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# A Novel Approximate Bayesian Inference Method for Compartmental Models in Epidemiology

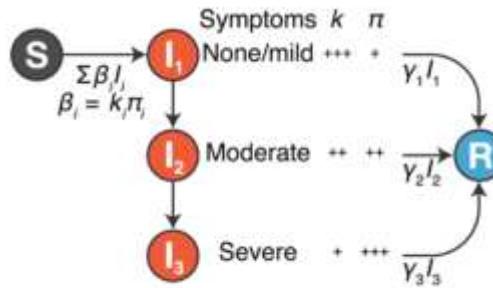
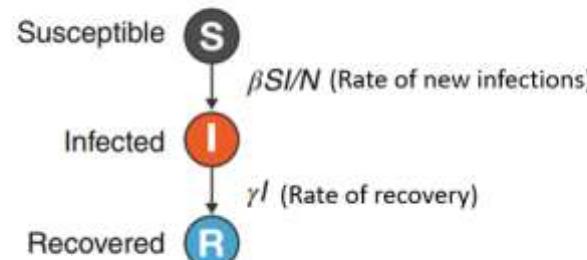
CPS 07  
Epidemiological Modelling

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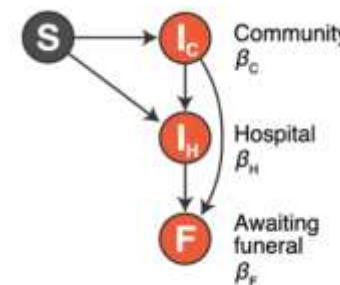
Ms. Xiahui Li, Dr Fergus Chadwick, Dr Ben Swallow  
University of St Andrews  
Monday 06 October, 4:00PM - 5:00PM



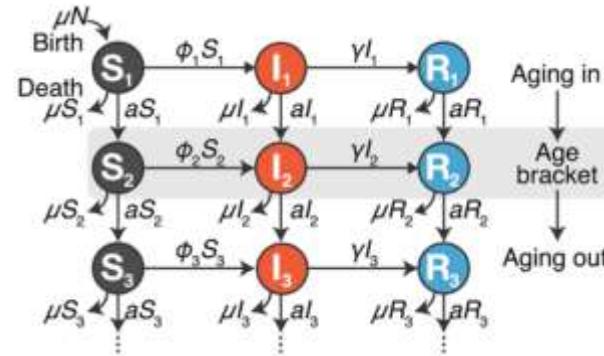
## SIR Model



**Symptoms and variable infectiousness**



**Multiple routes of transmission**



**Age-specific transmission**

### High-dimensional parameter spaces

(e.g., transmission rates,  
contact rates, recovery rates,  
birth and death)

**Incomplete or noisy data**  
(e.g., Missing case reports  
Underreporting  
Imprecise observations)

**Challenges** in  
Parameter  
Inference

### Latent variables

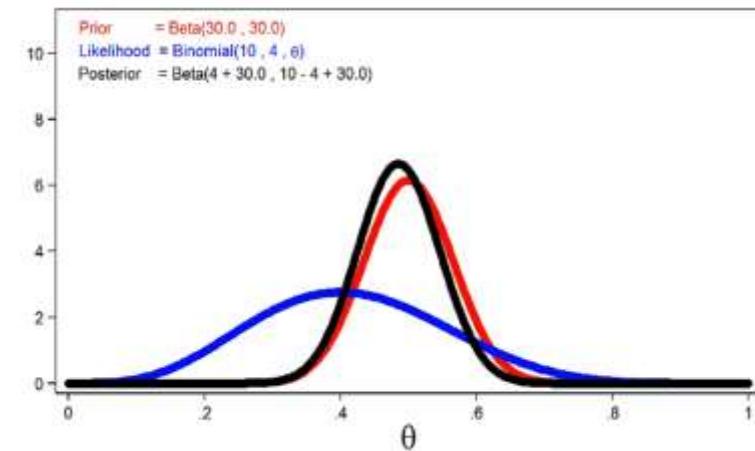
(e.g., infection time,  
transmission events)

**Uncertainties in model structure**  
(e.g., model assumptions, mixing  
patterns, inclusion of environmental  
factors)

### Bayes' rule

$$\pi(\theta|y_{obs}) \propto \text{Likelihood} \cdot \text{Prior}$$

↑ Likelihood  
 $\pi(y_{obs}|\theta) \pi(\theta)$   
↓ Prior  
↓ Posterior



GIF Animation 1: The effect of larger sample sizes on the posterior distribution. Adapted from "Introduction to Bayesian statistics," by Chuck Huber, 2016.  
<https://blog.stata.com/2016/11/01/introduction-to-bayesian-statistics-part-1-the-basic-concepts/>

## 1.4 Asymptotically Exact and Approximate Bayesian Method

$$\pi(\theta|y_{obs}) \propto \pi(y_{obs}|\theta)P(\theta) \text{ in Complex Model}$$

TRACTABLE Likelihood

Asymptotically Exact Bayesian Method

Directly sample from the true posterior distribution

**Popular Examples**

Metropolis-Hastings Algorithm  
Hamiltonian Monte Carlo (HMC)

INTRACTABLE Likelihood

Approximate Bayesian Method

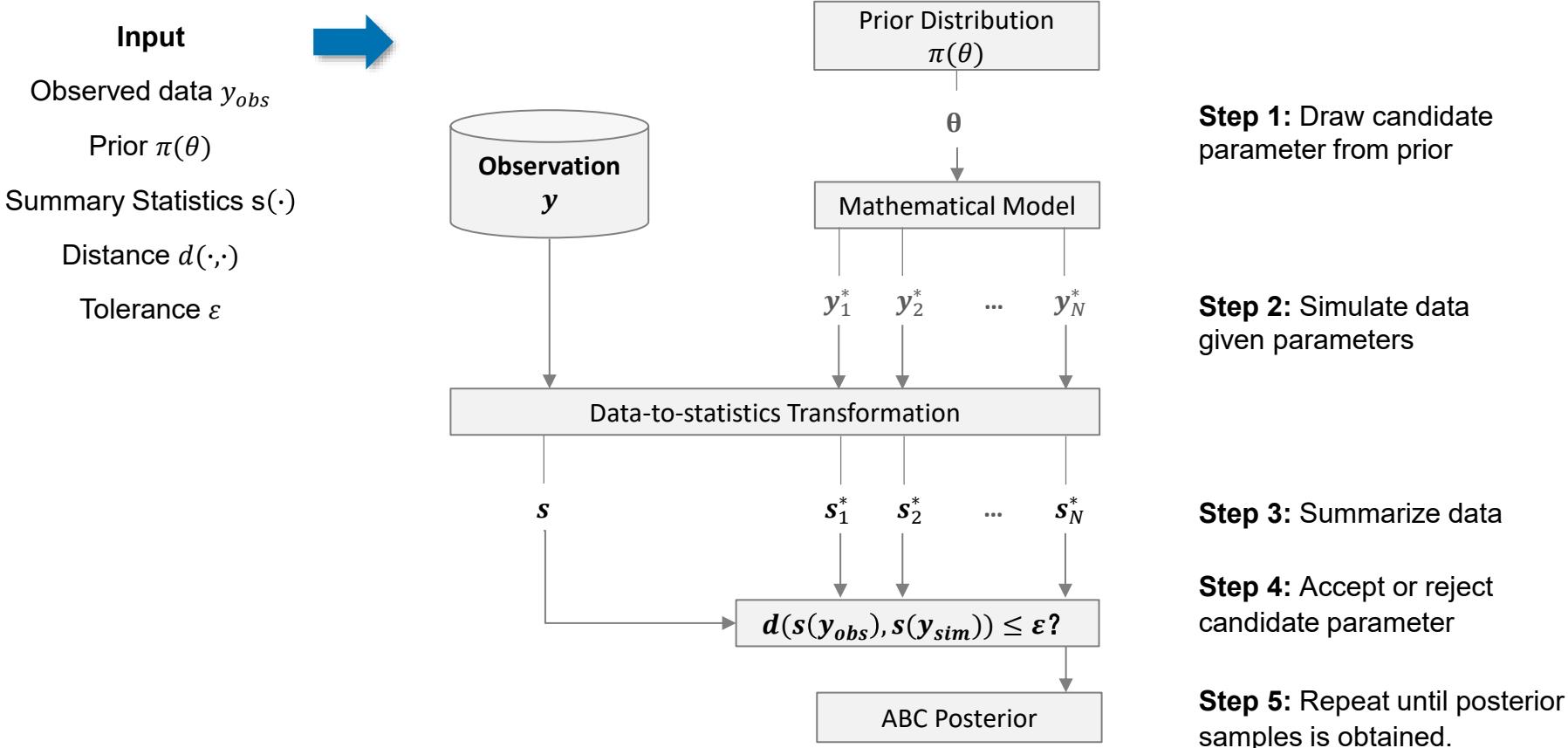
Bypass direct likelihood computation by matching simulated data to observations.

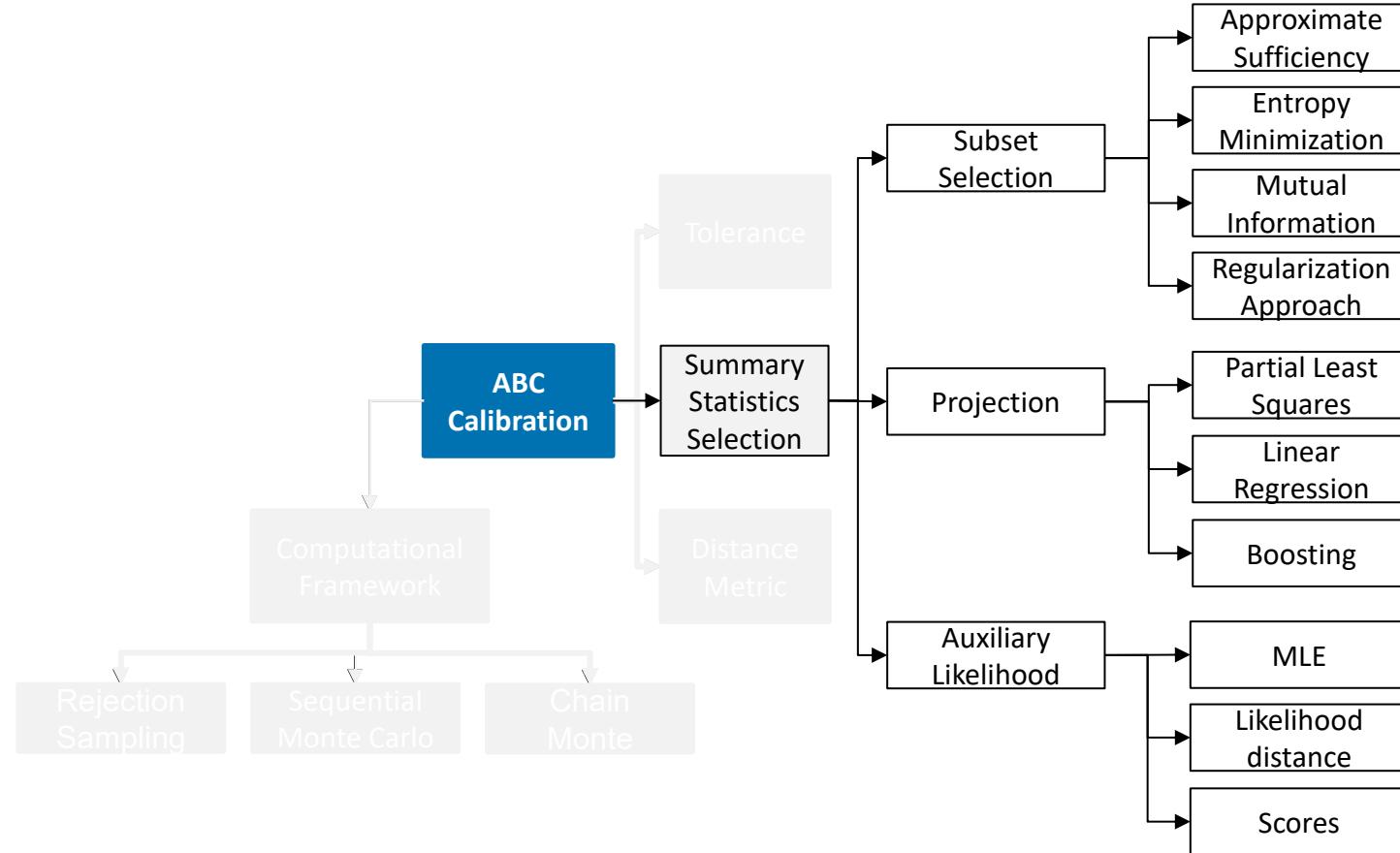
**Popular Examples**

Approximate Bayesian Computation  
Synthetic Likelihood Method

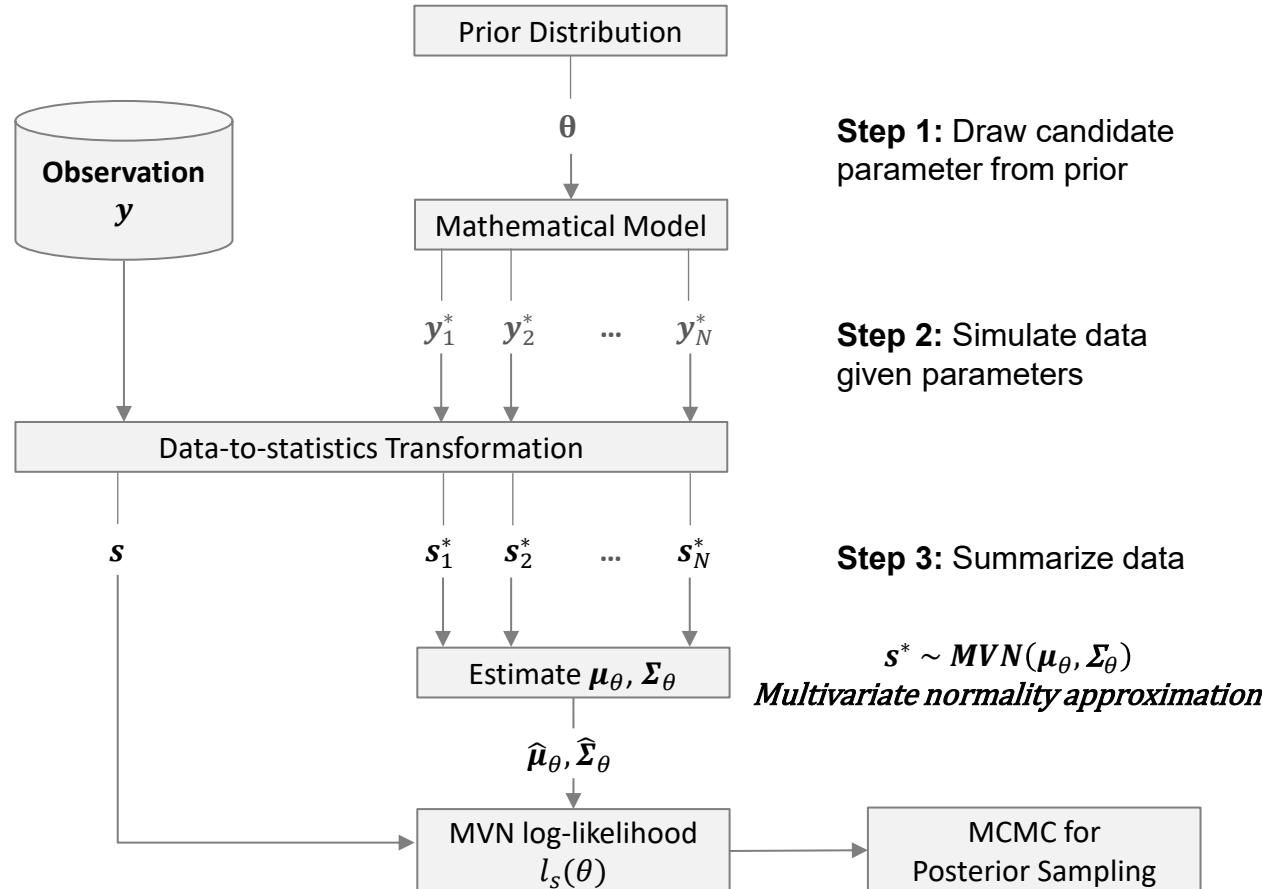
Computationally  
Prohibitive

Analytically  
Impossible

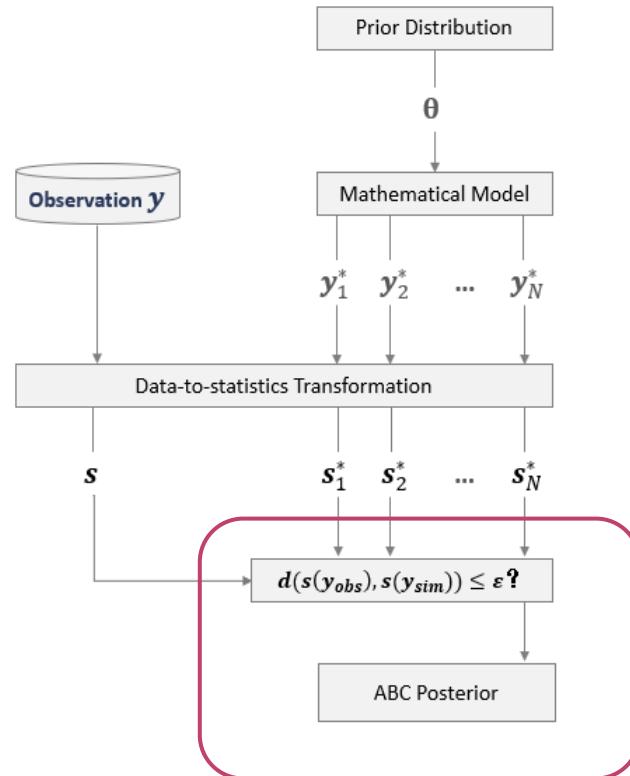




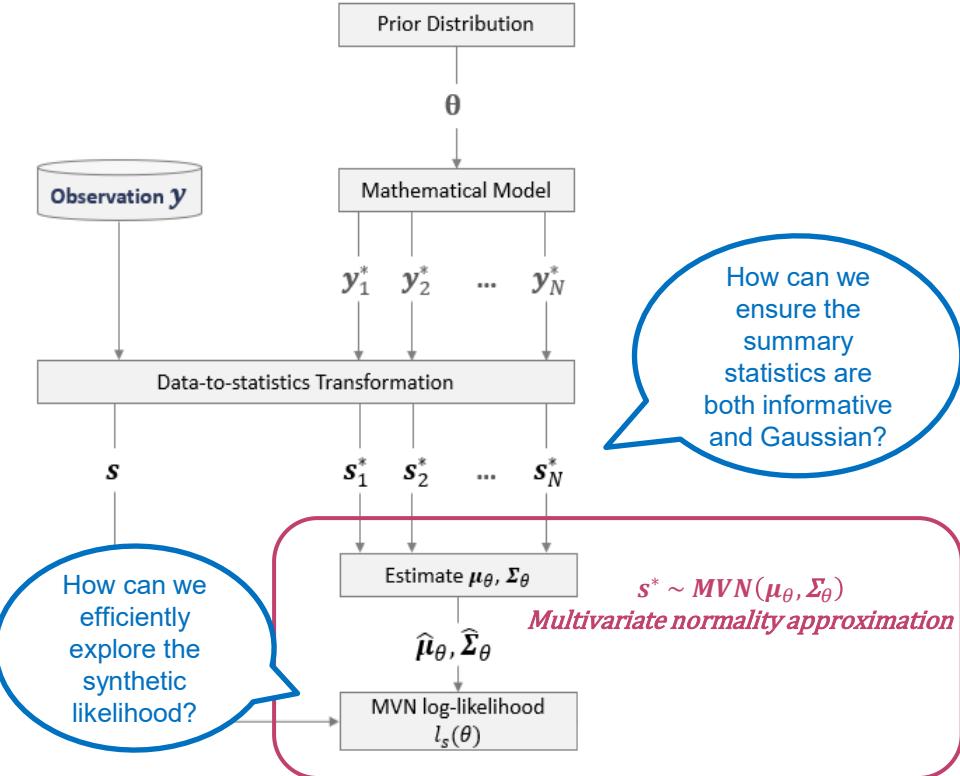
## 2.2 Synthetic Likelihood Method (Wood, 2010)

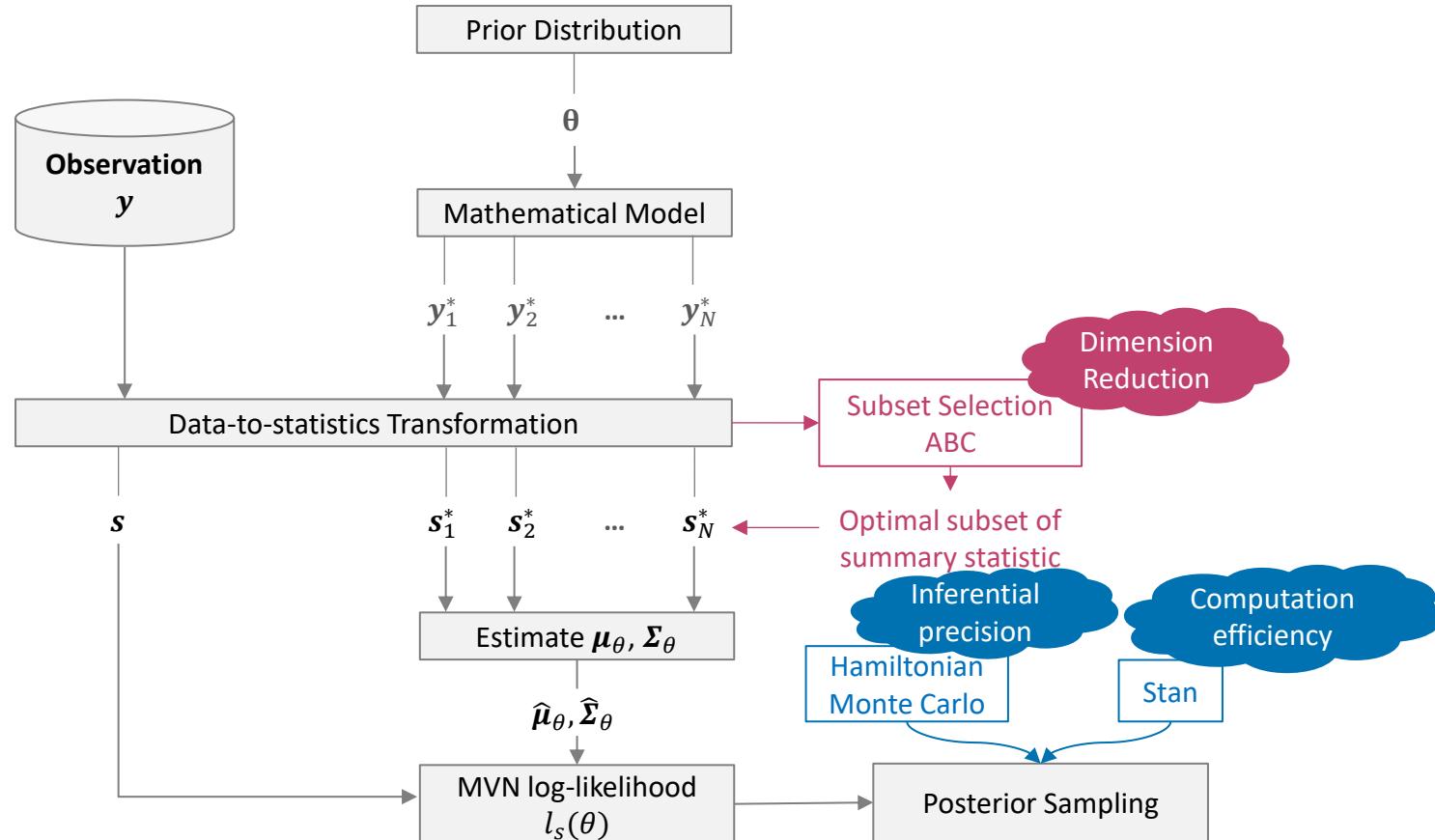


### ABC Workflow

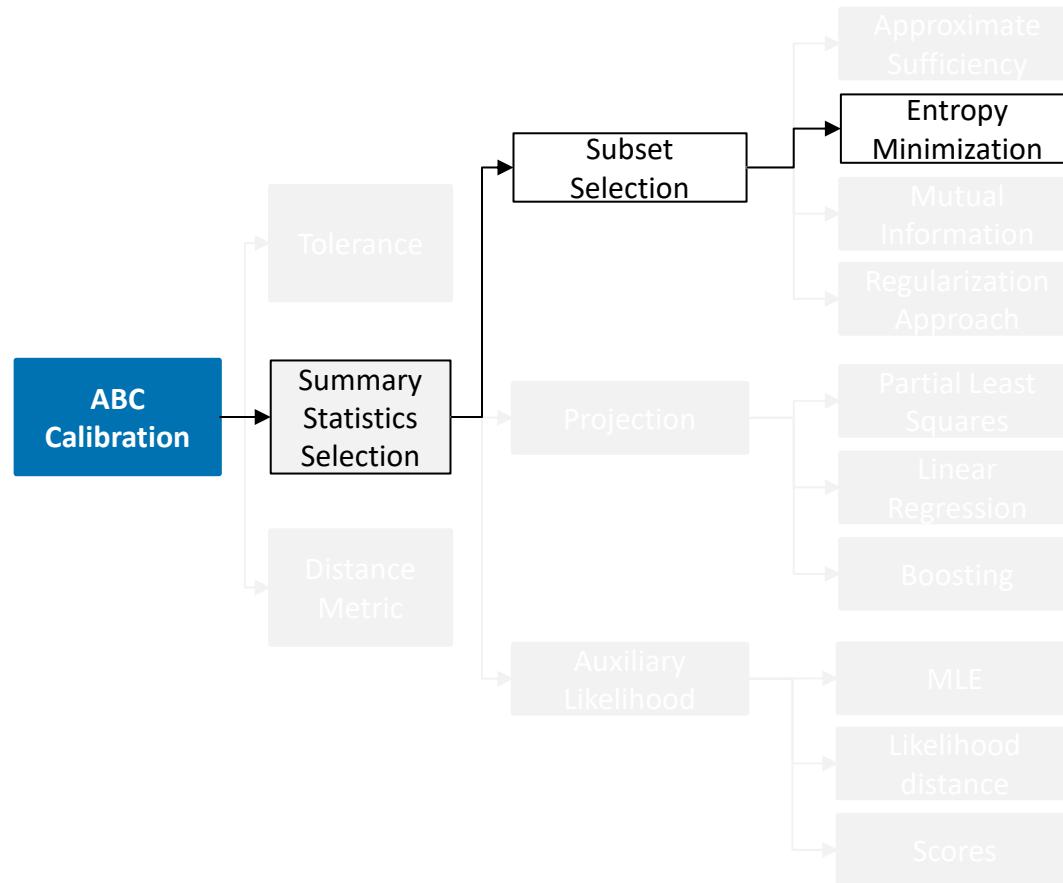


### BSL Workflow





## 3.1 Key Innovation 1: ABC for Summary Statistics Selection



### Entropy Minimisation

- Perform rejection-ABC and compute the  $k$ th nearest neighbour of entropy on the ABC posterior sample.

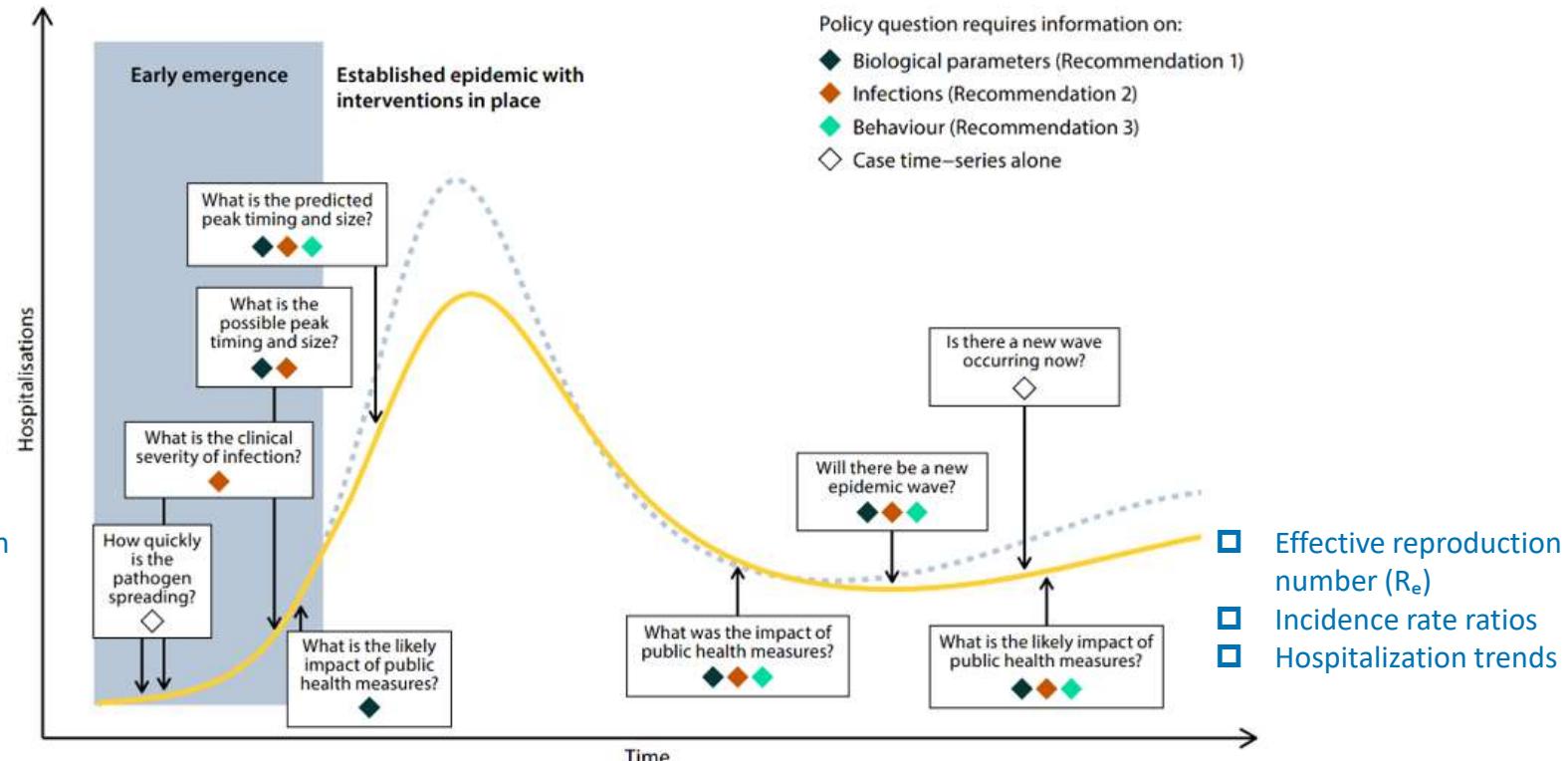
$$\hat{E} = \log \left[ \frac{\pi^{p/2}}{\Gamma(\frac{p}{2} + 1)} \right] - \Psi(\kappa) + \log(n) + \frac{p}{n} \sum_{i=1}^n \log(R_{i,\kappa})$$

### Why this method?

- balance of flexibility and interpretability—especially when data are limited during early outbreak

## 3.2 Key Innovation 2: Policy-informed Candidate Statistics

Figure 1: Exemplar policy questions and transmission-related surveillance needs



Data Sources for Strategic Decision-making. Adapted from "Opportunities to strengthen respiratory virus surveillance systems in Australia: Lessons learned from the COVID-19 response", by Shearer, F. M., Edwards, L., Kirk, M., Eales, O., Golding, N., Hassall, J., ... & McCaw, J. M. , 2024.

## 3.3 Key Innovation 3: HMC for Posterior Sampling (using Stan)

### Random Walk Metropolis Hastings

↑

### Hamiltonian Monte Carlo

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GIF Animation 1: Random walk Metropolis Hastings

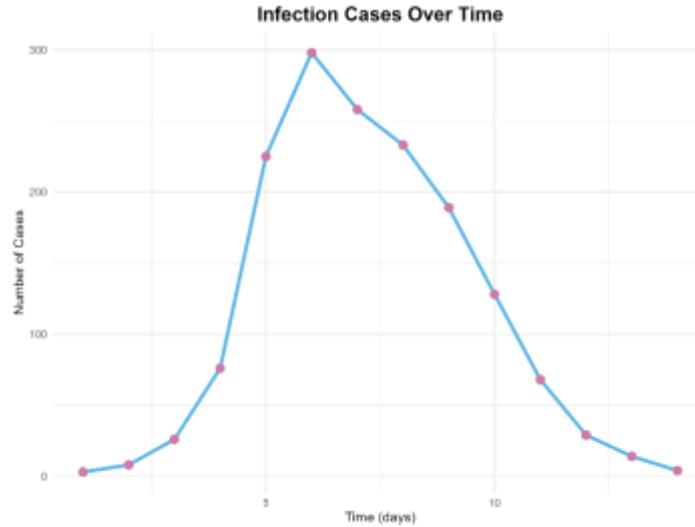
Adapted from

[https://bookdown.org/danbarch/psy\\_207\\_advanced\\_stats\\_I/M\\_CMC-methods.html](https://bookdown.org/danbarch/psy_207_advanced_stats_I/M_CMC-methods.html)

GIF Animation 2: Hamiltonian Monte Carlo sampling

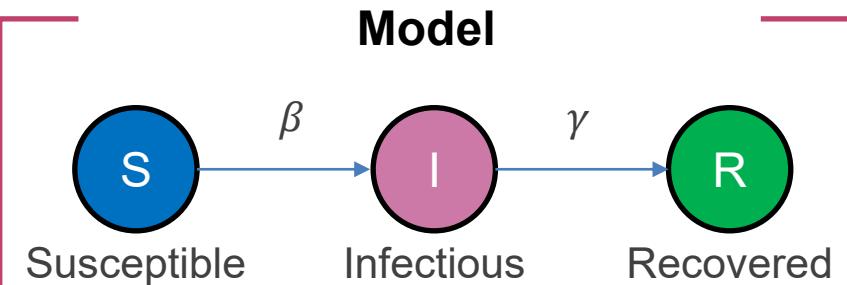
Adapted from Justinkunimune - Own work

using: [github.com/jkunimune/hamiltonian-mc](https://github.com/jkunimune/hamiltonian-mc), CC0



### About data

- 1978 influenza A outbreak at a British boarding school.
- During the outbreak, 512 out of 763 students became ill.



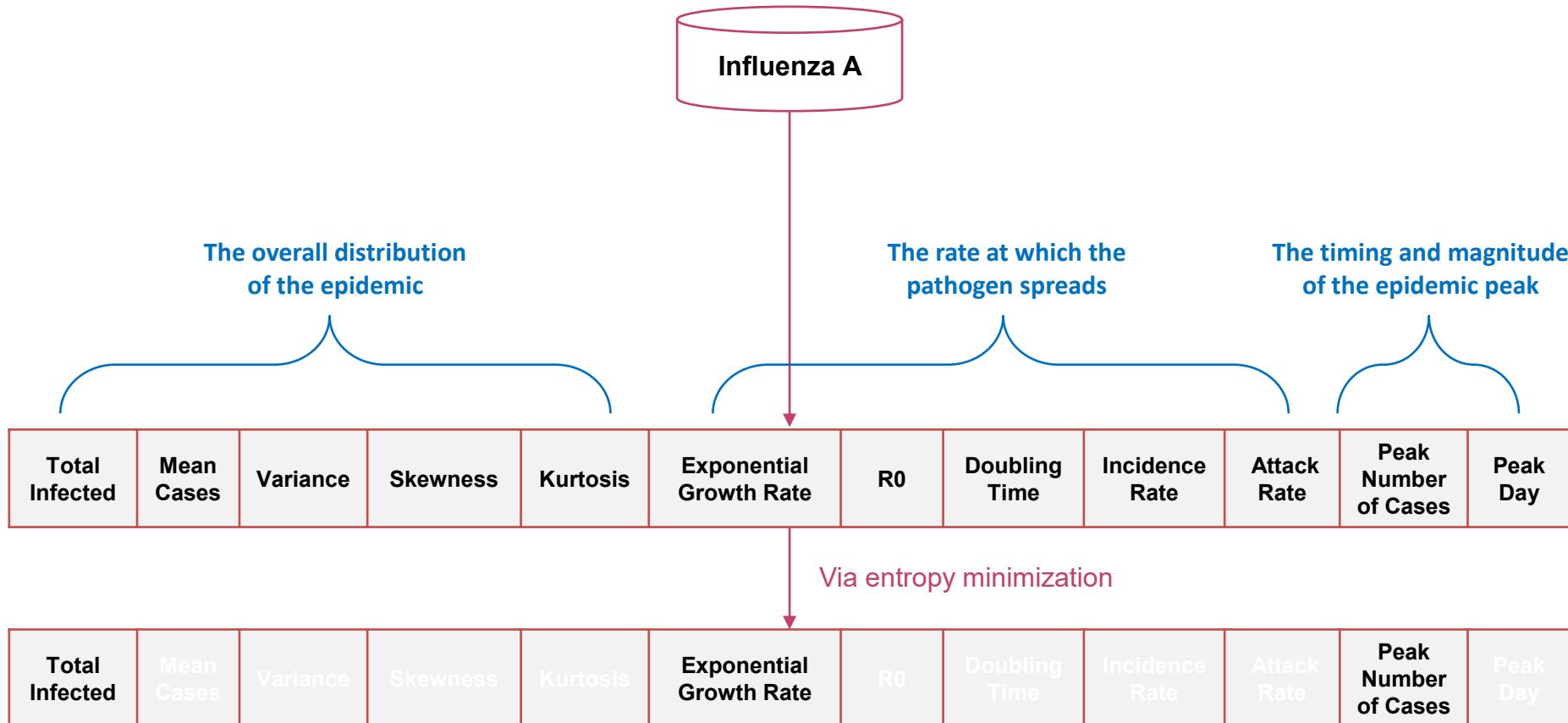
$$\frac{dS}{dt} = -\beta \frac{I(t)}{N} S(t)$$

$$\frac{dI}{dt} = \beta \frac{I(t)}{N} S(t) - \gamma I(t)$$

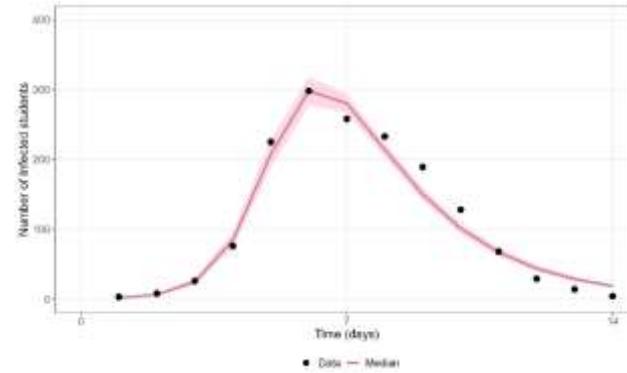
$$\frac{dR}{dt} = \gamma I(t)$$

$$Y_t \sim \text{Poisson}(\lambda_t)$$

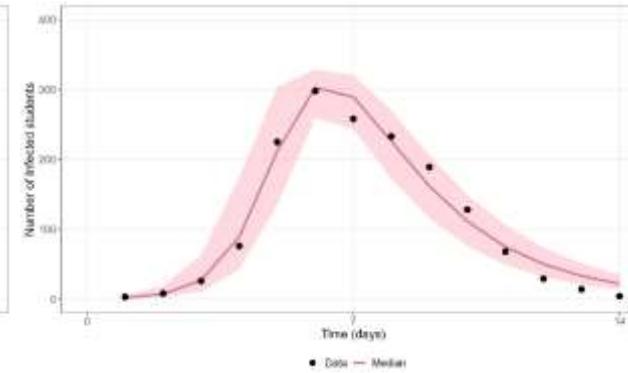
## 4.2 Application – Summary Statistics Selection using ABC



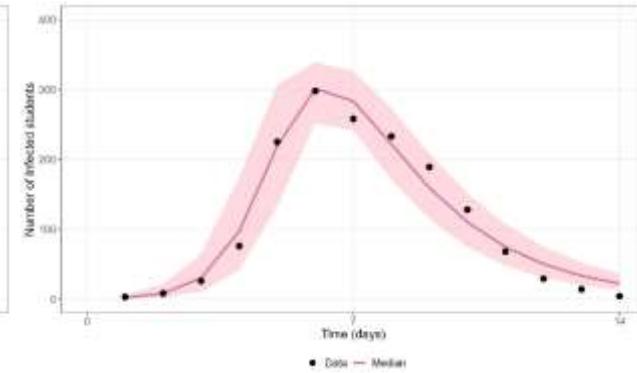
### Poisson Model



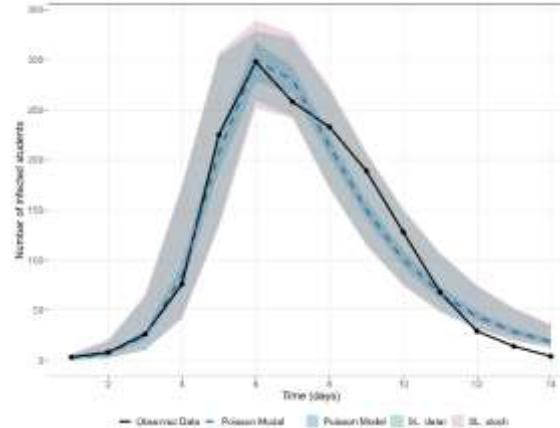
### SL Deterministic Model



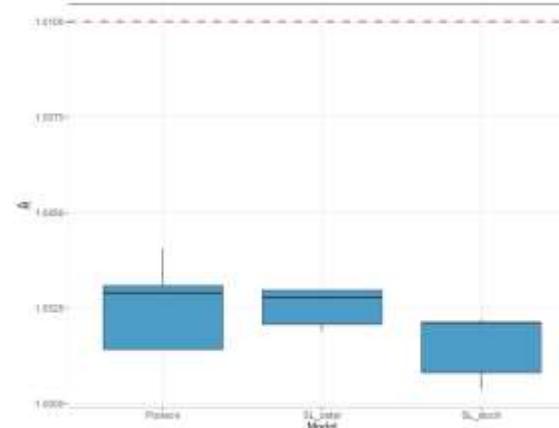
### SL Stochastic Model



Model Comparison: Epidemic Curve Fitting



R-hat Comparison Across Models



**Summary:**

- Introduced a **hybrid inference framework** for parameter estimation in epidemiological models with intractable likelihoods.
- Addressed challenges in Bayesian inference for compartmental models, by integrating
  - **ABC-based entropy minimization** for summary statistics selection
  - **BSL** for flexible likelihood approximation
  - **HMC** for posterior sampling

**Further Research:**

- Current numerical studies only focused on **compartmental models**, can extend to **more complex models**.
- **Gaussian approximation** might be restrictive for extreme cases, can explore **alternative assumptions** to improve **robustness**.
- Apply to **large-scale, real-world datasets** to test scalability and performance.
- **Theoretical investigation** on method under varying degrees of model complexity and data quality.



# THANK YOU.



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