Overbuilding and Recession: A new Drawback of Housing Market Boom-and-Bust Cycle

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Abstract

In this paper, I unveil a novel mechanism through which a housing market boom leads to a recession following the burst of a housing market bubble. Overbuilding, characterized by increased residential construction driven by optimism or misinformation rather than sound economic foundations, crowds out physical investment during the boom due to the general equilibrium effect. The crowded-out physical investment subsequently induces a recession (or amplifies the losses and prolongs the duration of the recession) through a scarcity of physical capital. The relative intratemporal elasticity of substitution (compared to intertemporal elasticity), financial frictions, and idiosyncratic shocks can exacerbate this crowding-out effect via consumption substitution, liquidity easing, and precautionary saving. Furthermore, wealth distribution plays a crucial role in catalyzing these effects and contributes to the problem of inequality.

JEL classification: E21, E22, E30, E51, E58

Keywords: Heterogeneous Household, Consumption, Expectations, Great Recession, Business Cycle, VAR

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1 Introduction

The Great Recession in US, starting in December 2007, created the largest retrogradation since the Great Depression, which was nearly a central ago. This recession caused a sharp increase in the unemployment rate and a drop in output, consumption, and investment as discussed by Mian and Sufi (2010) and Grusky et al. (2011). The long-lasting recession from the end of 2007 to 2009 came to the end after the central bank and governement introduced unconventional monetary policy and fiscal policy. A lot of scholars have tried to understand the origin of this recession and answer the questions such as where it was born and how it spread throughout the whole economy.

Most of them agree that the housing market boom and bust agitated the financial market's collapse and caused a demand-driven recession after the collapse propagated to the real economy. After this collapse, the Great Recession persisted for a long time and someone[1] argued that the long-lasting drop could result from self-fulling and animal spirit. In addition to the animal spirit, there are other channels which scholars has proposed[2,3] to explain this long-lasting recession. People are focusing more and more on the housing market, as these channels are mainly triggered by the housing market bust and generate real effects through financial friction. The household lost a lot in the wealth of real estate, which previously acted as collateral to borrow money and smoothed their consumption, yet now infected the real economy via the demand side.[4] However, is it true that only the financial market crisis could incur such a large descending in real economy? The answer is no and other aspects of the economy also contribute to the failure in economy.

Earilier's research[5] has argued that a considerable contribution to the Great Recession stems from the investment market, with the supply-side effect accounting for nearly 40% of the recession. This impact is far from negligible and warrants careful investigation. Previous studies have sought to reconcile Hayek's theory, which posits that recessions are caused by a fundamental scarcity of resources such as physical capital or technology, and Keynes's theory, which attributes recessions to economic frictions such as capital misallocation, search-and-match costs, or liquidity traps. These researchers have argued that a lack of capital generated the Great Recession, focusing primarily on the point at which scarcity had already occurred and was treated as an exogenous factor. In contrast, this paper takes a different direction by examining

[1] Islam and Verick (2011) and Cochrane (2011) discussed this problem.

[2] Brunnermeier (2009), Ivashina and Scharfstein (2010) and Jermann and Quadrini (2012)argued that the lack of liquidity of financial institution, mostly referring to the commercial bank, helped the crisis diffuse around and induce large recession.

[3] Christiano et al. (2015) and Fisher (2015) did an extension to the liquidity trap happened in great recession and argued that the prolonged trap caused the ZLB later. Recent works such as Guerrieri and Lorenzoni (2017) and Bayer et al. (2019) focused on the heterogeneous agent model and drew the conclusion that idiosyncratic shock and distribution channel are also important to explain the lack of liquidity.

[4] Eggertsson and Krugman (2012), Mian and Sufi (2010), Mian and Sufi (2014) and Qian (2023) discussed this problem. Household extracted their equity via collateral during the boom period which increased the consumption a lot. This constructed a mirage through general equilibrium. When the bust came, people struggled against the rapid constraint tightening and led to the Great Recession.

[5] A lot of people contributed to this direction such as Justiniano et al. (2010) and Justiniano et al. (2011).

the process through which capital scarcity is created, specifically by overbuilding. Throughout this paper, *overbuilding* is defined as an increase in residential construction not supported by underlying fundamentals, essentially representing a housing market bubble. Few theoretical lenses[6] have been applied to explain how a housing market boom can absorb a significant amount of liquidity. When this boom is a bubble caused by imperfect information and misguided beliefs of households rather than changes in economic fundamentals, the available liquidity, which could otherwise support firms' investments in capital such as factories, equipment, and R&D, is instead directed to the residential sector. This results in inefficiencies when compared to a perfect information scenario. Given a constant amount of liquidity held by financial institutions, a housing market boom attracts these institutions to lean more heavily on the household sector as opposed to the firm sector. They tend to prefer lending money to households as mortgages or subordinated debt rather than lending to firms. More liquidity flowing into the housing market implies less liquidity allocated to the supply side, as long as the supply of liquidity is sticky and cannot be freely expanded. Additionally, a positive correlation between house prices and nondurable consumption suggests that investment in the nondurable production sector will decrease due to general equilibrium effects.[7] This paper first employs a simple model with detailed analytical results to explicitly elaborate on the formation of scarcity, and then uses a comprehensive heterogeneous agent model to quantitatively examine the overbuilding process. My primary contribution lies in uncovering a new mechanism, the crowding-out effect, through which overbuilding exacerbates the scarcity of physical capital and in turn, worsens the recession, whether triggered by an investment hangover (supply-side recession) or demand contraction (demand-side recession). The analysis also reveals that the relative intratemporal elasticity of substitution to intertemporal elasticity of substitution, financial frictions, idiosyncratic shocks, and wealth distribution influence the extent of crowded-out investment.

Academic attention to the relative intratemporal elasticity of substitution and non-separable utility functions has been limited, with most researchers opting for separable utility functions[8] for the sake of simplicity, yet intratemporal and intertemporal elasticity of substitution are interconnected and should not be overlooked. In particular, intratemporal elasticity of substitution plays a critical role in general equilibrium models with flexible housing supply. Suppose a model shuts down the pecuniary effect and assumes there is no collateral constraint. In this case, the only channel through which house prices could affect nondurable consumption would be the intratemporal channel. General equilibrium ensures that all wealth effects are eliminated, as

[6]except Beaudry et al. (2018), Rognlie et al. (2018) and J Caballero and Farhi (2018) recently

[7]It is easy to understand this effect as the goods market cleaning condition in non-friction model should be $Y_t = F(L_t, K_t) = C_t + I_t^{residential} + I_t^{nonresidential}$ where K_t is predetermined. For simplicity if labor is fixed such that $L_t = \bar{L}$, higher $I_t^{residential}$, together with its coordinated C_t will return a lower $I_t^{nonresidential}$ which I call *crowd-out effect.*

[8]Iacoviello (2005), Liu et al. (2019) and Greenwald (2018) used the separable utility function to analyze the problem. However because their models lack of intratemporal channel they can only put weight on other elements such as bubbles, self-fulling and multiple credit constraints to generate enough consumption response to house price. On the contrary Berger et al. (2018) and Kaplan et al. (2020) used the nonseparable utility function to discuss the housing problem and they focus on the consumption response more, which requires the intratemporal effect.

the change in wealth caused by inflation in house prices is offset by rebated profits earned from construction firms. The elimination of the wealth effect implies that the intertemporal substitution effect(for housing service) also vanishes. Concurrently, flexible housing supply guarantees that intratemporal substitution is significant enough to influence nondurable consumption. Otherwise, the housing market would have no effect if the holding of residential estate remained fixed and this finding is supported by recent empirical work.[9] As long as housing services and consumption are weakly complementary within a short period, households will also increase their consumption, in turn crowding out investment. The stronger the intratemporal substitution relative to the intertemporal substitution, the less investment is crowded out by overbuilding, because the complementarity between nondurable goods and housing services weakens as the substitution effect becomes more pronounced.

In addition to the relative intratemporal elasticity of substitution, financial friction also plays a significant role in affecting the crowd-out effect, which has been well-established in the literature.[10] If the housing supply sector does not experience any fundamental cost shocks, the supply function remains unchanged, and house prices will rise, on top of overbuilding which is driven by shifts in demand function. Consequently, the residential property market will boom, easing credit constraints. Households may then allocate a larger portion of their budget to nondurable consumption through equity extraction and this increase in nondurable consumption contributes to the crowding-out of physical capital, which intensifies the subsequent bust and recession. In other words, the greater the financial friction, the more nondurable consumption is stimulated by a housing market boom via the wealth effect. This is because more households are financially constrained in the steady state, exhibiting a larger marginal propensity to consume (MPC), and would increase their nondurable consumption through equity extraction, as demonstrated by Bhutta and Keys (2016).

Additionally, aside from the relative intratemporal substitution and liquidity easing, household heterogeneity is another factor that amplifies the crowd-out effect, working through idiosyncratic income shocks and wealth distribution. If household income cannot be fully insured and everyone must bear idiosyncratic income shocks, households will have a precautionary saving motive, resulting in a larger portion of income(both wage income and asset returns) being saved compared to the representative agent model. When income risk is countercyclical[11], overbuilding often coincides with economic booms and smaller variance in idiosyncratic shocks. Lower risk implies that households are less cautious about accumulating wealth and will spend more on nondurable consumption. Consequently, overbuilding has a stronger effect on crowding out investment, as households have a lower demand for saving income. Forbye the second-order uncertainty

[9] Khorunzhina (2021) did this significant work.

[10] Garriga and Hedlund (2020),Hurst et al. (2016),Bailey et al. (2019), Garriga et al. (2017), Gorea and Midrigin (2017) and Chen et al. (2020) contribute a lot on this strand of literature.

[11] Debortoli and Galí (2017), Acharya and Dogra (2020) and Bilbiie and Ragot (2021) analyzed this problem linked with monetary policy theoretically. Storesletten et al. (2004), Schulhofer-Wohl (2011) and Guvenen et al. (2014) analyzed the countercyclical idiosyncratic shock empirically.

channel, heterogeneous wealth holding is important because wealth distribution is heavily *right-skewed*. Those who have more disposable cash to buy a new house are the ones who hold a larger share of total wealth.[12] Therefore, those who *can* contribute the most to overbuilding are indeed the ones who *contribute* the most to the crowd-out effect at the aggregate level. On the other hand, those with the tightest budget constraints (in the steady state) have a larger marginal propensity to consume (MPC). Despite the right-skewed wealth distribution, the MPC is left-skewed (Orchard et al. (2022)). The poor households, who would spend most of their money (resulting from looser budgets) on nondurable goods, make up a large share of the population and this large inequality also amplifies the crowd-out effect in the pass-through effect (from residential assets to nondurable consumption) and at the aggregate level.

Meanwhile, overbuilding may significantly contribute to a recession through the labor market and general equilibrium. A lack of investment initially can lead to a severe recession, as the total capital available is insufficient to support production in the end. Moreover, the presence of hand-to-mouth households can exacerbate the recession due to their high marginal propensity to consume (MPC) and low labor income (as labor and capital are complementary to each other). Furthermore, since residential property serves not only as a wealth function but also as a source of utility, its durable and irreversible (high transaction cost) characteristics may render a feedback loop from underinvestment to overinvestment during the burst period.[13] As a result, the span of the recession may be extended, and the overall impact of the recession may be magnified.

This paper makes several contributions to the literature. My first contribution is establishing a new connection between the pre-recession housing market boom (overbuilding) and the recession itself. A considerable amount of research has concluded that the boom in the housing market, as well as the nondurable goods market in 2007, was more of a mirage driven by expectation and speculation, as demonstrated by Landvoigt (2017), McQuinn et al. (2021) and Kaplan et al. (2020). Other studies have argued that credit supply also played an important role, as seen in works done by Campbell and Cocco (2007), Justiniano et al. (2019), Favara and Imbs (2015), Mian and Sufi (2022) and Favilukis et al. (2017). This expansion was built on sand, without the sustainability provided by investment and R&D, and could easily collapse due to a contractionary demand shock. The panic and pessimistic expectations, or tightened credit constraints, triggered a drop in demand and an increase in precautionary saving, yet the real estate only served as an asset in collateral constraint in these studies. Moreover, lack of investment and the complementarity between capital and labor magnified the demand-driven recession related to self-fulfilling or multiple equilibrium. In other words, the boom in the housing market not only affected investment in the construction sector (Boldrin et al. (2013)) but also crowded out physical investment in other sectors, when only large companies could make extensive margin

[12] "In 2019, the top 10% of U.S. households controlled more than 70 percent of total household wealth" argued by Batty et al. (2020) and related data can be found in Distributional Financial Accounts in federal reserve web.

[13] McKay and Wieland (2019) refined this channel penetratingly and argued that this channel is important to explain the persistent ZLB and negative real interest rate after the Great Recession. This channel can also explain the low interest rate after the implementing of unconventional monetary policy, as Sterk and Tenreyro (2018) did.

investments through self-finance, as discussed in Bachmann et al. (2013) and Winberry (2016). This shortage in investment amplified the recession in general equilibrium and resulted in high unemployment and low production. Some literature also uses the term “crowd-out” to describe the investment trade-off between housing and non-housing sectors (labor, physical asset and intangible asset, etc.), such as Dong et al. (2022) and Dong et al. (2023). However their concept of “crowd-out” is closer to portfolio adjustment in firms’ balance sheets and operates in partial equilibrium, which is far from the reality[14], as neither most of the residential asset is held by enterprises nor are residential assets of crucial importance in production activity. They just replaced one type of asset in asset misallocation literature of firms’ problem by residential asset cursorily.

In addition to explaining the reasons behind the Great Recession, this mechanism can also account for part of the policy failure, such as the Home Affordable Modification Program (HAMP), which is discussed by Mitman (2016) and Antunes et al. (2020). In this sense, this paper also holds a place in the literature related to long-lasting recessions and the ZLB. Because the recession is fueled by both supply and demand sides, the one-dimensional stimulus in the demand sector is not strong or effective enough to curb the declining economy. On top of that, both of these studies do not consider the supply of housing services, and their models are overly simplistic, even though Khan and Thomas (2008) have shown that a general equilibrium setting would generate entirely different results. My work extends the findings of Chodorow-Reich et al. (2021), Chahrour and Gaballo (2021) and Beaudry et al. (2018), while the former two primarily explained the causes of the Great Recession and real estate issues through over-optimism, the latter focused on labor market frictions and multiple equilibrium. The financial institutions in both sets of studies functioned as brokers who only provided liquidity to households and helped clear the bonds market, thus limiting the role of the housing market to a liquidity trap and demand-driven recession. In contrast, my work focuses on the investment in the nondurable sector and argues that overbuilding exacerbated the crowd-out effect and fueled a deeper recession. The closest literature to my paper is Rognlie et al. (2018), who used their partial (in financial market) model to explain the investment hangover via higher real interest rates. They argued that, given overbuilding at time zero, a high real interest rate results in a demand-driven recession due to nominal rigidity and the zero lower bound in monetary policy. However, the real interest rate and demand contraction is not the only reason for the recession, and even in the absence of nominal rigidity, overbuilding can also contribute to a supply-driven recession with significant welfare loss.

Furthermore, I also contribute to improving the numerical solution method for tackling the complicated heterogeneous agent model. Previous literature either uses a guess-and-verify method, such as Lorenzoni (2009) and Barsky and Sims (2012), or a reconstruction method, such as Baxter et al. (2011), Blanchard et al. (2013) and Hürtgen (2014) to solve the imperfect

[14] Kaplan et al. (2014) shows that “Housing equity forms the majority of illiquid wealth for households in every country with the exception of Germany”.

information DSGE model. The latter requires specific analytical equations regulating the unobserved state variable with other state variables, which is impossible to derive from a heterogeneous agent model that contains too many state variables.

Numerous studies emphasize the importance of household heterogeneity in explaining the housing boom-and-bust cycle, either empirically, such as Etheridge (2019), Mian et al. (2013), Li et al. (2016) and Díaz and Luengo-Prado (2010), or theoretically, such as Kaplan et al. (2020), Favilukis et al. (2017) and Garriga and Hedlund (2020). However, there are hardly any papers that incorporate heterogeneity in capital holding, housing, and income with information, animal spirits, learning, and anticipated shocks. This paper builds a model that demonstrates the distribution of wealth and income is pivotal in determining the strength of overbuilding and supplements the literature on how expectations and animal spirits can fuel a boom. Simultaneously, imperfect information and a slow learning process will inflate the bubble further.

In section 2 I use two identification strategy separately lay out the crowd-out effect generated by a contemporaneous and news shock to housing price. Later in section 3 I analytically demonstrate the crowd-out effect is driven by relative intratemporal elasticity of substitution, financial friction, income inequality and wealth distribution. In section 4 I quantitatively investigate the drawback of crowd-out effect spawned by a fake news shock through the lens of a full fledged heterogeneous agent model. In the last section I conclude the result.

2 Empirical evidence

Firstly we take a glance at the statistic property of the data which conceals the mechanism we want to argue in this paper. Figure 1 shows the amount of nonresidential investment (valued as the share of GDP) from 1960 to 2016. We can see that the total investment gradually increased when economy boomed and dropped when economy busted. Meanwhile the peak has the raised trend not only because the advanced technology required more physical mechanism but also because the increased wage cost required more mechanical equipment to replace physical labor. However, the increased trend starting at 2003 was broken by the great recession happened at the end of 2007. The trend is, even though looks same as before, different with what happened at 1970s and 1990s, because of investment hangover.

The increased trend in 1970s, from 1975 to 1981, created 25.6% increasing from 11.7% of GDP in 1975 to 14.7% of GDP in 1981. It is 4.27% each year on average . The trend in 1990s, from 1992 to 2000 created 3.53% increasing in investment each year on average. The latest trend, from 2010 to 2014, created 4.05% increasing each year on average. While the trend before the great recession, from 2003 to 2007, only created 2.94% increasing each year on average, which is summarized by table 1.

Enlightened by the statistic difference, there must be some reason that caused this distinct

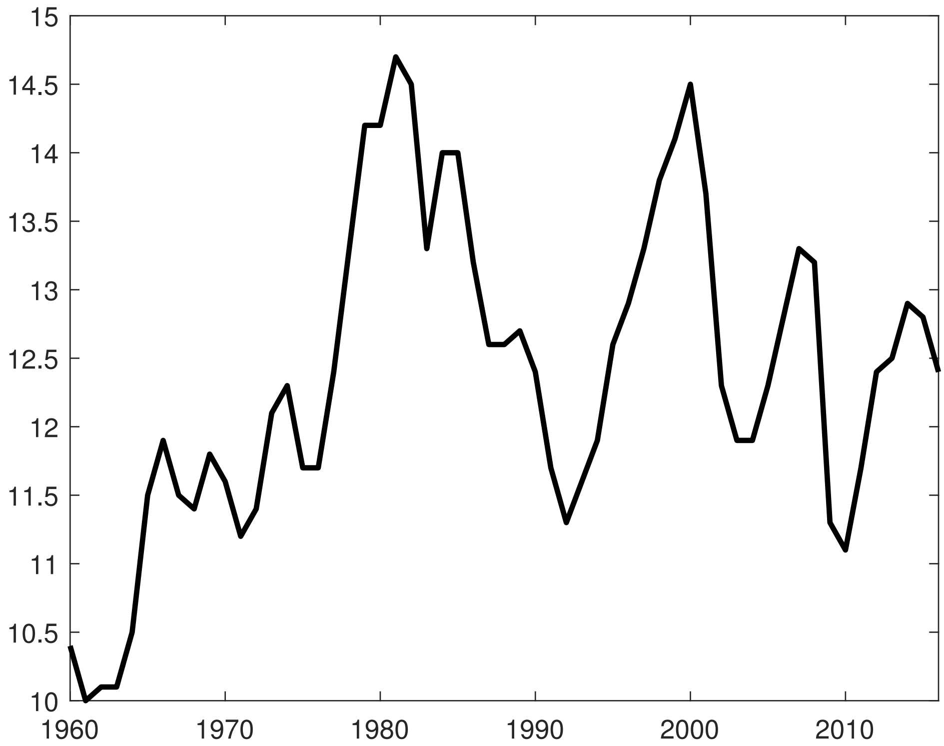


Figure 1: Nonresidential Investment (share of GDP)

Table 1: Extent of increased investment

Trend range	1975-1981	1992-2000	2003-2007	2010-2014
Increased investment	4.27%	3.53%	2.94%	4.05%

drop in investment before great recession and contributed part of the output loss during recession. The house market boom, at least under our investigation, takes some responsible for the investment drop. It is the house market boom that crowed out some investment at the demand sector. Financial institution may put more weight on household and preferred lending money to household for real estates to lending to companies for investment. On the other hand, household may prefer spending more money or liquidity on durable goods to saving at the bank who together with companies can in the end transfer these liquidity to investment and physical capital. Because of the long-lasting property of durable goods and precautionary motivate, household would like to occupy first when the goods price is increasing or has propensity to increase, like what happened from 2005 to 2007. This helps crowed out some part of the investment at the supply side. In summary both the demand and supply side help to elbow out investment and general equilibrium helps amplify this effect. The importance of general equilibrium which helps explain investment activities is widely accepted as Khan and Thomas (2008) proved previous partial analysis such as Caballero et al. (1995) maybe misleading. As given $Y = C_{nd} + I + C_d$, an increased C_{nd} and C_d will have effect on I since Y is concave at predetermined capital and labor which cannot increased too much as it is complementary to capital. After detrending the growth elements in per capita real GDP, real nonresidential investment and new construction housing units the data shows that there is a significant negative correlation between the relative physical investment and residential estate investment.[15] In this sense this paper can also be seen as a complement to Berger and Vavra (2015).

2.1 Contemporaneous real price shock

Figure 14 sheds light on the crowd-out effect crated by house market boom. However since the IRF goes back to steady-state so quickly that may not generate low investment enough, the crowd-out effect may not be important. Additionally the identification method we used, Sims et al. (1986), has been criticized that it is too strong to identify and sometimes could be artificially unreliable. Because of these problem I use another identification method, cholesky decomposition to identify effect of contemporaneous house price shock to other variables. Following Bernanke and Mihov (1998), Cholesky decomposition ensures that the shock can only take its effect on the variable after itself in order. The variable before itself in order will not be influenced directly by this shock. Therefore we put the house price in the last to how the other elements within economy response to the change in house price which is increased by an exogenous shock. Inspired by the literature I identify the house price effect by the model

$$Y_t = [y_t, c_t, i_t, cpi_t, r_t, p_t^a, hs_t, md_t, p_t^h]' \tag{1}$$

where y_t is real GDP; c_t is real non-durable consumption; i_t is real investment in non-residential sector; cpi_t is consumer price index without residential market; r_t is the nominal interest rate; p_t^a is the real capital price which we use stock market index as a proxy; hs_t is the house supply; md_t is the total amount of mortgage debt in real value; p_t^h is the real house price. I pick the time interval between 1987Q1 and 2007Q2. The left boundary is decided by the dataset I used, S&P/Case-Shiller U.S. National Home Price Index, whose earliest record was on the Q1 of 1987. I choose the right boundary because I want to investigate the overbuilding and crowd-out investment which occurred before the Great Recession. Meanwhile the tremendous drop in economy resulted in persistent abidance close to ZLB which let the data after Great Recession hardly unveil the mechanism. All the variable are in logarithm form. They are detrended by taking first order difference. All of them are placed in Y_t's order and an exogenous shock arrived at the end of vector will stimulate a small jump of house price. Then this price evokes other variables' movement following the intrinsic relationship and mechanism.

Figure 2 tells us the impulses response of different variables to one unit house price shock with 90% confidence band. We can see that a small, only one unit, jump in house price p_t^h at period 0, agitates a large over 9% climb 20 quarters later. People without house asset before are

[15] The relative correlation between relative physical investment and residential estate investment, $\text{corr}(\frac{I_{t,c}}{y_{t,c}}, \frac{I_{t,c}^H}{y_{t,c}})$ is -0.873 and $\text{corr}(\frac{I_t}{y_t}, \frac{I_t^H}{y_t}) = -0.17764$.(The subscript c denotes the cyclical data detrended from HP filter)

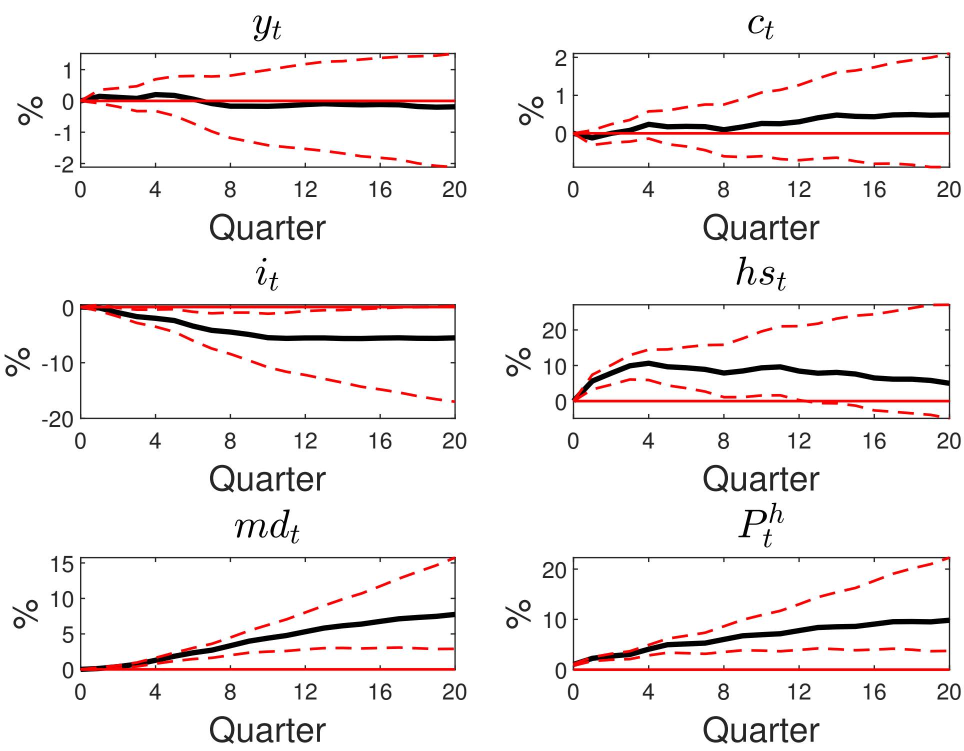


Figure 2: IRF to one unit house price jump

eager and optimistic to buy house as the price has been increased and it can either provide utility or relax the budget constraint. People who already holding the house will use the increased house price to extracting equity and free their liquidity. These two forces push up the mortgage debt up to 7% larger than before. This rapid expended credit stimulate the economy and builds a prosperity in mirror. Real output increases and sustains nearly two years. At the same time consumption of non-durable goods increases even higher than the real output and persists over the same time. This uncovers the crowd-out effect clearly. The investment in non-durable sector declines throughout the whole period and becomes stable after 3 years around 6% annualized. It shows that the crowd-out effect is strong and sensitive to the house-price stimulation as one unit ascending in house price generates 6% descending in investment. This over-reaction indicates that there are underground rivers passing and magnifying the flow from house price to investment. We can observe that the house price coordinates with increased house supply on nearly the same degree around 10%. This verifies two key argument we discussed before: overbuilding and crowd-out effect. The same extent of volatility shields light on the supply function that supply function in residential sector is not fully inelastic, as large amount of scholars assumed. As I discussed before and supported by VAR here, this elastic supply function, along with the overbuilding magnifies the corwd-out effect via general equilibrium.

2.2 Real price news shock

Although I successfully identify the house market boom, overbuilding and crowd-out effect in previous section, there is still an important question left: where does this "contemporaneous real price shock" come from? Even though I empirically test that an exogenous house price could trigger the economy move as the overbuilding and crowd-out effect predicted, the doubt about the reality of this shock arises naturally. It could be the truth that the mechanism I proposed along this paper is correct yet is not the reality that happened during the Great Recession since the source the house market boom before the recession is not simply caused by the exogenous real price shock. There is something other than price that induce the boom such as optimistic expectation, credit supply, secular decline in interest rates. Therefore in this section I identify the news shock using a sVAR model. Given the news of house price inflation in the future, what would other elements in economy response to this expectation shock. Following the method proposed by Barsky and Sims (2011) with some minor adjustment, I define the news shock as the component which can explain future forecast error but at the same time do not have contemporaneous effect.

Alternatively I use another detrend method to process the data because the variables Barsky and Sims (2011) used in their model and identification method are level data instead of what I used in previous section. I get the population of us Q_t and divide all aggregate variable by the population which returns the per capita GDP, nondurable consumption, nonresidential investment, house supply and mortgage debt. Then I use these data to do the same estimation process that I described above.

I define the "news" vector

$$R = [r_1, r_2, ..., r_{N-1}, r_N]'$$

where r_i is the unknown parameters which need to be estimated. It measures the effect of house-price-change news. It is constraint by two condition. The first one is $RR' = 1$ since the identification of shocks(estimation errors) should be orthogonal with each other. The second one is that the last row of matrix $\hat{P}R$ should be zero. The reason is that the news of house price inflation in the future should not have contemporaneous effect to house price itself.

Following the same sVAR model (equation 1) above and the same notation in equation 23 and 25, I estimate and identify the model by Cholesky decomposition to construct the estimated $\hat{\Phi}$ and $\hat{P}$.

Meanwhile I define the forecast error along the horizontal up to time h as

$$\text{fevd}_{n,h} = \frac{e_n' \left(\sum_{\tau=0}^{h} \hat{\Phi}^\tau \hat{P} R R' \hat{P}' \hat{\Phi}'^\tau \right) e_n}{e_n' \left(\sum_{\tau=0}^{h} \hat{\Phi}^\tau \hat{P} \hat{P}' \hat{\Phi}'^\tau \right) e_n}$$

where $\boldsymbol{e}_i$ is the selection vector.

Respectively the total forecast error from 0 to period H should be

$$\text{fevd}_n = \sum_{h=0}^{H} \text{fevd}_{n,h}$$

where $H = 8$

To estimate the parameters r_i, I should solve the problem that

$$R^* = \text{argmax} \sum_{h=0}^{H} \text{fevd}_{N,h} = \text{argmax} \sum_{h=0}^{H} \frac{e_n' \left(\sum_{\tau=0}^{h} \hat{\Phi}^\tau \hat{P} R R' \hat{P}' \hat{\Phi}'^\tau \right) e_n}{e_n' \left(\sum_{\tau=0}^{h} \hat{\Phi}^\tau \hat{P} \hat{P}' \hat{\Phi}'^\tau \right) e_n} \tag{2}$$

s.t

$$R'R = 1$$

Proposition 1. *The identification to a news shock R^* through equation 2 is unique up to sign.*

After estimating the effect of new shock $\hat{R}$ I can calculate the IRF based on the formula

$$\text{IRF}_t = \hat{\Phi}^t \hat{P} \hat{R}$$

I further do the sign restriction on the vector IRF_t based on the rule

$$\text{IRF}_t^{sign} = \begin{cases} \text{IRF}_t & \text{if } e_N' \text{IRF}_t e_N \geq 0 \\ -\text{IRF}_t & \text{if } e_N' \text{IRF}_t e_N < 0 \end{cases}$$

Proposition 2. *The identification to a news shock R^* through equation 2 is unique to covariance of the residual $\Omega = PP'$ from VAR's DGP 23.*

Proof. Give the covariance matrix of the residual from the DGP 23, the Cholesky P is unique to the covariance matrix Ω. Following Rubio-Ramirez et al. (2010), we know that any identification to the DGP is unique to PQ where Q is an orthogonal matrix. To identify the news shock I solve the maximization problem 2 to get the news shock R^* and the impulse response of news shock is PR^*. However for any different response matrix PQ, the identified news shock $\widetilde{R}^*$ must satisfy $\widetilde{R}^* = Q'R$ because of equation 2 and the impulse response of news shock is same to the Cholesky identification PR^*. $\square$

Proposition 2 is intuitive as news or information is neutral to the fundamental and people will response to it according to their own perception or belief about the reliability of the news. Whether the news is fake or true can only be known after the fundamental shock is realized and observed by agents in economy, several periods later. Therefore the response to news at the beginning, time 0, is unique to the covariance matrix and whether the news is fake or true and the corresponding responses cannot be identified.

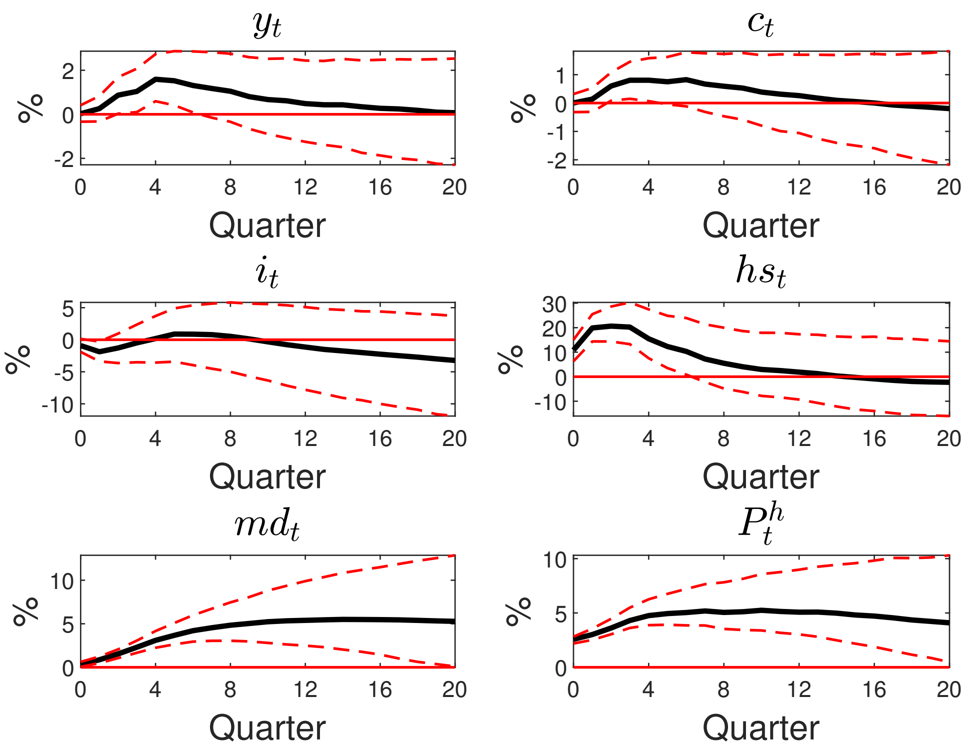


Figure 3: IRF to one unit housing price news shock at 90% confidence band

Figure 3 is the IRF of one unit of news shock which tells that the house price would increase in the future. Similarly I return the one standard deviation[16] band which shows that the crowd-out effect is significant. The response of house price is close to what in the contemporaneous shock. House price climbs gradually from origin up to 9% where the house price stays under contemporaneous shock. This firmly prove that identification of news shock works well and all the result is reliable and transparent. Furthermore it demonstrates that house market is sensitive and fragile during the pre-recession period. The house market could be triggered to boom only by the expectation and reaches to a high peak in the end, without any hesitation or drop. This house market boom also coordinate with overbuilding happened. What's more, the extent of overbuilding is even higher than that under contemporaneous shock and arrives at nearby 15%. The large amount of overbuilding verifies the novel that optimistic expectation on house price could result in a house market boom and overbuilding. In addition to house market boom and overbuilding, there also exists a consumption boom which in the end causes the investment crowd-out via general equilibrium. The consumption pertain to expectation and news shock is ten times larger than that pertain to contemporaneous shock. This may result from the illusory flourish, optimistic future condition and PIH as household feel richer than before. It is noticeable that consumption firstly dropped a litter with fluctuation but increased monotonically a lot later.

[16] I also return the confidence band under 90% interval which also significantly shows the crowd-out effect. I degrade it in the appendix.

The drop could be explained by the estimation error or illiquid mortgage debt market. Household could not adjust their mortgage debt quickly and freely. Thus they may choose to save money at first for down payment or adjustment cost. Even though there are frictions in the financial market, household still borrow a lot and push up the mortgage debt variation to 7%. It opens the veil to us that financial market and financial institution is also important. It is them that may worked as fuel which was ignited by expectation and burned up the boom later. Same as before, a lot of investment is crowded out during the house market boom, overbuilding and consumption boom. Comparing to the response to contemporaneous shock, investment is not crowded a lot while still drop over 4% after a year. This is reasonable as the boom starts at expectation without fundamental support. People would learn the truth in the end which revealed by the little crowd-out retrieval. Dropped investment goes back nearly half and stays around -2% after 4 years as people need time to learn and update their information.

2.3 Real price fake news shock

Due to proposition 2, canonical identification strategies such as sign restriction(Uhlig (2005)) and short-run restriction(Sims (1980) and Basu et al. (2006)) are insufficient. In response, I propose the "set of shock" restriction, where not one unique shock but a set of shocks represents the fake news and true news about future housing prices.

In section 2.2 I introduced the news shock R^*, which is the shock that can most explain the expectation error H periods later from period 0. However it is independent of the shock's status and cannot provide any information about whether the shock is fake or true (because it is identified based on expectation error without any proxy to the "fundamental situation"). In spite of the neutrality of the news shock R^* and our inability to directly identify the fake news before its realization, I introduce a "set of shock" as a new tool that can be used to distinguish fake news and true news (as well as the intermediate). Before introducing the "set shock", which helps me to separately identify fake news, I first add two assumptions with micro foundations as the cornerstones of set shock.

Assumption 1. *Denote the response to fake news with perfect information as $\overline{R}_1$ and the response to true news with perfect information as $\overline{R}_2$. The response to news with imperfect information, R^*, is a linear combination of fake news response $\overline{R}_1$ and true news response $\overline{R}_2$.*

This assumption is not overly restrictive, as I do not impose any restriction on the norms of $\overline{R}_1$ and $\overline{R}_2$. Further, there is no restrictions related to the coefficients of the combination, and the news shock R^* does not need to lie within the interval between true news and fake news. It is not limited to being mild, compared with true news response $\overline{R}_2$. In other words, the bubble or overreaction could exist, and the coefficient in front of $\overline{R}_2$ could be larger than 1. If the model behind is nonlinear, the assumption of linear combination will be strong because agents make decisions nonlinearly, and their response to news R^* will be a nonlinear combination of $\overline{R}_1$ and

$\overline{R}_2$. However the DGP of VAR, as shown in equation 23, is linear to shock u_t, so the assumption of linear combination is not too strong to my VAR identification scheme.

Assumption 2. *The response to fake news with perfect information is zero.*

This assumption is reasonable because, as long as the agents know the future perfectly, they will not respond to any shock without real effect. Later in section 4.3.1 I use a model to quantitatively show that it is optimal for households to only respond to the "real" shocks. To further simplify the problem and based on proposition 2, I assume the news shock is a scalar (thus, based on equation 2 and proposition 2 R^* is the corresponding column of the response matrix), which follows

$$\widetilde{w}_{t-\tau} = w_{t-\tau} + \nu_{t-\tau} \tag{3}$$

where $w_{t-\tau}$ is the true news shock observed by agents τ periods ahead as the observation $\widetilde{w}_{t-\tau}$ and $\nu_{t-\tau}$ is the noise or fake news shock. These two shocks are independent with each other and follow

$$\begin{bmatrix} \nu_t \\ w_t \end{bmatrix} \overset{iid}{\sim} \mathcal{N}\left(0, \begin{bmatrix} \sigma_\nu^2 & 0 \\ 0 & \sigma_w^2 \end{bmatrix}\right)$$

It is easy to demonstrate that this "noisy shock" setting satisfies the two assumptions above, at least under rational expectation, and I provide the basic process of proof below. Under the rational expectation assumption, household will update their belief about the true news shock using Bayesian rule and Kalman filter. Denote $\widetilde{w}_t^i$ as an observation to shock w_{t-i}. For example, a news shock w_t will have effect on G at $t+\tau$. At time $t+1$ household gets a new observation related to w_t, $\widetilde{w}_{t+1}^1$, in addition to the old observation of w_t at time t $\widetilde{w}_t$. I further assume

$$\widetilde{w}_{t-\tau+1}^1 = \widetilde{w}_{t-\tau+2}^2 = \cdots = \widetilde{w}_{t-1}^{\tau-1} = 0$$

holds. There is no further information between $t-\tau$ and t, and thus the agents do not learn and update their belief between time $t-\tau$ and t. This results in the relationship $w_{t-\tau|t-\tau} = w_{t-\tau|t-\tau+1} \cdots = w_{t-\tau|t-1} = \kappa \widetilde{w}_{t-\tau}$ where κ is the stationary Kalman gain.[17] Following Hamilton (2020), I can write the stationary Kalman gain as $\kappa = \sigma_w^2 \left(\sigma_\nu^2 + \sigma_w^2\right)^{-1}$ in one dimension case. When the household has perfect foresight or nearly perfect foresight, the variance of noise $\sigma_\nu^2 = 0$ must hold, as most of the variation in the observation $\widetilde{w}_{t-\tau}$ now comes from the true news $w_{t-\tau}$, and the noise is almost stuck at its mean, 0, without any variation. Therefore, the Kalman gain is now one ($\kappa = 1$), and households have faith in what they observe. Contrarily, when the household knows that most of their observations come from the noise and the news are just fake every time, the variance of noise $\sigma_\nu^2 \to \infty$ will hold as most of the variation in the observation $\widetilde{w}_{t-\tau}$ now comes from the noise $\nu_{t-\tau}$. In this scenario, the Kalman gain is zero

[17] The non-learning process is purely an assumption to provide the micro foundation of the two assumptions aforementioned. During the empirical identification I cannot distinguish whether the agents get extra information or not, after they were first announced by the news because all the update processes hide under the lagged coefficient Φ in equation 23.

($\kappa = 0$), and the household's belief to the true news shock is zero, leading them not to response to any observation.

Now I define the set $\mathbb{U} = \{R^*, R^*_{f,\tau}, \alpha, g(\cdot)\}$ as a set shock that represents the fake news. R^* in set $\mathbb{U}$ denotes the news shock I identified from problem 2 in the previous section. If the distribution news shock ν and w are independent and identical to each other, the linear combination will be a scalar in front of $\overline{R}_2$, which I denote as α. In this sense, α is the coefficient that agents use to update their beliefs about the news. Moreover, because of assumption 1 and assumption 2, it is easy to deduce that $R^* = \alpha\overline{R}_2$, which can be used to identify $\overline{R}_2$ as long as we know the coefficient that agents use to update their belief.

$R^*_{f,\tau}$ is the fake shock realization at time τ when the household expects that the shock announced by news would materialize. This type of "fake news" is the setting following Christiano et al. (2008), Schmitt-Grohé and Uribe (2012), Barsky et al. (2015) and Sims (2016) in which household gets a news about a shock ν_τ realized at time τ which is true for sure. However after the household reaches at time τ there is an identical negative unexpected shock $-\nu$ just offsetting the effect of positive shock ν_τ. Comparing to the setting in equation 3, in which household gets a news about ν_τ via ϵ (and totally believe it) but is misled because the observation ϵ is generated by noise w, Anderson and Moore (2012) and Chahrour and Jurado (2018) shows that this type of "fake news" shock is *observational equivalent.*[18] To theoretically formulate this type of fake news shock, we can consider the shock series $\epsilon_t = \nu_{0,t} + \nu_{1,t-\tau}$ where $\nu_{0,t}$ and $\nu_{1,t-\tau}$ are iid over time and follow

$$\begin{bmatrix} \nu_{0,t} \\ \nu_{1,t} \end{bmatrix} \overset{iid}{\sim} \mathcal{N}\left(0, \begin{bmatrix} \sigma^2_{\nu,0} & 0 \\ 0 & \sigma^2_{\nu,1} \end{bmatrix}\right)$$

To further clarify my identification, I am trying to find the two columns of response coefficient Q ($P\overline{R}_{2,s}$ and $PR^*_{f,\tau,s}$ where P is the Cholesky of $\Omega = QQ'$) with respect to the two shocks in

$$y_t = \Phi y_{t-1} + Q\varepsilon_t$$

where $\varepsilon_t = \begin{bmatrix} \cdots & \nu_{0,t} & \nu_{1,t-\tau} \end{bmatrix}'$. $\overline{R}_{2,s}$ and $R^*_{f,\tau,s}$ are normalization of $\overline{R}_2$ and $R^*_{f,\tau}$ with unit norm.

To further link these two assumptions to state space model, I assume there is a housing demand shock a_t follows AR1 process

$$\tilde{a}_t = a_t + \nu_t^\tau$$

where ν_t^τ is the contemporaneous shock to the observation of house demand shock a_t. The perception of demand shock $a_{t|t}$ that is the element in economy through which the news shock

[18] They call this representation to fundamental and belief as *news representation* and the representation in equation 3 as a *noise representation.*

$w_{t-\tau}$ affects the economy, will be

$$a_{t|t} = \gamma_1 a_{t-1|t-1} + \gamma_2 w_{t-\tau|t-\tau} + \gamma_3 a_{t-1} + \gamma_4 w_{t-\tau} + \gamma_5 \nu_t^\tau + \gamma_6 w_t^\tau \tag{4}$$

where $\gamma_1 = \rho \left[1 - \frac{z_{11}}{z_{11}+\sigma^2_{\nu^\tau}}\right]$, $\gamma_2 = 1 - \frac{z_{11}}{z_{11}+\sigma^2_{\nu^\tau}}$ and $a_{t|t} = \gamma_1 a_{t-1|t-1} + \gamma_2 w_{t-\tau|t-\tau} + \gamma_7 \widetilde{a}_t$. Because $\widetilde{a}_t = a_t + \nu_t^\tau$ and $a_t = \rho a_{t-1} + w_{t-\tau} + w_t^\tau$, I have $\widetilde{a}_t = \rho a_{t-1} + w_{t-\tau} + w_t^\tau + \nu_t^\tau$. Therefore $\gamma_3 = \gamma_7 \rho$ and $\gamma_4 = \gamma_5 = \gamma_6 = \gamma_7 = \frac{z_{11}}{z_{11}+\sigma^2_{\nu^\tau}}$ will hold. z_{11} can be solved from the positive root of quadratic equation

$$z_{11}^2 + \left(\sigma^2_{\nu^\tau} - \rho^2\sigma^2_{\nu^\tau} - \sigma^2_w + \sigma^2_{w^\tau} - \frac{\sigma^4_w}{\sigma^2_w + \sigma^2_\nu}\right) z_{11} - \sigma^2_{\nu^\tau}\left(\sigma^2_w + \sigma^2_{w^\tau} - \frac{\sigma^4_w}{\sigma^2_w + \sigma^2_\nu}\right) = 0$$

If you think R^* is a response to an observation triggered by true news, $\widetilde{w}_{t-\tau} = 1 = w_{t-\tau}$ will hold. According to equation 4, $\alpha = \gamma_2\theta + \gamma_4$. If you think R^* is a response to an observation triggered by fake news, $\widetilde{w}_{t-\tau} = 1 = \nu_{t-\tau}$ will hold. According to equation 4, $\alpha = \gamma_2\theta$. If you are an agnostic and think the identification of R^* cannot distinguish the true news $w_{t-\tau}$ and $\nu_{t-\tau}$ (as they have the same response throughout $t-\tau$ to t yet different scalar on y-axis), a mixture realization of $w_{t-\tau}$ and $\nu_{t-\tau}$ will posteriorly imply $\widetilde{w}_{t-\tau} = 1$. According to equation 4, $\alpha = \frac{\sigma^2_w}{\sigma^2_w+\sigma^2_\nu}(\gamma_2\theta + \gamma_4) + \frac{\sigma^2_\nu}{\sigma^2_w+\sigma^2_\nu}\gamma_2\theta$, which is a posterior mixed effect of true news and fake news. The fake shock realization at time τ, $R^*_{f,\tau}$, works as a purifier to wish out the effect of true news in mixed identification result R^* whose component $\frac{\sigma^2_w}{\sigma^2_w+\sigma^2_\nu}\gamma_4$ emerges in α.

To sum, in section 2.2 I identify the column of response $PR^*$to a unit realization of news $\widetilde{w}_{t-\tau}$ (or equivalently $w_{t-\tau|t-\tau}$). Given this unit realization of $\widetilde{w}_{t-\tau}$, I can solve the column of response $P\overline{R}_{2,s}$ and $PR^*_{f,\tau,s}$ to a *non-unit* shock realization $w_{t-\tau|t-\tau} = \frac{1}{\alpha}$ and $\nu_t^\tau = -\frac{1}{\alpha}$ because an unit perception to $w_{t-\tau}$ correspond to a non-unit realization.

Since it is an unexpected shock with respect to the expect news, which is almost certain, this shock would generate the most variation in variable i around the time of the shock realization, which I assume to be at time τ.

$$R^*_{f,\tau} = \text{argmax} e_i' \Psi_\tau \Psi_\tau' e_i \tag{5}$$

s.t.

$$R^{*\prime}_{f,\tau} R^*_{f,\tau} = \overline{R}_2' \overline{R}_2 \tag{6}$$

and

$$R^{*\prime}_{f,\tau} \overline{R}_2 = 0 \tag{7}$$

where

$$\Psi_\tau = \Phi^2 X_{\tau-1} + \Phi PR + \Phi X_{\tau-1} + PR - X_{\tau-1} - \Phi^{-1} X_{\tau-1}$$

and $X_{\tau-1}$ is the impulse triggered by expected news shock $\overline{R}_2$ at time $\tau - 1$.

Equation 6 constraints the norm of the fake shock, making the fake shock comparable and reasonable to the news shock. Essentially, I only want to know the response before the shock that is announced by news at time 0; in fact, neither do I care nor can I identify what would happen if the news is true and the shock announced by news materializes at time τ.

W.O.L.G I assume the variance of news shock follows $\sigma^2_{\widetilde{w}} = 1$, and the realized observation is one standard derivation with respect to it, so $\widetilde{w} = 1$. This is standard in literature as its "true" variance is absorbed in matrix P, and holds during the identification process. If the agent can perfectly observe the shock, the variance of fake news must follows $\sigma^2_\nu = 0$ and $\sigma^2_w = \sigma^2_{\widetilde{w}} = 1$. However, since α is not zero, the variance of fake news is absolutely not zero. Therefore, a one-unit news about the future can come from true news with possibility $\frac{\sigma^2_w}{\sigma^2_\nu+\sigma^2_w}$ and from fake news with possibility $\frac{\sigma^2_\nu}{\sigma^2_\nu+\sigma^2_w}$. Meanwhile, I define the mapping from the variance of fake news to the impulse response as $g : \sigma^2_\nu \to R_{f,\tau}$ where $R_{f,\tau}$ is the response to fake news at time τ, when the household realizes that they have been misled. It is easy to understand the existence of function $g(\cdot)$. The fake news shock $R^*_{f,\tau}$ we solved from problem 5 is based on the MIT shock or a shock with zero variance $R^*_{f,\tau} = \underset{\sigma^2_\nu \to 0}{lim}\, g(\sigma^2_\nu)$, whose realization is $\nu = 1$ because the observation $\widetilde{w}$ is 1. When $\sigma^2_\nu > 0$ holds, the same realized observation $\widetilde{w} = 1$ generated by fake news ν represents a smaller impact because now the shock size is relatively slighter (given the same realization, a larger variance denotes a smaller effect). Therefore, we can know that the impact function $g(\cdot)$ is a monotonic increasing function satisfying $\underset{\sigma^2_\nu \to 0}{lim}\, g(\sigma^2_\nu) = R^*_{f,\tau}$ and $\underset{\sigma^2_\nu \to \infty}{lim}\, g(\sigma^2_\nu) = 0$. Because of the standard variance assumption of news shock, any σ^2_ν can uniquely pin down the variance of true news σ^2_w, and thus the belief adjustment coefficient α is a function of σ^2_ν. To further simplify the problem, I rewrite the fake news impact function $g(\cdot)$ as $g(\alpha)$, which is monotonic decreasing with boundary condition $\underset{\alpha \to 1}{lim} g(\alpha) = R^*_{f,\tau}$ and $\underset{\alpha \to 0}{lim} g(\alpha) = 0$.

There are two elements of the fake news shock set $\mathbb{U}$, the belief adjustment coefficient α and the fake news impact function $g(\alpha)$, which are free to identify and should be pinned down exogenously. However, it is easy to notice that the adjustment coefficient α scales up the identified shock R^*, while the fake news impact function $g(\alpha)$ scales down the effect of fake news $R^*_{f,\tau}$. If the extents of their scaling are same, the pure effect may be offset, and the identification could be neutralized from the adjustment coefficient α. I demonstrate this counteraction in the proposition below.

Proposition 3. *When the fake news impact function $g(\alpha)$ is a linear function such that $g(\alpha) = \alpha R^*_{f,\tau}$, the identification of news shock set $\mathbb{U}$ is independent with the belief adjustment coefficient α and the fake news shock is fully identified.*

(Discussion to Figure 4. TBA)

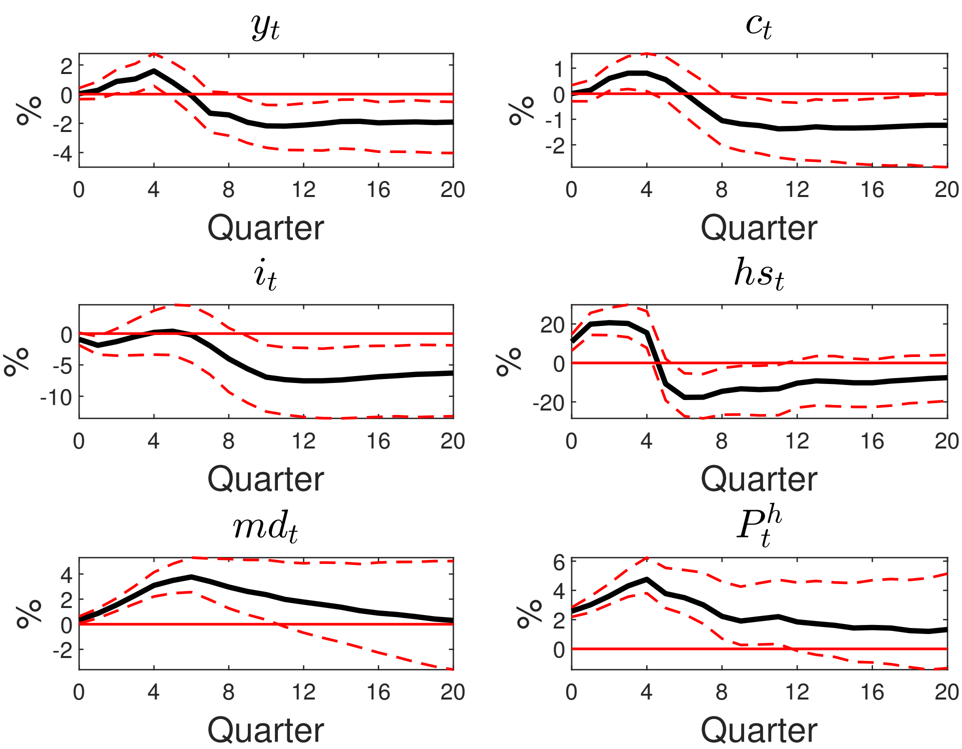


Figure 4: IRF to one unit housing price fake news shock at 90% confidence band

3 Crowd-out effect of overbuilding: insight from a simple model

Optimistic expectation about future house price incurs an upward jump of household demand function of real estate. This upward jump induces the housing market boom with inflation in housing price and overbuilding. If there is semi-inelastic supply existing in the economy, any demand-side change will not result in overbuilding problem a lot. On the contrary, if the supply function is elastic enough, a little demand boom could trigger a large overbuilding. The extent of overbuilding, and in turn the extent of crowded physical capital, is decided by the shape of supply function and demand function because the main mechanism through which crowd-out effect works is the general equilibrium in the end, and we need the supply function as well as the demand function to work together. In this section I first introduce a simple Aiyagari-Huggett model beneath an incomplete market. Then I use this model to illustrate that overbuilding leads to crowd-out effect which is influenced by intratemporal substitution, liquidity, precautionary saving and wealth inequality.

3.1 A simple Aiyagari-Huggett model

It is a standard Aiyagari-Huggett model where households use wage income and asset return to fulfill their demand for consumption and real estate. The durable good, house, is produced by real estate companies in complete market with land, capital and labor. Similarly the non-durable good is produced in complete market with capital and labor.

For simplicity I assume that household i provides inelastic labor supply with 1 unit exogenously to solve the problem

$$\max_{c_t^i, h_t^i, a_t^i} \sum_{t=0}^{\infty} \beta^t U^i \left(c_t^i, h_t^i\right) \tag{8}$$

s.t.

$$c_t^i + a_t^i + p_t^H h_t^i = R_t a_{t-1}^i + w_t \varepsilon_t^i + \left(1 - \delta^H\right) p_t^H h_{t-1}^i + T_t \tag{9}$$

$$-a_t^i \leq \gamma p_t^H h_t^i \tag{10}$$

where equation 9 is the budget constraint and equation 10 is the collateral constraint. a_t^i could either be positive or negative but in aggregate level is positive as it is the supply of capital which is used to produce durable and non-durable goods. w_t is wage and household earns productivity-weighted wage income from which ε_t^i is corresponded idiosyncratic shock. p_t^H is the real house price. h_t^i is the house amount hold by household i. T_t is the lump-sum transfer to household. For simplicity I further assume the real interest rate is fixed at $\overline{R}$.[19]

The production sector is a complete market where firms produce non-durable good via $Y_{N,t} = A_{N,t} K_{N,t-1}^{\alpha} L_{N,t}^{1-\alpha}$ and durable good via $Y_{H,t} = A_{H,t} \overline{L}_H^{\theta} K_{H,t-1}^{\nu} L_{H,t}^{1-\nu-\theta}$. The labor market is closed by inelastic labor supply such that $L_{N,t} + L_{H,t} = 1$. The capital is provided by household such that $K_{N,t-1} + K_{H,t-1} = K_{t-1} = \int a_{t-1}^i dG_{t-1}$ where G_{t-1} is the cumulative distribution function of household. The non-durable good is used either to consume or to invest so that $Y_{N,t} = K_t - (1-\delta) K_{t-1} + C_t$. Meanwhile all the increment in house is produced by real estate companies so that $Y_{H,t} = H_t - (1 - \delta^H) H_{t-1}$ where $H_{t-1} = \int h_{t-1}^i dG_{t-1}$.

Proposition 4. *Household will adjust their consumption of non-durable goods based on overbuilding and precautionary saving. The extent of adjustment is decided by*

$$\widetilde{c}_t = \underbrace{\Phi_H \widetilde{h}_t}_{\text{substitution effect}} - \underbrace{\Phi_\mu \widetilde{\mu}_t}_{\text{credit effect}} + \underbrace{\Phi_{p^H} \left[\frac{1}{1 - (1-\delta^H)\frac{1}{R}} F^H(\widetilde{H}_t) - \frac{\left(1-\delta^H\right)\frac{1}{R}}{1-(1-\delta^H)\frac{1}{R}} F^H(\widetilde{H}_{t+1}) \right]}_{\text{wealth effect}} \tag{11}$$
$$- \underbrace{\Phi_{cov} \widetilde{cov}_t}_{\text{precautionary saving effect}}$$

where $F^H(\cdot)$ is the inverse supply function,

$$\Phi_H = \frac{\frac{\lambda}{\lambda-\mu}\eta_{c,p^H} - \eta_{c,p^c}}{\eta_{h,p^c} - \frac{\lambda}{\lambda-\mu}\eta_{h,p^h}} \tag{12}$$

[19] It is not a too strong assumption since this could happen in many scenarios. For instance the nominal interest rate reaches the ZLB and the price is fixed. Or an open economy where the real interest rate is bounded by the international financial market.

$$\Phi_{\mu} = \frac{\mu}{\lambda - \mu} \frac{\eta_{ch}}{\eta_{h,p^c} - \frac{\lambda}{\lambda-\mu}\eta_{h,p^h}} \tag{13}$$

$$\Phi_{p^H} = \frac{\eta_{ch}}{\eta_{h,p^c} - \frac{\lambda}{\lambda-\mu}\eta_{h,p^h}} \tag{14}$$

$$\Phi_{cov} = \frac{\eta_c}{\eta_{h,p^c} - \frac{\lambda}{\lambda-\mu}\eta_{h,p^h}} \frac{\beta\left(1-\delta^H\right)\overline{cov}}{h}$$

and $\eta_{c,p^H} = \frac{u_{ch}u_h}{u_{ch}^2 - u_{cc}u_{hh}}\frac{1}{c}, \eta_{c,p^c} = \frac{u_{hh}u_c}{u_{ch}^2 - u_{cc}u_{hh}}\frac{1}{c}$, $\eta_{h,p^c} = \frac{u_{ch}u_c}{u_{ch}^2 - u_{cc}u_{hh}}\frac{1}{h}$, $\eta_{h,p^h} = \frac{u_{cc}u_h}{u_{ch}^2 - u_{cc}u_{hh}}\frac{1}{h}$, $\eta_{ch} = \frac{u_c u_h}{u_{ch}^2 - u_{cc}u_{hh}}\frac{1}{ch}$, $\eta_c = \frac{u_c}{u_{ch}^2 - u_{cc}u_{hh}}\frac{1}{c}$.

Proposition 4 shows that any perturb occurred in real estate market could be passed to non-durable consumption through 4 channels: substitution effect, wealth effect, credit effect and precautionary saving effect.[20] The directions of these four channels through which the housing market boom affects the consumption of non-durable goods are determined by the relative strength of intertemporal and intratemporal elasticity of substitutions between non-durable and durable goods, as well as the relative feature that housing wealth played in budget constraint and credit constraint. When overbuilding happened, a positive $\widetilde{h}_t$ and $\widetilde{H}_t$ will generate a variation in nondurable consumption through substitution and wealth effect. Meanwhile it may also affect consumption endogenously through the credit effect and precautionary saving effect. This variation in consumption triggered by a boom in house market will influence physical investment ultimately and induce a recession in the future as long as the total effect is positive.

It is worth noticing that η_{x,p^y} denotes the standard Frisch elasticity of variable x with respect to the relative price of y, which is pivotal regulating the clouts of the four effects. If nondurable consumption responses to housing price more than the nondurable goods price, a derivation in the holding of housing service will spark a larger echo in nondurable goods consumption which is unveiled in Φ_H. Contrariwise, if the response of household holding of housing service to nondurable goods price was larger(than to housing price), the elasticity of substitution would attenuate all four channels because now the consumption of durable housing is more stable and household does not variate their consumption a lot, which implies a minor pass through from the consumption of housing servicing to the consumption of nondurable goods.

3.2 Crowd-out effect of overbuilding

I will discuss how intratemporal elasticity of substitution, credit constraint, precautionary saving and wealth inequality amplify the crowd-out effect sparked by overbuilding. Intuitively overbuilding will affect the consumption of non-durable goods and crowd out physical investment as consumption and house are closer linked via complement(in aggregate level). Likely overbuilding will also relax the collateral constraint and this relaxation benefits household as they

[20] Berger et al. (2018) only discussed two of them meticulously but not focused on credit effect and precautionary saving effect. Additionally their goals about decomposition is related to analyze the inequality problem caused by house price inflation.

can borrow more to smooth their consumption demand. Similarly overbuilding will also pass to the consumption response because the inverse supply function of residential assset $F^H(\cdot)$ is monotonic increasing in complete market and more new construction leads to higher housing price in equilibrium. Because the house price entries into the budget constraint of household which alters their income, an increased price makes household feel wealthier as house works not only as utilitarian goods but also as an asset in budget constraint. This increased price derived from monotonic increasing supply function stands that overbuilding will also correspond with house price inflation through the supply side.

By aggregating the consumption decision of household from equation 11 and combining the FOC in supply sectors I can obtain the relationship between overbuilding and physical investment which is summarized in proposition 5.

Proposition 5. *The aggregate investment is driven by overbuilding and precautionary saving following*

$$\begin{aligned} I\widetilde{I}_t = -\Bigg\{ & \left(\Phi_H + \frac{\nu}{\alpha}p^H H\right) \int \widetilde{h}_t^i dG_i - \Phi_\mu \int \widetilde{\mu}_t^i dG_i \\ & + \Phi_{p^H}\left[\frac{1}{1-(1-\delta^H)\frac{1}{R}} F^H(\widetilde{H}_t) - \frac{(1-\delta^H)\frac{1}{R}}{1-(1-\delta^H)\frac{1}{R}} \mathbb{E}_t F^H(\widetilde{H}_{t+1})\right] \\ & - \Phi_{cov}^i \int \widetilde{cov}_t^i dG_i + \frac{\nu}{\alpha} Y_H p^H F^H(\widetilde{H}_t) \Bigg\} \end{aligned} \tag{15}$$

The overbuilding, $\widetilde{H}_t = \int \widetilde{h}_t^i dG_i > 0$, will crowd out physical investment as long as the substitution effect Φ_H and wealth effect Φ_{p^H} are not negative enough.

Equation 15 shows that the overbuilding will lead to a smaller physical investment and a lower physical capital afterwards throguh different story in demand and supply side, at least within a range of parameters. The term Φ_x relates to the contribution of pass-through from housing service to the consumption of nondurable goods and the term $\frac{\nu}{\alpha}$ relates to the supply side effect. Next I am going to discuss detailedly how intratemporal elasticity of substitution, credit constraint, precautionary saving and wealth inequality influence the crowd-out effect of overbuilding.

3.2.1 Intratemporal elasticity of substitution

Intertemporal substitution has been widely studied as it related to the Euler equation and monetary policy. However intratemporal substitution between durable and non-durable goods consumption is still in barren not only theoretically but also empirically. In this section I argue that intratemporal substitution is also important to the decision making of household, at least in analyzing the crowd-out effect created by overbuilding. Empirically in housing market, intratemporal substitution is much more important and powerful than intertemporal substitution[21] as household are mostly myopic or financial constrained so they do not pay much attention or simply cannot weight future consumption on decision today. Focusing on the coefficients of crowd-out effect in proposition 11, it comes to a conclusion in corollary 1 that the intratemporal substitution could theoretically enlarge the crowd-out effect across demand side of housing market.

Firstly I define the intertemporal and intratemporal elasticity of substitution as

Definition 1. The intratemporal elasticity of substitution is

$$\mathrm{ES} = -\frac{\partial \ln \frac{h}{c}}{\partial \ln \frac{U_h}{U_c}} \tag{16}$$

and the intertemporal elasticity of substitution to consumption bundle is

$$\mathrm{EIS} = -\frac{U_{BB}}{U_B}$$

Then based on the definition I obtain following corollary.

Corollary 1. *Ceteris paribus, household with larger intratemporal elasticity of substitution relative to intertemporal elasticity of substitution, as well as the standard CRRA utility function, will crowd out less investment through substitution and wealth effect.*

It is easy to understand corollary 1 that non-durable goods and housing services are both normal goods and if they are substituted more with each other, the crowd-out effect will be further muted because more consumption of housing servicing leads to less consumption of non-durable goods. The intratemporal elasticity of substitution measures the extent to which increased house could be substituted with increased consumption in intraperiod utility level.[22] Conversely the intertemporal elasticity of substitution is the metric of propensity to substitute the total consumption bundle over different period. If $\mathrm{ES} > \mathrm{EIS}$ holds, the household will prefer adjusting their consumption between durable and nondurable goods within a period, to adjusting their consumption interperiodicly. The larger the intratemporal elasticity of substitution is relative to intertemporal elasticity of substitution, the less increased consumption responses to the overbuilding within this period, as now they are more substitute rather than complementary. Intratemporal substitution is so powerful enough that decreased relative elasticity will magnify substitution and wealth effect as it directly affects the marginal benefit in utility instead of involving the budget constraint and endowment. However the credit effect derived from financial friction is ambiguous to the decreased relative elasticity[23] because the willingness to substitute

[21] Khorunzhina (2021) did this vital work empirically.

[22] It is intuitive to focus on U_{ch} which is closely related to the complementarity between house and non-durable good.

[23] It depends on the sign of Φ_μ whose sign cannot be derived analytically. However it is always positive within a range of reasonable parameters.

nondurable goods for durable goods is constrained by collateral requirement, which will also change the extent to accommodate consumption portfolio.

I solve the model 8 with unit intratemporal elasticity such that $\mathrm{ES} = 1$ but different the intermporal elasticity from 0.67 to 0.5, which is equivalent to increase the relative intratemporal elasticity. Figure 5a shows that the substitution effect shrinks, along with the larger relative intratemporal elasticity. Intuitively if there is a preference shock which increases the relative intratemporal elasticity of substitution relative to intertemporal elasiticity of substitution, the same amount of overbuilding will decrease the consumption response and then crowd out less investment as the complementarity between consumption and housing service is diluted by the stronger substitution. Meanwhile more propensity to substitution will also relax the collateral constraint because the demand to consume nondurable goods is smaller. Yet this higher relative elasticity exacerbates the consumption bundle and forces more household to stay financially constrained in steady state. To illustrate above argument mathematically, we can assume there are two economies a and b whose intertemporal elasticity of substitution satisfy $\frac{\mathrm{ES}_a}{\mathrm{EIS}_{c,a}} < \frac{\mathrm{ES}_b}{\mathrm{EIS}_{c,b}}$ and there are two extra exogenous tax rebate to household which generate the same jump in nondurable consumption $\Delta C_a = \Delta C_b = 0.5$. Because the intratemporal elasticity in economy a is smaller than that in economy b, people in economy a will increase their holding of durable consumption more, for instance, $\Delta H_a = 0.5 > \Delta H_b = 0.3$. These increased holding of residential asset slacks the collateral constraint and the extent of slackness should be proportional to the change of residential asset. Therefore the Karush–Kuhn–Tucker multiplier of equation 10 follow the relationship $\Delta\mu_a < \Delta\mu_b < 0$ which implies $\Phi^a_\mu > \Phi^b_\mu > 0$ in equation 11. This is shown in figure 5b in which the credit constraint grows larger and larger.

In addition to substitution effect and credit effect, overbuilding will also be passed to the consumption response through the inverse supply function $F^H(\cdot)$ because the residential asset is also a type of asset which enters into the budget constraint, except for acting as consumables in utility function. An inflation(of housing price) in housing market, inspired by overbuilding, also provide liquidity to household as long as they previously hold some amount of house because of the asset's pecuniary character. This wealth effect is amplified as the value, that one unit of housing service provides, now can be transferred to utilitarian value more with a smaller intratemporal elasticity of substitution. The intratemporal consumption decision between durable and nondurable goods, which comes from wealth effect, follows the relative marginal utility equation $\frac{U_{h,t}}{U_{c,t}} = f\left(p_t^+, p_{t+1}^-\right)$. This equation is intuitive and easy to understand. Household can use money to marginally increase one unit of housing servicing at time t and get $U_{h,t}$ unit of extra utility. Alternatively the household can also use the money that affords the housing servicing to buy nondurable consumption and get $U_{c,t}f\left(p_t^+, p_{t+1}^-\right)$ unit of extra utility. The extra unit of nondurable goods is rescaled by the price of housing servicing as the money that affords one unit of housing servicing does not afford the same unit of nondurable goods. If I given a same jump in housing price $\Delta p^H_{a,t} = \Delta p^H_{b,t} > 0$ on RHS and held the housing servicing, there would be an jump in nondurable goods consumption which results in a positive Φ_{p^H} in equation 11. A jump in nondurable goods consumption $\Delta C_t > 0$ will fulfill household's demand for nondurable goods with smaller marginal utility of nondurable goods $\Delta U_{c,t} < 0$ but a higher demand for durable goods(because of complementarity) with larger marginal utility of durable goods $\Delta U_{h,t} > 0$. A larger relative intratemporal elasticity of substitution allows larger variation between marginal utility of housing service and nondurable goods consumption, so a smaller nondurable goods consumption jump can support a given variation($\Delta f\left(p_t, p_{t+1}\right) > 0$) in relative marginal utility. The crowded-out effect is amplified further through the wealth effect and the pass-through from durable goods to nondurable goods. Figure 5c exhibits the decreased strength of wealth channel to crowd-out effect as the relative intratemporal elasticity rises and the one unit housing service becomes less important (can be replaced by nondurable consumption easier). Although in this section I do not quantitatively introduce the aggregate shock into the model and investigate the magnitude change of the precautionary saving effect, it is easy to comprehend that a higher relative intratemporal elasticity of substitution will conduce a smaller precautionary saving effect because the household cherishes the balanced consumption portfolio within a period more than that over periods. To sum up, overbuilding affects crowd-out effect via four channels while three of them are influenced by relative intratemporal elasticity of substitution.

3.2.2 Credit constraint and Liquidity

Overbuilding and house market boom push household to spend more money on nondurable goods through substitution effect as now their holding of real estate jumped. Additionally if the economy is incomplete and household cannot fully insure their idiosyncratic shock through financial market, the consumption of household may be constrained by an incomplete market where they cannot borrow as much as they want to confront the bad shock. This credit constraint generates the liquidity problem and some households may be constrained from time to time and not fulfill their consumption demand even though they could repay the amount they borrow in the future. Overbuilding offers more asset that household could use to borrow as collateral and hence it relaxes the previous credit constraint. In figure 6a the extend of financial friction is decreased by increasing the proposition of housing services whose value can be used to borrow money from 0.5 to 0.8. It verifies the argument that a tighter collateral constraint induces a higher substitution effect since one-unit-increased housing servicing becomes more valuable in utility in steady state.

Additionally, a marginal relaxation on the binded collateral constraint represents a smaller K-T multiplier, $\Delta\mu < 0$ in equation 11, and a tighter constraint connects with a smaller nondurable consumption response Φ_μ which ensues a smaller crowded out effect. To understand the credit effect I assume that the unexpected tax rebate spawns the same increased nondurable goods consumption $\Delta C_{a,t} = \Delta C_{b,t}$ and the collateral constraint γ in economy a is tighter than that in economy b and accordingly $\gamma_a < \gamma_b$ will hold in equation 10 as well as in figure 6. A tighter financial constraint reveals a larger K-T multiplier response ergo the absolute change of multiplier

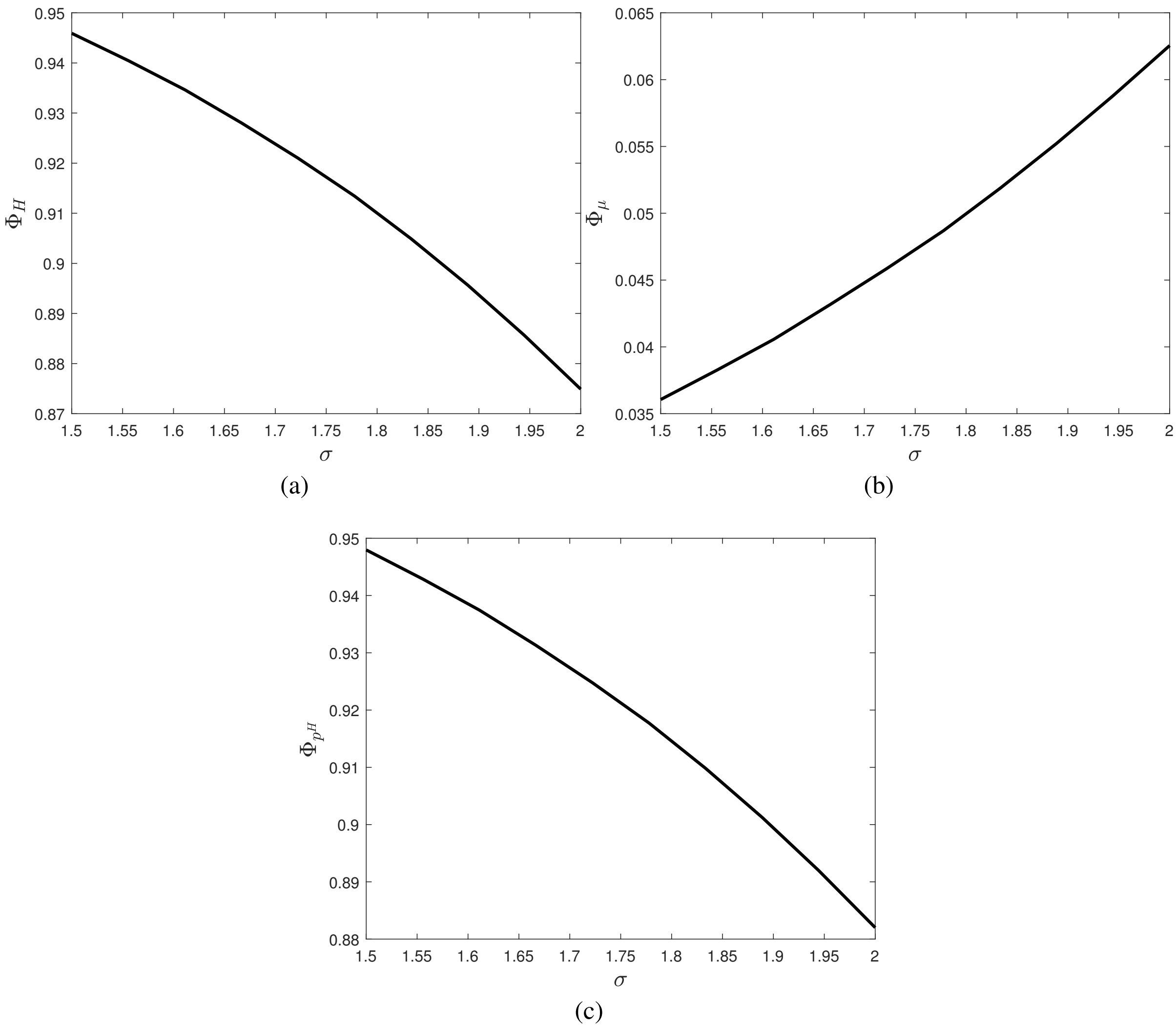


Figure 5: Elasticity of Substitution

in economy a is larger than that in economy $b(\Delta\mu_a < \Delta\mu_b < 0)$. This delineates that a unit change in marginal value of housing servicing in financial constraint performs feebly when the constraint is tight because the unit change in marginal value is now “cheaper” than that in steady state. Figure 5b tells us explicitly the credit crunch (a positive $\widetilde{\mu}_t$) inspired by overbuilding decreases less consumption (or crowd out more investment) when the financial friction is larger.

Comparing to the credit effect, the financial friction works in the opposite direction in wealth effect(but the same in substitution effect). Mathematically a larger financial friction results in a larger K-T multiplier and a larger μ in 14 therefore a larger wealth effect as shown in figure 6c. The mechanism in backdrop is the same as substitution aforementioned since the housing services itself and its price play the same role in collateral constraint 10 and their effect to the pass through should be the same.

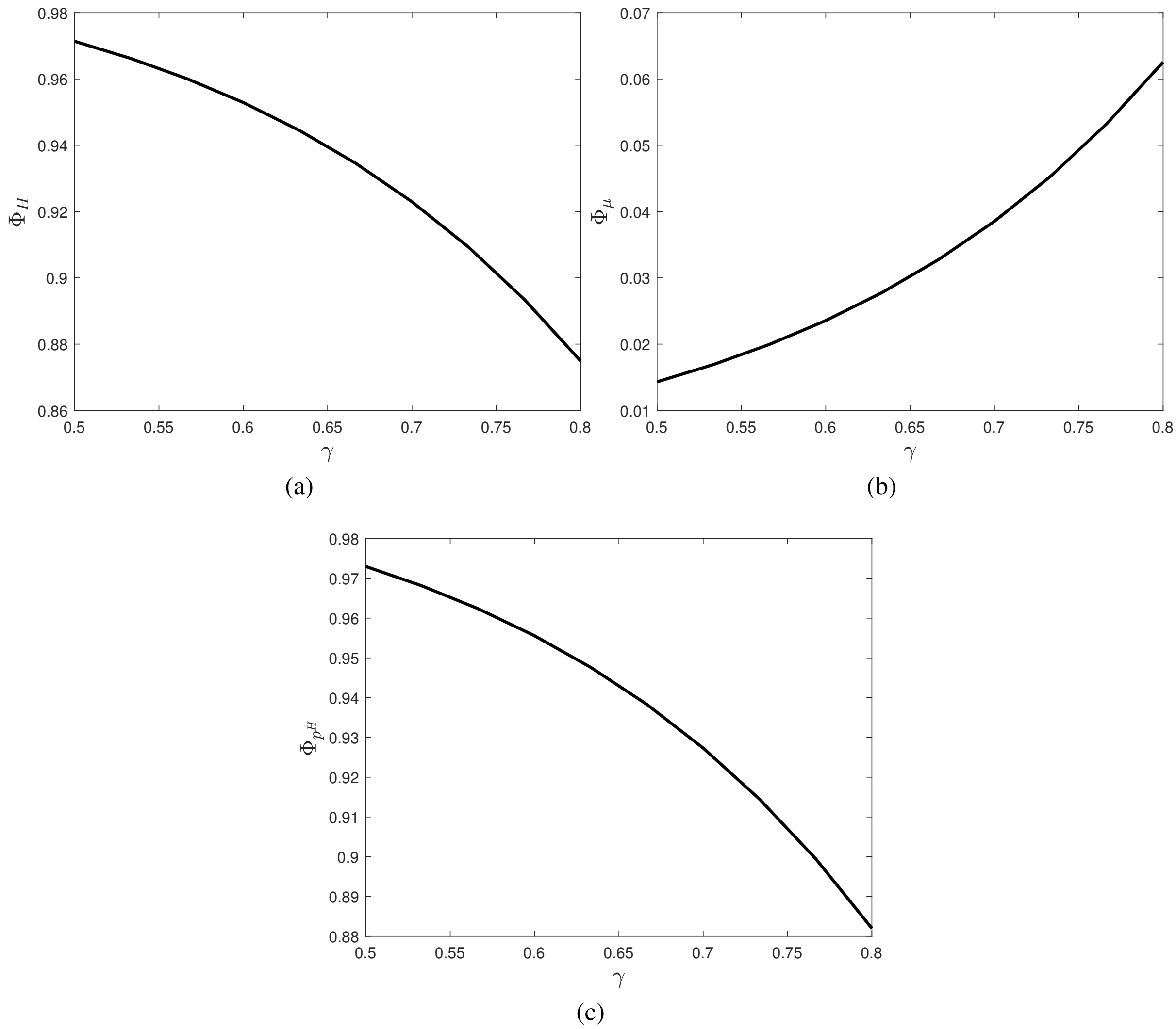


Figure 6: Financial friction

3.2.3 Precautionary saving and Wealth inequality

Household usually will not consume as much as they would do under the scenario without any idiosyncratic shock or they can perfectly insure the idiosyncratic shock. Household have the propensity to put more income into pocket to save for insuring idiosyncratic shock which we call precautionary saving motive. The last term of equation 15 shows that precautionary saving decreases the consumption adjustment as household save extra $\Phi_{cov}\widetilde{cov}_t$ amount instead of spending out when facing the uncertainty in income.

In addition to the four effects discussed above, substitution effect, credit effect, wealth effect and precautionary saving effect, overbuilding can amplify the crowd-out effect through the lens of business cycle. It is well known that idiosyncratic shock is countercyclical while overbuilding is mostly procyclical. Therefore when the overbuilding happens household are less precautionary since aggregate economic conditions are better and there are less large idiosyncratic shock. Boom and lower variation of idiosyncratic shock persuades household that economy is going to be better and they become optimistic to consume more and save less. $\widetilde{cov}_t$ in equation 15 will drop which indicates that household save less and consume more when overbuilding and boom arrival. However this amplification is not covered in my numerical experiment which is left for future study.

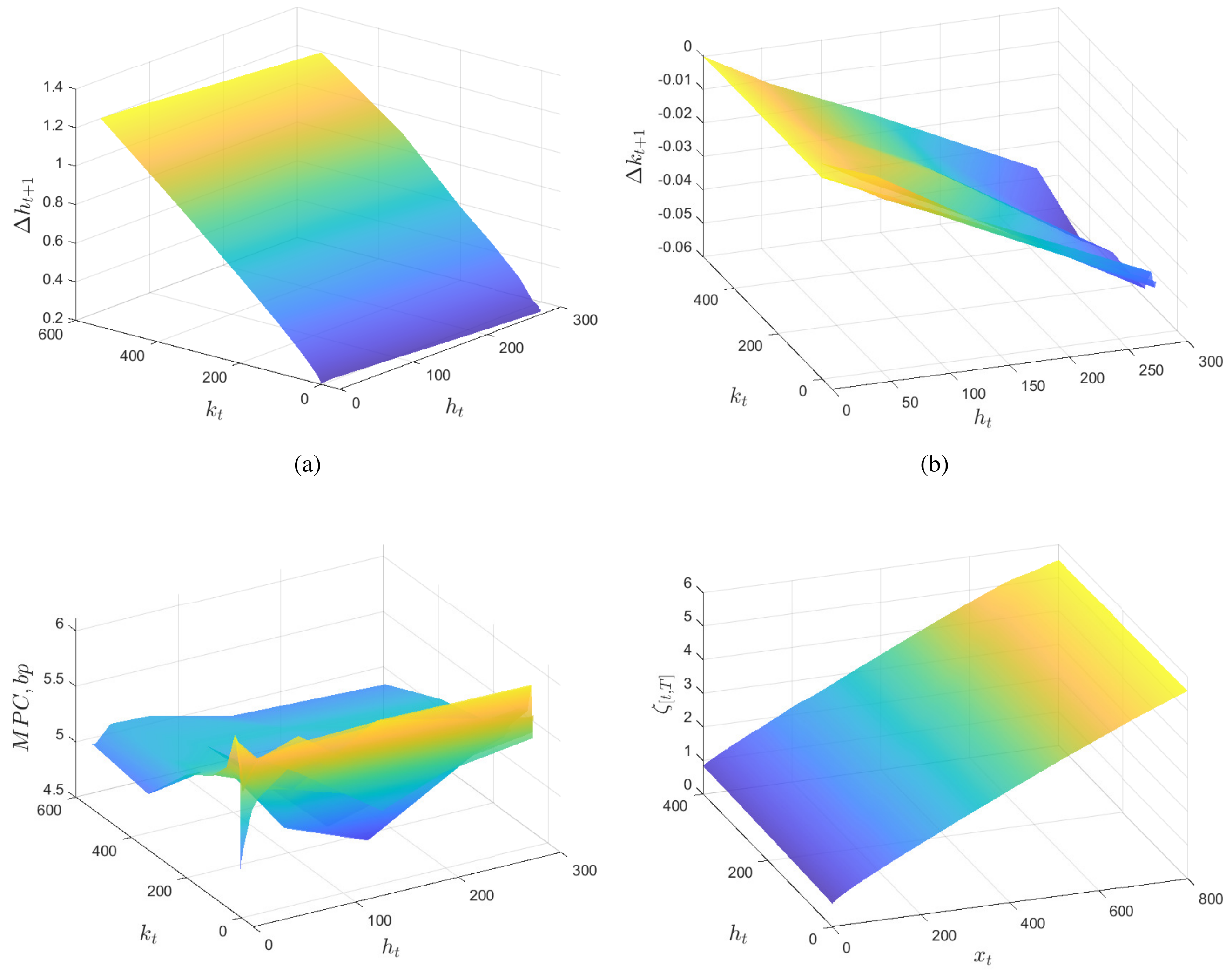
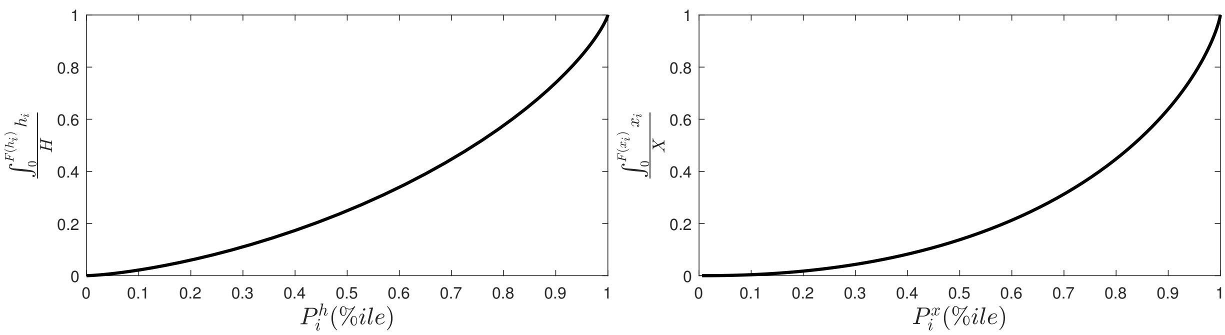


Figure 7: Wealth Distribution

Additionally, the wealth distribution may also manipulate the crowd-out effect triggered by overbuilding in aggregate level. Since the increased holding of housing service is funded by liquid asset and wage income, the absolute amount of large jump per capita in holding of housing service comes from those household who hold a lot of liquid asset and earned high wage income at steady state. After aggregating the consumption decision over household which is shown in equation 15, I can conclude that the distribution of wealth is important as it affects the distribution of coefficient and in turn affect the aggregate crowd-out effect. Figure 7a plots the distribution of changing in holding of housing servicing facing a decrease in house price.

The wealthier household who hold a lot of liquid asset is the household who buys more unit of housing service and then who decreases the physical investment as shown in figure 7b. The the cohort mass of the wealth people is small whereas the wealth distribution is right-skewed and the skewness is shown in figure 8a for residential asset and figure 8b for effective liquid asset. The most wealth in economy is held by the least people in the top and this right-skewed wealth distribution amplifies the crowd-out effect of overbuilding through the term $\int \widetilde{h}_t^i dG_i$ in equation 15. Furthermore, the distribution of MPC is left-skewed(figure 7c) and the standard general equilibrium effect of hand-to-mouth household will also be effective as it works in the pass through of monetary policy. This left-skewed MPC likewise amplifies the crowd-out effect of overbuilding but through the term $\int \widetilde{\mu}_t^i dG_i$ in equation 15. Figure 7d exhibits the wealth distribution effect of a demand-driven boom, which is triggered by an expectation of housing price inflation as I argued in corollary 2, instead of a supply-driven which I use in figure 7a and 7b. The result does not change the attenuation direction formed by wealth distribution which demonstrates that it is independent with type of housing market boom and direction of the change of house price.



(a) Lorenz curve of $F(h_i)$

(b) Lorenz curve of $F(x_i)$

Figure 8: Lorenz curve

3.2.4 Optimistic expectation and overbuilding

Previous arguments are focus on the crowd-out effect generated by overbuilding and we discussed different mechanisms through which this effect works depending on the assumption that overbuilding is already happened. Here I demonstrate that the existence of overbuilding is not a strong assumption and it can easily be created by an optimistic expectation about housing market in the future. When household have a positive expectation about the change of housing price in the future, they will increase their holding of real estate in this period which is similar to the change in consumption induced by intertemporal new keynesian cross. Corollary 2 shows that an increase in the expectation of the housing price in time $T+1$ will marginally provoke $-\left[\beta\left(1-\delta^H\right)\right]^T \Pi_{s=1}^T \frac{\lambda_{t+s}}{\lambda_{t+s}-\mu_{t+s}} \lambda_{t+T+1}/u''_{h^i}$ unit of increase in demand of housing service. If the expectation is driven by optimism or fake news about future, the increased new buildings will become "over"-building as it is not support by the fundamental change in economy but support by a mirage. After this mirage vanishes, the crowd-out effect engenders a recession because of the lack of physical capital produced by the illusion in housing market boom.

Corollary 2. *Ceteris paribus, an positive expectation about the housing price change in time* $T+1$ *will induce a jump in demand of housing service in time t. The response extend follows*

$$\widetilde{h}_t^i\Big|_{h_{t+i},\mu_{t+i},\lambda_{t+i},,i\in[1,T]} = \zeta_t^i dp_{t+T+1}^H \tag{17}$$

where $\zeta_t^i = -\frac{1}{u''_{h^i}}\mathbb{E}_t\left[\beta\left(1-\delta^H\right)\right]^T \Pi_{s=1}^T \frac{\lambda_{t+s}}{\lambda_{t+s}-\mu_{t+s}}\lambda_{t+T+1}$

4 Crowd-out effect of overbuilding: Full fledged model

In last section I use a simple model analytically show that an expectation in future housing market boom will inspire household to increase their holding of durable goods' consumption which in turn crowd out the physical investment. This crowd-out effect is influenced by relative intratemporal elasticity of substitution, credit constraint and wealth distribution. In this section I use a full fledged model to analyze the crowd-out effect quantitatively. By linking the model to data I show that news about future can generate a boom-burst cycle in housing market. When the news is fake and the fraud is not realized by household until several periods later, the boom which is supported by a fake news instead of fundamental creates the overbuilding, that induces a large loos in output and consumption during the burst period. I will first introduce the model I used to quantify the drawback of crowd-out effect. Then I use calibration and SMM connect the model with data. In the end I show the large break in economy caused by overbuilding in mirage via some impose response functions.

4.1 Model Setting

4.1.1 Household

Continue household[24] holds housing servicing h and liquid asset b at time t which he takes from last period. He chooses the non-durable consumption c, labor supply l, housing service h' and liquid asset holding b' at time t to solve the optimization problem

$$V(h_{t-1}, b_{t-1}, \varepsilon_{t-1}) = \max_{c,l,b',h'} U(c_t, h_t, l_t) + \beta(1-\theta^d)EV(h_t, b_t, \varepsilon_t)$$

$$\begin{aligned}\text{s.t.}c_t + Q_t b_t + p_t^h\left[h_t - (1-\delta^h)h_{t-1}\right] = {}& R_t Q_{t-1} b_{t-1} + (1-\tau)w_t l_t \varepsilon_t + \Pi_t^h \\ & - p_t^h C\left(h_t, h_{t-1}\right) + T_t\end{aligned} \tag{18}$$

[24]Here for simplicity I omit the index for specific household i.

$$-Q_t b_t \leq \gamma p_t^h h_t \tag{19}$$

where p_t^h is the relative price of housing unit at time t. R_t is the gross real return of liquid asset which follows $R_t = \frac{Q_t(1-\delta)+r_t}{Q_{t-1}}$. $C(h_t, h_{t-1})$ is the adjustment cost function when household want to adjust their holding of housing servicing. γ is the parameter governing the slackness of collateral constraint. δ^h is the depreciation rate. τ is the wage income. Π_t^h is the restitution from construction companies. T is the lump-sum tax transfer payed by government. θ^d is the death rate. ε is the idiosyncratic income shock which follows logarithmic AR1 process with coefficient ρ_ε and standard derivation σ_ε.

The adjustment function follows the canonical form

$$C(h_t, h_{t-1}) = \frac{\kappa_1}{\kappa_2}(h_{t-1}+\kappa_0)\left|\frac{h_t - h_{t-1}}{h_{t-1}+\kappa_0}\right|^{\kappa_2}$$

The utility function follows the CRRA form[25]

$$U(c_t, h_t, l_t) = \frac{\left(c_t^{\phi} h_t^{1-\phi}\right)^{1-\sigma}}{1-\sigma} + \kappa \frac{l_t^{1+\psi}}{1+\psi}$$

4.1.2 Firm

There are two types of firms, construction firms who produce the housing servicing and the non-durable goods producers. All of these two types of producers are staying in complete market but because the construction firms also use exogenous land supply as an input to construct house, they earn non-zero profit which in the end refunded back to their holder, household.

Non-durable goods producer use

$$\begin{aligned} Y_{N,t} &= C_t + I_t + C(h_t, h_{t-1}) \\ &= A_{n,t} K_{n,t}^{\alpha} L_{n,t}^{1-\alpha} \end{aligned} \tag{20}$$

to maximizes profit with the cost from real rental rate of capital used by non-durable goods producer K_n and related wage payment to labor L_n .

Similarly, durable goods (housing services) producer use

$$\begin{aligned} Y_{H,t} &= \left[H_t - (1-\delta^h)H_{t-1}\right] \\ &= A_{h,t} \overline{LD}_t^{\theta} K_{h,t}^{\nu} L_{h,t}^{\iota} \end{aligned} \tag{21}$$

to maximizes profit with the cost from real rental rate of capital used by durable goods producer $K_{h,t}$ and related wage payment to labor $L_{h,t}$. The$\overline{LD}_t$ in production function is the exogenous

[25] Piazzesi et al. (2007) use CEX data suggest that intratemporal elasticity of substitution is close to 1. In other words the utility function form of durable and nondurable goods is close to standard Cobb-Douglas case.

land supply follows $\overline{LD}_t = \overline{LD} A_{L,t}$ and the new construction is homogeneous to each production factor therefore the share of input satisfies $\theta + \nu + \iota = 1$.

4.1.3 Capital Producer

The capital producer uses final nondurable goods Y_N to produce capital following the maximization problem

$$\begin{aligned}
&\max E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \Lambda_{t,t+\tau} \left\{ \left(Q_\tau - 1\right) I_\tau - f\left(I_\tau, I_{\tau-1}\right) I_\tau \right\} \\
&\text{s.t. } f\left(I_\tau, I_{\tau-1}\right) = \frac{\psi_I}{2}\left(\frac{I_\tau}{I_{\tau-1}} - 1\right)^2
\end{aligned}$$

By solving above optimization problem I could get the capital price as a convex function of investment which is shown below

$$\begin{aligned}
Q_t = 1 + \frac{\psi_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 + \psi_I \left(\frac{I_t}{I_{t-1}} - 1\right)\frac{I_t}{I_{t-1}} - \\
E_t \beta \Lambda_{t,t+1} \psi_I \left(\frac{I_{t+1}}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2
\end{aligned} \tag{22}$$

So the goods market clearing condition will become

$$Y_{N,t} = C_t + I_t + f\left(I_t, I_{t-1}\right) I_{t-1} + p^h C(h', h)$$

4.1.4 Market cleaning

Capital is supplied by household with their liquid asset and labor is supplied in effective form

$$K = \left(1 - \theta^d\right) \int b dG = K_n + K_h$$

$$L = L_h + L_n = \int \varepsilon l dG$$

$$H = \left(1 - \theta^d\right) \int h dG$$

The goods market cleaning condition is

$$C + I + p^h C(h', h) = A_n K_n^{\alpha} L_n^{1-\alpha}$$

where $K' = (1-\delta)K + I$

Similarly, the housing market cleaning condition is

$$\left[H' - (1-\delta^h) H\right] = A_h \bar{L}^{\theta} K_h^{\nu} L_h^{\iota}$$

The return of gross liquid asset b comes from two component: capital return from firms r and capital gain $\frac{Q'(1-\delta)}{Q}$.

In the end the government close the economy by $T = \tau wL + \theta^d \left(K + p^h H\right)$ and $\Pi^h = p^h Y_H - wL_h - (r - 1 + \delta)K_h$ as all the new born household hold zero liquid asset and housing servicing.

The model contains three types of shock: *contemporaneous unexpected shock, news shock and noise shock* which I introduce detailedly in appendix F.7.1. I introduce the news and noise shock following Chahrour and Jurado (2018) who introduced the news and noise representation to overcome the observational equivalence problem in previous literature such as Schmitt-Grohé and Uribe (2012), Barsky and Sims (2012) and Blanchard et al. (2013).

4.1.5 Shocks

There are two fundamental shocks on the TFP of the two production function 20 and 21 respectively. These two shocks a_t^i follows the standard logarithm AR(1) process $\log(a_t^i) = \rho_a^i \log(a_{t-1}^i) + \varepsilon_t^{a^i}$ where $i \in \{h, n\}$. Thus the TFP of these two production functions follow $A_{n,t} = a_t^n \overline{A}_n$ and $A_{h,t} = a_t^h \overline{A}_h$.

Meanwhile I introduce a preference shock Φ_t^ϕ and the shock to its growth rate $\Phi_{g,t}^\phi$ to the preference ϕ in utility function in the demand side, cooperating with a land supply shock Φ_t^L and the shock to its growth rate $\Phi_{g,t}^L$ in the supply side to determinate the housing market.

Meanwhile to incorporate the noise and news into the model I assume that the household can get a news related to the shocks up to 8 periods before they realize and I defined them in companion form in equation 64. However the agents cannot perfectly observe these shocks but mixed with noisy observation shock to $\widetilde{\Phi}_t^i$ and $\widetilde{\Phi}_{g,t}^i$ in equation 66.[26]

4.2 Calibration

4.2.1 Parameter

Most of the parameters I used in production side comes from literature which is standard and robust. I relegate them into appendix F.1 which is summarized in table 5. I use the discount factor, disutility to labor supply, and three parameters in production side to match the gross real interest rate at 1.015 quarterly, labor supply at 1, physical investment over GDP at 0.13 and new construction over GDP at 0.05. The physical investment over GDP is estimated from Private Non-Residential Fixed Investment over Gross Domestic Product and the new construction over GDP is estimated from Private Residential Fixed Investment over Gross Domestic Product. The parameters in adjustment cost function is in line with Kaplan et al. (2018) and Auclert et al. (2021). The intertemporal elasticity of substitution and preference between durable and

[26] I define the news and noise shocks following the suggestion made by Chahrour and Jurado (2018) because this form does not suffer from the observational equivalence problem.

nondurable goods are borrowed from Kaplan et al. (2020). The AR1 coefficient and standard derivation of idiosyncratic shock follow the estimation by McKay et al. (2016). The death rate is estimated from the Underlying Cause of Death provided by Centers for Disease Control and Prevention from 1999 to 2020. All the value of corresponding parameters I used are summarized in table 2.

Table 2: Key Parameter Values

Parameter	**Value**	**Description**
β	0.9749	Discount factor
τ	0.20	Labor income tax
κ	-1.28	Disutility to supply labor
θ^d	0.21%	Death rate
γ	0.8	Slackness of collateral constraint
κ_0	0.25	Adjustment cost silent set
κ_1	1.3	Adjustment cost slope
κ_2	2	Adjustment cost curvature
σ	2	Inverse of intertemporal elasticity of substitution
ϕ	0.88	Preference between durable and nondurable
ρ_ε	0.966	AR1 coefficient of income shock
σ_ε	0.25	SD of income shock

4.2.2 Data to Model: Moment Matching

Even though I do not specifically match the moments in distribution, my model generates a lot of merits to replicate the moments extracted from data. Table 3 shows that my model has some nature ability to unveil the reality which I compare the data estimated by Kaplan et al. (2014) and Kaplan et al. (2018) and the moments calculated from model.

Table 3: Distribution Moments

Description	**Data**	**Model**
Poor Hand-to-Mouth Household	0.121	0.1102
Wealthy Hand-to-Mouth Household	0.192	0.2059
Top 10 percent share of Liquid asset	0.8	0.5
Top 10 percent share of Iliquid asset	0.7	0.3

To build the bridge between the model and data, I use GMM to estimate the parameters pertaining to the dynamic and business cycle. Particularly I match 34 moments with 31 parameters such as the persistence of shocks, observation matrix and standard derivations of noise shock. For similarity I further assume the covariance matrix of shocks is a diagonal matrix hence all the shocks are independent. The moments in data is calculated by detrending the trend from quarterly time series via hp-filter. I also follow the method proposed by Uhlig et al. (1995) and Ravn and Uhlig (2002) to calculate the moments of model in frequency space so that it is a comparative calculation akin to the filtered data. Table 4 summarizes the primary moments related to the housing market and physical capital investment on which I focus in this paper. The result shows that the model is in line with the reality and can be used to estimate the economic destruction caused by the real estate over-construction. All the details are relegated to the appendix.

Table 4: Real Business Cycle Moments

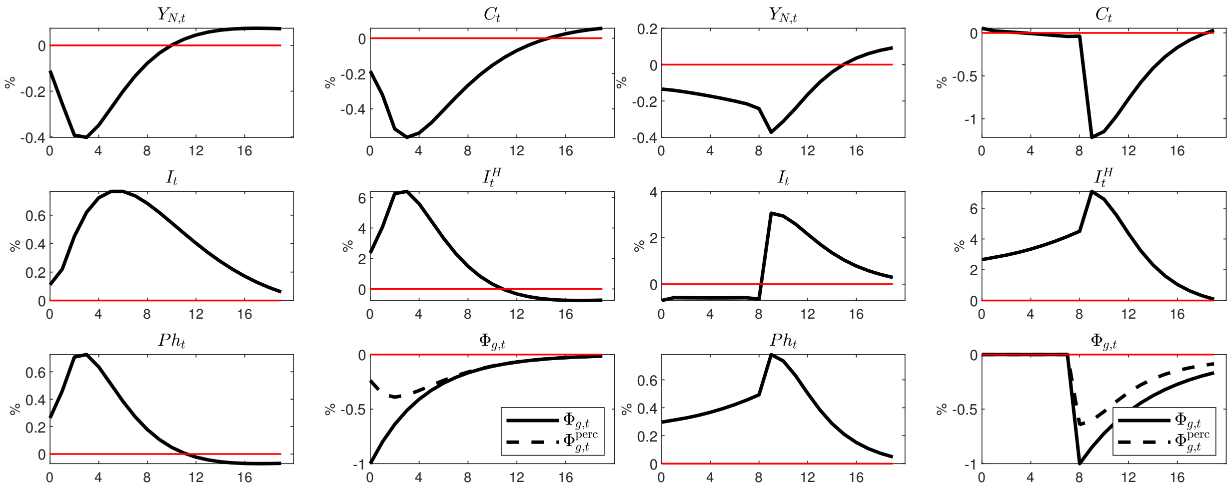
Moments	**Description**	**Data**	**Model**
σ_Y	Standard Derivation of output (non-durable goods production)	0.01	0.018
σ_{p^H}	Standard Derivation of real estate price	0.018	0.017
$\frac{\sigma_I}{\sigma_Y}$	Relative Standard Derivation between physical investment and output	4.74	4.28
$\frac{\sigma_{I^H}}{\sigma_Y}$	Relative Standard Derivation between new construction and output	8.66	8.59
$\text{cov}(p^H, I^H)$	Covariance between real estate price and new construction	0.00088	0.00097
$\text{cov}(I, Y)$	Covariance between physical investment and output	0.00023	0.00074

4.3 Quantitative Analysis

4.3.1 Overbuilding and Boom-Burst Cycle: News to the Future and Inefficiency of imperfect information

When a contemporaneous preference shock realized, household will decrease their nondurable goods consumption to exchange for more durable goods consumption, housing servicing because they prefer the real estate to the nondurable goods now. This altered preference draws the housing price up because of a demand curve shift to the right, which in the end generates a housing market boom which is shown in figure 9a. A one unit growth shock to preference is only perceived with 0.5 at the peak by the household. The household increase their consumption to housing servicing and this jump in demand increases the construction to the peak of 3 and house price to the peak of 0.6.

However if the shock is known by the household ahead of time when it realized, the household will response to this future shock when he known the realization news. They will increase the holding of house right away which pushes up the house price immediately. This will crowd



(a) (b)

Figure 9: Contemporaneous and News shock

out the physical investment through general equilibrium cycle if the nondurable consumption does not change. Further the household will also increase their consumption either because they are wealthier now fueled by the real estate appreciation or because they can borrow more fund from bank. This will magnify the crowd-out effect as nondurable consumption also entries into the goods market cleaning condition. Figure 9b shows this crowd-out effect triggered by a new shock. After observing a news about preference shock 8 periods later, household increase their holding of housing and there is a boom in housing market. They also increase the nondurable consumption but crowd out the physical investment.

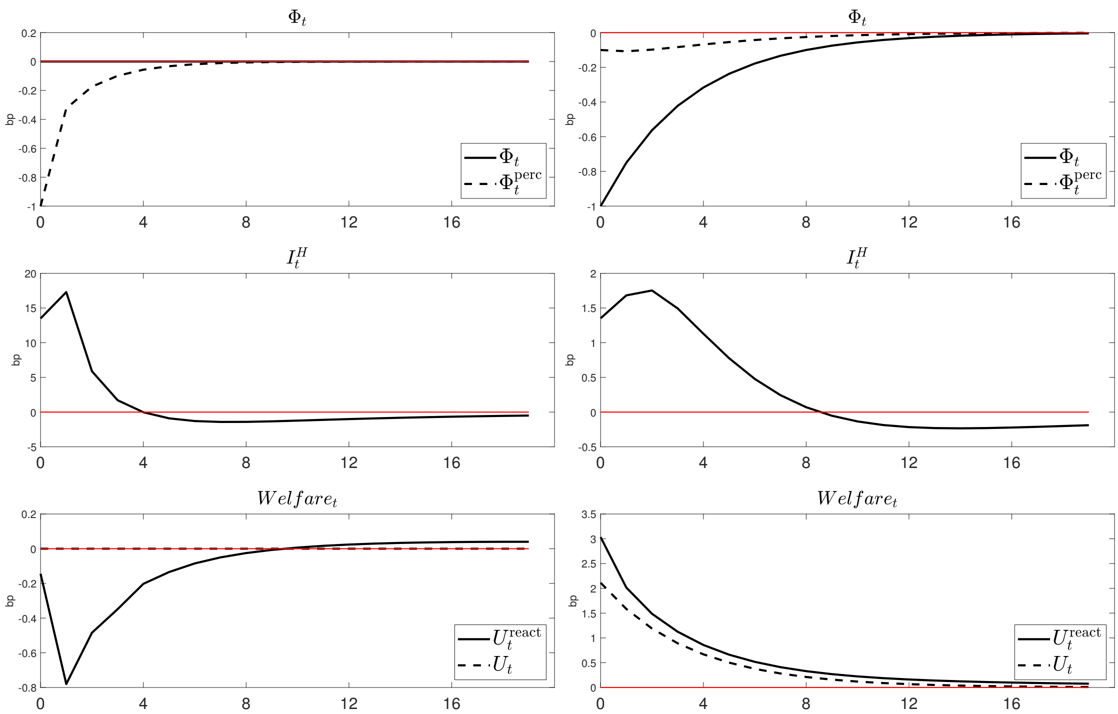


Figure 10: Welfare Loss in Imperfect Information

When the housing market boom is a bubble that is blown by a phantasm, the inefficiency of imperfect information could incur a welfare loss. Figure 10 illustrates the welfare loss caused by imperfect information. RHS is the response of investment and aggregate utility(with unit weight) to a preference shock on nondurable goods. By observing the decrease in contribution of nondurable goods to utility, the household has a perception of this preference shock as which I denotes the dash line in the first row. Because the housing service provides more utility now the household will increase their consumption to housing service and the aggregate welfare jump to 3bp which is shown in the solid line below. If the household does not response to the shock with zero derivation all the time, they will have a relative loss in welfare comparing to the situation that they react because the shock really happened and it is optimal to response to it. Although the household has an absolute increase in welfare because of the distribution effect and existence of hand-to-mouth household. Opposite to the realized preference shock, LHS of figure 10 shows the response of investment and aggregate welfare to a noise shock or observation shock. The household still increase their consumption to housing service because they thought that a preference shock has happened and they loss in welfare from this inappropriate reaction which I denote the solid line in the last row. If they did not react to the noise shock their welfare would have no change at all because nothing had happened which is shown by dash line. The experiment above corroborates the inefficiency of imperfect information as people misleadingly proceed housing market boom and I show that the noise in news, or fake news, can induce a further loss in output and consumption because of crowded-out physical capital.

4.3.2 Overbuilding and Boom-Burst Cycle: Fake News

When a pure noise(observation) shock instead of fundamental shock is informed to household, they would response to this shock as what they did to the fundamental shock because of the existence of information friction. Household cannot know the exact magnitude of the shock but a signal contaminated with noise. They response to what they perceived, or in other words their belief, instead of the fundamental shock. Therefore as long as the household believe there is an housing market boom in the future, they will increase their holding of housing service and crowd out the physical investment, which is a chronic poison to them as long as their belief is incorrect and the housing market boom is built on the Babylon tower. When the household across the manifest they need invest more physical capital because they temerariously exchange the physical capital to real estate just before. This large demand to physical capital results in a huge drop in nondurable consumption which follows a heavy loss in welfare. Additionally because the real estate is also a type of wealth which the household used to borrow money from bank, a housing market burst and a deep deflation in house price break the consumption pattern of low-income household and leave them at financial constrained edge, which leads to a further welfare loss.

Figure 11 compares the impulse response to the fake-news preference shock with and without

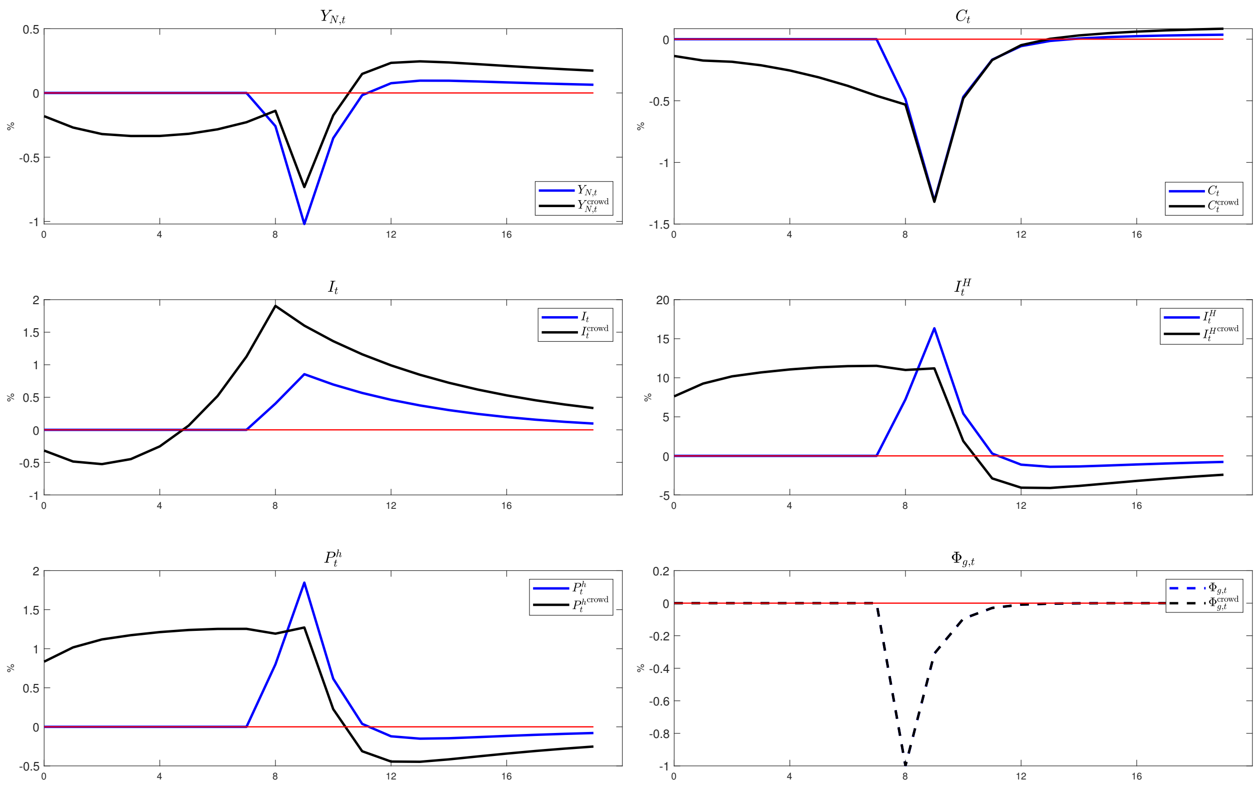


Figure 11: Fake news shock

pre-crowded physical capital which demonstrates the large output and welfare loss engendered by crowd-out effect. The blue solid lines are the responses to a contemporaneous noise shock $\widetilde{\Phi}^{\phi}_{g,t}$ of non-durable goods' production, non-durable goods consumption, physical investment, new construction and real housing price. The black solid lines represent the responses of them to noisy news $\widetilde{\Phi}^{\phi}_{g,t+8}$ which is informed to household 8 period ago. When the household knows that there will be an economic boom in the future, they increase the investment in real estate and induce a housing market boom immediately. Because all the household already hold some amount of real estate, this housing market boom spurs higher non-durable goods consumption because of the wealth effect, which is in line with Mian et al. (2013). This further crowds out the physical investment which is shown by the negative response in 11. After the shock "should" realized two years later, at period 9, household are gradually aware the true and increase the physical capital investment a lot to compensate the scarcity of capital caused by crowd-out effect from negative 2 percentage to positive 6 percentage. The burst in housing market leads to a 2.5 percentage drop in housing price and 2 percentage drop in non-durable goods consumption. On the other hand, if the physical capital is not pre-crowded, the economy response is mild and moderate with smaller output loss, consumption privation and housing market bust. There are only half of the loss in non-crowded scenario relative to crowded scenario. Similarly there are only two third of the boom-burst cycle in housing price and new constructions in the non-crowded situation. The difference in impulse response demonstrates the non-negligible drawback of the crowded-out effect in the housing market boom-and-burst cycle.

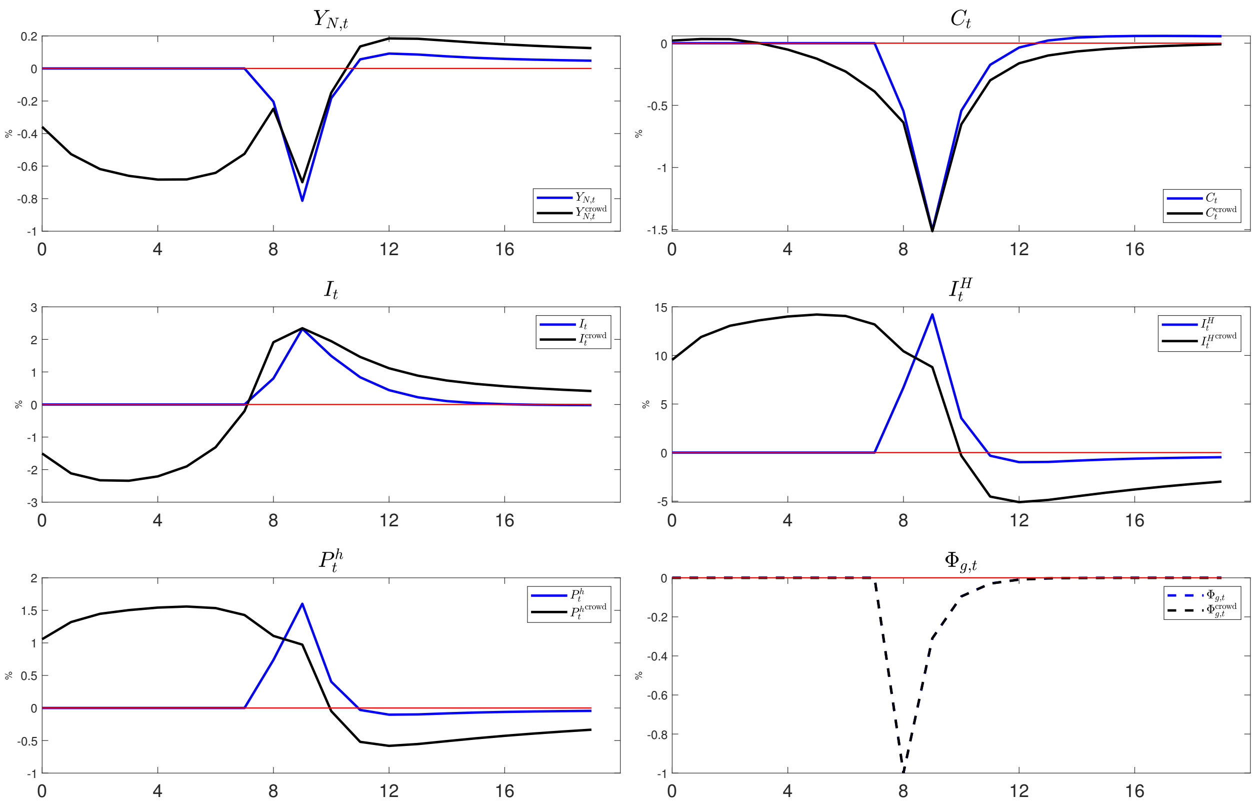


Figure 12: Fake news shock

4.3.3 Idiosyncratic Income shock, Financial Friction, Relative intratemporal elasticity of substitution

To investigate how the crowded-out effect is influenced by the idiosyncratic income shock, financial friction and relative intratemporal elasticity of substitution, I fix the expected jump in house price and change the relative parameters in model in this section. By decreasing the relative intratempral elasticity of substitution with the same amount in section 3.2.1, the blue dash line in figure 13 illustrates the attenuation caused by the relative intratemporal elasticity of substitution. A smaller relative intratemporal elasticity of substitution, from $\text{ES} - \text{EIS} = \frac{1}{2}$ to $\frac{1}{3}$, results in a huge physical investment drop, roughly 3 times larger than that in baseline model. Given this smaller intratemporal elasticity of substitution, household will care less about the substitution in utility between non-durable goods and housing service (in other words more complementarity) which result in a larger increase non-durable goods consumption in figure 13. These lead to a lower investment in physical capital via general equilibrium.

The red dash line in figure 13 depicts the response under a tight credit constraint, which implies an important role of wealth inequality. As shown in section 3.2.2, if we do not consider the wealth distribution (i.e. $\int \widetilde{h_t}^i dG_i$ and $\int \widetilde{\mu}_t^i dG_i$ in equation 15) a tighter financial constraint will result in a severer crowded-out problem because the real estate is more valuable now. However, as shown in section 3.2.3, household cannot increase their non-durable consumption and housing service as much as they want because of financial constraint and wealth inequality.

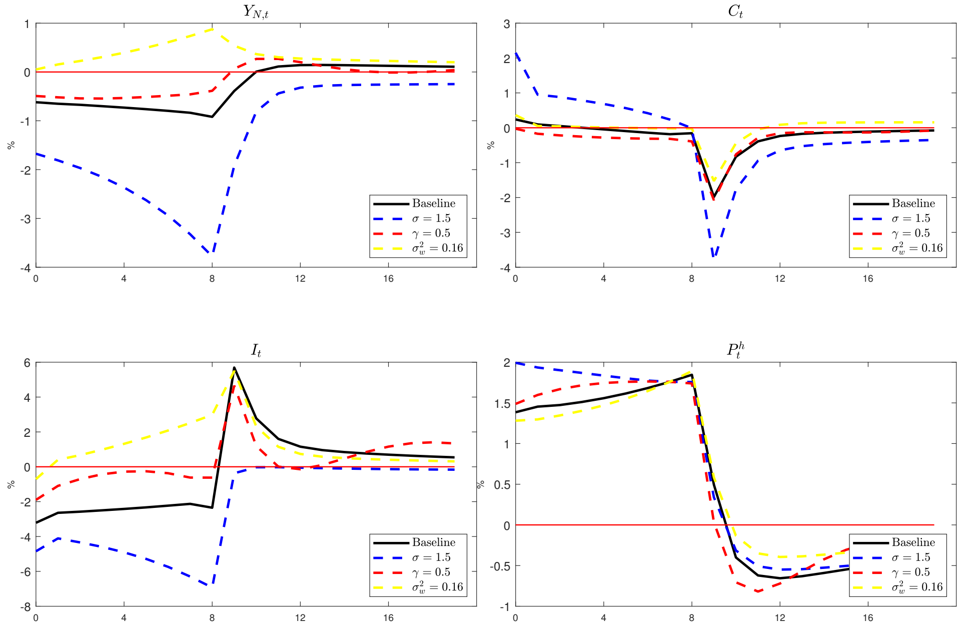


Figure 13: Crowded-out effect comparison

The larger $\widetilde{h}_t^i$ can only be realized in a smaller dG_i and figure 13 shows that this inequality channel dominates other channels. The physical capital is crowded out less than that in baseline model as there are more overwhelmed household who cannot increase their consumption as much as they want.

Additionally I increase the variance of idiosyncratic income shock from $\sigma_w^2 = 0.06$ in baseline model to 0.16 which I characterize with yellow dash line in figure 13. Facing a massive income shock, household will have more precautionary saving motive to hold the asset (to fulfill their consumption demand against potential low income and cash flow state) instead of borrowing money to buy housing services. Even though the household expect a housing market boom they only slightly decrease the physical capital at the first period and then increase until the shock realized. The reason why the physical capital further jumps is that household want to hold more housing services under the effect of expected shock. However they do not want to borrow money and decrease their holding of asset to buy real estate. They can only increase their labor supply to earn more wage income to buy housing services. The complement between labor and physical capital tempts the household to increase their asset instead of decreasing them with a higher asset return, which triggers a positive feedback loop on the boom in physical capital.

5 Conclusion

This paper documents a new mechanism through which the housing market boom magnifies the recession. An unnecessary jump in residential construction arouse by fake news and imperfect information will blow up a bubble in housing market which is a boom without solid inner filler and not supported by economic foundation. This overbuilding in housing market crowds out physical capital which is used to produce both durable and nondurable goods. The crowd-out effect in physical capital market aggravates the decline in output when the housing market bubble bursts because of the deficiency of physical capital. Firms do not have as much as capital they can use to support the optimal production under a specific level of TFP so they will decrease production and labor demand when facing a higher real interest rate and marginal production cost. I use a simple model to argue theoretically that the crowd-out effect of overbuilding is affected by relative intratemporal elasticity of substitution, financial friction, idiosyncratic income shock and wealth distribution. Later the quantitative result from a full-fledged model verifies the argument and demonstrates that the output loss caused by overbuilding is large.

However there are still some problems left for future studies. Even though the imperfect information does not exist the overbuilding and crowd-out effect may still be a significant drawback in the perspective of business cycle as it increases the economic volatility and household leave their first-best equilibrium further. Additionally how can the government introduce an optimal fiscal, monetary, or macroprudential policy to alleviate the crowd-out effect of overbuilding? Is there any complementarity between overbuilding and nominal rigidity in New Keynesian model which will further exacerbate the defect of overbuilding and crowd-out effect?

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A Data Description

Real GDP Y_t is directly downloaded listing "Real Gross Domestic Product" with seasonally adjusted. Real consumption C_t is directly downloaded listing "Real personal consumption expenditures: Nondurable goods" with seasonally adjusted. GDP deflator gdp_{def} is downloaded listing "GDP Implicit Price Deflator in United States" with seasonally adjusted. Nominal nondurable investment I_t^{nom} is downloaded listing "Private Nonresidential Fixed Investment" with seasonally adjusted. I get the real nondurable investment I_t by the formula $I_t = I_t^{\text{nom}}/\text{gdp}_{def}*100$. The CPI which we take is "Consumer Price Index for All Urban Consumers: All Items Less Shelter in U.S. City Average" since we should consider the correlation between house price and normal CPI. Thus we downloaded the CPI without shelter term. I take the nominal interest rate R_t^{nom} as "Effective Federal Funds Rate". The inflation rate is calculated from the GDP defltor in the form that $\pi_t = \frac{def_t - def_{t-1}}{def_{t-1}}$ (Since we solve the inflation from deflator in quarterly data, the inflation is measured within one quarter instead of annually). Combining the inflation π_t and nominal interest rate R_t^{nom} we can construct the real interest rate $R_t = (\frac{R_t^{\text{nom}}}{100} + 1)/(1 + \pi_t) - 1$ (I divided 100 because the original data is in percentage unit). The house supply H_t is measured by "New Privately-Owned Housing Units Started: Total Units". The nominal mortgage debt MD_t^{nom} comes from "Mortgage Debt Outstanding, All holders (DISCONTINUED)". Since the nominal mortgage debt is in money unit, I can directly get the real mortgage debt value from $MD_t = MD_t^{\text{nom}}/\text{gdp}_{def} * 100$ which is same as we did to get real investment. The real stock price P_t^a is calculated from "NASDAQ Composite Index" and normalized by GDP deflator as I did in constructing real investment and real mortgage debt. The real house price P_t^h is calculated from "All-Transactions Indexes" collected by Federal Housing Finance Agency.

B VAR and identification

B.1 Identification with sign and zero restricution

Based on the observation and argument, I use a simple SVAR model to decompose the effect of raised house price to investment. Given the model which is similar to Sims et al. (1986)

$$\boldsymbol{y}_t = \boldsymbol{c} + \boldsymbol{\Phi}_1 \boldsymbol{y}_{t-1} + ... + \boldsymbol{\Phi}_p \boldsymbol{y}_{t-p} + \boldsymbol{e}_t \tag{23}$$

where

$$\boldsymbol{y_t} = \begin{bmatrix} r_t \\ m_t \\ y_t \\ p_t \\ i_t \\ p_t^h \\ c_t \end{bmatrix} \tag{24}$$

r_t is the nominal interest rate; m_t is the money supply; y_t is the real output; p_t is the price level; i_t is the nominal investment; p_t^h is the nominal price of house; c_t is the real consumption of non-durable goods. Most the data comes from FRED, Federal Reserve Bank of St. Louis. I use treasury bill rate represents the nominal interest and GDP deflator for the price level. The price of house comes from FHFA house price index. The detail about it will be discussed at appendix. Meanwhile I use the short-run restriction as well as corresponding sign restriction to decompose the shock term $\boldsymbol{e_t}$ from $\boldsymbol{v}_t$ that

$$\boldsymbol{Pe_t} = \boldsymbol{v_t} \tag{25}$$

or detailedly

$$Pe_t \equiv \begin{vmatrix} 1 & b_{11} & 0 & 0 & 0 & 0 & 0 \\ b_{21} & 1 & b_{23} & b_{24} & 0 & 0 & 0 \\ b_{31} & 0 & 1 & 0 & b_{35} & 0 & b_{37} \\ b_{41} & b_{42} & b_{43} & 1 & b_{45} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & b_{56} & 0 \\ b_{61} & 0 & b_{63} & b_{64} & 0 & 1 & 0 \\ b_{71} & 0 & b_{73} & 0 & 0 & b_{76} & 0 \end{vmatrix} \begin{bmatrix} e_{rt} \\ e_{mt} \\ e_{yt} \\ e_{pt} \\ e_{it} \\ e_{p^h t} \\ e_{ct} \end{bmatrix} = \begin{bmatrix} v_{rt} \\ v_{mt} \\ v_{yt} \\ v_{pt} \\ v_{it} \\ v_{p^h t} \\ v_{ct} \end{bmatrix}$$

Figure 14 shows the IRF of one unite positive house price shock to output, investment, house price and non-durable goods consumption. The black line is the path of related variable up to 20 period. The read dash line is their related confidence band under 90% calculating by monte-carlo method. We can inspect from IRF that, house price inflation could stimulate the consumption of durable goods as it is long-lasting goods and household could derive out utility by just holding it. The household could feel satisfy and pleased either via living in this house or via owning the house which is valuable every period. Meanwhile the household can obtain utility not only from just holding and enjoying it each period, but also from financial market. The house is a goods that could be consumed. While at the same time it is also a asset that could be collateral and offers more liquidity to household. Household would use this liquidity to smooth their non-durable consumption leisurely, which provide extra benefit to household.

Therefore after observing one unit positive shock in house price, household snap up the house as house it not only a goods but also an asset which we discuss before. This increased demand

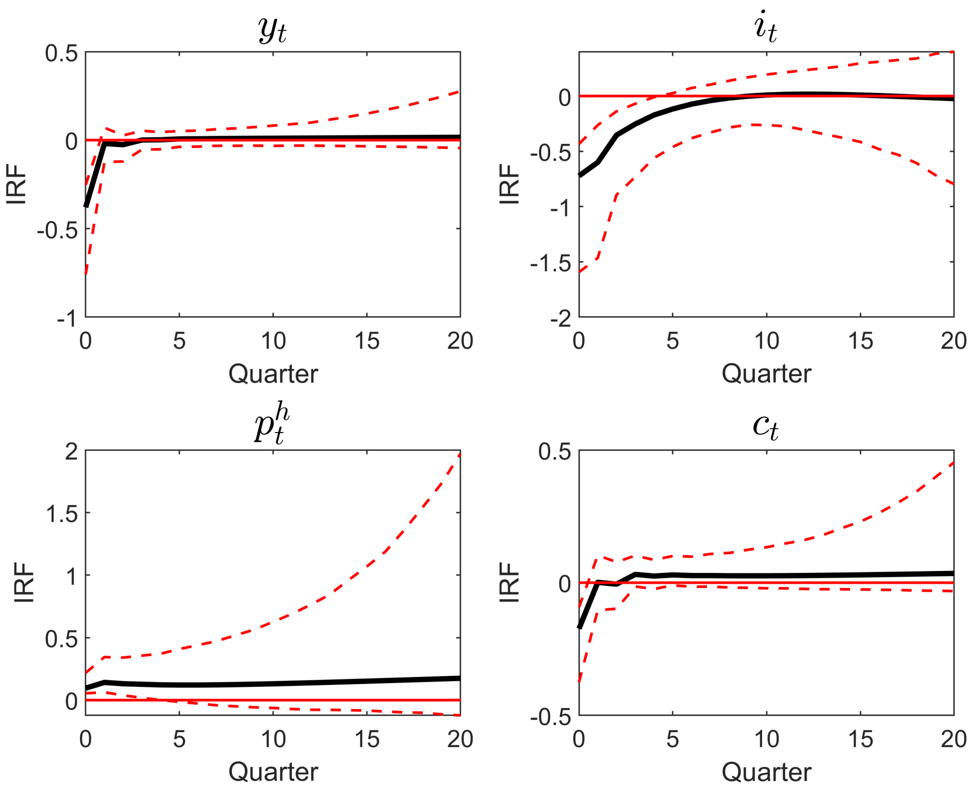

Figure 14: IRF of house price inflation

draw up the house price even more which we can see the house price is raising not only lat the beginning but also later. The house price in the end permanently increased because of increased household demand. This increased house price stimulates household who would borrow more from bank to buy house (the house supply discontinuity will aggravate this channel) or borrow more to help them share the risk as collateral is more expensive. Firms will be more difficult to borrow money to invest and the decreased demand in non-durable goods will also weaken firms' propensity to invest or R&D. Investment is crowded out by this two effects and this is what we can observe from the IRF. Investment drops the most and also spends longest time to recover. Output and non-durable consumption stands behind it. However both of them go back to steady state quickly which indicates that only the first jump in house price affects them. Later household use their more valuable collateral to smooth the consumption as well as output. Thus these two variable converge back quickly while because of strong and amplified effect both in demand and supply side, investment converges much slower than other two variables. This portends that there would be much larger drop in output if recession occurs because the accumulated decreased investment will pass its influence through the capital, a long-lasting things, later.

B.2 Contemporaneous real price shock

B.2.1 Process of estimation and identification

I detrend the main variable by taking logarithm first and first-order difference later. Then I get the detrended real GDP, real consumption, real investment, cpi, house supply, real mortgage debt, stock price and house price in lower-case letter. Then I ordered them in the vector

$$Y_t = [y_t, c_t, i_t, cpi_t, r_t, p_t^a, hs_t, md_t, p_t^h]'$$

I use the data period between 1987Q2 and 2006Q4. Then I add lagged term into the model up to 4 quarter and estimate the model

$$Y = [Y_5, Y_6 ...]$$

$$X_{t-1} = [y_{t-1}, c_{t-1}, i_{t-1}, cpi_{t-1}, r_{t-1}, p_{t-1}^a, hs_{t-1}, md_{t-1}, p_{t-1}^h, y_{t-2}, c_{t-2}, ..., p_{t-4}^h]'$$

$$X = [\mathbf{1}, X_4, X_5, ...]$$

Then use the projection matrix we can solve the factor that

$$\hat{\Phi} = YX'(XX')^{-1}$$

The residue is

$$\hat{e} = Y - \Phi X$$

and the variance of estimation error would be

$$\hat{\Omega} = cov(\hat{e}')$$

To simulate the model we can rewrite the variables into companion form such that

$$\boldsymbol{Y_t} = [y_t, c_t, i_t, cpi_t, r_t, p_t^a, hs_t, md_t, p_t^h, y_{t-1}, c_{t-1}, ..., p_{t-3}^h]'$$

Denote $\hat{P} = \text{chol}(\hat{\Omega})$ and

$$\hat{\boldsymbol{\Phi}} = \begin{bmatrix} \Phi_1 & \Phi_2 & \Phi_3 & \dots & \Phi_p \\ I_n & 0 & 0 & \dots & 0 \\ 0 & I_n & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & I_n & 0 \end{bmatrix}$$

where Φ(:,2:end) $= [\Phi_1 \Phi_2 \Phi_3 \dots \Phi_p]$ since I have intercept coefficient term with $\mathbf{1}$ in X.

Meanwhile we define

$$\hat{\boldsymbol{P}} = \begin{bmatrix} \hat{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$$

The shock term is

$$\nu_{n\times 1} = [0, 0, ..., 1]'$$

which means there is only one unit shock happened at house price row.

Similarly I should write it in companion form such that

$$\boldsymbol{\nu} = [\nu, \mathbf{0}]$$

Then we can get the IRF that

$$\mathrm{IRF}_t = \hat{\boldsymbol{\Phi}}^t \hat{\boldsymbol{P}} \boldsymbol{\nu}$$

where $t = 0, 1, 2, ..., 20$.

Finally we only take first 1 to n items in IRF_t. Since I take first-order difference to most of the data, at this stage I also calculate the cumsum of IRF to return the accumulated response.

B.2.2 Contemporaneous shock under larger confidence band

B.2.3 News shock under larger confidence band

B.3 Analytical Solution to Identification Problem

In this section I provide the derivative process related to news-shock identification in section 2.2.

B.3.1 Proof to Proposition 1

The problem I want to solve is

$$\underset{R}{max} \sum_{h=0}^{H} \frac{e_N' \left(\sum_{\tau=0}^{h} \Phi^\tau P R R' P' \Phi'^\tau\right) e_N}{e_N' \left(\sum_{\tau=0}^{h} \Phi^\tau P P' \Phi'^\tau\right) e_N} \tag{26}$$

s.t

$$R'R = 1$$
$$e_N' \hat{P} R = 0$$

The Lagrangian of above problem is

$$\mathcal{L}(R) = \sum_{h=0}^{H} \frac{e_N' \left(\sum_{\tau=0}^{h} \Phi^\tau P R R' P' \Phi'^\tau\right) e_N}{e_N' \left(\sum_{\tau=0}^{h} \Phi^\tau P P' \Phi'^\tau\right) e_N} + \lambda_1 (R'R - 1) + \lambda_2 P_N' R$$

The FOC is

$$R'\Xi + 2\lambda_1 R' = 0 \tag{27}$$

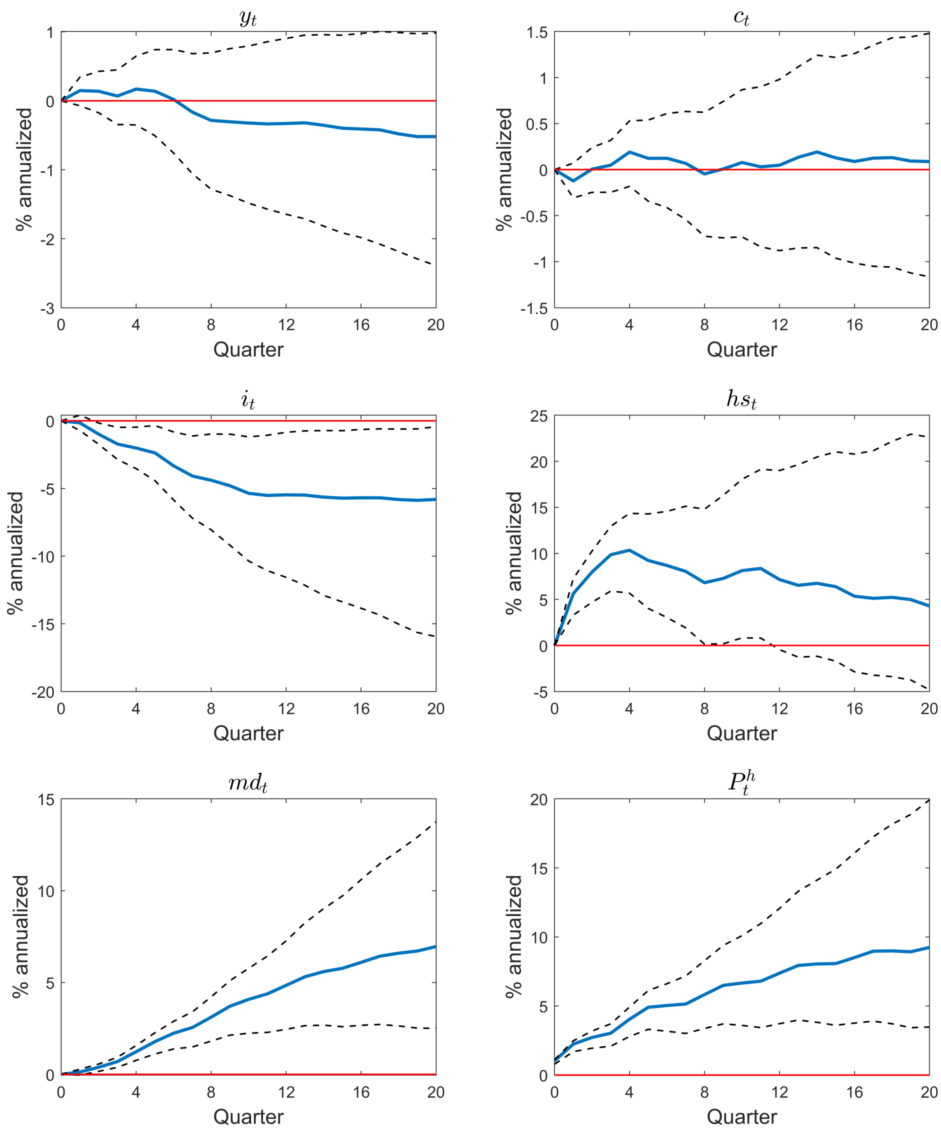

Figure 15: IRF with 90% confidence band

where

$$\Xi = \sum_{h=0}^{H} \frac{2\sum_{\tau=0}^{h} P'\Phi'^{\tau} e_N e_N' \Phi^{\tau} P}{\sum_{\tau=0}^{h} e_N' \Phi^{\tau} P P' \Phi'^{\tau} e_N}$$

Multiplying R on RHS of equation 27 yields

$$\lambda_1 = -\frac{1}{2} R'\Xi R \tag{28}$$

Finally plugging equation 28 back to equation 27 will give us

$$R'\Xi\left(I - RR'\right) = 0 \tag{29}$$

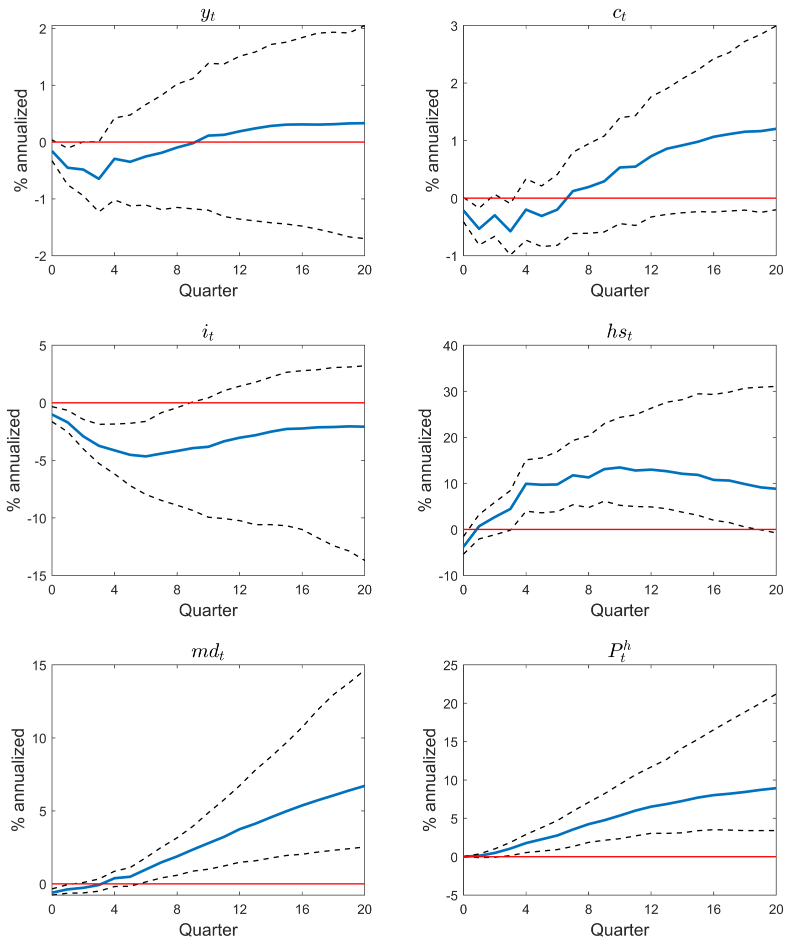

Figure 16: IRF with 90% confidence band

It is worth to notice that R is the eigenvector of RR' whose corresponded eigenvalue is 1 as $\|R\| = 1$. Therefore equation 29 can be written as a NARE(non-symmetric algebraic continuous time Riccati equation)

$$\Theta\Lambda - \Theta\Xi\Theta = 0 \tag{30}$$

where $\Theta = RR'$, $\Lambda = \Xi$. This NARE 30 cannot be solved by algorithms such as Maarouf (2017) and Shirilord and Dehghan (2022). However it could become a standard CARE(symmetric continuous time algebraic Riccati equation) $\Theta\Lambda + \Lambda'\Theta - 2\Theta\Xi\Theta = 0$ because Θ, Λ and Ξ are all symmetric matrix which requires less time to be derived.

To show the identification problem 26 is unique, I just need to show that equation 29 has an unique solution R. The uniqueness of solution to equation 29 is also equivalent to the uniqueness of orthogonal matrix $Z = I$ that satisfies $R'Z'\Xi\left(I - ZRR'Z'\right) = 0$. Otherwise there is a new solution $\tilde{R} = ZR$, on top of R, satisfying the equation 29.

Lemma 1. *There is an unique orthogonal matrix Z that satisfies $R'Z'\Xi\left(I - ZRR'Z'\right) = 0$.*

Proof. Multiplying any orthogonal matrix $Z \in \mathbb{R}^N$ on RHS of equation 29 yields

$$R'\Xi\left(Z' - RR'Z'\right) = 0$$

To ensure that there exist a new $\tilde{R} = ZR$ which makes equation 29 hold, we just rearrange above equation to $R'Z'Z\Xi Z'\left(I - ZRR'Z'\right) = 0$ which is equivalent to the condition

$$\left(Z \otimes Z\right) \text{vec}\left(\Xi\right) = \text{vec}\left(\Xi\right)$$

Therefore if $Z \otimes Z \neq I$, $\text{vec}\left(\Xi\right)$ should be the kernel of $Z \otimes Z - I$ because Lemma 2 shows that Ξ is well-defined. However because Z is an orthogonal matrix $Z \otimes Z - I$ should be well-defined either and its null space is empty, which contradict the well-defined Ξ. Therefore we must have $Z \otimes Z = I$ which implies $Z = I$ and Z is unique. $\square$

Lemma 2. *The matrix Ξ is well defined as long as $\frac{(H+1)(H+2)}{2} \geq N$*

Proof. It is easy to see the matrix Ξ is well defined as long as $\frac{(H+1)(H+2)}{2} \geq N$. Even through $\text{rank}(e_N e_N') = \text{rank}(P'\Phi'^{\tau} e_N e_N' \Phi^{\tau} P) = 1$, we have $\text{rank}\left(P'\Phi'^{\tau} e_N e_N' \Phi^{\tau} P | P'\Phi'^{\tau+i} e_N e_N' \Phi^{\tau+i} P\right) = 2, \forall i \neq \tau$. Therefore as long as $\frac{(H+1)(H+2)}{2} \geq N$ we must have $\text{rank}(\Xi) = N$ which means the matrix Ξ is well defined. $\square$

B.4 Alternative detrend Method

Alternatively I also use another method to deal with the data. I add the year number into the model to try to detrend the data. I marked the year with its "number" and add 0.1 to 0.4 on it as the label of quarter. Then I divided these "number" by 1000 to get a comfortable scalar. Specifically we take

$$Y_t = [t, t^2, t^3, y_t, c_t, i_t, cpi_t, r_t, p_t^a, hs_t, md_t, p_t^h]'$$

B.5 Confidence Band-MC Method

Here I explain the detailed steps that I used to calculate the confidence band of t he estimation using Monte Carlo method. Since there is no difference in steps between I estimate the confidence band in method I and method II, I only show the first part for simplicity.

I can calculate the estimated variance of the coefficient by

$$\hat{\sigma}_{\hat{\Phi}}^2 = \frac{\hat{\Omega} \bigotimes \left(\frac{XX'}{T}\right)^{-1}}{T}$$

Then I draw the coefficient simple $\tilde{\Phi}^{(b)}$ from the distribution

$$vec(\hat{\Phi}) \sim N\left(\text{vec}\left(\hat{\Phi}'\right), \hat{\sigma}_{\hat{\Phi}}^2\right)$$

At the same time the estimated variance of the coefficient variance would be

$$\hat{\sigma}^2_{\hat{\Omega}} = \frac{2D_n^+ \left(\hat{\Omega} \bigotimes \hat{\Omega}\right) D_n^{+\prime}}{T}$$

where $D_n^+ = \left(D_n' D_n\right)^{-1} D_n$ is the Moore-Penrose generalized inverse of duplication matrix D_n

I generate the variance simple $\tilde{\Omega}^{(b)}$ from the distribution

$$\text{vech}(\hat{\Omega}) \sim N\left(\text{vech}(\hat{\Omega}), \hat{\sigma}^2_{\hat{\Omega}}\right)$$

Then use the duplication matrix to transfer back to

$$\text{vec}(\tilde{\Omega}^{(b)}) = D_n \text{vech}(\tilde{\Omega}^{(b)})$$

C Perturbation result around the Simple Model

C.1 Proof of Proposition 4

The Lagrangian of the problem 8 could be written as

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t U^i\left(c_t^i, h_t^i\right) + \sum_{t=0}^{\infty} \lambda_t^i \left[R_t a_{t-1}^i + w_t \varepsilon_t^i + \left(1-\delta^H\right) p_t^H h_{t-1}^i + \pi_t^i + \pi_t^{H,i} - c_t^i - a_t^i - p_t^H h_t^i\right]$$
$$+ \sum_{t=0}^{\infty} \mu_t^i \left(p_t^H h_t^i + a_t^i\right)$$

I omit the superscript i henceforth for convenience. Then the first order condition would be

$$U_{c_t} = \lambda_t \tag{31}$$

$$-\lambda_t + \mu_t + \beta E_t R_{t+1} \lambda_{t+1} = 0 \tag{32}$$

$$U_{h_t} - \lambda_t p_t^H + \mu_t p_t^H + \beta\left(1-\delta^H\right) E_t \lambda_{t+1} p_{t+1}^H = 0 \tag{33}$$

To break the expectation I can rearrange the equation 33 as

$$U_{h_t} = \left(\lambda_t - \mu_t\right) p_t^H - \left(1-\delta^H\right)\left(\lambda_t - \mu_t\right) \frac{1}{E_t R_{t+1}} E_t p_{t+1}^H + \beta\left(1-\delta^H\right) \frac{cov\left(\lambda_{t+1}, R_{t+1}\right)}{E_t R_{t+1}} E_t p_{t+1}^H \tag{34}$$
$$- \beta\left(1-\delta^H\right) cov\left(\lambda_{t+1}, p_{t+1}^H\right)$$

Since the interest rate here is not related to the issue we want to solve, I further assume the exogenous TFP of non-durable goods production function is constant. Together with some assumption on the production function of durable and non-durable goods[27], $R_{t+1} = R_t = \overline{R}$ and $cov\left(\lambda_{t+1}, R_{t+1}\right) = 0$ will hold. Combining this assumption I log linearize equation 34 to get

$$\widetilde{U}_{h_t} = \frac{(\lambda-\mu)\left[p^H - \left(1-\delta^H\right)p^H\frac{1}{R}\right]}{U_h}\left\{\frac{\lambda}{\lambda-\mu}\widetilde{\lambda}_t - \frac{\mu}{\lambda-\mu}\widetilde{\mu}_t + \frac{p^H}{p^H - \left(1-\delta^H\right)p^H\frac{1}{R}}\widetilde{p}_t^H - \right. \tag{35}$$
$$\left. \frac{\left(1-\delta^H\right)p^H\frac{1}{R}}{p^H - \left(1-\delta^H\right)p^H\frac{1}{R}}\widetilde{p}_{t+1}^H\right\} - \frac{\beta\left(1-\delta^H\right)\overline{cov}}{U_h}\widetilde{cov}_t$$

where $\widetilde{cov}_t$ is the percentage derivation from steady state of $cov\left(\lambda_t, p_t^H\right)$

Then following Etheridge (2019) I expand U_{c_t} around its steady-state value U_c to get

$$U_{c_t} \approx U_c + U_{cc}c\widetilde{c}_t + U_{ch}h\widetilde{h}_t$$

I rearrange above equation to get

$$\frac{U_{c_t} - U_c}{U_c} = d\ln u_{c_t} = \widetilde{U}_{c_t} = \frac{U_{cc}c}{U_c}\widetilde{c}_t + \frac{U_{ch}h}{U_c}\widetilde{h}_t \tag{36}$$

Similarly expanding U_{h_t}around its steady-state value U_h gives

$$\frac{U_{h_t} - U_h}{U_h} = d\ln u_{h_t} = \widetilde{U}_{h_t} = \frac{U_{hc}c}{U_h}\widetilde{c}_t + \frac{U_{hh}h}{U_h}\widetilde{h}_t \tag{37}$$

Perturbing around its steady state for equation 31 returns

$$\widetilde{U}_{c_t} = \widetilde{\lambda}_t \tag{38}$$

Combining equation 35, 36, 37 and 38 I can solve out

$$\widetilde{c}_t = \left(\frac{\lambda}{\lambda-\mu}\eta_{c,p^H} - \eta_{c,p^c}\right)\widetilde{\lambda}_t - \frac{\mu}{\lambda-\mu}\eta_{c,p^H}\widetilde{\mu}_t + \eta_{c,p^H}\left[\frac{1}{1-\left(1-\delta^H\right)\frac{1}{R}}\widetilde{p}_t^H - \right.$$
$$\left.\frac{\left(1-\delta^H\right)\frac{1}{R}}{1-\left(1-\delta^H\right)\frac{1}{R}}\widetilde{p}_{t+1}^H\right] - \frac{U_{ch}}{U_{ch}^2 - U_{cc}U_{hh}}\frac{\beta\left(1-\delta^H\right)\overline{cov}}{c}\widetilde{cov}_t$$

Then plugging back equation 31 gives

$$\widetilde{c}_t = \frac{\frac{\lambda}{\lambda-\mu}\eta_{c,p^H} - \eta_{c,p^c}}{\eta_{h,p^c} - \frac{\lambda}{\lambda-\mu}\eta_{h,p^h}}\widetilde{h}_t - \frac{\mu}{\lambda-\mu}\frac{\eta_{ch}}{\eta_{h,p^c} - \frac{\lambda}{\lambda-\mu}\eta_{h,p^h}}\widetilde{\mu}_t + \frac{\eta_{ch}}{\eta_{h,p^c} - \frac{\lambda}{\lambda-\mu}\eta_{h,p^h}}\left[\frac{1}{1-\left(1-\delta^H\right)\frac{1}{R}}\widetilde{p}_t^H - \right.$$
$$\left.\frac{\left(1-\delta^H\right)\frac{1}{R}}{1-\left(1-\delta^H\right)\frac{1}{R}}\widetilde{p}_{t+1}^H\right] - \frac{\eta_c}{\eta_{h,p^c} - \frac{\lambda}{\lambda-\mu}\eta_{h,p^h}}\frac{\beta\left(1-\delta^H\right)\overline{cov}}{h}\widetilde{cov}_t$$

[27] The related assumptions are described at appendix E.1.1.

where η_{h,p^c}, η_{h,p^h}, η_{c,p^H}, η_{c,p^c}, η_{ch} and η_c are

$$\eta_{c,p^H} = \frac{u_{ch}u_h}{u_{ch}^2 - u_{cc}u_{hh}} \frac{1}{c}$$

$$\eta_{c,p^c} = \frac{u_{hh}u_c}{u_{ch}^2 - u_{cc}u_{hh}} \frac{1}{c}$$

$$\eta_{h,p^c} = \frac{u_{ch}u_c}{u_{ch}^2 - u_{cc}u_{hh}} \frac{1}{h}$$

$$\eta_{h,p^h} = \frac{u_{cc}u_h}{u_{ch}^2 - u_{cc}u_{hh}} \frac{1}{h}$$

$$\eta_{ch} = \frac{u_c u_h}{u_{ch}^2 - u_{cc}u_{hh}} \frac{1}{ch}$$

$$\eta_c = \frac{u_c}{u_{ch}^2 - u_{cc}u_{hh}} \frac{1}{c}$$

C.2 Derivation of the Definition of Intratemporal Elasticity of substitution 16

Firstly, following the standard procedure I first define the optimization problem

$$\underset{c,h}{max} u(c,h)$$

$$\text{s.t. } c + p^h h = y$$

where c is the consumption,p^h is the relative price of housing services and y is the exogenous income. The interior solution implies

$$p^h = \frac{u_h}{u_c}$$

which is used to define the intratemporal elasticity of substitution

$$\begin{aligned} \text{ES} &= -\frac{d\ln\left(\frac{c}{h}\right)}{d\ln\left(p^h\right)} \\ &= -\frac{d\ln\left(\frac{c}{h}\right)}{d\ln\left(\frac{U_c}{U_h}\right)} \end{aligned}$$

C.3 Proof of Proposition 5

I first use the same production function 20 and 21 which I defined at section 4. Since the sample model in section 3 is frictionless in adjusting housing and physical capital, the goods market clearing condition should be

$$\begin{aligned} Y &= Y_H + Y_N \\ &= C + I_N + I_H \end{aligned}$$

where $Y_H = I_H$ and $Y_N = C + I_N$

Combining equation 47 and the market clearing condition of capital I can get

$$\alpha Y_{N,t} + \nu P_t^H Y_{H,t} = (r_t + \delta)\, K_{t-1}$$

Taking differential on both side of above equation around their steady state will yield

$$\alpha dY_{N,t} + \nu Y_H dP_t^H + \nu P^H dY_{H,t} = 0$$

because the total capital K_{t-1} is predetermined and r_t is fixed by assumption. Further because the amount of total housing service at time $t-1$, H_{t-1} is predetermined, above equation can be rewritten to

$$\alpha dY_{N,t} + \nu Y_H dP_t^H + \nu P^H dH_t = 0$$

Plugging this back to goods market clearing condition will return the general equilibrium condition of crowd-out effect

$$-I_N \widetilde{I}_{N,t} = C\widetilde{C}_t + \frac{\nu}{\alpha} Y_H P^H \widetilde{P}_t^H + \frac{\nu}{\alpha} P^H H \widetilde{H}_t$$

Finally the equation 15 can be obtained by plugging equation 11 into above equation.

C.4 Proof of Corollary 1

If the household utility function follows the standard CRRA form

$$u_t = \frac{\left(\phi c_t^{\gamma} + (1-\phi) s_t^{1-\gamma}\right)^{\frac{1-\sigma}{1-\gamma}}}{1-\sigma}$$

Therefore the intratempral elasticity of substitution will be $\mathrm{ES} = \frac{1}{\gamma}$ and the intertemporal elasticity of substitution will be $\mathrm{EIS} = \frac{1}{\sigma}$ and $u_{ch} = \phi(1-\phi)(\gamma-\sigma)c^{\gamma-\sigma-1}h^{-\gamma}\left[\phi + (1-\phi)(\frac{h}{c})^{1-\gamma}\right]^{\frac{\gamma-\sigma}{1-\gamma}}$. Then based on the definition of relative force of substitution effect Φ_H and wealth effect Φ_{p^H} the prove process is straightforward.

C.5 Proof of Corollary 2

Iterating equation 33 forward with expectation at t on both side, I can eliminate the intra-price term until time $T+1$ with the chain rule of expectation

$$\begin{aligned} U_{h_t} + (\mu_t - \lambda_t) p_t^H + \beta \left(1-\delta^H\right) E_t \lambda_{t+1} p_{t+1}^H &= 0 \\ U_{h_{t+1}} + (\mu_{t+1} - \lambda_{t+1}) p_{t+1}^H + \beta \left(1-\delta^H\right) E_{t+1} \lambda_{t+2} p_{t+2}^H &= 0 \\ U_{h_{t+2}} + (\mu_{t+2} - \lambda_{t+2}) p_{t+2}^H + \beta \left(1-\delta^H\right) E_{t+2} \lambda_{t+3} p_{t+3}^H &= 0 \\ &\vdots \\ U_{h_{t+T}} + (\mu_{t+T} - \lambda_{t+T}) p_{t+T}^H + \beta \left(1-\delta^H\right) E_{t+T} \lambda_{t+T+1} p_{t+T+1}^H &= 0 \end{aligned} \tag{39}$$

Multiple $\frac{\beta\left(1-\delta^H\right)\lambda_{t+i}}{\lambda_{t+i}-\mu_{t+i}}$ on both side of above equation will yield (here I only take equation 39 as an example)

$$\frac{\beta\left(1-\delta^H\right)\lambda_{t+1}}{\lambda_{t+1}-\mu_{t+1}} U_{h_{t+1}} - \beta\left(1-\delta^H\right)\lambda_{t+1} p_{t+1}^H + \beta\left(1-\delta^H\right) \frac{\beta\left(1-\delta^H\right)\lambda_{t+1}}{\lambda_{t+1}-\mu_{t+1}} E_{t+1}\lambda_{t+2} p_{t+2}^H = 0$$

The last term can be rearranged to $\left[\beta\left(1-\delta^H\right)\right]^2 E_{t+1} \frac{\lambda_{t+1}}{\lambda_{t+1}-\mu_{t+1}} \lambda_{t+2} p_{t+2}^H$ because the term $\frac{\lambda_{t+1}}{\lambda_{t+1}-\mu_{t+1}}$ only contains the term at time $t+1$ which is known at time $t+1$. Then take expectation with the information at time t on both side of this equation to aggregate as

$$U_{h_t} + \mathbb{E}_t \sum_{i=1}^{T} \left[\beta\left(1-\delta^H\right)\right]^i \left[\prod_{s=1}^{i} \frac{\lambda_{t+s}}{\lambda_{t+s}-\mu_{t+s}}\right] U_{h_{t+i}} + \mathbb{E}_t \left[\beta\left(1-\delta^H\right)\right]^T \prod_{s=1}^{T} \frac{\lambda_{t+s}}{\lambda_{t+s}-\mu_{t+s}} \lambda_{t+T+1} p_{t+T+1}^H = 0$$

Equation 17 can be derived by take total differential on both side to above equation.

D Toy model with global solution

Given the budget constraint of household

$$c_0 + a_1 + p_0\left[s_1 - (1-\delta^h)s_0\right] = (1+R_0)a_0 + w_0 + \pi_0^h + \pi_0$$

$$c_1 + a_2 + p_1\left[s_2 - (1-\delta^h)s_1\right] = (1+R_1)a_1 + w_1 + \pi_1^h + \pi_1$$

$$c_2 = (1+R_2)a_2 + p_2(1-\delta^h)s_2 + w_2 + \pi_2^h + \pi_2$$

From utility function and FOC of household we can get the key equation

$$u_{c_0}\left[p_0 - \frac{1}{1+R_1}(1-\delta^h)p_1\right] = u_{s_1} \tag{40}$$

Then if we assume the utility function is non-separable such that

$$u_t = \frac{\left(c_t^{\nu} s_t^{1-\nu}\right)^{1-\sigma}}{1-\sigma}$$

By using the Euler equation of consumption as well as housing we can simplify equation 40 to

$$\left[p_0 - \frac{1}{1+R_1}(1-\delta^h)p_1\right] = \frac{c_1}{s_1^{\Phi} s_0^{\Psi}}$$

D.1 General equilibrium is important

A perturb happened at p_1 will decrease c_1 which in tern decrease c_2, If p_0, s_1 and R_1 not change.(This is the total effect of substitution and income as we derive from max utility which means from Marshallian demand function. This is pseudo-effect as we assume s_1 fixed)

However this analysis is based on the assumption that p_0, s_1 and R_1 will not change. Now we assume s_1 is not changed. Meanwhile the production is $Y_t = Aa_t$ so that $R_t = MPK = A$ which means R_1 will also be fixed. Which direction of p_0 changed?

The answer is that any small perturb increased happened in p_1 which returns $\tilde{p_1} = p_1 + \varepsilon$, p_0 will increase relative amount to make sure $p_0 - (1-\delta^h)p_1$ is fixed. This tells us that c_1 will in fact not change at all.[28]

Later we can also proof that given the decreasing return to scale production function such as $Y_t = Aa_t^{\alpha}$ will not change the result.

Intuition: Given p_1 increased, the household want to buy more s_1 at period 0. The fixed s_1 will caused p_0 increases a lot to even offset the wealth effect. If we assume s_1 increases and p_0 not change ($s_1$supply increased to the level that just fulfill the demand and p_0 does not change) the direction of c_1 will depends on the extent of increased s_1 and intratemperal substitution and intertemperal substitution). Another condition, p_0 increases more than related to $\frac{1}{1+R_1}(1-\delta^h)p_1$ is somehow less likely as an expectation causes a much higher inflation this period.

D.2 House supply is the key to determine non-durable consumption

Now we losse the assumption that s_1 does not change. From last section we know that under general equilibrium as long as the house supply does not increase, then no matter how large changed in p_1, c_1 will not change anymore because p_0 will adjusted one-to-one with it.

This give us the argument that the house supply or elasticity of house supply is much more important than scholar's focusing, as most of time we just take it as an IV in empirical research.

[28]The proof process is simple using induction. Given p_0 increases little but not enough to offset total decreased c_1. Then c_1 and c_0 will decreases little. Then using budget constraint, a_1 and $a_2$will relatively changed. Then to the final period we can get a contradiction. Inversely given p_0 increases a lot to result in c_1 increassing, we can get similar contradiction.

A right-hand shift in period 0 house demand(caused by a perturb in p_1) happened, the elasticity of house supply then determine the equilibrium changed in s. We have prove at previous section that when $e_1 = 0$, the increased p_0 will caused c_0 not change. In other words, under the most increased p_0, c_0 not changed. Then assume $e_1 > 0$, Δp_0 will decrease. LHS of equation 40 decrease. But because the intratemporal effect is larger than intertemporal effect, c_1 and c_0 will increase. In other words, the degree of elasticity of house supply determinate the non-durable consumption.

D.3 Unseparable utility function

D.3.1 partial effect

If the utility function is

$$u_t = \frac{\left(c_t^{\nu} s_t^{1-\nu}\right)^{1-\sigma}}{1-\sigma}$$

then we will have

$$s_0^{(1-\nu)(1-\sigma)} c_0^{\nu(1-\sigma)-1} = \beta R_1 s_1^{(1-\sigma)(1-\nu)} c_1^{\nu(1-\sigma)-1}$$

$$s_1^{(1-\nu)(1-\sigma)} c_1^{\nu(1-\sigma)-1} = \beta R_2 s_2^{(1-\sigma)(1-\nu)} c_2^{\nu(1-\sigma)-1}$$

$$\nu s_0^{(1-\nu)(1-\sigma)} c_0^{\nu(1-\sigma)-1} p_0 = \beta \nu s_1^{(1-\sigma)(1-\nu)} c_1^{\nu(1-\sigma)-1} p_1 (1-\delta^h) + \beta(1-\nu) c_1^{\nu(1-\sigma)} s_1^{\nu(\sigma-1)-\sigma}$$

$$\nu s_1^{(1-\nu)(1-\sigma)} c_1^{\nu(1-\sigma)-1} p_1 = \beta \nu s_2^{(1-\sigma)(1-\nu)} c_2^{\nu(1-\sigma)-1} p_2 (1-\delta^h) + \beta(1-\nu) c_2^{\nu(1-\sigma)} s_2^{\nu(\sigma-1)-\sigma}$$

Then we will solve out c_1, c_2, s_1, s_2 by these four equations

$$c_1 = \left[\frac{1}{\beta R_1}\right]^{\frac{(1-\nu)(1-\sigma)-1}{\sigma}} \left\{\frac{\nu}{1-\nu}\frac{1}{\beta}\left[p_0 - \frac{1}{R_1} p_1 \left(1-\delta^h\right)\right]\right\}^{-\frac{(1-\nu)(1-\sigma)}{\sigma}} \left[s_0^{(1-\nu)(1-\sigma)} c_0^{\nu(1-\sigma)-1}\right]^{-\frac{1}{\sigma}}$$

$$\begin{aligned}
s_1 &= \left\{\frac{\nu}{1-\nu}\frac{1}{\beta}\frac{s_0^{(1-\nu)(1-\sigma)} c_0^{\nu(1-\sigma)-1}}{c_1^{\nu(1-\sigma)}}\left[p_0 - \frac{1}{R_1} p_1 \left(1-\delta^h\right)\right]\right\}^{\frac{1}{(1-\nu)(1-\sigma)-1}} \\
&= \left[s_0^{(1-\nu)(1-\sigma)} c_0^{\nu(1-\sigma)-1}\right]^{-\frac{1}{\sigma}} \\
&\left\{\frac{\nu}{1-\nu}\frac{1}{\beta}\left[p_0 - \frac{1}{R_1} p_1 \left(1-\delta^h\right)\right]\right\}^{\frac{(1-\nu)(1-\sigma)}{(1-\nu)(1-\sigma)-1}\frac{\nu(1-\sigma)}{\sigma} + \frac{1}{(1-\nu)(1-\sigma)-1}} \left[\frac{1}{\beta R_1}\right]^{\frac{\nu(1-\sigma)}{\sigma}}
\end{aligned}$$

$$c_2 = \left[\frac{1}{\beta^2 R_1 R_2}\right]^{\frac{(1-\nu)(1-\sigma)-1}{\sigma}}$$
$$\left\{\frac{\nu}{1-\nu}\frac{1}{\beta^2 R_1}\left[p_1 - \frac{1}{R_2}p_2\left(1-\delta^h\right)\right]\right\}^{-\frac{(1-\nu)(1-\sigma)}{\sigma}} \left[s_0^{(1-\nu)(1-\sigma)} c_0^{\nu(1-\sigma)-1}\right]^{-\frac{1}{\sigma}}$$

$$s_2 = \left\{\frac{\nu}{1-\nu}\frac{1}{\beta^2 R_1}\frac{s_0^{(1-\nu)(1-\sigma)} c_0^{\nu(1-\sigma)-1}}{c_2^{\nu(1-\sigma)}}\left[p_1 - \frac{1}{R_2}p_2\left(1-\delta^h\right)\right]\right\}^{\frac{1}{(1-\nu)(1-\sigma)-1}}$$
$$= \left[s_0^{(1-\nu)(1-\sigma)} c_0^{\nu(1-\sigma)-1}\right]^{-\frac{1}{\sigma}}$$
$$\left\{\frac{\nu}{1-\nu}\frac{1}{\beta^2 R_1}\left[p_1 - \frac{1}{R_2}p_2\left(1-\delta^h\right)\right]\right\}^{\frac{(1-\nu)(1-\sigma)}{(1-\nu)(1-\sigma)-1}\frac{\nu(1-\sigma)}{\sigma}+\frac{1}{(1-\nu)(1-\sigma)-1}} \left[\frac{1}{\beta^2 R_1 R_2}\right]^{\frac{\nu(1-\sigma)}{\sigma}}$$

Under infinite horizon we will have

$$c_t = \left[\frac{1}{\beta^t \prod_{i=1}^t R_i}\right]^{\frac{(1-\nu)(1-\sigma)-1}{\sigma}}$$
$$\left\{\frac{\nu}{1-\nu}\frac{1}{\beta^t \prod_{i=1}^{t-1} R_i}\left[p_{t-1} - \frac{1}{R_t}p_t\left(1-\delta^h\right)\right]\right\}^{-\frac{(1-\nu)(1-\sigma)}{\sigma}} \left[s_0^{(1-\nu)(1-\sigma)} c_0^{\nu(1-\sigma)-1}\right]^{-\frac{1}{\sigma}}$$

$$s_t = \left[\frac{1}{\beta^t \prod_{i=1}^t R_i}\right]^{\frac{\nu(1-\sigma)}{\sigma}}$$
$$\left\{\frac{\nu}{1-\nu}\frac{1}{\beta^t \prod_{i=1}^{t-1} R_i}\left[p_{t-1} - \frac{1}{R_t}p_t\left(1-\delta^h\right)\right]\right\}^{\frac{(1-\nu)(1-\sigma)}{(1-\nu)(1-\sigma)-1}\frac{\nu(1-\sigma)}{\sigma}+\frac{1}{(1-\nu)(1-\sigma)-1}} \left[s_0^{(1-\nu)(1-\sigma)} c_0^{\nu(1-\sigma)-1}\right]^{-\frac{1}{\sigma}}$$

D.3.2 Other utility function

If the utility function is

$$u_t = log\left(c_t^\nu s_t^{1-\nu}\right)$$

then no GE effect

If the utility function is

$$u_t = log\left(c_t^\nu + s_t^{1-\nu}\right)$$

still unsolvable.

D.4 Standard utility function

D.4.1 general effect

No we assume that the utility function is no longer logarithmic such that

$$u_t = \frac{\left(c_t^{\nu} s_t^{1-\nu}\right)^{1-\sigma}}{1-\sigma}$$

Then we have two key market cleaning condition that

$$a_2 = A_1 a_1^{\alpha} - c_1 + (1-\delta)a_1 = A_1 a_1^{\alpha} - c_0 \left(\beta R_1\right)^{\frac{1}{1-\nu(1-\sigma)}} \left(\frac{s_0}{s_1}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}} + (1-\delta)a_1$$

$$(1-\delta)a_2 + A_2 a_2^{\alpha} = c_2 = c_0 \left(\beta^2 R_1 R_2\right)^{\frac{1}{1-\nu(1-\sigma)}} \left(\frac{s_0}{s_2}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}}$$

Based on these two equations we can rewrite equation as

$$(1-\delta)\left[A_1\left(A_0 a_0^{\alpha} + (1-\delta)a_0 - c_0\right)^{\alpha} - c_0\left(\beta\alpha A_1\left(A_0 a_0^{\alpha} + (1-\delta)a_0 - c_0\right)^{\alpha-1}\right)^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{s_1}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}} + \right.$$
$$\left.(1-\delta)\left(A_0 a_0^{\alpha} + (1-\delta)a_0 - c_0\right)\right] +$$
$$A_2\left[A_1\left(A_0 a_0^{\alpha} + (1-\delta)a_0 - c_0\right)^{\alpha} - c_0\left(\beta\alpha A_1\left(A_0 a_0^{\alpha} + (1-\delta)a_0 - c_0\right)^{\alpha-1}\right)^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{s_1}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}\alpha} + \right. \tag{41}$$
$$\left.(1-\delta)\left(A_0 a_0^{\alpha} + (1-\delta)a_0 - c_0\right)\right] =$$
$$c_0\left\{\beta^2\alpha^2 A_1 A_2\left(A_0 a_0^{\alpha} + (1-\delta)a_0 - c_0\right)^{\alpha-1}\right.$$
$$\left[A_1\left(A_0 a_0^{\alpha} + (1-\delta)a_0 - c_0\right)^{\alpha} - c_0\left(\beta\alpha A_1\left(A_0 a_0^{\alpha} + (1-\delta)a_0 - c_0\right)^{\alpha-1}\right)^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{s_1}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}}\right. \tag{42}$$
$$\left.\left.+(1-\delta)\left(A_0 a_0^{\alpha} + (1-\delta)a_0 - c_0\right)\right]^{\alpha-1}\right\}^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{s_2}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}}$$

Similarly we set $\alpha = 1$, equation 41 becomes

$$(1-\delta)\left[A_1\left(A_0a_0+(1-\delta)a_0-c_0\right)-c_0\left(\beta\alpha A_1\right)^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{s_1}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}}+\right.$$
$$\left.(1-\delta)\left(A_0a_0+(1-\delta)a_0-c_0\right)\right]+$$
$$A_2\left[A_1\left(A_0a_0+(1-\delta)a_0-c_0\right)-c_0\left(\beta\alpha A_1\right)^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{s_1}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}\alpha}+\right.$$
$$\left.(1-\delta)\left(A_0a_0+(1-\delta)a_0-c_0\right)\right]=$$
$$c_0\left(\beta^2\alpha^2A_1A_2\right)^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{s_2}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}}$$

Now we can solve the c_0 as

$$c_0=\frac{\left(A_2+1-\delta\right)\left(A_1+1-\delta\right)\left(A_0a_0+(1-\delta)a_0\right)}{\left(A_2+1-\delta\right)\left[A_1+1-\delta+\left(\beta\alpha A_1\right)^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{s_1}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}}\right]+\left(\beta^2\alpha^2A_1A_2\right)^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{s_2}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}}}$$
$$=\frac{\left(A_2+1-\delta\right)\left(A_1+1-\delta\right)\left(A_0a_0+(1-\delta)a_0\right)}{\left(A_2+1-\delta\right)\left[A_1+1-\delta+\left(\beta\alpha A_1\right)^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{(1-\delta^h)s_0+\bar{s}_1}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}}\right]+\left(\beta^2\alpha^2A_1A_2\right)^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{\bar{s}_2+(1-\delta^h)\bar{s}_1+(1-\delta^h)^2s_0}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}}}$$

Under the GE and determined economy, c_0 can only be decided by the equalized house stock. It is intuitive as in the end because all excess profit are payback by construction companies and consumption is mainly determined by IES & market cleaning condition. If we assume that good market clean does not involve construction industry, the house market can only affect the consumption via the Euler equation of asset. Here $\bar{s}_2$ decreases will lead p_2 increase, but it increase c_0 at the same time.

D.4.2 Infinite horizon condition

The market cleaning condition will be

$$a_1 = A_0a_0^\alpha + (1-\delta)a_0 - c_0$$

$$a_2 = A_1a_1^\alpha - c_1 + (1-\delta)a_1$$
$$a_3 = A_2a_2^\alpha - c_2 + (1-\delta)a_2$$
$$(1-\delta)a_\infty + A_\infty a_\infty^\alpha = c_\infty = c_0\left(\beta^3R_1R_2R_3\right)^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{s_3}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}}$$

$$
\begin{aligned}
c_0 &= \frac{\left(A_0 a_0 + (1-\delta)a_0\right)\prod_{t=1}^{\infty}\left(A_t + 1 - \delta\right)}{\sum_{t=1}^{T}\left[\prod_{i=t}^{T}\left(A_i + 1 - \delta\right)\right]\left(\beta^{t-1}\alpha^{t-1}\prod_{i=0}^{t-1}A_i\right)^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{s_{t-1}}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}} + \left(\beta^T\alpha^T\prod_{t=0}^{T}A_t\right)^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{s_T}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}}} \\
&= \frac{\left(A_0 a_0 + (1-\delta)a_0\right)\prod_{t=1}^{T}\left(A_t + 1 - \delta\right)}{\sum_{t=1}^{T}\left[\prod_{i=t}^{T}\left(A_i + 1 - \delta\right)\right]\left(\beta^{t-1}\alpha^{t-1}\prod_{i=0}^{t-1}A_i\right)^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{\sum_{i=0}^{t-1}(1-\delta^h)^i\bar{s}_i}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}} + \left(\beta^T\alpha^T\prod_{t=0}^{T}A_t\right)^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{\sum_{i=0}^{T}(1-\delta^h)^i\bar{s}_i}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}}}
\end{aligned}
$$

when normalizes $A_0 = 1$

D.5 Separable utility function

D.5.1 partial effect

$$
\begin{aligned}
c_1 &= c_0\left(\beta R_1\right)^{\frac{1}{\sigma}} \\
c_2 &= c_0\left(\beta^2 R_1 R_2\right)^{\frac{1}{\sigma}} \\
s_1 &= \left[p_0 R_1 - p_1(1-\delta^h)\right]^{-\frac{1}{\nu}} \\
s_2 &= \left[p_1 R_2 - p_2(1-\delta^h)\right]^{-\frac{1}{\nu}}
\end{aligned}
$$

$$
\begin{aligned}
c_0\left(\beta^2 R_1 R_2\right)^{\frac{1}{\sigma}} + R_2 c_0\left(\beta R_1\right)^{\frac{1}{\sigma}} + R_1 R_2 c_0 + \\
R_2 p_1\left\{\left[p_1 R_2 - p_2(1-\delta^h)\right]^{-\frac{1}{\nu}} - (1-\delta^h)\left[p_0 R_1 - p_1(1-\delta^h)\right]^{-\frac{1}{\nu}}\right\} + \\
R_1 R_2 p_0\left\{\left[p_0 R_1 - p_1(1-\delta^h)\right]^{-\frac{1}{\nu}} - p_0(1-\delta^h)\right\} = \\
R_0 R_1 R_2 a_0 + R_1 R_2\left(w_0 + \pi_0\right) + R_2\left(w_1 + \pi_1\right) + w_2 + \pi_2 \\
+ p_2(1-\delta^h)\left[p_1 R_2 - p_2(1-\delta^h)\right]^{-\frac{1}{\nu}}
\end{aligned}
$$

$$
\begin{aligned}
F_{p_1} = {} & R_2\left\{\left[p_1 R_2 - p_2(1-\delta^h)\right]^{-\frac{1}{\nu}} - (1-\delta^h)\left[p_0 R_1 - p_1(1-\delta^h)\right]^{-\frac{1}{\nu}}\right\} \\
& + R_2 p_1\left\{-\frac{1}{\nu}R_2\left[p_1 R_2 - p_2(1-\delta^h)\right]^{-\frac{1+\nu}{\nu}} - \frac{1}{\nu}(1-\delta^h)^2\left[p_0 R_1 - p_1(1-\delta^h)\right]^{-\frac{1+\nu}{\nu}}\right\} \\
& + \frac{(1-\delta^h)}{\nu}R_1 R_2 p_0\left[p_0 R_1 - p_1(1-\delta^h)\right]^{-\frac{1+\nu}{\nu}} + \frac{1}{\nu}p_2 R_2(1-\delta^h)\left[p_1 R_2 - p_2(1-\delta^h)\right]^{-\frac{1+\nu}{\nu}}
\end{aligned}
$$

$$
F_{c_0} = \left(\beta^2 R_1 R_2\right)^{\frac{1}{\sigma}} + R_2\left(\beta R_1\right)^{\frac{1}{\sigma}} + R_1 R_2
$$

D.5.2 general effect

$$
a_1 = A_0 a_0^{\alpha} + (1-\delta)a_0 - c_0
$$

$$
\begin{aligned}
a_2 = {} & A_1\left[A_0 a_0^{\alpha} + (1-\delta)a_0 - c_0\right]^{\alpha} - c_0\left[\beta\alpha A_1\left(A_0 a_0^{\alpha} + (1-\delta)a_0 - c_0\right)^{\alpha-1}\right]^{\frac{1}{\sigma}} \\
& + (1-\delta)\left[A_0 a_0^{\alpha} + (1-\delta)a_0 - c_0\right]
\end{aligned}
$$

we can solve c_0 by

$$(1-\delta)a_2 + A_2 a_2^{\alpha} = c_0 \left(\beta^2 \alpha^2 A_1 A_2 \left(a_1 a_2\right)^{\alpha-1}\right)^{\frac{1}{\sigma}}$$

which means it is predetermined.

E Equilibrium condition of the full fledged model

E.1 Focs

E.1.1 Focs in production sector

The non-durable goods producer solve the problem

$$\underset{K_n,L_n}{max} A_n K_{n,t}^{\alpha} L_{n,t}^{1-\alpha} - (r_t+\delta)K_{n,t} - wL_{n,t}$$

to yield the Foc

$$(1-\alpha)\, A_n K_{n,t}^{\alpha} L_{n,t}^{-\alpha} = w_t \tag{43}$$

and

$$\alpha A_n K_{n,t-1}^{\alpha-1} L_{n,t}^{1-\alpha} = r_t + \delta \tag{44}$$

Similarly the durable goods producer solve the problem

$$\underset{K_h,L_h}{max} \Pi^h = p_t^h A_h \overline{L}_t^{\theta} K_{h,t}^{\nu} L_{h,t}^{\iota} - (r_t+\delta)K_{h,t} - wL_h$$

to yield the Foc

$$\iota A_h p_t^h \overline{L}_t^{\theta} K_{h,t}^{\nu} L_{h,t}^{\iota-1} = w_t \tag{45}$$

and

$$\nu A_h p_t^h \overline{L}_t^{\theta} K_{h,t}^{\nu-1} L_{h,t}^{\iota} = r_t + \delta \tag{46}$$

Combine equation 44 and 46 will yield

$$\frac{\nu p_t^h Y_{H,t}}{K_{h,t}} = r_t + \delta = \frac{\alpha Y_{N,t}}{K_{n,t}} \tag{47}$$

It is easy to check that when $\frac{\iota}{\nu} = \frac{1-\alpha}{\alpha}$ the real rental rate and wage at time t is fixed, as long as the total capital used at time t, K_{t-1} and labor L_t is fixed. I attach the proof process below.

By dividing equation 43, 44, 45 and 46 with each other I can get the relative input sharing condition

$$\frac{\iota\alpha}{\nu\,(1-\alpha)} \frac{K_{h,t}}{K_{n,t}} \frac{L_{n,t}}{L_{h,t}} = 1$$

when $\frac{\iota}{\nu} = \frac{1-\alpha}{\alpha}$ holds, above equation will change to $\frac{K_{h,t}}{K_{n,t}} = \frac{L_{n,t}}{L_{h,t}}$.

Furthermore, the relative value of $K_{n,t}$ and $L_{n,t}$ can be pinned down with the market clearing condition $K_{H,t-1} = K_{h,t} + K_{n,t}$ and $L_t = L_{h,t} + L_{n,t}$. In section 3 I assume that the labor supply is exogenous which will help to demonstrate that the relative value of $K_{n,t}$ and $L_{n,t}$ follows

$$\frac{K_{n,t}}{L_{n,t}} = \frac{K_{H,t-1}}{L} \frac{1 + \frac{K_{n,t}}{L_{n,t}}}{1 + \frac{K_{h,t}}{L_{h,t}}}$$

Because $K_{H,t-1}$ is predetermined and $\frac{K_{h,t}}{K_{n,t}} = \frac{L_{n,t}}{L_{h,t}}$, the $\frac{K_{n,t}}{L_{n,t}}$ is fixed. Therefore r_t is fixed from equation 47.

E.1.2 Focs in consumer sector

The household solve the problem

$$V(h_{t-1}, x_{t-1}, \varepsilon_{t-1}) = \underset{h_t, x_t, l_t, c_t}{max} U(c_t, h_t, l_t) + \beta EV(h_t, x_t, \varepsilon_t)$$

$$\begin{aligned} \text{s.t.} c_t + x_t + (1-\gamma) p_t^h h_t = &\left[\left(1-\delta^h\right) p_t^h - \gamma R_t p_{t-1}^h\right] h_{t-1} + R_t x_{t-1} \\ &+ (1-\tau) w_t l_t \varepsilon_{t-1} - p_t^h C(h_t, h_{t-1}) + T_t \end{aligned} \tag{48}$$

and

$$x_t \geq 0$$

The related Lagrange is

$$\begin{aligned} \mathcal{L} = &U(c_t, h_t, l_t) + \beta E_t V(h_t, x_t, \varepsilon_t) \\ &+ \lambda_t \left[c_t + x_t + (1-\gamma) p_t^h h_t - \left[\left(1-\delta^h\right) p_t^h - \gamma R_t p_{t-1}^h\right] h_{t-1}\right. \\ &\left. - R_t x_{t-1} - (1-\tau) w_t l_t \varepsilon_{t-1} + p_t^h C(h_t, h_{t-1}) - T_t\right] \\ &+ \mu_t x_t \end{aligned}$$

Then the FOCs related to consumer's problem will be

$$U_{c,t} + \lambda_t = 0 \tag{49}$$

$$U_{h,t} + \beta E_t V_{h,t} + \lambda_t (1 - \gamma + C_{h,t}) p_t^h = 0 \tag{50}$$

$$\beta E_t V_{x,t} + \lambda_t + \mu_t = 0 \tag{51}$$

$$U_{l,t} - \lambda_t (1-\tau) w_t \varepsilon_{t-1} = 0 \tag{52}$$

The envelop conditions are

$$V_h(h_{t-1}, x_{t-1}, \varepsilon_{t-1}) = -\lambda_t \left[\left(1-\delta^h\right) p_t^h - \gamma R_t p_{t-1}^h - C_{h_{t-1}}\left(h_t, h_{t-1}\right) p_t^h\right] \tag{53}$$

$$V_x(h_{t-1}, x_{t-1}, \varepsilon_{t-1}) = -\lambda_t R_t \tag{54}$$

E.2 Alternative Setting to Capital Producer

E.2.1 Capital Producer(Setting I)

The capital producer uses final nondurable goods Y_N to produce capital following the maximization problem

$$\begin{aligned} &\max\left(Q_t-1\right) I_t - f\left(I_t, K_{t-1}\right) K_{t-1} \\ \text{s.t. } f\left(I_t, K_{t-1}\right) =& \frac{\psi_{I,1}}{\psi_{I,2}}\left(\frac{I_t}{K_{t-1}} - \overline{\delta}\right)^{\psi_{I,2}} \end{aligned}$$

where $\overline{\delta}$ is the steady-state investment rate.

By solving above optimization problem I could get the capital price as a convex function of investment which is shown below

$$Q_t = 1 + \psi_{I,1}\left(\frac{I_t}{K_{t-1}} - \overline{\delta}\right)^{\psi_{I,2}-1}$$

So the goods market clearing condition will become

$$Y_{N,t} = C_t + I_t + f\left(I_t, K_{t-1}\right) K_{t-1} + p^h C(h', h)$$

E.2.2 Capital Producer(Setting II)

The capital producer uses final nondurable goods Y_N to produce capital following the maximization problem

$$\begin{aligned} &\max Q_t I_t - f\left(I_t, K_{t-1}\right) K_{t-1} \\ \text{s.t. } f\left(I_t, K_{t-1}\right) =& \frac{\overline{\delta}^{-1/\phi}}{1+1/\phi}\left(\frac{I_t}{K_{t-1}}\right)^{1+1/\phi} + \frac{\overline{\delta}}{\phi+1} \end{aligned}$$

where $\overline{\delta}$ is the steady-state investment rate following $\overline{\delta} = \frac{\overline{I}}{\overline{K}}$

By solving above optimization problem I could get the capital price as a convex function of investment which is shown below

$$Q_t = \left(\frac{I_t}{K_{t-1}\overline{\delta}}\right)^{1+1/\phi}$$

So the goods market clearing condition will become

$$Y_{N,t} = C_t + f\left(I_t, K_{t-1}\right) K_{t-1} + p^h C\left(h', h\right)$$

E.2.3 Capital Producer(Setting III)

The capital producer uses final nondurable goods Y_N to produce capital following the maximization problem

$$\begin{aligned} &\max Q_t f\left(I_t, K_{t-1}\right) K_{t-1} - I_t \\ &\text{s.t. } f\left(I_t, K_{t-1}\right) = \frac{\overline{\delta}^{1/\phi}}{1-1/\phi}\left(\frac{I_t}{K_{t-1}}\right)^{1-1/\phi} - \frac{\overline{\delta}}{\phi+1} \end{aligned}$$

where $\overline{\delta}$ is the steady-state investment rate.

By solving above optimization problem I could get the capital price as a convex function of investment which is shown below

$$Q_t = \left(\frac{I_t}{K_{t-1}\overline{\delta}}\right)^{1-1/\phi}$$

and the law of motion of capital will become

$$K_t = (1-\delta)K_{t-1} + f\left(I_t, K_{t-1}\right) K_{t-1}$$

The goods market clearing condition will become

$$Y_{N,t} = C_t + I_t + p^h C\left(h', h\right)$$

E.2.4 Capital Producer(Setting IV)

The capital producer uses final nondurable goods Y_N to produce capital following the maximization problem

$$\begin{aligned} &\max E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t}\Lambda_{t,t+\tau}\left\{\left(Q_\tau - 1\right) I_\tau - f\left(I_\tau, I_{\tau-1}\right) I_\tau\right\} \\ &\text{s.t. } f\left(I_\tau, I_{\tau-1}\right) = \frac{\psi_{I,1}}{\psi_{I,2}}\left(\frac{I_\tau}{I_{\tau-1}} - 1\right)^{\psi_{I,2}} \end{aligned}$$

By solving above optimization problem I could get the capital price as a convex function of investment which is shown below

$$Q_t = 1 + \frac{\psi_{I,1}}{\psi_{I,2}} \left(\frac{I_t}{I_{t-1}} - 1 \right)^{\psi_{I,2}} + \psi_{I,1} \left(\frac{I_t}{I_{t-1}} - 1 \right)^{\psi_{I,2}-1} \frac{I_t}{I_{t-1}} -$$
$$E_t \beta \Lambda_{t,t+1} \psi_{I,1} \left(\frac{I_{t+1}}{I_t} - 1 \right)^{\psi_{I,2}-1} \left(\frac{I_{t+1}}{I_t} \right)^2$$

So the goods market clearing condition will become

$$Y_{N,t} = C_t + I_t + f\left(I_t, I_{t-1}\right) I_{t-1} + p^h C(h', h)$$

F Numerical solution

F.1 Calibration to full fledged model

All the parameters related to production sector are selected from literature. The depreciation rate of physical capital is 0.03 which implies 12% annually. The depreciation rate of housing service is estimated from data which is constructed by Rognlie et al. (2018) as my model in supply side is too simple to use the gross GDP in NIPA. Therefore I use the GDP constructed by Rognlie et al. (2018) which is more suitable to this simple supply side. The depreciation rate of housing service is roughly 1.9% quarterly which is in line with Kaplan et al. (2020). The relative share of production factors in construction function ν, θ and ι comes from Favilukis et al. (2017). The last three parameters, exogenous land supply, TFP in production function and TFP in construction function, together with other parameters in household problem, are selected to match the real gross rate, labor demand, liquid asset over GDP and iliquid asset over GDP.

Table 5: Parameter Values

Parameter	**Value**	**Description**
δ	0.03	Depreciation rate of physical capital
δ^h	0.01873	Depreciation rate of housing service
α	0.36	Capital share in production function
ν	0.27	Capital share in construction function
ι	0.36	Labor share in construction function
θ	0.1	Land share in construction function
$\overline{LD}$	4.95	Land supply
A_n	0.67	TFP in production function
A_h	2.75	TFP in construction function

F.2 Bayesian estimation to full fledged model

I use Bayesian method to estimate the parameters that control the impulse response and transition path such as the AR1 coefficients ρ_a^i, the observation matrix H and related covariance matrix $\eta\eta'$ and $\epsilon\epsilon'$. Since the data process itself is not stationary it is not appropriate to use the full-information Bayesian and if we used the statistic method to detrend such as first-order difference and hp filter, the Bayesian update rule would not be further used and the posterior $p\left(\theta|Y^T\right) \propto p\left(Y^T|\theta\right)p\left(\theta\right)$ would be unsolvable as $p\left(Y^T|\theta\right)$ was unknown. Therefore I use GMM to match the moments in data and model to proceed the estimation. In this subsection I first introduce the moments I used to match the data and then explain the Bayesian estimation strategy in detail.

F.2.1 Moments Selection and Theoretical moments after filter

I impose hp filter on the data and calculate moments from the cyclical elements such as the autocovariance of output, standard derivation of output, physical investment, new constructed residential estate, relative housing price and their related covariance. The covariance between output and physical investment $\text{cov}(y_t, I_t)$ captures the general equilibrium $Y = C + I$. Similarly the covariance between residential investment and physical investment $\text{cov}(I_t^H, I_t)$ captures the crowded-out effect. The covariance between new constructed residential estate and relative housing price capture the demand and supply equilibrium in the housing market. All these eight moments are summarized in vector $g(\cdot) = \Psi$ following

$$\Psi = \left[\begin{array}{ccc} \varrho_m' & \sigma_{m,m}' & \sigma_{m,n}' \end{array}\right]'$$

where ϱ_m is the vector that contains the autocovariance moments (ρ_m^i represents the $\text{AR}(i)$'s coefficient of variable m)

$$\varrho_m = \left[\begin{array}{cccccc} \rho_y^1 & \rho_c^1 & \rho_I^1 & \rho_{I_H}^1 & \rho_{p_H}^1 & \rho_Q^1 \end{array}\right]'$$

$\sigma_{m,m}$ is the vector that contains the standard derivation moments

$$\sigma_{m,m} = \left[\begin{array}{ccccc} \sigma_y & \sigma_c & \sigma_I & \sigma_{p_H} & \sigma_Q \end{array}\right]'$$

$\sigma_{m,n}$ is the vector that contains the covariance moments of variables $\phi_v = \left[\begin{array}{ccccccc} y & c & I & I_H & p_H & Q & R \end{array}\right]'$

$$\sigma_{m,n} = \left[\begin{array}{ccccccccc} \sigma_{y,c} & \sigma_{y,I} & \sigma_{y,I_H} & \sigma_{y,p_H} & \sigma_{y,Q} & \sigma_{y,R} & \sigma_{c,I_H} & \cdots & \sigma_{Q,R} \end{array}\right]'$$

Moreover I solve the theoretical moments from model after hp filter by switching to frequency domain and the spectrum. After some algebra I can solve the covariance matrix

$$\mathbb{E}\left[\widetilde{Y}_t\widetilde{Y}_{t-1}\right] = \int_{-\pi}^{\pi} g^{\mathrm{HP}}(\omega)e^{i\omega k}d\omega$$

where $\widetilde{Y}_t = \begin{bmatrix} s'_t & s'_{t|t} & Ec'_{t+1} \end{bmatrix}'$ in equation 73. The spectral density of HP filter $g^{\mathrm{HP}}(\omega)$ follows $g^{\mathrm{HP}}(\omega) = h^2(\omega)g(\omega)$. $h(\omega) = \frac{4\lambda(1-\cos(\omega))^2}{1+4\lambda(1-\cos(\omega))^2}$ is the transfer function of HP derived from King and Rebelo (1993). The spectral density of state and control variables Y_t is solved by

$$g(\omega) = \begin{bmatrix} I_{ns} & 0_{ns,nq} \\ M_{21}\mathrm{e}^{-i\omega} & D_2 \\ 0_{nq,ns} & I_{nq} \end{bmatrix} f(\omega) \begin{bmatrix} I_{ns} & M'_{21}e^{i\omega} & 0_{ns,nq} \\ 0_{nq,ns} & D'_2 & I_{nq} \end{bmatrix} = Wf(\omega)W' \tag{55}$$

$$f(\omega) = \frac{1}{2\pi}\begin{bmatrix} \left(I_{ns} - M_{11}\mathrm{e}^{-i\omega}\right)^{-1} D_1 \\ I_{nq} \end{bmatrix} \Sigma \left[D'_1\left(I_{ns} - M'_{11}e^{i\omega}\right)^{-1}, I_{nq}\right] \tag{56}$$

where ns is the number of state variables and nq is the number of shocks. M and D come from the policy function 79 and Σ is the covariance matrix of shocks. Because I assume the shock term Ξ_t in system 73 follows standard normal distribution and all the covariance terms are absorbed in η and ϵ, Σ in equation 56 is an identity matrix.

W.L.O.G, I assume the shock Ξ_t in equation 79 is independent with each other and all the covariance term is stored in response D. Therefore the covariance term Σ in equation 56 is an identity matrix and the equation can be further simplified as

$$f(\omega) = \frac{1}{2\pi}\begin{bmatrix} \left(I_{ns} - M_{11}\mathrm{e}^{-i\omega}\right)^{-1} D_1 D'_1 \left(I_{ns} - M'_{11}e^{i\omega}\right)^{-1} & \left(I_{ns} - M_{11}\mathrm{e}^{-i\omega}\right)^{-1} D_1 \\ D'_1\left(I_{ns} - M'_{11}e^{i\omega}\right)^{-1} & I_{nq} \end{bmatrix}$$

Then equation 55 becomes

$$\begin{aligned} g(\omega) &= \frac{1}{2\pi}\begin{bmatrix} \left(I_{ns} - M_{11}\mathrm{e}^{-i\omega}\right)^{-1} D_1 D'_1 \left(I_{ns} - M'_{11}e^{i\omega}\right)^{-1} & \left(I_{ns} - M_{11}\mathrm{e}^{-i\omega}\right)^{-1} D_1 \\ M_{21}\mathrm{e}^{-i\omega}\left(I_{ns} - M_{11}\mathrm{e}^{-i\omega}\right)^{-1} D_1 D'_1 \left(I_{ns} - M'_{11}e^{i\omega}\right)^{-1} & M_{21}\mathrm{e}^{-i\omega}\left(I_{ns} - M_{11}\mathrm{e}^{-i\omega}\right)^{-1} D_1 \\ D'_1\left(I_{ns} - M'_{11}e^{i\omega}\right)^{-1} & I_{nq} \end{bmatrix} W' \\ &+ \frac{1}{2\pi}\begin{bmatrix} 0 & 0 \\ D_2 D'_1\left(I_{ns} - M'_{11}e^{i\omega}\right)^{-1} & D_2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} I_{ns} & M'_{21}e^{i\omega} & 0_{ns,nq} \\ 0_{nq,ns} & D'_2 & I_{nq} \end{bmatrix} W' \\ &= \frac{1}{2\pi}\left(\Upsilon_1 + \Upsilon_2 + \Upsilon_3 + \Upsilon_4\right) \end{aligned}$$

where

$$\Upsilon_1 = \begin{bmatrix} (I_{ns} - M_{11}\mathrm{e}^{-i\omega})^{-1} D_1 D_1' (I_{ns} - M_{11}' e^{i\omega})^{-1} & (I_{ns} - M_{11}\mathrm{e}^{-i\omega})^{-1} D_1 D_1' (I_{ns} - M_{11}' e^{i\omega})^{-1} M_{21}' e^{i\omega} & (I_{ns} - M_{11}\mathrm{e}^{-i\omega})^{-1} D_1 \\ M_{21}\mathrm{e}^{-i\omega} (I_{ns} - M_{11}\mathrm{e}^{-i\omega})^{-1} D_1 D_1' (I_{ns} - M_{11}' e^{i\omega})^{-1} & M_{21} (I_{ns} - M_{11}\mathrm{e}^{-i\omega})^{-1} D_1 D_1' (I_{ns} - M_{11}' e^{i\omega})^{-1} M_{21}' & M_{21}\mathrm{e}^{-i\omega} (I_{ns} - M_{11}\mathrm{e}^{-i\omega})^{-1} D_1 \\ D_1' (I_{ns} - M_{11}' e^{i\omega})^{-1} & D_1' (I_{ns} - M_{11}' e^{i\omega})^{-1} M_{21}' e^{i\omega} & I_{nq} \end{bmatrix}$$

$$\Upsilon_2 = \begin{bmatrix} 0 & (I_{ns} - M_{11}\mathrm{e}^{-i\omega})^{-1} D_1 D_2' & 0 \\ 0 & M_{21}\mathrm{e}^{-i\omega} (I_{ns} - M_{11}\mathrm{e}^{-i\omega})^{-1} D_1 D_2' & 0 \\ 0 & D_2' & 0 \end{bmatrix}$$

$$\Upsilon_3 = \begin{bmatrix} 0 & 0 & 0 \\ D_2 D_1' (I_{ns} - M_{11}' e^{i\omega})^{-1} & D_2 D_1' (I_{ns} - M_{11}' e^{i\omega})^{-1} M_{21}' e^{i\omega} & D_2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Upsilon_4 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & D_2 D_2' & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

To further decrease the computation burden it is easy to show that $M_{21} (I_{ns} - M_{11}\mathrm{e}^{-i\omega})^{-1} = \mathrm{e}^{i\omega} M_{21} U_M (\mathrm{e}^{i\omega} I_{ns} - T_M)^{-1} U_M'$ where $M_{11} = U_M T_M U_M'$ is prederived from Schur decomposition.

F.2.2 Bayesian GMM

Following Rotemberg and Woodford (1997), Christiano et al. (2005) and Barsky and Sims (2012), to construct the asymptotic properties of the moments which I select to conduct the Bayesian GMM, I first construct the auxiliary variable ψ_t

$$\psi_t = \begin{bmatrix} y_t & c_t & I_t & I_{t,H} & p_{t,H} & Q_t & R_t & y_t y_{t-1} & c_t c_{t-1} & \cdots & y_t^2 & c_t^2 & \cdots & p_{t,H}^2 & y_t c_t & y_t I_t & \cdots & Q_t R_t \end{bmatrix}'$$

Additionally I define the moment function as $g(\cdot)$ which yields the moments

$$g(\psi_t) = \Psi$$

If the sample estimation of ψ_t is $\widehat{\psi}$ the moment function is well defined as

$$g(\widehat{\psi}) = \begin{bmatrix} \hat{\psi}_{y_t y_{t-1}} - \hat{\psi}_y^2 \\ \hat{\psi}_{c_t c_{t-1}} - \hat{\psi}_c^2 \\ \vdots \\ \sqrt{\hat{\psi}_{y^2} - \hat{\psi}_y^2} \\ \sqrt{\hat{\psi}_{c^2} - \hat{\psi}_c^2} \\ \vdots \\ \hat{\psi}_{yc} - \hat{\psi}_y \hat{\psi}_c \\ \hat{\psi}_{yI} - \hat{\psi}_y \hat{\psi}_I \\ \vdots \\ \hat{\psi}_{QR} - \hat{\psi}_Q \hat{\psi}_R \end{bmatrix}$$

Therefore the Jacobian of moment function $\Gamma_g(\cdot)$ should be

$$\Gamma_g(\widehat{\psi}) = \frac{\partial g}{\partial \psi} = \begin{bmatrix} -2\mu_y & 0 & \cdots & 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2\mu_c & 0 & \cdots & 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -\frac{\mu_y}{\sigma_y} & 0 & \cdots & 0 & 0 & \frac{1}{2\sigma_y} & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\mu_c}{\sigma_c} & 0 & \cdots & 0 & 0 & \frac{1}{2\sigma_c} & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & -\frac{\mu_Q}{\sigma_Q} & 0 & \cdots & 0 & 0 & \frac{1}{2\sigma_Q} & 0 & \cdots & 0 & 0 & 0 \\ -\mu_c & -\mu_y & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ -\mu_I & 0 & -\mu_y & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & -\mu_R & -\mu_Q & 0 & \cdots & 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

By applying the Delta Method the sample estimation of moments $\widehat{\Psi}$ has the following asymptotic properities

$$\sqrt{T}\left(\widehat{\Psi} - \Psi\right) \xrightarrow{d} N\left(0, \Gamma_g \Sigma \Gamma_g'\right)$$

where Σ is the LRV of ψ_t.

F.3 Solution method to simple model

F.3.1 Reconstruction

Similar to the section F.7.1, I replace the saving a_t by the effective asset holding x_t which follows $x_t = \gamma p_t^H h_t + a_t$. Then the problem 3 change to

$$\max_{c_t,h_t,x_t} \sum_{t=0}^{\infty} \beta^t U\left(c_t, h_t\right) \tag{57}$$

s.t.

$$c_t + x_t + (1-\gamma)\, p_t^H h_t = R_t x_{t-1} + w_t \varepsilon_t + \left[(1-\delta^H) p_t^H - \gamma R_t p_{t-1}^H\right] h_{t-1} + T_t \tag{58}$$

$$x_t \geq 0$$

The related FOCs 31, 32 and 33 will become

$$U_{c_t} = \lambda_t \tag{59}$$

$$-\lambda_t + \mu_t + \beta E_t R_{t+1} \lambda_{t+1} = 0 \tag{60}$$

$$U_{h_t} - (1-\gamma)\, \lambda_t p_t^H + \beta E_t \lambda_{t+1} \left[(1-\delta^H) p_{t+1}^H - \gamma R_{t+1} p_t^H\right] = 0 \tag{61}$$

Similar to the full fledged model, I assume the utility function $U\left(c_t, h_t\right)$ follows the Cobb-Douglas formula

$$U\left(c_t, h_t\right) = \frac{\left(c_t^{\phi} h_t^{1-\phi}\right)^{1-\sigma}}{1-\sigma} \tag{62}$$

Since I assume there is no aggregate shock existing in the simple model, R_{t+1}, p_{t+1}^H and p_t^H can be perfectly expected. Therefore for non-constrained household there exists a static relationship between c_t and h_t from the combining of equation 59, 60 and 61

$$c_t = \frac{\phi}{1-\phi} h_t \left[p_t^H - (1-\delta^H) \frac{p_{t+1}^H}{R_{t+1}}\right] \tag{63}$$

When the collateral constraint is binding, it is worth to notice that the two FOC 32 and 60 have the same form. Therefore the Khun-Tucker multiplier is the same between the two model, the original one and the reconstructed one. To sum up, the problem 57 degenerates to a one state x_t problem which can be solved easily by value function iteration.

F.3.2 Solution Steps

Since in this simple problem I use Cobb-Douglas utility function where intratemporal elasticity of substitution between housing service and non-durable consumption is constant at 1, the consumption and housing servicing is homogeneous in degree 1 (linear) in the frictionless scenario. Therefore it is solvable to use value function iteration method.

1. Take an initial guess about value function $V(h_{-1}, x_{-1}, \varepsilon_{-1}) = \sum_{t=0}^{\infty} \beta^t U(c_t, h_t)$. If h_0, x_0 is still on grid I can remove the expectation with $\widetilde{V}(h_0, x_0, \varepsilon_{-1}) = E_0 V(h_0, x_0, \varepsilon_0) = \Pi V(h_0, x_0, \varepsilon_0)$ as h_0, x_0 is determined at time 0.

2. If the budget constraint is not binding, equation 63 will always hold. Therefore given an initial guess of $h_0(h_{-1}, x_{-1}, \varepsilon_{-1})$, I can get the unique mapping $x_0(h_0, h_{-1}, x_{-1}, \varepsilon_{-1})$ and $c_0(h_0, h_{-1}, x_{-1}, \varepsilon_{-1})$ through budget constraint 58 and equation 63. Then it is easy to find

$$h_0^{uc}(h_{-1}, x_{-1}, \varepsilon_{-1}) = \underset{h_0}{argmax} U\left[c_0(h_0, h_{-1}, x_{-1}, \varepsilon_{-1}), h_0\right] + \beta \widetilde{V}\left[h_0, x_0(h_0, h_{-1}, x_{-1}, \varepsilon_{-1}), \varepsilon_{-1}\right]$$

   where $\widetilde{V}\left[h_0, x_0(h_0, h_{-1}, x_{-1}, \varepsilon_{-1}), \varepsilon_{-1}\right]$ can be solved from linear interpolation on the on-grid value $\widetilde{V}(h_0, x_0, \varepsilon_{-1})$ in last step. I also define and save the value

$$\text{RHS}^{UC} = \max U\left[c_0(h_0, h_{-1}, x_{-1}, \varepsilon_{-1}), h_0\right] + \beta \widetilde{V}\left[h_0, x_0(h_0, h_{-1}, x_{-1}, \varepsilon_{-1}), \varepsilon_{-1}\right]$$

   .

3. If the budget constraint is binding, the Euler equation does not hold anymore. Therefore the mapping between h_0 and c_0 is no longer useful. However the effective wealth is known as now the household is constrained so $x_0(h_{-1}, x_{-1}, \varepsilon_{-1}) = 0$. Given any guess of $h_0(h_{-1}, x_{-1}, \varepsilon_{-1})$ the consumption $c_0(h_0, h_{-1}, x_{-1}, \varepsilon_{-1})$ can be solved from budget constraint 58. Then it is easy to find

$$h_0^{c}(h_{-1}, x_{-1}, \varepsilon_{-1}) = \underset{h_0}{argmax} U\left[c_0(h_0, h_{-1}, x_{-1}, \varepsilon_{-1}), h_0\right] + \beta \widetilde{V}\left[h_0, 0, \varepsilon_{-1}\right]$$

   where $\widetilde{V}\left[h_0, 0, \varepsilon_{-1}\right]$ can be solved from linear interpolation on the on-grid value $\widetilde{V}(h_0, 0, \varepsilon_{-1})$ in step 1. I also define and save the value

$$\text{RHS}^{C} = \max U\left[c_0(h_0, h_{-1}, x_{-1}, \varepsilon_{-1}), h_0\right] + \beta \widetilde{V}\left[h_0, 0, \varepsilon_{-1}\right]$$

   .

4. Because the result of constrained optimization in convex function optimization problem is always inferior than that of unconstrained optimization, the updated value function $V(h_{-1}, x_{-1}, \varepsilon_{-1})$ will follows

$$V(h_{-1}, x_{-1}, \varepsilon_{-1}) = \begin{cases} \text{RHS}^{UC} & x_0^{uc} \geq 0 \\ \text{RHS}^{C} & x_0^{c} < 0 \end{cases}$$

Update the value function and go back to step 1.

F.4 Solution method to simple model with separable utility function

F.4.1 Reconstruction and new FOCs

Change the utility function from 62 to the separable utility function

$$U(c_t, h_t) = \frac{\phi c_t^{1-\sigma} + (1-\phi) h_t^{1-\sigma}}{1-\sigma}$$

Then the mapping from c_t to h_t under the frictionless scenario changes to

$$c_t = \left(\frac{\phi}{1-\phi}\right)^{\frac{1}{\sigma}} \left[p_t^H - (1-\delta^H)\frac{p_{t+1}^H}{R_{t+1}}\right]^{\frac{1}{\sigma}} h_t$$

F.5 Expected news shock

Then denote the "fundamental" variable X_t as

$$X_t = \begin{bmatrix} \log \Phi_t^i & \log \Phi_{g,t}^i & \varepsilon_t^8 & \varepsilon_{t-1}^8 & \varepsilon_{t-2}^8 & \varepsilon_{t-3}^8 & \varepsilon_{t-4}^8 & \varepsilon_{t-5}^8 & \varepsilon_{t-6}^8 & \varepsilon_{t-7}^8 \end{bmatrix}' \tag{64}$$

Then X_t follows

$$X_t = B^s X_{t-1} + \eta w_t \tag{65}$$

where

$$B^s = \begin{bmatrix} \rho_a & 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & \rho_g & 0 & 0 & 0 & \cdots & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 0 \end{bmatrix}_{10\times 10}$$

$$\eta = \begin{bmatrix} \sigma_a & 0 & 0 \\ 0 & \sigma_g & 0 \\ 0 & 0 & \sigma_g^8 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix}_{10\times 3}$$

$$\boldsymbol{w}_t = \begin{bmatrix} w_t^a \\ w_t^g \\ w_t^8 \end{bmatrix}$$

However household can only observe the variable $\widetilde{X}_t$ such that

$$\widetilde{X}_t = \begin{bmatrix} \log\widetilde{\Phi}_t & \log\widetilde{\Phi}_{g,t} & \widetilde{\varepsilon}^8_t & \widetilde{\varepsilon}^8_{t-1} & \widetilde{\varepsilon}^8_{t-2} & \widetilde{\varepsilon}^8_{t-3} & \widetilde{\varepsilon}^8_{t-4} & \widetilde{\varepsilon}^8_{t-5} & \widetilde{\varepsilon}^8_{t-6} & \widetilde{\varepsilon}^8_{t-7} \end{bmatrix}' \tag{66}$$

which follows

$$\widetilde{X}_t = HX_t + \epsilon v \tag{67}$$

where

$$H = \begin{bmatrix} H^{11}_{3\times 3} & 0_{3\times 5} \\ 0_{5\times 3} & m_4 I_{5\times 5} \end{bmatrix}$$

$$H^{11} = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}$$

$$m \in \mathbb{R}^+$$

$$\epsilon = \begin{bmatrix} \sigma^s_a & 0 & 0 & \cdots & 0 \\ 0 & \sigma^s_g & 0 & \cdots & 0 \\ 0 & 0 & \sigma^s_{g1} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \sigma^s_{g8} \end{bmatrix}_{10\times 10}$$

$$v_t = \begin{bmatrix} v^a_t \\ v^g_t \\ v^{g1}_t \\ \vdots \\ v^{g8}_t \end{bmatrix}$$

F.6 Kalman Filter

Even though the household can successfully observe A_t at time t, he cannot observe g_t at time t. This make the household harder to estimate the A_{t+1} as $E_t\log(A_{t+1}) = \rho_a \log A_t + E_t \log g_t$. Thus we need get $g_{t|t}$ to get the expectation of A_{t+1}. Based on the Kalman filter and equation 65 and 67, we can solve out the perception of g_t by household as[29]

$$X_{t+1|t+1} = A^s X_{t|t} + P^s \widetilde{X}_{t+1} \tag{68}$$

where P^s is the Kalman gain and $A^s = (I - P^s H)B^s$

[29]For the reference Hamilton (2020) provides rigorous proof to this equation.

F.7 Model Reconstruction and Solution Process

The computation process follows the augmented endogenous gird method which is proposed by Auclert et al. (2021).

F.7.1 Preliminaries

I define the risk-adjusted expected value function as

$$\widetilde{V}(h_t, b_t, \varepsilon_{t-1}) = \beta EV(h_t, b_t, \varepsilon_t)$$

Therefore the marginal risk-adjusted expected value should be

$$\widetilde{V}_h(h_t, b_t, \varepsilon_{t-1}) = \beta EV_h(h_t, b_t, \varepsilon_t)$$

and

$$\widetilde{V}_b(h_t, b_t, \varepsilon_{t-1}) = \beta EV_b(h_t, b_t, \varepsilon_t)$$

To simplify the computation process, I further define the auxiliary variable x_t as t he effective asset holding which follows $x_t = \gamma p_t^h h_t + b_t$. Therefore the budget constraint 9 becomes

$$\begin{aligned} c_t + x_t + (1-\gamma)\, p_t^h h_t = & \left[\left(1-\delta^h\right) p_t^h - \gamma R_t p_{t-1}^h\right] h_{t-1} + R_t x_{t-1} \\ & + (1-\tau) w_t l_t \varepsilon_{t-1} - p_t^h C\left(h_t, h_{t-1}\right) + T_t \end{aligned} \tag{69}$$

Correspondingly collateral constraint becomes

$$x_t \geq 0$$

F.7.2 Decision Problems

The household solve the problem

$$V(h_{t-1}, x_{t-1}, \varepsilon_{t-1}) = \underset{h_t, x_t, l_t, c_t}{max} U\left(c_t, h_t, l_t\right) + \beta EV(h_t, x_t, \varepsilon_t)$$

$$\begin{aligned} \text{s.t.} c_t + x_t + (1-\gamma)\, p_t^h h_t = & \left[\left(1-\delta^h\right) p_t^h - \gamma R_t p_{t-1}^h\right] h_{t-1} + R_t x_{t-1} \\ & + (1-\tau) w_t l_t \varepsilon_{t-1} - p_t^h C\left(h_t, h_{t-1}\right) + T_t \end{aligned}$$

and

$$x_t \geq 0$$

F.7.3 Solve step

1. Take the initial guess to marginal value function at time $t+1$ as $V_h(h_t, x_t, \varepsilon_t)$ and $V_x(h_t, x_t, \varepsilon_t)$

2. Solve the expectation problem on marginal value function to get risk-adjusted expected value function

$$\widetilde{V}_h(h_t, x_t, \varepsilon_{t-1}) = \beta \Pi V_h(h_t, x_t, \varepsilon_t)$$

and

$$\widetilde{V}_x(h_t, x_t, \varepsilon_{t-1}) = \beta \Pi V_x(h_t, x_t, \varepsilon_t)$$

3. Assuming the collateral constraint is unconstrained, I can combine equation 49, 50 and 51 to get

$$F\left(h_t, x_t, \varepsilon_{t-1}, h_{t-1}\right) = \frac{U_{h,t} + \widetilde{V}_h}{p_t^h \widetilde{V}_x} - \left(1 - \gamma + C_{h,t}\right) = 0$$

Further because the unseparable utility function $U\left(c_t, h_t, l_t\right)$ is homogeneous between c_t and h_t, $U_{h,t}$ can be written as a function of $\widetilde{V}_x$

$$U_{h,t} = (1-\phi)\left(\frac{\widetilde{V}_x}{\phi}\right)^{\frac{\phi(1-\sigma)}{\phi(1-\sigma)-1}} h_t^{\frac{\phi(1-\phi)(1-\sigma)^2}{1-\phi(1-\sigma)} + (1-\phi)(1-\sigma) - 1} \tag{70}$$

This can be used to solve $h_t\left(h_{t-1}, x_t, \varepsilon_{t-1}\right)$. The related mapping weight can also be used to map $\widetilde{V}_x(h_t, x_t, \varepsilon_{t-1})$ into $\widetilde{V}_x(h_{t-1}, x_t, \varepsilon_{t-1})$. Then $c\left(h_{t-1}, x_t, \varepsilon_{t-1}\right)$ and $l\left(h_{t-1}, x_t, \varepsilon_{t-1}\right)$ can be solved straightforward from

$$c\left(h_{t-1}, x_t, \varepsilon_{t-1}\right) = \left(\frac{\widetilde{V}_x(h_{t-1}, x_t, \varepsilon_{t-1})}{\phi}\right)^{\frac{1}{\phi(1-\sigma)-1}} h_t\left(h_{t-1}, x_t, \varepsilon_{t-1}\right)^{\frac{(1-\phi)(1-\sigma)}{1-\phi(1-\sigma)}} \tag{71}$$

and

$$l\left(h_{t-1}, x_t, \varepsilon_{t-1}\right) = \left(-\phi \frac{(1-\tau) w_t \varepsilon_{t-1}}{\kappa}\right)^{\frac{1}{\psi}} c\left(h_{t-1}, x_t, \varepsilon_{t-1}\right)^{\frac{\phi(1-\sigma)-1}{\psi}} h_t\left(h_{t-1}, x_t, \varepsilon_{t-1}\right)^{\frac{(1-\phi)(1-\sigma)}{\psi}} \tag{72}$$

4. Then the effective asset holding can be solved from budget constraint

$$\begin{aligned} x_{t-1}\left(h_{t-1}, x_t, \varepsilon_{t-1}\right) = & \frac{c\left(h_{t-1}, x_t, \varepsilon_{t-1}\right) + x_t + (1-\gamma)\, p_t^h h_t\left(h_{t-1}, x_t, \varepsilon_{t-1}\right)}{R_t} \\ & - \frac{\left[\left(1-\delta^h\right) p_t^h - \gamma R_t p_{t-1}^h\right] h_{t-1} + (1-\tau)\varepsilon_{t-1} w_t l\left(h_{t-1}, x_t, \varepsilon_{t-1}\right) + T_t}{R_t} \\ & + \frac{p_t^h C\left(h_t\left(h_{t-1}, x_t, \varepsilon_{t-1}\right), h_{t-1}\right)}{R_t} \end{aligned}$$

Now invert above function $x_{t-1}(h_{t-1}, x_t, \varepsilon_{t-1})$ to $x_t(h_{t-1}, x_{t-1}, \varepsilon_{t-1})$. After this invert process the function $h_t(h_{t-1}, x_t, \varepsilon_{t-1})$ can be mapped to $h_t(h_{t-1}, x_{t-1}, \varepsilon_{t-1})$ by the function $x_t(h_{t-1}, x_{t-1}, \varepsilon_{t-1})$.

5. Assuming the collateral constraint is constrained, I further define the relative Khun-Tucker multiplier as $\widetilde{\mu}_t(h_t, 0, \varepsilon_{t-1}) = \frac{\mu_t}{\widetilde{V}_x(h_t, 0, \varepsilon_{t-1})}$ so that equation 51 becomes

$$U_{c,t} = (1 + \widetilde{\mu}_t)\, \widetilde{V}_x$$

Therefore the equation 70 changes to

$$U_{h,t} = (1-\phi)\left(\frac{(1+\widetilde{\mu}_t)\,\widetilde{V}_x}{\phi}\right)^{\frac{\phi(1-\sigma)}{\phi(1-\sigma)-1}} h_t^{\frac{\phi(1-\phi)(1-\sigma)^2}{1-\phi(1-\sigma)}+(1-\phi)(1-\sigma)-1}$$

Similar to the process in step 3 this can be used to solve $h_t(h_{t-1}, \widetilde{\mu}_t, \varepsilon_{t-1})$ from

$$F(h_t, \widetilde{\mu}_t, \varepsilon_{t-1}, h_{t-1}) = \frac{1}{1+\widetilde{\mu}_t}\frac{U_{h,t} + \widetilde{V}_h}{p_t^h \widetilde{V}_x} - (1 - \gamma + C_{h,t}) = 0$$

and equation 71 changes to

$$c(h_{t-1}, \widetilde{\mu}_t, \varepsilon_{t-1}) = \left(\frac{(1+\widetilde{\mu}_t)\,\widetilde{V}_x(h_t, 0, \varepsilon_{t-1})}{\phi h_t(h_{t-1}, \widetilde{\mu}_t, \varepsilon_{t-1})^{(1-\phi)(1-\sigma)}}\right)^{\frac{1}{\phi(1-\sigma)-1}}$$

and corresponded optimal labor supply $l(h_{t-1}, \widetilde{\mu}_t, \varepsilon_{t-1})$ from equation 72.

6. The effective asset holding under the constraint scenario can be solved from budget constraint

$$\begin{aligned} x_{t-1}(h_{t-1}, \widetilde{\mu}_t, \varepsilon_{t-1}) = & \frac{c(h_{t-1}, \widetilde{\mu}_t, \varepsilon_{t-1}) + (1-\gamma)\, p_t^h h_t(h_{t-1}, \widetilde{\mu}_t, \varepsilon_{t-1})}{R_t} \\ & - \frac{\left[(1-\delta^h)\, p_t^h - \gamma R_t p_{t-1}^h\right] h_{t-1} + (1-\tau)\varepsilon_{t-1} w_t l(h_{t-1}, \widetilde{\mu}_t, \varepsilon_{t-1}) + T_t}{R_t} \\ & + \frac{p_t^h C(h_t(h_{t-1}, \widetilde{\mu}_t, \varepsilon_{t-1}), h_{t-1})}{R_t} \end{aligned}$$

Now invert above function $x_{t-1}(h_{t-1}, \widetilde{\mu}_t, \varepsilon_{t-1})$ to $\widetilde{\mu}_t(h_{t-1}, x_{t-1}, \varepsilon_{t-1})$. After this invert process the function $h_t(h_{t-1}, \widetilde{\mu}_t, \varepsilon_{t-1})$ can be mapped to $h_t^c(h_{t-1}, x_{t-1}, \varepsilon_{t-1})$.[30] It is worth to notice that $x_t^c(h_{t-1}, x_{t-1}, \varepsilon_{t-1})$ is already known such that $x_t^c(h_{t-1}, x_{t-1}, \varepsilon_{t-1}) = 0$.

7. Compare $x_t(h_{t-1}, x_{t-1}, \varepsilon_{t-1})$ and $x_t^c(h_{t-1}, x_{t-1}, \varepsilon_{t-1})$ to select the largest elemental value. Then replace the unconstrained optimal housing service choice $h_t(h_{t-1}, x_{t-1}, \varepsilon_{t-1})$ with

[30] Here I use c in superscript as the notation to "constrained".

$h_t^c\left(h_{t-1}, x_{t-1}, \varepsilon_{t-1}\right)$. Then for each grid point solve the nonlinear equation

$$\begin{aligned}
c\left(h_{t-1}, x_{t-1}, \varepsilon_{t-1}\right)= & {\left[\left(1-\delta^h\right) p_t^h-\gamma R_t p_{t-1}^h\right] h_{t-1}+R_t x_{t-1} } \\
& +(1-\tau) w_t \varepsilon_{t-1}\left(-\phi \frac{(1-\tau) w_t \varepsilon_{t-1}}{\kappa}\right)^{\frac{1}{\psi}} \\
& c\left(h_{t-1}, x_{t-1}, \varepsilon_{t-1}\right)^{\frac{\phi(1-\sigma)-1}{\psi}} h_t\left(h_{t-1}, x_{t-1}, \varepsilon_{t-1}\right)^{\frac{(1-\phi)(1-\sigma)}{\psi}} \\
& -p_t^h C\left(h_t\left(h_{t-1}, x_{t-1}, \varepsilon_{t-1}\right), h_{t-1}\right)+T_t \\
& -x_t\left(h_{t-1}, x_{t-1}, \varepsilon_{t-1}\right)-(1-\gamma) p_t^h h_t\left(h_{t-1}, x_{t-1}, \varepsilon_{t-1}\right)
\end{aligned}$$

Then update the marginal value function through the envelop condition 53 and 54

$$V_h(h_{t-1}, x_{t-1}, \varepsilon_{t-1})=U_{c,t}\left[\left(1-\delta^h\right) p_t^h-\gamma R_t p_{t-1}^h-C_{h_{t-1}}\left(h_t, h_{t-1}\right) p_t^h\right]$$

$$V_x(h_{t-1}, x_{t-1}, \varepsilon_{t-1})=U_{c,t} R_t$$

F.8 Solve Rational Expectation model with imperfect information

Following Baxter et al. (2011) and Hürtgen (2014), I first solve perfect information model

$$A Y_t=B Y_{t-1}+C^{\text{pseo}} \Xi_t \tag{73}$$

where $Y_t=\left[\begin{array}{cc} s_t^{\prime} & E c_{t+1}^{\prime}\end{array}\right]^{\prime}$ where s_t is the vector of state variable and c_t is the vector of control variable. Ξ_t is the vector of pseudo-shock and composed with fundamental shock w_t and noisy shock v_t such that $\Xi_t=\left[\begin{array}{cc} w_t^{\prime} & v_t^{\prime}\end{array}\right]^{\prime}$. The effect of shock C^{pseo} naturally becomes $C^{\text{pseo}}=\left[\begin{array}{c} P^s H \eta \\ P^s \epsilon\end{array}\right]$ where P^s is the Kalman gain from equation 68. This linear model can be easily solved by Klein (2000) to yield $Y_t=P Y_{t-1}+Q \Xi_t$. Take partition on P as

$$P=\left[\begin{array}{cc} P_{11} & P_{12} \\ P_{21} & P_{22}\end{array}\right]$$

It is widely known that to solve the linear rational expectation model we pre-impose the restriction that $P_{12}=0$ and $P_{22}=0$. Further because of the holding of CEQ under first-order perturbation method, the policy function of control variables c_t will follow

$$c_t=P_{21} s_{t-1 \mid t-1}+Q_2^w w_t+Q_2^v v_t \tag{74}$$

where Q_2^w and Q_2^v are subset of Q^w and Q^v which comes from Q such that $Q=\left[\begin{array}{cc} Q^w & Q^s\end{array}\right]$. Plug equation 74 into partition of equation 73 but replace $C^{\text{pseo}} \Xi_t$ with true fundamental shock process ηw_t such that

$$A_{11}s_t + A_{12}Ec_{t+1} = B_{11}s_{t-1} + B_{12}c_t + \eta w_t$$

$$A_{11}s_t + A_{12}P_{21}s_{t|t} = B_{11}s_{t-1} + B_{12}\left(P_{21}s_{t-1|t-1} + Q_2^w w_t + Q_2^v v_t\right) + \eta w_t \tag{75}$$

It is worth to notice that here I use the first ns linear equations of equation 73 which is not free of choice yet a simplification in notation. The basic purpose now is to solve the law of motion of perceived state variable $s_{t|t}$ therefore we need ns "core" linear equations related to state variables to pin down ns state variable $s_{t|t}$. The word "core" refers to those equations that affect state variables directly, or more specifically, the law of motion of state variables. For instance, if we want to select one out of two linear equations in 73, 1) Euler equation $-\sigma\widetilde{c}_t = \widetilde{R}_t - \sigma\widetilde{c}_{t+1}$ and 2) Law of Motion of Capital $K\widetilde{k}_t = I\widetilde{I}_t + K\widetilde{k}_{t-1}$, which is used in equation 75, we should select the equation 2 because the equation 1 is implicitly comprised in the mapping from $s_{t-1|t-1}$ to c_t in equation 74. Otherwise we redundantly use the linear constraints and the matrix $A_{11} + A_{12}P_{21}G$ in equation 78 will not be well-defined.

Furthermore, the law of motion of perception of unobservable variables could be derived through plugging equation 67 into equation 68 to yield

$$X_{t|t} = A^s X_{t-1|t-1} + P^s H X_t + P^s \epsilon v_t \tag{76}$$

However, It is not all the state variables s_t that is unobservable, so I rewrite the law of motion of perceived state variable $s_{t|t}$ below. Without loss of generality, I assume the unobservable state variables lay on the last nx row (in this paper $nx = 10$ as equation 64 shows).

$$s_{t|t} = Fs_{t-1|t-1} + Gs_t + G_{P^s}\epsilon v_t \tag{77}$$

where $F = \begin{bmatrix} 0 & 0 \\ 0 & A^s \end{bmatrix}$, $G = \begin{bmatrix} I & 0 \\ 0 & P^s H \end{bmatrix}$ and $G_{P^s} = \begin{bmatrix} 0 \\ P^s \end{bmatrix}$.

And then plug equation 77 back to above equation 75

$$A_{11}s_t + A_{12}P_{21}\left(Fs_{t-1|t-1} + Gs_t + G_{P^s}\epsilon v_t\right) = B_{11}s_{t-1} + B_{12}\left(P_{21}s_{t-1|t-1} + Q_2^w w_t + Q_2^v v_t\right) + \eta w_t$$

$$\begin{aligned}\left(A_{11} + A_{12}P_{21}G\right)s_t = {} & B_{11}s_{t-1} + \left(B_{12}P_{21} - A_{12}P_{21}F\right)s_{t-1|t-1} + \left(B_{12}Q_2^w + \eta\right)w_t \\ & + \left(B_{12}Q_2^v - A_{12}P_{21}G_{P^s}\epsilon\right)v_t\end{aligned} \tag{78}$$

Simplify above equation to

$$\widetilde{Y}_t = M\widetilde{Y}_{t-1} + D\Xi_t \tag{79}$$

where

$$\widetilde{Y}_t = \begin{bmatrix} s_t \\ s_{t|t} \\ c_t \end{bmatrix}$$

$$A_L = \begin{bmatrix} I & 0 & 0 \\ -G & I & 0 \\ 0 & 0 & I \end{bmatrix}$$

$$B_L = \begin{bmatrix} \widetilde{P}_{11} & \widetilde{P}_{12} & 0 \\ 0 & F & 0 \\ 0 & P_{21} & 0 \end{bmatrix}$$

$$C_L = \begin{bmatrix} \widetilde{Q}_{11} & \widetilde{Q}_{12} \\ 0 & P^s\epsilon \\ Q_2^w & Q_2^v \end{bmatrix}$$

$M = A_L^{-1}B_L$, $D = A_L^{-1}C_L$, $\widetilde{P}_{11} = (A_{11} + A_{12}P_{21}G)^{-1} B_{11}$,
$\widetilde{P}_{12} = (A_{11} + A_{12}P_{21}G)^{-1} (B_{12}P_{21} - A_{12}P_{21}F)$, $\widetilde{Q}_{11} = (A_{11} + A_{12}P_{21}G)^{-1} (B_{12}Q_2^w + \eta)$
and
$\widetilde{Q}_{12} = (A_{11} + A_{12}P_{21}G)^{-1} (B_{12}Q_2^v - A_{12}P_{21}G_{P^s}\epsilon)$.

F.9 Arguments to fake news and inefficiency

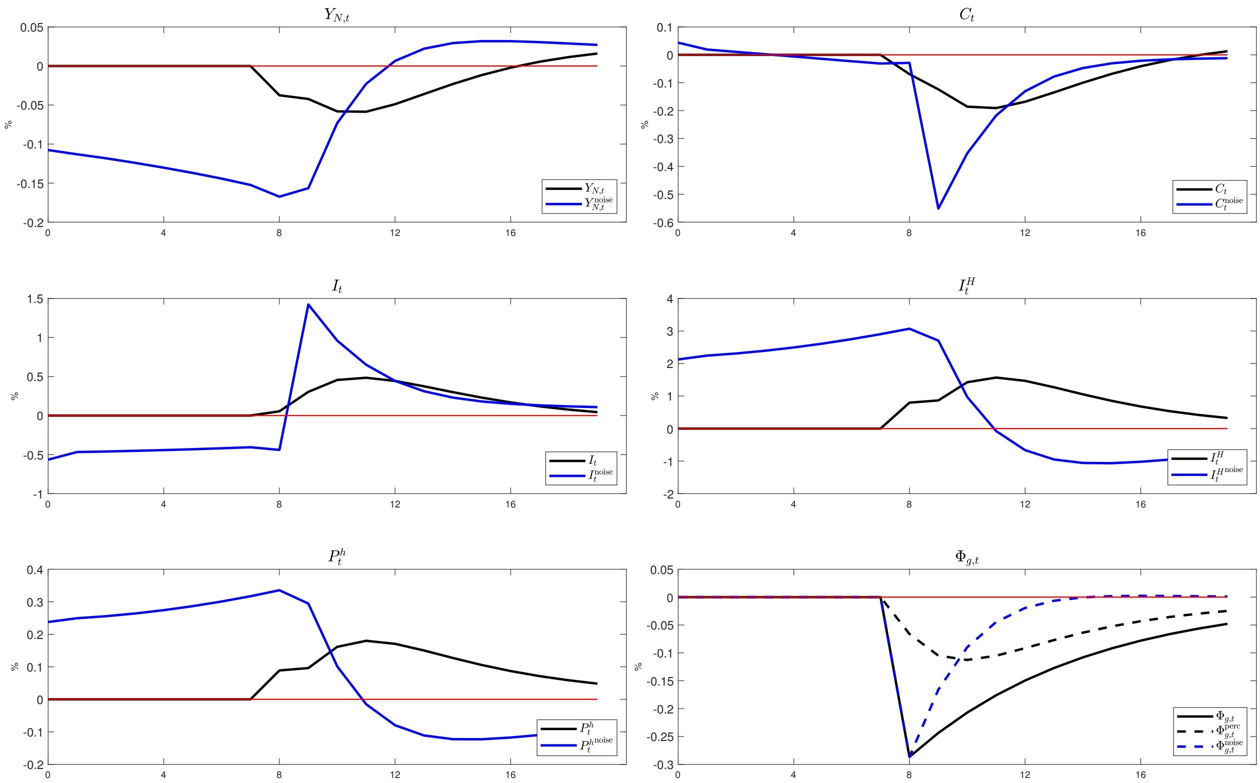

Figure 17: Fake news and True shock

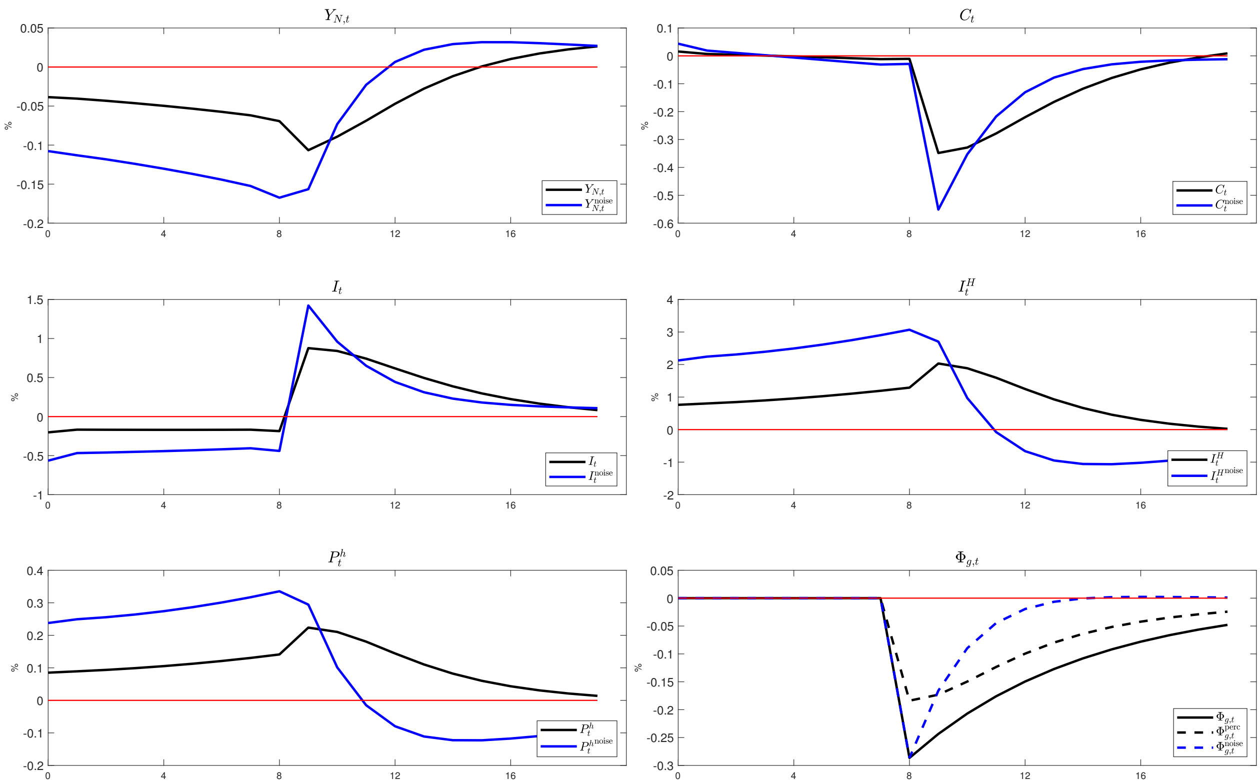

Figure 18: Fake news and True News