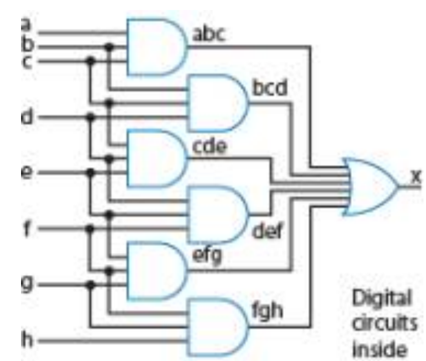
# Topic 1

# Introduction to Digital Design

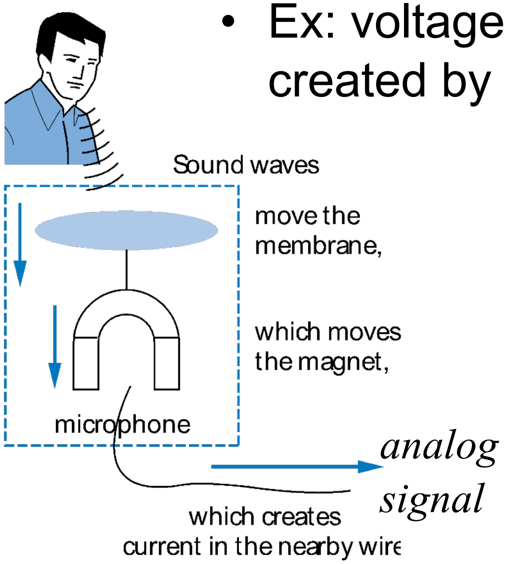
# Why Study Digital Design?

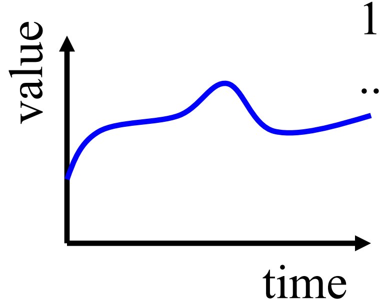
- Many elements of our lives are or are to become digital
  - Computer, camera, cell phone, TV, car...
- Solid understanding benefits ECE engineers
  - For computer engineer – fundamental
  - For electrical engineer – many times necessary
  - Even for software engineer – confident and insightful when aware of hardware issues



# What Does “Digital” Mean?

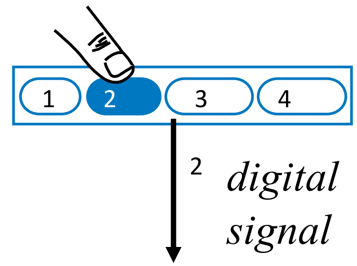
- Analog signal
  - Infinite possible values
    - Ex: voltage on a wire created by microphone

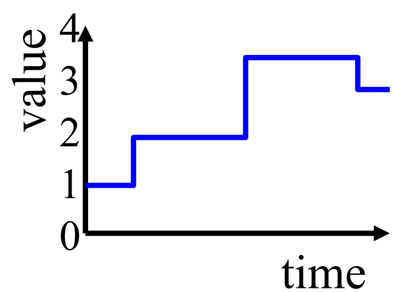



Possible values:
1.00, 1.01, 2.0000009,
... infinite possibilities

- Digital signal
  - Finite possible values
    - Ex: button pressed on a keypad

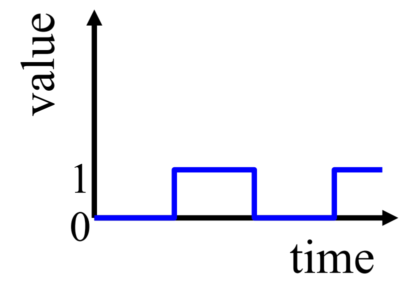



Possible values:
0, 1, 2, 3, or 4.
That’s it.

# Digital Signals with Only Two Values: Binary

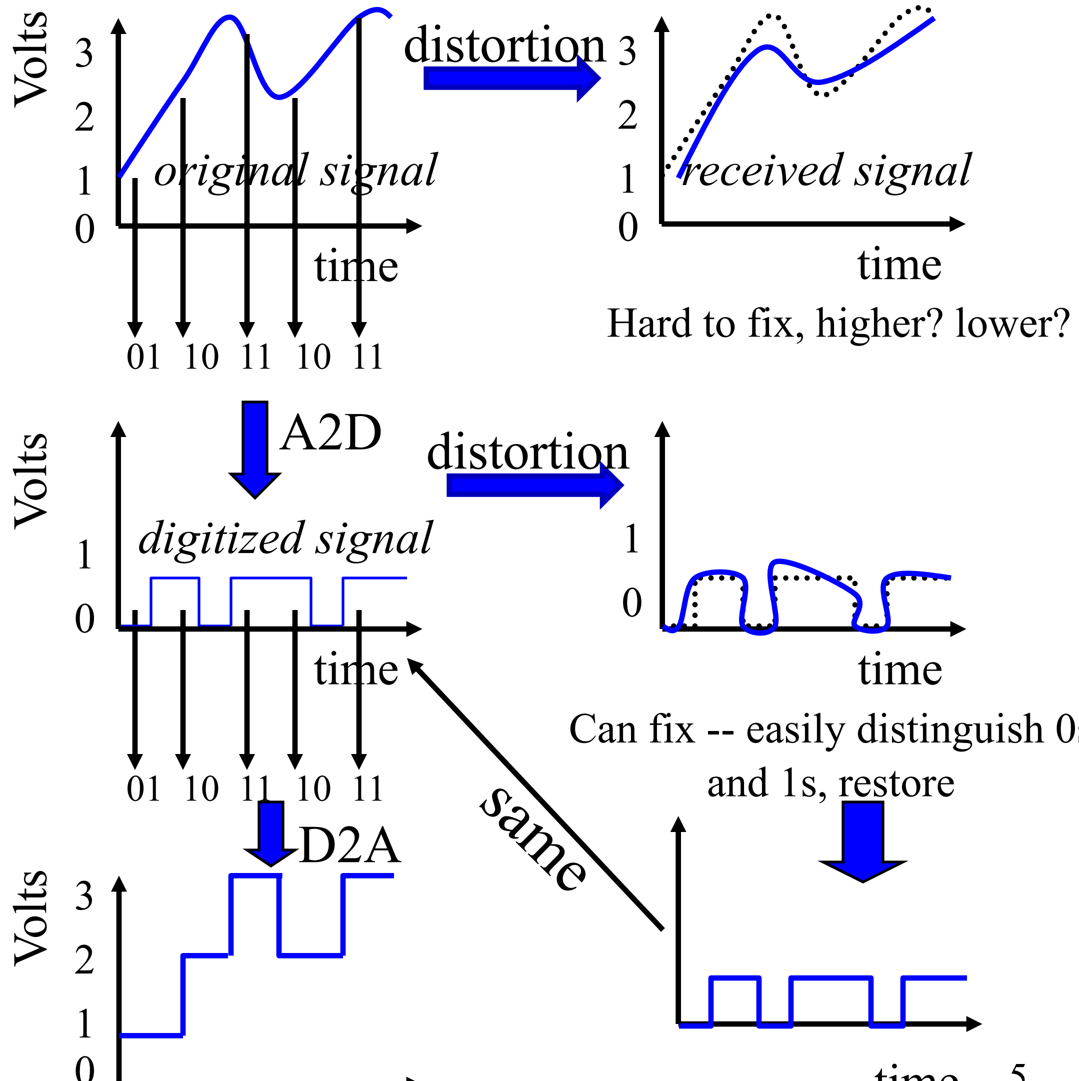
- ***Binary*** digital signal -- only *two* possible values
  - low voltage (e.g. 0V or -5V) and high voltage (e.g. 3.3V or 5V)
  - Typically represented as **0** and **1**, respectively
  - All values are represented as combinations of 0's and 1's, e.g. 1011, 11010
    - Called binary value or binary number
    - Each binary digit is a ***bit***
  - We'll only consider binary digital signals
    - Although there are other types of digital signals
  - Binary is popular because
    - Transistors, the basic digital electric component, operate using *two* voltages
    - Storing/transmitting one of *two* values is easier than three or more



# From Analog to Digital – Digitization

- Analog signal (e.g., audio) may lose quality
  - Voltage levels not saved/copied/transmitted perfectly
  - Hard to recover
- Digitized version:
  - "Sample" voltage at particular rate
  - Easy to distinguish 0s from 1s, thus easy to recover
  - Increase sample rate to improve quality

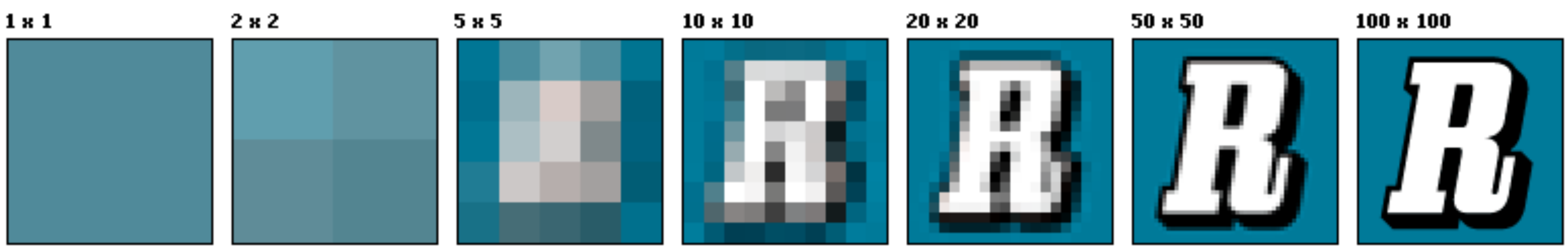
Example:
if only 4 sampled values
let binary representation be:
0 V: "00"
1 V: "01"
2 V: "10"
3 V: "11"



# Resolution

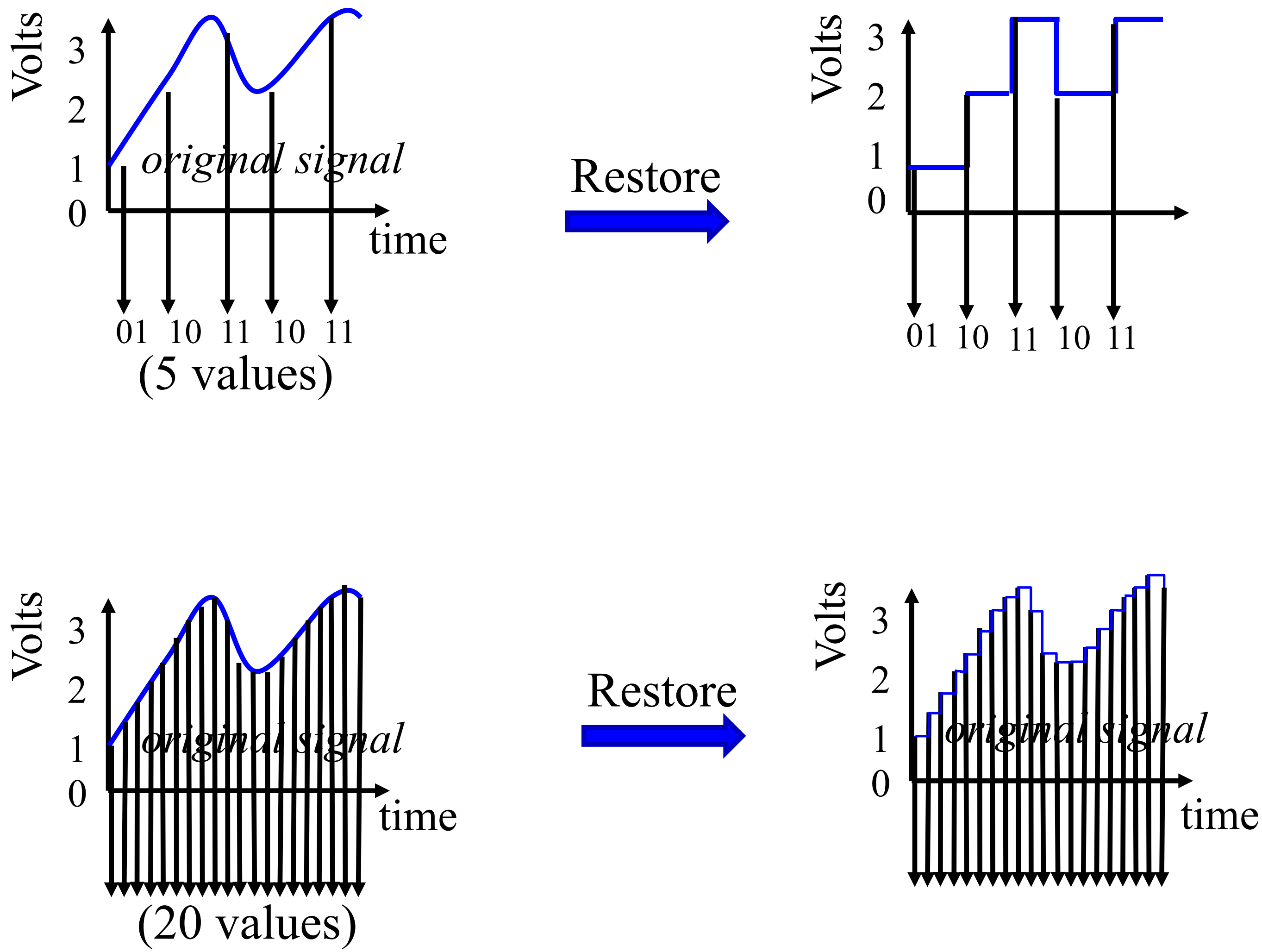
What happened?

(*source: cntv.cn*)



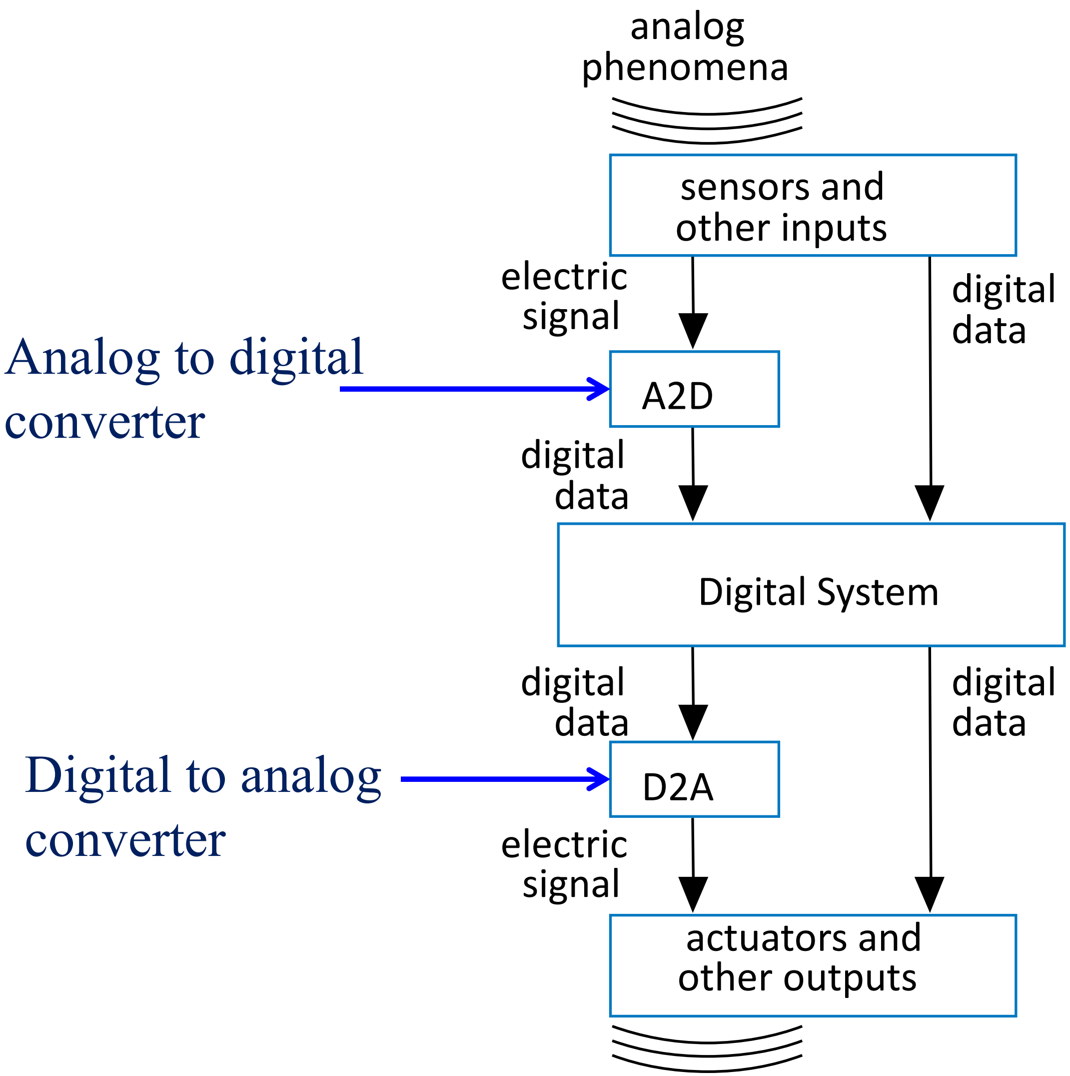

(*source: wikipedia.org*)

# From Analog to Digital – Digitization




Higher resolution (sample rate)

# Typical Digital System



# How Do We Encode Numbers with Bits

- Number systems: decimal, binary, octal, hexadecimal, …
- Each position of a number is associated with a weight quantity
  - Base ten (***decimal***)

| | | | | |
|---|---|---|---|---|
| __ | __ | 5 | 2 | 3 |
| $10^4$ | $10^3$ | $10^2$ | $10^1$ | $10^0$ |

  - Base two (***binary***)

| | | | | |
|---|---|---|---|---|
| __ | __ | 1 | 0 | 1 |
| $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$ |

# Binary System

- The Binary System is a base 2 (modulo 2) number system:
  - 2 digits: 0 or 1
- Counting beyond 1 requires additional place
- In a binary number, each position has a decimal weight in power of 2, **10011.01**

| | | | | | | . | | |
|---|---|---|---|---|---|---|---|---|
| | **1** | **0** | **0** | **1** | **1** | **.** | **0** | **1** |
| | $\times$**16** | $\times$**8** | $\times$**4** | $\times$**2** | $\times$**1** | | **½** | **¼** |
| **weight** | **($2^4$)** | **($2^3$)** | **($2^2$)** | **($2^1$)** | **($2^0$)** | | **($2^{-1}$)** | **($2^{-2}$)** |
| **position** | **4** | **3** | **2** | **1** | **0** | | **-1** | **-2** |

# Find Equivalent Decimal for Binary Numbers

- Example: Convert binary number $10011.01_2$ to decimal

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Number:** | **1** | **0** | **0** | **1** | **1** . | **0** | **1** |
| **Position:** | **4** | **3** | **2** | **1** | **0** | **-1** | **-2** |
| **Weight:** | $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$ | $2^{-1}$ | $2^{-2}$ |

$$1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + 0 \times 2^{-1} + 1 \times 2^{-2}$$

$$= 16 + 0 + 0 + 2 + 1 + 0 + 1/4$$

$$= 19.25_{10} = 10011.01_2$$

# Encode Decimal as Binary Numbers:
## Subtraction Method (Easy for Humans)

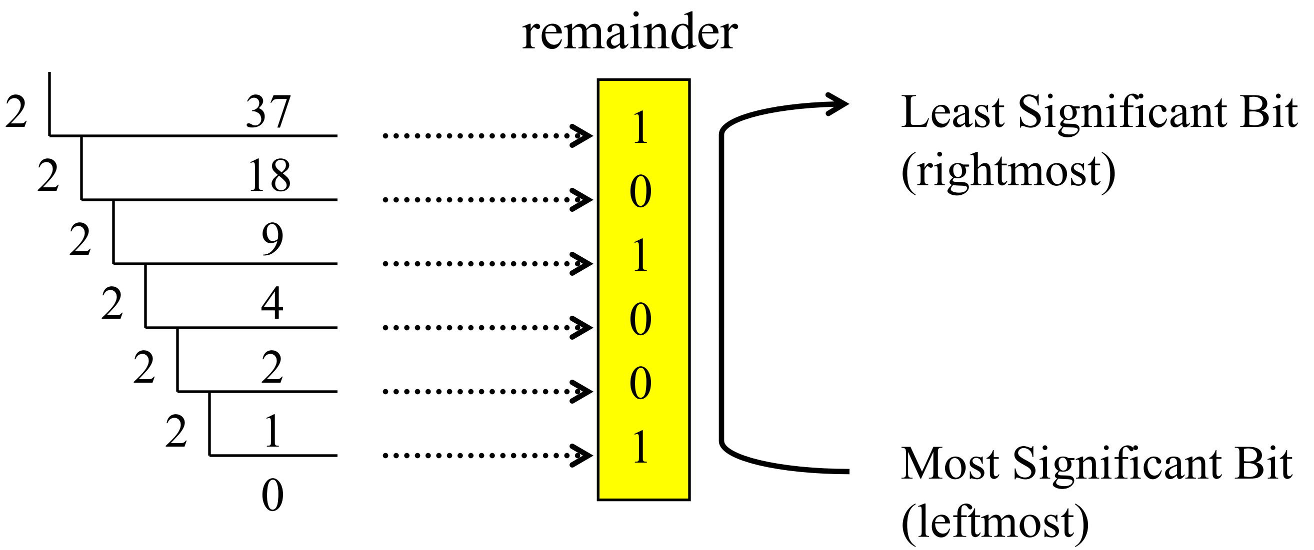
- Subtraction method
  - To make the job easier (especially for big numbers), we can just subtract a selected binary weight from the (remaining) quantity
    - Then, we have a new remaining quantity, and we start again (from the present binary position)
    - Stop when remaining quantity is 0

Remaining quantity: 12

| 32 | 16 | 8 | 4 | 2 | 1 | |
|---|---|---|---|---|---|---|
| _ | _ | _ | _ | _ | _ | |
| 1 | _ | _ | _ | _ | _ | 32 is too much |
| 0 | 1 | _ | _ | _ | _ | 16 is too much |
| 0 | 0 | 1 | _ | _ | _ | **12 – 8 = 4** |
| 0 | 0 | 1 | 1 | _ | _ | **4-4=0** DONE |
| **0** | **0** | **1** | **1** | **0** | **0** | answer |

# Encode Decimal in Binary Numbers: Division Method (Good for Computers)

- Example: Convert decimal number 37 to binary
  - Repeated-division-by-base (here, base 2)




$(37)_{10} = (100101)_2$

# Encode Fractional Decimal in Binary

- Example: Convert fractional part $0.715_{10}$ to binary
  - Repeated-multiplication-by-base (here, base 2)

| | | |
|---|---|---|
| $0.715 \times 2 = 1\ .430$ | 1 | Most Significant Bit (leftmost) |
| $0.430 \times 2 = 0\ .860$ | 0 | |
| $0.860 \times 2 = 1\ .720$ | 1 | |
| $0.720 \times 2 = 1\ .440$ | 1 | |
| $0.440 \times 2 = 0\ .880$ | 0 | Least Significant Bit (rightmost) |

$(0.715)_{10} \approx (0.10110\ldots\ldots)_2$

# Encode Numbers with Bits

- Bigger number needs more bits to encode
  - $37_{10} = 100101_2$ (6 bits)
  - $137_{10} = 10001001_2$ (8 bits)
  - $10307_{10} = 10100001000011_2$ (14 bits)
- N bits can represent $2^N$ non-negative integers
  - 0, 1, 2, …, $2^N$-1
  - Negative numbers will be discussed later

# Encode Decimal Numbers by Binary Bits

| Binary | | | | Decimal |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 2 |
| 0 | 0 | 1 | 1 | 3 |
| 0 | 1 | 0 | 0 | 4 |
| 0 | 1 | 0 | 1 | 5 |
| 0 | 1 | 1 | 0 | 6 |
| 0 | 1 | 1 | 1 | 7 |
| 1 | 0 | 0 | 0 | 8 |
| 1 | 0 | 0 | 1 | 9 |
| 1 | 0 | 1 | 0 | 10 |
| 1 | 0 | 1 | 1 | 11 |
| 1 | 1 | 0 | 0 | 12 |
| 1 | 1 | 0 | 1 | 13 |
| 1 | 1 | 1 | 0 | 14 |
| 1 | 1 | 1 | 1 | 15 |
| | | | | 16 |
| …… | | | | …… |

……

# Hexadecimal System

- The Hexadecimal system is a base 16 (modulo 16) number system:
  - 16 digits: 0 1 2 3 4 5 6 7 8 9 A B C D E F
- Letters A ~ F represent decimal 10 through decimal 15
- Each position has a decimal weight in power of 16, e.g. **E3A**

| | **E** | **3** | **A** |
|---|---|---|---|
| | $\times$**256** | $\times$**16** | $\times$**1** |
| weight | **($16^2$)** | **($16^1$)** | **($16^0$)** |

$$(E3A)_{16} = E \times 256 + 3 \times 16 + A \times 1 = 3584 + 48 + 10 = 3642$$

# Encode Decimal to Hexadecimal

- Example: Convert decimal number 58 to hexadecimal
  - Repeated-division-by-base (here, base 16)

| | | | |
|---|---|---|---|
| 16 \| 58 | → | 10 (A) | Least Significant Digit |
| 16 \| 3 | → | 3 | Most Significant Digit |
| 0 | | | |

$(58)_{10} = (3A)_{16}$

# Summary

| Binary | | | | Decimal | Hexaecimal |
|---|---|---|---|---|---|
| **0** | **0** | **0** | **0** | **0** | **0** |
| **0** | **0** | **0** | **1** | **1** | **1** |
| **0** | **0** | **1** | **0** | **2** | **2** |
| **0** | **0** | **1** | **1** | **3** | **3** |
| **0** | **1** | **0** | **0** | **4** | **4** |
| **0** | **1** | **0** | **1** | **5** | **5** |
| **0** | **1** | **1** | **0** | **6** | **6** |
| **0** | **1** | **1** | **1** | **7** | **7** |
| **1** | **0** | **0** | **0** | **8** | **8** |
| **1** | **0** | **0** | **1** | **9** | **9** |
| **1** | **0** | **1** | **0** | **10** | **A** |
| **1** | **0** | **1** | **1** | **11** | **b** |
| **1** | **1** | **0** | **0** | **12** | **C** |
| **1** | **1** | **0** | **1** | **13** | **d** |
| **1** | **1** | **1** | **0** | **14** | **E** |
| **1** | **1** | **1** | **1** | **15** | **F** |

# Convert Binary to Hexadecimal

- Look for groups of 4 bits starting from the LSB
- Example: Convert 11 1011 0101.11 to hexadecimal:

$11\ 1011\ 0101.11 = (0011\ 1011\ 0101.1100)_2$

3 b 5 c

$(11\ 1011\ 0101.11)_2 = (3b5.c)_{16}$

| Binary | | | | Decimal | Hexaecimal |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 |
| 0 | 0 | 1 | 0 | 2 | 2 |
| 0 | 0 | 1 | 1 | 3 | 3 |
| 0 | 1 | 0 | 0 | 4 | 4 |
| 0 | 1 | 0 | 1 | 5 | 5 |
| 0 | 1 | 1 | 0 | 6 | 6 |
| 0 | 1 | 1 | 1 | 7 | 7 |
| 1 | 0 | 0 | 0 | 8 | 8 |
| 1 | 0 | 0 | 1 | 9 | 9 |
| 1 | 0 | 1 | 0 | 10 | A |
| 1 | 0 | 1 | 1 | 11 | b |
| 1 | 1 | 0 | 0 | 12 | C |
| 1 | 1 | 0 | 1 | 13 | d |
| 1 | 1 | 1 | 0 | 14 | E |
| 1 | 1 | 1 | 1 | 15 | F |

# Convert Hexadecimal to Binary

- Each digit is converted to 4 bits in binary
- Arrange the groups of 4 bits in the same order
- Example: convert $(3F7)_{16}$ to binary:

$$(3\ F\ 7)_{16}$$

$$(\ \mathbf{0011}\quad \mathbf{1111}\quad \mathbf{0111}\ )_2$$

- Drop the initial 0's to simplify

$(3F7)_{16}$ = $(11\ 1111\ 0111)_2$

| Binary | Decimal | Hexaecimal |
|---|---|---|
| 0 0 0 0 | 0 | 0 |
| 0 0 0 1 | 1 | 1 |
| 0 0 1 0 | 2 | 2 |
| 0 0 1 1 | 3 | 3 |
| 0 1 0 0 | 4 | 4 |
| 0 1 0 1 | 5 | 5 |
| 0 1 1 0 | 6 | 6 |
| 0 1 1 1 | 7 | 7 |
| 1 0 0 0 | 8 | 8 |
| 1 0 0 1 | 9 | 9 |
| 1 0 1 0 | 10 | A |
| 1 0 1 1 | 11 | b |
| 1 1 0 0 | 12 | C |
| 1 1 0 1 | 13 | d |
| 1 1 1 0 | 14 | E |
| 1 1 1 1 | 15 | F |

# Octal System

- The Octal number system is a base 8 (modulo 8) number system:
  - 8 digits: 0 1 2 3 4 5 6 7
- Each position has a decimal weight in power of 8
- Each octal digital corresponds to a 3-bit binary number

$$(3\ 1\ 7)_8$$

$$(\ \mathbf{011}\quad \mathbf{001}\quad \mathbf{111}\ )_2$$

# Convert Base-M System to Base-N System

- Decimal can always be used as the intermediate number system
- Generally, the rule “divide/multiply by the base of destination system” applies to all the number system conversions
  - Example, to convert a Hex number to base-3 number, just divide the Hex number by 3

# Binary Arithmetic

- Example:

```
carry   11110111                 borrow  11000010
         10110101                          10110101
      +  10110011                        - 11010011
-------------------------        ------------------
        110001000                          11100010
```

# Hexadecimal Arithmetic

- Example:

```
carry     111                              borrow    11
          8F5A                                      8F5A
        + 11BC                                    - 11BC
-----------------------                   ----------------
          A116                                      7D9E
```

# How Do We Encode Text with Binary Bits

- A popular code: ASCII (American Standard Code for Information Interchange)
  - 7- (or 8-) bit encoding of each letter, number, or symbol

| Symbol | Encoding |
|---|---|
| R | 1010010 |
| S | 1010011 |
| T | 1010100 |
| L | 1001100 |
| N | 1001110 |
| E | 1000101 |
| 0 | 0110000 |
| . | 0101110 |
| <tab> | 0001001 |

| Symbol | Encoding |
|---|---|
| r | 1110010 |
| s | 1110011 |
| t | 1110100 |
| l | 1101100 |
| n | 1101110 |
| e | 1100101 |
| 9 | 0111001 |
| ! | 0100001 |
| <space> | 0100000 |

- Unicode: Increasingly popular 16-bit encoding
  - Encodes characters from various world languages

Question:
What does this ASCII bit sequence represent?

1010010 1000101 1010011 1010100

R E S T

# ASCII Coding Chart

| Decimal | Binary | Octal | Hex | ASCII | Decimal | Binary | Octal | Hex | ASCII |
|---|---|---|---|---|---|---|---|---|---|
| 64 | 01000000 | 100 | 40 | @ | 96 | 01100000 | 140 | 60 | ` |
| 65 | 01000001 | 101 | 41 | A | 97 | 01100001 | 141 | 61 | a |
| 66 | 01000010 | 102 | 42 | B | 98 | 01100010 | 142 | 62 | b |
| 67 | 01000011 | 103 | 43 | C | 99 | 01100011 | 143 | 63 | c |
| 68 | 01000100 | 104 | 44 | D | 100 | 01100100 | 144 | 64 | d |
| 69 | 01000101 | 105 | 45 | E | 101 | 01100101 | 145 | 65 | e |
| 70 | 01000110 | 106 | 46 | F | 102 | 01100110 | 146 | 66 | f |
| 71 | 01000111 | 107 | 47 | G | 103 | 01100111 | 147 | 67 | g |
| 72 | 01001000 | 110 | 48 | H | 104 | 01101000 | 150 | 68 | h |
| 73 | 01001001 | 111 | 49 | I | 105 | 01101001 | 151 | 69 | i |
| 74 | 01001010 | 112 | 4A | J | 106 | 01101010 | 152 | 6A | j |
| 75 | 01001011 | 113 | 4B | K | 107 | 01101011 | 153 | 6B | k |
| 76 | 01001100 | 114 | 4C | L | 108 | 01101100 | 154 | 6C | l |
| 77 | 01001101 | 115 | 4D | M | 109 | 01101101 | 155 | 6D | m |

⋮

# Signed Binary Numbers

- To represent negative numbers
  - Cannot use minus sign: binary systems work with only two values, 0 and 1
  - The left-most bit of a binary number represents the sign of a number – sign bit
    - Sign bit 0 indicates positive numbers
    - Sign bit 1 indicates negative number

# Representation of Negative Numbers

- Negative numbers are represented by
  - Sign and magnitude
  - 1’s complement code
  - 2’s complement code
- Sign and magnitude
  - MSB is the sign bit: 0 → positive, 1 → negative
- 1’s complement representation of –N
  - Negation of every bit of N
  - Example, 1’s complement representation of -3
    - N = 3 = 0011
    - -N = -3 = 1100
- 2’s complement representation of –N is
  - Negation of every bit of N, then plus 1
  - Example, 2’s complement representation of -3
    - N = 3 = 0011
    - -N = -3 = 1100 + 1 = 1101

# Signed 2's Complement Number

- **Signed numbers are represented as 2's complement numbers in computers**
- **Recognize a signed 2's complement number**
  - Sign bit = 0, positive number, recognize as a regular binary number
    - **0**101 = +5;
  - Sign bit = 1, negative number, the magnitude of the number is obtained by 2's complement operation
    - **1**011
      - Sign: negative number
      - Magnitude: 2's complete operation of (1011) = 0100+1 = 0101 = 5
      - So 1011 = –5

# Ranges of Signed 2's Complement Number

- In general the 2's complement values range from $-2^{n-1}$ to $2^{n-1}-1$
- For n = 4, the 2's complement values range from –8 to 7
- For n = 8, the 2's complement values range from –128 to 127
- For n = 16, the 2's complement values range from $-2^{15}$ to $2^{15}-1$

- Overflow
  - If an n-bit 2's complement number is greater than $2^{n-1}-1$ or less than $-2^{n-1}$, we say there is an overflow

# Detecting Overflow: Method 1

- Overflow detection logic
  - Two numbers' sign bits are the same but are different from the result's sign bit
  - If the two numbers' sign bits are different, overflow is impossible
    - Adding a positive and negative can't exceed largest magnitude positive or negative
- 4-bit example

sign bits

| | | |
|---|---|---|
| 0 1 1 1 | 1 1 1 1 | 1 0 0 0 |
| + 0 0 0 1 | + 1 0 0 0 | + 0 1 1 1 |
| 1 0 0 0 | 0 1 1 1 | 1 1 1 1 |
| overflow | overflow | no overflow |
| (a) | (b) | (c) |

# Binary Number Subtraction

- Using two's complement representation

  A – B = A + (-B)

  = A + (two's complement of B)

  = A + invert_bits(B) + 1

- Example:

```
      11000010                 00111101
       10110101                 10110101   -75
     - 11010011               + 00101100    44
--------------------          +        1    1
       11100010    same       ----------------
                                11100010
```