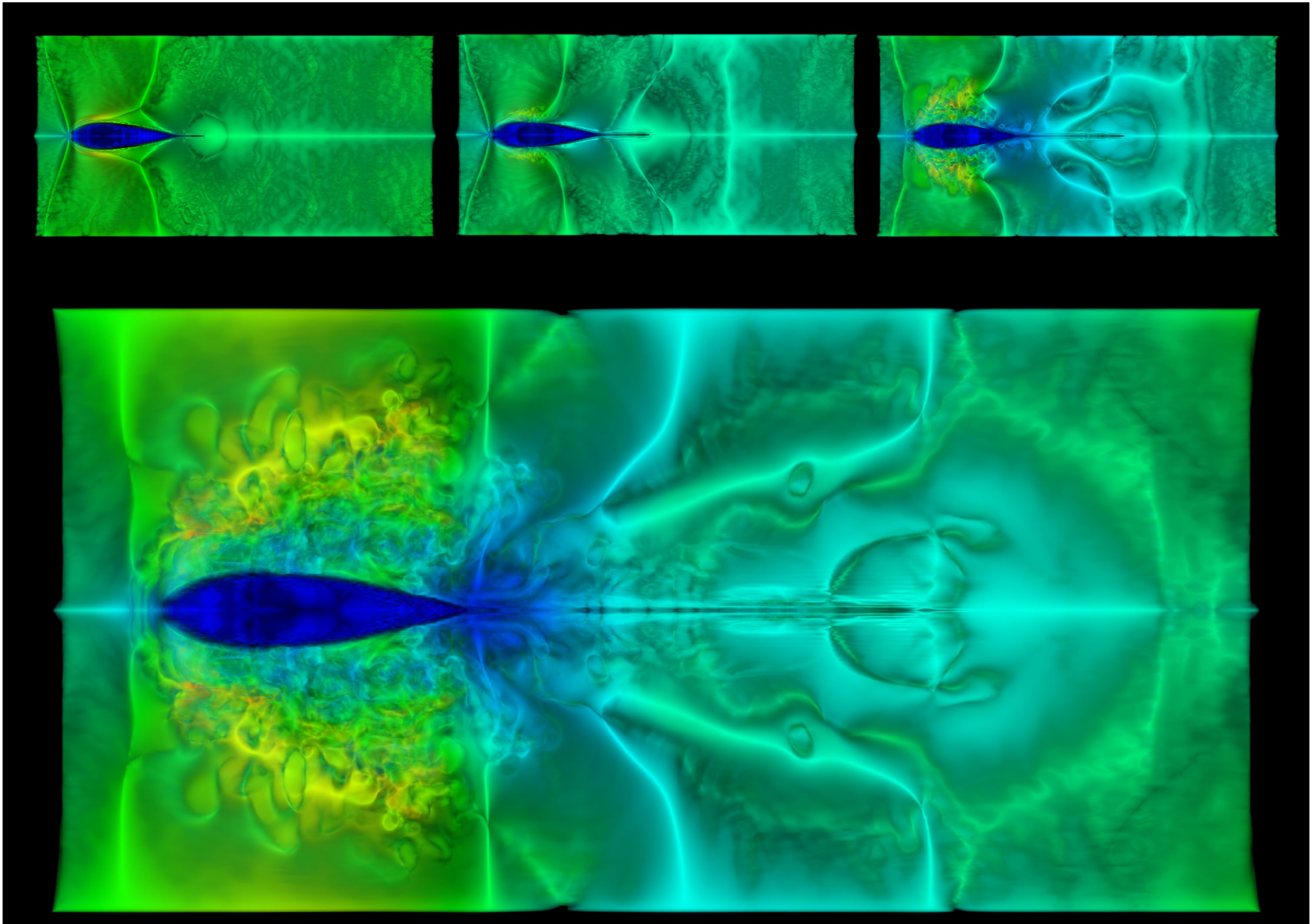


Breaking waves • simulation by D. Dommermuth • TACC vis gallery

PHYS 8750

CLASS # 20
ATMOSPHERIC MODELS AND COMMUNITY MODELS

CLASS #18, 19
ATMOSPHERIC MODELS

Outline

1. Atmospheric Models
2. Model types
   - Empirical models/Physics models
   - Data assimilative models
3. NRLMSIS and IRI
4. GSWM and CTMT
5. Physics models
   - TIEGCM (difference scheme)
   - CESM (piecewise finite volume method)
6. Model run and supercomputing

Atmospheric Models

- **Empirical Models (data based, mathematical, statistical)**
  - Good for climatology and background atmosphere
  - NRLMSIS, IRI, HWM

- **Physics Models (physical models)**
  - **Steady-state (external forcing or boundary condition)**
    - GSWM, CTMT, Stationary Planetary Waves (week of Nov 17, 19)
  - **Time-evolving (initial condition)**
    - General circulation models (TIEGCM, CESM, KMCM, CTIPe)
    - Barotropic/baroclinic planetary wave model (week of Nov 17, 19)
    - Gravity wave models (MAGIC)

- **Data Assimilative Models**
  - Underlying Physics models + data assimilation
  - WACCM-DART, ECMWF, MERRA

NRLMSIS

Naval Research Lab Mass Spectrometer Incoherent Scatter Model

NRLMSIS 2.0: A whole-atmosphere empirical model of temperature and neutral species densities

J. T. Emmert[1], D. P. Drob[1], J. M. Picone[2], D. E. Siskind[1], M. Jones Jr.[1], M. G. Mlynczak[3], P. F. Bernath[4,5], X. Chu[6,7], E. Doornbos[8], B. Funke[9], L. P. Goncharenko[10], M. E. Hervig[11], M. J. Schwartz[12], P. E. Sheese[13], F. Vargas[14], B. P. Williams[15], and T. Yuan[16]

NRLMSIS 2.0 is an empirical atmospheric model that extends from the ground to the exobase and describes the **average observed behavior** of temperature, 8 species densities, and mass density via a parametric analytic formulation. The model inputs are location, day of year, time of day, solar activity, and geomagnetic activity.

NRLMSIS

Parametric analytic formulation + Fitting Using Data

$$\frac{1}{T(\zeta)} = \begin{cases} \left\{T_{ex} - (T_{ex} - T_B)\exp\left[-\sigma(\zeta - \zeta_B)\right]\right\}^{-1} & ; \zeta \geq \zeta_B \\ \sum_{i=0}^{N_S - 1} \alpha_i S_i(\zeta) & ; \zeta < \zeta_B \end{cases}$$

$T(\zeta)$	Temperature profile as a function of geopotential height
$\zeta_B =$ 122.5 km	Bates profile reference height and joining height
T_{ex}	Exospheric temperature (fitting parameter)
$T_B = T(\zeta_B)$	Temperature at ζ_B (fitting parameter)
$\sigma = T'_B / (T_{ex} - T_B)$	Shape parameter
$T'_B = \left.\frac{dT}{d\zeta}\right\rvert_{\zeta=\zeta_B}$	Temperature gradient at ζ_B (fitting parameter)
$N_S = 24$	Number of B-spline basis functions
α_i	Coefficients on B-spline basis functions (fitting parameters)
S_i	Cubic B-splines with nodes at heights $\zeta_{S,i}$; $i = 0$ to $N_S + 3$

$\zeta_{S,i} = \{-15, -10, -5, 0, 5, \ldots, 80, 85, 92.5, 102.5, 112.5, 122.5, 132.5, 142.5, 152.5\}$ km

$$\ln n(\zeta) = \ln n_0 - \frac{g_0}{k}\int_{\zeta_0}^{\zeta} \frac{M(\zeta')}{T(\zeta')} d\zeta' - \ln\frac{T(\zeta)}{T(\zeta_0)} - Ce^{-(\zeta-\zeta_C)/H_C} + R\left[1 + \tanh\left(\frac{\zeta - \zeta_R}{\gamma(\zeta) H_R}\right)\right]$$

$n(\zeta)$	Number density of a particular species
$n_0 = n(\zeta_0)$	Reference density (defined below)
ζ_0	Reference geopotential height
g_0	Reference gravitational acceleration (see equation (A3))
k	Boltzmann constant
$M(\zeta)$	Effective mass profile (defined below)
C, ζ_C, H_C	Chemical loss term parameters
R, ζ_R, H_R	Chemical/dynamical correction parameters

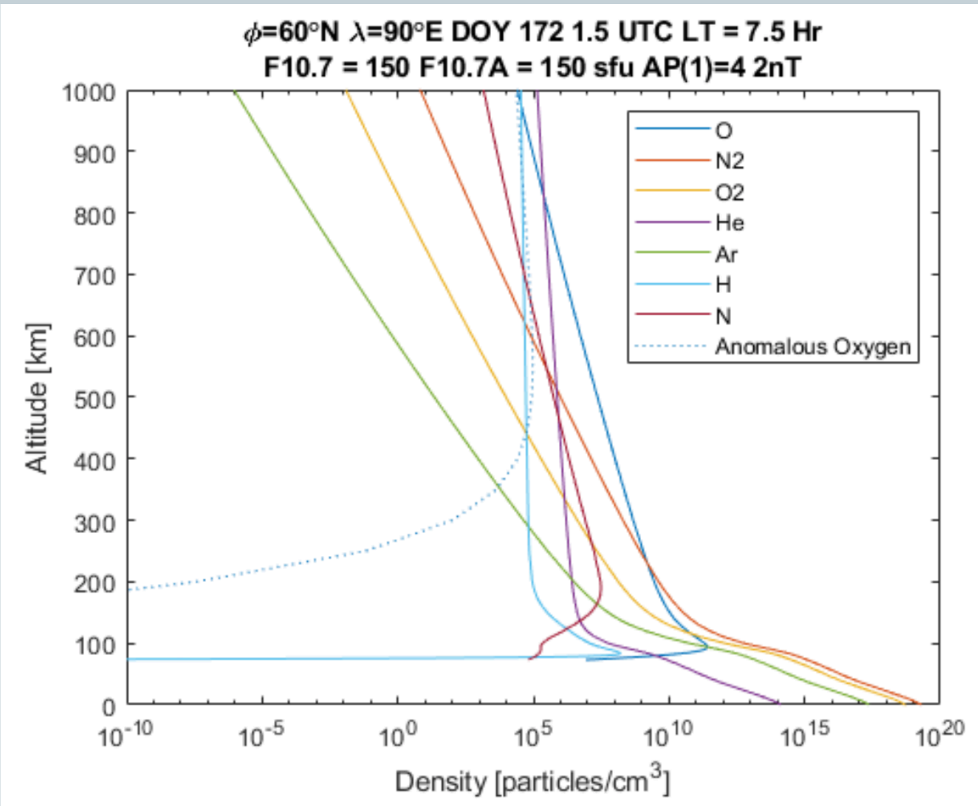
$$\gamma(\zeta) = \frac{1}{2}\left\{1 + \tanh\left(\frac{\zeta - \zeta_\gamma}{H_\gamma}\right)\right\}$$

$\zeta_\gamma = 70$ km

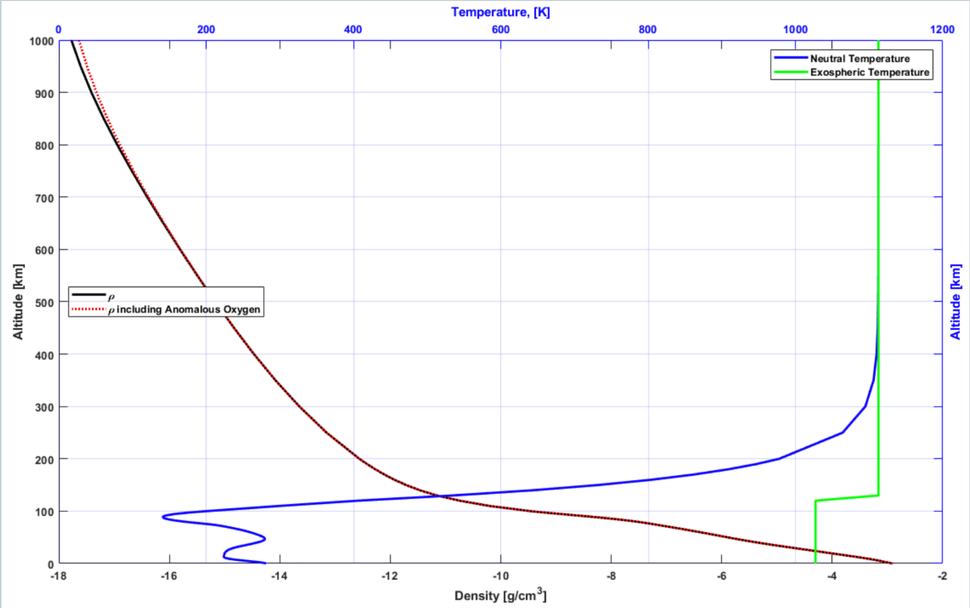
$H_\gamma = 40$ km

Outputs of NRLMSIS

Density of Species



Density & Temperature



IRI (International Reference Ionosphere)

The International Reference Ionosphere (IRI) is an international project sponsored by the Committee on Space Research (COSPAR) and the International Union of Radio Science (URSI).

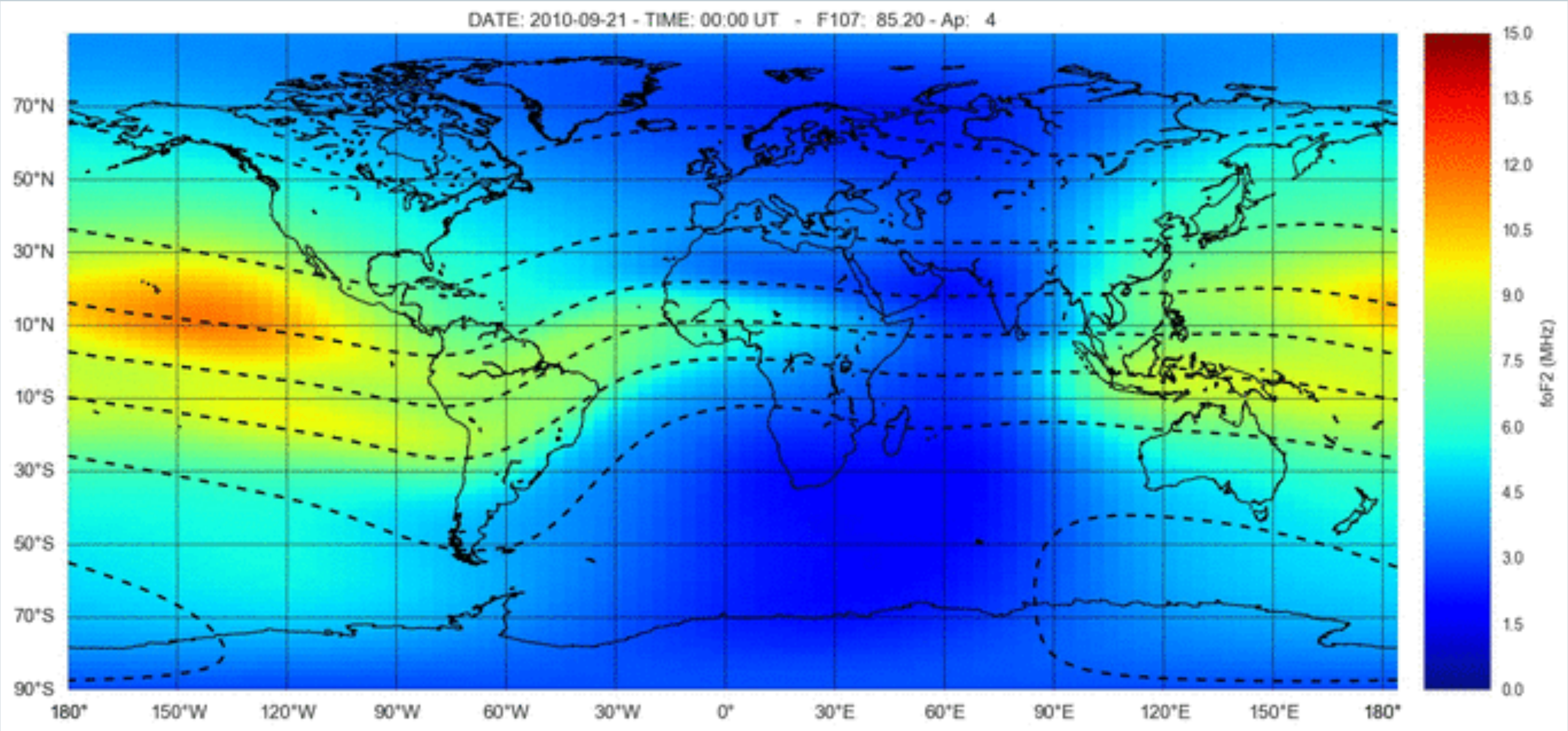
PARAMETERS:

Ne, Te, Ti, ion composition (O^+, H^+, He^+, N^+, NO^+, O^+_2, Cluster ions), equatorial VI, vertical TEC (vTEC), spread-F probability, auroral boundaries, effects of ionospheric storms on F and E peak densities.

INPUTS:

Required: solar indices (F10.7), ionospheric index (ionosonde-based IG index 12-month running mean), magnetic index (ap).

IRI Outputs


DATE: 2010-09-21 - TIME: 00:00 UT - F107: 85.20 - Ap: 4
70°N
50°N
30°N
10°N
10°S
30°S
50°S
70°S
90°S
180°
150°W
120°W
90°W
60°W
30°W
0°
30°E
60°E
90°E
120°E
150°E
180°
15.0
13.5
12.0
10.5
9.0
7.5
6.0
4.5
3.0
1.5
0.0
foF2 (MHz)

- **Steady-State**

Mean values or waves amplitude do not change with time.

Large-scale Wave models

Tides: GSWM, CTMT

Planetary Waves: stationary wave model

Global Scale Wave Model

$$\frac{\partial u}{\partial t} - 2\Omega \sin\theta v + \frac{1}{a\cos\theta}\frac{\partial \Phi}{\partial \lambda} = 0 \qquad (1)$$

$$\frac{\partial v}{\partial t} + 2\Omega \sin\theta u + \frac{1}{a}\frac{\partial \Phi}{\partial \theta} = 0 \qquad (2)$$

$$\frac{\partial}{\partial t}\Phi_z + N^2 w = \frac{\kappa J}{H} \qquad (3)$$

$$\frac{1}{a\cos\theta}\left[\frac{\partial u}{\partial \lambda} + \frac{\partial}{\partial \theta}(v\cos\theta)\right] + \frac{1}{\rho_o}\frac{\partial}{\partial z}(\rho_o w) = 0 \qquad (4)$$

where

u	eastward velocity
v	northward velocity
w	upward velocity
Φ	perturbation geopotential
N^2	buoyancy frequency squared $= \kappa g/H$
Ω	angular velocity of Earth
ρ_o	basic state density $\propto e^{-z/H}$
z	altitude
λ	longitude
θ	latitude
κ	$R/c_p \approx 2/7$
J	heating per unit mass
a	radius of Earth
g	acceleration due to gravity
H	constant scale height
t	time

Forbes, 1995

By assuming the wave form

$$\{u, v, w, \Phi\} = \{\hat{u}, \hat{v}, \hat{w}, \hat{\Phi}\}\exp[i(s\lambda - \sigma t)] \qquad (5)$$

Then 4D problem becomes 2D

$$\frac{\partial}{\partial t} = -i\sigma, \frac{\partial}{\partial \lambda} = is$$

Separation of variable and solve for latitude and altitude directions

$$\hat{J} = \sum_n \dot{\Theta}_n(\theta) J_n(z) \qquad (7)$$

$$\hat{\Phi} = \sum_n \Theta_n(\theta) G_n(z) \qquad (6)$$

Global Scale Wave Model

Equation along altitude (z direction)

$$i\sigma H\left[\frac{1}{\rho_o}\frac{\partial}{\partial z}\rho_o\frac{\partial}{\partial z}G_n\right]+\frac{1}{\rho_o}\frac{\partial}{\partial z}(\rho_o\kappa J_n)=-\frac{i\sigma\kappa}{h_n}G_n \quad (12)$$

Equation along latitude (y direction)

$$\frac{d}{d\mu}\left[\frac{(1-\mu^2)}{(f^2-\mu^2)}\frac{d\Theta_n}{d\mu}\right]-\frac{1}{f^2-\mu^2}$$

$$\left[-\frac{s}{f}\frac{(f^2+\mu^2)}{(f^2-\mu^2)}+\frac{s^2}{1-\mu^2}\right]\Theta_n+\epsilon\Theta_n=0 \quad (14)$$

For each tidal component, decompose heating term into different Hough modes, and solve for the tidal responses in different modes. The full solution is the superposition of all the Hough modes.

Global Scale Wave Model

Hough Modes

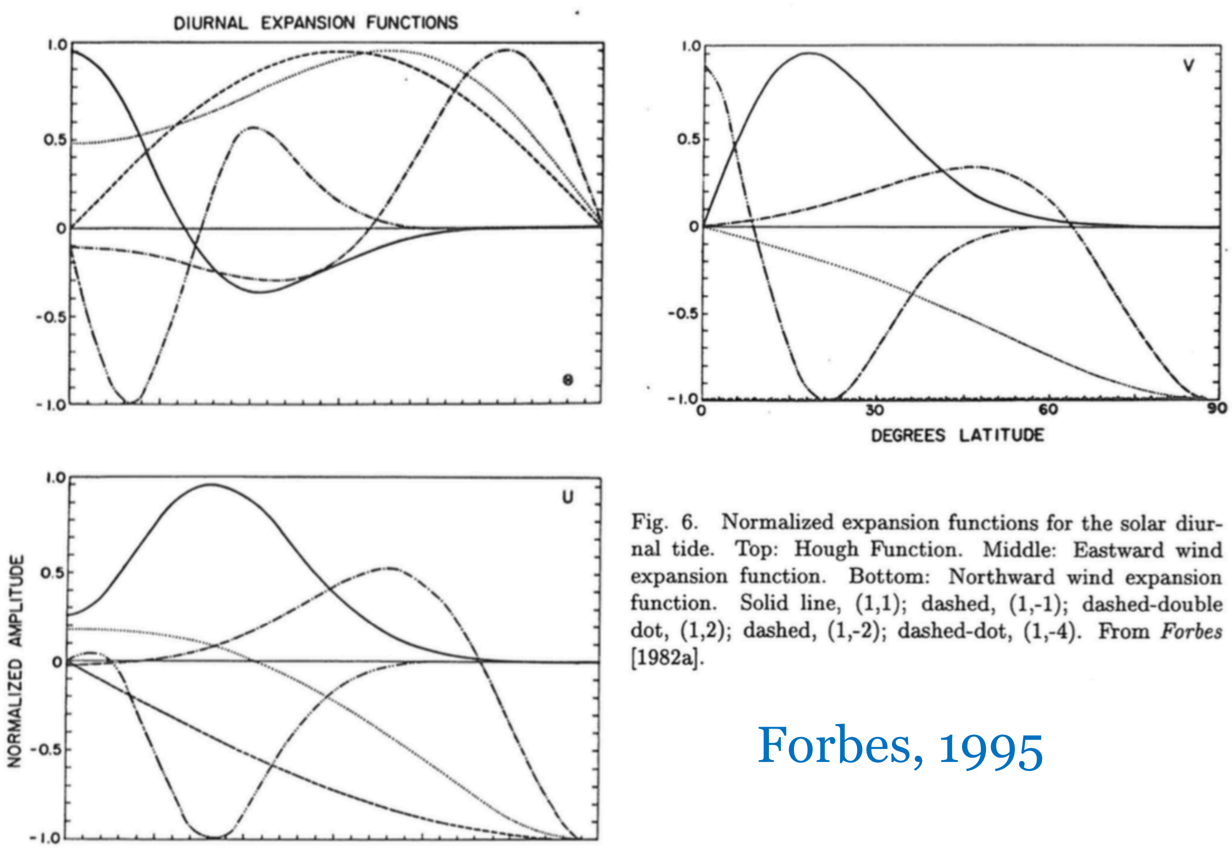
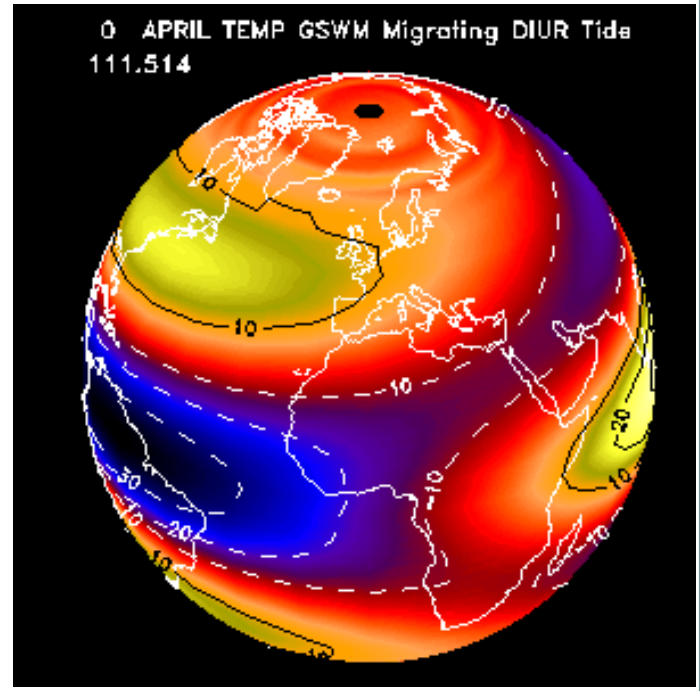


Fig. 6. Normalized expansion functions for the solar diurnal tide. Top: Hough Function. Middle: Eastward wind expansion function. Bottom: Northward wind expansion function. Solid line, (1,1); dashed, (1,-1); dashed-double dot, (1,2); dashed, (1,-2); dashed-dot, (1,-4). From *Forbes* [1982a].

Forbes, 1995

Temperature DW1



Spherical coordinate: Legendre functions

Climatological Tidal Model of the Thermosphere (CTMT)

Hough Mode Extensions

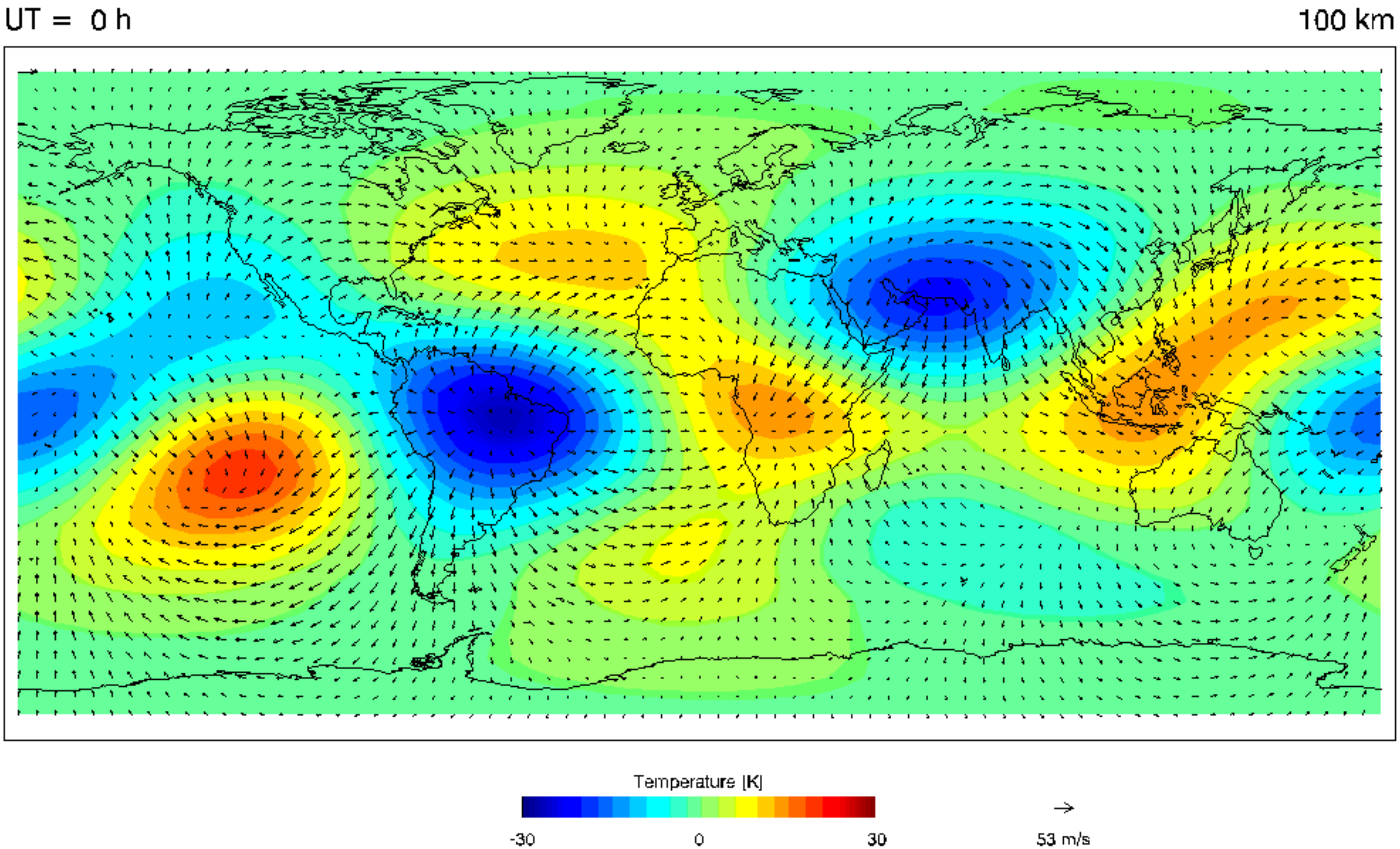
- Generalized form of Hough Modes; account for dissipation

Model concept

- Fit HMEs to TIMED tides in the MLT region; 2002-2008 averages
- Use fit coefficients to reconstruct the tides in the thermosphere, F10.7 = 110 sfu

Oberheide, J., J. M. Forbes, X. Zhang, and S. L. Bruinsma, Climatology of upward propagating diurnal and semidiurnal tides in the thermosphere, *J. Geophys. Res., 116, A11306, doi:10.1029/2011JA016784, 2011.*

Climatological Tidal Model of the Thermosphere (CTMT)



TIEGCM
(Thermosphere Ionosphere Electrodynamics General Circulation Model)

1 Continuity (Neutral and Ion)

$$\frac{\partial n}{\partial t} - Q + Ln = -\boldsymbol{\nabla} \cdot (n\mathbf{v})$$

Q: Production, Ln: Lose, $-\boldsymbol{\nabla} \cdot (n\mathbf{v})$: Transport

2 Neutral Dynamics

mean mass calculation

$$\overline{m} = \left(\frac{\Phi_{\mathrm{O}_2}}{m_{\mathrm{O}_2}} + \frac{\Phi_{\mathrm{O}}}{m_{\mathrm{O}}} + \frac{\Phi_{\mathrm{N}_2}}{m_{\mathrm{N}_2}} \right)^{-1}$$

continuity equation

$$\frac{1}{R\cos\lambda}\frac{\partial}{\partial\lambda}(v_n\cos\lambda) + \frac{1}{R\cos\lambda}\frac{\partial u_n}{\partial\phi} + \mathrm{e}^z\frac{\partial}{\partial z}\left(\mathrm{e}^{-z}W\right) = 0$$

momentum equation zonal direction

$$\frac{\partial u_n}{\partial t} = \frac{g\mathrm{e}^z}{p_0}\frac{\partial}{\partial z}\left(\frac{\mu}{H}\frac{\partial u_n}{\partial z}\right) + f^{\mathrm{cor}}v_n + \lambda_{xx}(v_{\mathrm{ExB,x}} - u_n) + \lambda_{xy}(v_{\mathrm{ExB,y}} - v_n) - \mathbf{v}_n \cdot \boldsymbol{\nabla} u_n + \frac{u_n v_n}{R_E}\tan\lambda - \frac{1}{R_E\cos\lambda}\frac{\partial\Phi}{\partial\phi} - W\frac{\partial u_n}{\partial z} - hd_u$$

momentum equation meridional direction

$$\frac{\partial v_n}{\partial t} = \frac{g\mathrm{e}^z}{p_0}\frac{\partial}{\partial z}\left(\frac{\mu}{H}\frac{\partial v_n}{\partial z}\right) - f^{\mathrm{cor}}u_n + \lambda_{yy}(v_{\mathrm{ExB,x}} - u_n) + \lambda_{yx}(v_{\mathrm{ExB,y}} - v_n) - \mathbf{v}_n \cdot \boldsymbol{\nabla} v_n + \frac{u_n v_n}{R_E}\tan\lambda - \frac{1}{R_E}\frac{\partial\Phi}{\partial\phi} - W\frac{\partial v_n}{\partial z} - hd_v$$

3 Neutral Thermodynamics

$$\frac{\partial T_n}{\partial t} = \frac{g e^z}{p_0 C_p}\frac{\partial}{\partial z}\left[\frac{K_T}{H}\frac{\partial T_n}{\partial z} + K_E H^2 C_p \rho_n\left(\frac{g}{C_p} + \frac{1}{H}\frac{\partial T_n}{\partial z}\right)\right] - \mathbf{v}_n \cdot \boldsymbol{\nabla} T_n - W\left(\frac{\partial T_n}{\partial z} + \frac{R^* T_n}{C_p \overline{m}}\right) + \frac{Q^{\exp} - e^z L^{\exp}}{C_p} - L^{\mathrm{imp}} T_n$$

T_n: neutral temperature
$Q^{\exp}$: heating term including Joule heating
L^{imp}: cooling coefficient
first term on RHS: thermal conductance, damping term
second term on RHS: eddy diffusion, damping term
third term on RHS: heat transport or convection term

4 Major Species Dynamics

$$\frac{\partial \Phi}{\partial t} = -e^z \tau^{-1}\frac{\partial}{\partial z}\frac{\overline{m}\alpha^{-1} L\Phi}{m_{\mathrm{N}_2}(T_{00}/T_n)^{1/4}} + e^z \frac{\partial}{\partial z}\left(K(z)e^{-z}\frac{\partial \Phi}{\partial z}\right) - \mathbf{v}_n \cdot \boldsymbol{\nabla}\Phi - W\frac{\partial \Phi}{\partial z} + S - R$$

5 Minor Species Dynamics

$$\frac{\partial \Phi}{\partial t} = -e^z \frac{\partial}{\partial z}\left[A\left(\frac{\partial}{\partial z} - E\right)\Phi\right] + S\Phi - R - \left(\mathbf{v}_n \cdot \boldsymbol{\nabla}\Phi + W\frac{\partial \Phi}{\partial z}\right) + K_E\left(\frac{\partial}{\partial z} + \frac{1}{\overline{m}}\frac{\partial \overline{m}}{\partial z}\right)\Phi + H_{\mathrm{sub}}$$

6 Electron Thermodynamics

$$\frac{3}{2}N_e k_B \frac{\partial T_e}{\partial t} = -N_e k_B T_e \boldsymbol{\nabla}\cdot \mathbf{u}_e - \frac{3}{2}N_e k_B \mathbf{u}_e \cdot \boldsymbol{\nabla} T_e - \boldsymbol{\nabla}\cdot\left(-\beta_e \mathbf{J} - K^e \boldsymbol{\nabla}_{\parallel} T_e\right) + \sum Q_e - \sum L_e$$

7 Ionization

$$f(\lambda) = f_{\mathrm{ref}}(\lambda)[1 + A(\lambda)((F_{10.7} + \overline{F_{10.7}})/2 - 80)]$$

$$I(\lambda, z) = I(\lambda, \infty)\exp(\tau(\lambda, z)) \qquad \tau(\lambda, z) = \sum_j \sigma_j(\lambda) n_j(z)\mathrm{Ch}$$

$$Q_J = \lambda_1(\mathbf{v}_{\mathrm{ExB}} - \mathbf{v}_{n\perp})^2 \qquad \lambda_1 = \sigma_P B^2/\rho$$

TIEGCM
(Numerical Scheme Examples)

Using a Leapfrog scheme leads to

ZONAL MOMENTUM

$$\frac{u_n^{t+\Delta t} - u_n^{t-\Delta t}}{2\Delta t} = \frac{ge^z}{p_0}\frac{d}{dz}\left[\frac{\mu du_n^{t+\Delta t}}{Hdz}\right] + fv_n^{t+\Delta t} + \lambda_{xx}(v_{ExB,x}^t - u_n^{t+\Delta t}) + \lambda_{xy}(v_{ExB,y}^t - v_n^{t+\Delta t}) - \mathbf{v}_n^t \cdot \nabla u_n^t + \frac{u_n^{t+\Delta t} v_n^t}{R_E}\tan\lambda - \frac{1}{R_E cos\lambda}\frac{d\Phi^{t+\Delta t *}}{d\phi} - W^{t+\Delta t}\frac{du_n^{t+\Delta t}}{dZ} \tag{3.17}$$

MERIDIONAL MOMENTUM

$$\frac{v_n^{t+\Delta t} - v_n^{t-\Delta t}}{2\Delta t} = \frac{ge^z}{p_0}\frac{d}{dZ}\left[\frac{\mu dv_n^{t+\Delta t}}{HdZ}\right] - fu_n^{t+\Delta t} + \lambda_{yy}(v_{ExB,y}^t - v_n^{t+\Delta t}) + \lambda_{yx}(v_{ExB,x}^t - u_n^{t+\Delta t}) - \mathbf{v}_n^t \cdot \nabla v_n^t - \frac{u_n^{t+\Delta t} u_n^t}{R_E}\tan\lambda - \frac{1}{R_E}\frac{d\Phi^{t+\Delta t *}}{d\lambda} - W^{t+\Delta t}\frac{dv_n^{t+\Delta t}}{dZ} \tag{3.18}$$

Spatial Filtering (Shapiro constant c = 0.3)

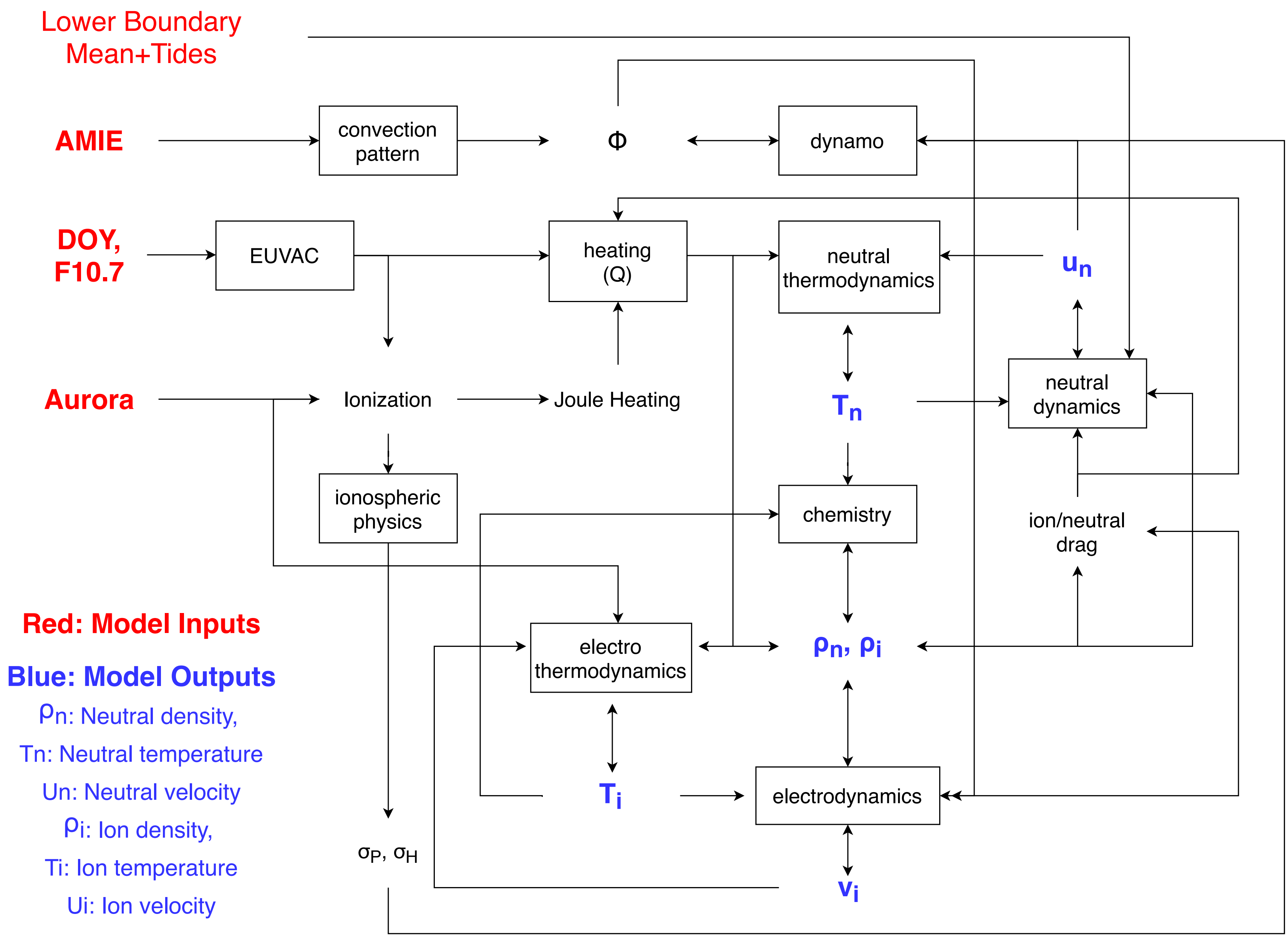
$$f^{smooth}_{merid} = f(\phi,\lambda) - c_{shapiro}\{f(\phi,\lambda+2\Delta\lambda) + f(\phi,\lambda-2\Delta\lambda) - 4\left[f(\phi,\lambda+\Delta\lambda) + f(\phi,\lambda-\Delta\lambda)\right] + 6f(\phi,\lambda)\} \quad (3.21)$$

$$f^{smooth}_{zonal} = f^{smooth}_{merid} - c_{shapiro}\{f^{smooth}_{merid}(\phi+2\Delta\phi,\lambda) + f^{smooth}_{merid}(\phi-2\Delta\phi,\lambda) - 4\left[f^{smooth}_{merid}(\phi+\Delta\phi,\lambda) + f^{smooth}_{merid}(\phi-\Delta\phi,\lambda)\right] + 6f^{smooth}_{merid}(\phi,\lambda)\} \quad (3.22)$$

Vertical Direction (Diffusion)

$$\frac{\partial}{\partial z}\left(f_{vis}\frac{\partial u_n^{t+\Delta t}}{\partial z}\right)(z+\frac{1}{2}\Delta z) = \frac{\left[f^{int}_{vis}\frac{\partial u_n^{t+\Delta t}}{\partial z}\right](z+\Delta z) - \left[f^{int}_{vis}\frac{\partial u_n^{t+\Delta t}}{\partial z}\right](z)}{\Delta z} =$$
$$= \frac{1}{\Delta z}(f_{vis}(z+\Delta z)\frac{u_n(z+\frac{3}{2}\Delta z) - u_n(z+\frac{1}{2}\Delta z)}{\Delta z}$$
$$- f_{vis}(z)\frac{u_n(z+\frac{1}{2}\Delta z) - u_n(z-\frac{1}{2}\Delta z)}{\Delta z}) = \quad (3.25)$$
$$= \frac{1}{\Delta z^2}(f_{vis}(z)u_n(z+\frac{3}{2}\Delta z)$$
$$- (f_{vis}(z+\Delta z) + f_{vis}(z))u_n(z+\frac{1}{2}\Delta z) + f_{vis}(z)u_n(z-\frac{1}{2}\Delta z))$$

TIEGCM Model Structure



Palmetto Login Page

```
Enter a passcode or select one of the following options:

 1. Duo Push to XXX-XXX-2552
 2. Phone call to XXX-XXX-2552
 3. SMS passcodes to XXX-XXX-2552

Passcode or option (1-3): 1
Success. Logging you in...
Last login: Mon Nov  5 20:08:19 2018 from 99.5.125.84

 ------------------------------------------------------------
           Welcome to the PALMETTO CLUSTER at CLEMSON UNIVERSITY

  * Email ithelp@clemson.edu with questions or to report proble

  * Palmetto "office hours" are every Wednesday 8am-11am in 412

  * Quarterly maintenance periods:  May (followed by Top 500 be
    August, November and Feb.  Email will be sent before each p
    details of cluster availability.

      User guide:  http://www.palmetto.clemson.edu/palmetto
  Sample programs:  https://github.com/clemsonciti/palmetto-exa
      JupyterHub:  https://www.palmetto.clemson.edu/jupyterhub

  Useful commands:
    module avail              - list available software packages
    qstat -xf jobid           - check status of your job
    qstat -Qf queuename       - check status of a queue
    checkquota                - check your disk quota
    checkqueuecfg             - check general workq max running
    cat /etc/hardware-table   - list node hardware: ram,cores,ch
    qpeek                     - look at a running job's stdout a
    whatsfree                 - see what nodes are free right no

  Please do not use /home as your PBS working directory.  Jobs
  as working directory may be killed as performance deteriorate

  DO NOT RUN JOBS/PROGRAMS/TESTS/PRE-OR-POST PROCESSING ON THE
  They will be terminated without notice. No exceptions.

 ------------- This file is: /etc/motd -------- Last Updated: 2

[xianl@login001 ~]$
```

TIEGCM Code

```
      module lbc
!
! This software is part of the NCAR TIE-GCM.  Use is go
! Open Source Academic Research License Agreement conta
! tiegcmlicense.txt.
!
! Calculate lower boundary conditions for T,U,V,Z
!
      use params_module,only: nlonp4,nlat,nlev,dz
      use cons_module,only: pi,atm_amu,gask,grav,freq_s
     |  dt,re,dlamda,tgrad,cs,cor,tn
      use cons_module,only: tbound,zbound
      use addfld_module,only: addfld
      implicit none
!
! Total lower boundary conditions returned by this modu
! (dimensioned at full global grid, but defined at subd
!
      real,dimension(nlonp4,nlat) :: t_lbc, u_lbc, v_lbc
!
! Diurnal and semi-diurnal tidal perturbations using Ho
!
      complex,dimension(nlat) ::
     |  t_di  , u_di  , v_di  , z_di, t_sdi, u_sdi, v_sdi,
      complex,parameter :: ci=(0.,1.), expta=1.
      complex :: bnd_sdi(nlonp4), bnd_di(nlonp4)
!
! For bndcmp:
      real :: b(nlonp4,3,3),fb(nlonp4,3)
!
! This t0 is different than the t0 in cons.
      real :: t0(nlev+1)
!
! Lower boundary for helium (mmr):
      real,parameter :: pshelb=0.1154E-5
      contains
!-------------------------------------------------------
      subroutine init_lbc
!
! Called once per run from tgcm.
```

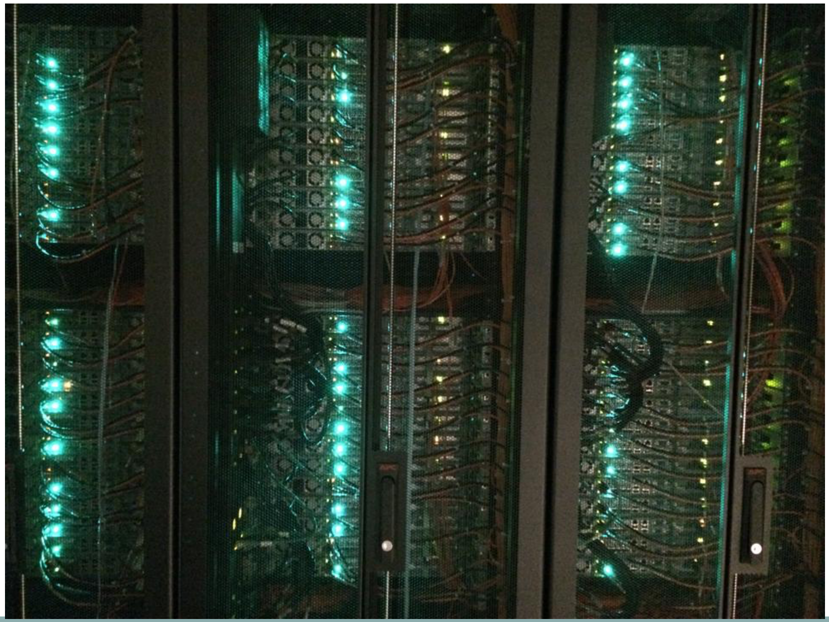
Clemson Palmetto Supercomputer

Community Models:

- CESM
- WACCM
- TIEGCM,
- TIMEGCM

Mechanistic Models:

- Gravity Wave
- Planetary Wave



NCAR Cheyenne Supercomputer

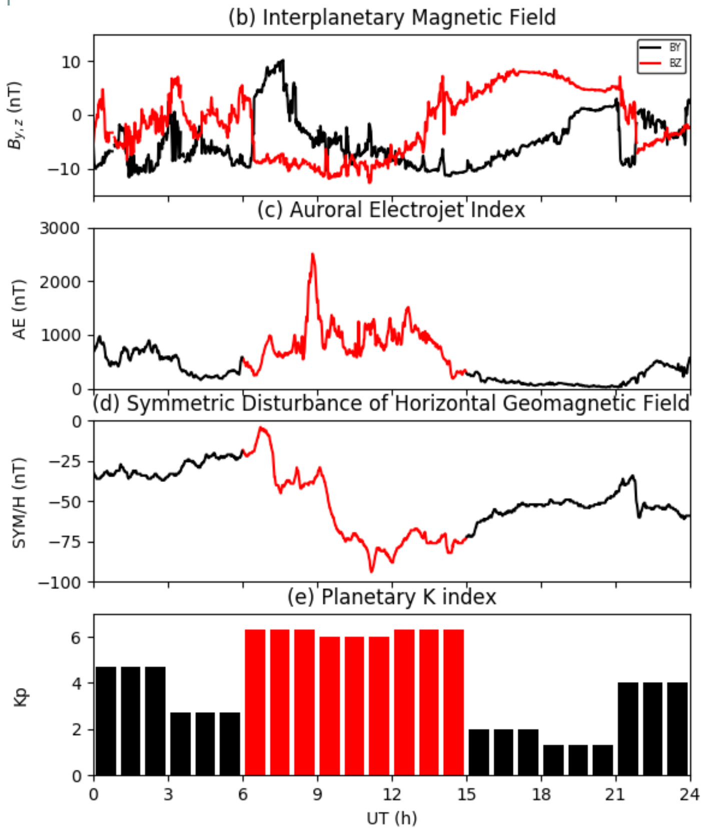
Computing Units

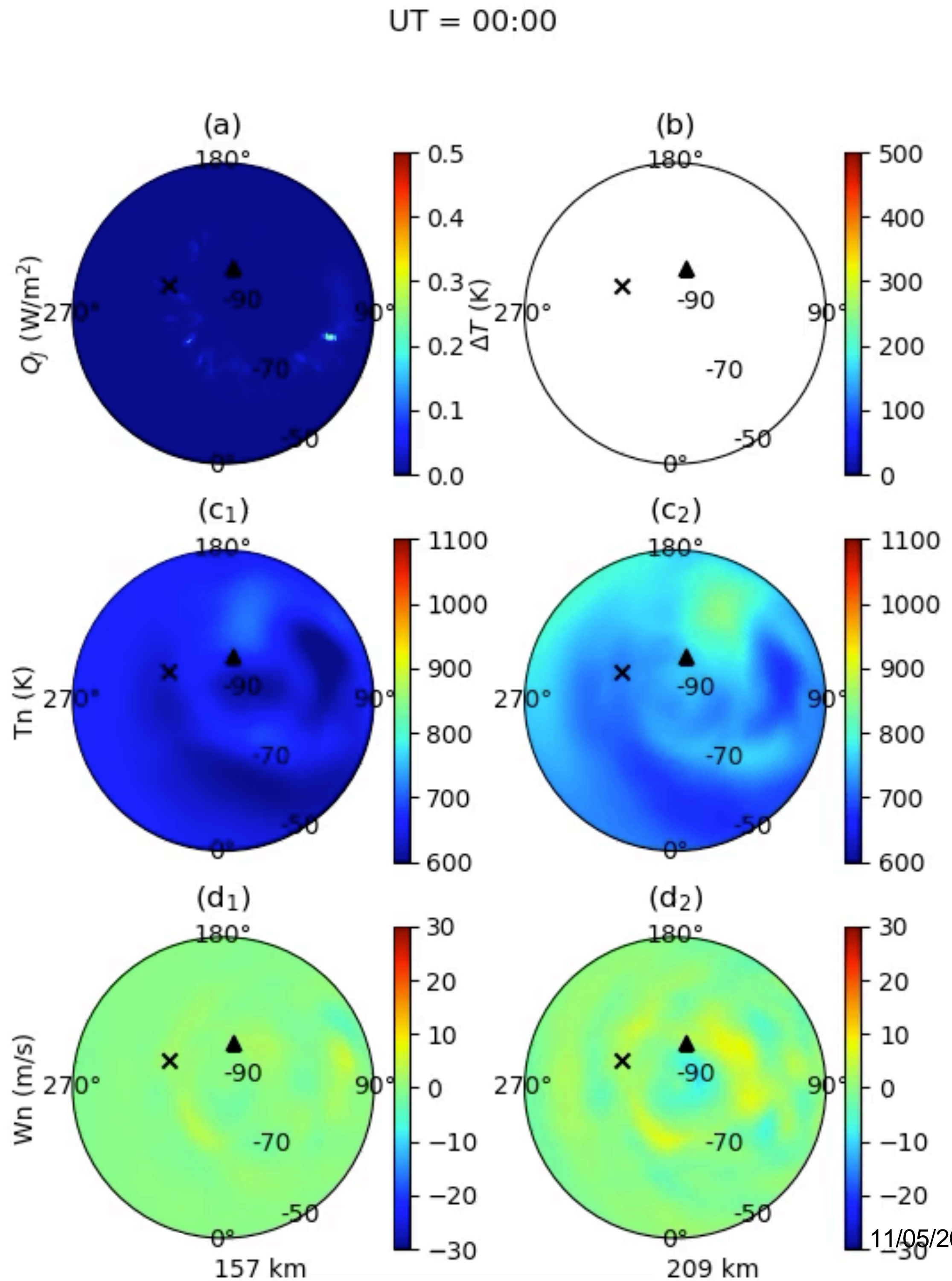
Storage Units

Cooling System

Monitor Room

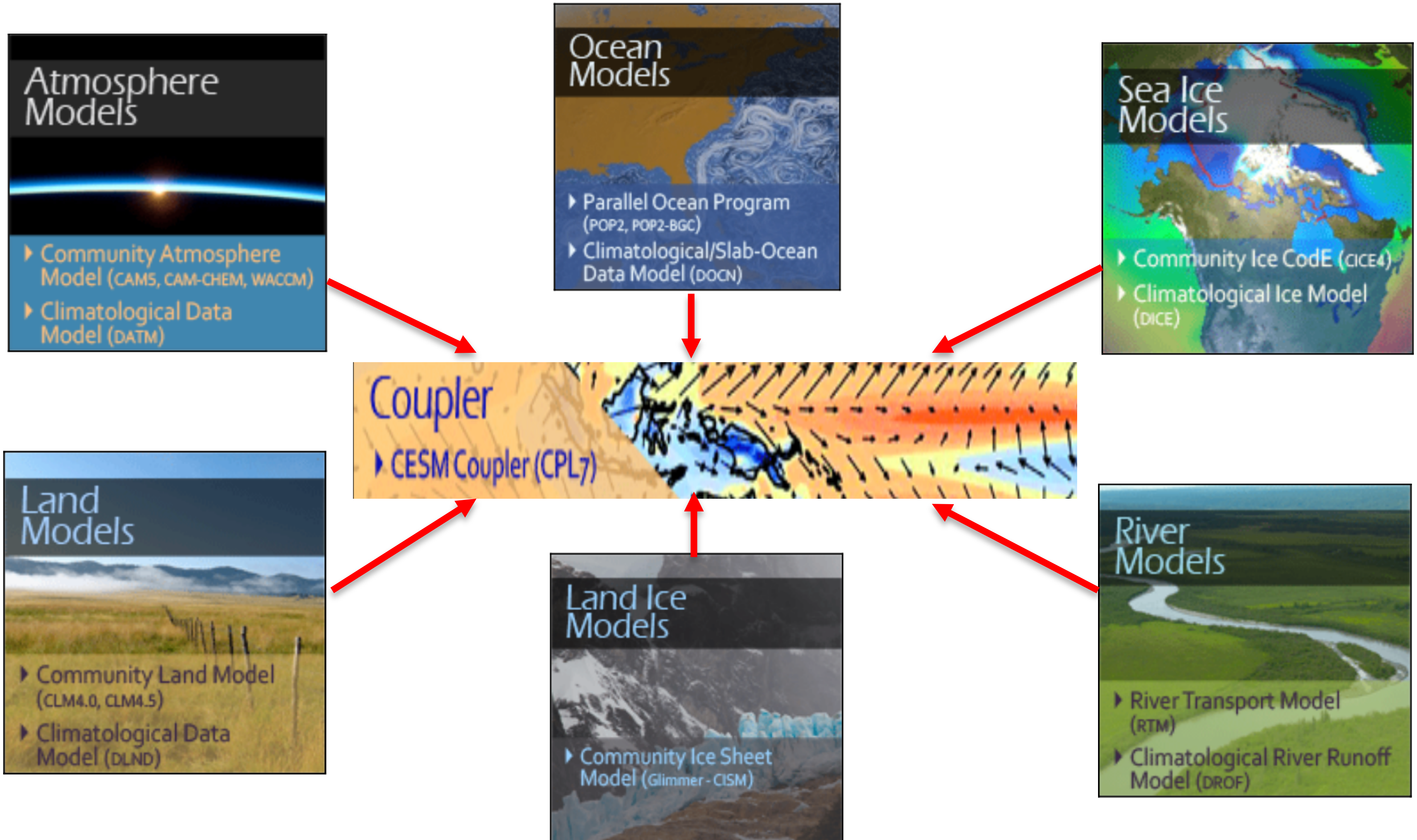
TIEGCM Simulation of Space Storm





CESM
(The Community Earth System model)

Fully-coupled, global climate model that provides state-of-the-art computer simulations of the Earth's past, present, and future climate states.



http://www.cesm.ucar.edu/models/cesm1.2/

Finite Volume Method

Mass conservation: $$\frac{\partial}{\partial t}\pi + \nabla \cdot \left(\vec{V}\pi\right) = 0,$$

The finite-volume (*integral*) representation of the continuous π field is defined as follows:

$$\widetilde{\pi}(t) \equiv \frac{1}{A^2 \Delta\theta \Delta\lambda cos\theta} \iint \pi(t;\lambda,\theta) A^2 cos\theta \, d\theta d\lambda. \qquad (3.9)$$

Given the *exact* 2D wind field $\vec{V}(t;\lambda,\theta) = (U,V)$ the 2D integral representation of the conservation law for $\widetilde{\pi}$ can be obtained by integrating (3.8) in time and in space

$$\widetilde{\pi}^{n+1} = \widetilde{\pi}^{n} - \underbrace{\frac{1}{A^2 \Delta\theta \Delta\lambda cos\theta} \int_t^{t+\Delta t} \left[\oint \pi(t;\lambda,\theta)\vec{V} \cdot \vec{n}\, dl\right] dt}_{\text{Convergence of Flux}}. \qquad (3.10)$$

1D transport of flux

$$F(u^*, \Delta t, \widetilde{\pi}) = -\frac{1}{A \Delta\lambda cos\theta} \delta_\lambda \underbrace{\left[\int_t^{t+\Delta t} \pi U \, dt\right]}_{\text{Flux Term}} = -\frac{\Delta t}{A\Delta\lambda cos\theta} \delta_\lambda \underbrace{\left[\chi(u^*, \Delta t; \pi)\right]}_{\text{Flux Term}}, \qquad (3.11)$$

DescriptionofCAM5

Finite Volume Method

Key issue: estimate fluxes coming in and out of the grid cell

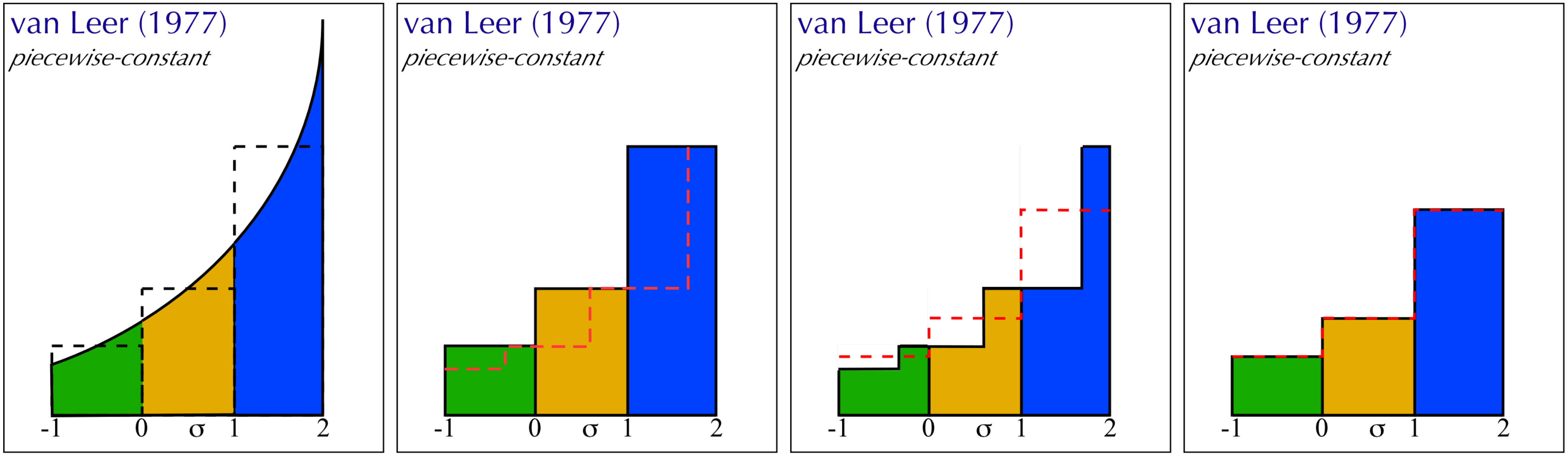
- Step 1 –
- identify grid zone averages…
- X coordinate is Courant number σ
- Grid box runs from [0-to-1]•Δx

- Step 2 –
- look at distribution **before** and after advection takes place

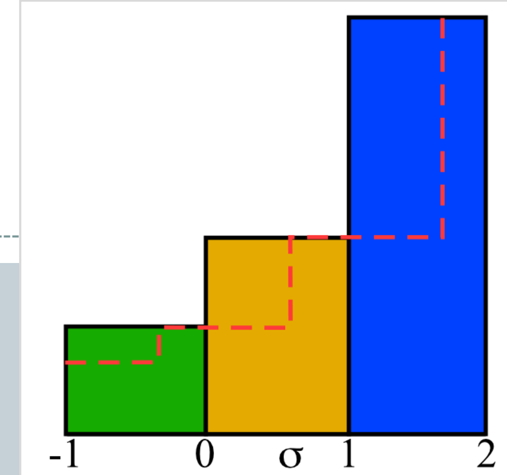
- Step 3 -
- Compute new grid zone averages.

- Step 4 -
- The averages *are* the function here (for piecewise constant)
- These are the initial values for the next time step.

Upstream – piecewise *constant*



van Leer (1977)



- New value from integrating under piecewise constant function at time *t* that *will be in the grid zone [0,Δx] at t+Δt.*

$$q^{n+1} \equiv \bar{q}^{1/2} = \int_{0}^{1-\sigma} q_{1/2}dx + \int_{-\sigma}^{0} q_{-1/2}dx \qquad \sigma = \frac{u\Delta t}{\Delta x}$$

Grid-point value f(j) represents the average of the function over the grid cell (see Durran, § 1.3.1, p. 27)

- Piecewise *constant* in each zone, so:

$$\begin{aligned}\bar{q}^{1/2} &= \bar{q}_{1/2}(1-\sigma) + \bar{q}_{-1/2}\sigma \\ &= \bar{q}_{1/2} - \sigma(\bar{q}_{1/2} - \bar{q}_{-1/2})\end{aligned} \quad \longleftrightarrow \quad \bar{q}^{1/2} = \bar{q}_{1/2} - \left[u\bar{q}_{1/2} - u\bar{q}_{-1/2}\right]\frac{\Delta t}{\Delta x}$$

van Leer (1977)

- van Leer notation ...

$$\overline{q}^{1/2} = \overline{q}_{1/2} - \sigma\left(\overline{q}_{1/2} - \overline{q}_{-1/2}\right)$$
$$= \overline{q}_{1/2} - \left[\sigma\overline{q}_{1/2} - \sigma\overline{q}_{-1/2}\right]$$
$$= \overline{q}_{1/2} - \left[Flux_{1/2} - Flux_{-1/2}\right]$$

- Fluxes ...

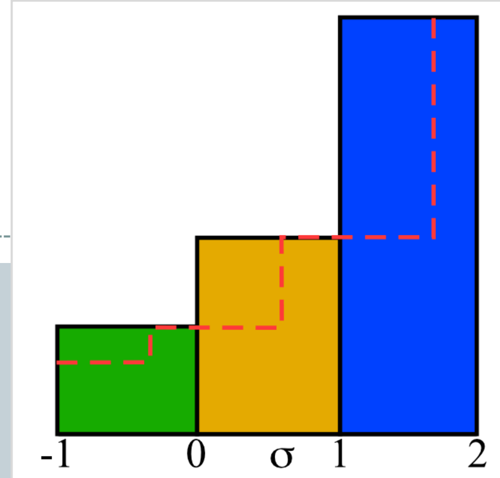
$$Flux(i) \equiv Flux_{-1/2}$$
$$= \sigma q_{-1/2} = \sigma q(i-1)$$

  - note Δt is already included in (is part of) the fluxes.

- Coding:

$$q^{n+1} = q^n - \left(Flux_{1/2} - Flux_{-1/2}\right) \; + q_i^n \Delta t \delta_x u$$
$$= q^n - \left[\left(\sigma q_i\right) - \left(\sigma q_{i-1}\right)\right] \; + q_i^n \frac{\Delta t}{\Delta x}\left(u_{i+1} - u_i\right)$$
$$= q^n - \sigma\left(q_i - q_{i-1}\right) \quad \textit{if} \;\; \mathrm{u} = \text{constant}$$

Greater accuracy: *Piecewise*



- Our general update formula:

$$q^{n+1} \equiv \bar{q}^{1/2} = \int_0^{1-\sigma} q_{1/2}\,dx + \int_{-\sigma}^{0} q_{-1/2}\,dx \quad \sigma = \frac{u\Delta t}{\Delta x}$$

- Piecewise *constant*

$$q(x,t_0) = \bar{q}_{1/2}$$

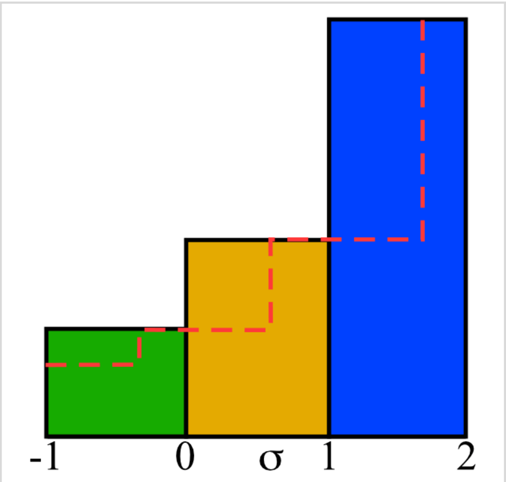
- Piecewise *linear*

$$q(x,t_0) = \bar{q}_{1/2} + \bar{\Delta}_{1/2} q\left(x - \frac{1}{2}\right)$$

- Includes:
  - Zone average $\bar{q}_{1/2} = \int_0^1 q(x,t^0)\,dx$
  - Average slope $\bar{\Delta}_{1/2} q \equiv \left(\overline{\partial q / \partial x}\right)_{1/2}$

Constant – why?

Piecewise linear & beyond



- Van Leer piecewise linear: *Scheme I*

$$\bar{q}^{1/2} = \bar{q}_{1/2} - \sigma\left(\bar{q}_{1/2} - \bar{q}_{-1/2}\right) - \frac{\sigma}{2}(1-\sigma)\left(\bar{\Delta}_{1/2}q - \bar{\Delta}_{-1/2}q\right)$$

LIKE PIECEWISE-CONSTANT:

FLUXES FROM ZONE AVERAGES.

$$\bar{q}_{1/2} = \int_0^1 q(x,t^0)\,dx$$

SLOPES IN ZONES.

- CAN BE EVALUATED MANY WAYS.

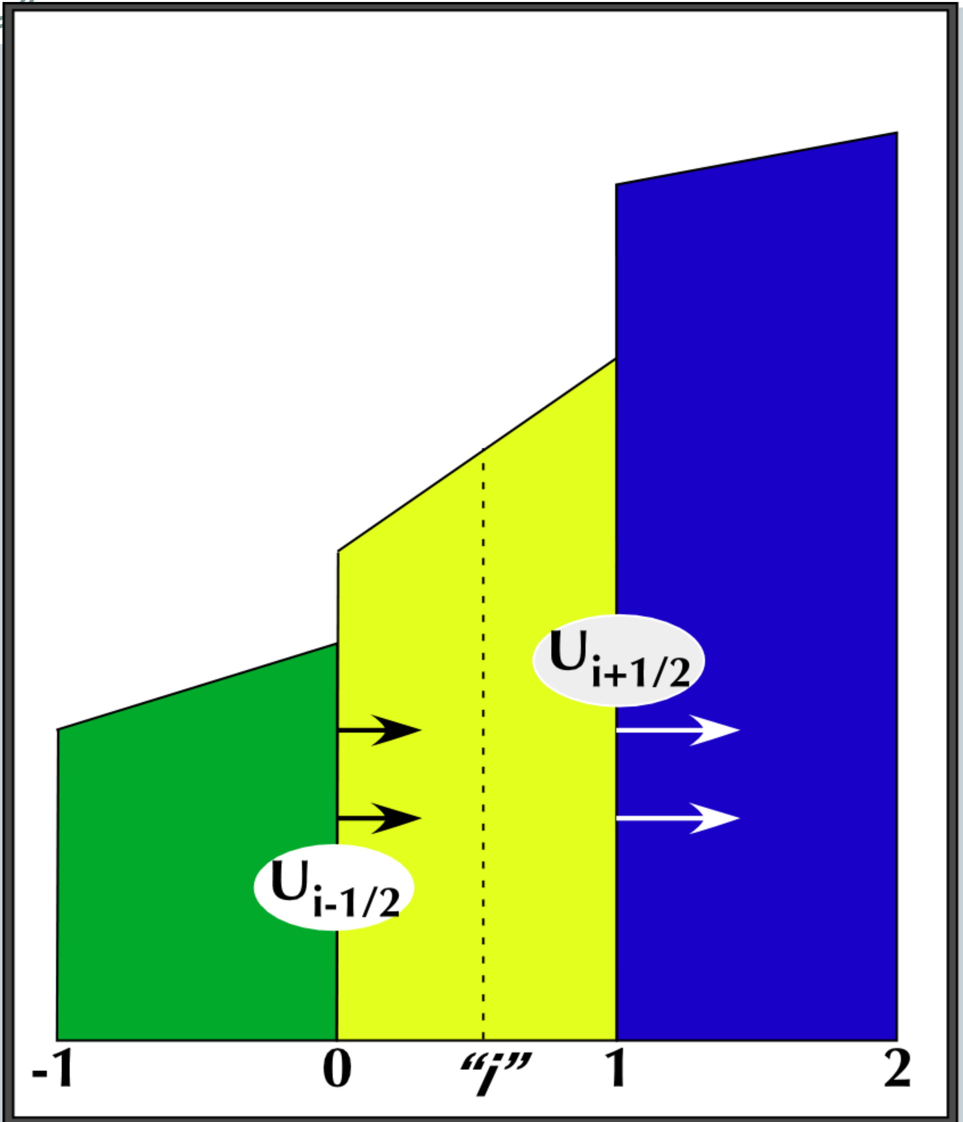
SCHEME 1: CENTERED DIFFERENCES

$$\bar{\Delta}_{1/2}q = \frac{1}{2}\left(\bar{q}_{3/2} - \bar{q}_{-1/2}\right)$$

Piecewise linear form

$$\bar{q}^{1/2} = \int_{0}^{1-\sigma} q_{1/2}\,dx + \int_{-\sigma}^{0} q_{-1/2}\,dx$$

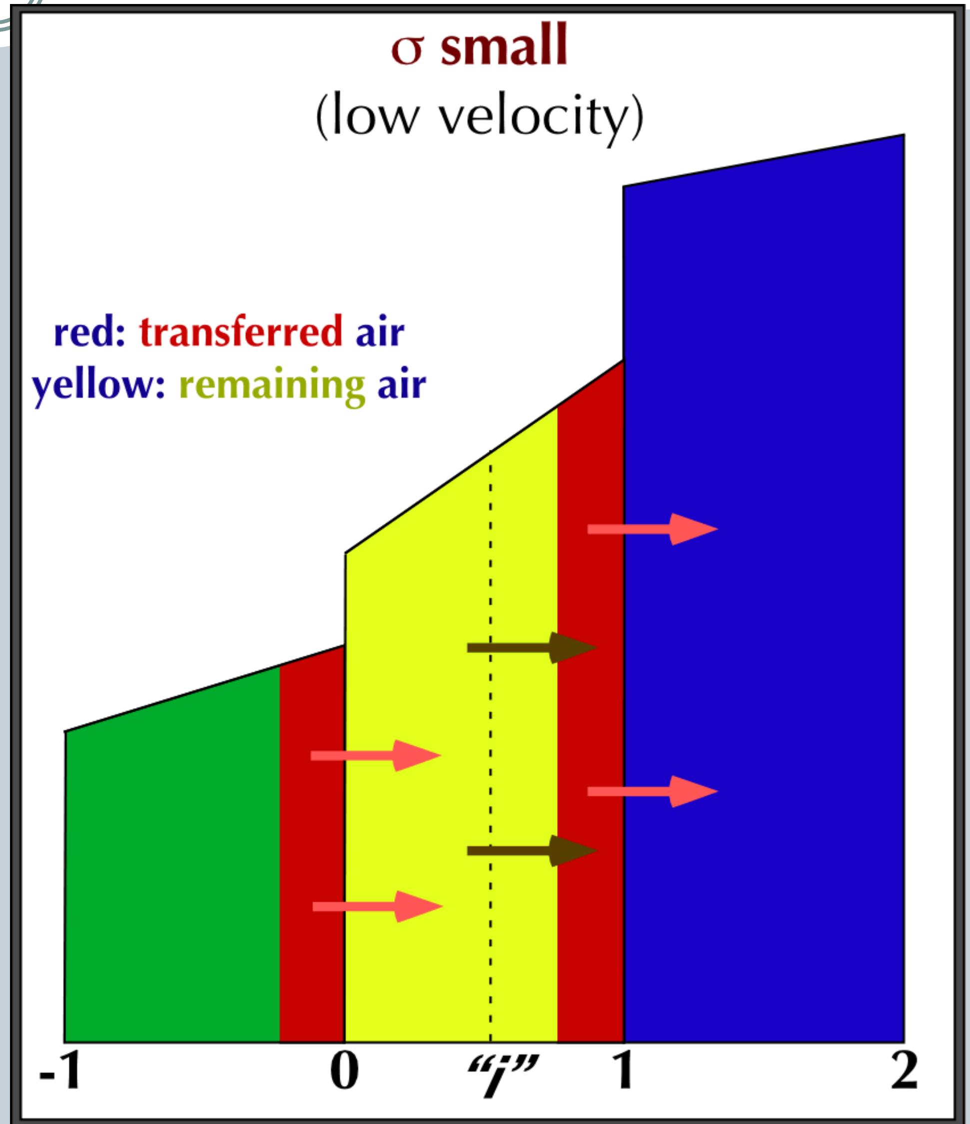
- Fluxes
  - integrate under function at time t *that will be in zone* [0,Δx] *at t+Δt*.
  - Grid zone boundary velocities (C-grid) shown



Piecewise linear form

$$\overline{q}^{1/2} = \int_0^{1-\sigma} q_{1/2}dx + \int_{-\sigma}^{0} q_{-1/2}dx$$

- Fluxes
  - Small σ: area in red transferred; yellow remains in the zone [0,Δx]

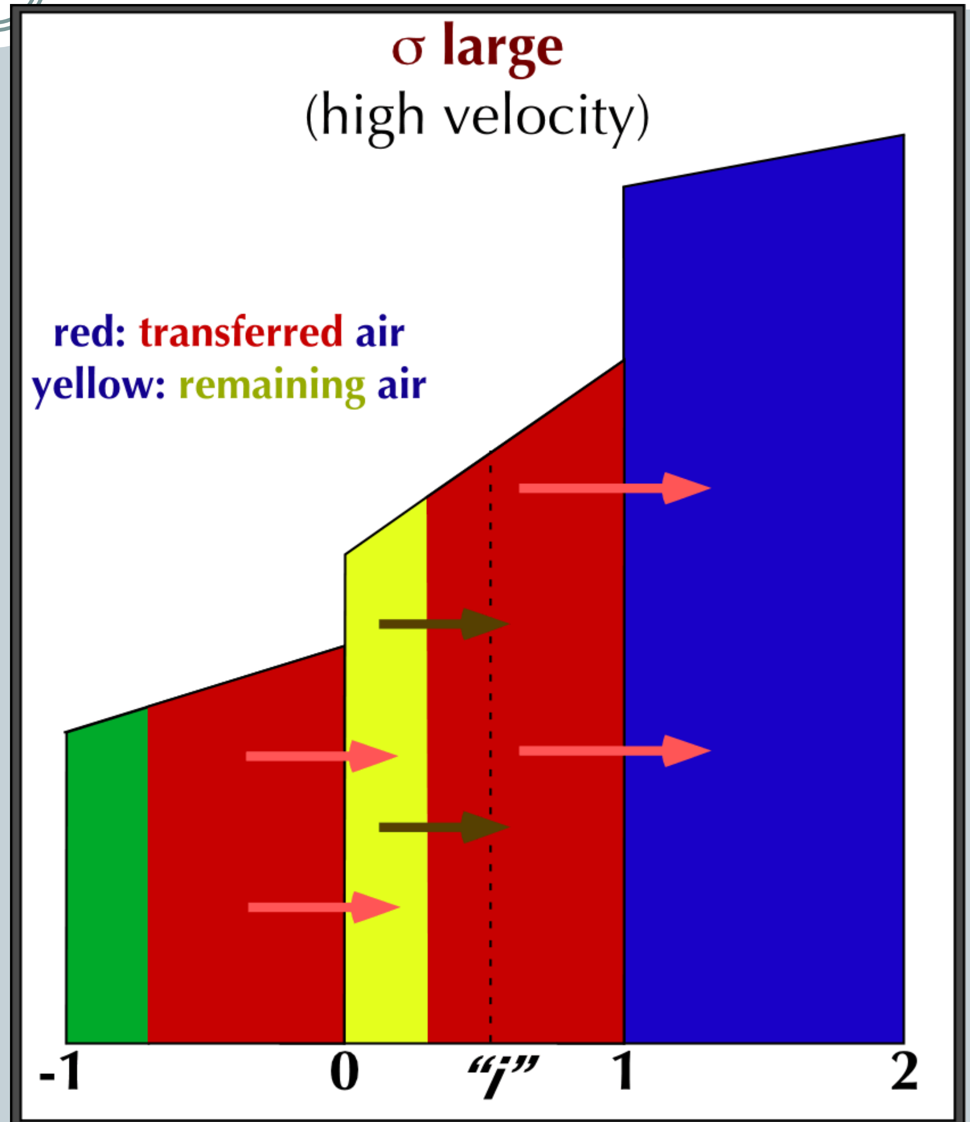


Credit: ATMOS 502, UUC

Piecewise linear form

$$\bar{q}^{1/2} = \int_0^{1-\sigma} q_{1/2}dx + \int_{-\sigma}^{0} q_{-1/2}dx$$

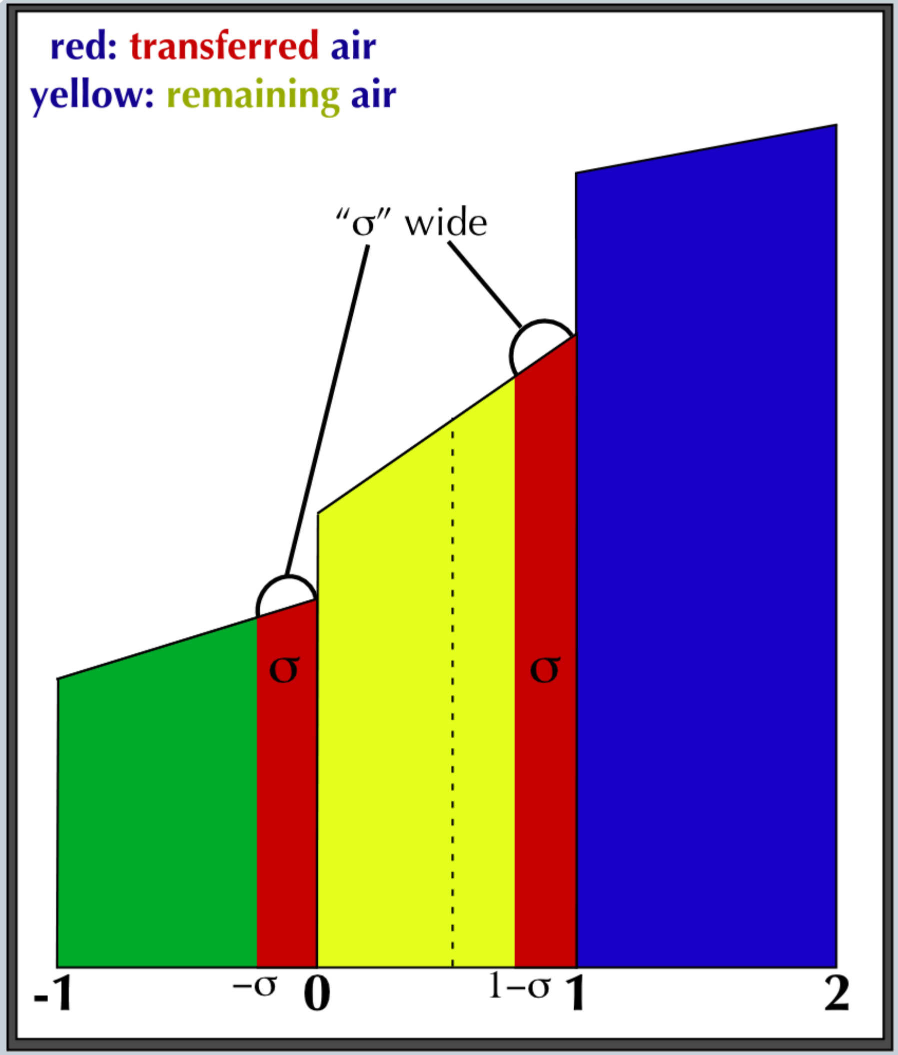
- Fluxes
  - High σ: area in red transferred; yellow remains in the zone [0,Δx]



van Leer, integration

$$\overline{q}^{1/2} = \int_{0}^{1-\sigma} q_{1/2}\,dx + \int_{-\sigma}^{0} q_{-1/2}\,dx$$

- Integration bounds
  - *Current* area [0,1-σ] remains in zone.
  - *Upstream* area [-σ,0] is transferred into the zone [0,1].
- Local function
  - is independently determined for each grid zone.



Implementing Piecewise Linear

- **Handling PL inside *advect1d*:**
  - In the x-advect or z-advect step: for the x-direction, prepare 1D arrays of a row of theta^n and u^n as input to your advect1D.
    - Inside advect1d, you compute Courant number, slopes and fluxes:

$$F_{i-1/2} = \begin{cases} r_{i-1/2}\left(q_{i-1}^n + \dfrac{1-r_{i-1/2}}{2}\Delta q_{i-1}^n\right), & \text{if } \mathrm{u}_{\mathrm{i-1/2}}^{\mathrm{n}} \geq 0; \\ r_{i-1/2}\left(-q_i^n + \dfrac{1-r_{i-1/2}}{2}\Delta q_i^n\right), & \text{if } \mathrm{u}_{\mathrm{i-1/2}}^{\mathrm{n}} < 0; \end{cases} \quad \text{where} \quad \begin{cases} r_{i-1/2} = \left|\dfrac{\Delta t}{\Delta x} u_{i-1/2}^n\right| \\ \Delta q_i = \dfrac{q_{i+1}^n - q_{i-1}^n}{2} \end{cases}$$

    - Then you can do the integration, still in the 1-D advection code:

$$q_i^{n+1} = q_i^n - \left(F_{i+1} - F_i\right) + \frac{\Delta t}{\Delta x} q_i^n \left(u_{i+1} - u_i\right)$$

PL: monotonic *slope limiter*

- Simple centered differences

$$\overline{\Delta}_{1/2} q = \frac{1}{2}\left(\overline{q}_{3/2} - \overline{q}_{-1/2}\right)$$

- In English:

$$\overline{\Delta}_{1/2} q = \frac{\left(q(i+1) - q(i-1)\right)}{2}$$

- This is *average slope* in the grid zone
  - *Not monotonic*

- ***monotonic*** *slope form*

$$IF \;\; \left(q_i - q_{i-1}\right)\left(q_{i+1} - q_i\right) \geq 0,$$

$$\Delta\theta_i = \operatorname{sgn}\left(q_{i+1} - q_{i-1}\right)$$

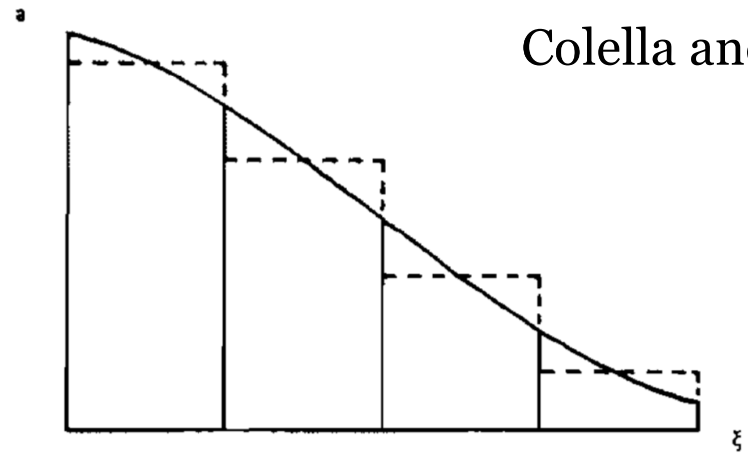
$$\times \min \begin{pmatrix} \left|q_i - q_{i-1}\right|, \\ \left|q_{i+1} - q_i\right|, \\ \left|q_{i+1} - q_{i-1}\right| / 2 \end{pmatrix}$$

$$Otherwise: \quad \Delta q_i = 0$$

This is known as the "minmod" flux limiter (see Durran, § 5.5.2, p. 230)

Piecewise Parabolic Method (PPM)

Adopted in CESM



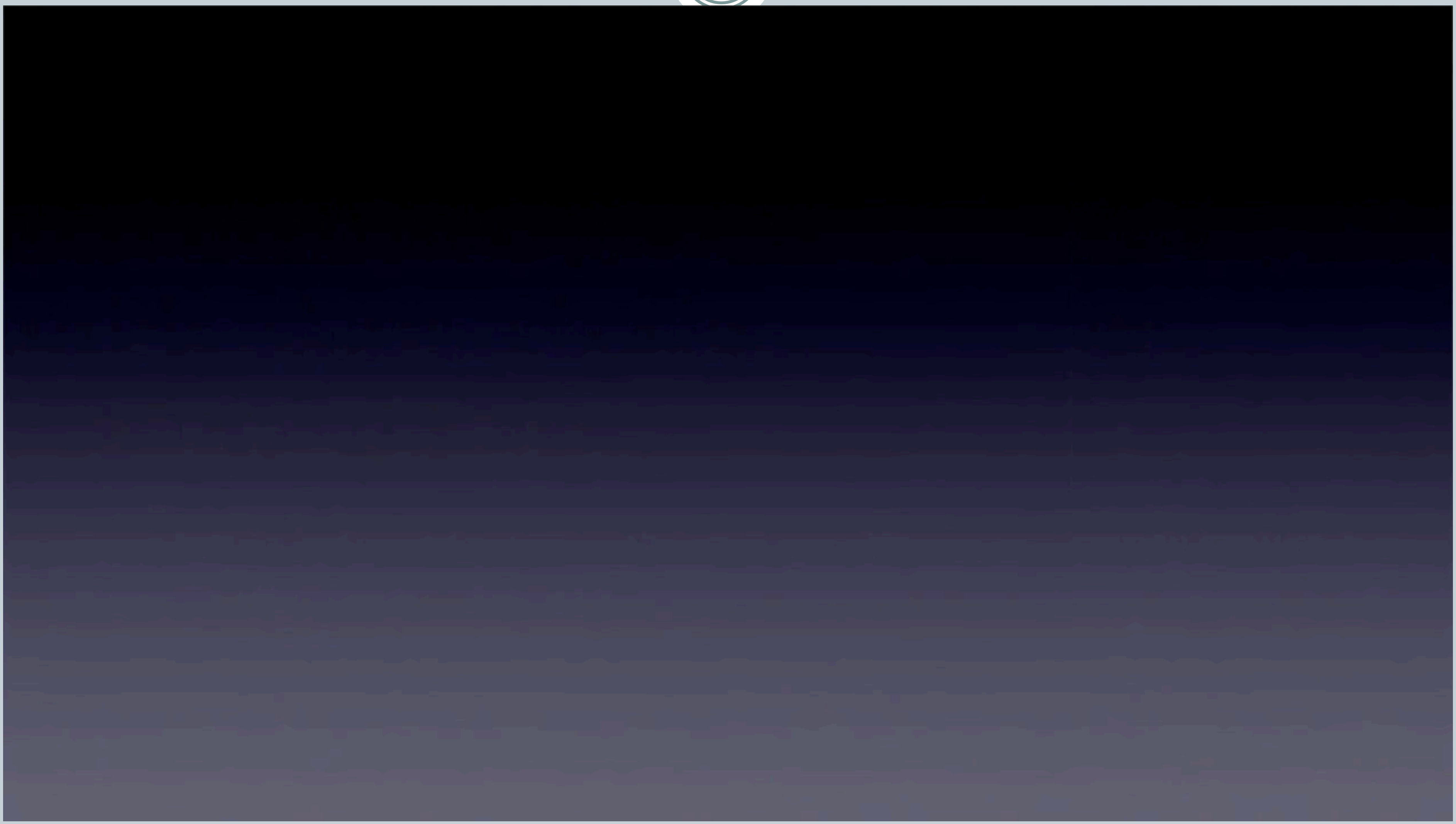
Colella and Woodward, 1984

FIG. 1. The interpolation step of the PPM advection scheme. The initial data are given as values of the variable a averaged over the four zones shown. These averaged values are represented by dashed lines. From this data values of the variable a are interpolated at zone edges, using cubic curves which have the prescribed average values in the four zones nearest the edge. The interpolation parabolae within the zones, which are shown as solid lines, connect these edge values and give back the initial data when averaged over the zones.

- *PPM (second-order polynomial within the volume.*
- Mass conserving, monotonicity preserving, and high-order accuracy
- *Good balance between computational efficiency and accuracy.*

Earth (Terrestrial) Weather

- A Compset of CESM (Community Earth System Model)
- Currently Running at Palmetto, Clemson



Liu et al., GRL, 2015