

Program Design (I)

Homework #3

Due: 2023/11/29 23:59

NOTE: Please upload your C program to DomJudge before the due date and time.

1. (30%) Yuji Itadori, a high school student, was entrusted by his grandfather before his passing to help others as much as possible and wished for him to die surrounded by people. Yuji Itadori encounters Megumi Fushiguro, who is looking for special-grade cursed objects at their school. In order to save his seniors who were in danger due to a cursed object, Yuji Itadori swallows a cursed finger, merging with the curse called "Ryomen Sukuna." After learning that only he can deal with Sukuna, he believes that it is his mission and joins the Tokyo Jujutsu High School, becoming a student of the strongest Jujutsu sorcerer, Satoru Gojo. He also became classmates with Yuji Itadori and Nobara Kugisaki.

Unfortunately, Yuji Itadori still has to study mathematics at Jujutsu High School. With his poor math skills, he was unable to determine the result of $x^y \bmod m$. If he couldn't solve the problem, he wouldn't be able to submit his assignment. Please use the advanced cursed technique, "Fast Power," to determine $x^y \bmod m$ for Yuji Itadori.

Please write a program using the Fast Power algorithm through a **recursive function** to compute $x^y \bmod m$.

Fast Power Algorithm

- To calculate x^{100} , we can use the formula $x^{100} = x^{50} \times x^{50}$. Then $x^{50} = x^{25} \times x^{25}$. Because the exponent of x^{25} is odd, we can compute x^{25} through $x^{25} = x \times x^{24}$. Since 24 is even, we can reuse the original formula $x^{24} = x^{12} \times x^{12}$, and so on.
- You can also module x by m after every step in the algorithm, such as $(x^{100} \bmod m) = (x^{50} \bmod m) \times (x^{50} \bmod m)$.

Input Format

- The input contains one line, which represents x , y , and m respectively, with space-separated.

Output Format

- An integer represents the result.

Technical Specifications

- $2 \leq x, y, m \leq 10^9$

The table below shows the example input and output. The underscored number is the input from users.

Input Example	Output Example
<u>3</u> 3 11	5
9 <u>18</u> 10	1

2. (30%) In one of Yuji Itadori's fights against the cursed spirit, a playing card, "Maxxx "C", in the game Yo-Gi-Oh! has dropped as a cursed object. The card creates certain cursed spirits based on the Fibonacci sequence when its controller is attacked. If Yuji Itadori is attacked n times, the number of cursed spirits on the field can be determined as follows:

0. Before the attack, no cursed spirit is on the field.
1. After the first attack, based on the Fibonacci sequence, 1 cursed spirit is generated since the first number of the Fibonacci sequence is 1.
2. After the second attack, another 1 cursed spirit is generated.
(the sum of 1 from the previous attack and 0 before the attack)
3. After the third attack, 2 more cursed spirits are generated
(the sum of 1 from the previous attack and 1 from the attack before the previous attack).
4. After the fourth attack, 3 more cursed spirits are generated
(the sum of 2 from the previous attacks and 1 from the attack before the previous attack).
5. After the fifth attack, 5 more cursed spirits are generated
(the sum of 3 from the previous attacks and 2 from the attack before the previous attack).

This pattern continues, following the Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, and so on.

Please determine the total number of spirits after Yuji Itadori is attacked n times.

Input Format

- A number n represents how many times Yuji Itadori is attacked.

Output Format

- A number represents the total number of cursed spirits that have been summoned.

Technical Specifications

- $1 \leq n \leq 100$

The table below shows the example input and output. The underscored number is the input from users.

Input Example	Output Example
<u>6</u>	20
<u>2</u>	2

3. (40%) Nevertheless, the world had to confront the Cursed King, Ryomen Sukuna. Satoru Gojo, being the strongest jujutsu sorcerer of modern times, became the first to challenge him. In their battle, Satoru Gojo utilized his advanced jujutsu technique, the "Purple" (Hollow Technique: Purple), to counter Ryomen Sukuna.

- Hollow Technique: Purple (虚式「茈」 Kyoshiki・Murasaki): An advanced technique that collides the Lapse and Reversal of the Limitless, resulting in an imaginary mass that is launched at the target.
- Cursed Technique Lapse: Blue (術式順転「蒼」 Jutsushiki Junten・Ao): The reinforced version of the neutral limitless; Blue is the byproduct of Gojo amplifying the Limitless and negative energy, producing a vacuum. Through these, Gojo creates a scenario where the world itself is forced to correct itself and fill in the negative space, causing all matter in the range of the commanded space to be pulled together.
- Cursed Technique Reversal: Red (術式反転「赫」 Jutsushiki Hanten・Aka): Being an inversion of Blue, Red has the opposite effect. Using his reversal technique to invert the properties of Blue, Satoru activates the divergence of his infinity and causes an extremely powerful repulsive force, repelling all matter in its vicinity. Being an ability that is powered by positive cursed energy and formed as a reverse curse technique, the ability has two times the destructive ability of Blue.
- Limitless (無下限 Mukagen): At the base level, Limitless is an innate technique that is derived from the Gojo family. This technique grants its users high-absolute control over space through cursed energy manipulation at an atomic level, resulting in multiple subsequent results and techniques within the overall ability. With the manipulation of space as just the base level of the technique, Limitless has three standard and one "non-standard" form beyond this.

As mentioned, Purple requires the collision of Red and Blue. In order to control the power of "Purple", Satoru Gojo must accurately determine the ratio of "Blue" and "Red". The equation to calculate the power of "Purple" can be represented as follows: $P = R^8 + 7 \cdot B^3$. Please compute the number of R and B when Satoru Gojo wants to release "Purple" with a certain power P to avoid himself from being .

Input Format

- A number P represents the power of "Purple".

Output Format

- R and B with space-separated.

Technical Specifications

- $1 \leq P \leq 10^{18}$

Hint:

- The idea to solve the formula:
 - Reorganize the formula to the form of $7B^3 = P - R^8$ and use the range of P to determine the range of R , hence deciding the data type to use in the program.
 - Observe the reorganized formula, search the range of R to find a value of R that lets $P - R^8$ fulfills the characteristic of the reorganized formula.
 - Suppose we found a value K from the previous step, now, we need to find the value of B .
 - One of the ideas is to use brute force that searches every possibility of B to find B . The time complexity of brute force is $O(n^2)$, hence this method is unable to find B under the time limit.
 - The alternative way is to use **binary search** to compute the value of B .
 - The idea of writing a binary search for this situation:
 1. Suppose $L = 0, R = (P - R^8)/7$.
 2. If $Mid > B^3, R = Mid - 1$
 3. If $Mid < B^3, L = Mid + 1$
 4. If $Mid == B$ return Mid
 - Precise form of binary search:
(<https://www.geeksforgeeks.org/binary-search/>)
- The range of `pow()` in `math.h` is unable to compute the value required.

The table below shows the example input and output. The underscored number is the input from users.

Input Example	Output Example
<u>8</u>	1 1
<u>99927</u>	4 17
<u>697575744113026069</u>	170 123