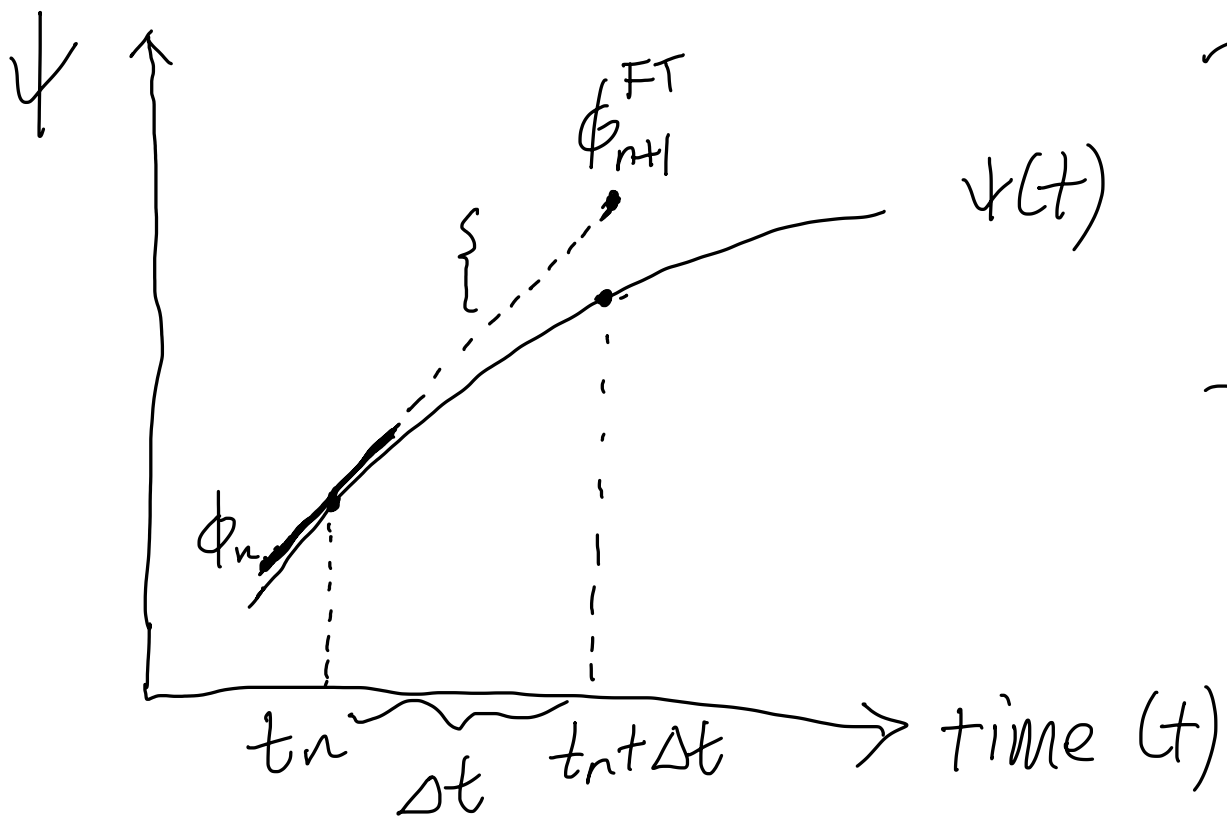
8-27-2020 PHYS 8750

Multi-stage method:

$$\frac{d\psi}{dt} \approx F(\psi, t)$$



FT: $\left[\frac{\phi_{n+1}-\phi_n}{\Delta t}\right] = \left[F(\phi_n, t_n)\right]$

$\phi_{n+1} = \phi_n + F(\phi_n, t_n)\cdot\Delta t$

one-stage

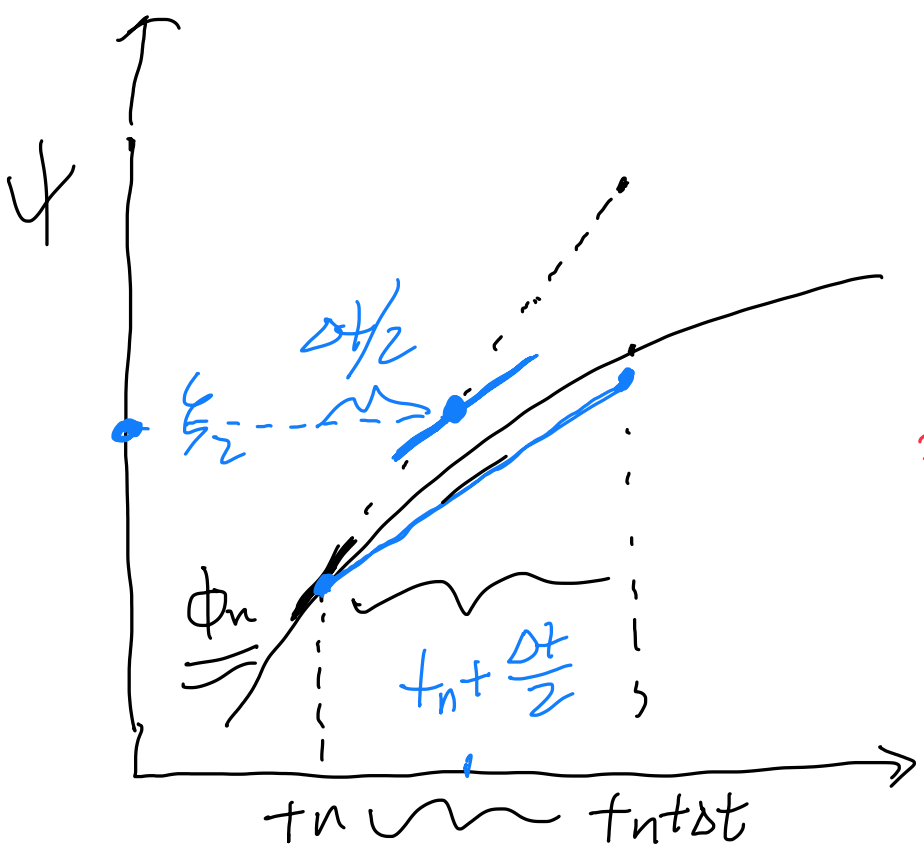
Multi-stage: how many times that from t_n to $t_n+\Delta t$, derivatives are evaluated.

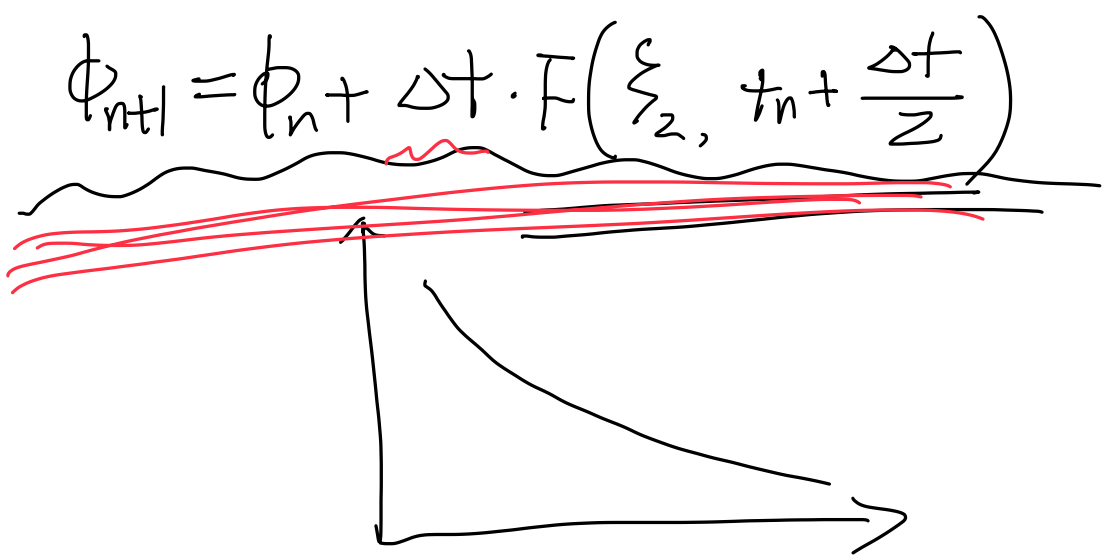
2-stage Runge-kutta

$\xi_1 = \phi_n$

$\xi_2 = \phi_n + \frac{\Delta t}{2} F(\phi_n, t_n)$

$\phi_{n+1} = \phi_n + \Delta t \cdot F\left(\xi_2, t_n + \frac{\Delta t}{2}\right)$





Δt

2-stage RK : 2-nd order of accuracy.

FT: $\underbrace{\frac{\psi_{n+1}-\psi_n}{\Delta t}} - \underbrace{\frac{d\psi}{dt}} = O(\Delta t)$

$\psi|_{t_n+\Delta t}$

$\textcircled{1}\ \psi_{n+1} = \psi\Big|_{t_n+\frac{\Delta t}{2}} + \frac{d\psi}{dt}\Big|_{t_n+\frac{\Delta t}{2}} \cdot \left(\frac{\Delta t}{2}\right) + \frac{d^2\psi}{dt^2}\Big|_{t_n+\frac{\Delta t}{2}} \cdot \frac{\left(\frac{\Delta t}{2}\right)^2}{2}$

$+ \frac{d^3\psi}{dt^3}\Big|_{t_n+\frac{\Delta t}{2}} \cdot \frac{\left(\frac{\Delta t}{2}\right)^3}{6} + O(\Delta t^4)$

ψ_n

$\psi_n = \psi\Big|_{t_n+\frac{\Delta t}{2}} + \frac{d\psi}{dt}\Big|_{t_n+\frac{\Delta t}{2}} \left(-\frac{\Delta t}{2}\right) + \frac{d^2\psi}{dt^2}\Big|_{t_n+\frac{\Delta t}{2}} \frac{\left(\frac{-\Delta t}{2}\right)^2}{2} + \frac{d^3\psi}{dt^3}\frac{\left(\frac{-\Delta t}{2}\right)^3}{6} + O(\Delta t^4)$

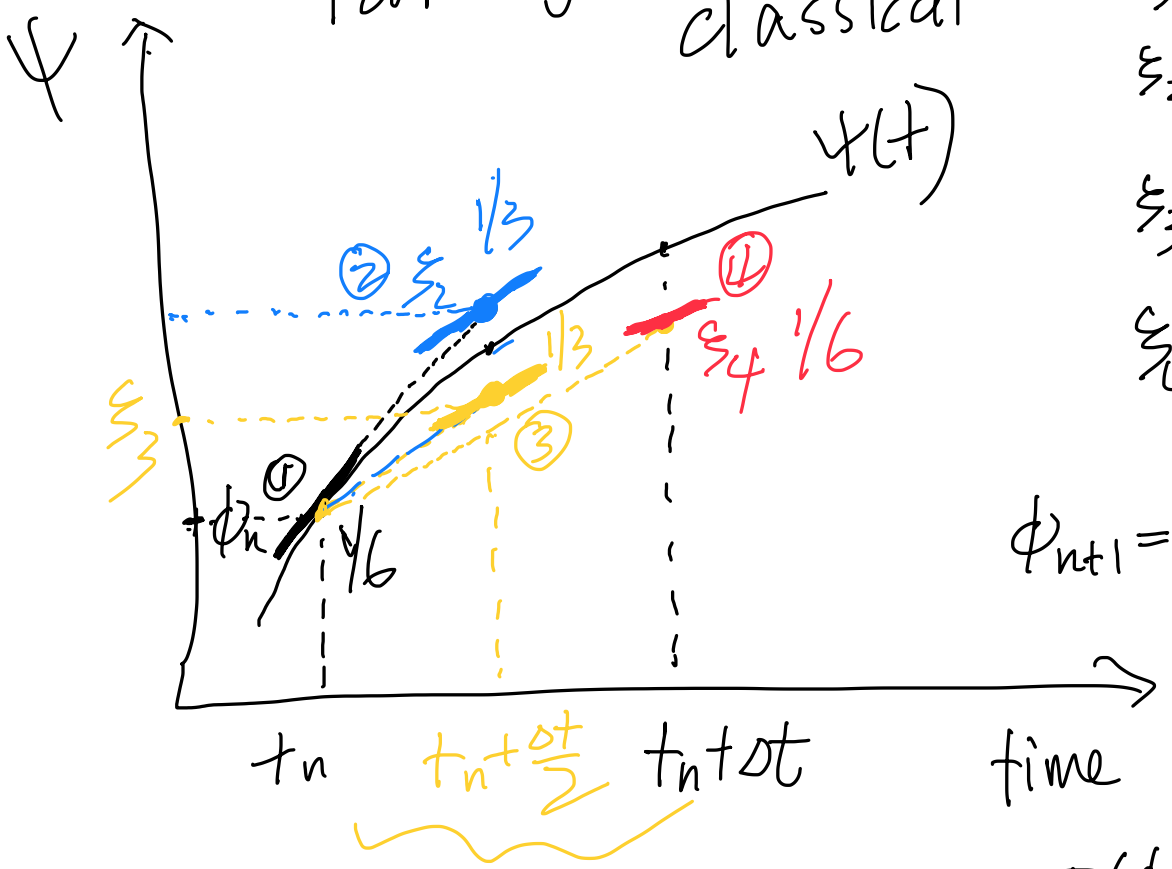
$\textcircled{2}$

$\textcircled{1}-\textcircled{2}$: $\psi_{n+1} - \psi_n = \frac{d\psi}{dt}\Big|_{t_n+\frac{\Delta t}{2}} \cdot \Delta t + \frac{d^3\psi}{dt^3}\Big|_{t_n+\frac{\Delta t}{2}} \cdot \frac{\Delta t^3}{24} + O(\Delta t^5)$

$\frac{\psi_{n+1}-\psi_n}{\Delta t} - \frac{d\psi}{dt}\Big|_{t_n+\frac{\Delta t}{2}} = \frac{d^3\psi}{dt^3}\Big|_{t_n+\frac{\Delta t}{2}} \cdot \frac{(\Delta t)^2}{24} + O(\Delta t^4)$

second order accuracy

Fourth-order RK method.
Four-stage.
classical



$\frac{d\psi}{dt} = F$ ✓

$\xi_1 = \phi_n$

$\xi_2 = \phi_n + \frac{\Delta t}{2} F(\phi_n, t_n)$ ✓

$\xi_3 = \phi_n + \frac{\Delta t}{2} F(\xi_2, t_n + \frac{\Delta t}{2})$ ✓

$\xi_4 = \phi_n + \Delta t \; F(\xi_3, t_n + \frac{\Delta t}{2})$

$\phi_{n+1} = \phi_n + \Delta t \; F_{general}$ ⇒ F: derivative,

$= \phi_n + \frac{\Delta t}{6}(F_1 + 2F_2 + 2F_3 + F_4)$

F_1: $F(\phi_n, t_n)$, $F_2 = F(\xi_2, t_n + \frac{\Delta t}{2})$

$\phi_{n+1} = \phi_n + \frac{\Delta t}{6}(2F_1 + F_2 + F_3 + 2F_4)$