Feynman Rules Brillouin

(Written by Maximilian Ammer, 2023)

This notebook generates the fermionic Feynman rules for the qqg-, qqgg-, and qqggg-vertices in lattice perturbation theory for the Brillouin fermion action.

The resulting functions are saved to files, which can be loaded by other notebooks for further calculations.

```
SetDirectory["~path~"];
```

In[•]:=
```
Get["myLPT.m"]
Clear[p, q, k, 𝕀, 𝔵, σ]
```

Brillouin action

In[•]:=
```
w2[mu_, nu_, x_] := U[1, mu, x] × U[2, nu, x + e[mu]]
w3[mu_, nu_, rho_, x_] :=
 U[1, mu, x] × U[2, nu, x + e[mu]] × U[3, rho, x + e[mu] + e[nu]]
w4[mu_, nu_, rho_, sigma_, x_] := U[1, mu, x] × U[2, nu, x + e[mu]] ×
  U[3, rho, x + e[mu] + e[nu]] × U[4, sigma, x + e[mu] + e[nu] + e[rho]]
```

In[•]:=
```
W1[mu_, x_] := U[1, mu, x]
W2[mu_, nu_, x_] := 1 / 2 (w2[mu, nu, x] + w2[nu, mu, x])
W3[mu_, nu_, rho_, x_] :=
 1 / 6 (w3[mu, nu, rho, x] + w3[mu, rho, nu, x] + w3[rho, mu, nu, x] +
       w3[nu, mu, rho, x] + w3[nu, rho, mu, x] + w3[rho, nu, mu, x])
W4[mu_, nu_, rho_, sigma_, x_] :=
 1 / 24 (w4[mu, nu, rho, sigma, x] + w4[mu, nu, sigma, rho, x]
      + w4[mu, rho, nu, sigma, x] + w4[mu, rho, sigma, nu, x]
      + w4[mu, sigma, nu, rho, x] + w4[mu, sigma, rho, nu, x]
      + w4[nu, mu, rho, sigma, x] + w4[nu, mu, sigma, rho, x]
      + w4[nu, rho, mu, sigma, x] + w4[nu, rho, sigma, mu, x]
      + w4[nu, sigma, mu, rho, x] + w4[nu, sigma, rho, mu, x]
      + w4[rho, mu, nu, sigma, x] + w4[rho, mu, sigma, nu, x]
      + w4[rho, nu, mu, sigma, x] + w4[rho, nu, sigma, mu, x]
      + w4[rho, sigma, nu, mu, x] + w4[rho, sigma, mu, nu, x]
      + w4[sigma, mu, nu, rho, x] + w4[sigma, mu, rho, nu, x]
      + w4[sigma, nu, mu, rho, x] + w4[sigma, nu, rho, mu, x]
      + w4[sigma, rho, mu, nu, x] + w4[sigma, rho, nu, mu, x])
```

```
(*
S1=(ϱ1   ϒ[mu] - r λ1/2);
S2=(ϱ2   ϒ[mu] - r λ2/4);
S3=(ϱ3/2 ϒ[mu] - r λ3/12);
S4=(ϱ4/6 ϒ[mu] - r λ4/48);
*)
BrillouinDiracOp = -r λ0 / 2 δ[x, y] +
    Sum[S1[i mu] × W1[i mu, x] × δ[x + i e[mu], y], {i, {-1, 1}}] +
    Sum[S2[i mu] × W2[i mu, j nu1, x] × δ[x + i e[mu] + j e[nu1], y] ×
      SUM[nu1] Boole[nu1 ≠ mu], {i, {-1, 1}}, {j, {-1, 1}}] +
    Sum[S3[i mu] × W3[i mu, j nu1, k nu2, x] × δ[x + i e[mu] + j e[nu1] + k e[nu2], y] ×
      SUM[nu1] × SUM[nu2] Boole[nu1 ≠ nu2 ≠ mu],
     {i, {-1, 1}}, {j, {-1, 1}}, {k, {-1, 1}}] +
    Sum[S4[i mu] × W4[i mu, j nu1, k nu2, l nu3, x] ×
      δ[x + i e[mu] + j e[nu1] + k e[nu2] + l e[nu3], y] ×
      SUM[nu1] × SUM[nu2] × SUM[nu3] Boole[nu1 ≠ nu2 ≠ nu3 ≠ mu],
     {i, {-1, 1}}, {j, {-1, 1}}, {k, {-1, 1}}, {l, {-1, 1}}];
```

```
(* Brillouin action in position space *)
SBrillouin = ψbar[x] BrillouinDiracOp ψ[y];
```

```
(* Brillouin action in momentum space *)
SBrillouinFT = SBrillouin // myExpandU // myFourierTransform
```

Out[ ]=

$$-\frac{1}{2}\, e^{-i\,q\,x+i\,s\,(x-y)+i\,p\,y}\, r\,\lambda 0\, \text{Int}[p]\times \text{Int}[q]\times \text{Int}[s]\times \Psi[p]\times \Psi\text{bar}[q] +$$
$$e^{-i\,q\,x+i\,p\,y+i\,s\,(x-y-e[mu])}\, \text{Int}[p]\times \text{Int}[q]\times \text{Int}[s]\times S1[-mu]\times \Psi[p]\times \Psi\text{bar}[q] +$$
$$e^{\langle\!\langle 1 \rangle\!\rangle}\, \text{Int}[p]\times \text{Int}[q]\times \text{Int}[s]\times S1[mu]\times \Psi[p]\times \Psi\text{bar}[q] + \langle\!\langle 13\,994 \rangle\!\rangle$$

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Vertices

QQG Vertex

```
V1 = g0 Coefficient[SBrillouinFT, g0];
```

```
V11 = (ϱ1 ϒ[mu] - r 𝕀 λ1 / 2) Coefficient[V1, S1[mu]] +
    (-ϱ1 ϒ[mu] - r 𝕀 λ1 / 2) Coefficient[V1, S1[-mu]];
V12 = (ϱ2 ϒ[mu] - r 𝕀 λ2 / 4) Coefficient[V1, S2[mu]] +
    (-ϱ2 ϒ[mu] - r 𝕀 λ2 / 4) Coefficient[V1, S2[-mu]];
V13 = (ϱ3 / 2 ϒ[mu] - r 𝕀 λ3 / 12) Coefficient[V1, S3[mu]] +
    (-ϱ3 / 2 ϒ[mu] - r 𝕀 λ3 / 12) Coefficient[V1, S3[-mu]];
V14 = (ϱ4 / 6 ϒ[mu] - r 𝕀 λ4 / 48) Coefficient[V1, S4[mu]] +
    (-ϱ4 / 6 ϒ[mu] - r 𝕀 λ4 / 48) Coefficient[V1, S4[-mu]];
```

In[ ]:=
```
V1f1 = myFRintegrate[V11] // myFRbreakdown
```

Out[ ]=

$$-\text{g0}\,\text{r}\,\mathbb{I}\,\lambda 1\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a]}-2\,\text{i}\,\text{g0}\,\varrho 1\,\text{Cos}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a]}\times\gamma[\text{mu}]$$

In[ ]:=
```
V1f2 = Collect[myFRintegrate[V12] // myFRbreakdown, {g0 r λ2 , g0 ϱ2 }]
```

Out[ ]=

$$\begin{aligned}
&\text{g0}\,\text{r}\,\lambda 2\\
&\Bigg(-\frac{1}{2}\,\mathbb{I}\,\text{Boole[1}\neq\text{mu]}\,\text{Cos[p[1]]}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a]}-\frac{1}{2}\,\mathbb{I}\,\text{Boole[mu}\neq\text{1]}\,\text{Cos[p[1]]}\\
&\quad\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a]}-\frac{1}{2}\,\mathbb{I}\,\text{Boole[2}\neq\text{mu]}\,\text{Cos[p[2]]}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a]}-\\
&\quad\frac{1}{2}\,\mathbb{I}\,\text{Boole[mu}\neq\text{2]}\,\text{Cos[p[2]]}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a]}-\\
&\quad\frac{1}{2}\,\mathbb{I}\,\text{Boole[3}\neq\text{mu]}\,\text{Cos[p[3]]}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a]}-\\
&\quad\frac{1}{2}\,\mathbb{I}\,\text{Boole[mu}\neq\text{3]}\,\text{Cos[p[3]]}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a]}-\\
&\quad\frac{1}{2}\,\mathbb{I}\,\text{Boole[4}\neq\text{mu]}\,\text{Cos[p[4]]}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a]}-\\
&\quad\frac{1}{2}\,\mathbb{I}\,\text{Boole[mu}\neq\text{4]}\,\text{Cos[p[4]]}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a]}-\\
&\quad\frac{1}{2}\,\mathbb{I}\,\text{Boole[1}\neq\text{mu]}\,\text{Cos[q[1]]}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a]}-\\
&\quad\frac{1}{2}\,\mathbb{I}\,\text{Boole[mu}\neq\text{1]}\,\text{Cos[q[1]]}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a]}-\\
&\quad\frac{1}{2}\,\mathbb{I}\,\text{Boole[2}\neq\text{mu]}\,\text{Cos[q[2]]}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a]}-\\
&\quad\frac{1}{2}\,\mathbb{I}\,\text{Boole[mu}\neq\text{2]}\,\text{Cos[q[2]]}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a]}-\\
&\quad\frac{1}{2}\,\mathbb{I}\,\text{Boole[3}\neq\text{mu]}\,\text{Cos[q[3]]}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a]}-\\
&\quad\frac{1}{2}\,\mathbb{I}\,\text{Boole[mu}\neq\text{3]}\,\text{Cos[q[3]]}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a]}-\\
&\quad\frac{1}{2}\,\mathbb{I}\,\text{Boole[4}\neq\text{mu]}\,\text{Cos[q[4]]}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a]}-\\
&\quad\frac{1}{2}\,\mathbb{I}\,\text{Boole[mu}\neq\text{4]}\,\text{Cos[q[4]]}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a]}\Bigg)+\\
&\text{g0}\,\varrho 2\,\Bigg(2\,\text{i}\,\text{Boole[mu}\neq\text{1]}\,\text{Sin[p[1]]}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a]}\times\gamma[1]+\\
&\quad 2\,\text{i}\,\text{Boole[mu}\neq\text{1]}\,\text{Sin[q[1]]}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a]}\times\gamma[1]+\\
&\quad 2\,\text{i}\,\text{Boole[mu}\neq\text{2]}\,\text{Sin[p[2]]}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a]}\times\gamma[2]+\\
&\quad 2\,\text{i}\,\text{Boole[mu}\neq\text{2]}\,\text{Sin[q[2]]}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a]}\times\gamma[2]+\\
&\quad 2\,\text{i}\,\text{Boole[mu}\neq\text{3]}\,\text{Sin[p[3]]}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a]}\times\gamma[3]+
\end{aligned}$$

$$2\,i\,\text{Boole}[\text{mu}\neq 3]\,\text{Sin}[q[3]]\,\text{Sin}\left[\frac{p[\text{mu}]}{2}+\frac{q[\text{mu}]}{2}\right]\,T[a]\times\gamma[3]+$$
$$2\,i\,\text{Boole}[\text{mu}\neq 4]\,\text{Sin}[p[4]]\,\text{Sin}\left[\frac{p[\text{mu}]}{2}+\frac{q[\text{mu}]}{2}\right]\,T[a]\times\gamma[4]+$$
$$2\,i\,\text{Boole}[\text{mu}\neq 4]\,\text{Sin}[q[4]]\,\text{Sin}\left[\frac{p[\text{mu}]}{2}+\frac{q[\text{mu}]}{2}\right]\,T[a]\times\gamma[4]-$$
$$2\,i\,\text{Boole}[1\neq \text{mu}]\,\text{Cos}[p[1]]\,\text{Cos}\left[\frac{p[\text{mu}]}{2}+\frac{q[\text{mu}]}{2}\right]\,T[a]\times\gamma[\text{mu}]-$$
$$2\,i\,\text{Boole}[2\neq \text{mu}]\,\text{Cos}[p[2]]\,\text{Cos}\left[\frac{p[\text{mu}]}{2}+\frac{q[\text{mu}]}{2}\right]\,T[a]\times\gamma[\text{mu}]-$$
$$2\,i\,\text{Boole}[3\neq \text{mu}]\,\text{Cos}[p[3]]\,\text{Cos}\left[\frac{p[\text{mu}]}{2}+\frac{q[\text{mu}]}{2}\right]\,T[a]\times\gamma[\text{mu}]-$$
$$2\,i\,\text{Boole}[4\neq \text{mu}]\,\text{Cos}[p[4]]\,\text{Cos}\left[\frac{p[\text{mu}]}{2}+\frac{q[\text{mu}]}{2}\right]\,T[a]\times\gamma[\text{mu}]-$$
$$2\,i\,\text{Boole}[1\neq \text{mu}]\,\text{Cos}[q[1]]\,\text{Cos}\left[\frac{p[\text{mu}]}{2}+\frac{q[\text{mu}]}{2}\right]\,T[a]\times\gamma[\text{mu}]-$$
$$2\,i\,\text{Boole}[2\neq \text{mu}]\,\text{Cos}[q[2]]\,\text{Cos}\left[\frac{p[\text{mu}]}{2}+\frac{q[\text{mu}]}{2}\right]\,T[a]\times\gamma[\text{mu}]-$$
$$2\,i\,\text{Boole}[3\neq \text{mu}]\,\text{Cos}[q[3]]\,\text{Cos}\left[\frac{p[\text{mu}]}{2}+\frac{q[\text{mu}]}{2}\right]\,T[a]\times\gamma[\text{mu}]-$$
$$2\,i\,\text{Boole}[4\neq \text{mu}]\,\text{Cos}[q[4]]\,\text{Cos}\left[\frac{p[\text{mu}]}{2}+\frac{q[\text{mu}]}{2}\right]\,T[a]\times\gamma[\text{mu}]\Big)$$

In[•]:=
```
V1f3 = Collect[myFRintegrate[V13] // myFRbreakdown, {g0 r λ3 , g0 ϱ3 }]
```

Out[•]=

$$g0\,r\,\lambda 3\left(-\frac{1}{9}\,\mathbb{I}\,\text{Boole}[\text{mu}\neq 1\neq \text{mu}]\,\text{Cos}[p[1]]\,\text{Sin}\left[\frac{p[\text{mu}]}{2}-\frac{q[\text{mu}]}{2}\right]\,T[a]-\right.$$
$$\frac{1}{9}\,\mathbb{I}\,\text{Boole}[\text{mu}\neq 2\neq \text{mu}]\,\text{Cos}[p[2]]\,\text{Sin}\left[\frac{p[\text{mu}]}{2}-\frac{q[\text{mu}]}{2}\right]\,T[a]-$$
$$\frac{1}{9}\,\mathbb{I}\,\text{Boole}[\text{mu}\neq 3\neq \text{mu}]\,\text{Cos}[p[3]]\,\text{Sin}\left[\frac{p[\text{mu}]}{2}-\frac{q[\text{mu}]}{2}\right]\,T[a]-$$
$$\frac{1}{9}\,\mathbb{I}\,\text{Boole}[\text{mu}\neq 4\neq \text{mu}]\,\text{Cos}[p[4]]\,\text{Sin}\left[\frac{p[\text{mu}]}{2}-\frac{q[\text{mu}]}{2}\right]\,T[a]+$$
$$\frac{1}{9}\,\mathbb{I}\,\text{Boole}[\text{mu}\neq 1\neq \text{mu}]\,\text{Cos}[q[1]]\,\text{Sin}\left[\frac{p[\text{mu}]}{2}-\frac{q[\text{mu}]}{2}\right]\,T[a]+\langle\!\langle 371\rangle\!\rangle+$$
$$\frac{2}{9}\,\mathbb{I}\,\text{Boole}[\text{mu}\neq 1\neq \text{mu}]\,\text{Cos}[q[1]]\,\text{Sin}\left[\frac{p[\text{mu}]}{2}+\frac{3\,q[\text{mu}]}{2}\right]\,T[a]+$$
$$\frac{2}{9}\,\mathbb{I}\,\text{Boole}[\text{mu}\neq 2\neq \text{mu}]\,\text{Cos}[q[2]]\,\text{Sin}\left[\frac{p[\text{mu}]}{2}+\frac{3\,q[\text{mu}]}{2}\right]\,T[a]+$$
$$\frac{2}{9}\,\mathbb{I}\,\text{Boole}[\text{mu}\neq 3\neq \text{mu}]\,\text{Cos}[q[3]]\,\text{Sin}\left[\frac{p[\text{mu}]}{2}+\frac{3\,q[\text{mu}]}{2}\right]\,T[a]+$$
$$\left.\frac{2}{9}\,\mathbb{I}\,\text{Boole}[\text{mu}\neq 4\neq \text{mu}]\,\text{Cos}[q[4]]\,\text{Sin}\left[\frac{p[\text{mu}]}{2}+\frac{3\,q[\text{mu}]}{2}\right]\,T[a]\right)+g0\,\varrho 3\,(\langle\!\langle 1\rangle\!\rangle)$$

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In[•]:=
```
V1f4 = Collect[myFRintegrate[V14] // myFRbreakdown, {g0 r λ4 , g0 ϱ4 }] /.
    Boole[a_ ≠ a_ ≠ mu] → 0 /. Boole[a_ ≠ mu ≠ a_] → 0 /. Boole[mu ≠ a_ ≠ a_] → 0
```

Out[•]=

$$g0\, r\, \lambda 4 \left(\cdots 3195 \cdots + \frac{1}{12}\, \mathbb{I}\, \text{Boole}[\text{mu} \neq 4 \neq 2 \neq \text{mu}]\, \text{Cos}[q[2]]\, \text{Cos}[q[4]]\, \text{Sin}\left[\frac{p[\text{mu}]}{2} + \frac{3\, \cdots 1 \cdots}{2}\right] T[a] + \frac{1}{12}\, \mathbb{I}\, \text{Boole}[\text{mu} \neq 3 \neq 4 \neq \text{mu}]\, \text{Cos}[q[3]]\, \text{Cos}[q[4]]\, \text{Sin}\left[\frac{p[\text{mu}]}{2} + \frac{3\, q[\text{mu}]}{2}\right] T[a] + \frac{1}{12}\, \mathbb{I}\, \text{Boole}[\text{mu} \neq 4 \neq 3 \neq \text{mu}]\, \text{Cos}[q[3]]\, \text{Cos}[q[4]]\, \text{Sin}\left[\frac{p[\text{mu}]}{2} + \frac{3\, q[\text{mu}]}{2}\right] T[a] + \frac{1}{12}\, \mathbb{I}\, \text{Boole}[\text{mu} \neq 4 \neq 4 \neq \text{mu}]\, \text{Cos}[q[4]]^2\, \text{Sin}\left[\frac{p[\text{mu}]}{2} + \frac{3\, q[\text{mu}]}{2}\right] T[a] \right) + g0\, \varrho 4 \left(\frac{2}{3}\, \mathbb{i}\, \text{Boole}[1 \neq \text{mu} \neq 1 \neq 1]\, \text{Cos}[p[1]]^2\, \text{Sin}[p[1]]\, \text{Sin}\left[\frac{p[\text{mu}]}{2} + \frac{q[\text{mu}]}{2}\right] T[a] \times \gamma[1] + \frac{2}{3}\, \mathbb{i}\, \text{Boole}[\text{mu} \neq 1 \neq 1 \neq 1]\, \text{Cos}[p[1]]^2\, \text{Sin}[p[1]]\, \text{Sin}\left[\frac{p[\text{mu}]}{2} + \frac{q[\text{mu}]}{2}\right] T[a] \times \gamma[1] + \cdots 2181 \cdots + \frac{2}{3}\, \mathbb{i}\, \text{Boole}[\text{mu} \neq 4 \neq 4 \neq \text{mu}]\, \text{Cos}[q[4]]^2\, \text{Cos}\left[\frac{p[\text{mu}]}{2} + \frac{3\, q[\text{mu}]}{2}\right] T[a] \times \gamma[\text{mu}] \right)$$

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In[•]:=
```
V1f = V1f1 + V1f2 + V1f3 + V1f4;
```

In[•]:=
```
(* Preparing for saving and loading *)
V1Out = V1f /. T[a__] :> 1 // ExpandAll;
```

In[•]:=
```
(* Saving *)
V1Brillouin[mu_, p_, q_] := Evaluate[V1Out]
DumpSave["V1_Brillouin.mx", V1Brillouin]
```

Out[•]=
```
{V1Brillouin}
```

QQGG Vertex

In[•]:=
```
V2 = g0^2 Coefficient[SBrillouinFT, g0^2];
```

In[•]:=
```
V21 = (ϱ1 γ[mu] - r I λ1 / 2) Coefficient[V2, S1[mu]] +
    (-ϱ1 γ[mu] - r I λ1 / 2) Coefficient[V2, S1[-mu]];
V22 = (ϱ2 γ[mu] - r I λ2 / 4) Coefficient[V2, S2[mu]] +
    (-ϱ2 γ[mu] - r I λ2 / 4) Coefficient[V2, S2[-mu]];
V23 = (ϱ3 / 2 γ[mu] - r I λ3 / 12) Coefficient[V2, S3[mu]] +
    (-ϱ3 / 2 γ[mu] - r I λ3 / 12) Coefficient[V2, S3[-mu]];
V24 = (ϱ4 / 6 γ[mu] - r I λ4 / 48) Coefficient[V2, S4[mu]] +
    (-ϱ4 / 6 γ[mu] - r I λ4 / 48) Coefficient[V2, S4[-mu]];
```

In[•]:=
```
V2f1 = myFRintegrate[V21] // myFRbreakdown
```

Out[•]=

$$-\frac{1}{2}\, g0^2\, r\, \mathbb{I}\, \lambda 1\, \text{Cos}\left[\frac{p[\text{mu}]}{2} + \frac{q[\text{mu}]}{2}\right] \delta_{\text{mu,nu}}\, T[a, b] + \mathbb{i}\, g0^2\, \varrho 1\, \delta_{\text{mu,nu}}\, \text{Sin}\left[\frac{p[\text{mu}]}{2} + \frac{q[\text{mu}]}{2}\right] T[a, b] \times \gamma[\text{mu}]$$

In[•]:=
```
V2f2 = Collect[myFRintegrate[V22] // myFRbreakdown, {λ2 , ϱ2 }]
```

Out[•]=

$$
\begin{aligned}
&\lambda 2 \left(-\frac{1}{4}\, g0^2\, r\, \mathbb{I}\, \text{Boole}[1 \neq mu]\, \text{Cos}[p[1]]\, \text{Cos}\left[\frac{p[mu]}{2} + \frac{q[mu]}{2}\right] \delta_{mu,nu}\, T[a, b] - \right. \\
&\quad \frac{1}{4}\, g0^2\, r\, \mathbb{I}\, \text{Boole}[mu \neq 1]\, \text{Cos}[p[1]]\, \text{Cos}\left[\frac{p[mu]}{2} + \frac{q[mu]}{2}\right] \delta_{mu,nu}\, T[a, b] - \\
&\quad \frac{1}{4}\, g0^2\, r\, \mathbb{I}\, \text{Boole}[2 \neq mu]\, \text{Cos}[p[2]]\, \text{Cos}\left[\frac{p[mu]}{2} + \frac{q[mu]}{2}\right] \delta_{mu,nu}\, T[a, b] - \\
&\quad \frac{1}{4}\, g0^2\, r\, \mathbb{I}\, \text{Boole}[mu \neq 2]\, \text{Cos}[p[2]]\, \text{Cos}\left[\frac{p[mu]}{2} + \frac{q[mu]}{2}\right] \delta_{mu,nu}\, T[a, b] - \\
&\quad \frac{1}{4}\, g0^2\, r\, \mathbb{I}\, \text{Boole}[3 \neq mu]\, \text{Cos}[p[3]]\, \text{Cos}\left[\frac{p[mu]}{2} + \frac{q[mu]}{2}\right] \delta_{mu,nu}\, T[a, b] - \\
&\quad \frac{1}{4}\, g0^2\, r\, \mathbb{I}\, \text{Boole}[mu \neq 3]\, \text{Cos}[p[3]]\, \text{Cos}\left[\frac{p[mu]}{2} + \frac{q[mu]}{2}\right] \delta_{mu,nu}\, T[a, b] - \\
&\quad \frac{1}{4}\, g0^2\, r\, \mathbb{I}\, \text{Boole}[4 \neq mu]\, \text{Cos}[p[4]]\, \text{Cos}\left[\frac{p[mu]}{2} + \frac{q[mu]}{2}\right] \delta_{mu,nu}\, T[a, b] - \\
&\quad \frac{1}{4}\, g0^2\, r\, \mathbb{I}\, \text{Boole}[mu \neq 4]\, \text{Cos}[p[4]]\, \text{Cos}\left[\frac{p[mu]}{2} + \frac{q[mu]}{2}\right] \delta_{mu,nu}\, T[a, b] - \\
&\quad \frac{1}{4}\, g0^2\, r\, \mathbb{I}\, \text{Boole}[1 \neq mu]\, \text{Cos}[q[1]]\, \text{Cos}\left[\frac{p[mu]}{2} + \frac{q[mu]}{2}\right] \delta_{mu,nu}\, T[a, b] - \\
&\quad \frac{1}{4}\, g0^2\, r\, \mathbb{I}\, \text{Boole}[mu \neq 1]\, \text{Cos}[q[1]]\, \text{Cos}\left[\frac{p[mu]}{2} + \frac{q[mu]}{2}\right] \delta_{mu,nu}\, T[a, b] - \\
&\quad \frac{1}{4}\, g0^2\, r\, \mathbb{I}\, \text{Boole}[2 \neq mu]\, \text{Cos}[q[2]]\, \text{Cos}\left[\frac{p[mu]}{2} + \frac{q[mu]}{2}\right] \delta_{mu,nu}\, T[a, b] - \\
&\quad \frac{1}{4}\, g0^2\, r\, \mathbb{I}\, \text{Boole}[mu \neq 2]\, \text{Cos}[q[2]]\, \text{Cos}\left[\frac{p[mu]}{2} + \frac{q[mu]}{2}\right] \delta_{mu,nu}\, T[a, b] - \\
&\quad \frac{1}{4}\, g0^2\, r\, \mathbb{I}\, \text{Boole}[3 \neq mu]\, \text{Cos}[q[3]]\, \text{Cos}\left[\frac{p[mu]}{2} + \frac{q[mu]}{2}\right] \delta_{mu,nu}\, T[a, b] - \\
&\quad \frac{1}{4}\, g0^2\, r\, \mathbb{I}\, \text{Boole}[mu \neq 3]\, \text{Cos}[q[3]]\, \text{Cos}\left[\frac{p[mu]}{2} + \frac{q[mu]}{2}\right] \delta_{mu,nu}\, T[a, b] - \\
&\quad \frac{1}{4}\, g0^2\, r\, \mathbb{I}\, \text{Boole}[4 \neq mu]\, \text{Cos}[q[4]]\, \text{Cos}\left[\frac{p[mu]}{2} + \frac{q[mu]}{2}\right] \delta_{mu,nu}\, T[a, b] - \\
&\quad \frac{1}{4}\, g0^2\, r\, \mathbb{I}\, \text{Boole}[mu \neq 4]\, \text{Cos}[q[4]]\, \text{Cos}\left[\frac{p[mu]}{2} + \frac{q[mu]}{2}\right] \delta_{mu,nu}\, T[a, b] + \\
&\quad \frac{1}{2}\, g0^2\, r\, \mathbb{I}\, \text{Boole}[mu \neq nu]\, \text{Sin}\left[\frac{k2[nu]}{2} + p[nu]\right] \text{Sin}\left[\frac{k2[mu]}{2} + \frac{p[mu]}{2} + \frac{q[mu]}{2}\right] T[a, b] + \\
&\quad \frac{1}{2}\, g0^2\, r\, \mathbb{I}\, \text{Boole}[nu \neq mu]\, \text{Sin}\left[\frac{k2[nu]}{2} + p[nu]\right] \\
&\quad \left. \text{Sin}\left[\frac{k2[mu]}{2} + \frac{p[mu]}{2} + \frac{q[mu]}{2}\right] T[a, b] \right) + \\
&\varrho 2 \left(\mathbb{i}\, g0^2\, \text{Boole}[mu \neq 1]\, \text{Cos}\left[\frac{p[mu]}{2} + \frac{q[mu]}{2}\right] \delta_{mu,nu}\, \text{Sin}[p[1]]\, T[a, b] \times \gamma[1] + \right. \\
&\quad \mathbb{i}\, g0^2\, \text{Boole}[mu \neq 1]\, \text{Cos}\left[\frac{p[mu]}{2} + \frac{q[mu]}{2}\right] \delta_{mu,nu}\, \text{Sin}[q[1]]\, T[a, b] \times \gamma[1] + \\
&\quad \mathbb{i}\, g0^2\, \text{Boole}[mu \neq 2]\, \text{Cos}\left[\frac{p[mu]}{2} + \frac{q[mu]}{2}\right] \delta_{mu,nu}\, \text{Sin}[p[2]]\, T[a, b] \times \gamma[2] +
\end{aligned}
$$

$$\begin{aligned}
&\mathrm{i}\, g0^2\, \mathrm{Boole}[\mathrm{mu} \neq 2]\, \mathrm{Cos}\left[\frac{p[\mathrm{mu}]}{2} + \frac{q[\mathrm{mu}]}{2}\right] \delta_{\mathrm{mu},\mathrm{nu}}\, \mathrm{Sin}[q[2]]\, T[a, b] \times \gamma[2] + \\
&\mathrm{i}\, g0^2\, \mathrm{Boole}[\mathrm{mu} \neq 3]\, \mathrm{Cos}\left[\frac{p[\mathrm{mu}]}{2} + \frac{q[\mathrm{mu}]}{2}\right] \delta_{\mathrm{mu},\mathrm{nu}}\, \mathrm{Sin}[p[3]]\, T[a, b] \times \gamma[3] + \\
&\mathrm{i}\, g0^2\, \mathrm{Boole}[\mathrm{mu} \neq 3]\, \mathrm{Cos}\left[\frac{p[\mathrm{mu}]}{2} + \frac{q[\mathrm{mu}]}{2}\right] \delta_{\mathrm{mu},\mathrm{nu}}\, \mathrm{Sin}[q[3]]\, T[a, b] \times \gamma[3] + \\
&\mathrm{i}\, g0^2\, \mathrm{Boole}[\mathrm{mu} \neq 4]\, \mathrm{Cos}\left[\frac{p[\mathrm{mu}]}{2} + \frac{q[\mathrm{mu}]}{2}\right] \delta_{\mathrm{mu},\mathrm{nu}}\, \mathrm{Sin}[p[4]]\, T[a, b] \times \gamma[4] + \\
&\mathrm{i}\, g0^2\, \mathrm{Boole}[\mathrm{mu} \neq 4]\, \mathrm{Cos}\left[\frac{p[\mathrm{mu}]}{2} + \frac{q[\mathrm{mu}]}{2}\right] \delta_{\mathrm{mu},\mathrm{nu}}\, \mathrm{Sin}[q[4]]\, T[a, b] \times \gamma[4] + \\
&2\, \mathrm{i}\, g0^2\, \mathrm{Boole}[\mathrm{nu} \neq \mathrm{mu}]\, \mathrm{Cos}\left[\frac{k2[\mathrm{mu}]}{2} + \frac{p[\mathrm{mu}]}{2} + \frac{q[\mathrm{mu}]}{2}\right] \mathrm{Sin}\left[\frac{k2[\mathrm{nu}]}{2} + p[\mathrm{nu}]\right] T[a, b] \times \\
&\gamma[\mathrm{mu}] + \mathrm{i}\, g0^2\, \mathrm{Boole}[1 \neq \mathrm{mu}]\, \mathrm{Cos}[p[1]]\, \delta_{\mathrm{mu},\mathrm{nu}}\, \mathrm{Sin}\left[\frac{p[\mathrm{mu}]}{2} + \frac{q[\mathrm{mu}]}{2}\right] T[a, b] \times \gamma[\mathrm{mu}] + \\
&\mathrm{i}\, g0^2\, \mathrm{Boole}[2 \neq \mathrm{mu}]\, \mathrm{Cos}[p[2]]\, \delta_{\mathrm{mu},\mathrm{nu}}\, \mathrm{Sin}\left[\frac{p[\mathrm{mu}]}{2} + \frac{q[\mathrm{mu}]}{2}\right] T[a, b] \times \gamma[\mathrm{mu}] + \\
&\mathrm{i}\, g0^2\, \mathrm{Boole}[3 \neq \mathrm{mu}]\, \mathrm{Cos}[p[3]]\, \delta_{\mathrm{mu},\mathrm{nu}}\, \mathrm{Sin}\left[\frac{p[\mathrm{mu}]}{2} + \frac{q[\mathrm{mu}]}{2}\right] T[a, b] \times \gamma[\mathrm{mu}] + \\
&\mathrm{i}\, g0^2\, \mathrm{Boole}[4 \neq \mathrm{mu}]\, \mathrm{Cos}[p[4]]\, \delta_{\mathrm{mu},\mathrm{nu}}\, \mathrm{Sin}\left[\frac{p[\mathrm{mu}]}{2} + \frac{q[\mathrm{mu}]}{2}\right] T[a, b] \times \gamma[\mathrm{mu}] + \\
&\mathrm{i}\, g0^2\, \mathrm{Boole}[1 \neq \mathrm{mu}]\, \mathrm{Cos}[q[1]]\, \delta_{\mathrm{mu},\mathrm{nu}}\, \mathrm{Sin}\left[\frac{p[\mathrm{mu}]}{2} + \frac{q[\mathrm{mu}]}{2}\right] T[a, b] \times \gamma[\mathrm{mu}] + \\
&\mathrm{i}\, g0^2\, \mathrm{Boole}[2 \neq \mathrm{mu}]\, \mathrm{Cos}[q[2]]\, \delta_{\mathrm{mu},\mathrm{nu}}\, \mathrm{Sin}\left[\frac{p[\mathrm{mu}]}{2} + \frac{q[\mathrm{mu}]}{2}\right] T[a, b] \times \gamma[\mathrm{mu}] + \\
&\mathrm{i}\, g0^2\, \mathrm{Boole}[3 \neq \mathrm{mu}]\, \mathrm{Cos}[q[3]]\, \delta_{\mathrm{mu},\mathrm{nu}}\, \mathrm{Sin}\left[\frac{p[\mathrm{mu}]}{2} + \frac{q[\mathrm{mu}]}{2}\right] T[a, b] \times \gamma[\mathrm{mu}] + \\
&\mathrm{i}\, g0^2\, \mathrm{Boole}[4 \neq \mathrm{mu}]\, \mathrm{Cos}[q[4]]\, \delta_{\mathrm{mu},\mathrm{nu}}\, \mathrm{Sin}\left[\frac{p[\mathrm{mu}]}{2} + \frac{q[\mathrm{mu}]}{2}\right] T[a, b] \times \gamma[\mathrm{mu}] + 2\, \mathrm{i}\, g0^2 \\
&\mathrm{Boole}[\mathrm{mu} \neq \mathrm{nu}]\, \mathrm{Cos}\left[\frac{k2[\mathrm{nu}]}{2} + p[\mathrm{nu}]\right] \mathrm{Sin}\left[\frac{k2[\mathrm{mu}]}{2} + \frac{p[\mathrm{mu}]}{2} + \frac{q[\mathrm{mu}]}{2}\right] T[a, b] \times \gamma[\mathrm{nu}]\Big)
\end{aligned}$$

In[•]:=
```
V2f3 = Collect[myFRintegrate[V23] // myFRbreakdown, {λ3 , ϱ3 }]
```

Out[•]=

$$\begin{aligned}
&\lambda 3 \left(\frac{1}{18}\, g0^2\, r\, \mathbb{I}\, \mathrm{Boole}[\mathrm{mu} \neq 1 \neq \mathrm{mu}]\, \mathrm{Cos}[p[1]]\, \mathrm{Cos}\left[\frac{p[\mathrm{mu}]}{2} - \frac{q[\mathrm{mu}]}{2}\right] \delta_{\mathrm{mu},\mathrm{nu}}\, T[a, b] - \right. \\
&\quad \frac{1}{9}\, g0^2\, r\, \mathbb{I}\, \mathrm{Boole}[\mathrm{mu} \neq 1 \neq \mathrm{mu}]\, \mathrm{Cos}[k2[1] + p[1]]\, \mathrm{Cos}\left[\frac{p[\mathrm{mu}]}{2} - \frac{q[\mathrm{mu}]}{2}\right] \delta_{\mathrm{mu},\mathrm{nu}}\, T[a, b] + \\
&\quad \frac{1}{18}\, g0^2\, r\, \mathbb{I}\, \mathrm{Boole}[\mathrm{mu} \neq 2 \neq \mathrm{mu}]\, \mathrm{Cos}[p[2]]\, \mathrm{Cos}\left[\frac{p[\mathrm{mu}]}{2} - \frac{q[\mathrm{mu}]}{2}\right] \delta_{\mathrm{mu},\mathrm{nu}}\, T[a, b] - \\
&\quad \left. \frac{1}{9}\, g0^2\, r\, \langle\!\langle 4 \rangle\!\rangle\, \delta_{\mathrm{mu},\mathrm{nu}}\, T[a, b] + \langle\!\langle 482 \rangle\!\rangle\right) + \\
&\varrho 3 \left(\frac{2}{3}\, \mathrm{i}\, g0^2\, \mathrm{Boole}[1 \neq \mathrm{mu} \neq 1]\, \mathrm{Cos}[p[1]]\, \mathrm{Cos}\left[\frac{p[\mathrm{mu}]}{2} + \frac{q[\mathrm{mu}]}{2}\right] \delta_{\mathrm{mu},\mathrm{nu}}\, \mathrm{Sin}[p[1]]\right. \\
&\quad T[a, b] \times \gamma[1] + \frac{2}{3}\, \mathrm{i}\, g0^2\, \mathrm{Boole}[\mathrm{mu} \neq 1 \neq 1]\, \mathrm{Cos}[p[1]]\, \mathrm{Cos}\left[\langle\!\langle 1 \rangle\!\rangle\right] \delta_{\mathrm{mu},\mathrm{nu}} \\
&\quad \mathrm{Sin}[p[1]]\, T[a, b] \times \gamma[1] + \langle\!\langle 359 \rangle\!\rangle + \langle\!\langle 1 \rangle\!\rangle - \frac{2}{3}\, \mathrm{i}\, g0^2\, \mathrm{Boole}[\mathrm{nu} \neq \mathrm{mu} \neq \mathrm{nu}] \\
&\quad \left. \mathrm{Cos}\left[\frac{k2[\mathrm{nu}]}{2} + p[\mathrm{nu}] + q[\mathrm{nu}]\right] \mathrm{Sin}\left[\frac{k2[\mathrm{mu}]}{2} + \frac{p[\mathrm{mu}]}{2} + \frac{q[\mathrm{mu}]}{2}\right] T[a, b] \times \gamma[\mathrm{nu}]\right)
\end{aligned}$$

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In[•]:=
```
V2f4 = Collect[myFRintegrate[V24] // myFRbreakdown, {λ4 , ϱ4 }]
```

Out[•]=

$$\lambda 4 \left(\frac{1}{36}\, g0^2\, r\, \mathbb{I}\, \text{Boole}[\text{mu} \neq 1 \neq 1 \neq \text{mu}]\, \text{Cos}[p[1]]^2\, \text{Cos}\left[\frac{p[\text{mu}]}{2} - \frac{q[\text{mu}]}{2}\right] \delta_{\text{mu,nu}}\, T[a, b] - \frac{1}{36}\, g0^2\, r\, \mathbb{I}\, \text{Boole}[\text{mu} \neq 1 \neq 1 \neq \text{mu}]\, \text{Cos}[p[1]]\, \text{Cos}[k2[1] + p[1]]\, \text{Cos}\left[\frac{p[\text{mu}]}{2} - \frac{q[\text{mu}]}{2}\right] \delta_{\text{mu,nu}}\, T[a, b] - \frac{1}{36}\, g0^2\, r\, \mathbb{I}\, \text{Boole}[\text{mu} \neq 1 \neq 1 \neq \text{mu}]\, \text{Cos}[k2[1] + p[1]]^2\, \text{Cos}\left[\frac{p[\text{mu}]}{2} - \frac{q[\text{mu}]}{2}\right] \delta_{\text{mu,nu}}\, T[a, b] + \frac{1}{36}\, g0^2\, \langle\langle 6 \rangle\rangle\, \delta_{\text{mu,nu}}\, T[a, b] + \langle\langle 5610 \rangle\rangle\right) + \varrho 4 \left(\frac{1}{3}\, i\, g0^2\, \text{Boole}[1 \neq \text{mu} \neq 1 \neq 1]\, \text{Cos}[p[1]]^2\, \text{Cos}\left[\frac{p[\text{mu}]}{2} + \frac{q[\text{mu}]}{2}\right] \delta_{\text{mu,nu}}\, \text{Sin}[p[1]]\, T[a, b] \times \gamma[1] + \frac{1}{3}\, i\, g0^2\, \langle\langle 4 \rangle\rangle\, \text{Sin}[p[1]]\, T[a, b] \times \gamma[1] + \langle\langle 5182 \rangle\rangle\right)$$

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In[•]:=
```
V2f = V2f1 + V2f2 + V2f3 + V2f4;
```

In[•]:=
```
(* Preparing for saving and loading *)
V2Out = V2f /. T[a__] :> 1 // ExpandAll;
```

In[•]:=
```
(* Saving *)
V2Brillouin[mu_, nu_, p_, q_, k1_, k2_] := Evaluate[V2Out]
DumpSave["V2_Brillouin.mx", V2Brillouin]
```

Out[•]=
```
{V2Brillouin}
```

QQGGG Vertex

In[•]:=
```
V3 = g0^3 Coefficient[SBrillouinFT, g0^3];
```

In[•]:=
```
V31 = (ϱ1 γ[mu] - r 𝕀 λ1 / 2) Coefficient[V3, S1[mu]] +
    (-ϱ1 γ[mu] - r 𝕀 λ1 / 2) Coefficient[V3, S1[-mu]];
V32 = (ϱ2 γ[mu] - r 𝕀 λ2 / 4) Coefficient[V3, S2[mu]] +
    (-ϱ2 γ[mu] - r 𝕀 λ2 / 4) Coefficient[V3, S2[-mu]];
V33 = (ϱ3 / 2 γ[mu] - r 𝕀 λ3 / 12) Coefficient[V3, S3[mu]] +
    (-ϱ3 / 2 γ[mu] - r 𝕀 λ3 / 12) Coefficient[V3, S3[-mu]];
V34 = (ϱ4 / 6 γ[mu] - r 𝕀 λ4 / 48) Coefficient[V3, S4[mu]] +
    (-ϱ4 / 6 γ[mu] - r 𝕀 λ4 / 48) Coefficient[V3, S4[-mu]];
```

In[•]:=
```
V3f1 = myFRintegrate[V31] // myFRbreakdown
```

Out[•]=

$$\frac{1}{6}\, g0^3\, r\, \mathbb{I}\, \lambda 1\, \delta_{\text{mu,nu}}\, \delta_{\text{mu,rho}}\, \text{Sin}\left[\frac{p[\text{mu}]}{2} + \frac{q[\text{mu}]}{2}\right] T[a, b, c] + \frac{1}{3}\, i\, g0^3\, \varrho 1\, \text{Cos}\left[\frac{p[\text{mu}]}{2} + \frac{q[\text{mu}]}{2}\right] \delta_{\text{mu,nu}}\, \delta_{\text{mu,rho}}\, T[a, b, c] \times \gamma[\text{mu}]$$

In[•]:=
```
V3f2 = Collect[myFRintegrate[V32] // myFRbreakdown, {λ2 , ϱ2 }]
```

Out[•]=

$$\lambda 2 \left(\frac{1}{4}\, g0^3\, r\, \mathbb{I}\, \text{Boole}[\text{mu} \neq \text{rho}]\, \text{Cos}\left[\frac{k3[\text{mu}]}{2} + \frac{p[\text{mu}]}{2} + \frac{q[\text{mu}]}{2}\right]\right.$$

$$\delta_{\text{mu,nu}}\,\text{Sin}\left[\frac{\text{k3[rho]}}{2}+\text{p[rho]}\right]\text{T[a, b, c]}+\frac{1}{4}\,\text{g0}^3\,\text{r}\,\mathbb{I}\,\text{Boole[rho}\neq\text{mu]}$$

$$\text{Cos}\left[\frac{\text{k3[mu]}}{2}+\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\delta_{\text{mu,nu}}\,\text{Sin}\left[\frac{\text{k3[rho]}}{2}+\text{p[rho]}\right]\text{T[a, b, c]}+$$

$$\frac{1}{12}\,\text{g0}^3\,\text{r}\,\mathbb{I}\,\text{Boole[1}\neq\text{mu]}\,\text{Cos[p[1]]}\,\delta_{\text{mu,nu}}\,\delta_{\text{mu,rho}}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a, b, c]}+$$

$$\frac{1}{12}\,\text{g0}^3\,\text{r}\,\mathbb{I}\,\text{Boole[mu}\neq\text{1]}\,\text{Cos[p[1]]}\,\delta_{\text{mu,nu}}\,\delta_{\text{mu,rho}}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a, b, c]}+$$

$$\frac{1}{12}\,\text{g0}^3\,\text{r}\,\mathbb{I}\,\text{Boole[2}\neq\text{mu]}\,\text{Cos[p[2]]}\,\delta_{\text{mu,nu}}\,\delta_{\text{mu,rho}}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a, b, c]}+$$

$$\frac{1}{12}\,\text{g0}^3\,\text{r}\,\mathbb{I}\,\text{Boole[mu}\neq\text{2]}\,\text{Cos[p[2]]}\,\delta_{\text{mu,nu}}\,\delta_{\text{mu,rho}}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a, b, c]}+$$

$$\frac{1}{12}\,\text{g0}^3\,\text{r}\,\mathbb{I}\,\text{Boole[3}\neq\text{mu]}\,\text{Cos[p[3]]}\,\delta_{\text{mu,nu}}\,\delta_{\text{mu,rho}}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a, b, c]}+$$

$$\frac{1}{12}\,\text{g0}^3\,\text{r}\,\mathbb{I}\,\text{Boole[mu}\neq\text{3]}\,\text{Cos[p[3]]}\,\delta_{\text{mu,nu}}\,\delta_{\text{mu,rho}}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a, b, c]}+$$

$$\frac{1}{12}\,\text{g0}^3\,\text{r}\,\mathbb{I}\,\text{Boole[4}\neq\text{mu]}\,\text{Cos[p[4]]}\,\delta_{\text{mu,nu}}\,\delta_{\text{mu,rho}}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a, b, c]}+$$

$$\frac{1}{12}\,\text{g0}^3\,\text{r}\,\mathbb{I}\,\text{Boole[mu}\neq\text{4]}\,\text{Cos[p[4]]}\,\delta_{\text{mu,nu}}\,\delta_{\text{mu,rho}}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a, b, c]}+$$

$$\frac{1}{12}\,\text{g0}^3\,\text{r}\,\mathbb{I}\,\text{Boole[1}\neq\text{mu]}\,\text{Cos[q[1]]}\,\delta_{\text{mu,nu}}\,\delta_{\text{mu,rho}}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a, b, c]}+$$

$$\frac{1}{12}\,\text{g0}^3\,\text{r}\,\mathbb{I}\,\text{Boole[mu}\neq\text{1]}\,\text{Cos[q[1]]}\,\delta_{\text{mu,nu}}\,\delta_{\text{mu,rho}}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a, b, c]}+$$

$$\frac{1}{12}\,\text{g0}^3\,\text{r}\,\mathbb{I}\,\text{Boole[2}\neq\text{mu]}\,\text{Cos[q[2]]}\,\delta_{\text{mu,nu}}\,\delta_{\text{mu,rho}}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a, b, c]}+$$

$$\frac{1}{12}\,\text{g0}^3\,\text{r}\,\mathbb{I}\,\text{Boole[mu}\neq\text{2]}\,\text{Cos[q[2]]}\,\delta_{\text{mu,nu}}\,\delta_{\text{mu,rho}}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a, b, c]}+$$

$$\frac{1}{12}\,\text{g0}^3\,\text{r}\,\mathbb{I}\,\text{Boole[3}\neq\text{mu]}\,\text{Cos[q[3]]}\,\delta_{\text{mu,nu}}\,\delta_{\text{mu,rho}}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a, b, c]}+$$

$$\frac{1}{12}\,\text{g0}^3\,\text{r}\,\mathbb{I}\,\text{Boole[mu}\neq\text{3]}\,\text{Cos[q[3]]}\,\delta_{\text{mu,nu}}\,\delta_{\text{mu,rho}}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a, b, c]}+$$

$$\frac{1}{12}\,\text{g0}^3\,\text{r}\,\mathbb{I}\,\text{Boole[4}\neq\text{mu]}\,\text{Cos[q[4]]}\,\delta_{\text{mu,nu}}\,\delta_{\text{mu,rho}}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a, b, c]}+$$

$$\frac{1}{12}\,\text{g0}^3\,\text{r}\,\mathbb{I}\,\text{Boole[mu}\neq\text{4]}\,\text{Cos[q[4]]}\,\delta_{\text{mu,nu}}\,\delta_{\text{mu,rho}}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a, b, c]}+$$

$$\frac{1}{4}\,\text{g0}^3\,\text{r}\,\mathbb{I}\,\text{Boole[mu}\neq\text{nu]}\,\text{Cos}\left[\frac{\text{k2[nu]}}{2}+\frac{\text{k3[nu]}}{2}+\text{p[nu]}\right]$$

$$\delta_{\text{nu,rho}}\,\text{Sin}\left[\frac{\text{k2[mu]}}{2}+\frac{\text{k3[mu]}}{2}+\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a, b, c]}+$$

$$\frac{1}{4}\,\text{g0}^3\,\text{r}\,\mathbb{I}\,\text{Boole[nu}\neq\text{mu]}\,\text{Cos}\left[\frac{\text{k2[nu]}}{2}+\frac{\text{k3[nu]}}{2}+\text{p[nu]}\right]\delta_{\text{nu,rho}}$$

$$\text{Sin}\left[\frac{\text{k2[mu]}}{2}+\frac{\text{k3[mu]}}{2}+\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a, b, c]}\Bigg)+\varrho 2$$

$$\left(-\frac{1}{3}\,\mathbb{i}\,\text{g0}^3\,\text{Boole[mu}\neq\text{1]}\,\delta_{\text{mu,nu}}\,\delta_{\text{mu,rho}}\,\text{Sin[p[1]]}\,\text{Sin}\left[\frac{\text{p[mu]}}{2}+\frac{\text{q[mu]}}{2}\right]\text{T[a, b, c]}\times\gamma\text{[1]}-\right.$$

$$\frac{1}{3}\, i\, g0^3\, \text{Boole}[\text{mu} \neq 1]\, \delta_{\text{mu,nu}}\, \delta_{\text{mu,rho}}\, \text{Sin}[\text{q}[1]]\, \text{Sin}\left[\frac{\text{p}[\text{mu}]}{2} + \frac{\text{q}[\text{mu}]}{2}\right] \text{T}[\text{a, b, c}] \times \gamma[1] -$$
$$\frac{1}{3}\, i\, g0^3\, \text{Boole}[\text{mu} \neq 2]\, \delta_{\text{mu,nu}}\, \delta_{\text{mu,rho}}\, \text{Sin}[\text{p}[2]]\, \text{Sin}\left[\frac{\text{p}[\text{mu}]}{2} + \frac{\text{q}[\text{mu}]}{2}\right] \text{T}[\text{a, b, c}] \times \gamma[2] -$$
$$\frac{1}{3}\, i\, g0^3\, \text{Boole}[\text{mu} \neq 2]\, \delta_{\text{mu,nu}}\, \delta_{\text{mu,rho}}\, \text{Sin}[\text{q}[2]]\, \text{Sin}\left[\frac{\text{p}[\text{mu}]}{2} + \frac{\text{q}[\text{mu}]}{2}\right] \text{T}[\text{a, b, c}] \times \gamma[2] -$$
$$\frac{1}{3}\, i\, g0^3\, \text{Boole}[\text{mu} \neq 3]\, \delta_{\text{mu,nu}}\, \delta_{\text{mu,rho}}\, \text{Sin}[\text{p}[3]]\, \text{Sin}\left[\frac{\text{p}[\text{mu}]}{2} + \frac{\text{q}[\text{mu}]}{2}\right] \text{T}[\text{a, b, c}] \times \gamma[3] -$$
$$\frac{1}{3}\, i\, g0^3\, \text{Boole}[\text{mu} \neq 3]\, \delta_{\text{mu,nu}}\, \delta_{\text{mu,rho}}\, \text{Sin}[\text{q}[3]]\, \text{Sin}\left[\frac{\text{p}[\text{mu}]}{2} + \frac{\text{q}[\text{mu}]}{2}\right] \text{T}[\text{a, b, c}] \times \gamma[3] -$$
$$\frac{1}{3}\, i\, g0^3\, \text{Boole}[\text{mu} \neq 4]\, \delta_{\text{mu,nu}}\, \delta_{\text{mu,rho}}\, \text{Sin}[\text{p}[4]]\, \text{Sin}\left[\frac{\text{p}[\text{mu}]}{2} + \frac{\text{q}[\text{mu}]}{2}\right] \text{T}[\text{a, b, c}] \times \gamma[4] -$$
$$\frac{1}{3}\, i\, g0^3\, \text{Boole}[\text{mu} \neq 4]\, \delta_{\text{mu,nu}}\, \delta_{\text{mu,rho}}\, \text{Sin}[\text{q}[4]]\, \text{Sin}\left[\frac{\text{p}[\text{mu}]}{2} + \frac{\text{q}[\text{mu}]}{2}\right] \text{T}[\text{a, b, c}] \times \gamma[4] +$$
$$\frac{1}{3}\, i\, g0^3\, \text{Boole}[1 \neq \text{mu}]\, \text{Cos}[\text{p}[1]]\, \text{Cos}\left[\frac{\text{p}[\text{mu}]}{2} + \frac{\text{q}[\text{mu}]}{2}\right] \delta_{\text{mu,nu}}\, \delta_{\text{mu,rho}}\, \text{T}[\text{a, b, c}] \times \gamma[\text{mu}] +$$
$$\frac{1}{3}\, i\, g0^3\, \text{Boole}[2 \neq \text{mu}]\, \text{Cos}[\text{p}[2]]\, \text{Cos}\left[\frac{\text{p}[\text{mu}]}{2} + \frac{\text{q}[\text{mu}]}{2}\right] \delta_{\text{mu,nu}}\, \delta_{\text{mu,rho}}\, \text{T}[\text{a, b, c}] \times \gamma[\text{mu}] +$$
$$\frac{1}{3}\, i\, g0^3\, \text{Boole}[3 \neq \text{mu}]\, \text{Cos}[\text{p}[3]]\, \text{Cos}\left[\frac{\text{p}[\text{mu}]}{2} + \frac{\text{q}[\text{mu}]}{2}\right] \delta_{\text{mu,nu}}\, \delta_{\text{mu,rho}}\, \text{T}[\text{a, b, c}] \times \gamma[\text{mu}] +$$
$$\frac{1}{3}\, i\, g0^3\, \text{Boole}[4 \neq \text{mu}]\, \text{Cos}[\text{p}[4]]\, \text{Cos}\left[\frac{\text{p}[\text{mu}]}{2} + \frac{\text{q}[\text{mu}]}{2}\right] \delta_{\text{mu,nu}}\, \delta_{\text{mu,rho}}\, \text{T}[\text{a, b, c}] \times \gamma[\text{mu}] +$$
$$\frac{1}{3}\, i\, g0^3\, \text{Boole}[1 \neq \text{mu}]\, \text{Cos}[\text{q}[1]]\, \text{Cos}\left[\frac{\text{p}[\text{mu}]}{2} + \frac{\text{q}[\text{mu}]}{2}\right] \delta_{\text{mu,nu}}\, \delta_{\text{mu,rho}}\, \text{T}[\text{a, b, c}] \times \gamma[\text{mu}] +$$
$$\frac{1}{3}\, i\, g0^3\, \text{Boole}[2 \neq \text{mu}]\, \text{Cos}[\text{q}[2]]\, \text{Cos}\left[\frac{\text{p}[\text{mu}]}{2} + \frac{\text{q}[\text{mu}]}{2}\right] \delta_{\text{mu,nu}}\, \delta_{\text{mu,rho}}\, \text{T}[\text{a, b, c}] \times \gamma[\text{mu}] +$$
$$\frac{1}{3}\, i\, g0^3\, \text{Boole}[3 \neq \text{mu}]\, \text{Cos}[\text{q}[3]]\, \text{Cos}\left[\frac{\text{p}[\text{mu}]}{2} + \frac{\text{q}[\text{mu}]}{2}\right] \delta_{\text{mu,nu}}\, \delta_{\text{mu,rho}}\, \text{T}[\text{a, b, c}] \times \gamma[\text{mu}] +$$
$$\frac{1}{3}\, i\, g0^3\, \text{Boole}[4 \neq \text{mu}]\, \text{Cos}[\text{q}[4]]\, \text{Cos}\left[\frac{\text{p}[\text{mu}]}{2} + \frac{\text{q}[\text{mu}]}{2}\right] \delta_{\text{mu,nu}}\, \delta_{\text{mu,rho}}\, \text{T}[\text{a, b, c}] \times \gamma[\text{mu}] +$$
$$i\, g0^3\, \text{Boole}[\text{nu} \neq \text{mu}]\, \text{Cos}\left[\frac{\text{k2}[\text{nu}]}{2} + \frac{\text{k3}[\text{nu}]}{2} + \text{p}[\text{nu}]\right]$$
$$\text{Cos}\left[\frac{\text{k2}[\text{mu}]}{2} + \frac{\text{k3}[\text{mu}]}{2} + \frac{\text{p}[\text{mu}]}{2} + \frac{\text{q}[\text{mu}]}{2}\right] \delta_{\text{nu,rho}}\, \text{T}[\text{a, b, c}] \times \gamma[\text{mu}] -$$
$$i\, g0^3\, \text{Boole}[\text{rho} \neq \text{mu}]\, \delta_{\text{mu,nu}}\, \text{Sin}\left[\frac{\text{k3}[\text{rho}]}{2} + \text{p}[\text{rho}]\right] \text{Sin}\left[\frac{\text{k3}[\text{mu}]}{2} + \frac{\text{p}[\text{mu}]}{2} + \frac{\text{q}[\text{mu}]}{2}\right]$$
$$\text{T}[\text{a, b, c}] \times \gamma[\text{mu}] - i\, g0^3\, \text{Boole}[\text{mu} \neq \text{nu}]\, \delta_{\text{nu,rho}}\, \text{Sin}\left[\frac{\text{k2}[\text{nu}]}{2} + \frac{\text{k3}[\text{nu}]}{2} + \text{p}[\text{nu}]\right]$$
$$\text{Sin}\left[\frac{\text{k2}[\text{mu}]}{2} + \frac{\text{k3}[\text{mu}]}{2} + \frac{\text{p}[\text{mu}]}{2} + \frac{\text{q}[\text{mu}]}{2}\right] \text{T}[\text{a, b, c}] \times \gamma[\text{nu}] + i\, g0^3\, \text{Boole}[\text{mu} \neq \text{rho}]$$
$$\text{Cos}\left[\frac{\text{k3}[\text{rho}]}{2} + \text{p}[\text{rho}]\right] \text{Cos}\left[\frac{\text{k3}[\text{mu}]}{2} + \frac{\text{p}[\text{mu}]}{2} + \frac{\text{q}[\text{mu}]}{2}\right] \delta_{\text{mu,nu}}\, \text{T}[\text{a, b, c}] \times \gamma[\text{rho}]\Big)$$

In[•]:= **V3f3 = Collect[myFRintegrate[V33] // myFRbreakdown, {λ3 , ϱ3 }]**

Out[ ]=

$$\lambda 3 \left(\frac{1}{9}\, g0^3\, r\, \mathbb{I}\, \text{Boole}[\text{mu} \neq \text{nu} \neq \text{mu}] \right.$$
$$\text{Cos}\left[\frac{\text{k2[mu]}}{2} - \frac{\text{p[mu]}}{2} + \frac{\text{q[mu]}}{2}\right] \delta_{\text{mu,rho}}\, \text{Sin}\left[\frac{\text{k2[nu]}}{2} + \text{k3[nu]} + \text{p[nu]}\right] T[a, b, c] -$$
$$\frac{1}{9}\, g0^3\, r\, \mathbb{I}\, \text{Boole}[\text{mu} \neq \text{nu} \neq \text{mu}]\, \text{Cos}\left[\frac{\text{k2[mu]}}{2} + \text{k3[mu]} + \frac{3\,\text{p[mu]}}{2} + \frac{\text{q[mu]}}{2}\right]$$
$$\delta_{\text{mu,rho}}\, \text{Sin}\left[\frac{\text{k2[nu]}}{2} + \text{k3[nu]} + \text{p[nu]}\right] T[a, b, c] -$$
$$\frac{1}{18}\, g0^3\, r\, \mathbb{I}\, \text{Boole}[\text{mu} \neq \text{rho} \neq \text{mu}]\, \text{Cos}\left[\frac{\text{k3[mu]}}{2} - \frac{\text{p[mu]}}{2} + \frac{\text{q[mu]}}{2}\right] \delta_{\text{mu,nu}}$$
$$\left.\text{Sin}\left[\frac{\text{k3[rho]}}{2} + \text{p[rho]}\right] T[a, b, c] + \langle\langle 502 \rangle\rangle\right) + \varrho 3 \left(\langle\langle 1 \rangle\rangle\right)$$

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In[ ]:=
```
V3f4 = Collect[myFRintegrate[V34] // myFRbreakdown, {λ4 , ϱ4 }]
```

Out[ ]=

$$\lambda 4 \left(-\frac{1}{144}\, g0^3\, r\, \mathbb{I}\, \text{Boole}[\text{rho} \neq 1 \neq \text{mu} \neq \text{rho}]\, \text{Cos}[\text{p[1]}]\, \text{Cos}\left[\frac{\text{k3[mu]}}{2} + \frac{\text{p[mu]}}{2} + \frac{\text{q[mu]}}{2}\right] \delta_{\text{mu,nu}} \right.$$
$$\text{Sin}\left[\frac{\text{k3[rho]}}{2}\right] T[a, b, c] - \frac{1}{144}\, g0^3\, r\, \mathbb{I}\, \text{Boole}[\text{rho} \neq \text{mu} \neq 1 \neq \text{rho}]\, \text{Cos}[\text{p[1]}]$$
$$\text{Cos}\left[\frac{\text{k3[mu]}}{2} + \frac{\text{p[mu]}}{2} + \frac{\text{q[mu]}}{2}\right] \delta_{\text{mu,nu}}\, \text{Sin}\left[\frac{\text{k3[rho]}}{2}\right] T[a, b, c] + \langle\langle 8280 \rangle\rangle +$$
$$\frac{1}{72}\, g0^3\, r\, \mathbb{I}\, \text{Boole}[\text{rho} \neq \text{mu} \neq \text{nu} \neq \text{rho}]\, \text{Sin}\left[\frac{\text{k2[nu]}}{2} + \text{k3[nu]} + \text{p[nu]}\right]$$
$$\text{Sin}\left[\frac{\text{k2[mu]}}{2} + \frac{\text{k3[mu]}}{2} + \frac{\text{p[mu]}}{2} + \frac{\text{q[mu]}}{2}\right] \text{Sin}\left[\frac{\text{k3[rho]}}{2} + \text{p[rho]} + \text{q[rho]}\right] T[a, b, c] +$$
$$\frac{1}{72}\, g0^3\, r\, \mathbb{I}\, \text{Boole}[\text{rho} \neq \text{nu} \neq \text{mu} \neq \text{rho}]\, \text{Sin}\left[\frac{\text{k2[nu]}}{2} + \text{k3[nu]} + \text{p[nu]}\right]$$
$$\left.\text{Sin}\left[\frac{\text{k2[mu]}}{2} + \frac{\text{k3[mu]}}{2} + \frac{\text{p[mu]}}{2} + \frac{\text{q[mu]}}{2}\right] \text{Sin}\left[\frac{\text{k3[rho]}}{2} + \text{p[rho]} + \text{q[rho]}\right] T[a, b, c]\right) +$$
$$\varrho 4 \left(\langle\langle 10\,822 \rangle\rangle + \langle\langle 1 \rangle\rangle\right)$$

Size in memory: 22.5 MB | + Show more | Show all | Iconize | Store full expression in notebook

In[ ]:=
```
V3f = V3f1 + V3f2 + V3f3 + V3f4;
```

In[ ]:=
```
(* Preparing for saving and loading *)
V3Out = V3f /. T[a__] :> 1 // ExpandAll;
```

In[ ]:=
```
(* Saving *)
V3Brillouin[mu_, nu_, rho_, p_, q_, k1_, k2_, k3_] := Evaluate[V3Out]
DumpSave["V3_Brillouin.mx", V3Brillouin]
```

Out[ ]=
```
{V3Brillouin}
```