The University of Texas at Austin
Department of Electrical and Computer Engineering

EE381K: Convex Optimization — Fall 2019

Problem Set 5

Due: Sunday, October 13, 2019.

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1. Prove that the set $\{x : ||Ax + b||_2 \leq c^T x + d\}$ is a convex set.

2. Suppose you are given a matrix $M \in R^{n\times n}$. Prove that, or give a counter-example for each of the following.

   (a) If $M \succeq 0$, then for every i, j, $M_{ii} \geq |M_{ij}|$.

   (b) If $M_{ii} \geq |M_{ij}|$ for for alli, j, and M is symmetric, then $M \succeq 0$.

   (c) If $M = \sum_i a_i a_i^T$ where $a_i \in R^n$ are arbitrary vectors, $M \succeq 0$.

   (d) If $M = \begin{bmatrix} M_1 & M_2 \\ M_2 & M_3 \end{bmatrix} \succeq 0$, then $\begin{bmatrix} M_1 & 0 \\ 0 & M_3 \end{bmatrix} \succeq 0$ where M_1, M_3 are square block matrices.

3. Recall that $\langle A, B\rangle = Tr(A^\top B)$ for matrices A and B. Prove that for a symmetric matrix M, $M \succeq 0$ if and only if $\langle M, Z\rangle \geq 0$ for all $Z \succeq 0$.

4. Consider the set $\mathcal{C} = \{\mathbf{x} \in \mathbb{R}^n | \mathbf{x}^\top \mathbf{A}\mathbf{x} + \mathbf{b}^\top \mathbf{x} + c \leq 0\}$ with $\mathbf{A} \in \mathbf{S}^n$, $\mathbf{b} \in \mathbb{R}^n$, and $c \in \mathbb{R}$.

   (a) Show that $\mathcal{C}$ is convex if $\mathbf{A} \succeq \mathbf{0}$.

   (b) Is the converse of this statement true? (If $\mathcal{C}$ is convex, then $\mathbf{A} \succeq \mathbf{0}$)

5. Show that if $\mathcal{S}_1$ and $\mathcal{S}_2$ are convex sets in $\mathbb{R}^{m\times n}$, then so is their partial sum defined as

$$\mathcal{S} = \{(\mathbf{x}, \mathbf{y}_1 + \mathbf{y}_2) \mid \mathbf{x} \in \mathbb{R}^m,\ \mathbf{y}_1, \mathbf{y}_2 \in \mathbb{R}^n,\ (\mathbf{x}, \mathbf{y}_1) \in \mathcal{S}_1,\ (\mathbf{x}, \mathbf{y}_2) \in \mathcal{S}_2\}.$$