目录

8.6 数值微分

#############2####diff()###

8.6.1 问题的提出

#########(#####)###$f'(x_0)=\dfrac{f(x_0+h)-f(x_0)}{h}$，############ ######$f'(x_0)=\dfrac{f(x_0)-f(x_0-h)}{h}$，############$f'(x_0)=\dfrac{f(x_0+h)-f(x_0-h)}{2h}$。###########h############ #######h####$f(x_0+h)$#$f(x_0)$#$f(x_0-h)$###

8.6.2 中心差分算法

###################$o(\Delta t^4)$########################

$$y_i'=\frac{-y_{i+2}+8y_{i+1}-8y_{i-1}+y_{i-2}}{12\Delta t}$$

$$y_i''=\frac{-y_{i+2}+16y_{i+1}-30y_i+16y_{i-1}-y_{i-2}}{12\Delta t}$$

$$y_i'''=\frac{-y_{i+3}+8y_{i+2}-13y_{i+1}+13y_{i-1}-8y_{i-2}+y_{i-3}}{8\Delta t^3}$$

$$y_i^{(4)}=\frac{-y_{i+3}+12y_{i+2}-39y_{i+1}+56y_i-39y_{i-1}+12y_{i-2}-y_{i-3}}{6\Delta t^4}$$

#########$o(\Delta t^4)$########Δt###0####################MATLAB############MATLAB###

#MATLAB#######diff()### ##########

```
Y=diff(x,n)
```

```
Y=diff(A,n,dim)
```

##x######;A######;n#############1;dim########,dim=1(####) ######;dim=2#######;###Y######## ####Y###(##)#x(A)#(#)#

########################diff_ctr.m.##########

```
function [dy,dx]=diff_ctr(y,dt,n)

% ######
```

% #####

%   ---y#################

%   ---dt#######

%   ---n#########1~4#

% #####

%   ---dy########

%   ---dx##dy######

yx1=[y 0 0 0 0 0];yx2=[0 y 0 0 0 0];yx3=[0 0 y 0 0 0];

yx4=[0 0 0 y 0 0];yx5=[0 0 0 0 y 0];yx6=[0 0 0 0 0 y];

switch n

case 1

dy=(-diff(yx1)+7*diff(yx2)+7*diff(yx3)-diff(yx4))/(12*dt);

L0=3;

case 2

dy=(-diff(yx1)+15*diff(yx2)-15*diff(yx3)+diff(yx4))/(12*dt^2);

L0=3;

case 3

dy=(-diff(yx1)+7*diff(yx2)-6*diff(yx3)-6*diff(yx4)+...

7*diff(yx5)-diff(yx6))/(8*dt^3);L0=5;

case 4

dy=(-diff(yx1)+11*diff(yx2)-28*diff(yx3)+28*diff(yx4)-...

11*diff(yx5)+diff(yx6))/(6*dt^4);L0=5;

end

dy=dy(L0+1:end-L0);

dx=([1:length(dy)]+L0-2-(n>2))*dt;

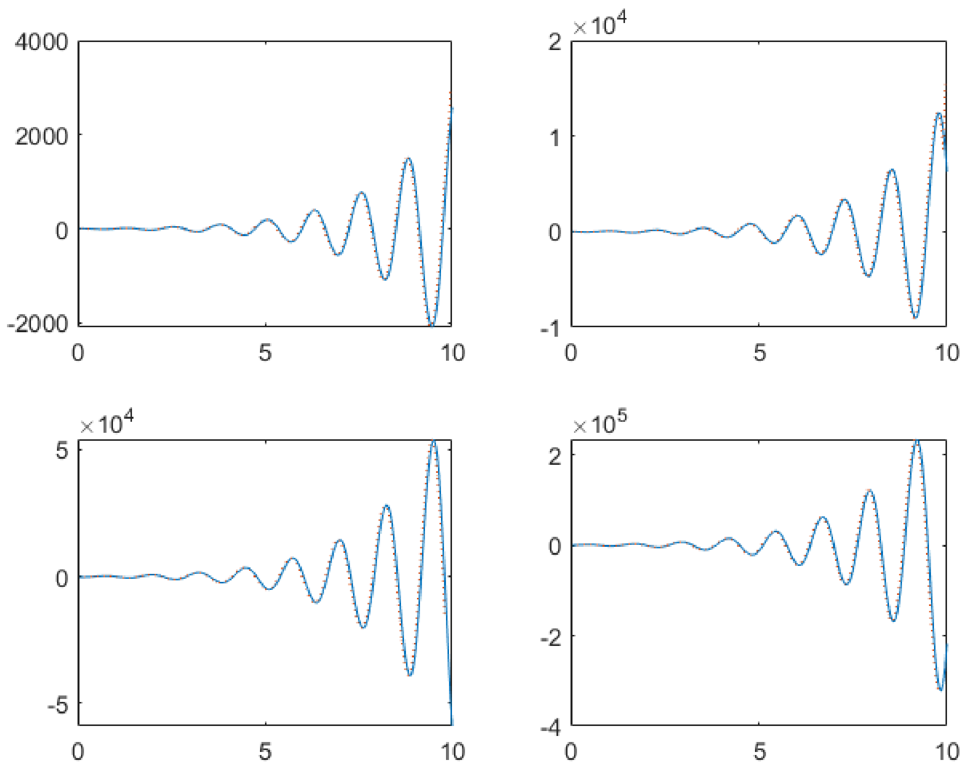
##########[dy,dx] =diff_ctr(y,dt,n)

##8-29############### $y = e^{x^{0.8}}\sin 5x$ #1~4############

```
syms x;
y=exp(x^0.8)*sin(5*x)%求1~4阶导数的解析解
```

y = $\sin(5\,x)\,\mathrm{e}^{x^{4/5}}$

```
y1=diff(y,x); y2=diff(y,x,2); y3=diff(y,x,3); y4=diff(y,x,4);
h=0.05;x=0.05:h:10;
Y=eval(subs(y)); y11=subs(y1); y22=subs(y2); y33=subs(y3); y44=subs(y4);

y=[y11;y22;y33;y44];%求1~4阶导数的数值解，并和解析解比较
for k=1:4
    [dy,dx]=diff_ctr(Y,h,k);
    subplot(2,2,k)
    plot(x,y(k,:),dx,dy,':')
end
```



8.6.3 梯度和法矢量的数值计算

n###$F(x_1,x_2,\ldots,x_n)$########$\nabla F = \dfrac{\partial F}{\partial x_1}e_1 + \dfrac{\partial F}{\partial x_2}e_2 + \ldots + \dfrac{\partial F}{\partial x_n}e_n$

##n###$F(x_1,x_2,\ldots,x_n)$##############diff()#jacobian()##
#MATLAB##########MATLAB###########gradient()###########

[Fx1,Fx2,Fx3,...,Fxn]=gradient(F,h)

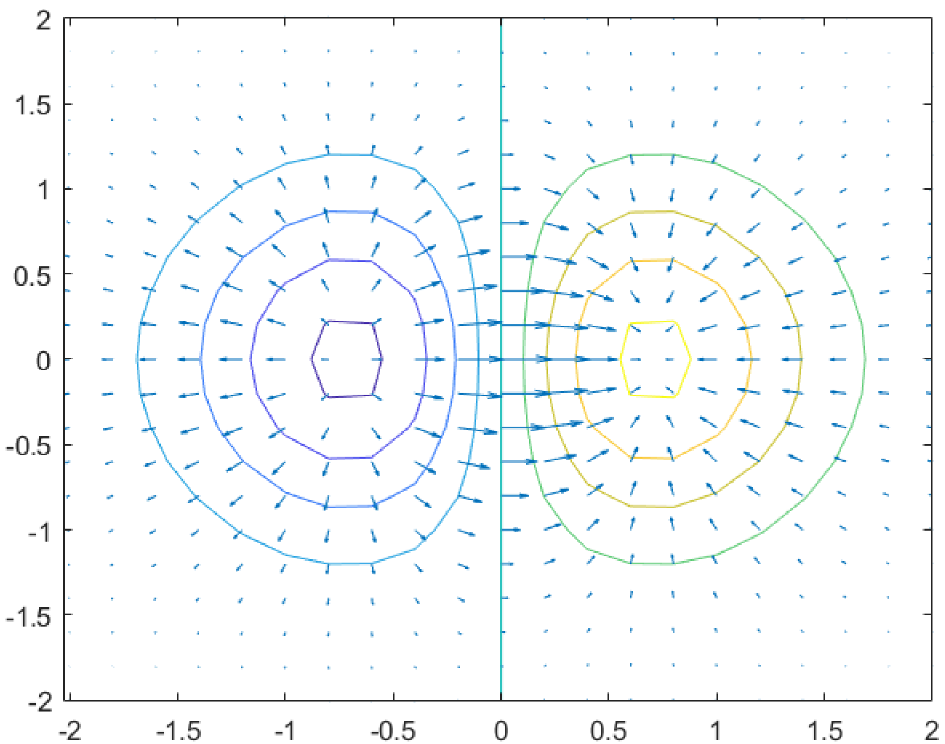
[Fx1,Fx2,Fx3,...,Fxn]=gradient(F,hx1,hx2,...,hxn)

##F####n###$F(x_1,x_2,\ldots,x_n)$######h#######################1#hx1#hx2#...#hxn#################

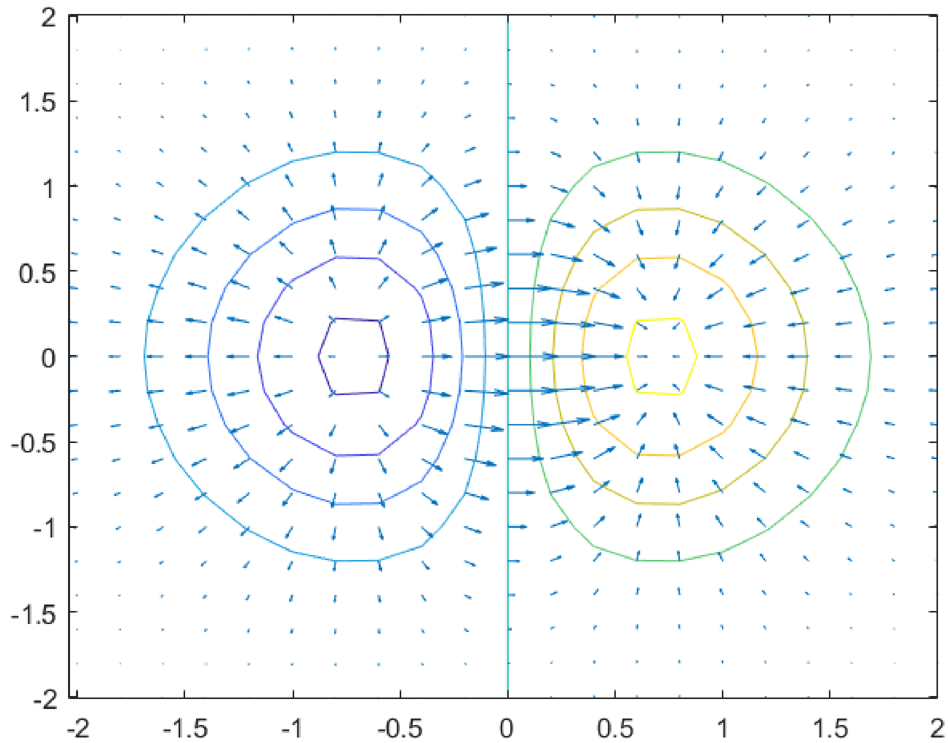
###Fx1#Fx2#Fx3#...#Fxn##############################$F_{x_i} = \frac{F_{x_i} h_{x_i}}{d_{x_i}} (i = 1, 2, \ldots, n)$###$d_{x_i}$#$x_i$########

##8-30#####$z = xe^{-x^2-y^2}$####$[-2, 2] \times [-2, 2]$####################################

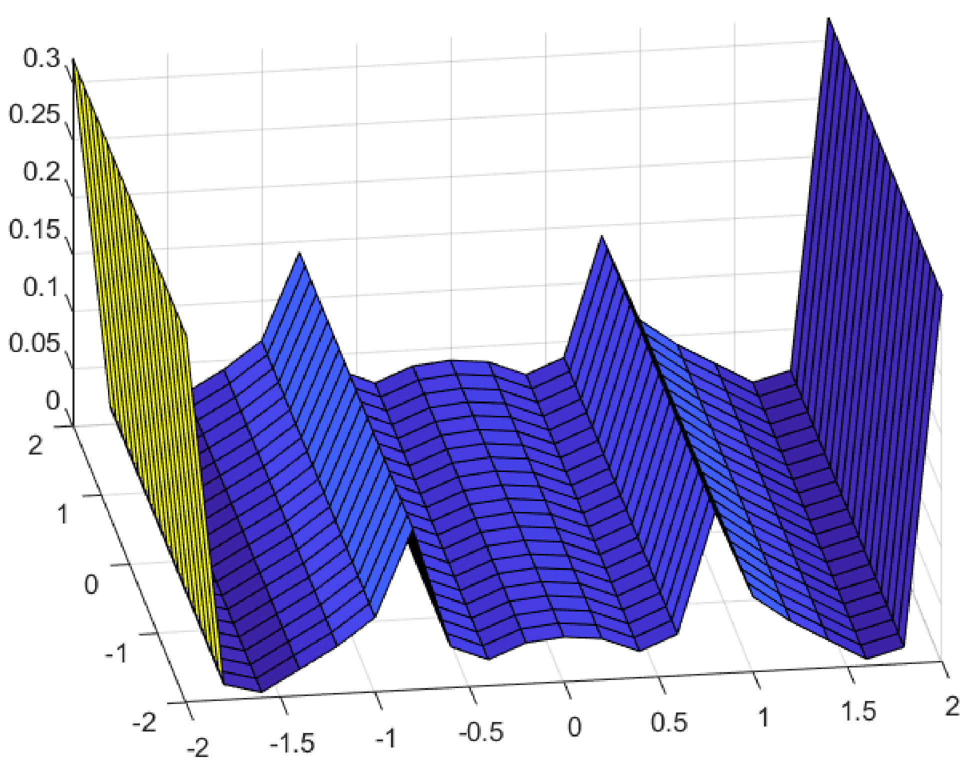
```
%------求梯度的解析解-.-- -%
clf
syms x y;
z=x*exp(-x^2-y^2);
zx=diff(z,x);
zy=diff(z,y);

% --绘图比较解析解和数值解---------%
[x,y]=meshgrid(-2:.2: 2);
z=x.*exp(-x.^2-y.^2);
[px,py]=gradient(z,.2,.2);
contour(x, y, z) , hold on , quiver(x, y, eval(zx), eval(zy)) , hold off
```



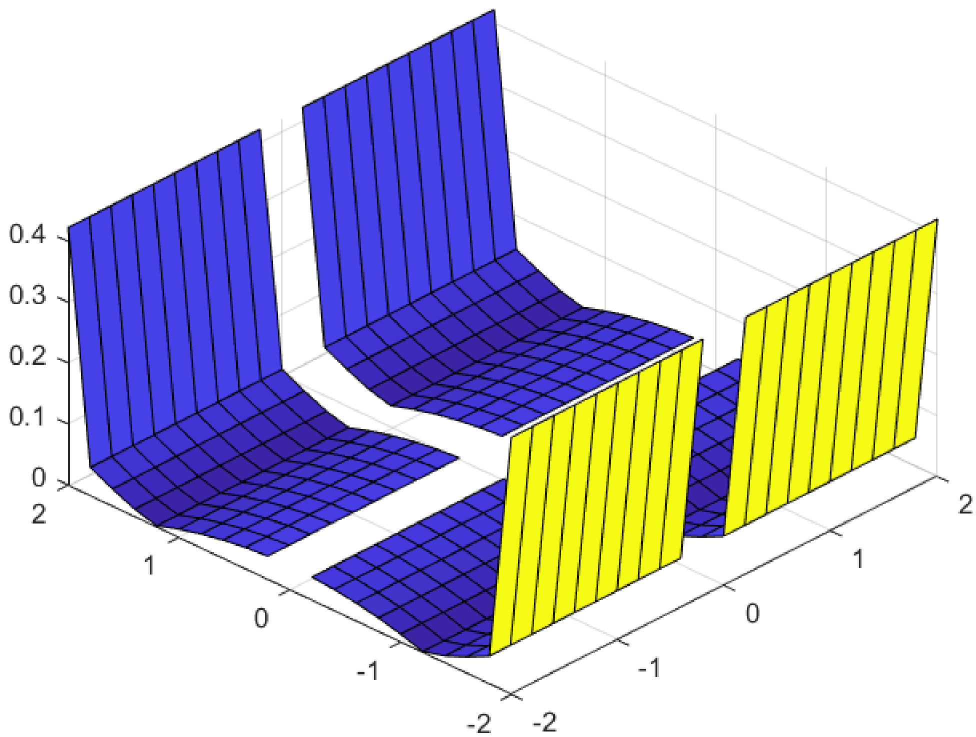

```
figure
contour(x, y, z) , hold on , quiver(x, y, px,py) , hold off
```




```
figure
surf(x,y,abs((px-eval(zx))./eval(zx))) %绘制dz/dx的误差曲面
```



```
figure
surf(x,y,abs((py-eval(zy))./eval(zy))) %绘制dz/dx的误差曲面
```



##MATLAB###########

#surfnorm()############

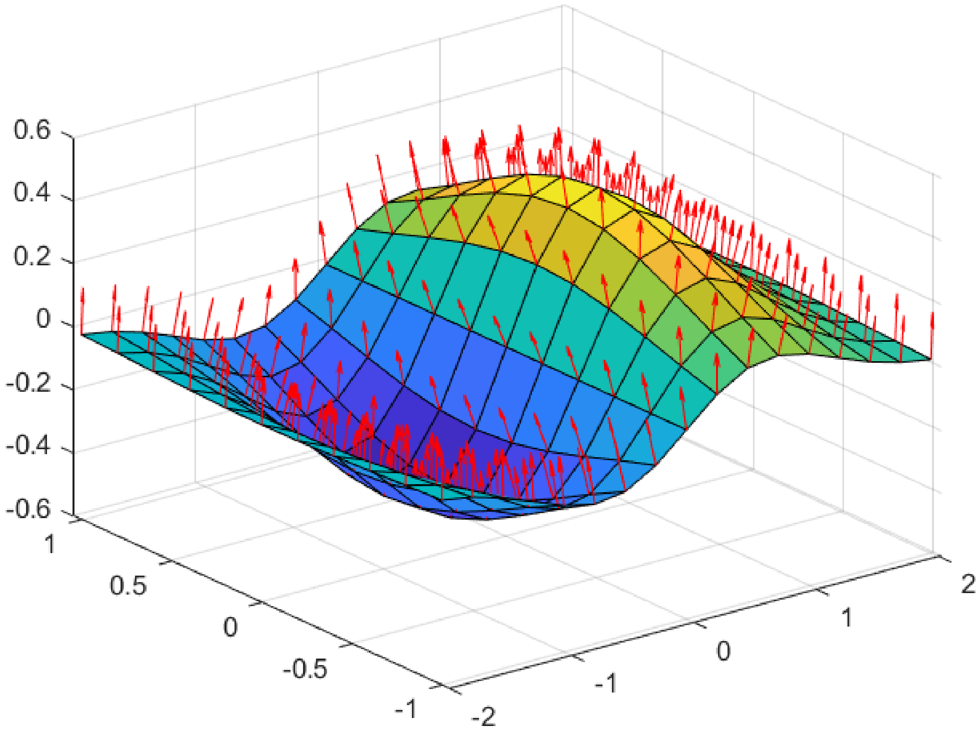
[Nx#Ny#Nz] =surfnorm(X#Y#Z) %#######

surfnorm(X#Y#Z) %#####

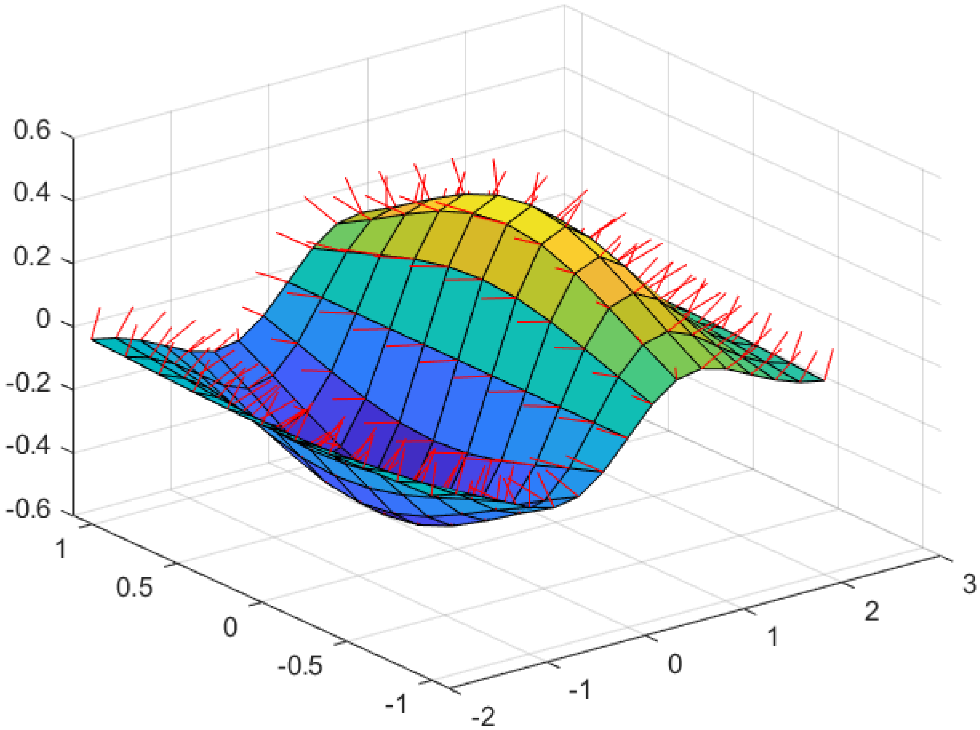
##X#Y#Z###########Nx#Ny#Nz#########

##8-31####8-30#############$[-2,3]\times[-2,2]$################

```
[X,Y]=meshgrid(-2:0.25:2,-1:0.2:1);
Z=X.*exp(-X.^2-Y.^2);
[U,V,W] =surfnorm(X,Y,Z); %计算法矢量
quiver3(X,Y,Z,U,V,W,0.5,'r'); %绘制箭头图形
hold on
surf(X,Y,Z)
```




```
%axis([-2 3 -2 2 -1 1]) %下面绘制的法矢量图和上面的图形效果是差不多的
figure
surfnorm(X,Y,Z) %绘制法矢量图
```



8.7 实验范例：自行车轮式物的运动轨迹

1)#####

##
################################

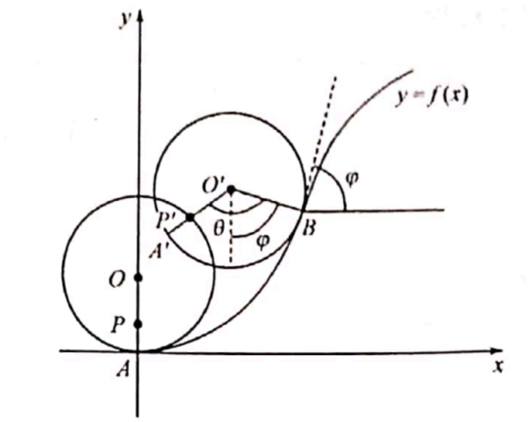
2)#####

#############

- ############### $f(x)$ ###########*R*#######*O*######## $f(x)$ ############
- ################# $f(x)$ ########
- ##P####### $\overline{\mathrm{OP}} = r$ #### $r \le R$ #
- ##############

3)#####

#OP####*A*####*A*##########(###)##*θ*##############*B*####*O*## O' ###*P*######## P' ######



$B(x_0,y_0), O'(X,Y), P'(x,y)$,# $\widehat{BA'}=R\theta=\widehat{BA}=\int_0^{x_0}\sqrt{1+[f'(x)]^2}\mathrm{dx}$

因此$\theta=\frac{1}{R}\int_0^{x_0}\sqrt{1+[f'(x)]^2}\,\mathrm{dx}$

##$O'(X,Y)$###$y=f(x)$#B#########$\overline{O'B}=R$##

$$\begin{cases} Y-f(x_0)=-\dfrac{1}{f'(x_0)}(X-x_0) \\ (X-x_0)^2+(Y-f(x_0))^2=R^2 \end{cases}$$

解得$\begin{cases} X=x_0-\dfrac{\mathrm{Rf'}(x_0)}{\sqrt{1+[f'(x_0)]^2}} \\ Y=f(x_0)+\dfrac{R}{\sqrt{1+[f'(x_0)]^2}} \end{cases}$

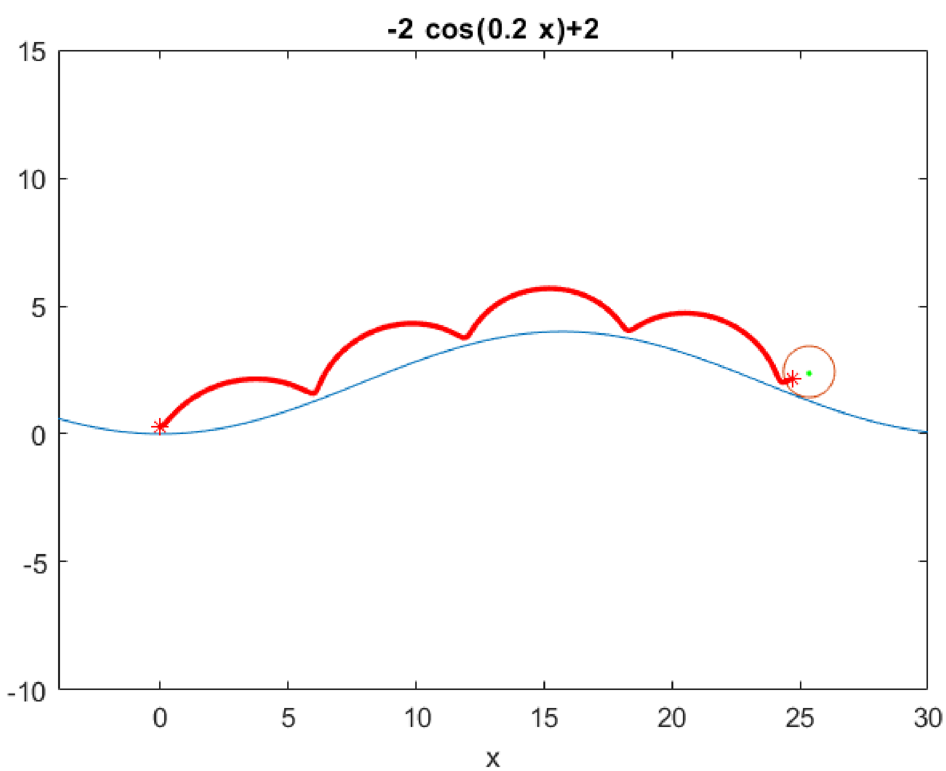
又对P'(x,y)有$\begin{cases} x=X-\mathrm{rsin}(\theta-\varphi) \\ y=Y-\mathrm{rcos}(\theta-\varphi) \end{cases}$

其中$\begin{cases} \varphi=\arctan(f'(x_0)) \\ \theta=\dfrac{1}{R}\int_0^{x_0}\sqrt{1+[f'(x)]^2}\,\mathrm{dx} \end{cases}$

所以饰物的运动轨迹为$\begin{cases} x=x_0-\dfrac{\mathrm{Rf'}(x_0)}{\sqrt{1+[f'(x_0)]^2}}-\mathrm{rsin}(\theta-\varphi) \\ Y=f(x_0)+\dfrac{R}{\sqrt{1+[f'(x_0)]^2}}-\mathrm{rcos}(\theta-\varphi) \end{cases}$

```
R=1;r=0.7*R;
f=@(x)-2*cos(0.2*x) +2; %定义函数f(x)
df=@(x)0.4*sin(0.2*x); %定义函数df(x)/dx
x0=eps:0.1:25;
set(gcf,'DoubleBuffer','on'); %设置图形窗口的渲染效果
h=ezplot('-2*cos(0.2*x)+2',[-4,30]);
axis equal
```

```
ylim([-10,15])
hold on
t=linspace(0,2*pi,length(x0));
h1=plot(R*(exp(i*t) +i));
phi=atan(df(x0(1))); %计算phi
theta=1/R*quadl(@(x)sqrt(1+df(x).^2),0,x0(1)); %计算theta
x=x0(1)-R*df(x0(1))/sqrt(1+df(x0(1))^2)-r*sin(theta-phi);
y=f(x0(1))+R/sqrt(1+df(x0(1))^2) -r*cos(theta-phi);
X=x0(1)-R*df(x0(1))/sqrt(1+df(x0(1))^2);
Y=(x0(1))+R/sqrt(1+df(x0(1))^2);
h2=plot(X,Y,'g.');
h3=plot(x,y,'r*');
h4=plot(x,y,'r','LineWidth',2);
Z1=[x];Z2=[y];
for k=2:length(x0) %制作动画
    phi1=atan(df(x0(k))); theta1=1/R*quadl(@(x)sqrt(1+df(x).^2),0,x0(k));
    C1=x0(k)-R*df(x0(k))/sqrt(1+df(x0(k))^2)-r*sin(theta1-phi1);
    C2=f(x0(k))+R/sqrt(1+df(x0(k))^2)-r*cos(theta1-phi1);
    X=x0(k)-R*df(x0(k))/sqrt(1+df(x0(k))^2);
    Y=f(x0(k))+R/sqrt(1+df(x0(k))^2);
    %更新车轮的坐标
    set(h1,'Xdata',real(R*(exp(i*t)+i))+x0(k)-R*sin(phi1),'Ydata',imag(R*(exp(i*t)+i))+f(x0(k)
    set(h2,'Xdata',X,'Ydata',Y) %更新O'点的坐标
    Z1=[Z1,C1];Z2=[Z2,C2];
    set(h3,'Xdata',[x,C1],'Ydata',[y,C2]) %更新P点的坐标
    set(h4,'Xdata',Z1,'Ydata',Z2)
    pause(0.1) %暂停0.1s
end
```



########### $f(x) = -2\cos 0.2x + 2$ ########