The University of Texas at Austin
Department of Electrical and Computer Engineering

EE381K: Convex Optimization — Fall 2019

PROBLEM SET I

Due: Friday, September 13, 2019.

Preliminary. Download the set of exercises, "Lieven_LP_problems.pdf" (LLP), from canvas.

1. Solve Exercise 1 in LLP.
2. Solve Exercise 2 in LLP.
3. Solve Exercise 4 in LLP.
4. Solve Exercise 6 in LLP.
5. Solve Exercise 9 in LLP.

   Note that L_1 norm and infinity norm of $x \in \mathbb{R}^n$ are defined as

   $$||x||_1 = \sum_{i=1}^{n} |x_i|, \qquad ||x||_\infty = \max_{i=1,\ldots,n} |x_i|$$

6. Solve Exercise 10 in LLP.
7. Solve Exercise 12 in LLP.
8. Show if the polyhedron $P = \{(x, y, z) \mid |x| \leq z, |y| \leq z\}$ is pointed or not.
9. Prove that the following definitions of an extreme point are equivalent.

   Definition 1: A minimal face of a pointed pointed polyhedron
   Definition 2: A point in the polyhedron that cannot be written as a convex combination of any two other points in the polyhedron.

10. Let $P = \{x : Ax \leq b\}$ be a polyhedron. Recall the definitions of extreme point, vertex and basic feasible solutions as done in class. Show that the following statements are equivalent.

    (1) v is a an extreme point

    (2) v is a vertex

    (3) v is a basic feasible solution

11. (Birkhoff's Theorem) An $n \times n$ matrix X is doubly stochastic if

    $$X_{ij} \geq 0 \text{ for all i,j}, \qquad X\mathbf{1} = \mathbf{1}, \qquad X^\top \mathbf{1} = \mathbf{1}.$$

    (a) Show that the set of doubly stochastic matrices is a pointed polyhedron in $\mathbb{R}^{n \times n}$.

    (b) Show that the extreme points are the permutation matrices. [a permutation matrix is a doubly stochastic matrix with elements 0 or 1]