The University of Texas at Austin
Department of Electrical and Computer Engineering

EE381K: Convex Optimization — Fall 2019

PROBLEM SET 7

Due: Sunday, November 3, 2019.

1. Prove that the convolution of log-concave functions is log-concave. Note that convolution of f and g is defined as

$$(f * g)(x) = \int f(x - y)g(y) \ dy$$

2. Suppose p is a convex function, q is a concave function, with $p(\mathbf{x}) \geq 0$ and $q(\mathbf{x}) > 0$ on a convex set $\mathcal{C}$.

   (a) Show that the function f defined by $f(\mathbf{x}) = p(\mathbf{x})/q(\mathbf{x})$ is quasi-convex on $\mathcal{C}$.

   (b) Show that for this specific function f we can show that

   $$f(\mathbf{x}) \leq t \quad \Leftrightarrow \quad p(\mathbf{x}) - tq(\mathbf{x}) \leq 0.$$

   (c) Show that the function $\phi_t(\mathbf{x}) := p(\mathbf{x}) - tq(\mathbf{x})$ is convex and $\phi_t(\mathbf{x})$ is decreasing in t.

3. Solve Problem 3.46 in the Convex Optimization book (Boyd-Vandenberghe).

4. Solve Problem 4.9 in the Convex Optimization book (Boyd-Vandenberghe).

5. Solve Problem 4.21 in the Convex Optimization book (Boyd-Vandenberghe).