PHYS 8750
HW2
Due September 22, 2020

Consider the advection problem

$$\frac{\partial \psi}{\partial t} + c\frac{\partial \psi}{\partial \mathrm{x}} = 0$$

1. Using von Neumann stability analysis, derive the stability condition for the following difference scheme:

$$\delta_{2t}\phi + c\left(\frac{4}{3}\delta_{2x}\phi - \frac{1}{3}\delta_{4x}\phi\right) = 0$$

Here $\delta_{nt}f(t) = \frac{f(t+n\Delta t/2)-f(\mathrm{t}-n\Delta t/2)}{n\Delta t}, \delta_{nx}f(x) = \frac{f(x+n\Delta x/2)-f(x-n\Delta x/2)}{n\Delta x}$

2. Read Takacs (1985) paper, derive the five coefficients (a_1, a_2, a_3, a_4, and a_5) for the fourth-order Takacs method. Examine and understand the function F_phin_4s_Takacs.m. Are the coefficients correctly set up?

3. Use the code "Advection_PDE_RK_2rdSpace_LaxWen_Takacs_stability_2.m" and answer the following questions:

1) For "waveform = 3 (select one wave component to start)", Try CFL = 0.2, 0.5, 0.9, 1.01, 1.05, 1.1, run the model for the same amount of time (not the same amount of time step), comment on the CFL stability condition.
Mark the maximum amplitudes and compare these among different schemes and different CFL numbers for each individual scheme.

Note: you can name the time you will stop the model, as long as you can make up your story and put forward reasonings. (stopping time is flexible).

2) For "waveform = 3", turn on two waves. Comment on dispersion errors of different schemes.

3) Which scheme leads to the best amplitude performance? Comment on amplitude and phase errors, respectively?

4) Replace the wave function to triangle and step functions, refer to questions 1) and 3), and comment on what you find. Discuss shared principles among these three functions, and differences.

Some of the answers will be quantitative with respect to the settings you choose. The comments from you will learn from these exercises can be qualitative.