Some theoretical model (Eshelby or autocohesent) can be used to complete this evaluation (see Chapter 16).

3.4 Experimental determination of diffraction elastic constants (DECs)

Finally, it is generally necessary to measure the diffraction elastic constants (DECs) in order to verify model results, or when the parameters are unknown for a particular peak. The simplest way to measure DECs is to apply a uniaxial stress to a sample of the material and to measure the strain evolution $\varepsilon_{\phi\psi}$ for different values of the applied stress. The macroscopic stress state is the sum of the residual stress and the applied stress.

Generally, we apply a tensile or compressive stress by a stress rig or a four-point bending device. We consider the effect of the applied stress only, taking any residual stress to be our reference state. With $\phi = 0$, the general formula is:

$$\varepsilon_{\psi} = \frac{1}{2} S_2 \overline{\sigma_{xx}} \sin^2 \psi + S_1 (\overline{\sigma_{xx}} + \overline{\sigma_{yy}})$$

where $(1/2)S_2$ and $(1/2)S_1$ are the DECs with:

$$\frac{1}{2} S_2 = \frac{1 + \nu}{E}; \quad S_1 = -\frac{\nu}{E}$$

where E is the Young's modulus and ν the Poisson coefficient of the plane (hkl) .

3.4.1 First method: $\sin^2 \psi$ method

For different values of the applied load ($\bar{\sigma}_{\text{app}} = \bar{\sigma}_{xx}$) and with different determination of $\varepsilon_{\psi} = f(\sin^2 \psi)$ we have the general result given by [Figure 3.2](#).

If we consider for different $\bar{\sigma}_{\text{app}}$

$$\alpha = \frac{\partial \varepsilon_{\psi}}{\partial \sin^2 \phi} = \frac{1}{2} S_2 \bar{\sigma}_{\text{app}}$$

the slope of the graph gives S_2

$$\left(\frac{1}{2} S_2 = \frac{\partial \alpha}{\partial \bar{\sigma}_{\text{app}}} \right)$$

and also for different $\bar{\sigma}_{\text{app}}$ the origin β ($\beta = \varepsilon_{(\psi=0)}$) gives S_1 ($S_1 = \partial \beta / \partial \bar{\sigma}_{\text{app}}$).

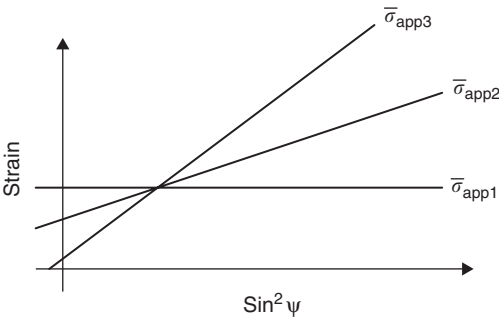


Figure 3.2 $\varepsilon_\psi = f(\sin^2 \psi)$ for different applied stresses.

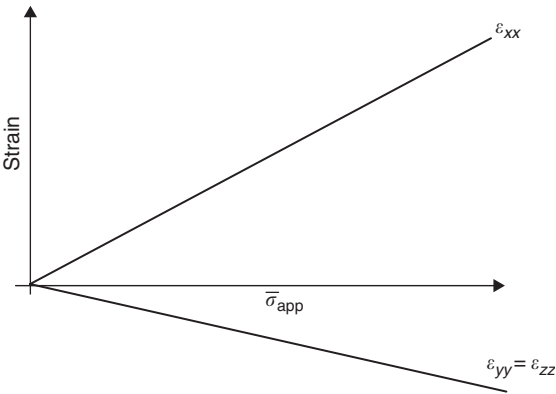


Figure 3.3 ε_{xx} , ε_{yy} and ε_{zz} for different applied stresses.

3.4.2 Second method: 3D method

We measure the three principal strains for different applied stress

$$\begin{aligned}\varepsilon_{xx} &= \varepsilon_{0,90} \\ \varepsilon_{yy} &= \varepsilon_{90,90} \\ \varepsilon_{zz} &= \varepsilon_{0,0}\end{aligned}$$

The slope of the graph gives directly $E_{(hkl)}$ and $\nu_{(hkl)}$ (see Figure 3.3).
It is easy to calculate S_1 and $(1/2)S_2$. This method is used with the neutron diffraction technique.

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References

- [1] Mura T., *Micromechanics of Defects in Solids*, 1982, Martinus Nijhoff Publishers, The Hague, Netherlands.
- [2] Macherauch E. and Kloss K. H., *Proceedings of the International Conference on Residual Stresses*, Garmisch-Partenkirchen, FRG, 1986, pp. 167–174.
- [3] Lodini A., Perrin M., *Analyse des contraintes mécaniques par diffraction des rayons X et des neutrons*, 1996, Editions du CEA, Saclay.
- [4] Lodini A., *Analysis of Residual Stresses from Materials to Biomaterials*, 1997, Editions IITT, Marne la Vallée.
- [5] Hirschi K., *Thesis Université de Reims*, Reims, France, 1999.
- [6] Levy R., *Thesis Université de Reims*, Reims, France, 1999.
- [7] Ceretti M., *Thesis Université de Reims*, Reims, France, 1993.
- [8] Voigt W., *Lehrbuch der Kristallphysik*, Teubner, Leipzig/Berlin, 1928.
- [9] Voigt W., *Wied. Ann.* **38**, 573–587 (1889).
- [10] Reuss A., *Z. Angew. Math. Mech.* **9**, 49–58 (1929).
- [11] Kröner E., *J. Mech. Phys. Solids* **15**, 319–329 (1967).
- [12] Hill R., *Proc. Phy. Soc.* **A65**, 349–354, (1952).
- [13] Neerfeld H., *Mitt. K.W.I. Eisen Düsseldorf* **24**, 64–70 (1942).
- [14] Clyne T. W. and Withers P. J., *An Introduction to Metal Matrix Composites*, 1993, Cambridge Solid State Sciences Series, Cambridge University Press, Cambridge.
- [15] Baczmanski A., Braham C., Lodini A. and Wierzbanski K., *Mater. Sci. Forum* **404–7**, 729–734 (2002).