Feynman Rules Wilson

(Written by Maximilian Ammer, 2023)
This notebook generates the fermionic Feynman rules for the qqg-, qqgg-, and qqggg-vertices in lattice perturbation theory for the Wilson fermion action.
The resulting functions are saved to files, which can be loaded by other notebooks for further calculations.

```
SetDirectory["~path~"];
```

In[•]:=
```
Get["myLPT.m"]
Clear[p, q, k, 𝕀, 𝛄, σ]
```

Wilson action

In[•]:=
```
derstd[mu_] := 1 / 2 (U[1, mu, x] × δ[x + e[mu], y] - U[1, -mu, x] × δ[x - e[mu], y])
```

In[•]:=
```
Δstd[mu_] := U[1, mu, x] × δ[x + e[mu], y] - 2 δ[x, y] + U[1, -mu, x] × δ[x - e[mu], y]
```

In[•]:=
```
WilsonDiracOp = 𝛄[mu] × derstd[mu] - r / 2 Δstd[mu];
```

In[•]:=
```
(* Wilson action in position space *)
SWilson = ψbar[x] WilsonDiracOp ψ[y]
```

Out[•]=

$$\psi[y] \times \psi\text{bar}[x] \left(-\frac{1}{2} r \left(-2\,\delta[x, y] + U[1, -mu, x] \times \delta[x - e[mu], y] + U[1, mu, x] \times \delta[x + e[mu], y]\right) + \frac{1}{2} \left(-U[1, -mu, x] \times \delta[x - e[mu], y] + U[1, mu, x] \times \delta[x + e[mu], y]\right) \gamma[mu]\right)$$

In[•]:=
```
(* Wilson action in momentum space *)
SWilsonFT = SWilson // myExpandU // myFourierTransform
```

Out[ ]=

$$
\begin{aligned}
&e^{-i\,q\,x+i\,s\,(x-y)+i\,p\,y}\ r\ \text{Int}[p]\times\text{Int}[q]\times\text{Int}[s]\times\Psi[p]\times\Psi\text{bar}[q]\ -\\
&\frac{1}{2}\ e^{-i\,q\,x+i\,p\,y+i\,s\,(x-y-e[mu])}\ r\ \text{Int}[p]\times\text{Int}[q]\times\text{Int}[s]\times\Psi[p]\times\Psi\text{bar}[q]\ -\\
&\frac{1}{2}\ e^{-i\,q\,x+i\,p\,y+i\,s\,(x-y+e[mu])}\ r\ \text{Int}[p]\times\text{Int}[q]\times\text{Int}[s]\times\Psi[p]\times\Psi\text{bar}[q]\ +\\
&\frac{1}{2}\ i\ e^{-i\,q\,x+i\,p\,y+i\,s\,(x-y-e[mu])+i\,k1\left(x-\frac{e[mu]}{2}\right)}\ g0\ r\ A[a,\text{mu},k1]\times\text{Int}[k1]\times\text{Int}[p]\times\\
&\text{Int}[q]\times\text{Int}[s]\times T[a]\times\Psi[p]\times\Psi\text{bar}[q]\ -\ \frac{1}{2}\ i\ e^{-i\,q\,x+i\,p\,y+i\,k1\left(x+\frac{e[mu]}{2}\right)+i\,s\,(x-y+e[mu])}\\
&g0\ r\ A[a,\text{mu},k1]\times\text{Int}[k1]\times\text{Int}[p]\times\text{Int}[q]\times\text{Int}[s]\times T[a]\times\Psi[p]\times\Psi\text{bar}[q]\ +\\
&\frac{1}{4}\ e^{-i\,q\,x+i\,p\,y+i\,s\,(x-y-e[mu])+i\,k1\left(x-\frac{e[mu]}{2}\right)+i\,k2\left(x-\frac{e[mu]}{2}\right)}\ g0^2\ r\ A[a,\text{mu},k1]\times A[b,\text{mu},k2]\times\\
&\text{Int}[k1]\times\text{Int}[k2]\times\text{Int}[p]\times\text{Int}[q]\times\text{Int}[s]\times T[a,b]\times\Psi[p]\times\Psi\text{bar}[q]\ +\\
&\frac{1}{4}\ e^{-i\,q\,x+i\,p\,y+i\,k1\left(x+\frac{e[mu]}{2}\right)+i\,k2\left(x+\frac{e[mu]}{2}\right)+i\,s\,(x-y+e[mu])}\ g0^2\ r\ A[a,\text{mu},k1]\times A[b,\text{mu},k2]\times\\
&\text{Int}[k1]\times\text{Int}[k2]\times\text{Int}[p]\times\text{Int}[q]\times\text{Int}[s]\times T[a,b]\times\Psi[p]\times\Psi\text{bar}[q]\ -\\
&\frac{1}{12}\ i\ e^{-i\,q\,x+i\,p\,y+i\,s\,(x-y-e[mu])+i\,k1\left(x-\frac{e[mu]}{2}\right)+i\,k2\left(x-\frac{e[mu]}{2}\right)+i\,k3\left(x-\frac{e[mu]}{2}\right)}\ g0^3\ r\\
&A[a,\text{mu},k1]\times A[b,\text{mu},k2]\times A[c,\text{mu},k3]\times\text{Int}[k1]\times\text{Int}[k2]\times\\
&\text{Int}[k3]\times\text{Int}[p]\times\text{Int}[q]\times\text{Int}[s]\times T[a,b,c]\times\Psi[p]\times\Psi\text{bar}[q]\ +\\
&\frac{1}{12}\ i\ e^{-i\,q\,x+i\,p\,y+i\,k1\left(x+\frac{e[mu]}{2}\right)+i\,k2\left(x+\frac{e[mu]}{2}\right)+i\,k3\left(x+\frac{e[mu]}{2}\right)+i\,s\,(x-y+e[mu])}\ g0^3\ r\\
&A[a,\text{mu},k1]\times A[b,\text{mu},k2]\times A[c,\text{mu},k3]\times\text{Int}[k1]\times\text{Int}[k2]\times\\
&\text{Int}[k3]\times\text{Int}[p]\times\text{Int}[q]\times\text{Int}[s]\times T[a,b,c]\times\Psi[p]\times\Psi\text{bar}[q]\ -\\
&\frac{1}{2}\ e^{-i\,q\,x+i\,p\,y+i\,s\,(x-y-e[mu])}\ \text{Int}[p]\times\text{Int}[q]\times\text{Int}[s]\times\Psi[p]\times\Psi\text{bar}[q]\times\gamma[mu]\ +\\
&\frac{1}{2}\ e^{-i\,q\,x+i\,p\,y+i\,s\,(x-y+e[mu])}\ \text{Int}[p]\times\text{Int}[q]\times\text{Int}[s]\times\Psi[p]\times\Psi\text{bar}[q]\times\gamma[mu]\ +\\
&\frac{1}{2}\ i\ e^{-i\,q\,x+i\,p\,y+i\,s\,(x-y-e[mu])+i\,k1\left(x-\frac{e[mu]}{2}\right)}\ g0\ A[a,\text{mu},k1]\times\text{Int}[k1]\times\text{Int}[p]\times\text{Int}[q]\times\\
&\text{Int}[s]\times T[a]\times\Psi[p]\times\Psi\text{bar}[q]\times\gamma[mu]\ +\ \frac{1}{2}\ i\ e^{-i\,q\,x+i\,p\,y+i\,k1\left(x+\frac{e[mu]}{2}\right)+i\,s\,(x-y+e[mu])}\ g0\\
&A[a,\text{mu},k1]\times\text{Int}[k1]\times\text{Int}[p]\times\text{Int}[q]\times\text{Int}[s]\times T[a]\times\Psi[p]\times\Psi\text{bar}[q]\times\gamma[mu]\ +\\
&\frac{1}{4}\ e^{-i\,q\,x+i\,p\,y+i\,s\,(x-y-e[mu])+i\,k1\left(x-\frac{e[mu]}{2}\right)+i\,k2\left(x-\frac{e[mu]}{2}\right)}\ g0^2\ A[a,\text{mu},k1]\times A[b,\text{mu},k2]\times\\
&\text{Int}[k1]\times\text{Int}[k2]\times\text{Int}[p]\times\text{Int}[q]\times\text{Int}[s]\times T[a,b]\times\Psi[p]\times\Psi\text{bar}[q]\times\gamma[mu]\ -\\
&\frac{1}{4}\ e^{-i\,q\,x+i\,p\,y+i\,k1\left(x+\frac{e[mu]}{2}\right)+i\,k2\left(x+\frac{e[mu]}{2}\right)+i\,s\,(x-y+e[mu])}\ g0^2\ A[a,\text{mu},k1]\times A[b,\text{mu},k2]\times\\
&\text{Int}[k1]\times\text{Int}[k2]\times\text{Int}[p]\times\text{Int}[q]\times\text{Int}[s]\times T[a,b]\times\Psi[p]\times\Psi\text{bar}[q]\times\gamma[mu]\ -\\
&\frac{1}{12}\ i\ e^{-i\,q\,x+i\,p\,y+i\,s\,(x-y-e[mu])+i\,k1\left(x-\frac{e[mu]}{2}\right)+i\,k2\left(x-\frac{e[mu]}{2}\right)+i\,k3\left(x-\frac{e[mu]}{2}\right)}\ g0^3\ A[a,\text{mu},k1]\times\\
&A[b,\text{mu},k2]\times A[c,\text{mu},k3]\times\text{Int}[k1]\times\text{Int}[k2]\times\text{Int}[k3]\times\\
&\text{Int}[p]\times\text{Int}[q]\times\text{Int}[s]\times T[a,b,c]\times\Psi[p]\times\Psi\text{bar}[q]\times\gamma[mu]\ -\\
&\frac{1}{12}\ i\ e^{-i\,q\,x+i\,p\,y+i\,k1\left(x+\frac{e[mu]}{2}\right)+i\,k2\left(x+\frac{e[mu]}{2}\right)+i\,k3\left(x+\frac{e[mu]}{2}\right)+i\,s\,(x-y+e[mu])}\ g0^3\\
&A[a,\text{mu},k1]\times A[b,\text{mu},k2]\times A[c,\text{mu},k3]\times\text{Int}[k1]\times\text{Int}[k2]\times\\
&\text{Int}[k3]\times\text{Int}[p]\times\text{Int}[q]\times\text{Int}[s]\times T[a,b,c]\times\Psi[p]\times\Psi\text{bar}[q]\times\gamma[mu]
\end{aligned}
$$

Vertices

QQG Vertex

In[•]:=
```
V1 = g0 Coefficient[SWilsonFT, g0]
```

Out[•]=

$$g0\left(\frac{1}{2}\,i\,e^{-i\,q\,x+i\,p\,y+i\,s\,(x-y-e[mu])+i\,k1\left(x-\frac{e[mu]}{2}\right)}\,r\,A[a,\,mu,\,k1]\times Int[k1]\times Int[p]\times Int[q]\times Int[s]\times T[a]\times \Psi[p]\times \Psi bar[q]-\frac{1}{2}\,i\,e^{-i\,q\,x+i\,p\,y+i\,k1\left(x+\frac{e[mu]}{2}\right)+i\,s\,(x-y+e[mu])}\,r\,A[a,\,mu,\,k1]\times Int[k1]\times Int[p]\times Int[q]\times Int[s]\times T[a]\times \Psi[p]\times \Psi bar[q]+\frac{1}{2}\,i\,e^{-i\,q\,x+i\,p\,y+i\,s\,(x-y-e[mu])+i\,k1\left(x-\frac{e[mu]}{2}\right)}\,A[a,\,mu,\,k1]\times Int[k1]\times Int[p]\times Int[q]\times Int[s]\times T[a]\times \Psi[p]\times \Psi bar[q]\times \gamma[mu]+\frac{1}{2}\,i\,e^{-i\,q\,x+i\,p\,y+i\,k1\left(x+\frac{e[mu]}{2}\right)+i\,s\,(x-y+e[mu])}\,A[a,\,mu,\,k1]\times Int[k1]\times Int[p]\times Int[q]\times Int[s]\times T[a]\times \Psi[p]\times \Psi bar[q]\times \gamma[mu]\right)$$

In[•]:=
```
V1f = myFRintegrate[V1] // myFRbreakdown // Simplify
```

Out[•]=

$$-g0\,T[a]\left(r\,Sin\left[\frac{1}{2}\,(p[mu]+q[mu])\right]+i\,Cos\left[\frac{1}{2}\,(p[mu]+q[mu])\right]\gamma[mu]\right)$$

In[•]:=
```
(* Preparing for saving and loading *)
V1Out = V1f 𝕀 /. T[a__] :> 1 /. γ[mu_] :> γ[mu] / 𝕀 // ExpandAll
```

Out[•]=

$$-g0\,r\,\mathbb{I}\,Sin\left[\frac{p[mu]}{2}+\frac{q[mu]}{2}\right]-i\,g0\,Cos\left[\frac{p[mu]}{2}+\frac{q[mu]}{2}\right]\gamma[mu]$$

In[•]:=
```
(* Saving *)
V1Wilson[mu_, p_, q_] := Evaluate[V1Out]
DumpSave["V1_Wilson.mx", V1Wilson]
```

Out[•]=

```
{V1Wilson}
```

QQGG Vertex

In[•]:=
```
V2 = g0^2 Coefficient[SWilsonFT, g0^2]
```

Out[•]=

$$g0^2 \left(\frac{1}{4} e^{-i\,q\,x+i\,p\,y+i\,s\,(x-y-e[mu])+i\,k1\left(x-\frac{e[mu]}{2}\right)+i\,k2\left(x-\frac{e[mu]}{2}\right)}\, r\, A[a, mu, k1] \times A[b, mu, k2] \times Int[k1] \times Int[k2] \times Int[p] \times Int[q] \times Int[s] \times T[a, b] \times \Psi[p] \times \Psi bar[q] + \frac{1}{4} e^{-i\,q\,x+i\,p\,y+i\,k1\left(x+\frac{e[mu]}{2}\right)+i\,k2\left(x+\frac{e[mu]}{2}\right)+i\,s\,(x-y+e[mu])}\, r\, A[a, mu, k1] \times A[b, mu, k2] \times Int[k1] \times Int[k2] \times Int[p] \times Int[q] \times Int[s] \times T[a, b] \times \Psi[p] \times \Psi bar[q] + \frac{1}{4} e^{-i\,q\,x+i\,p\,y+i\,s\,(x-y-e[mu])+i\,k1\left(x-\frac{e[mu]}{2}\right)+i\,k2\left(x-\frac{e[mu]}{2}\right)}\, A[a, mu, k1] \times A[b, mu, k2] \times Int[k1] \times Int[k2] \times Int[p] \times Int[q] \times Int[s] \times T[a, b] \times \Psi[p] \times \Psi bar[q] \times \gamma[mu] - \frac{1}{4} e^{-i\,q\,x+i\,p\,y+i\,k1\left(x+\frac{e[mu]}{2}\right)+i\,k2\left(x+\frac{e[mu]}{2}\right)+i\,s\,(x-y+e[mu])}\, A[a, mu, k1] \times A[b, mu, k2] \times Int[k1] \times Int[k2] \times Int[p] \times Int[q] \times Int[s] \times T[a, b] \times \Psi[p] \times \Psi bar[q] \times \gamma[mu]\right)$$

In[•]:=
```
V2f = myFRintegrate[V2] // myFRbreakdown // Simplify
```

Out[•]=

$$-\frac{1}{2} g0^2\, \delta_{mu,nu}\, T[a, b] \left(r\, Cos\left[\frac{1}{2}\,(p[mu] + q[mu])\right] - i\, Sin\left[\frac{1}{2}\,(p[mu] + q[mu])\right] \gamma[mu]\right)$$

In[•]:=
```
(* Preparing for saving and loading *)
V2Out = V2f 𝕀 /. T[a__] :> 1 /. γ[mu_] :> γ[mu] / 𝕀 // ExpandAll
```

Out[•]=

$$-\frac{1}{2} g0^2\, r\, \mathbb{I}\, Cos\left[\frac{p[mu]}{2} + \frac{q[mu]}{2}\right] \delta_{mu,nu} + \frac{1}{2} i\, g0^2\, \delta_{mu,nu}\, Sin\left[\frac{p[mu]}{2} + \frac{q[mu]}{2}\right] \gamma[mu]$$

In[•]:=
```
(* Saving *)
V2Wilson[mu_, nu_, p_, q_] := Evaluate[V2Out]
DumpSave["V2_Wilson.mx", V2Wilson]
```

Out[•]=

```
{V2Wilson}
```

QQGGG Vertex

In[•]:=
```
(* QQG Vertex *)
V3 = g0^3 Coefficient[SWilsonFT, g0^3]
```

Out[•]=

$$g0^3 \left(-\frac{1}{12}\, i\, e^{-i\,q\,x+i\,p\,y+i\,s\,(x-y-e[mu])+i\,k1\left(x-\frac{e[mu]}{2}\right)+i\,k2\left(x-\frac{e[mu]}{2}\right)+i\,k3\left(x-\frac{e[mu]}{2}\right)}\right.$$
$$r\,A[a,\,mu,\,k1]\times A[b,\,mu,\,k2]\times A[c,\,mu,\,k3]\times Int[k1]\times Int[k2]\times$$
$$Int[k3]\times Int[p]\times Int[q]\times Int[s]\times T[a,\,b,\,c]\times \Psi[p]\times \Psi bar[q]+$$
$$\frac{1}{12}\, i\, e^{-i\,q\,x+i\,p\,y+i\,k1\left(x+\frac{e[mu]}{2}\right)+i\,k2\left(x+\frac{e[mu]}{2}\right)+i\,k3\left(x+\frac{e[mu]}{2}\right)+i\,s\,(x-y+e[mu])}\, r$$
$$A[a,\,mu,\,k1]\times A[b,\,mu,\,k2]\times A[c,\,mu,\,k3]\times Int[k1]\times Int[k2]\times$$
$$Int[k3]\times Int[p]\times Int[q]\times Int[s]\times T[a,\,b,\,c]\times \Psi[p]\times \Psi bar[q]-$$
$$\frac{1}{12}\, i\, e^{-i\,q\,x+i\,p\,y+i\,s\,(x-y-e[mu])+i\,k1\left(x-\frac{e[mu]}{2}\right)+i\,k2\left(x-\frac{e[mu]}{2}\right)+i\,k3\left(x-\frac{e[mu]}{2}\right)}\, A[a,\,mu,\,k1]\times$$
$$A[b,\,mu,\,k2]\times A[c,\,mu,\,k3]\times Int[k1]\times Int[k2]\times Int[k3]\times$$
$$Int[p]\times Int[q]\times Int[s]\times T[a,\,b,\,c]\times \Psi[p]\times \Psi bar[q]\times \gamma[mu]-$$
$$\frac{1}{12}\, i\, e^{-i\,q\,x+i\,p\,y+i\,k1\left(x+\frac{e[mu]}{2}\right)+i\,k2\left(x+\frac{e[mu]}{2}\right)+i\,k3\left(x+\frac{e[mu]}{2}\right)+i\,s\,(x-y+e[mu])}\, A[a,\,mu,\,k1]\times$$
$$A[b,\,mu,\,k2]\times A[c,\,mu,\,k3]\times Int[k1]\times Int[k2]\times Int[k3]\times$$
$$\left.Int[p]\times Int[q]\times Int[s]\times T[a,\,b,\,c]\times \Psi[p]\times \Psi bar[q]\times \gamma[mu]\right)$$

In[•]:=
```
V3f = myFRintegrate[V3] // myFRbreakdown // Simplify
```

Out[•]=

$$\frac{1}{6}\, g0^3\, \delta_{mu,nu}\, \delta_{mu,rho}\, T[a,\,b,\,c]\left(r\,Sin\left[\frac{1}{2}\,(p[mu]+q[mu])\right]+i\,Cos\left[\frac{1}{2}\,(p[mu]+q[mu])\right]\gamma[mu]\right)$$

In[•]:=
```
(* Preparing for saving and loading *)
V3Out = V3f𝕀 /. T[a__] :> 1 /. γ[mu_] :> γ[mu] / 𝕀 // ExpandAll
```

Out[•]=

$$\frac{1}{6}\, g0^3\, r\, \mathbb{I}\, \delta_{mu,nu}\, \delta_{mu,rho}\, Sin\left[\frac{p[mu]}{2}+\frac{q[mu]}{2}\right]+\frac{1}{6}\, i\, g0^3\, Cos\left[\frac{p[mu]}{2}+\frac{q[mu]}{2}\right]\delta_{mu,nu}\, \delta_{mu,rho}\, \gamma[mu]$$

In[•]:=
```
(* Saving *)
V3Wilson[mu_, nu_, rho_, p_, q_] := Evaluate[V3Out]
DumpSave["V3_Wilson.mx", V3Wilson]
```

Out[•]=

```
{V3Wilson}
```