Exercise 2 – An Omnidirectional Robot

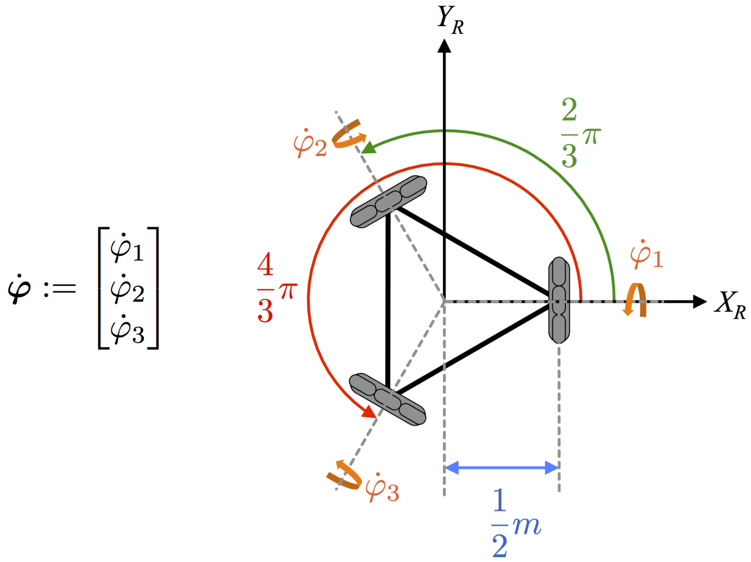

Figure 1: Omnidirectional ground robot.

Overview

In this problem set, we will derive the equations of motion for a three-wheeled robot. The content of this problem set closely follows the worked example in the online video, so it is a good idea to go through that video segment first.

This question considers the robot shown in Fig. 1. The robot has three non-steerable wheels. Each wheel is a "Swedish 90 degree wheel" as described in the first lecture segment of this section. The robot frame is placed at the center of the robot, and each wheel is 0.5 meters from the robot center. The current robot state ξ can be written as the position and orientation of the robot relative to a world frame, $\xi = \begin{bmatrix} x & y & \theta \end{bmatrix}^T$. Pay careful attention to the defined turning direction of each wheel. Making your equations match the defined turning directions will be key to success in this problem set.

Q1 The 90 Degree Swedish Wheel

First, we will consider the properties of a single wheel, and derive the equations for one of these wheels.

Consider the wheel equation for the 90 degree Swedish wheel in Fig. 2. Pay attention to the turning direction defined in the diagram. This is different than the turning direction defined in the worked example in the lecture. We will consider this wheel i, which lies on the axis X_S between the robot centre and the wheel centre. The wheel properties are defined by the angle α_i from the robot X_R axis to X_S, distance from robot centre to wheel centre l_i, and relative wheel orientation β_i between X_S and the large wheel rotation axis X_W. The radius of the wheel is r.

Recall that the Swedish wheel has one large wheel, and many small freely-spinning wheels around its circum-

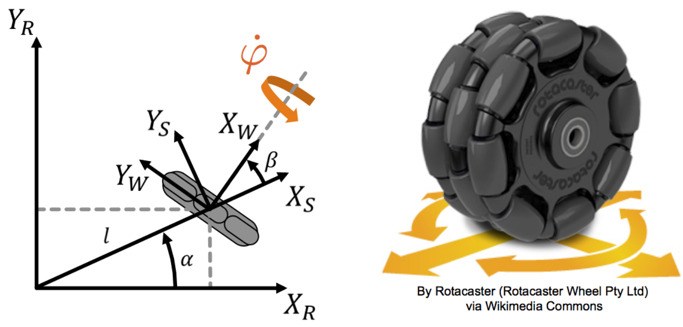

Figure 2: The 90 degree Swedish wheel.

ference. The only actuator/encoder available is the one that spins the large wheel. Movement orthogonal to the large wheel (along X_W) is free due to the rolling of the small wheels. Therefore, the no-sliding-constraint matrix, **C**, is empty and the wheel equation has the form

$$\mathbf{J}_i \dot{\xi_R} = r\dot{\varphi}_i, \tag{1}$$

where $\dot{\xi}_R = \begin{bmatrix}\dot{x} & \dot{y} & \dot{\theta}\end{bmatrix}^T$ is the robot's velocity expressed in the robot frame, r is the radius of the big wheel, and $\dot{\varphi}_i$ is the turning speed (radians per second) of the big wheel. The matrix $\mathbf{J}_i$ is 1×3 and has components $\mathbf{J}_i = \begin{bmatrix}j_{i,1} & j_{i,2} & j_{i,3}\end{bmatrix}$.

Questions:

1. How many degrees of freedom does this wheel have?
2. Write an expression for each component of $\mathbf{J}_i$ in terms of α_i, β_i and l_i.
3. (MATLAB) Write out the values and forward differential kinematic equations for this system in the associated script.

Answer:

1. The wheel has **three** degrees of freedom:
   (a) motion in the y direction of the wheel coordinate frame is explained by turning of the large wheel,
   (b) motion in the x direction of the wheel coordinate frame is explained by turning of the small wheels, and
   (c) rotation around the contact point.
2. $j_{i,1} = \sin(\alpha + \beta)$, $j_{i,2} = -\cos(\alpha + \beta)$, and $j_{i,3} = -l\cos(\beta)$.

Q2 The stacked wheel equations

In this problem, we will derive the stacked wheel equation for this robot, and then use these equations to simulate the robot motion. Consider now that we have three wheels, and that the model from the previous question, with appropriate choice of the α, β and l parameters, can be used to decribe all three wheels. The radius r of each wheel is 0.1, and the wheels are numbered counter-clockwise starting from the wheel 1 on the far right (i.e. wheel 1 is the one lying along the X_R axis). Let the wheel rotations be $\dot{\varphi} = \begin{bmatrix}\dot{\varphi}_1 & \dot{\varphi}_2 & \dot{\varphi}_3\end{bmatrix}^T$.

1. Calculate the α_i, β_i, l_i parameters for the three wheels, i.e. $\forall i \in \{1, 2, 3\}$.
2. Using the result from Q1, create the 1×3 $\mathbf{J}_i$ array for each wheel, and assemble the stacked matrix $\mathbf{J} = \begin{bmatrix}\mathbf{J}_1 \\ \mathbf{J}_2 \\ \mathbf{J}_3\end{bmatrix}$.

3. Write an expression for the relative rotation matrix $\mathbf{R}$ such that $\mathbf{J}\dot{\xi_R} = \mathbf{R}\dot{\varphi}$.
4. (MATLAB) Given the constraint equations and parameters from the last question, compute the forward differential kinematics matrix $\mathbf{F}$, such that $\dot{\xi}_R = \mathbf{F}\dot{\varphi}$. Then, use this to forward-propagate the vehicle dynamics for input wheel speeds $\dot{\varphi}(t)$.

Answer:

1. (a) $\alpha_1 = 0.0, \beta_1 = 0.0, l_1 = 0.5$
   (b) $\alpha_2 = \frac{2}{3}\pi, \beta_2 = 0.0, l_2 = 0.5$
   (c) $\alpha_3 = \frac{4}{3}\pi, \beta_3 = 0.0, l_3 = 0.5$
2. (a) $\mathbf{J}_1 = \begin{bmatrix} \sin(\alpha_1 + \beta_1) & -\cos(\alpha_1 + \beta_1) & -l_1 \cos(\beta_1) \end{bmatrix}$
   (b) $\mathbf{J}_2 = \begin{bmatrix} \sin(\alpha_2 + \beta_2) & -\cos(\alpha_2 + \beta_2) & -l_2 \cos(\beta_2) \end{bmatrix}$
   (c) $\mathbf{J}_3 = \begin{bmatrix} \sin(\alpha_3 + \beta_3) & -\cos(\alpha_3 + \beta_3) & -l_3 \cos(\beta_3) \end{bmatrix}$
3. $\mathbf{R} = r \cdot \mathbf{I}_3$
4. $\mathbf{F} = \mathbf{J}^{-1}\mathbf{R}$, See Ex2_answer.m

Q3 Driving the omnidirectional robot

(MATLAB) The associated code provides a framework for setting the wheel speeds $\dot{\varphi}$ as a function of time and then using the dynamics model to forward-propagate the dynamics and simulate the robot motion. The associated plotting script will plot the motion over time. Try changing the wheel speeds, and see what happens. What inputs would be required for:

1. Pure rotation of the robot at a constant rate $\dot{\theta}$?
2. Pure linear motion of the robot along the $\mathbf{X}_R$ axis?
3. A circular trajectory of the robot centre?

Answer:

1. Constant, equal rotation of all three wheels, $\dot{\varphi}_1 = \dot{\varphi}_2 = \dot{\varphi}_3 = -2\frac{l_1}{Tr}\pi$ for rotation period $T = \frac{2\pi}{\dot{\theta}}$.
2. $\dot{\varphi}_1 = 0$, $\dot{\varphi}_2 = -\dot{\varphi}_3 = c$.
3. One possible answer (with $\theta(t) = 0$): $\dot{\varphi}_1(t) = \cos(2\pi\frac{t}{T} - \alpha_1)$, $\dot{\varphi}_2(t) = \cos(2\pi\frac{t}{T} - \alpha_2)$, $\dot{\varphi}_3(t) = \cos(2\pi\frac{t}{T} - \alpha_3)$ (could add a constant to all rates to also rotate the system).