Sequential Probability Ratio Test

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1 Boundaries

I have calculated the Wald's boundaries, where

$$
\begin{cases}
a_0 = \frac{\alpha_1}{1-\alpha_0} \\
a_1 = \frac{1-\alpha_1}{\alpha_0}
\end{cases}
$$

Comparing with given boundaries, Wald's boundaries are much wider and conservative. We use given boundaries in this project for a better performance. Boundary of side with a smaller error rate would be wider, since it is more strict to conclude that the statistic has reached boundary.

alpha_0	alpha_1	a0_Wald	a1_Wald	a0_given	a1_given
0.001	0.001	6.906755	6.906755	6.611755	6.611755
0.001	0.0001	9.209340	6.906755	8.91434	6.611755
0.0001	0.001	6.906755	9.209340	6.611755	8.91434

Table 1: Comparison of Wald Boundaries and Given Boundaries

2 Average Sample Number

The Average Sample Number (ASN) is computed by solving the linear equation
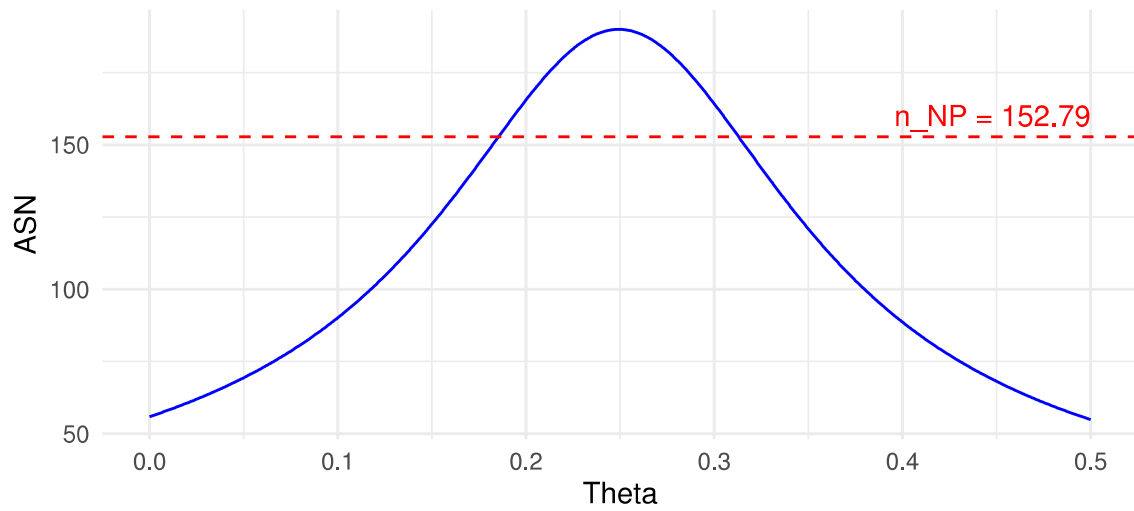
$$
\mathbf{u}_N = \boldsymbol{\nu} + \mathcal{K}_\vartheta \mathbf{u}_N,
$$

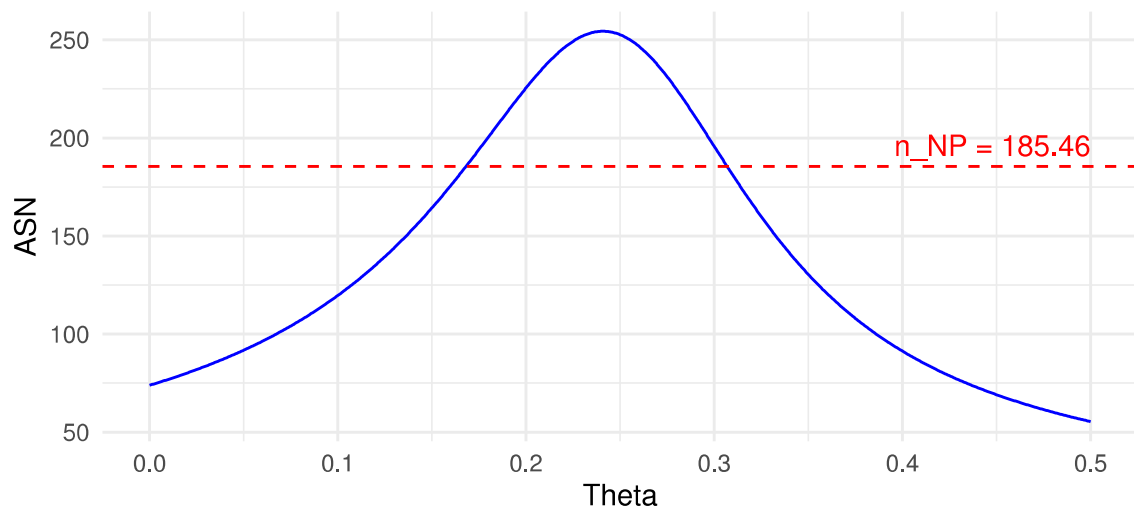then summing the product of $\mathbf{u}_N$'s with indicator list χ's, for each ϑ in $[\vartheta_0, \vartheta_1]$.

The ASN against ϑ plots are attached below. We can see for each senario, ASN would be small when ϑ is closer to end points ϑ_0 and ϑ_1, and very high when ϑ is in middle. The curve in senario 1 is symmetric, since $\alpha_1 = \alpha_2 = 0.001$. However, the curve in Senario 2 & 3 are skewed, since $\alpha_0 \neq \alpha_1$ in these senarios. The side with smaller α would need more observations to reject hypothesis.

The Neyman-Pearson test sample size is added to plots, as red dashed lines. From the plots, we can see that with most ϑ's, ASN is smaller than n_{NP}. When ϑ is in the middle, ASN would exceed n_{NP}. This means, in most times, SPRT is more efficient than NP-test.

ASN vs Theta (s = 1 , a0 = 6.611755 , a1 = 6.611755)

n_NP = 152.79

ASN vs Theta (s = 1 , a0 = 6.611755 , a1 = 8.91434)

n_NP = 185.46

ASN vs Theta (s = 1 , a0 = 6.611755 , a1 = 8.91434)

n_NP = 185.46