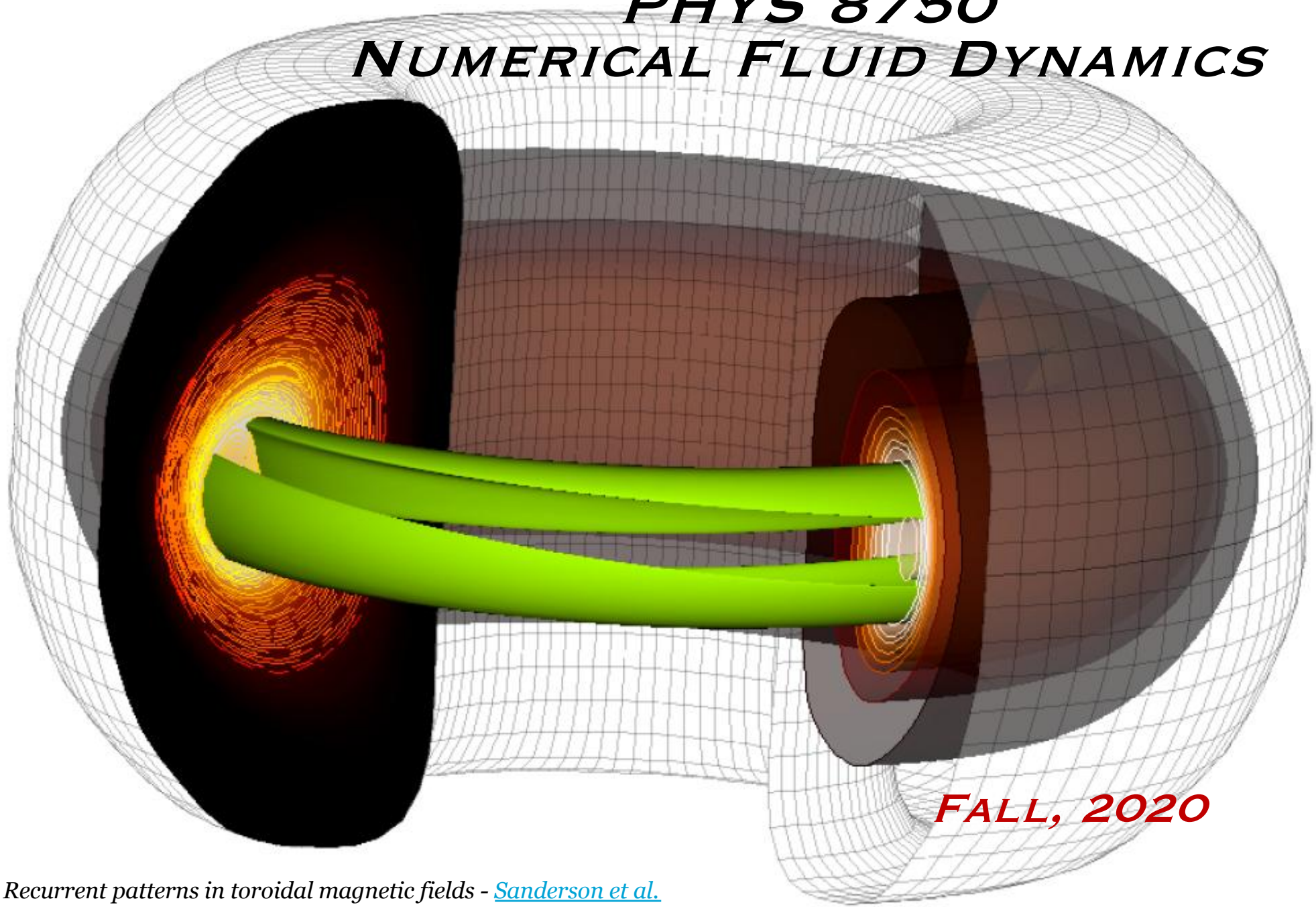
PHYS 8750
Numerical Fluid Dynamics



Fall, 2020

Recurrent patterns in toroidal magnetic fields - Sanderson et al.

PHYS 8750

Class #5 (Chapter 3.1)
1) Partial Differential Equations

Truncation errors

Stability, convergence, consistency

2) Von Neumann's method (Stability)

Class #7 (Takac's Schemes)

Outline

- Partial Differential Equations
  - 1) Truncation errors
  - 2) Stability, Convergence, Consistency
- Von Neumann's methods (Stability)
- Dispersion and Dissipation errors
- 1-st, 2-nd, 3-rh, and 4-th order space schemes (Runge-Kutta scheme).
- Lax-Wendroff Scheme (two time step)
- Takacs Scheme (two time step)

Forms of diffusion

- Behavior seen especially w/sharp gradients:

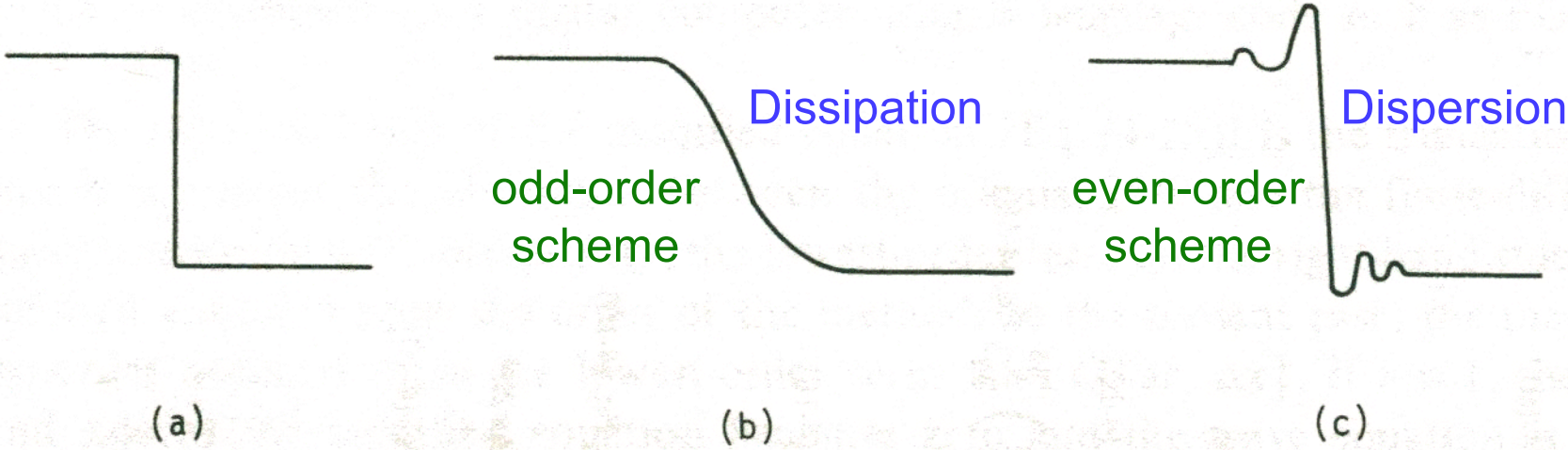


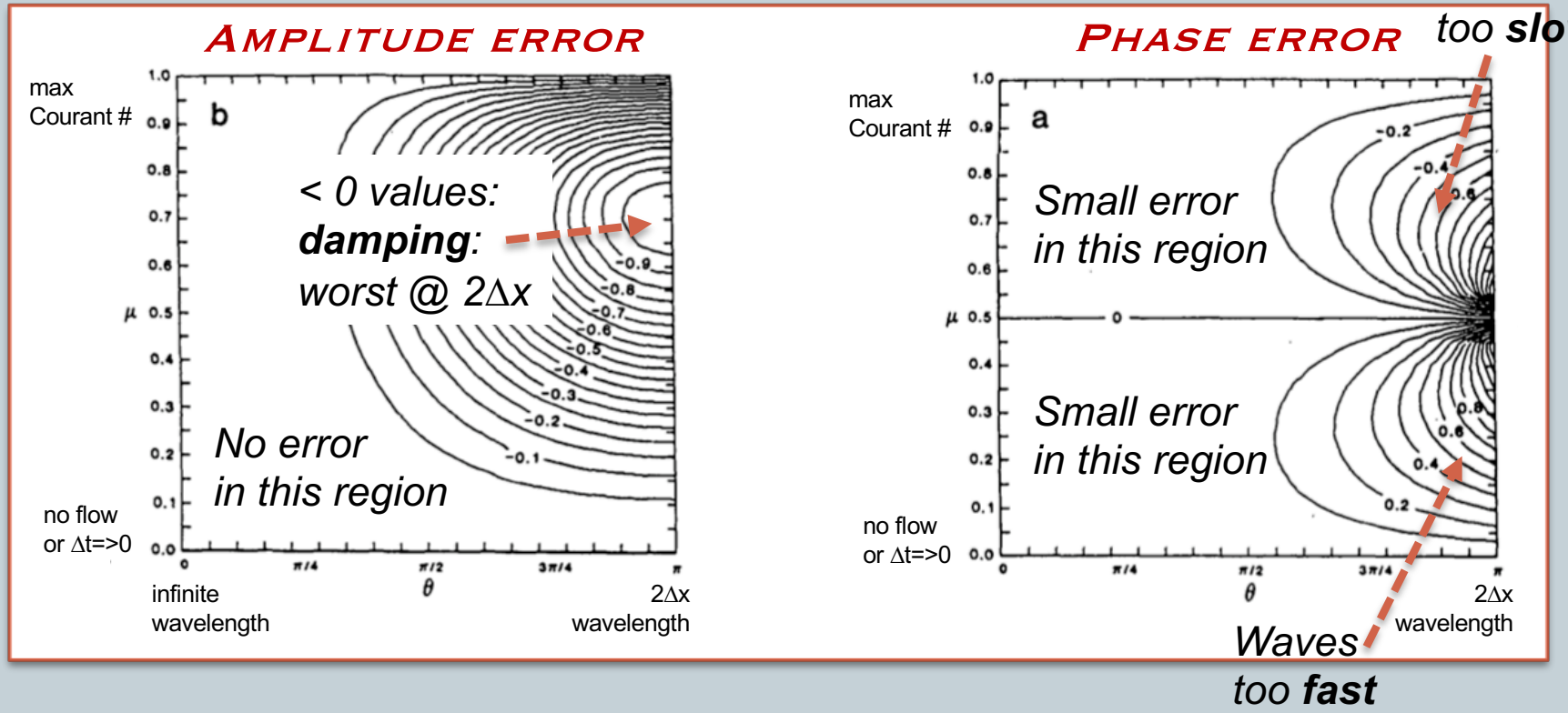
Figure 4-1 Effects of dissipation and dispersion. (a) Exact solution. (b) Numerical solution distorted primarily by dissipation errors (typical of first-order methods). (c) Numerical solution distorted primarily by dispersion errors (typical of second-order methods).

Anderson (1984)

Takacs' plots

TAKACS PP. 1053-1054

- Plots – Amplitude and Phase Error
  - θ = kΔx = nondimensional wavenumber
  - μ = Courant number c*dt/dx



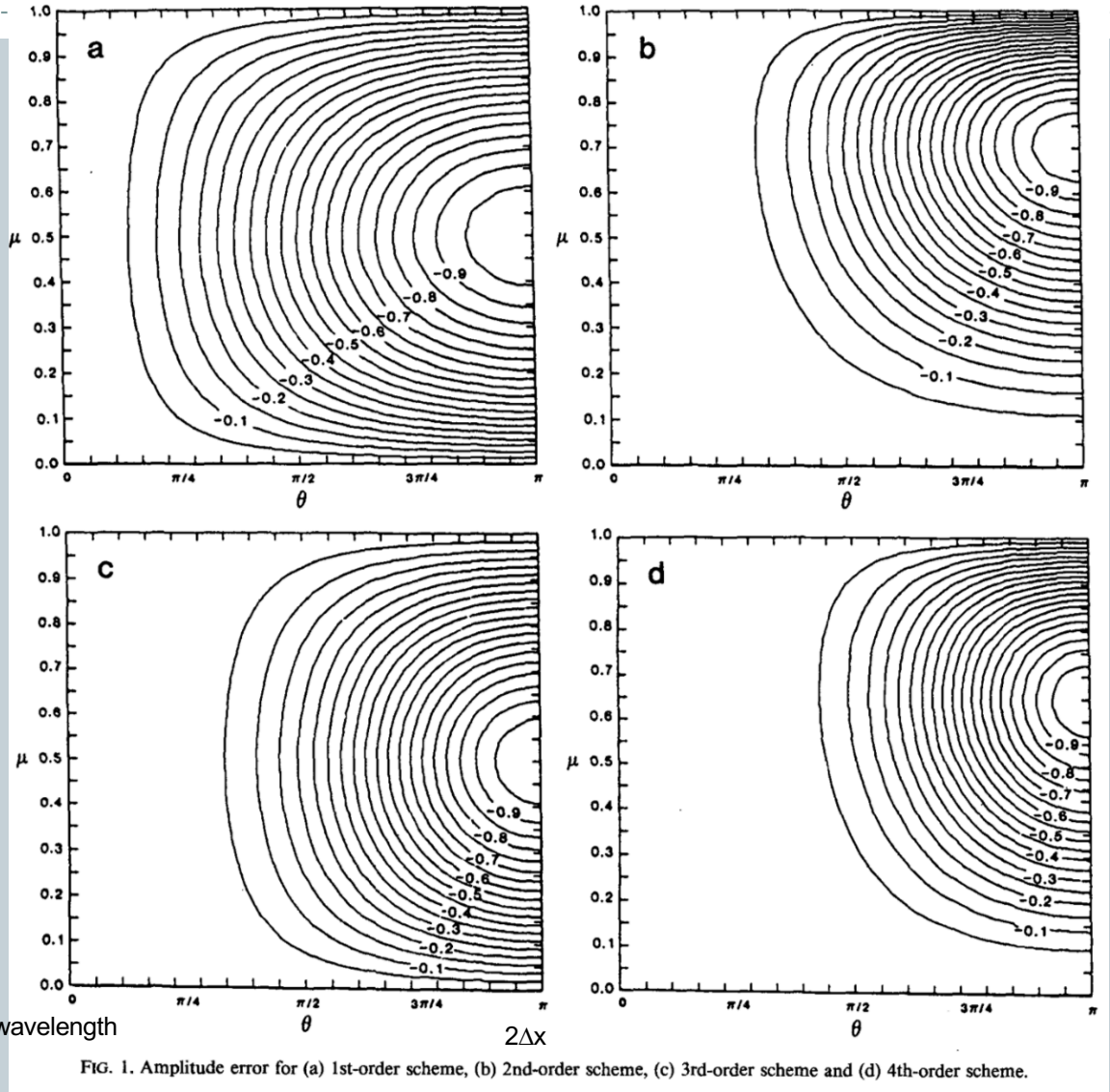
Takacs' plots – Amplitude Errors

1ST ERROR

2ND ERROR

3RD ERROR

4TH ERROR



Infinite wavelength

$2\Delta x$

FIG. 1. Amplitude error for (a) 1st-order scheme, (b) 2nd-order scheme, (c) 3rd-order scheme and (d) 4th-order scheme.

Takacs' plots – Phase Errors

1ST ERROR

2ND ERROR

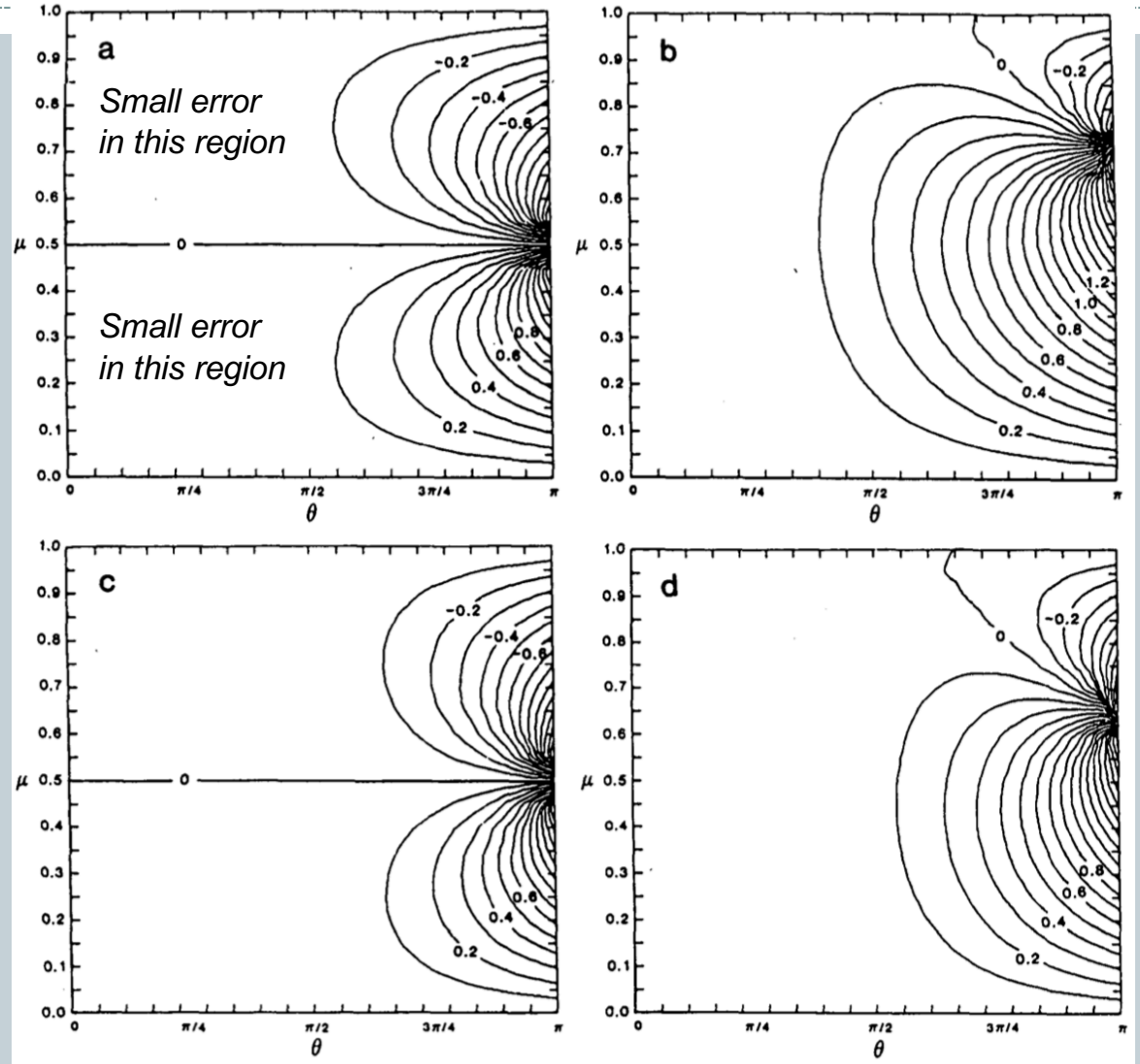


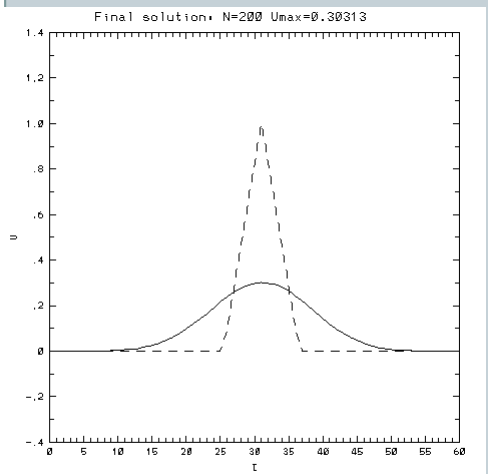
FIG. 2. Phase error for (a) 1st-order scheme, (b) 2nd-order scheme, (c) 3rd-order scheme and (d) 4th-order scheme.

3RD ERROR

4TH ERROR

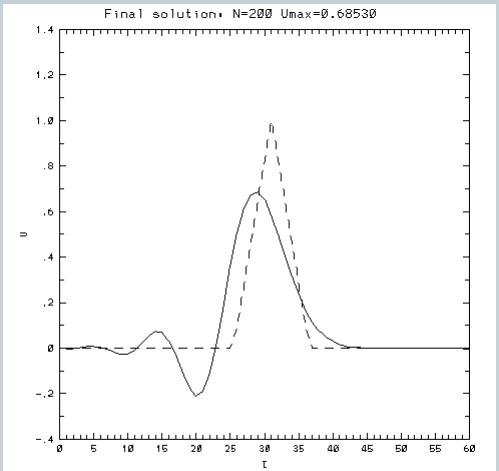
Takacs (1985)

- Considerations:
  - Odd-order schemes generally dissipative
    - *amplitude errors*
  - Even-order schemes generally dispersive
    - *phase errors*



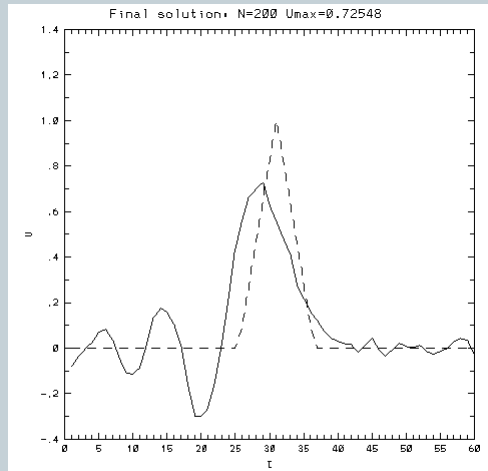
1ST ORDER

UPSTREAM



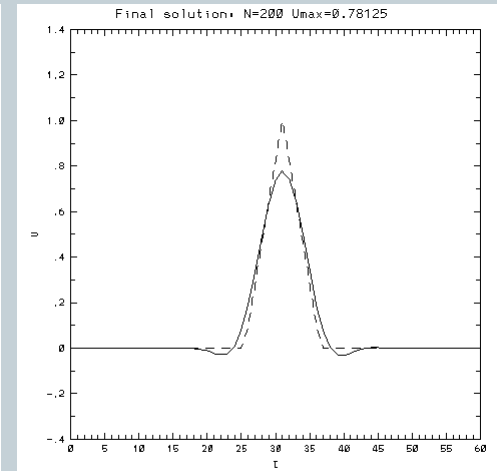
2ND ORDER

LAX-WENDROFF



2ND ORDER

LEAPFROG



3RD ORDER

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