The University of Texas at Austin
Department of Electrical and Computer Engineering

EE381K: Convex Optimization — Fall 2019

PROBLEM SET 11

Due: Sunday, December 8, 2019.

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1. Solve Problem 7.7 in the Convex Optimization book (Boyd-Vandenberghe). [You don't need to prove the hints]

2. Solve Problem 8.24 in the Convex Optimization book (Boyd-Vandenberghe).

3. Solve Problem 8.25 in the Convex Optimization book (Boyd-Vandenberghe).

4. Consider a differentiable function $f : \mathbb{R}^n \to \mathbb{R}$ that is m-strongly convex. Show that for any $\mathbf{x}$ and $\mathbf{y}$ in $\mathbb{R}^n$ we have

$$(\nabla f(\mathbf{x}) - \nabla f(\mathbf{y}))^\top (\mathbf{x} - \mathbf{y}) \geq m\|\mathbf{x} - \mathbf{y}\|^2$$

5. Consider a function $f : \mathbb{R}^n \to \mathbb{R}$ that is M-smooth.

   (a) Show that for any $\mathbf{x}$ and $\mathbf{y}$ in $\mathbb{R}^n$ and $\alpha \in [0, 1]$ we have

$$f(\alpha\mathbf{x} + (1-\alpha)\mathbf{y}) \geq \alpha f(\mathbf{x}) + (1-\alpha) f(\mathbf{y}) - \frac{\alpha(1-\alpha)M}{2}\|\mathbf{x} - \mathbf{y}\|^2$$

   (b) Show that if the function f is differentiable then for any $\mathbf{x}$ and $\mathbf{y}$ in $\mathbb{R}^n$ we have

$$(\nabla f(\mathbf{x}) - \nabla f(\mathbf{y}))^\top (\mathbf{x} - \mathbf{y}) \leq M\|\mathbf{x} - \mathbf{y}\|^2$$