

PHYS 8750
***NUMERICAL
FLUID
DYNAMICS***

FALL 2020



PHYS 8750

Class #2 (Chapter 2.1,
2.2)

1) Truncation errors &
order of accuracy

2) A-stability and
stability diagram

3) Amplitude and
phase error

CLASS #3
(CHAPTER 2.3,
2.4)

Outline

- Multi-stage methods
 - 1) Runge-Kutta method
 - 2) Matsuno method
- Multi-step methods
 - 1) Leap-frog method
- Amplitude and phase behavior of multi-stage(step) methods

Amplitude and Phase Behavior

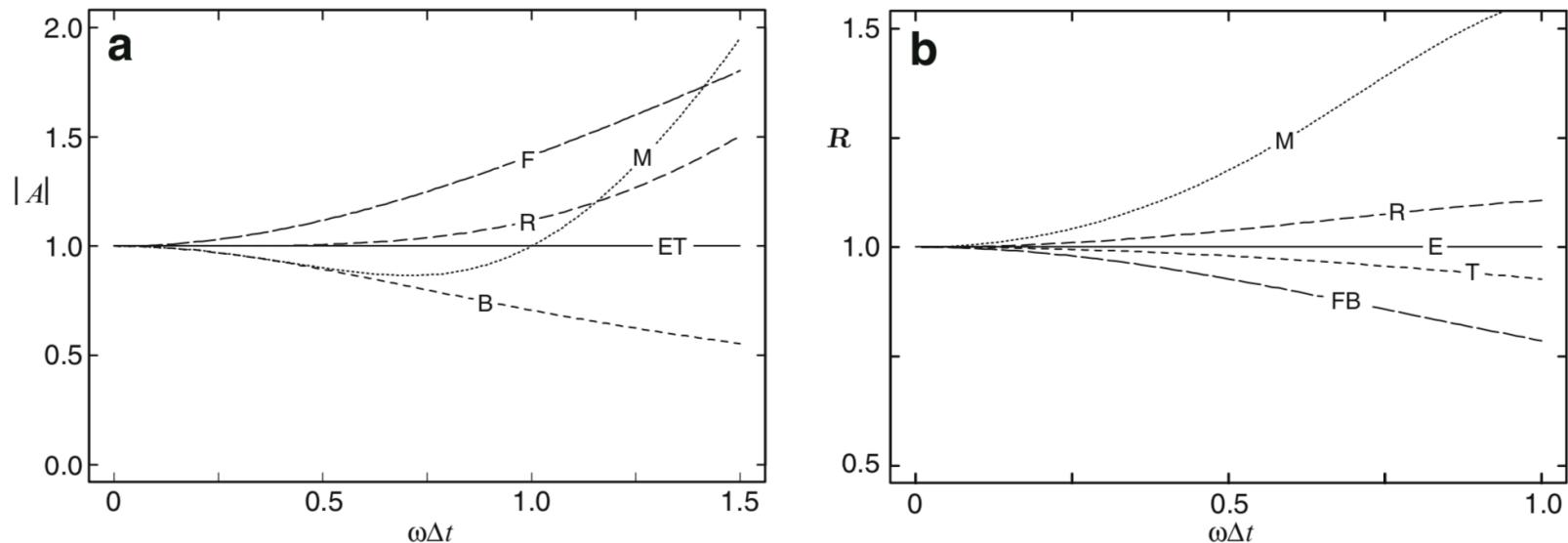


Fig. 2.2 The modulus of the amplification factor **(a)** and the relative phase change **(b)** as a function of temporal resolution $\omega\Delta t$ for the true solution and five two-level schemes: exact solution (*E*) and trapezoidal method (*T*), forward-Euler scheme (*F*), backward-Euler scheme (*B*), two-stage second-order Runge–Kutta scheme (*R*), and Matsuno scheme (*M*)

$$|A|_{\text{forward}} \approx 1 + \frac{1}{2}(\omega\Delta t)^2$$

$$|A|_{\text{backward}} \approx 1 - \frac{1}{2}(\omega\Delta t)^2$$

$$|A|_{\text{Matsuno}}^2 = 1 - (\omega\Delta t)^2 + (\omega\Delta t)^4$$

$$|A|_{\text{RKe2}} \approx 1 + \frac{1}{8}(\omega\Delta t)^4$$

$$R_{\text{trapezoidal}} \approx \frac{1}{\omega\Delta t} \arctan \left(\omega\Delta t \left(1 + \frac{\omega^2 \Delta t^2}{4} \right) \right) \approx 1 - \frac{\omega^2 \Delta t^2}{12}$$

$$R_{\text{forward}} = R_{\text{backward}} \approx 1 - \frac{(\omega\Delta t)^2}{3}$$

$$R_{\text{RKe2}} \approx 1 + \frac{1}{6}(\omega\Delta t)^2$$

$$R_{\text{Matsuno}} \approx 1 + \frac{2}{3}(\omega\Delta t)^2$$

Multi-Stage Runge-Kutta Method

Carl Runge



Carl David Tolm  Runge

Born	30 August 1856 Bremen, German Confederation
Died	3 January 1927 (aged 70) G�ttingen, Weimar Republic
Citizenship	German

Martin Kutta



Martin Kutta (1867–1944)

Born	3 November 1867 Pitschen, Upper Silesia
Died	25 December 1944 (aged 77) F�rstenfeldbruck
Nationality	German

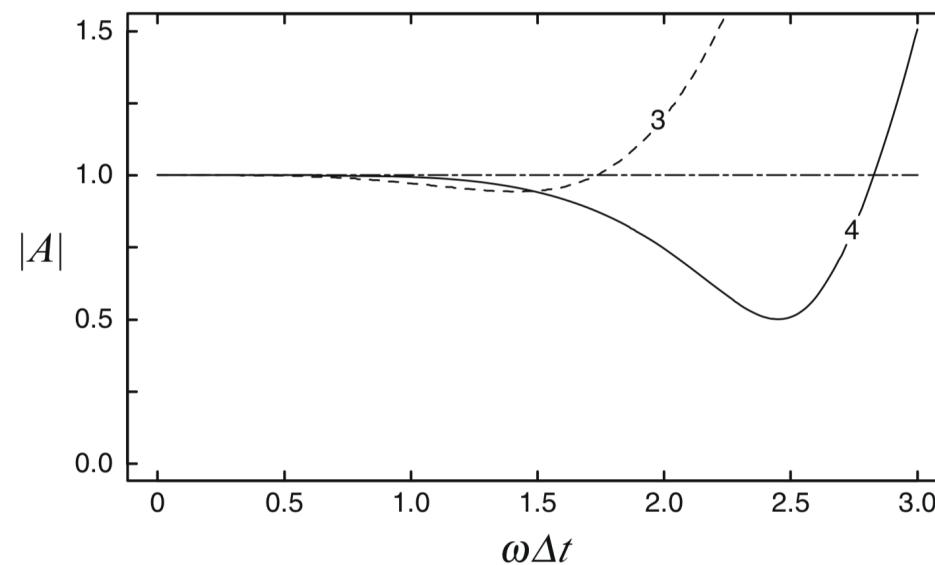


Fig. 2.3 Modulus of the amplification factor as a function of temporal resolution $\omega\Delta t$ for third-order three-stage (dashed line) and fourth-order four-stage (solid line) explicit Runge–Kutta solutions to the oscillation equation