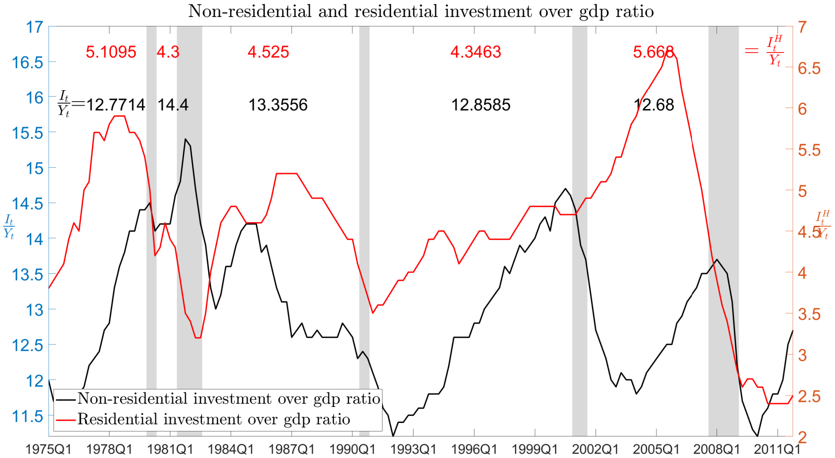
Overbuilding and Underinvestment over Housing Boom-Bust Cycles

Xiang SHI

Department of Economics, Hong Kong University of Science and Technology

November 24, 2023

Non-residential Investment crowds out Residential Investment



1990-1999: $\mathrm{corr}(\frac{I_t^H}{Y_t}, \frac{I_t}{Y_t}) = 0.76$; 2000-2007: $\mathrm{corr}(\frac{I_t^H}{Y_t}, \frac{I_t}{Y_t}) = -0.65$; 1947-2019:

Housing Market Boom and Crowded-out Capital

- Demand driven housing market boom
  - Residential investment and housing price jump up $I_t^H \uparrow$
  - Expansion in nondurable consumption $C_t \uparrow$
    - Wealth Effect: more asset works as collateral and equity extraction

Housing Market Boom and Crowded-out Capital

- Demand driven housing market boom
  - Residential investment and housing price jump up $I_t^H \uparrow$
  - Expansion in nondurable consumption $C_t \uparrow$
    - Wealth Effect: more asset works as collateral and equity extraction
- Physical Investment is crowded out
  - Macro: general equilibrium.

$$Y_t = C_t \uparrow + I_t + I_t^H \uparrow$$

  - Micro: investment portfolio reallocation $\rightarrow$ decrease holding of capital(equity); increase holding of residential asset

What if the boom is inefficient?

- Housing market boom without fundamental support
  - Increased new construction in housing market: **Overbuilding**

What if the boom is inefficient?

- Housing market boom without fundamental support
  - Increased new construction in housing market: **Overbuilding**
- Crowded-out physical investment: **Underinvestment**
  - Macro: Capital misallocation. Too much residential asset yet too little physical capital
  - Micro: A large capital distortion and higher real interest rate(relative to LR real interest rate)

What if the boom is inefficient?

- Housing market boom without fundamental support
  - Increased new construction in housing market: **Overbuilding**
- Crowded-out physical investment: **Underinvestment**
  - Macro: Capital misallocation. Too much residential asset yet too little physical capital
  - Micro: A large capital distortion and higher real interest rate(relative to LR real interest rate)
- Amplified economic loss during bust period Market Share

Imperfect Information and (Fake) News Shock

- Throughout this paper I use news and fake news shock to analyze the crowding-out effect
- Intuitive introduction to the (fake) news shock
  - what the shock is
  - why I use this shock
- Takeaway: the crowding-out effect is not limited to the (fake) news shock

Imperfect Information and (Fake) News Shock

- Intuitive example

$$p_t^h = \rho p_{t-1}^h + \beta x_t + w_{t-3} + u_t + \alpha w_t$$

  where w_t is the news shock; p_t^h is the housing price; x_t other macro variables
- Imperfect information and inefficiency: w_t cannot be perfectly observed
  - instead $\widetilde{w}_t = w_t + \nu_t$ where ν_t is the noisy to the news w_t
  - non-fundamental supported boom: ν_t instead of w_t
- News shock w_t; Fake news shock ν_t

Why (Fake) News Shock?

- Long-lasting housing market boom (in data)→significant amount of crowded-out physical capital Other Types of Shock
  - consecutive lower physical investment I_t
  - significant change in state variable: capital (we expect a scarcity in capital caused by crowd-out effect)
- Require assumption on the persistent of a standard shock such as credit shock or sentiment shock
- Imperfect information, news and fake (inefficiency) news shock
  - merit 1: one single shock can generate persistent housing market boom
  - merit 2: indeed explains a moderate share of variation of housing price in reality

Novelty in This Paper

- News and fake news shock $\widetilde{w}_t = w_t + \nu_t$

$$p_t^h = \rho p_{t-1}^h + \beta x_t + w_{t-3} + u_t + \alpha \widetilde{w}_t$$

- An improvement on Barsky and Sims (2012)
  - VAR identification to observation shock $\widetilde{w}_t$ (with contemporaneous endogeneity α)
- A new algorithm to identify fake news shock ν_t
- Empirically crowding-out effect is significant

Novelty in This Paper

- (Fake) News shock on preference
  - Utility function follows $U_t = \frac{\left(c_t^{\phi_t} h_t^{1-\phi_t}\right)^{1-\sigma}}{1-\sigma}$
  - Stochastic preference $\log(\phi_t) = \log(\overline{\phi}) + \rho_\phi \log(\phi_{t-1}) + w_{t-3} + u_t$
- Theoretically crowding-out effect is affected by three factors in demand side:
  - Relative intratemporal elasticity of substitution $\frac{U_H}{U_C} = f\left(p_t^H, p_{t+1}^H\right)$ (to intertemporal elasticity $U_C = \beta R U_{C'}$)
  - Financial friction
  - Idiosyncratic income shock and wealth distribution
- Quantitatively crowd-out effect can explain 13% welfare loss.
- After introducing a macroprudential policy which curbs the LTV ratio, the welfare loss is reduced significantly

Literature Review

- VAR Identification on news shock with contemporaneous endogeneity
  - Lorenzoni (2009), Barsky and Sims (2012)
- VAR Identification on fake news shock
  - Chahrour and Jurado (2018), Wolf and McKay (2022)
- Crowding-out effect works through general equilibrium and pay more attention on the pass-through between residential investment and consumption
  - Capital Misallocation: Rognlie et al. (2018), Chodorow-Reich et al. (2021), and Feng et al. (2022)
- Crowding-out effect amplifies on the supply side: scarcity of physical capital
  - Mian et al. (2013), Favilukis et al. (2017), and McQuinn et al. (2021)
- Solving heterogeneous agent model with imperfect information (noisy shock) and quantitative result
  - Etheridge (2019), Kaplan et al. (2020) and Auclert et al. (2021)

Outline

Empirical Evidence

- Introduction to VAR model and identification to news shock
- Algorithm to identify the fake news shock
- Empirical evidence
- Historical decomposition: significance in reality

VAR identification

- VAR
$$y_t = \Phi_1 y_{t-1} + \Phi_2 y_{t-2} + \ldots + \Phi_4 y_{t-4} + R\varepsilon_t \quad (1)$$
- y_t vector of macro variables
  - output, consumption, physical investment, housing price, housing supply and capital (equity) price
  - 1985-2007, Quarterly
- ε_t vector of shocks
  - monetary policy shock, productivity shock $\varepsilon_{i,t}$
  - land supply shock $\varepsilon_{s,t}$
  - housing market demand shock $\varepsilon_{u,t}$
  - housing market demand news shock $\varepsilon_{w,t}$
- Objective: identify the structure shocks' effect (column) R_w

VAR identification (conceptual framework)

- Objective: IRF of macro variables responding to fake news (noisy) shock ν_t
  - News and fake news shock $\widetilde{w}_t = w_t + \nu_t$

VAR identification (conceptual framework)

- Objective: IRF of macro variables responding to fake news (noisy) shock ν_t
  - News and fake news shock $\widetilde{w}_t = w_t + \nu_t$
- Variation of housing price is driven by $\mathrm{var}(p_t^h) = \alpha_i \mathrm{var}(\varepsilon_{i,t}) + \alpha_s \mathrm{var}(\varepsilon_{s,t}) + \alpha_u \mathrm{var}(u_t) + \alpha_w \mathrm{var}(\widetilde{w}_t) + \ldots$

VAR identification (conceptual framework)

- Objective: IRF of macro variables responding to fake news (noisy) shock ν_t
  - News and fake news shock $\widetilde{w}_t = w_t + \nu_t$
- Variation of housing price is driven by $\mathrm{var}(p_t^h) = \alpha_i \mathrm{var}(\varepsilon_{i,t}) + \cancel{\alpha_s \mathrm{var}(\varepsilon_{s,t})} + \alpha_u \mathrm{var}(u_t) + \alpha_w \mathrm{var}(\widetilde{w}_t) + \ldots$
- Sign restriction to rule out any shock on supply side

VAR identification (conceptual framework)

- Objective: IRF of macro variables responding to fake news (noisy) shock ν_t
  - News and fake news shock $\widetilde{w}_t = w_t + \nu_t$
- Variation of housing price is driven by
  $\mathrm{var}(p_t^h) = \alpha_i \mathrm{var}(\varepsilon_{i,t}) + \cancel{\alpha_s \mathrm{var}(\varepsilon_{s,t}) + \alpha_u \mathrm{var}(u_t)} + \alpha_w \mathrm{var}(\widetilde{w}_t) + \ldots$
- Sign restriction to rule out any shock on supply side
- Orthogonal restriction to rule out any contemporaneous housing market demand shock u_t Purification

VAR identification (conceptual framework)

- Objective: IRF of macro variables responding to fake news (noisy) shock ν_t
  - News and fake news shock $\widetilde{w}_t = w_t + \nu_t$
- Variation of housing price is driven by
  $\mathrm{var}(p_t^h) = \cancel{\alpha_I \mathrm{var}(\varepsilon_{I,t}) + \alpha_s \mathrm{var}(\varepsilon_{s,t}) + \alpha_u \mathrm{var}(u_t)} + \alpha_w \mathrm{var}(\widetilde{w}_t) + \ldots$
- Sign restriction to rule out any shock on supply side
- Orthogonal restriction to rule out any contemporaneous housing market demand shock u_t Purification
- Assumption: Housing market demand news shock explains the housing price most

VAR identification (conceptual framework)

- Chahrour and Jurado (2018): Theoretically fake news shock ν_t is observational equivalent with two incorporated shocks
  - $\widetilde{w}_t$ realizes first at t
  - τ period later, when w_t works at $t + \tau$, a negative $-u_{t+\tau}$ offsets its effect

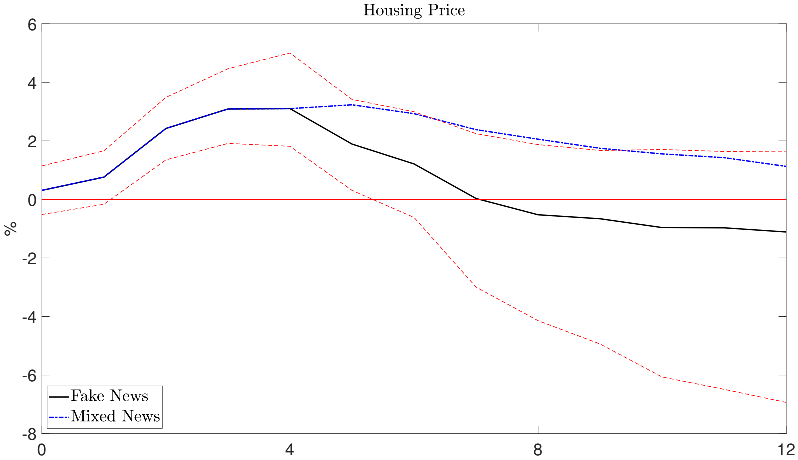
VAR identification (conceptual framework)

- Chahrour and Jurado (2018): Theoretically fake news shock ν_t is observational equivalent with two incorporated shocks
  - $\widetilde{w}_t$ realizes first at t
  - τ period later, when w_t works at $t+\tau$, a negative $-u_{t+\tau}$ offsets its effect
- Identification to fake news as Microfoundation

$$\widehat{y}_i^F = \begin{cases} y_i & i \leq \tau \\ y_i - \frac{e_k' y_{k,\tau+1}}{e_k' y_{k,0}^\tau} y_{i-\tau-1}^\tau & i > \tau \end{cases} \tag{2}$$

where $y_i \in U$ and y_i^τ is the response path to a contemporaneous shock on direct fundamental impact, variable k, in equation 10

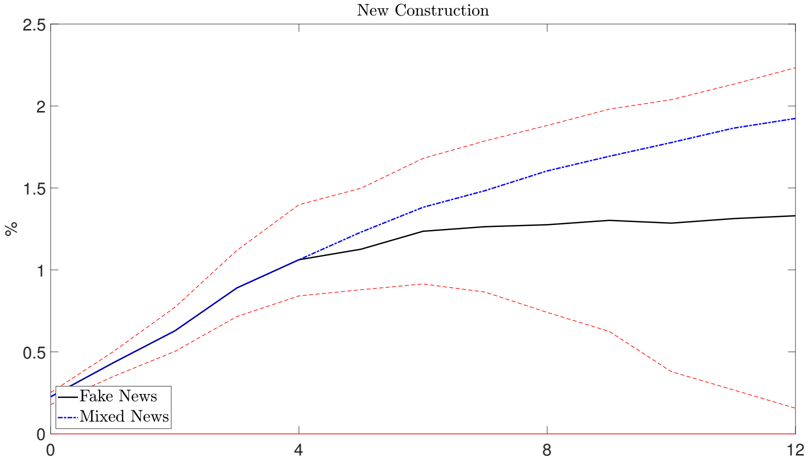
Housing Market Boom-Bust Cycle



90% confidence band; percentage derivation from mean

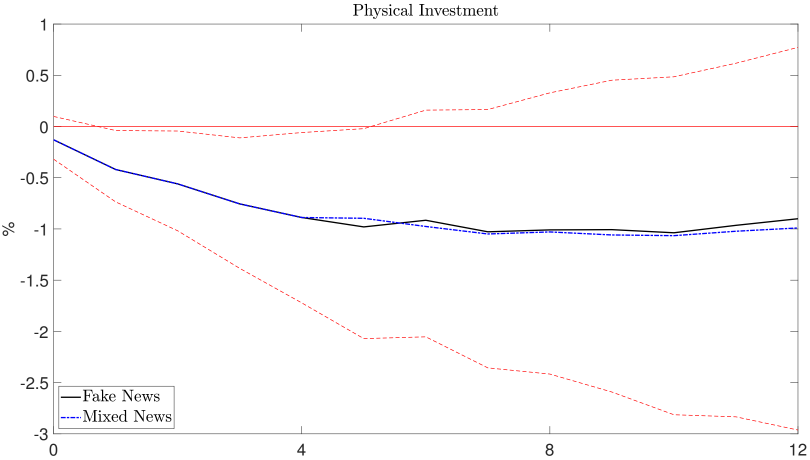
Housing market boom-bust cycles

Housing Market Boom-Bust Cycle



90% confidence band; percentage derivation from mean
Housing market boom-bust cycles

Crowded-out Physical Investment

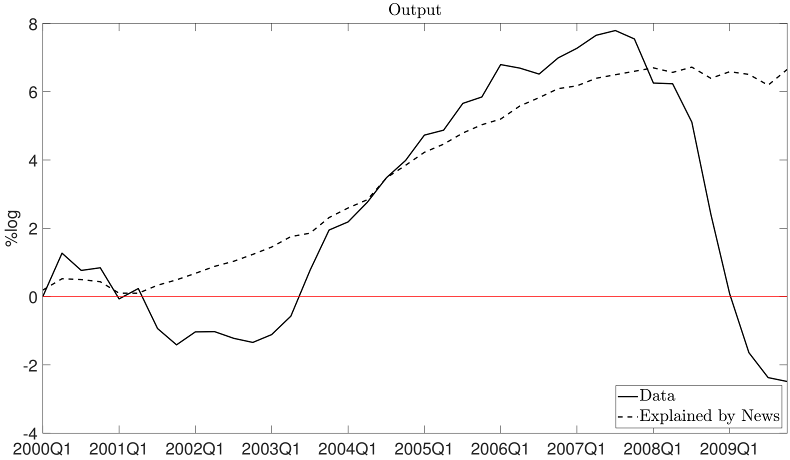


Inertia in physical investment

Recession period. Less demand on physical capital and production.

Is fake news or news important in reality?

Historical variance decomposition

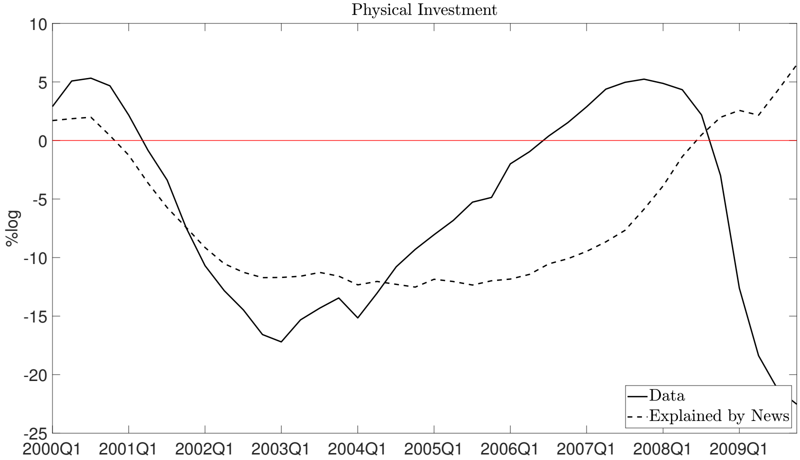


$\log(Y_t) - \log(Y_t^T)$ where Y_t^T is the trend component that calculated by average growth rate

Driven by news shock during boom period; significant divergence during bust period;

Is fake news or news important in reality?

Historical variance decomposition



$\log(I_t) - \log(I_t^T)$ where I_t^T is the trend component that calculated by average growth rate

Driven by news shock during boom period; significant divergence during bust period;

Empirically Significant

- A new algorithm to identify fake news shock
- Empirically crowding-out effect is significant
- Historical decomposition: news shock and fake news shock can explain part of the boom-bust cycle in housing market

Model Implication

- Model setting: Bewley-Huggett-Aiyagari model
    - heterogeneous household
    - representative producers
- Crowding-out effect is influenced by three factors on demand side

An Bewley-Huggett-Aiyagari model without aggregate shock

- Household maximize their utility by choosing housing services, non-durable consumption and liquid asset

$$\max_{c_t^i, h_t^i, a_t^i} \sum_{t=0}^{\infty} \beta^t U_t^i \left(c_t^i, h_t^i \right) \tag{3}$$

- Inelastic labor supply and idiosyncratic income shock

$$c_t^i + a_t^i + p_t^H h_t^i = R_t a_{t-1}^i + w_t \varepsilon_t^i + \left(1 - \delta^H \right) p_t^H h_{t-1}^i + T_t \tag{4}$$

- Financial Friction: occasional bound collateral constraint

$$-a_t^i \leq \gamma p_t^H h_t^i \tag{5}$$

An Bewley-Huggett-Aiyagari model without aggregate shock

- Two sector model (on supply side) with representative producers
- Construction Sector: $Y_{H,t} = A_{H,t}\mathscr{L}^{\theta}K_{H,t-1}^{\nu}L_{H,t}^{1-\nu-\theta}$
  - New construction is built through TFP on construction sector A_H, Land $\mathscr{L}$, physical capital (used by construction sector) K_H and labor L_H
  - Land supply $\mathscr{L}$ is fixed
- Final Goods production: $Y_{N,t} = A_{N,t}K_{N,t-1}^{\alpha}L_{N,t}^{1-\alpha}$
- Complete market

An Bewley-Huggett-Aiyagari model without aggregate shock

- Capital and labor are provided by household $K_{N,t-1} + K_{H,t-1} = K_{t-1} = \int a^i_{t-1} di$ and $L_{N,t} + L_{H,t} = \int \varepsilon^i_t di$
- Housing market clearing condition: $Y_{H,t} = H_t - (1 - \delta^H) H_{t-1}$ where $H_{t-1} = \int h^i_{t-1} di$
- Final goods can either be consumed or be invested

$$Y_{N,t} = K_t - (1 - \delta) K_{t-1} + C_t$$

Heuristic Analysis

- Expectation of housing prices in the future$p^H_{t+T+1} \uparrow$
- Residential investment and consumption will response to it
- Physical investment is crowded out
- Perturbation around s.s; Partial on housing price (fix other prices)
- Derivation: $\widetilde{h}_t$, $\widetilde{c}_t$ and $\widetilde{I}_t$

Crowding-out effect: increased in housing demand

Corollary 1

A positive expectation about the housing price change at time $T+1$ will induce a jump in demand for housing at time t. The response extends as follows:

$$\widetilde{h}_t \Big|_{h_{t+i},\mu_{t+i},\lambda_{t+i},,i\in[1,T]} = \zeta_t dp^H_{t+T+1} \tag{6}$$

where $\zeta_t = -\frac{1}{u_h''}\mathbb{E}_t\left[\beta\left(1-\delta^H\right)\right]^T \prod_{s=1}^T \frac{\lambda_{t+s}}{\lambda_{t+s}-\mu_{t+s}}\lambda_{t+T+1}$

Crowd-out effect of Overbuilding: Passthrough

Proposition 1

Household will adjust their consumption of non-durable goods based on overbuilding and precautionary saving. The extent of adjustment is decided by

$$\widetilde{c}_t = \underbrace{\Phi_H \widetilde{h}_t}_{\text{substitution effect}} - \underbrace{\Phi_\mu \widetilde{\mu}_t}_{\text{credit effect}} + \underbrace{\Phi_{p^H}\left[\frac{1}{1-(1-\delta^H)\frac{1}{R}} F^H(\widetilde{H}_t) - \frac{(1-\delta^H)\frac{1}{R}}{1-(1-\delta^H)\frac{1}{R}} F^H(\widetilde{H}_{t+1})\right]}_{\text{wealth effect}}$$

$$- \underbrace{\Phi_{cov}\widetilde{cov}_t}_{\text{precautionary saving effect}}$$

where $F^H(\cdot)$ is the inverse supply function

Crowd-out effect of Overbuilding

Proposition 2

The aggregate investment is driven by overbuilding and precautionary saving following

$$
\begin{aligned}
\widetilde{II_t} = -\Bigg\{ & \left(\Phi_H + \frac{\nu}{\alpha} p^H H\right) \int \widetilde{h}_t^{\,i} di - \Phi_\mu \int \widetilde{\mu}_t^i di \\
& + \Phi_{p^H} \left[\frac{1}{1-(1-\delta^H)\frac{1}{R}} F^H(\widetilde{H}_t) - \frac{(1-\delta^H)\frac{1}{R}}{1-(1-\delta^H)\frac{1}{R}} \mathbb{E}_t F^H(\widetilde{H}_{t+1}) \right] \\
& - \Phi_{cov}^i \int \widetilde{cov}_t^i di + \frac{\nu}{\alpha} Y_H p^H F^H(\widetilde{H}_t) \Bigg\}
\end{aligned}
$$

The overbuilding, $\widetilde{H}_t = \int \widetilde{h}_t^{\,i} di > 0$, will crowd out physical investment as long as the substitution effect Φ_H and wealth effect Φ_{p^H} are not negative enough.

Which elements affect this crowding-out effect?

- Smaller Relative Intratemporal elasticity of substitution Numerical Mechanism
  - Intratemporal elasticity of substitution $\frac{U_H}{U_C} = f\left(p_t^H, p_{t+1}^H\right)$: consumption bundle arrangement within this period
  - Intertemporal elasticity of substitution $U_C = \beta R U_{C'}$: consumption bundle arrangement over different period
  - "Smaller Relative": Intratemporal↑ and Intertemporal↓
  - Consumption responses to increased holding of housing less (for unconstrained households)
  - Crowding-out effect ↑

Which elements affect this crowding-out effect?

- Smaller Relative Intratemporal elasticity of substitution Numerical Mechanism
  - Intratemporal elasticity of substitution $\frac{U_H}{U_C} = f\left(p_t^H, p_{t+1}^H\right)$: consumption bundle arrangement within this period
  - Intertemporal elasticity of substitution $U_C = \beta R U_{C'}$: consumption bundle arrangement over different period
  - "Smaller Relative": Intratemporal↑ and Intertemporal↓
  - Consumption responses to increased holding of housing less (for unconstrained households)
  - Crowding-out effect ↑
- Larger financial friction Numerical Mechanism
  - Marginal value of house increases
  - More constrained households
  - Crowding-out effect ↑

Which elements affect this crowding-out effect?

Proposition 3

When the housing supply is fixed; initial housing distribution over dynamic path is exogenous and $\left(\frac{\frac{1-\beta}{\beta}}{\alpha A}\right)^{\frac{1}{\alpha-1}} > \frac{K}{L} > \left(\frac{\delta}{\alpha A}\right)^{\frac{1}{\alpha-1}}$ *holds, substitution effect* Φ_H *and wealth effect* Φ_{p^H} *will decrease as relative intratemporal elasticity of substitution (collateral constraint) is higher (slacker). Further, when the aggregate Khun-Tucker multiplier is not too large, credit effect* Φ_μ *will increase as relative intratemporal elasticity of substitution (collateral constraint) is higher (slacker).*

Which elements affect this crowding-out effect?

- Heterogeneous response along wealth distribution Numerical Mechanism Numerical(Inequality)
- Inequality and wealth distribution
  - Wealth people hold a large share of assets
  - Unconstrained households have the capacity and willingness to invest in real estate market
  - Amplification in $\int \widetilde{h_t}^i di$
- Marginal propensity to consume is right-skewed $\rightarrow$ Amplification in $\int \widetilde{\mu}_t^i di$

Theoretical Analysis

- An increased expectation on housing price in the future will trigger a housing market boom and crowd out physical investment
- Three factors on demand side influence the crowding-out effect
  - Relative Intratemporal elasticity of substitution
  - Financial friction
  - Wealth and MPC distribution

Quantitative Result

- Calibration and Bayesian estimation
- Welfare loss from crowding-out effect
- Distributional effect
- Macroprudential policy (countercyclical LTV ratio) reduces the welfare loss significantly

Full-fledged Model

- Two types of assets: liquid and illiquid asset

$$c_t + Q_t b_t + p_t^h \left[h_t - (1-\delta^h) h_{t-1} \right] = R_t Q_{t-1} b_{t-1} + (1-\tau) w_t l_t \varepsilon_t - p_t^h C\left(h_t, h_{t-1}\right) + T_t$$

where

$$C\left(h_t, h_{t-1}\right) = \frac{\kappa_1}{\kappa_2}\left(h_{t-1} + \kappa_0\right) \left| \frac{h_t - h_{t-1}}{h_{t-1} + \kappa_0} \right|^{\kappa_2}$$

Full-fledged Model

- Capital producer

$$\max E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \Lambda_{t,t+\tau} \left\{ Q_\tau I_\tau \eta_{I,t} - f\left(I_\tau, I_{\tau-1}\right) I_\tau \eta_{I,t} - I_\tau \right\}$$

$$\text{s.t. } f\left(I_\tau, I_{\tau-1}\right) = \frac{\psi_I}{2} \left(\frac{I_\tau}{I_{\tau-1}} - 1 \right)^2$$

- Capital price

$$Q_t \eta_{I,t} = 1 + \frac{\psi_I}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \eta_{I,t} + \psi_I \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \eta_{I,t} -$$
$$E_t \beta \Lambda_{t,t+1} \psi_I \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2$$

where $\eta_{I,t}$ is the marginal efficiency of the investment shock, following Justiniano et al. (2011)

Fake news shock on preference to houses

- Utility function

$$U_t\left(c_t, h_t, l_t\right) = \frac{\left(c_t^{\phi_t} h_t^{1-\phi_t}\right)^{1-\sigma}}{1-\sigma} - \kappa \frac{l_t^{1+\psi}}{1+\psi}$$

- Preference ϕ_t follows

$$\log(\phi_t) = \log(\overline{\phi}) + \rho_\phi \log(\phi_{t-1}) + w_{t-8} + u_t$$

From Model to Data (Steady State): Calibration

- Most parameters come from literature
- Use discount factor, disutility to labor supply, TFP of two production to match real interest rate, labor supply, physical investment over GDP ratio, and new construction over GDP ratio
- Distributional Moments are out of the scope of pre-matching

From Model to Data (Steady State): Calibration

- Most parameters come from literature
- Use discount factor, disutility to labor supply, TFP of two production to match real interest rate, labor supply, physical investment over GDP ratio, and new construction over GDP ratio
- Distributional Moments are out of the scope of pre-matching

Table: Distribution Moments

Description	**Data**	**Model**
Poor Hand-to-Mouth Household	0.121	0.1102
Wealthy Hand-to-Mouth Household	0.192	0.2059
Top 10 percent share of Liquid asset	0.8	0.5
Top 10 percent share of Iliquid asset	0.7	0.3

From Model to Data (Dynamic): Bayesian Estimation

- Full Information Bayesian Estimation (do not match specific moments) Solution Method
- 7 shock series with 7 variables
  - TFP shocks
  - Land supply shock
  - Preference (on residential asset) shock
  - Marginal efficiency of the investment shock
  - News shocks on land supply and preference

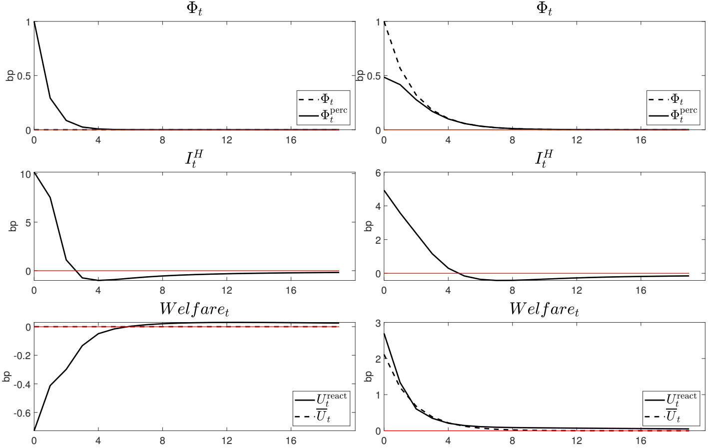
From Model to Data (Dynamic): Bayesian Estimation

- Full Information Bayesian Estimation (do not match specific moments) Solution Method
- 7 shock series with 7 variables
  - TFP shocks
  - Land supply shock
  - Preference (on residential asset) shock
  - Marginal efficiency of the investment shock
  - News shocks on land supply and preference

Moments	**Data**	**Model**
$\mathrm{corr}(p^H, I^H)$	0.42	0.23
$\mathrm{corr}(I, I^H)$	-0.15	-0.28
$\mathrm{corr}(I, Y)$	0.06	0.19
$\mathrm{corr}(I, Q)$	0.40	0.32

Fake News and Inefficient reaction $\rightarrow$ Welfare loss

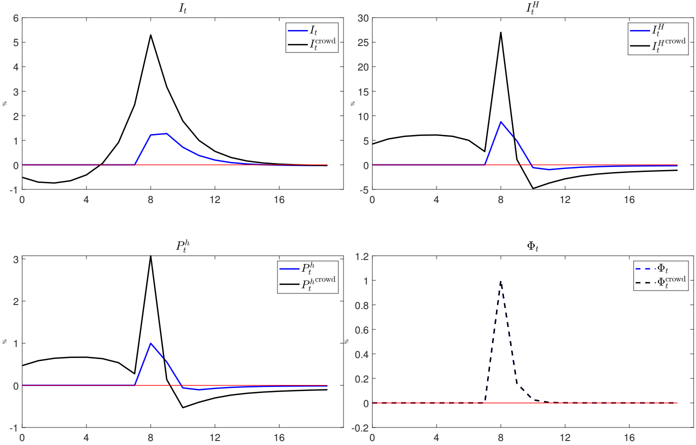
- Response to a fake news: welfare loss



Φ_t Preference Shock to Residential Asset; I_t^H New Construction of Residential Asset; Welfare is calculated from summing up the utility across households with unit weight

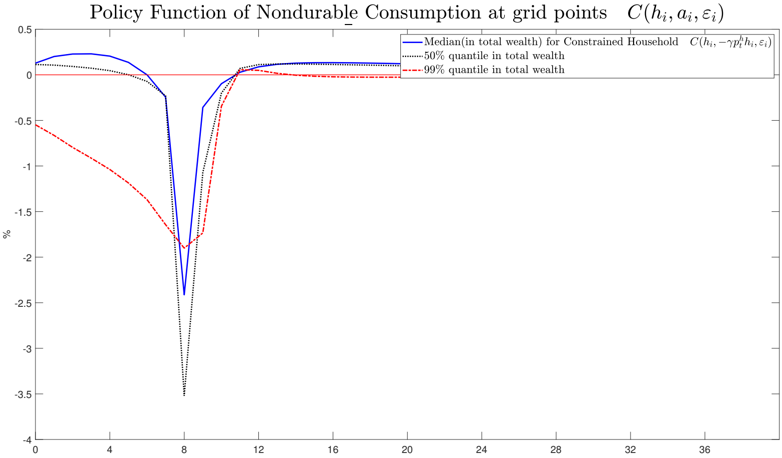
Crowd-out effect and boom-bust cycle

- Fake news shock at $t = 0$ and contemporaneous noise shock at $t = 8$
- The only difference: crowding-out effet



Distributional Effect

Right-Skewed distribution of MPC



Macroprudential Policy

- procyclical collateral constraint on capital-output ratio (countercyclical restriction on housing market boom)

$$\frac{\gamma_t}{\overline{\gamma}} = \left(\frac{\gamma_{t-1}}{\overline{\gamma}}\right)^{\rho_\gamma} \left(\frac{\upsilon_t}{\overline{\upsilon}}\right)^{\eta_\gamma(1-\rho_\gamma)}$$

  where γ_t is the collateral constraint in equation and υ_t is the capital-output ratio. $\overline{\gamma}$ and $\overline{\upsilon}$ are their corresponding value in steady state.

- The welfare loss of crowd-out effect decrease from 13% to 7%

Conclusion

- Document a new mechanism through which the housing market boom magnifies the recession.
- Empirically identify the crowding-out effect of overbuilding.
- Crowding-out effect of overbuilding is affected by relative intratemporal elasticity of substitution, financial friction and wealth distribution.
- A fake news that results in housing market boom can explain 13% welfare loss.

Kaplan, Greg, Giovanni L Violante, and Justin Weidner. 2014. "The wealthy hand-to-mouth."Technical report, National Bureau of Economic Research.

Smets, Frank, and Raf Wouters. 2003. "An estimated dynamic stochastic general equilibrium model of the euro area." *Journal of the European economic association* 1 (5): 1123–1175.

Smets, Frank, and Rafael Wouters. 2007. "Shocks and frictions in US business cycles: A Bayesian DSGE approach." *American economic review* 97 (3): 586–606.

Why Important?

What is the difference between this drawback and other capital misallocation? (Other Literature such as Firm Dynamic, Uncertainty, Asset Pricing?)

Why Important?

What is the difference between this drawback and other capital misallocation? (Other Literature such as Firm Dynamic, Uncertainty, Asset Pricing?)

- Real Estate is an important type of asset of household
  - "Housing equity forms the majority of illiquid wealth for households in every country with the exception of Germany" Kaplan et al. (2014)
  - Median Values of Household's Asset Holdings: 70% is real estate (Equity in own home, Rental property and other real estate)
  - Mean Values of Household's Asset Holdings: Nearly 50% is real estate (Excludes households in the top 1 percent of wealth)

Why Important?

What is the difference between this drawback and other capital misallocation? (Other Literature such as Firm Dynamic, Uncertainty, Asset Pricing?)

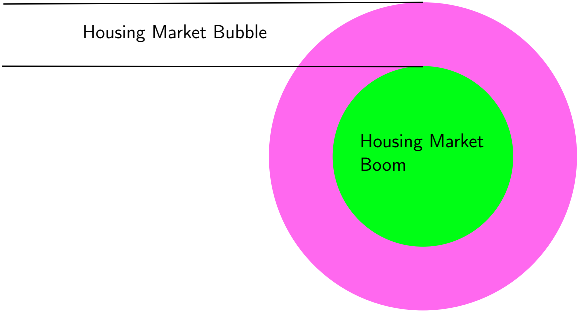
- Real Estate is an important type of asset of household
  - "Housing equity forms the majority of illiquid wealth for households in every country with the exception of Germany" Kaplan et al. (2014)
  - Median Values of Household's Asset Holdings: 70% is real estate (Equity in own home, Rental property and other real estate)
  - Mean Values of Household's Asset Holdings: Nearly 50% is real estate (Excludes households in the top 1 percent of wealth)
- General Equilibrium Effect: Powerful Back

Boom to Bubble: Inefficiency Problem

- Frictions in economy blow the boom further to a bubble

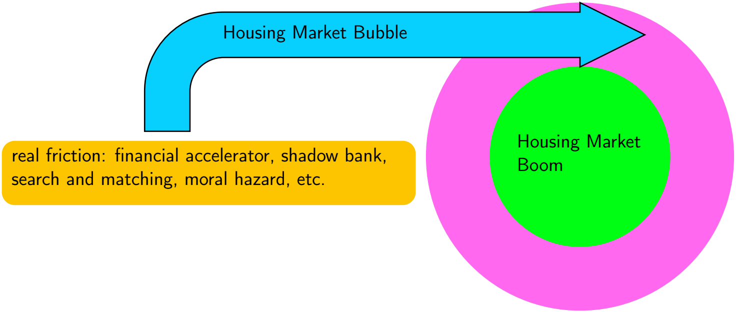
Boom to Bubble: Inefficiency Problem

- Frictions in economy blow the boom further to a bubble



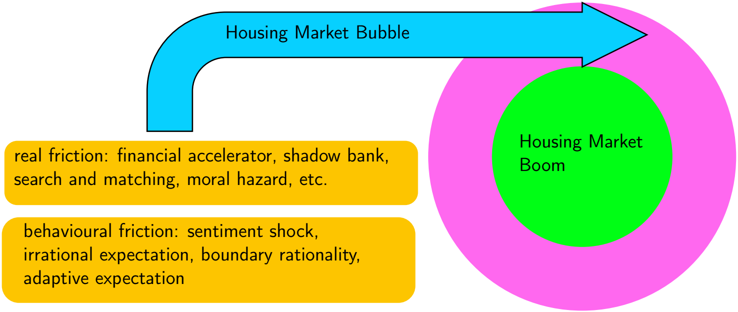
Boom to Bubble: Inefficiency Problem

- Frictions in economy blow the boom further to a bubble



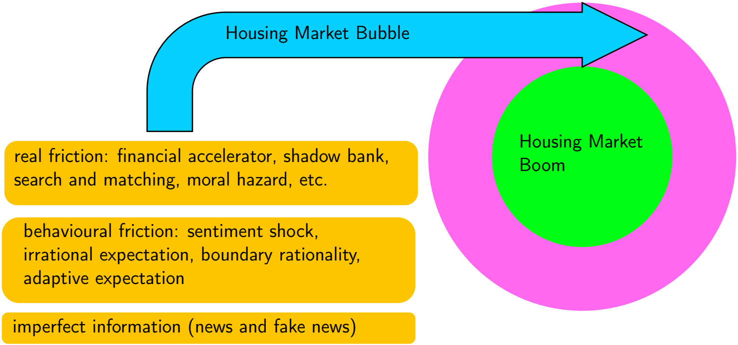
Boom to Bubble: Inefficiency Problem

- Frictions in economy blow the boom further to a bubble



Boom to Bubble: Inefficiency Problem

- Frictions in economy blow the boom further to a bubble



VAR identification

- VAR

$$y_t = \Phi_1 y_{t-1} + \Phi_2 y_{t-2} + ... + \Phi_q y_{t-q} + R\varepsilon_t \tag{7}$$

- R cannot be directly identified as what we get from data is the residual $u_t = R\varepsilon_t$
  - we can get $\mathrm{cov}(u_t) = RR' = \Omega$ yet not R itself
  - Cholesky decomposition of $\mathrm{cov}(u_t)$ will uniquely pin down Cholesky $P = \mathrm{chol}(\Omega)$
  - R can be identified up to rotation (orthogonal matrix Q) i.e. $R = PQ$

VAR identification: news shock

- we do not need to identify all the shocks but the one that we focus
  - Identify one column of rotation Q instead of the whole matrix
- Notation:
  - column related to shock i of Q: Qe_i
  - "identify the effect of news shock"≡"find the column Qe_w"≡"find the column $R^* = PQe_w$"≡"the shock of news"
  - $y^j_{i,t}$: the variation (IRF) of variable i that triggered by shock j at time t

VAR Identification Strategy: News shock

- solve the problem that Purification

$$R^{*} = \text{argmax}\frac{\mathrm{var}\left(y_{i,t}^{w}\right)}{\mathrm{var}\left(y_{i,t}\right)} = \text{argmax}\sum_{h=0}^{H}\frac{e_n'\left(\sum_{s=0}^{h}\Phi^{s}PRR'P'\Phi'^{s}\right)e_n}{e_n'\left(\sum_{s=0}^{h}\Phi^{s}PP'\Phi'^{s}\right)e_n} \tag{8}$$

s.t

$$R'R = 1 \tag{9}$$

$$e_k'PR = 0 \tag{10}$$

- The identified shock R^{*} can explain most of the forecast error of variable n.

VAR identification: news shock

- Housing price news shock q^*: A shock that can explain the variation of expectation error of housing price most, yet subject to
- Orthogonality (a constraint on the scalar (norm) of shock): $q'q = 1$
- Non-contemporaneous effect on variable k:
  $e_k' P q = 0$ Orthogonal Requirement Purification

VAR identification: news shock

$$y_{p^H,t+3} - E_t y_{p^H,t+3} = \left.\underbrace{R_{w^d}}_{w^d_{t+3}} + \underbrace{\Phi R_{w^d}}_{w^d_{t+2}} + \underbrace{\Phi^2 R_{w^d}}_{w^d_{t+1}}\right\} \text{housing price news shock on demand}$$

$$\left. + \underbrace{R_{w^s}}_{w^s_{t+3}} + \underbrace{\Phi R_{w^s}}_{w^s_{t+2}} + \underbrace{\Phi^2 R_{w^s}}_{w^s_{t+1}}\right\} \text{housing price news shock on supply}$$

VAR identification: news shock

$$
\begin{aligned}
y_{p^H,t+3} - E_t y_{p^H,t+3} = {} & \left.\underbrace{R_{w^d}}_{w^d_{t+3}} + \underbrace{\Phi R_{w^d}}_{w^d_{t+2}} + \underbrace{\Phi^2 R_{w^d}}_{w^d_{t+1}}\right\} \text{housing price news shock on demand} \\
& \left.+ \underbrace{R_{w^s}}_{w^s_{t+3}} + \underbrace{\Phi R_{w^s}}_{w^s_{t+2}} + \underbrace{\Phi^2 R_{w^s}}_{w^s_{t+1}}\right\} \text{housing price news shock on supply} \\
& \left.+ \underbrace{R_{u}}_{u_{t+3}} + \underbrace{\Phi R_{u}}_{u_{t+2}} + \underbrace{\Phi^2 R_{u}}_{u_{t+1}}\right\} \text{housing price contemporaneous shock} \\
& \left.+ \underbrace{R_{\varepsilon}}_{\varepsilon_{t+3}} + \underbrace{\Phi R_{\varepsilon}}_{\varepsilon_{t+2}} + \underbrace{\Phi^2 R_{\varepsilon}}_{\varepsilon_{t+1}}\right\} \text{other macro shocks} \\
& \left.+ \underbrace{R_{\xi}}_{\xi_{t+3}} + \underbrace{\Phi R_{\xi}}_{\xi_{t+2}} + \underbrace{\Phi^2 R_{\xi}}_{\xi_{t+1}}\right\} \text{other macro news shocks}
\end{aligned}
$$

VAR identification: news shock

Sign Restriction rules out supply shock

$$
\begin{aligned}
y_{p^H,t+3} - E_t y_{p^H,t+3} = & \left.\underbrace{R_{w^d}}_{w^d_{t+3}} + \underbrace{\Phi R_{w^d}}_{w^d_{t+2}} + \underbrace{\Phi^2 R_{w^d}}_{w^d_{t+1}}\right\} \text{housing price news shock on demand} \\
& \left.+ \underbrace{R_u}_{u_{t+3}} + \underbrace{\Phi R_u}_{u_{t+2}} + \underbrace{\Phi^2 R_u}_{u_{t+1}}\right\} \text{housing price contemporaneous shock} \\
& \left.+ \underbrace{R_\varepsilon}_{\varepsilon_{t+3}} + \underbrace{\Phi R_\varepsilon}_{\varepsilon_{t+2}} + \underbrace{\Phi^2 R_\varepsilon}_{\varepsilon_{t+1}}\right\} \text{other macro shocks} \\
& \left.+ \underbrace{R_\xi}_{\xi_{t+3}} + \underbrace{\Phi R_\xi}_{\xi_{t+2}} + \underbrace{\Phi^2 R_\xi}_{\xi_{t+1}}\right\} \text{other macro news shocks}
\end{aligned}
$$

VAR identification: news shock

Mild Assumption on the effects of macro shock (i.e. most of the variation of housing price cannot come from monetary policy, TFP variation or marginal cost shock

$$
\begin{aligned}
y_{p^H,t+3} - E_t y_{p^H,t+3} = & \left.\underbrace{R_{w^d}}_{w^d_{t+3}} + \underbrace{\Phi R_{w^d}}_{w^d_{t+2}} + \underbrace{\Phi^2 R_{w^d}}_{w^d_{t+1}}\right\} \text{housing price news shock on demand} \\
& + \left.\underbrace{R_u}_{u_{t+3}} + \underbrace{\Phi R_u}_{u_{t+2}} + \underbrace{\Phi^2 R_u}_{u_{t+1}}\right\} \text{housing price contemporaneous shock}
\end{aligned}
$$

VAR identification: news shock

Mild Assumption on the effects of macro shock (i.e. most of the variation of housing price cannot come from monetary policy, TFP variation or marginal cost shock

$$
\begin{aligned}
y_{p^H,t+3} - E_t y_{p^H,t+3} = &\left.\underbrace{R_{w^d}}_{w^d_{t+3}} + \underbrace{\Phi R_{w^d}}_{w^d_{t+2}} + \underbrace{\Phi^2 R_{w^d}}_{w^d_{t+1}}\right\}\text{housing price news shock on demand} \\
&\left.+ \underbrace{R_u}_{u_{t+3}} + \underbrace{\Phi R_u}_{u_{t+2}} + \underbrace{\Phi^2 R_u}_{u_{t+1}}\right\}\text{housing price contemporaneous shock}
\end{aligned}
$$

Solve a shock R_w that can explain the variation of expectation error of housing price **most**

VAR identification: news shock

Zero Restriction: Find a variable k which satisfies $y^u_{p^H,0} = 0$ and $y^w_{p^H,0} \neq 0$

$$y_{p^H,t+3} - E_t y_{p^H,t+3} = \left.\underbrace{R_{w^d}}_{w^d_{t+3}} + \underbrace{\Phi R_{w^d}}_{w^d_{t+2}} + \underbrace{\Phi^2 R_{w^d}}_{w^d_{t+1}}\right\} \text{housing price news shock on demand}$$

VAR identification: news shock

Zero Restriction: Find a variable k which satisfies $y^{u}_{p^H,0} = 0$ and $y^{w}_{p^H,0} \neq 0$

$$y_{p^H,t+3} - E_t y_{p^H,t+3} = \left.\underbrace{R_{w^d}}_{w^d_{t+3}} + \underbrace{\Phi R_{w^d}}_{w^d_{t+2}} + \underbrace{\Phi^2 R_{w^d}}_{w^d_{t+1}}\right\} \text{housing price news shock on demand}$$

subject the constraint $e'_k R_w = 0$

Purification

- Endogenous response to News

$$\Upsilon_t = \rho_g \Upsilon_{t-1} + \alpha_1 x_t + w_{t-3} + u_t + \alpha_2 w_t \tag{11}$$

where x_t denotes any macro variable such as interest rate, GDP, unemployment rate, etc. α_1 measures the cross-connection effect between macro economics and perception about fundamental. In other words a monetary policy shock may driven the interest rate and output first and then Υ_t comoves with the output and interest rate. w_{t-3} is just the news shock which announced 3 period ahead. u_t is the contemporaneous shock and α_2 measures the endogenous contemporaneous effect induced by news shock w_t.

- Purification objective: Construct a time series

$$y_{k,t} = \widehat{\Upsilon}_t = \Upsilon_t - \alpha_2 w_t$$

Purification: Heuristic Explanation

- Expectation: survey data

$$E_t\Upsilon_{t+6} = \rho^6\Upsilon_t + \rho^5 w_{t-2} + \rho^4 w_{t-1} + \rho^3 w_t$$

- Regression of $E_t\Upsilon_{t+6}$ on Υ_t

$$\gamma_t^E = \rho^5 w_{t-2} + \rho^4 w_{t-1} + \rho^3 w_t$$

- Regression of Υ_t on γ_t^E

$$\gamma_t = \Upsilon_t - \beta\gamma_t^E$$

Back

VAR Identification Strategy: News shock

- Fundamental impact indicator k: an index to the fundamental element that drives the demand function of housing demand function
  - i.e. the preference ϕ_t in Cobb–Douglas utility function $U(c_t, h_t, l_t) = \frac{\left(c_t^{\phi_t} h_t^{1-\phi_t}\right)^{1-\sigma}}{1-\sigma} + \kappa \frac{l_t^{1+\psi}}{1+\psi}$ which follows $\phi_t = (1-\rho_\phi)\overline{\phi} + \rho_\phi \phi_{t-1} + w_{t-\tau} + w_t^\tau$ and $w_{t-\tau}$ is the news shock to housing demand.
- NAHB/Wells Fargo Housing Market Index (HMI)
  - a monthly survey on NAHB members about their perception about the housing market right now him_t, and expectation over the next six month $E_t \mathrm{him}_{t+6}$
  - pure contemporaneous impact u_t^{him} from him_t

$$\mathrm{him}_t = \alpha_0 + \alpha_1 E_{t-6}\mathrm{him}_t + \alpha_2 E_t \mathrm{him}_{t+6} + u_t^{\mathrm{him}}$$

VAR Identification Strategy: News shock

Proposition 4

The identification to a news shock R^* through equation 8 is unique to covariance of the residual $\Omega = PP'$ from VAR's DGP 1.

VAR Identification Strategy: News shock

- Since my interest is to explore the house market boom, I further do the sign restriction on the vector IRF_t based on the rule

$$\text{IRF}_t^{sign} = \begin{cases} \text{IRF}_t & \text{if } e_n' \text{IRF}_{\boldsymbol{t}} e_n \geq 0 \\ -\text{IRF}_t & \text{if } e_n' \text{IRF}_{\boldsymbol{t}} e_n < 0 \end{cases}$$

Back

VAR Identification Strategy: Fake News shock

- Household

$$c_t^{-\sigma} = \beta R_{t+1} c_{t+1}^{-\sigma}$$

$$h_t^{\varphi} = w c_t^{-\sigma}$$

- Firm

$$R_t = \alpha \frac{y_t}{k_{t-1}} + \delta - 1$$

$$w_t = (1-\alpha)\frac{y_t}{h_t}$$

$$y_t = A_t k_{t-t}^{\alpha} h_t^{1-\alpha}$$

- Market Cleaning

$$y_t = c_t + I_t + \log(G_t)$$

$$I_t = k_t - (1-\delta)k_{t-1}$$

$$g_t = \rho_g g_{t-1} + w_{t-\tau} + w_t^{\tau} \tag{12}$$

VAR Identification Strategy: Fake News shock

- The household cannot know the value of G_t and w_t but a signal to then

$$\widetilde{g}_t = g_t + \nu_t^\tau$$

$$\widetilde{w}_{t-\tau} = w_{t-\tau} + \nu_{t-\tau}$$

- Household at time $t-\tau$ will have a perception of $w_{t-\tau}$ given the observation $\widetilde{w}_{t-\tau}$ and I denote it as $w_{t-\tau|t-\tau} = \theta \widetilde{w}_{t-\tau}$
- Denote $\widetilde{w}_t^i$ as an observation to shock w_{t-i}. For example, a news shock w_t will have effect on G at $t+\tau$. At time $t+1$ household gets a new observation related to w_t, $\widetilde{w}_{t+1}^1$, in addition to the old observation of w_t at time t $\widetilde{w}_t$. I further assume

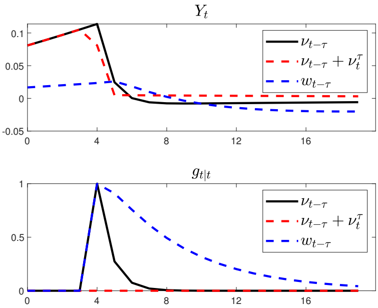
$$\widetilde{w}_{t-\tau+1}^1 = \widetilde{w}_{t-\tau+2}^2 = \cdots = \widetilde{w}_{t-1}^{\tau-1} = 0$$

holds. Therefore

$$w_{t-\tau|t-\tau} = w_{t-\tau|t-\tau+1} = w_{t-\tau|t-\tau+2} = \cdots = w_{t-\tau|t-1}$$

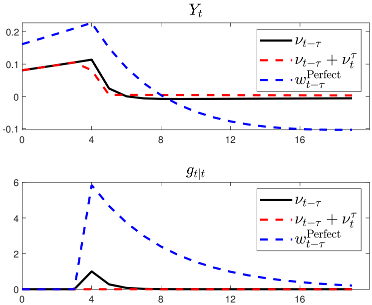
VAR Identification Strategy: Fake News shock

- Throughout exercise 1 to 3, imperfect information holds.
- Same perception: $g^{\nu}_{t|t} = g^{w}_{t|t} = g^{\nu+\nu^{\tau}}_{t|t}$
  1. Only noisy shock $\nu_{t-\tau}$;
  2. Fake news shock. A noisy shock on $w_{t-\tau}$ at time $t-\tau$, $\nu_{t-\tau}$, as well as a negative noisy shock on g_t at time t, ν^{τ}_{t};
  3. A news shock $w_{t-\tau}$.



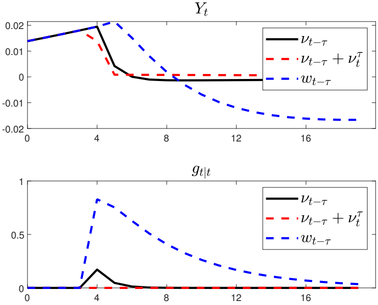
VAR Identification Strategy: Fake News shock

- Throughout exercise 1 to 2, imperfect information holds. In exercise 3, it is the type of perfect news.
- Same observation at time $t-\tau$: $\widetilde{w}_{t-\tau}$
    1. Only noisy shock $\nu_{t-\tau}$;
    2. Fake news shock. A noisy shock on $w_{t-\tau}$ at time $t-\tau$, $\nu_{t-\tau}$, as well as a negative noisy shock on g_t at time t, ν_t^τ;
    3. A perfect news shock $w_{t-\tau}$.



VAR Identification Strategy: Fake News shock

- Notation: Throughout exercise 1 to 3, imperfect information holds.
- Same observation at time $t - \tau$: $\widetilde{w}_{t-\tau}$
  1. Only noisy shock $\nu_{t-\tau}$;
  2. Fake news shock. A noisy shock on $w_{t-\tau}$ at time $t - \tau$, $\nu_{t-\tau}$, as well as a negative noisy shock on g_t at time t, ν_t^τ;
  3. A news shock $w_{t-\tau}$.

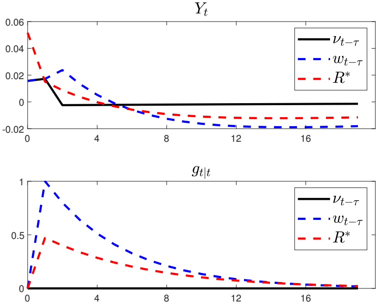


Back

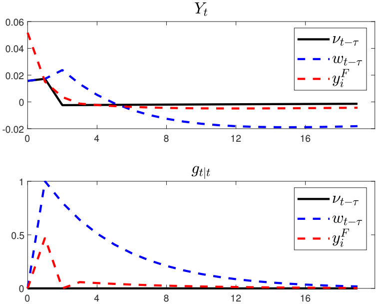
VAR Identification Strategy: Fake News shock

- The fundamental impact g_t is observable.

Identification to news shock R^*



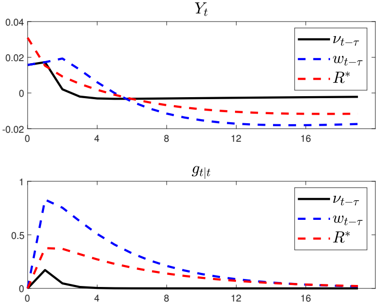
Identification to fake news shock R_F^*



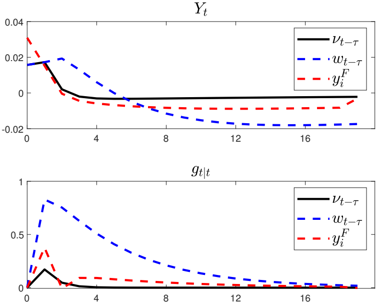
VAR Identification Strategy: Fake News shock

- The fundamental impact g_t is unobservable.

Identification to news shock R^*



Identification to fake news shock R_F^*



Back

VAR Identification Strategy: Fake News shock

- The fundamental impact g_t is observable
  - $g_t = \rho_g g_{t-1} + w_{t-\tau} + w_t^{\tau}$ and $\widetilde{w}_{t-\tau} = w_{t-\tau} + \nu_{t-\tau}$
  - whether the news $\widetilde{w}_{t-\tau}$ is true or fake is informed to household via g_t at time t without any delay
  - $y^{\tau}_{i-\tau-1}$ in equation 2 works as a contemporaneous shock w_t^{τ} offsets the true shock realized at t, $w_{t-\tau}$ and generates $g_t = 0$
- The fundamental impact g_t is unobservable.
  - there is no other signal that household can use to infer whether $\widetilde{w}_{t-\tau}$ comes from $w_{t-\tau}$ or $\nu_{t-\tau}$ but learn through observation gradually
  - $g_{t|t} = \gamma_1 g_{t-1|t-1} + \gamma_2 w_{t-\tau|t-\tau} + \gamma_3 g_{t-1} + \gamma_4 w_{t-\tau} + \gamma_5 \nu_t^{\tau} + \gamma_6 w_t^{\tau}$ where $\gamma_1 = \rho\left[1 - \frac{z_{11}}{z_{11}+\sigma^2_{\nu^{\tau}}}\right]$, $\gamma_2 = 1 - \frac{z_{11}}{z_{11}+\sigma^2_{\nu^{\tau}}}$, $\gamma_3 = \gamma_7\rho$ and $\gamma_4 = \gamma_5 = \gamma_6 = \gamma_7 = \frac{z_{11}}{z_{11}+\sigma^2_{\nu^{\tau}}}$. z_{11} can be solved from a positive root of quadratic equation
  - $y^{\tau}_{i-\tau-1}$ in equation 2 works as a contemporaneous shock w_t^{τ} which offsets the effect of true shock $w_{t-\tau}$ at time t.

Back

VAR identification to fake news shock

Assumption 1

The response to a news shock, either a fake news or a true news, under imperfect information, will be the same before the shock realized. In other words $\overline{R}_1 = \overline{R}_2 = R^*$ *and* $y_i^F = y_i^T = y_i, \forall y^F \in U^F, y^T \in U^T, y \in U, i \in [0, \tau]$ *will hold where* U^F *is the response to fake news and* U^T *is response to true news.* Numerical Example

VAR identification to fake news shock

Assumption 1

The response to a news shock, either a fake news or a true news, under imperfect information, will be the same before the shock realized. In other words $\overline{R}_1 = \overline{R}_2 = R^*$ *and* $y_i^F = y_i^T = y_i, \forall y^F \in U^F, y^T \in U^T, y \in U, i \in [0, \tau]$ *will hold where* U^F *is the response to fake news and* U^T *is response to true news.* Numerical Example

Takeaway

We do not need to separately identify the response before τ, when the news realizes.

VAR identification to fake news shock

Assumption 2

The empirically identified news shock U lies on the medial of response to fake news U^F and response to true news U^T. In other words, $y_i \in \left[y_i^F, y_i^T\right], \forall y^F \in U^F, y^T \in U^T, y \in U, i \in [\tau + 1, \infty]$ will hold. Furthermore, the news shock U is a linear combination of U^F and U^T and $y_i = \alpha y_i^F + \beta y_i^T$ holds. Numerical Example

VAR identification to fake news shock

Assumption 2

The empirically identified news shock U lies on the medial of response to fake news U^F and response to true news U^T. In other words, $y_i \in \left[y_i^F, y_i^T\right], \forall y^F \in U^F, y^T \in U^T, y \in U, i \in [\tau+1, \infty]$ will hold. Furthermore, the news shock U is a linear combination of U^F and U^T and $y_i = \alpha y_i^F + \beta y_i^T$ holds. Numerical Example

Takeaway

The mixed identification result can be refined to fake news by subtracting the realization βy_i^T

Crowd-out effect of Overbuilding

$$
\begin{aligned}
I\widetilde{I}_t = -\Bigg\{ & \left(\Phi_H + \frac{\nu}{\alpha} p^H H\right) \int \widetilde{h}_t{}^i dG_i - \Phi_\mu \int \widetilde{\mu}_t^i dG_i \\
& + \Phi_{p^H} \left[\frac{1}{1-\left(1-\delta^H\right)\frac{1}{R}} F^H(\widetilde{H}_t) - \frac{\left(1-\delta^H\right)\frac{1}{R}}{1-\left(1-\delta^H\right)\frac{1}{R}} \mathbb{E}_t F^H(\widetilde{H}_{t+1}) \right] \\
& - \Phi^i_{cov} \int \widetilde{cov}^i_t dG_i + \frac{\nu}{\alpha} Y_H p^H F^H(\widetilde{H}_t) \Bigg\}
\end{aligned}
$$

Crowd-out effect of Overbuilding

Demand-Side Effect

$$
\begin{aligned}
I\widetilde{I}_t = -\Bigg\{ & \left(\Phi_H + \frac{\nu}{\alpha} p^H H\right) \int \widetilde{h}_t{}^i dG_i - \Phi_\mu \int \widetilde{\mu}_t^i dG_i \\
& + \Phi_{p^H} \left[\frac{1}{1-\left(1-\delta^H\right)\frac{1}{R}} F^H(\widetilde{H}_t) - \frac{\left(1-\delta^H\right)\frac{1}{R}}{1-\left(1-\delta^H\right)\frac{1}{R}} \mathbb{E}_t F^H(\widetilde{H}_{t+1}) \right] \\
& - \Phi^i_{cov} \int \widetilde{cov}^i_t dG_i + \frac{\nu}{\alpha} Y_H p^H F^H(\widetilde{H}_t) \Bigg\}
\end{aligned}
$$

Crowd-out effect of Overbuilding

Supply-Side Effect

$$
\begin{aligned}
\widetilde{II_t} = -\Bigg\{ & \left(\Phi_H + \frac{\nu}{\alpha} p^H H\right) \int \widetilde{h_t}^i dG_i - \Phi_\mu \int \widetilde{\mu}_t^i dG_i \\
& + \Phi_{p^H} \left[\frac{1}{1-\left(1-\delta^H\right)\frac{1}{R}} F^H(\widetilde{H}_t) - \frac{\left(1-\delta^H\right)\frac{1}{R}}{1-\left(1-\delta^H\right)\frac{1}{R}} \mathbb{E}_t F^H(\widetilde{H}_{t+1}) \right] \\
& - \Phi^i_{cov} \int \widetilde{cov}^i_t dG_i + \frac{\nu}{\alpha} Y_H p^H F^H(\widetilde{H}_t) \Bigg\}
\end{aligned}
$$

Mechanism: Relative Intratemporal Elasticity

- A smaller intratemporal elasticity of substitution $\rightarrow$ A larger complement effect contemporaneously
- Financial friction works $\rightarrow$ marginal value of housing servicing $\uparrow$
  - One unit extra wealth $\Delta c_t^1 = 0.5$, $\Delta h_t^1 = 0.5$ or $\Delta c_t^2 = 0.5$, $\Delta h_t^2 = 0.2$
  - $\Delta\mu_t^1 < \Delta\mu_t^2 < 0$ as $\Delta h > 0$, more housing servicing used to slack the collateral constraint
- Wealth effect: $\Delta p_t^H > 0$, the value, that one unit of housing service provides, now can be transferred to utilitarian value more with a smaller intratemporal elasticity of substitution.

$$\frac{U_{h,t}}{U_{c,t}} = f\left(p_t^{+}, p_{t+1}^{-}\right)$$

Back

Mechanism: Financial Friction

- Marginal Value of housing is larger as now it plays a larger role in collateral constraint(through μ in ss)
- Financial friction works more silent $\rightarrow$ marginal value of housing servicing $\uparrow$
  - One unit extra wealth $\Delta c_t^1 = 0.5$, $\Delta h_t^1 = 0.5$, $\gamma = 0.5$ or $\Delta c_t^2 = 0.5$, $\Delta h_t^2 = 0.5$, $\gamma = 0.8$
  - $\Delta\mu_t^1 < \Delta\mu_t^2 < 0$ as $\Delta h > 0$, more housing servicing used to slack the collateral constraint
- Wealth effect: $\Delta p_t^H > 0$, Same as item 1 as in this scenario p_t^H and h_t are isomorphic in collateral constraint.

Back

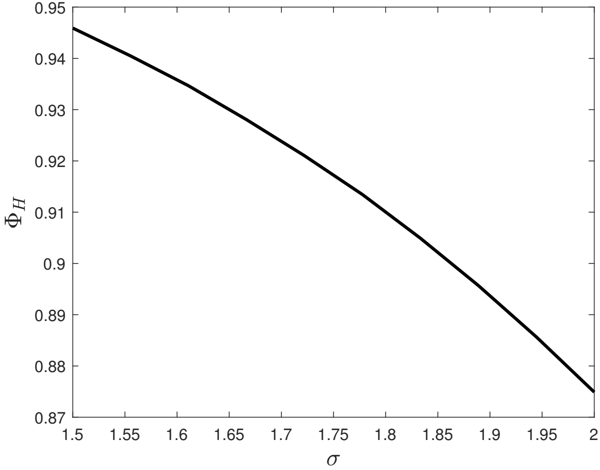
Mechanism: Wealth Distribution

- Left-skewed residential asset response:The more wealthy you are, the more response you would have.
- Left-skewed distribution of residential asset: The more response you have, the larger proportion of your wealth in distribution
- Right-skewed marginal propensity to consumption: Amplified passthrough from residential asset and nondurable consumption

Back

Relative Intratemporal Elasticity of Substitution

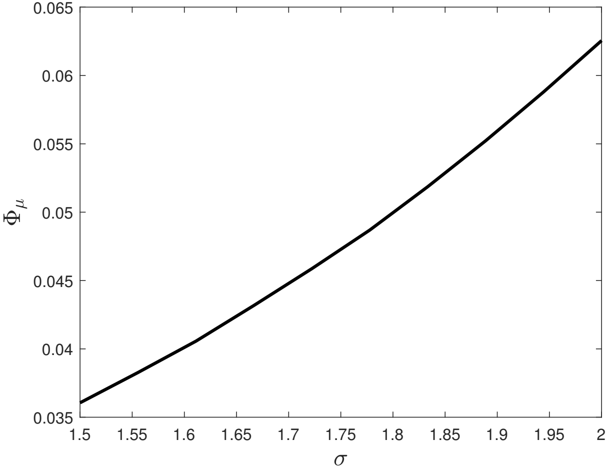
Relative Intratemporal Elasticity of Substitution $\sigma \uparrow$



Back

Relative Intratemporal Elasticity of Substitution

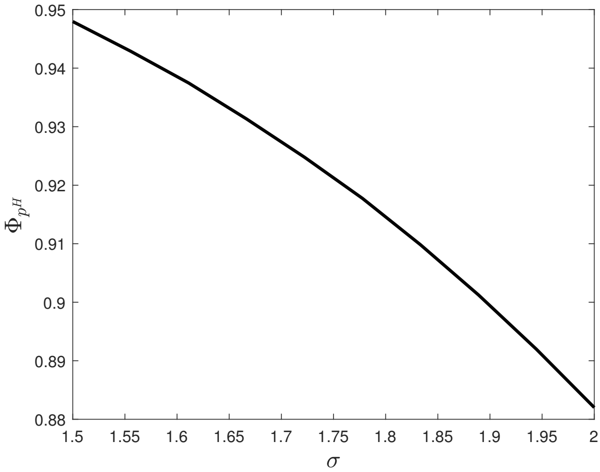
Relative Intratemporal Elasticity of Substitution $\sigma \uparrow$



Back

Relative Intratemporal Elasticity of Substitution

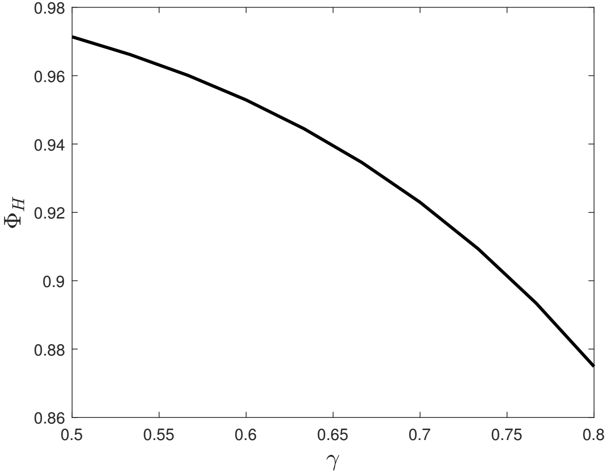
Relative Intratemporal Elasticity of Substitution $\sigma \uparrow$



Back

Financial Friction

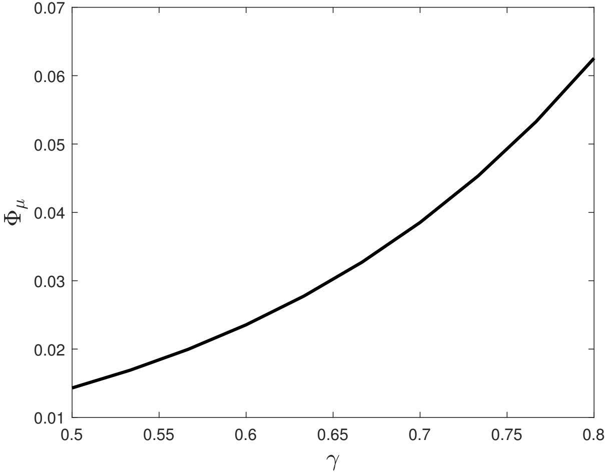
Looser Collateral Constraint $\gamma \uparrow$



Back

Financial Friction

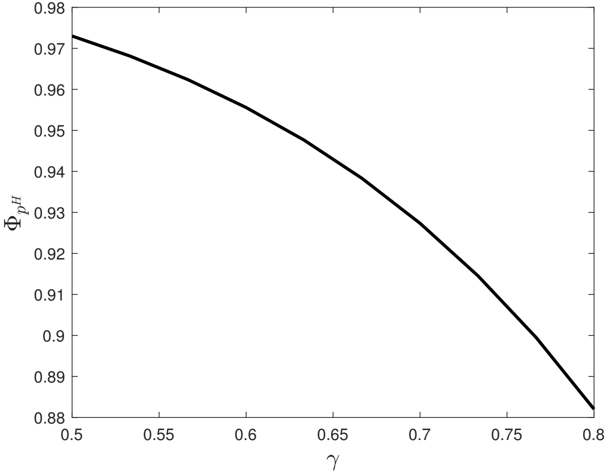
Looser Collateral Constraint $\gamma \uparrow$



Back

Financial Friction

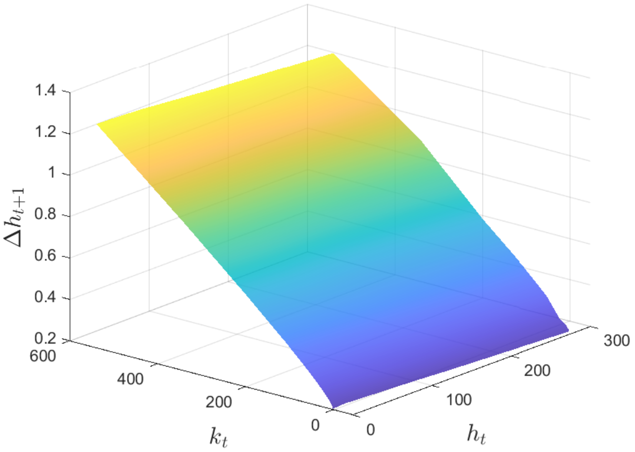
Looser Collateral Constraint $\gamma \uparrow$



Back

Wealth Distribution

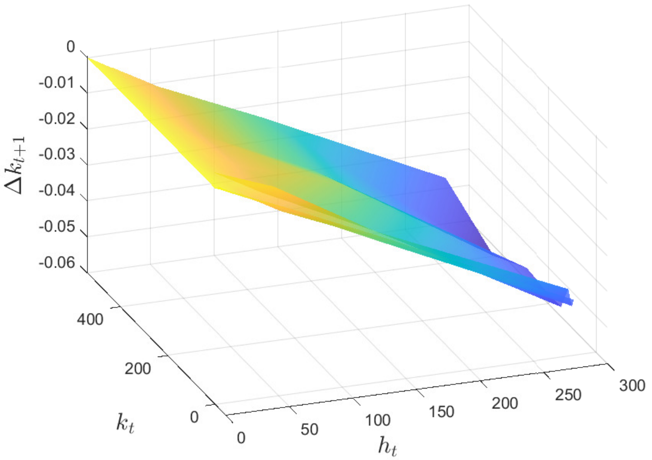
Heterogeneous response along wealth distribution



Back

Wealth Distribution

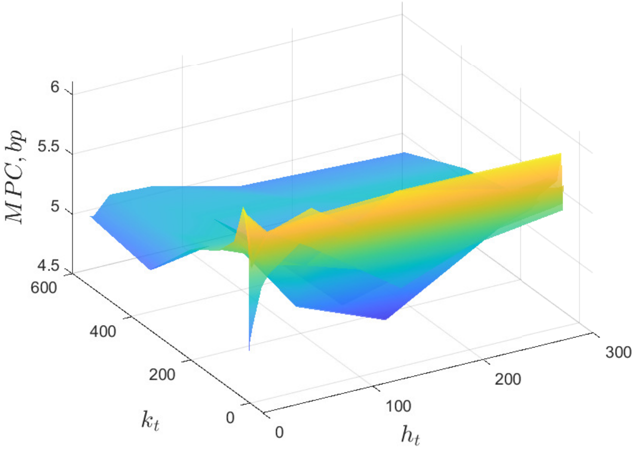
Heterogeneous response along wealth distribution



Back

Wealth Distribution

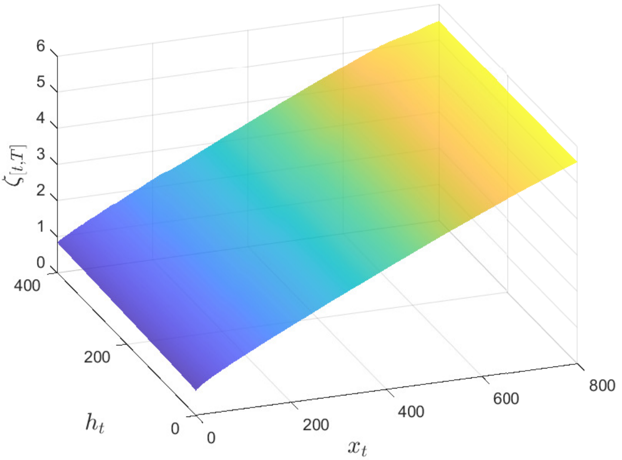
Heterogeneous response along wealth distribution



Back

Wealth Distribution

Heterogeneous response along wealth distribution



Back

Housing Expectation and Housing Market Boom

Corollary 2

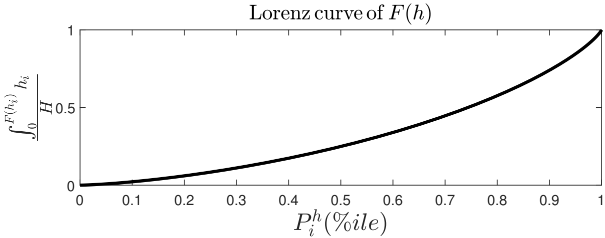
Ceteris paribus, an positive expectation about the housing price change in time $T+1$ will induce a jump in demand of housing service in time t. The response extend follows

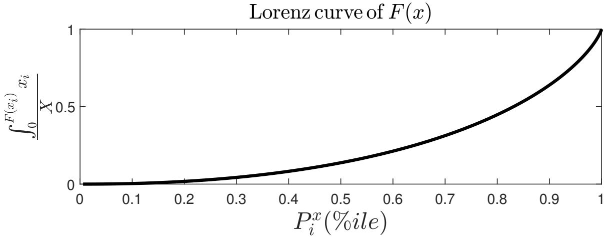
$$\left.\widetilde{h}_t^i\right|_{h_{t+i},\mu_{t+i},\lambda_{t+i},,i\in[1,T]} = \zeta_t^i dp_{t+T+1}^H \tag{13}$$

where $\zeta_t^i = -\frac{1}{u''_{h^i}}\mathbb{E}_t\left[\beta\left(1-\delta^H\right)\right]^T \prod_{s=1}^T \frac{\lambda_{t+s}}{\lambda_{t+s}-\mu_{t+s}}\lambda_{t+T+1}$

Wealth Distribution: Inequality

The Lorenz curve of residential asset h and real effective asset x





Back

Solution Method

- Two endogenous asset: liquid and illiquid asset
- Steady-state policy function: endogenous grid method; Carroll (2006) and Auclert et al. (2021)
- Steady-state distribution: Young (2010)
- Dimensionality reduction: Image Compression; Bayer et al. (2018)
- Perturbation with imperfect information: A new contribution based on Baxter et al. (2011) and Uhlig (1999)

Solution Method

- s_t state variable; c_t control variable; Ξ_t exogenous shocks
- prefect information
  - policy function (matrix) P_1, P_2, Q_1 and Q_2
  - $s_t = P_1 s_{t-1} + Q_1 \Xi_t$ and $c_t = P_2 c_{t-1} + Q_2 \Xi_t$
- imperfect information
  - perception on subset of state variables $s_{2,t}$ and $s_{2,t|t}$
  - law of motion of perception process (bayesian updating) $s_{2,t|t} = A^s s_{2,t-1|t-1} + P^s \widetilde{s}_{2,t}$ where $\widetilde{s}_{2,t}$ is the observation of $s_{2,t}$
  - we can still use P_2 and Q_2 because of CEQ (certainty equivalent) $c_t = P_2 s_{t|t} + \widetilde{Q}_2 \Xi_t$ ($\widetilde{Q}_2$ is adjusted from Q_2 by observation and perception function)
- solve new policy $\widetilde{P}_1$, new mapping $\widetilde{P}_3$ and $\widetilde{Q}_1$ satisfying $s_t = \widetilde{P}_1 s_{t-1} + \widetilde{P}_3 s_{t-1|t-1} + \widetilde{Q}_1 \Xi_t$

Solution Method: solve new policy $\widetilde{P}_1$

- $\widetilde{P}_1$ deviates from P_1 a lot (assuming $s_{t-1|t-1} = 0$)
  - $s_{t|t}$ deviates from $s_t \to c_t^{\text{imperfect}}$ deviates from $c_t^{\text{perfect}} \to$ investment $I_t^{\text{imperfect}}$ deviates from I_t^{perfect}
  - $s_t^{\text{imperfect}}$ deviates from s_t^{perfect}

Back

- literature: guess and verify
- new method: solving directly via technique in linear algebra

Full-information Bayesian Estimation

- Following Smets and Wouters (2003, 2007), data is detrended to growth rate with long-run trend $d\log(Y_t) = \overline{y} + y_t$
- State Space Model: To be comparable, implement the same augmentation
  - State Equation

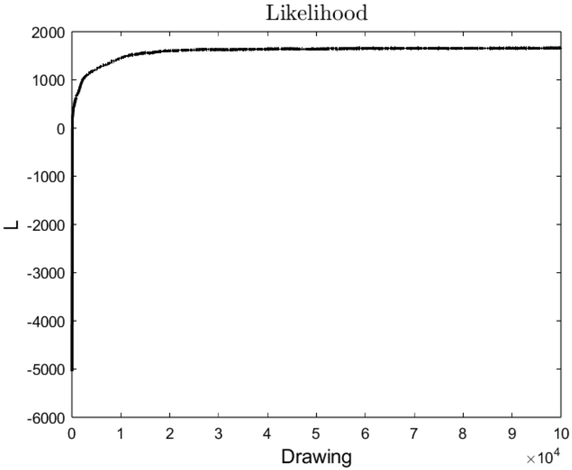
$$\begin{bmatrix} x_t \\ x_{t-1} \end{bmatrix} = \begin{bmatrix} P & 0 \\ I & 0 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ x_{t-2} \end{bmatrix} + \begin{bmatrix} Q \\ 0 \end{bmatrix} \varepsilon_t$$

  - Measurement Equation

$$y_t = \begin{bmatrix} H & -H \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_t \\ x_{t-1} \end{bmatrix} + S\zeta_t$$

Full-information Bayesian Estimation

- RWMH



Back