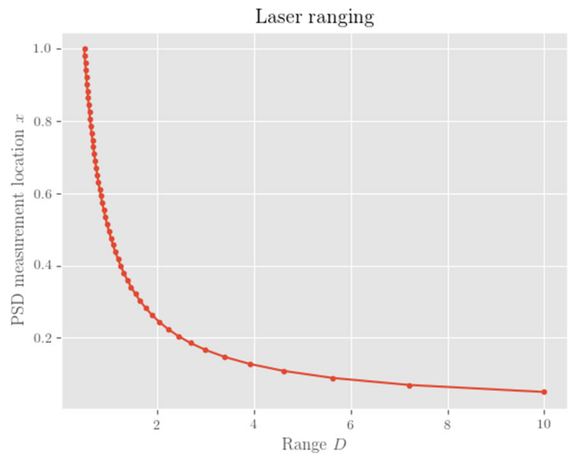
Solution Exercise 3 – Triangulation Sensor

Solution Q1:



$$D = f\frac{L}{x}$$ → See Matlab Code

For distances far away, the resolution gets very bad.

By the way, the same applies for stereo imaging.

Solution Q2:

Error propagation law:

$$Y_j = h_j(X_1 \cdots X_n)$$

$$C_Y = H_X C_X H_X^T$$

With:

C_X: covariance matrix representing the input uncertainty

C_Y: covariance matrix representing the propagated uncertainty for the output

H_X: is the Jacobian matrix defined as $H_X = \begin{bmatrix} \frac{dh_1}{dX_1} & \cdots & \frac{dh_1}{dX_n} \\ \vdots & \vdots & \vdots \\ \frac{dh_m}{dX_1} & \cdots & \frac{dh_m}{dX_n} \end{bmatrix}$

For the triangulation sensor (scaler case)

$Y_1 = h_1(X_1)$

$$Y_1 = D$$

$$h_1(X_1) = f\frac{L}{x}$$

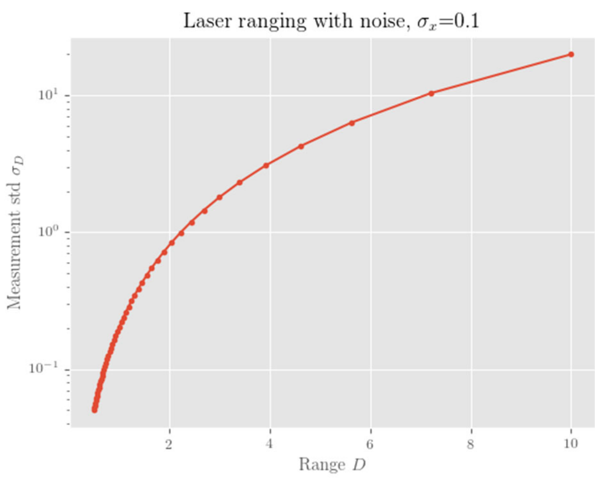
$$D = f\frac{L}{x}$$

$$C_X = \sigma_x^2 \quad ; \quad C_Y = \sigma_D^2$$

$$H_x = \frac{dh_1}{dX_1} = \frac{d}{dx} f\frac{L}{x} = -f\frac{L}{x^2}$$

$$\sigma_D^2 = C_Y = H_x C_X H_x^T = \left(-f\frac{L}{x^2}\right)\sigma_x^2\left(-f\frac{L}{x^2}\right) = \frac{f^2L^2}{x^4}\sigma_x^2$$

$$\sigma_D = \frac{fL}{x^2}\sigma_x \quad \text{and with } x = f\frac{L}{D}$$



$$\sigma_D = \frac{D^2}{fL}\sigma_x$$ See Matlab Code

For measurements far away, the uncertainty (standard deviation σ_D) becomes very big