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Identification of linear parameter varying models

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SUMMARY

We consider identification of a certain class of discrete-time nonlinear systems known as linear parameter varying system. We assume that inputs, outputs and the scheduling parameters are directly measured, and a form of the functional dependence of the system coefficients on the parameters is known. We show how this identification problem can be reduced to a linear regression, and provide compact formulae for the corresponding least mean square and recursive least-squares algorithms. We derive conditions on persistency of excitation in terms of the inputs and scheduling parameter trajectories when the functional dependence is of polynomial type. These conditions have a natural polynomial interpolation interpretation, and do not require the scheduling parameter trajectories to vary slowly. This method is illustrated with a simulation example using two different parameter trajectories. Copyright © 2002 John Wiley & Sons, Ltd.

KEY WORDS: identification; LPV models; persistence of excitation

1. INTRODUCTION

In this paper, a new technique is developed for the identification of discrete-time linear parameter varying (LPV) models of nonlinear or time-varying systems.

Let us assume that the system generating the data is described by the (possibly nonlinear) relationship

$$y = f(u, p, \eta) \tag{1}$$

where y, u and p are the measured experimental data, and η represents the unknown signals entering the system. In particular, y is the output, u is the input and p is the gain-scheduling parameter, a variable parameter that can be thought as determining the 'set-point' of the system.

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In many processes (e.g. autopilot and flight control, automotive engine control, chemical processes) it is a realistic assumption that the set point variable is measurable.

As a *model*, we consider a special class of discrete-time LPV models. A motivation for this type of model comes from the fact that in control applications the *true* system is not linear, yet LTI or LPV models are often satisfactory in industrial control design. In particular, we consider LPV models rather than generic linear time varying (LTV) models because of their connection to the industrial practice of gain-scheduling [1,2,3,4].

A discrete LPV system is represented in state-space as

$$x(t+1) = A(p(t))x(t) + B(p(t))u(t)$$
$$y(t) = C(p(t))x(t) + D(p(t))u(t)$$

The exogenous parameter $p(t)$ is assumed *a priori* unknown. However, it can be measured or estimated upon operation of the system. In Reference [4], the modelling of nonlinear systems via LPV models has been studied in detail.

Recently, methods to infer stability and construct stabilizing feedback controller for LPV systems have been developed. We refer the reader to References [5,6,7,8,9], and the recent survey [2] and references therein. For some applications of LPV modelling, gain-scheduling and identification, see References [9,10,11] where the stall and surge control of jet engine compressors has been considered.

Identification problems for models similar to LPV systems have been considered in the literature. For example, we refer the reader to Reference [12] where a survey of the literature has been presented for LTV models, and the exponentially weighted least-squares (LS) algorithm and its stochastic properties have been studied.

However, LPV model identification is a more recent topic, and is significantly different from LTV system identification. More closely related to our approach is the work of Reference [13], which addresses the problem of identifying a discrete LPV model

$$y_k = P(p, \theta)u_k + v_k$$

where p is a measured time-varying parameter. In Reference [13], a solution is presented using nonlinear programming. In Reference [14], a special case of the LPV problem with state measurements and one parameter was shown to be equivalent to a linear regression, and certain conservative conditions for persistency of excitation were given. In Reference [15], LPV models are identified with an hybrid linear/nonlinear procedure. The nonlinear part is identified through neural network and the linear parametric part trough LS algorithm.

In this paper, we pose an LPV identification problem in terms of input–output and parameter trajectory data. We show how this problem can be recast as linear regression. This allows us to formulate both a least mean square (LMS) and a recursive least-squares (RLS) algorithm. We also present a particular matrix representation of the data that significantly simplifies algorithm implementation. Finally, we consider persistency of excitation conditions for these algorithms. For the case of polynomial representation of systems coefficient, this conditions turns out to have a very natural interpolation interpretation. Furthermore, this condition does not require the scheduling parameter to vary slowly, allowing for identification experiments where rapid setpoint changes are possible.

2. PRELIMINARIES

We will need the notions of inner and tensor products of matrices. Given any two matrices A, B of equal dimensions, their inner product is defined as

$$\langle A, B \rangle := \text{trace } (A^* B) = \text{trace } (BA^*)$$

where A^* means complex conjugate transpose of the matrix A. For given integers (n, m), this inner product makes $\Re^{n \times m}$ (the space of $n \times m$) matrices into an inner product space.

Using the above inner product, one can define the tensor product of two matrices $U \in \Re^{n \times m}$ and $V \in \Re^{p \times q}$ as the linear operator $U \otimes V : \Re^{p \times q} \rightarrow \Re^{n \times m}$ defined by

$$(U \otimes V)(X) := U \langle V, X \rangle$$

This definition generalizes the notion of outer products of vectors to matrices.

3. PROBLEM FORMULATION

Our special class of discrete-time LPV models is parameterized as follows:

$$M(\delta, p) = \frac{B(\delta, p)}{A(\delta, p)} \tag{2}$$

where δ is the delay operator, i.e. $(\delta y)_k := y(k-1)$, and

$$B(\delta, p) := b_0(p) + b_1(p)\delta + \cdots + b_{n_b}(p)\delta^{n_b}$$

$$A(\delta, p) := 1 + a_1(p)\delta + \cdots + a_{n_a}(p)\delta^{n_a} \tag{3}$$

$n = n_a + n_b + 1$ is the number of parametric functions to be identified. Moreover, we assume that the varying parameter p is a function of discrete time $(p = p(k))$. The fractional representation in (2) is short hand for the operational notation

$$A(\delta, p)y(k) = B(\delta, p)u(k) \tag{4}$$

where y is the output and u is the input.

We assume that the functions $\{a_i\}$, $\{b_j\}$ in (3) are *linear combinations* of a set of known fixed basis functions $\{f_1, \ldots, f_N\}$, e.g.

$$a_1(p) = a_1^1 f_1(p) + \cdots + a_1^N f_N(p) \tag{5}$$

where the constants a_1^i are real numbers. Thus any particular model in our class is completely characterized by the real numbers $\{a_i^k\}$ and $\{b_j^l\}$. The goal of a parametric identification scheme is then to find these constants from data.

In general, the right basis functions may not be known. Our approach in this paper is to assume basis functions for which an algorithmic implementation is simple (e.g. polynomials). While for a particular LPV model, this may not yield an exact fit for the functions $\{a_i\}$, $\{b_j\}$, one typically can chose a set of basis functions that approximate well. Many choices are possible for the functions $f_l(p)$. A particularly interesting case is that of polynomial dependence, then the functions $f_l(p)$ are powers of p

$$f_l(p) = p^{l-1}, \quad l = 1, \ldots, N$$

In this case, the coefficient functions are

$$\begin{aligned} a_i(p) &= a_i^1 + a_i^2 p + \cdots + a_i^N p^{N-1} \\ b_j(p) &= b_j^1 + b_j^2 p + \cdots + b_j^N p^{N-1} \end{aligned} \tag{6}$$

Many other choices are possible. We list here two particularly interesting cases. The first is when f_l are periodic functions, e.g.

$$f_l(p) = \sin(l\omega_0 p), \quad l = 1, \ldots, N$$

This representation would be useful for problems where the coefficients and the parameters are cyclical (e.g. angles of rotation), such as in non-circular machining [16]. If the frequency ω_0 is known then Equation (5) represents the coefficients as a linear combination of the first N harmonics of a general periodic function of p. The second interesting case is related to neural network representations [15]. In this case, the constants c_l represent the network's weights, and the function f_l the sigmoidal functions.

Remark

In this paper we consider only the case of dependence on a single parameter. It seems that most of our results in this paper can be generalized to the case where the functions $\{a_i\}$ and $\{b_j\}$ are functions of several parameters.

We now observe that the input–output model in (4), together with (3) and (5) can be put into a nice linear regression form as follows. Let us define an $n \times N$ matrix Θ which contains all the coefficients to be identified:

$$\Theta := \begin{bmatrix} a_1^1 & \cdots & a_1^N \\ a_2^1 & \cdots & a_2^N \\ \vdots & \vdots & \vdots \\ a_{na}^1 & \cdots & a_{na}^N \\ b_0^1 & \cdots & b_0^N \\ \vdots & \vdots & \vdots \\ b_{nb}^1 & \cdots & b_{nb}^N \end{bmatrix} \tag{7}$$

We also define the *extended regressor* Ψ, which will be made up of past i/o data and parameter trajectories

$$\Psi_k := \phi_k \pi_k := \begin{bmatrix} -y_{k-1} \\ \vdots \\ -y_{k-n_a} \\ u_k \\ \vdots \\ u_{k-n_b} \end{bmatrix} [\, f_1(p_k) \quad f_2(p_k) \quad \ldots \quad f_N(p_k)] \tag{8}$$

where for notational simplicity we use a subscript for the time index (e.g. $p_k := p(k)$).

With the above definitions, it is easy to verify that models (3)–(5) can now be rewritten as

$$y_k = \langle \Theta, \Psi_k \rangle \tag{9}$$

where the inner product is the matrix inner product defined in the preliminaries.

The advantage of rewriting the model in the inner product form (9) is that the usual RLS and LMS algorithms can now be easily generalized to this case. To do so, one needs the correct notion of outer products (which is given by the tensor product), and to appropriately handle the issues of measurement noise and persistency of excitation.

We note that if $N = 1$, in the polynomial case $f_1(p_k) = 1$ and $\pi = 1$, then $\Theta = \theta \in \Re^{n\times 1}$ and $\langle \Theta, \Psi \rangle = \langle \theta, \phi \rangle = \text{trace}(\phi^* \theta) = \phi^{\mathrm{T}} \theta$. This is the standard product among the vector of parameters θ and the regressor vector ϕ of any classical identification scheme [17].

Uncertainty considerations

In this paper, we only consider measurement noise of the following form:

$$y_k = \langle \Theta, \Psi_k \rangle + \eta_k \tag{10}$$

where η is a white noise sequence uncorrelated with the input u.

4. IDENTIFICATION ALGORITHMS

Let us consider the following loss function:

$$J = J(\Theta) = \frac{1}{T} \sum_{k=0}^{T} E\{\varepsilon(k, \Theta)^2\} \tag{11}$$

where the *prediction error* ε is defined as

$$\varepsilon(k, \Theta) = y_k - \langle \Theta, \Psi_k \rangle \tag{12}$$

Let $\hat{\Theta}_k$ be the matrix of estimated parameters at time k. We recall that a steepest descent algorithm for updating the parameters, and minimizing (12) iteratively has the form

$$\delta\hat{\Theta} = \hat{\Theta}_k - \hat{\Theta}_{k-1} = -\tfrac{1}{2}\alpha g(\hat{\Theta}_{k-1})$$

where $g(\Theta_{k-1})$ is the mean square error (MSE) gradient

$$g(\Theta) = \frac{\mathrm{d}J(\Theta)}{\mathrm{d}\Theta} \tag{13}$$

and the parameter α is the step size.

An instantaneous approximation of the MSE algorithm is the least mean square (LMS) [18], where in Equation (13) the loss function (11) is replaced by

$$J(\Theta) = \varepsilon(k, \Theta)^2 \tag{14}$$

The updating of the parameters is then done according to the following recursion:

$$\hat{\Theta}_{k+1} = \hat{\Theta}_k - \frac{1}{2}\alpha\left(\frac{\mathrm{d}}{\mathrm{d}\Theta}(\varepsilon(k,\Theta)^2)\right)\Bigg|_{\Theta=\hat{\Theta}_k} \tag{15}$$

Recalling that $\varepsilon(k,\Theta)$ is given by $y_k - \langle\Theta, \Psi_k\rangle = y_k - \mathrm{trace}(\Theta^{\mathrm{T}}\Psi_k)$, and using the derivative property $(\mathrm{d}/\mathrm{d}X)$ trace $(X^{\mathrm{T}}B) = B$, we get

$$\begin{aligned}\frac{1}{2}\alpha\frac{\mathrm{d}}{\mathrm{d}\Theta}(\varepsilon^2) &= \alpha\left(\varepsilon\frac{\mathrm{d}}{\mathrm{d}\Theta}(y_k - \mathrm{trace}(\Theta^{\mathrm{T}}\Psi_k))\right)\\ &= -\alpha(\varepsilon\Psi_k)\end{aligned}$$

Then, evaluating it for $\Theta = \hat{\Theta}_k$ the following LMS algorithm is derived.

LMS Algorithm. For the model in (10), under assumptions A1–A3 the LMS identification algorithm is given by

$$\Psi_k = \phi_k\pi_k$$

$$\varepsilon_k = y_k - \langle\hat{\Theta}_k, \Psi_k\rangle = y_k - \mathrm{trace}\left(\hat{\Theta}_k^{\mathrm{T}}\Psi_k\right)$$

$$\hat{\Theta}_{k+1} = \hat{\Theta}_k + \alpha\varepsilon_k\Psi_k$$

where the extended regressor Ψ_k is defined by (8).

We note that all that is needed for the implementation of this LMS algorithm is the formation of the extended regressor, and taking the matrix inner product involved in forming ε_k. This is particularly simple using our matrix representation of the data, since a matrix inner product is performed using matrix multiplication and then taking a trace.

For the LSs algorithm, we begin with some general remarks. Since (10) is in the form of a linear regression, an immediate method to solve the LSs problem is to 'string out' the components of Ψ_k as a column vector, which then gives us the regression in its standard form. However, this is notationally very cumbersome, and would obscure the natural persistency of excitation condition that we derive later on. Instead, we find it very convenient to consider the LSs problem over a general finite dimensional inner product space, rather than the usual Euclidean space with its column vector notation. Consider the system equation in (10), where y_k and η_k are scalars, and Θ and Ψ_k are elements of an inner product space. Under our assumptions, it can be shown that the LSs problem for this system is solved by the usual formulae [17,18] with the Euclidean inner product replaced by the given inner product, and the outer vector product replaced by the tensor product. The details are as follows.

RLS Algorithm

The RLS algorithm for system (10) under assumptions A1–A3 is given by

$$\begin{aligned}\varepsilon_k &= y_k - \langle \hat{\Theta}_{k-1}, \Psi_k \rangle = y_k - \mathrm{trace}(\hat{\Theta}^*_{k-1}\Psi_k)\\ \hat{\Theta}_k &= \hat{\Theta}_{k-1} + K_k\varepsilon_k\end{aligned} \tag{16}$$

$$K_k = P_k\{\Psi_k\} \tag{17}$$

$$P_k = P_{k-1} - P_{k-1}\frac{\Psi_k \otimes \Psi_k}{1 + \langle \Psi_k, P_{k-1}\Psi_k \rangle}P_{k-1} \tag{18}$$

A little care is needed in interpreting the above notation. Since both Θ and Ψ_k are in $\Re^{n\times N}$, then the gain $K_k \in \Re^{n\times N}$. The correlation operator P_k is then an operator $P_k : \Re^{n\times N} \to \Re^{n\times N}$. In the standard Euclidean representation, it would require $(nN)^2$ real numbers to obtain its matrix representation (this number maybe reduced by almost a half due to the symmetry property of P). We will instead leave it in this abstract form for simplicity. Equation (18) expresses the recursive update of P_k in terms of operator compositions. The factor $1 + \langle \Psi_k, P_{k-1}\Psi_k \rangle$ is a scalar. The tensor product forms an operator $\Psi_k \otimes \Psi_k : \Re^{n\times N} \to \Re^{n\times N}$, and thus the last term of (18) represents the composition of three operators.

5. PERSISTENCY OF EXCITATION

For both the LMS and the RLS algorithm a *persistency of excitation* condition has to be satisfied in order to guarantee the consistence of the algorithm. We state the condition for the case of polynomial dependence of the coefficients on the parameters (i.e. when $f_l(p) = p^{l-1}$).

Let Θ_0 be the true value, then the identification algorithm converges if

$$\lim_{k\to\infty}\hat{\Theta}_k = \Theta_0 \tag{19}$$

Definition 1

The extended regressor $\{\Psi_k\}$ is said to be *persistently exciting* (*PE*) if

$$\lim_{T\to\infty}\frac{1}{T}\sum_{k=0}^{T} E\{\Psi_k \otimes \Psi_k\} > 0 \tag{20}$$

We will need the following result, which is a generalization of a similar result for vectors, and an immediate consequence of the tensor product definition.

Lemma 1

Ψ_k is PE if and only if, for any non-zero $V \in \Re^{n\times N}$,

$$\lim_{T\to\infty}\frac{1}{T}\sum_{k=0}^{T} E\{\langle V, \Psi_k \rangle^2\} > 0$$

Definition 2
Let $\mathscr{I}$ be a subsequence of the natural numbers. The sequence $\{\phi_k\}$ is said to be *persistently exciting relative to* $\mathscr{I}$ if

$$\lim_{T\to\infty}\frac{1}{T}\sum_{k\in\mathscr{I}}^{T} E\{\phi_k\phi_k^*\} > 0 \tag{21}$$

An example of such a sequence is a white noise vector process $\{\phi_k\}$, where the subsequence $\mathscr{I}$ goes to infinity not slower than its index, i.e.

$$\mathscr{I} := \{s_1, s_2, \ldots\}, \quad \lim_{i\to\infty}\frac{i}{s_i} > 0$$

The key to the PE condition for LPV systems is a natural interpolation-type condition which we now state.

Theorem 1
The extended regressor $\{\Psi_k\}$ is PE if and only if the following two conditions hold:

(1) There exists N distinct points $\bar{p}_i \in \Re$, $i = 1, \ldots, N$, that are limit points of $p(k)$. Let $\mathscr{I}_1, \ldots, \mathscr{I}_N$ be subsequence indices such that

$$\lim_{k\in\mathscr{I}_i} p(k) = \bar{p}_i$$

(2) The sequence $\{\phi_k\}$ is persistently exciting relative to all sets $\mathscr{I}_1, \ldots, \mathscr{I}_N$, i.e. for each $i = 1, \ldots, N$,

$$\lim_{T\to\infty}\frac{1}{T}\sum_{k\in\mathscr{I}_i}^{T} E\{\phi_k\phi_k^*\} = Q_i > 0 \tag{22}$$

Proof
We first use Lemma 1 to recast the PE condition for Ψ_k. We recall that Ψ_k is PE iff for any non-zero $V \in \Re^{n\times N}$,

$$\lim_{T\to\infty}\frac{1}{T}\sum_{k=0}^{T} E\{\langle V, \Psi_k\rangle^2\} > 0$$

Using the definition of the inner product, we compute

$$\begin{aligned} E\{\langle V, \Psi_k\rangle^2\} &= E\{\text{trace}(\Psi_k V^*)^2\} \\ &= E\{\text{trace}(\phi_k\pi_k V^*)^2\} \\ &= E\{\text{trace}(\pi_k V^*\phi_k)^2\} && (23) \\ &= E\{\pi_k V^*\phi_k\phi_k^* V\pi_k^*\} && (24) \end{aligned}$$

where in (23) we used the property trace (AB) = trace(BA), and (24) follows because $\pi_k V^*\phi_k$ is a scalar.

The above implies that the PE condition on $\{\Psi_k\}$ now becomes

$$\lim_{T\to\infty} \frac{1}{T} \sum_{k=0}^{T} E\{\pi_k V^* \phi_k \phi_k^* V \pi_k^*\} > 0 \tag{25}$$

Now using the existence of the limit points and the PE condition on ϕ_k relative to them, we compute

$$\lim_{T\to\infty} \frac{1}{T} \sum_{k=0}^{T} E\{\pi_k V^* \phi_k \phi_k^* V \pi_k^*\} \tag{26}$$

$$\geqslant \lim_{T\to\infty} \sum_{i=1}^{N} \frac{1}{T} \sum_{k\in\mathscr{I}_i}^{T} E\{\pi_k V^* \phi_k \phi_k^* V \pi_k^*\} \tag{27}$$

$$= \sum_{i=1}^{N} \lim_{T\to\infty} \frac{1}{T} \sum_{k\in\mathscr{I}_i}^{T} E\{\pi_i V^* \phi_k \phi_k^* V \pi_i^*\} \tag{28}$$

$$= \sum_{i=1}^{N} \bar{\pi}_i V^* Q_i V \bar{\pi}_i^* \tag{29}$$

where

$$\bar{\pi}_i := [1 \bar{p}_i, \dots, \bar{p}_i^{N-1}]$$

and $\bar{p}_i$ are the limit points. Now the expression in (29) can be rewritten as

$$\sum_{i=1}^{N} \bar{\pi}_i V^* Q_i V \bar{\pi}_i^* \tag{30}$$

$$\geqslant \sum_{i=1}^{N} \bar{\pi}_i \bar{\pi}_i^* \lambda_{\min}(V^* Q_i V) \tag{31}$$

$$\geqslant \left(\sum_{i=1}^{N} \bar{\pi} \bar{\pi}^* \right) \left(\min_i \lambda_{\min}(V^* Q_i V) \right) \tag{32}$$

$$\geqslant \operatorname{trace}(\bar{\Pi}\bar{\Pi}^*) \left(\min_i \lambda_{\min}(V^* Q_i V) \right) \tag{33}$$

where matrix $\bar{\Pi}$ is constructed from the limit points by

$$\bar{\Pi} := \begin{bmatrix} 1 & \bar{p}_1 & \cdots & \bar{p}_1^{N-1} \\ & \vdots & & \vdots \\ 1 & \bar{p}_N & \cdots & \bar{p}_N^{N-1} \end{bmatrix}$$

Note that $\bar{\Pi}$ is a Vandermonde matrix, and is non-singular since the points p_i are distinct. The non-singularity of $\bar{\Pi}$, the fact that V is non-zero, together with the positive definiteness of Q_i implies that the right-hand side of (33) is strictly positive. This in turn implies that condition (25) is satisfied, and the theorem is proved. □

Remark
Though the conditions of Theorem 1 may seem a little technical, they can be intuitively understood. Assuming that the input is sufficiently rich to insure that ϕ_k is PE in the above sense, what is needed is that the trajectory of p_k 'visit' N distinct points infinitely many times. Condition (22) means that the rate at which p_k revisits each of these limit points should *not slow down*. Thus, these revisits are sufficient to ergodically extract the correlation data of ϕ_k.

The above remark can be further elaborated as follows. The LPV model is essentially a parameterized family of LTI models. If the parameter determines a process set point, a possible identification scheme is to fix each set point, and collect enough small signal data to identify the LTI model at that set point. The identified LTI coefficients can then be used as interpolation points to find the coefficients as polynomial functions of the parameter p. This procedure would produce a similar model to ours, and would require at least N LTI models at distinct set points. The identification schemes we present in this paper essentially carry this process out in 'one shot'. Condition (22) insures that enough data is gathered around the N interpolation points. The advantage of our schemes is that the set point does not have to vary slowly. For example, periodic parameter trajectories of any period (as long as N points are covered) will be sufficient for identification. A case of non-periodic trajectories that are still sufficient for identification is illustrated in the following example.

6. EXAMPLE

We illustrate our algorithms using a simple example involving the following LPV model:

$$y_k + a_1(p)y_{k-1} + a_2(p)y_{k-2} = b_0(p)u_{k-1} + b_1u_{k-2} + \eta_k$$

where the functional dependence in p of the parameters is given by polynomials of order $N = 3$:

$$a_1(p) = 1 - 0.5p + 0.2p^2$$

$$a_2(p) = 1 - 0.7p - 0.1p^2$$

$$b_0(p) = 0.5 - 0.4p + 0.01p^2$$

$$b_1(p) = 0.2 - 0.3p + 0.02p^2$$

and η_k was chosen to be a random white noise sequences with variance 10^{-3}.

The *true* parameter matrix is then

$$\Theta_0 := \begin{bmatrix} 1 & -0.5 & 0.2 \\ 1 & -0.7 & -0.1 \\ 0.5 & -0.4 & 0.01 \\ 0.2 & -0.3 & 0.02 \end{bmatrix} \tag{34}$$

and the *extended regressor* Ψ is

$$\Psi_k := \phi_k \pi_k := \begin{bmatrix} -y_{k-1} \\ -y_{k-2} \\ u_{k-1} \\ u_{k-2} \end{bmatrix} [1 \quad p_k \quad p_k^2]$$

A simulation was carried out with $T = 4500$. The input u_k is chosen as random noise with uniform distribution in the interval $[-1, 1]$.

Two different parameter trajectory sequences $p_k = \sin((\pi/3)k)$ and $p_k = \sin(k)$, respectively were used. For $p_k = \sin((\pi/3)k)$ the conditions of Theorem 1 are clearly satisfied, visiting three points in the range $[-1, 1]$ infinitely many times at a linear rate. The second parameter trajectory $p_k = \sin(k)$ was chosen since 1 is not a rational multiple of the transcendental number π. Therefore, $\sin(k)$ is not a periodic function of k, however, its range of values is dense in $[-1, 1]$. Moreover, it can be shown that given any neighbourhood of a point in $[-1, 1]$, $\sin(k)$ will revisit this neighbourhood at a rate of order k. Thus the conditions of Theorem 1 are also satisfied in this case.

Using the RLS algorithm, the estimate of the parameter matrix $\hat{\theta}$ converged rapidly to θ_0 to several significant figures. The convergence of the LMS algorithm with $\alpha = 0.025$ was somewhat

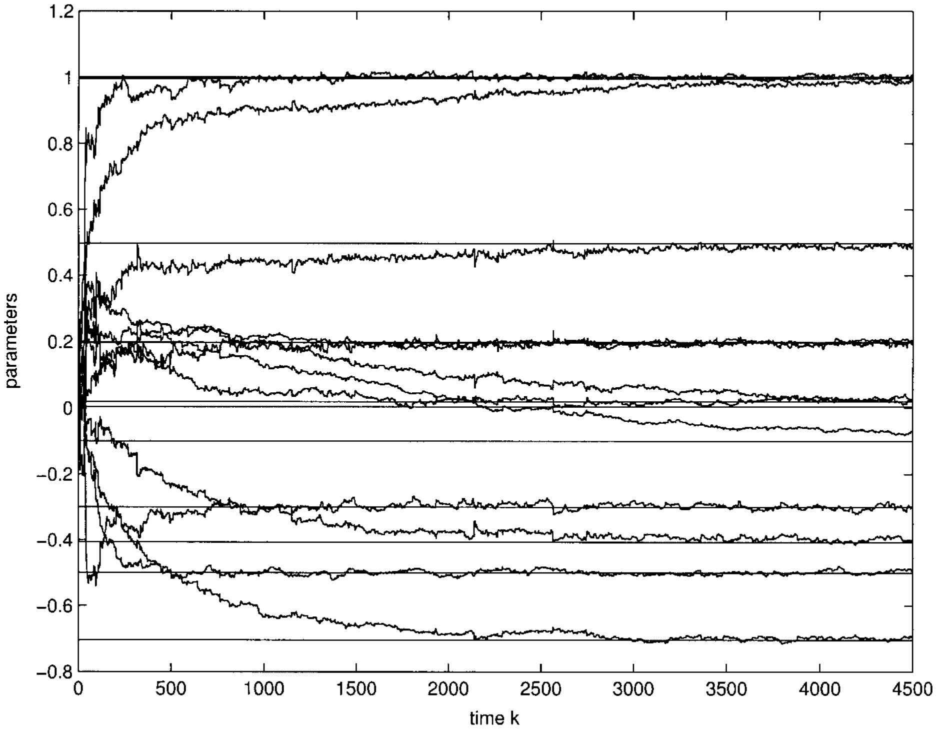

Figure 1. LMS algorithm: parameters' convergence.

slower. The estimated matrix at time $k = T$ for $p_k = \sin((\pi/3)k)$ was

$$\hat{\Theta}_k = \begin{bmatrix} 1.00 & -0.5 & 0.2 \\ 1.0 & -0.7 & -0.1 \\ 0.5 & -0.4 & 0.004 \\ 0.2 & -0.3 & 0.02 \end{bmatrix} \tag{35}$$

and the convergence of the parameters is shown in Figure 1.

The estimated matrix at time $k = T$ for $p_k = \sin(k)$ was

$$\hat{\Theta}_k = \begin{bmatrix} 1.00 & -0.5 & 0.2 \\ 0.99 & -0.7 & -0.1 \\ 0.49 & -0.4 & 0.007 \\ 0.2 & -0.3 & 0.02 \end{bmatrix} \tag{36}$$

and the convergence of the parameters is shown in Figure 2.

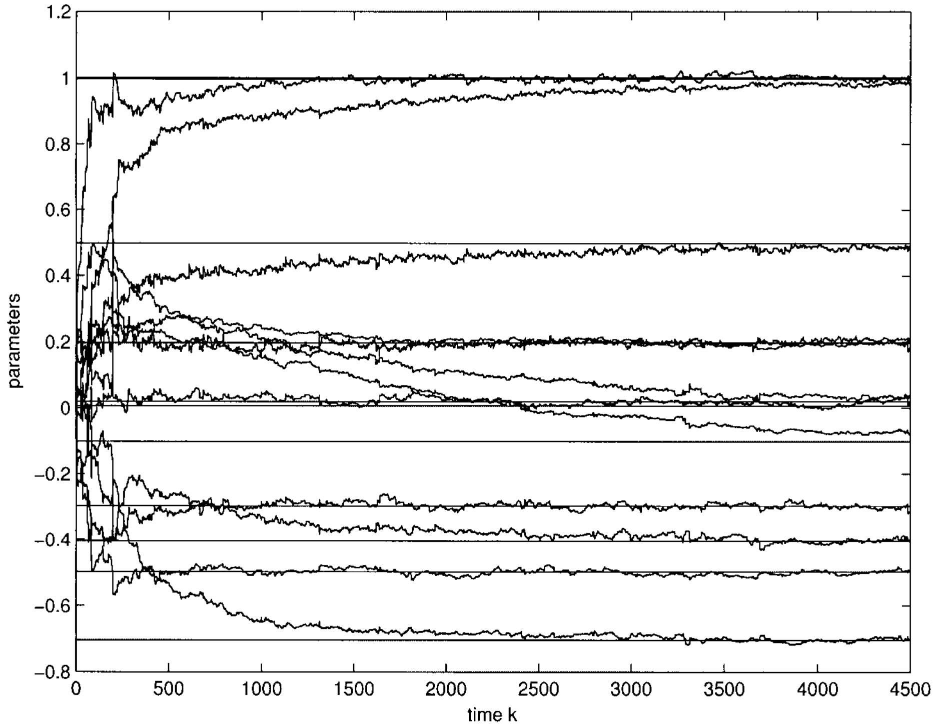

Figure 2. LMS algorithm: parameters' convergence.

7. CONCLUSIONS

We have considered LSs-type identification problems for LPV systems with polynomial dependence on the parameters. For this particular class, we have shown that the problem can be solved by linear regression. By representing the regression as a matrix inner product, we derived simple representation of the RLS and LMS algorithms, and conditions for persistency of excitation.

The persistency of excitation conditions have a natural interpretation in terms of function interpolation. The algorithms converge with arbitrary changes of the parameters representing the set points, as long as certain 'interpolating' values are visited regularly. This removes the typical restrictions of having to vary the set points slowly while collecting enough data to identify local LTI models.

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