In[•]:=

```
(*依次输出联络、黎曼张量、曲率张量、里奇张量、里奇标量*)
variable = {ψ, θ, ϕ};(*输入变量*)
gdown = DiagonalMatrix[{1, Sin[ψ]^2, Sin[ψ]^2 Sin[θ]^2}];(*输入度规g_μν*)
        对角矩阵
gup = Inverse[gdown];
      逆
a = Length[variable];
    长度
pd = Table[D[gdown, variable[[i]]], {i, a}];
     表格   偏导
Γ = Table[Sum[1/2 gup[[ρ, σ]] (pd[[μ, σ, ν]] + pd[[ν, μ, σ]] - pd[[σ, μ, ν]]), {σ, a}],
    表格   求和
   {ρ, a}, {μ, a}, {ν, a}] // FullSimplify;
                              完全简化
Print[Column[Table[ToString["Γ"^ToString[i]_ToString[μν], StandardForm] <>
打印  列     表格   转换为字符串                              标准格式
   "=" <> ToString[MatrixForm[Γ[[i]]], StandardForm] <> "\n", {i, a}]]];
          转换为…    矩阵格式                标准格式
pdΓ = Table[D[Γ, variable[[i]]], {i, a}];
      表格   偏导
RiemannTensor = Table[pdΓ[[μ, ρ, ν, σ]] - pdΓ[[ν, ρ, μ, σ]] +
                表格
   Sum[Γ[[ρ, μ, λ]] × Γ[[λ, ν, σ]] - Γ[[ρ, ν, λ]] × Γ[[λ, μ, σ]], {λ, a}],
   求和
   {ρ, a}, {σ, a}, {μ, a}, {ν, a}] // FullSimplify;
                                      完全简化
Print[Grid[Table[ToString["R"^ToString[i]_(" "<>ToString[j]<>ToString[μν]), StandardForm] <>
打印  格子 表格   转换为字符串                                                   标准格式
   "=" <> ToString[MatrixForm[RiemannTensor[[i, j]]], StandardForm] <> "\n",
          转换为…    矩阵格式                             标准格式
  {i, a}, {j, a}], Spacings → {2}, Alignment → Left]]
                   间隔              对齐          左
CurvatureTensor =
  TensorContract[TensorProduct[gdown, RiemannTensor], {2, 3}] // FullSimplify;
  张量缩并         张量乘积                                         完全简化
Print[Grid[Table[ToString["R"_(ToString[i]<>ToString[j]<>ToString[μν]), StandardForm] <>
打印  格子 表格   转换为字符串                                            标准格式
   "=" <> ToString[MatrixForm[CurvatureTensor[[i, j]]], StandardForm] <> "\n",
          转换为…    矩阵格式                               标准格式
  {i, a}, {j, a}], Spacings → {2}, Alignment → Left]]
                   间隔              对齐          左
RicciTensor = TensorContract[RiemannTensor, {{1, 3}}] // FullSimplify;
              张量缩并                                  完全简化
RicciScalar = Sum[gup[[i, j]] × RicciTensor[[i, j]], {i, a}, {j, a}] // FullSimplify;
              求和                                                       完全简化
Print["R_μν=", MatrixForm[RicciTensor]]
打印            矩阵格式
Print["\nR=", RicciScalar]
打印
```

$$\Gamma^{1}_{\mu\nu}=\begin{pmatrix} 0 & 0 & 0 \\ 0 & -\mathrm{Cos}[\psi]\,\mathrm{Sin}[\psi] & 0 \\ 0 & 0 & -\mathrm{Cos}[\psi]\,\mathrm{Sin}[\theta]^2\,\mathrm{Sin}[\psi] \end{pmatrix}$$

$$\Gamma^{2}_{\mu\nu}=\begin{pmatrix} 0 & \mathrm{Cot}[\psi] & 0 \\ \mathrm{Cot}[\psi] & 0 & 0 \\ 0 & 0 & -\mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta] \end{pmatrix}$$

$$\Gamma^{3}_{\mu\nu}=\begin{pmatrix} 0 & 0 & \mathrm{Cot}[\psi] \\ 0 & 0 & \mathrm{Cot}[\theta] \\ \mathrm{Cot}[\psi] & \mathrm{Cot}[\theta] & 0 \end{pmatrix}$$

$$R^{1}{}_{1\mu\nu}=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad R^{1}{}_{2\mu\nu}=\begin{pmatrix} 0 & \mathrm{Sin}[\psi]^2 & 0 \\ -\mathrm{Sin}[\psi]^2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad R^{1}{}_{3\mu\nu}=\begin{pmatrix} 0 & 0 & \mathrm{Sin}[\theta]^2\,\mathrm{Sin}[\psi]^2 \\ 0 & 0 & 0 \\ -\mathrm{Sin}[\theta]^2\,\mathrm{Sin}[\psi]^2 & 0 & 0 \end{pmatrix}$$

$$R^{2}{}_{1\mu\nu}=\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad R^{2}{}_{2\mu\nu}=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad R^{2}{}_{3\mu\nu}=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \mathrm{Sin}[\theta]^2\,\mathrm{Sin}[\psi]^2 \\ 0 & -\mathrm{Sin}[\theta]^2\,\mathrm{Sin}[\psi]^2 & 0 \end{pmatrix}$$

$$R^{3}{}_{1\mu\nu}=\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad R^{3}{}_{2\mu\nu}=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\mathrm{Sin}[\psi]^2 \\ 0 & \mathrm{Sin}[\psi]^2 & 0 \end{pmatrix} \quad R^{3}{}_{3\mu\nu}=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$R_{11\mu\nu}=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad R_{12\mu\nu}=\begin{pmatrix} 0 & \mathrm{Sin}[\psi]^2 & 0 \\ -\mathrm{Sin}[\psi]^2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad R_{13\mu}$$

$$R_{21\mu\nu}=\begin{pmatrix} 0 & -\mathrm{Sin}[\psi]^2 & 0 \\ \mathrm{Sin}[\psi]^2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad R_{22\mu\nu}=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad R_{23\mu}$$

$$R_{31\mu\nu}=\begin{pmatrix} 0 & 0 & -\mathrm{Sin}[\theta]^2\,\mathrm{Sin}[\psi]^2 \\ 0 & 0 & 0 \\ \mathrm{Sin}[\theta]^2\,\mathrm{Sin}[\psi]^2 & 0 & 0 \end{pmatrix} \quad R_{32\mu\nu}=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\mathrm{Sin}[\theta]^2\,\mathrm{Sin}[\psi]^4 \\ 0 & \mathrm{Sin}[\theta]^2\,\mathrm{Sin}[\psi]^4 & 0 \end{pmatrix} \quad R_{33\mu}$$

$$R_{\mu\nu}=\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2\,\mathrm{Sin}[\psi]^2 & 0 \\ 0 & 0 & 2\,\mathrm{Sin}[\theta]^2\,\mathrm{Sin}[\psi]^2 \end{pmatrix}$$

$R=6$