

(*以下这是《曾-钱量子力学习题剖析》的5.2题的推导*)

(*清除所有变量*) ClearAll[r, α, ρ, u, v, ħ, μ, λ, l, β, Eval]

清除全部

(*假设 α 和 β 不等于0*)

(*α=2;

β= $\frac{1-\alpha}{2\alpha}$ *)

Assuming[{α ≠ 0, ρ ≠ 0}, r = ρ^(1/α)];

假定

Assuming[{β ≠ 0, ρ ≠ 0}, u = ρ^β v[ρ]];

假定

(*计算 dr/dρ 和 dudr*)

drdρ = D[r, ρ];

偏导

dudr = D[u, ρ] / drdρ;

偏导

(*计算二阶导数*)

ddudrr = D[dudr, ρ] / drdρ;

偏导

(*定义径向方程*)

radialEquation =

(ħ² / (2 * μ)) * ddudrr + (Eval - λ * r² - (1 * (1 + 1) * (ħ²) / (2 * μ * r²))) * u;

rCoefficient = Coefficient[radialEquation, v''[ρ]];

系数

eq = Simplify[$\frac{\text{radialEquation } \hbar^2}{\text{rCoefficient } 2 \mu}$];

化简

vCoeff = Coefficient[eq, v[ρ]]

系数

Out[*]=

$$\frac{2 \text{ Eval } \mu \rho^{2/\alpha} - 2 \lambda \mu \rho^{3/\alpha} - (1 + 1^2 + \alpha \beta (1 - \alpha \beta)) \hbar^2}{2 \alpha^2 \mu \rho^2}$$

In[*]:= (*经分析α只能取1或者2才能满足径向方程的一般形式，其中分别对应各向同性谐振子势和库伦势*)

In[*]:= (*以下是以下这是《曾-钱量子力学习题剖析》的5.3题的推导*)

ClearAll[r, α, ρ, u, v, ħ, μ, λ, l, β, Eval]

[清除全部](#)

Assuming[{α ≠ 0, ρ ≠ 0}, r = ρ^(1/α)];

[假定](#)

Assuming[{β ≠ 0, ρ ≠ 0}, u = ρ^β v[ρ]];

[假定](#)

$$\beta = \frac{1 - \alpha}{2 \alpha};$$

(*计算 dr/dρ 和 dudr*)

drdρ = D[r, ρ];

[偏导](#)

dudr = D[u, ρ] / drdρ;

[偏导](#)

(*计算二阶导数*)

ddudrr = D[dudr, ρ] / drdρ;

[偏导](#)

(*定义径向方程*)

Assuming[v ≠ 0, radialEquation =

[假定](#)

$$(\hbar^2 / (2 * \mu)) * ddudrr + (Eval - \lambda * r^v - (1 * (1 + 1) * (\hbar^2) / (2 * \mu * r^2))) * u];$$

rCoefficient = Coefficient[radialEquation, v'[ρ]];

[系数](#)

$$eq = \text{Simplify}\left[\frac{\text{radialEquation} \hbar^2}{\text{rCoefficient} 2 \mu}\right];$$

[化简](#)

vCoeff = Coefficient[eq, v[ρ]]

[系数](#)

Out[*]=

$$\frac{8 \text{ Eval } \mu \rho^{2/\alpha} - 8 \lambda \mu \rho^{2/\alpha} \left(\rho^{\frac{1}{\alpha}}\right)^v + (-1 - 4 l - 4 l^2 + \alpha^2) \hbar^2}{8 \alpha^2 \mu \rho^2}$$

$$\text{In[*]:= } \frac{2}{\alpha} + \frac{v}{\alpha} - 2 == 0;$$

$$\alpha = \frac{v + 2}{2};$$

$$v_{\text{Prim}} = \frac{2}{\alpha} - 2$$

Out[*]=

$$-2 + \frac{4}{2 + v}$$