# The Power of Quantitative Easing: Liquidity Channel vs Interest Rate Channel

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**Abstract**

This paper disentangles the effectiveness of quantitative easing (QE) into two channels: Liquidity (injecting liquidity into the market) and interest rate (twisting the term yield). Through a general equilibrium model with household heterogeneity and financial friction, I demonstrate that they affect the power of QE by altering the liquidity and interest rate channel asymmetrically. Based on the model and calibration, I conclude that the effect of liquidity channel to stimulation power of QE on output is approximately 1.5 times larger than that of interest rate channel. Meanwhile, the complementarity between household heterogeneity and financial friction plays a vital role in determining the power of QE. In the end, I empirically identify these two channels and support the quantitative result by proposing a new instrument variable in IV-VAR.

**JEL classification:** C11, E21, E30, E52, E58

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# 1 Introduction

Quantitative easing is widely studied in Finance literature as it twists the term yield and drives up the price of long-term bonds. People agree that the term structure is affected mainly through three channels: imperfect sustainability, signal about future short-term rate and improvements in financial balance sheet[1], which is summarized by Kuttner (2018). Bauer and Rudebusch (2014) argue that 22% of the QE1 effect on term structure change was attributable to the interest rate channel and the left 78% comes from the term premium change (The balance sheet effect). Contrast to them, Krishnamurthy and Vissing-Jorgensen (2011) argued that interest rate channel is ascribed to the most important channel, compared to balance sheet channel. However, there are scarce researchers analyzing that how unconventional monetary policy stimulates the macroeconomy through different channels. All the literature focus on the channels empirically and in the financial market. Similar to those arguments in finance literature that a decreased long-term rate was triggered by QE via interest rate and fundamental liquidity status, I argue in this paper theoretically that the effect of QE to macroeconomics can also be divided into two channels: interest rate channel and liquidity channel.

These two channels are related to the asset purchasing process of the central bank. During the ZLB period, the federal reserve implemented unconventional monetary policy by increasing the holding of long-term treasury bond and mortgage-backed securities which we also call large asset purchasing policy. There are mainly two things that happened when the central bank bought long-term assets in financial market. One is that the central bank injected liquidity, the cash, into financial market through crowding out the holding of long-term bonds of financial institutions and the another one is that the interest rate of long-term bonds decreased as there is more demand to long-term bonds but constant supply in a short time. The unconventional monetary policy can work on stimulating economy either through injecting liquidity, which I outline as the liquidity channel, or through twisting the term yield and pulling down the long-term rate which I call the interest rate channel. The liquidity channel states that there is more liquidity in financial market and the underinvestment caused by financial friction is alleviated so that there are more investment and more output. Conversely the interest rate channel states that the drop in long-term rate changes people's expectation about future interest rate and the return of long-term bonds so that the financial institutions have a propensity to invest more in capital because of the non-arbitrage condition which generates portfolio adjustment and more output.

To understand these two channels on which unconventional monetary policy works, I first use a general equilibrium model with heterogeneous household and financial accelerator to investigate how the unconventional monetary policy stimulates economy through these two

[1]Imperfect sustainability states that there exists market segmentation so that long-term bonds with different maturities can not be substituted with each other freely. Signaling channel states that the change in expectation of future short-term interest rates will pass to the price of long-term bonds. Financial balance sheet channel implies that the change in balance sheet of financial institutions will affect their demand for long-term bonds with different maturity so that the price of long-term bonds will be changed.

channels and which element in economy regulate the power of unconventional monetary policy through affecting these channels analytically and quantitatively. The analytical result shows that financial friction and household heterogeneity play important role in determining the effect of two channels and increase the power of unconventional monetary policy from the supply and demand side of the economy. Similar to the argument under conventional monetary policy by Kaplan et al. (2018); Auclert (2019), the existence of heterogeneous household and hand-to-mouth agents also magnifies the effect of unconventional monetary policy through general equilibrium. and its amplification on liquidity and interest rate channel are the same. Even though at ZLB the interest rate is fixed and the consumption of household can not be spurred through Euler equation, the hand-to-mouth household with high MPC increases consumption and stimulates the economy as the Euler equation does not hold for them and the budget constraint governs their consumption. Meanwhile, there is an additional redistribution effect spawned by the heterogeneous household in liquidity channel as the liquidity that central bank used is funded from households and the households also hold the networth of the financial institutions and share the income and dividend of the financial institutions[2].

In addition to the amplification effect created by heterogeneous household at demand side, financial accelerator expands the effectiveness of unconventional monetary policy by affecting the liquidity and interest rate channel asymmetrically. Because of the financial constraint, financial institutions cannot borrow as much as they want to fulfil their ideal portfolio arrangement of assets. Therefore they will not increase their investment in capital even though the return on long-term bonds decreases when they do not have liquidity to adjust their investment portfolio. Conversely, because of the financial constraint and a large leverage ratio, they will increase more than one unit of money in capital by borrowing when they get one unit of money as liquidity injection. In this sense financial friction amplifies the effectiveness of unconventional monetary policy mainly through liquidity channel yet hardly through interest rate channel, albeit "not through" as the financial institutions still can get some extra liquidity through interest rate channel by pecuniary effect (they previously hold some amount of long-term bonds which now become more valuable).

Both of these two key elements, household heterogeneity and financial accelerator, can augment the power of unconventional monetary policy separately at demand side and supply side through the two channels. Moreover, they together engender a complementary effect to amplify the power of unconventional monetary policy through the two channels, akin to the discussion under conventional monetary policy by Bilbiie et al. (2022).

In order to quantify the relative effect of liquidity and interest rate channel and quantitatively explore which mechanism aforementioned mainly governs these two channels, I carefully calibrate the model and link the model to empirical evidence. After conducting several counterfactual experiments, I conclude that the effectiveness of liquidity channel is more significant than that of

[2]Di Maggio et al. (2020) empirically tested related effect and draw a conclusion that QE1 substantially increased refinancing activity and stimulate consumption while there is no theoretical investigation on this effect.

interest rate channel, and the formal one is roughly one and a half times larger than the latter one. Meanwhile, I also show that neither the household heterogeneity nor the financial accelerator alone helps amplify the power of unconventional monetary policy but together they contribute to the amplification.

Additionally, I use a VAR model to show the effects of unconventional monetary policy through liquidity and interest rate channel by proposing a new instrument variable and a new Bayesian method to identify IV-VAR, result of which makes my quantitative result more tractable and credible. I separately identify these two channels and empirically exhibit that the ratio of the effectiveness of unconventional monetary policy through liquidity channel and interest rate channel is almost the same as what I get quantitatively.

This paper contributes to the HANK and monetary policy literature such as McKay et al. (2016); Gornemann et al. (2016); Guerrieri and Lorenzoni (2017); Kaplan et al. (2018); Auclert (2019); Bayer et al. (2019); Hagedorn et al. (2019); Bilbiie (2020); Chang et al. (2021); Luetticke (2021). Most of them focus on investigating the conventional monetary policy or solving the forward guidance puzzle, yet this paper focuses on the cross effect between HANK and the most potent tools of unconventional monetary policy, QE. In this manner my work modifies and complements Cui and Sterk (2021); Sims et al. (2022). Cui and Sterk (2021) analyzed the effect of QE in a heterogeneous-household model while they did not specifically stress the importance of heterogeneity and compare the HANK model to RANK model. Meanwhile they do not incorporate financial friction in financial sector into their HANK model and only consider the precautionary effect and distributional MPC. Sims et al. (2022) inspected the contribution of heterogeneous households to the effectiveness of unconventional monetary policy, yet their discussion is not tractable without analytically solution. Additionally their work only focused on the gross effect of unconventional monetary in which they neither distinguished liquidity channel and interest rate channel nor the complementary effect between financial friction and household heterogeneity.

Further this paper also contributes to the literature related to unconventional monetary policy such as Gertler and Karadi (2011); Krishnamurthy and Vissing-Jorgensen (2011); Carlstrom et al. (2017); Harrison (2017); Sims and Wu (2021). However, most of them concentrated on the interaction between financial market and monetary policy or the optimal unconventional monetary policy. My paper combines these literature and HANK literature to clarify the cross effect of unconventional monetary policy.

This paper has three two contributions empirically. Firstly I make an improvement on methodology related to IV-VAR. Stock and Watson (2012) and Mertens and Ravn (2013) first proposed a new SVAR method incorporating instrument variables[3]. Later Arias et al. (2021) and Giacomini et al. (2021) first incorporated the Bayesian method into IV-VAR and proposed IV-BVAR. However they did not impose restrictions on the relationship between instrument variables

[3]Precisely they introduced proxy variables into SVAR. However, after several derivation, the estimation of related variables coincides with the formula used in IV estimation. Therefore their method is called IV-VAR.

and independent shock but estimated related relationships and wasted the degree of freedom on that. Then to identify the structure shock, they make some other assumptions on the coefficient matrix such as sign and zero restrictions. Conversely, I make an assumption on the relationship between instrument variables and shock, which saves the degree of freedom to estimate the coefficient matrix where I do not put any restriction. The new method I proposed will be more helpful and closer to reality as long as people find appropriate instrument variables. Secondly, this paper firstly identifies the liquidity channel and interest rate channel of unconventional monetary policy separately. It empirically demonstrates that the liquidity channel is more important than interest rate channel by introducing the new instrument variable which is identified by high-frequency data in financial market within treasury bonds issuing announcement windows.

There are five sectors remaining in this paper which are organized as follows. Section 2 shows the main body of the baseline model. Section 3 discusses how household heterogeneity and financial friction work together to influence the liquidity and interest rate channel of unconventional monetary policy. After the theoretical analysis section 4 provides the calibration and quantitative result related to the two channels. Then section 5 introduces the empirical support with the identification methodology and the description of instrument variables. In the end section 6 concludes the whole result.

# 2 Baseline Model

There are five sectors in the baseline model: households, financial institutions, firms, central bank and government. Households provide labour and earn wage income which they can use to invest in liquid and illiquid assets to earn asset return. Financial institutions are the supplier of illiquid asset which is their net worth and is used to invest in firm equity and long-term treasury bond funded by borrowing from central bank. Retailer collects the intermediate goods and differential them to the final goods produced with monopolistic power. Capital goods producer uses final goods to produce capital suffering from production cost and sell it to financial institutions. Central bank gets deposits from households and lends to financial institutions. Meanwhile they impose the conventional (set nominal interest rate) or unconventional (buy long-term treasury bonds from financial institutions) monetary policy to stabilize the economy. Finally the government issues long-term treasury bonds and provides the social welfare to hand-to-mouth households.

## 2.1 Household

There are three types of household in the economy and $i$ denotes their type following $i \in \{\mathrm{pHtM}, \mathrm{wHtM}, \mathrm{nHtM}\}$where $\mathrm{pHtM}$ states poor hand-to-mouth household.$\mathrm{wHtM}$ states wealthy hand-to-mouth household and respectively $\mathrm{nHtM}$ states non-hand-to-mouth household. Household $i$ supply labour $l_t^i$ to intermediate goods producers and earn real wage $w_t$ with idiosyncratic shock $\varepsilon_t^i$ at period $t$[4]. The government taxes $\tau$ portion of the total wage income to finance social welfare spending. Households can also hold two types of bond: short-term bonds $b$ and illiquid asset $a$. The liquid bonds $b_t$ brings a gross real return rate $R_t$ realized at time $t+1$; the illiquid asset $a_t$ brings a gross real return $R_{t+1}^a$ realized at time $t+1$. Households can freely adjust their holding of liquid bonds $b$ without transaction restriction but they cannot adjust the illiquid asset freely[5]. In addition to the interest rate and labour income households get lump sum tax transfer $T_t$ and public unemployment insurance subsidy $\Theta_t$.

Households at period $t$ maximize their future discounted utility

$$
\begin{aligned}
V(b_{t-1}^i, a_{t-1}^i, \varepsilon^i) &= \max_{c_t, b_t, X_t^i} U(c_t^i, l_t^i) + \beta \mathbb{E} V(b_t^i, a_t^i, \varepsilon^i) \\
\text{s.t. } c_t^i + b_t^i &= X_t^i + b_{t-1}^i R_{t-1} + (1-\tau_l) w_t l_t \varepsilon_t^i + \Theta_t^i 1_{\varepsilon_t^i = 0} + T_t \\
a_t^i &\geq 0 \\
R_t^a a_{t-1}^i - X_t^i &= a_t^i
\end{aligned}
$$

The utility function is given by standard CRRA form $U(c,l) = \frac{c^{1-\sigma}}{1-\sigma} - \kappa \frac{l^{1+\psi}}{1+\psi}$ where $\sigma$ is the inverse intertemporal elasticity of substitution and $\psi$ is the inverse Frisch elasticity of labour supply. $\kappa$ measures the extent of disutility that labour generates.

$X_t^i$ is the real illiquid asset extraction which is exogenous and fixed at the baseline model[6]. Furthermore I assume there are only two types of realization of the labour productivity shock $\varepsilon_t^i$ for tractability

$$
\varepsilon_t^i = \begin{cases} 0 & i \in \{\text{pHtM}, \text{wHtM}\} \\ 1 & i \in \{\text{nHtM}\} \end{cases}
$$

Because the hand-to-mouth household does not have wage income, they can not borrow as much as they want to satisfy their optimal consumption decision, which is governed by the Euler equation. Therefore their consumption level is only determined by their budget constraint and any change of real interest rate cannot stimulate their consumption as they are now financial constrained. Hence the consumption of $\text{pHtM}$ is pinned down by lump-sum tax transfer and unemployment insurance such that $c_t^{pHtM} = \Theta_t^{\text{HtM}} + T_t$. Similarly consumption of $\text{wHtM}$ is pinned down by $c_t^{\text{wHtM}} = X^{\text{wHtM}} + \Theta_t^{\text{HtM}} + T_t$. Compared to poor hand-to-mouth household, the wealthy hand-to-mouth household has an extra income resource from the illiquid asset they hold which is $X^{\text{wHtM}}$ in baseline model. For the non-hand-to-mouth household their consumption is

[4]For simplicity I omit the agent index of household $i$ henceforth since their optimization problem is isomorphic.

[5]As shown by Cui and Sterk (2021) the household selecting the illiquid asset withdrawing is equivalent to selecting the optimal illiquid asset level. Both of them can pin down the illiquid asset distribution path as long as they have the same starting point.

[6]As Cui and Sterk (2021) proved it is equivalent to pin down illiquid asset $a_t^i$ or pin down extraction $X_t^i$ as long as the economy has the same initial illiquid asset distribution. I use extraction instead of the illiquid asset in model henceforth because it is more tractable and helpful to simplify the model.

governed by the standard Euler equation except the precautionary saving part at the right-hand side below, as now their future income and consumption are uninsured.

$$U_c\left(c_t^{\mathrm{nHtM}}\right) = \mathbb{E}\beta R_t\left\{p^{\mathrm{nHtM}}U_c\left(c_{t+1}^{\mathrm{nHtM}}\right) + p^{\mathrm{pHtM}}U_c\left(c_{t+1}^{\mathrm{pHtM}}\right) + p^{\mathrm{wHtM}}U_c\left(c_{t+1}^{\mathrm{wHtM}}\right)\right\} \tag{1}$$

Since only the wealthy hand-to-mouth household and none hand-to-mouth household hold illiquid assets, these two agents can get access to the financial market to withdraw the asset. This will close the illiquid asset market by

$$X_t = h^{\mathrm{nHtM}}X_t^{\mathrm{nHtM}} + h^{\mathrm{wHtM}}X_t^{\mathrm{wHtM}}$$

where $X_t$ is the aggregate illiquid asset withdrawing.

## 2.2 Mutual funds

A continuum survived or newly entried mutual fund $j$ indexed from 0 to 1 selects the share of firm equity $s_{j,t}$ and the amount of long-term real treasury bonds holding $b_{j,t}^m$ at the end of period $t$ which will yield return at time $t+1$[7]. At the beginning of period $t+1$, the aggregate shock first realized before the production process happened when $R_{t+1}^k$ and $R_{t+1}^B$ realized. By choosing the optimal portfolio arrangement the mutual fund solves the problem

$$W\left(n_t|s_t^*, b_{,t}^{m*}\right) = \max_{s_{j,t}, b_{j,t}^m} V\left(s_t, b_t^m, n_t\right) \tag{2}$$

$$\text{s.t. } V\left(s_t, b_t^m, n_t\right) \geq \lambda^v Q_t s_t + \lambda^b\lambda^v q_t^B b_t^m \tag{3}$$

where the asterisk represents the variable evaluated at the optimal equilibrium level. $W_t$ is the value of survived mutual fund and $V_t$ is the value of mutual fund at the end of period $t$. $\lambda^v$ and $\lambda^b$ are parameters regulate the collateral constraint which implies that the market value of mutual fund $j$ itself should not be lower than the asset they hold. $\lambda^\nu$ is the strength of the collateral constraint and $\lambda^b$ is the relative strength between equity and treasury bonds that contributes to the collateral constraint.

After the mutual funds pay their borrowing cost they will survive to next period with probability $\theta^m$ and exit the financial market with probability $1-\theta^m$. Given the random exit-and-entry risk of mutual fund, you can take $W_t$ as the *ex-post* value of mutual and $V_t$ as the *ex-ante* value of mutual fund which is composed of two components: one is the return from exited and another one is the expected future value of surviving such that

$$V\left(s_t, b_t^m, n_t\right) = E_t\beta\Lambda_{t,t+1}\left[\left(1-\theta^m\right)n_{t+1} + \theta^m W\left(n_{t+1}|s_{t+1}^*, b_{t+1}^{m*}\right)\right]$$

[7]For simplicity I omit the subscript $j$ until aggregation.

The balance sheet of the mutual fund is

$$Q_t s_t + q_t^B b_t^m = n_t + d_t^m \tag{4}$$

and the law of motion of the net worth of mutual funds are

$$n_t = \left(R_t^k - R_{t-1}\right) Q_{t-1} s_{t-1} + \left(R_t^B - R_{t-1}\right) q_{t-1}^B b_{t-1}^m + R_{t-1} n_{t-1} \tag{5}$$

where $d_t^m$ is the money that investment bank borrows from central bank, $Q_t$ is the real price of capital, $q_t^B$ is the real price of long-term treasury bond, $R_t^k$ is the real return of equity, $R_t^B$ is the real return of long-term treasury bonds and $R_{t-1}$ is the short term real interest rate.

Since every period there are $1 - \theta^m$ proportion of mutual funds will exit the financial market, in aggregate level if there were no start-ups, the aggregate net worth would shrink and concentrate to the luckiest mutual fund. To ensure the stability of aggregate networth of the mutual funds, I assume that each period there are some new mutual fund companies entry into the financial market, total networth of which are a fraction $\varphi$ of aggregate effective asset $\phi_t N_{t-1}$. Aggregating the net worth of survived mutual funds provides the law of motion of aggregate net worth such that

$$N_t = \theta^m \left[\left(R_t^k - R_{t-1}\right) Q_{t-1} S_{t-1} + \left(R_t^B - R_{t-1}\right) q_{t-1}^B B_{t-1}^m + R_{t-1} N_{t-1}\right] + \varphi \phi_t N_{t-1}$$

where $N_t = \int n_{j,t} dj$.

Solve the optimization problem 2 can yield the non-arbitrage condition $\lambda^b E_t \beta \Omega_{t,t+1} \left(R_{t+1}^k - R_t\right) = E_t \beta \Omega_{t,t+1} \left(R_{t+1}^B - R_t\right)$ which shows that the excess return of firm equity should be same as the excess return of long-term bonds but adjusted by the collateral constraint parameter $\lambda^b$.

Define the effective leverage ratio of the mutual fund at time $t$ is $\phi_t$ as $\phi_t = \frac{Q_t s_t + \lambda^b q_t^B b_t^m}{n_t}$. Because of the collateral constraint 3 the mutual funds cannot borrow as much as they want which implies an upper boundary of the leverage ratio

$$\phi_t \leq \overline{\phi} = \frac{E_t \left[\beta \Omega_{t,t+1} R_t\right]}{\lambda^v - E_t \left[\beta \Omega_{t,t+1} \left(R_{t+1}^k - R_t\right)\right]} \tag{6}$$

and relatively the effective multiplier $\lambda_t$[8] is non-negative such that

$$\lambda_t = \max \left\{0, 1 - \frac{\zeta_t}{\lambda^v \phi_t}\right\} \tag{7}$$

where $\zeta_t$ is the marginal return of net worth and $\Omega_{t,t+1}$ is the augmented SDF.

Equation 6 entails that the larger the borrowing cost $\beta \Omega_{t,t+1} R_t$ is, the less money that mutual funds want to borrow. Therefore the smaller boundary of the leverage is as the mutual funds

[8]Denote $\nu^\lambda$ is the Lagrange multiplier of corollary constraint 3 and $\lambda_t$ is called the effective multiplier as $\lambda_t = \frac{\upsilon_t}{1+\upsilon_t}$

have smaller borrowing demand and smaller liability volume[9]. Additionally the larger excess return of firm equity $E_t\left[\beta\Omega_{t,t+1}\left(R^k_{t+1}-R_t\right)\right]$ is, the more valuable the mutual funds are and $V_t$ becomes larger. Therefore the mutual funds' collaterals become more valuable and they can borrow more from central bank which expands the upper boundary of the leverage ratio.

## 2.3 Intermediate good producer

Intermediate good producers use a production function with constant return to scale $Y^m_t = A^{\text{TFP}}_t\left(U_t\xi_t K_{t-1}\right)^\alpha L^{1-\alpha}_t$ to produce intermediate good at time $t$. $K_{t-1}$ is the total capital used; $L_t$ is the labour demand; $A^{\text{TFP}}_t$ is technology and $U_t$ is the capital utilization rate which is determined at time $t$[10]. $\xi_t$ is the effective capital shock and all the old capital after production will be $\xi_t K_{t-1}$[11]. Therefore the law of motion of capital is

$$K_t = \xi_t K_{t-1} + I_t - \delta\left(U_t\right)\xi_t K_{t-1} \tag{8}$$

The depreciation rate $\delta\left(U_t\right)$ is a function of capital utilization $U_t$ which is first-order convex and second-order semi-convex such that $\delta'\left(\cdot\right)>0$ and $\delta''\left(\cdot\right)\geq 0$[12][13].

The intermediate good producers choose $U_t$ and $L_t$ to produce good and pay wage cost, depreciation cost and real fixed cost. They solve the problem

$$\Pi^f_t = \underset{U_t,L_t}{max} P^m_t Y^m_t - W_t L_t - \delta\left(U_t\right)\xi_t K_{t-1} - \tau_{y^m} \tag{9}$$

where the $\tau_{y^m}$ is the real fixed production cost and is refunded to equity holders and will be refunded back to investment bank to make sure the fix cost is non-distortional[14]. It will be refunded back to investment bank to make sure the fix cost is non-distortional. Note that the real depreciation cost $\delta\left(U_t\right)\xi_t K_{t-1}$ instead of $Q_t\delta\left(U_t\right)\xi_t K_{t-1}$ which will isolate the inflation and price setting problem with capital fluctuation[15]. Therefore the gross equity return of the

[9]Rewrite equation 6 as $\overline{\phi} = \frac{1}{\frac{\lambda^v - E_t\left[\beta\Omega_{t,t+1}R^k_{t+1}\right]}{E_t\left[\beta\Omega_{t,t+1}R_t\right]}+1}$. Since $\lambda^v - E_t\left[\beta\Omega_{t,t+1}R^k_{t+1}\right]<0$ a larger $E_t\left[\beta\Omega_{t,t+1}R_t\right]$ induces a lower $\overline{\phi}$.

[10]This form of production function is proposed by Greenwood et al. (1988) to generate endogenous depreciation rate and more fluctuating real rental rate of capital.

[11]This is directly followed from Merton (1973) who uses effective capital to generate a component within asset return which is a pure "shock" and is not connected with fundamental economy. Incorporating this effectiveness shock into model talking about unconventional monetary policy is firstly proposed by Gertler and Karadi (2011) and Gertler and Karadi (2018). which helps to increase the volatility of the capital return $R^k$ and the power of financial accelerator.

[12]Incorporating this convex depreciation function can help to generate the investment response and equity return to shock which is argued by Jaimovich and Rebelo (2009); Christiano et al. (2014).

[13]The key element that is related to the response is the capital utilization elasticity of marginal depreciation $\frac{\delta''(u)u}{\delta'(u)}$. Therefore I use the depreciation function $\delta\left(u_t\right)=\overline{\delta}+\frac{Y_u}{1+v}u^{1+v}_t-\frac{Y_u}{1+v}$ where $Y_u=\alpha\frac{Y^m}{Qu^{1+\nu}K\xi}$.

[14]The only effect that $\tau_{y^m}$ does is to match $R^k_t$ at steady state as Favilukis et al. (2017) and Bianchi and Mendoza (2018) did.

[15]The intermediate good producers only suffer $\delta\left(U_t\right)\xi_t K_{t-1}$ unit of depreciation cost instead of the total depreciation cost $Q_t\delta\left(U_t\right)\xi_t K_{t-1}$ and the remained $\left(Q_{t+1}-1\right)\delta\left(U_{t+1}\right)\xi_{t+1}K_{t+1}$ part is borne by capital producer.

intermediate good producer is

$$R_t^k = \frac{\left[\frac{\Pi_t^f + \tau_{ym}}{\xi_t K_{t-1}} + Q_t\right]\xi_t}{Q_{t-1}}$$

The intermediate good producers do not have the balance sheet and are all owned by mutual funds so that the stock market clearing condition $S_t = K_t$ holds. The intertemporal optimization problem related to state variable $K_t$ is in fact solved by financial institutions, a setting that is equivalent to the environment where firms have balance sheet and can accumulate net worth and solve the intertemporal problem through borrowing from financial market[16].

## 2.4 Retailer and Final good producer

Retailer buys intermediate good and differentiates it with monopolic power which makes sure they can freely set the price. Final good producer uses differentiated goods from retailer to produce final goods via the standard CES function $Y_t = \left[\int_0^1 Y_{jt}^{(\sigma_p-1)/\sigma_p} dj\right]^{\sigma_p/(\sigma_p-1)}$ in competitive market.

Retailer sets the retail goods price based on the strategy proposed by Calvo (1983) such that

$$\begin{aligned} &\max E_t \sum_{\tau=0}^{\infty} (\theta\beta)^\tau \Lambda_{t,t+\tau} \left[\frac{P_{jt}^*}{P_{t+\tau}} \prod_{k=1}^{\tau} \Pi_{t+k-1}^{\gamma_p} - P_{t+\tau}^m\right] Y_{jt+\tau} \\ &\text{s.t. } Y_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\sigma_p} Y_t \end{aligned} \tag{10}$$

Denote $\mu_t$ as the price dispersion which is defined as

$$\mu_t = \int_0^1 \left(\frac{P_{jt}}{P_t}\right)^{-\sigma_p} dj = (1-\theta)\left(\frac{\Pi_t^*}{\Pi_t}\right)^{-\sigma_p} + \theta\left(\frac{\Pi_{t-1}^{\gamma_p}}{\Pi_t}\right)^{-\sigma_p} \mu_{t-1}$$

Therefore equation 10 can be written as $Y_t^m = \mu_t Y_t$.

## 2.5 Capital producers

Capital producer uses the final goods to produce the physical capital and bears production cost following the function $f(I_{n,\tau}, I_{n,\tau-1})$ which helps to match the investment response respect to monetary policy[17]. Capital producer may earn profit and the profit is transferred to household finally via lumpy-sum transfer.

The capital producer maximizes its present discounted real profit by choosing the net invest-

Therefore the capital producers optimize their objective function based on the net investment $I_{nt} = I_t - \delta(U_t)\xi_t K_t$ instead of $I_t$.

[16]Carlstrom et al. (2012),Coenen et al. (2018) and Sims et al. (2022) use this type of setting. Carceles-Poveda and Coen-Pirani (2010) did a deeper investigation on this equivalence.

[17]This is firstly argued by Christiano et al. (2005).

ment this period such that

$$\max E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \Lambda_{t,t+\tau} \left\{ \left(Q_\tau - 1\right) I_{n,\tau} - f\left(I_{n,\tau}, I_{n,\tau-1}\right)\left(I_{n,\tau} + I_{ss}\right) \right\}$$
$$\text{s.t. } f\left(I_{n,\tau}, I_{n,\tau-1}\right) = \frac{\psi_I}{2}\left(\frac{I_{n,\tau} + I_{ss}}{I_{n,\tau-1} + I_{ss}} - 1\right)^2$$

where $I_{n,\tau}$ is the net investment after depreciation such that $I_{n,\tau} = I_t - \delta\left(U_t\right)\xi_t K_{t-1}$.

This closes the supply market of capital and pins down the capital price as a convex function of investment following

$$Q_t = 1 + \frac{\psi_I}{2}\left(\frac{I_{n,t} + I_{ss}}{I_{n,t-1} + I_{ss}} - 1\right)^2 + \psi_I\left(\frac{I_{n,t} + I_{ss}}{I_{n,t-1} + I_{ss}} - 1\right)\frac{I_{n,t} + I_{ss}}{I_{n,t-1} + I_{ss}} - E_t \beta \Lambda_{t,t+1} \psi_I \left(\frac{I_{n,t+1} + I_{ss}}{I_{n,t} + I_{ss}} - 1\right)\left(\frac{I_{n,t+1} + I_{ss}}{I_{n,t} + I_{ss}}\right)^2 \tag{11}$$

## 2.6 Central Bank and Government

Either household or mutual funds can borrow money $D_t^i$ from central bank at time $t$ and then pay the gross interest rate $R_t$ at time $t+1$. Meanwhile it also buys long-term treasury bonds $B_t^{cb}$ and earns gross return $1 + \rho q_{t+1}^B$ realized at time $t+1$ which is paid by the government at geometric decay rate $\rho$[18] following the budget constraint 13.

The government issues the long-term treasury bonds $B_t^g$ at time $t$ and funded its budget constraint by seigniorage $T_t^s$ from central bank following 12.

$$T_t = T_t^s - \frac{\left(1 + \rho q_t^B\right)}{\Pi_t} B_{t-1}^g + q_t^B B_t^g \tag{12}$$

$$T_t^s + D_t^h - R_{t-1} D_{t-1}^h + D_t^m - R_{t-1} D_{t-1}^m = \frac{\left(1 + \rho q_t^B\right)}{\Pi_t} B_{t-1}^{cb} - q_t^B B_t^{cb} \tag{13}$$

where

$$B_t^g = B_t^{cb} + B_t^m = 0 \tag{14}$$

$$B_t^m = \int b_{i,t}^m di \tag{15}$$

$$D_t^h = -h^{\text{nHtM}} b_t^{\text{nHtM}} \tag{16}$$

where $m$ states mutual fund company and $h$ represents household. Further I assume the supply of long-term bonds from treasury department is constant and zero, which helps isolate the effect of monetary policy. I can simplify the equation 12 and 13 via zero bonds supply assumption to

$$T_t + D_t^h - R_{t-1} D_{t-1}^h + D_t^m - R_{t-1} D_{t-1}^m = q_t^B B_t^m - \frac{\left(1 + \rho q_t^B\right)}{\Pi_t} B_{t-1}^m \tag{17}$$

[18]This type of modeling long-term bonds is followed by Woodford (2001)

Moreover, the government is also in charge of providing income subsidy to the hand-to-mouth households, which is funded by the labour income tax $\tau$ following the clearing condition $\tau w_t L_t = \left(h^{\text{pHtM}} + h^{\text{wHtM}}\right)\Theta_t^{\text{HtM}}$.

During the non-ZLB episode central bank stabilizes the economy via conventional monetary policy such that it directly sets the nominal interest rate $\mathcal{R}_t$ following Taylor rule

$$\mathcal{R}_t = \max\left\{\mathcal{R}_{t-1}^{\theta_r}\mathbb{E}_t\left[\left(\frac{\Pi_{t+1}}{\Pi}\right)^{\theta_\pi}\left(\frac{Y_t}{Y}\right)^{\theta_y}\right]^{1-\theta_r}\gamma_t^{MP}, 1\right\} \tag{18}$$

where $\gamma_t^{\text{MP}}$ is the monetary policy shock. The variables without time subscripts denote steady-state value of corresponding variables. The real interest rate is pinned down by nominal interest rate $R_t = \frac{\mathcal{R}_t}{\Pi_{t+1}}$.

In addition to the conventional monetary policy, central bank can also impose unconventional monetary policy by increasing the holding of long-term treasury bonds. Since the supply of treasury bonds is fixed as equation 14 states, controlling the central bank's holding of long-term bonds is equivalent to controlling the holding of long-term bonds of mutual funds. Therefore I set the QE policy rule[19] as

$$\frac{B_t^m}{\overline{B^m}} = \frac{B_{t-1}^m}{\overline{B^m}}^{\theta_r^{QE}}\left[\left(\frac{\Pi_{t+1}}{\Pi}\right)^{\theta_\pi^{QE}}\left(\frac{Y_t}{Y}\right)^{\theta_y^{QE}}\right]^{1-\theta_r^{QE}}\gamma_t^{QE} \tag{19}$$

Following Cui and Sterk (2021) and Sims et al. (2022) I assume the money used to implement QE policy is funded by household which implies

$$q_t^B B_t^m - D_t^h = \overline{T}_{\text{cb}}$$

# 3 The power of QE: Decomposition

The unconventional monetary policy stimulates the economy mainly through two channels: liquidity channel and interest rate channel. Long-term yield drops after large scale asset purchasing, which also increases the cash held by the financial institutions as central bank buys long-term treasury bonds from their hands. This extra cash slacks their collateral constraint (equation 3) and pushes them to invest more in corporate equities which induces more physical capital and generates a supply-side driven stimulation. The decreased yield will also motivate the financial institutions to invest more in equity because of the pecuniary effect as they hold some long-term bonds from previous period which are now more valuable. Additionally, the general equilibrium effect resourced from demand side is always held. The larger demand for investment

[19]Since $B_t^{cb} + B_t^m = 0$ where $B_t^m$ is opposite to $B_t^{cb}$, the coefficient o reaction should be negative such that $\theta_\pi^{QE} < 0, \theta_y^{QE} < 0$.

stimulates the demand for output, which in turn spires firms to hire more labour to produce final goods. More labour demand results in more income earned by household which passes to more consumption demand as long as there exists hand-to-mouth household whose consumption is driven by income and budget constraint instead of future expectation and Euler equation. This consumption demand further stimulates the economy and magnifies the power of QE policy.

## 3.1 Transmission mechanism

What we can observe after the central bank implemented unconventional monetary policy is that $\Delta q_t^B B_t^m$ becomes negative since central bank holds more long-term treasury bonds. At the meantime $\Delta q_t^B$ is positive as the long-term yield decreases because there is more demand for the long-term bonds. The endpoint of unconventional monetary policy is that $\Delta Y_t$ is positive so that the output is stimulated which is widely investigated empirically[20]. While there are several roads we can walk from the starting point (negative $\Delta q_t^B B_t^m$ and positive $\Delta q_t^B$) to the endpoint (positive $\Delta Y_t$) which I summarized in Table 1.

Table 1: QE Decomposition

| QE effect | Liquidity channel | Interest rate channel |
|---|---|---|
| Supply side | liquidity easing | pecuniary easing |
| Demand side | redistribution effect | substitution effect |

### 3.1.1 Liquidity channel

The liquidity channel states that the unconventional monetary policy stimulates the economy through injecting liquidity into the market. In this sense the liquidity that central bank injected is just $\left|\Delta q_t^B B_t^m\right|$ as central bank provides this amount of money to financial institutions (to exchange for the long-term treasury bonds which financial institutions previously held). Financial institutions will use this liquidity to invest in supply side (firms) to induce more investment and production. In other words unconventional monetary policy *crowds out* the holding of long-term bonds of financial institutions, which forces them to readjust their investment portfolios and put more investment on production side to stimulate the economy[21].

Financial friction ensures that this liquidity injection works so that the financial institutions will indeed use this extra liquidity to invest instead of paying their debt and accumulating new

[20] The output-stimulation effect of unconventional monetary policy is identified by Baumeister and Benati (2012); Kapetanios et al. (2012); Stock and Watson (2012); Weale and Wieladek (2016); Wu and Xia (2016); Di Maggio et al. (2020); Bauer and Rudebusch (2014); Swanson and Williams (2014); Engen et al. (2015); Hesse et al. (2018) and is summarized by Kuttner (2018); Lombardi et al. (2018); Borio and Zabai (2018)

[21] It works like the increased government spending contemporaneously as it expands the output by more demand through which the government increases its expenditure even though it does nothing more and drops this expenditure into the sea. However this supply-side stimulation is persistent because capital is a state variable and complementary to labour which will generate a long-lasting effect.

worth. From equation 6 and 7 we can know that if there did not exist financial friction or the financial institutions were not staying at constrained states, it would be suboptimum for financial institutions to spend this liquidity on investing equities. When $\phi_t$ is not fixed at the upper boundary, it is optimal for the financial institutions to decrease their debt $D_t^m$ and increase their net worth $N_t^m$ as leverage ratio is not bound so that financial institutions have the propensity to decrease their debt. This partially explains why quantitative easing *cannot always works*, for instance, at Japan. This effectiveness shows that financial friction is a double-edged sword to the economy and central bank. During normal time it causes the under-investment problem because financial institutions cannot borrow as much as they want to invest so the capital level is in fact below the optimal level. However during the ZLB period it contributes to the effectiveness of the unconventional monetary policy. It makes sure that the monetary policy works to stimulate the economy and decrease the real interest rate.

Additionally financial frictions can also amplify the power of unconventional monetary policy as it may work as an accelerator to the economic activities, which we called the financial accelerator[22]. If the leverage ratio and net worth were fixed so that there was no endogenous portfolio adjustment, $\left|\Delta q_t^B B_t^m\right|$ amount of crowded long-term bonds would in the end generate $\lambda^b \left|\Delta q_t^B B_t^m\right|$ amount of new investment in equity given the leverage ratio is $\phi_t = \frac{Q_t s_t + \lambda^b q_t^B b_t^m}{n_t}$. The endogenous leverage ratio will further enlarge the effect which is the heart of financial accelerator such that financial market takes response more to shock compared with the production side. This overresponse amplifies the shock's effect and induces the economy to become fluctuant.

In addition to the stimulation generated by financial institutions, there will always exist the general equilibrium effect to which demand side contributes. Take the standard Euler equation $u_t' = \beta R_t E_t u_{t+1}'$ as an example. The real rental rate of capital $R_t^k$ is closely linked with the real interest rate $R_t$ as more investment will result in more production and higher inflation $\Pi_{t+1}$. Because the unconventional monetary policy is imposed at ZBL when the nominal interest rate is fixed, $R_t$ will decrease which is driven by Fisher equation. The decreased real interest rate will encourage the households to consume more following Euler equation. The money they used to consume is the extra wage income which is paid by intermediate goods producer as there is more demand of final good.

Moreover, except for the standard Euler equation mechanism, demand-side general equilibrium effect will be further magnified by heterogeneous household. If there existed some households who were financial constrained so that their consumption did not follow Euler equation, the general equilibrium effect would be amplified as these hand-to-mouth households consumed all the increased wage income which generated a strong feedback loop to the production sector. Except for this Keynesian-corss effect, there is also a redistribution effect amplifying the power of unconventional monetary, akin to the redistribution channel analyzed by Auclert (2019) and Luetticke (2021). It is the real credit borrowed from households that central bank

[22]This is first proposed by Bernanke et al. (1999)

uses to buy long-term treasury bonds from financial institutions. Non-hand-to-mouth household pays the cost of unconventional monetary policy whereas they only earn some of the profit prompted by economic stimulation. This redistribution mechanism enlarges the effect of unconventional monetary policy as it redistributes the wealth from non-hand-to-mouth household to hand-to-mouth household who has a larger marginal propensity to consume.

### 3.1.2 Interest rate channel

The unconventional monetary policy not only decreases the holding of long-term treasury bonds of financial institutions, but also twists the term yield and decreases the long-term interest rate. A positive $\Delta q_t^B$ will also stimulate the economy albeit through a different mechanism, interest rate channel. The elements related to long-term bonds within the budget constraint of the financial institutions is $(1+\rho^B q_t^B)B_{t-1}^m - q_t^B B_t^m$ which represents that at time $t$ financial institutions hold $(1+\rho^B q_t^B)B_{t-1}^m$ amount of market-value-based long-term bonds. In addition to the bonds they have held since last period, they choose $B_t^m$ to take to next period which they spend $q_t^B B_t^m$ of money to buy. Even though there is no liquidity injected by central bank as $q_t^B B_t^m$ is fixed, the economy is still stimulated because financial institutions still can get some liquidity from the bonds they have held since last period as now the price of long-term bonds is higher. After this pecuniary easing, financial institutions will invest more in corporate equity, leading to more investment and production. The non-arbitrage condition $\lambda^b E_t \beta \Omega_{t,t+1}\left(R_{t+1}^k - R_t\right) = E_t \beta \Omega_{t,t+1}\left(R_{t+1}^B - R_t\right)$ inspires financial institutions to invest more in production sector withal because the expected return of long-term bonds drops as $R_{t+1}^B = \frac{1+\rho^B q_{t+1}^B}{q_t^B}$. Similarly the financial friction will strengthen the interest rate channel notwithstanding the response may not be as large as liquidity channel since now there is no liquidity injection and all the amplification comes from endogenous leverage decisions.

Likewise, the unconventional monetary policy can also stimulate the economy at demand side via the interest rate channel yet work on a totally different story. The unconventional monetary policy changes the return and portfolio of financial institutions, which varies the return of illiquid asset afterwards as the financial institutions is also the supplier of the illiquid asset. Taking a glance on equation 5 we can know that the return of financial institutions will increase because $R_t^B$ and $R_t^k$ will increase. The increased long-term treasury bonds' price firstly drives up the return of long-term bonds this period (but drives down the expected return next period which is also the current yields this period[23]). Because of non-arbitrage condition and financial friction the financial institutions increase their investment in equity, which in turn increases $R_t^k$ as more investment means larger capital price (though it also decreases the expected return of equity next period as more capital means lower real rental rate).

**Proposition 1.** *Denote the PE effect of unconventional monetary policy to illiquid asset return is*

[23] It is worth to notice that the yield to maturity this period $q_t^{-1} + \rho^B - 1$ also decreases.

$\frac{\partial \widehat{R}_t^a}{\partial \widehat{q}_t^B}\Big|_{\widehat{q}_t^B+\widehat{B}_t^m=q^B+B^m}^{PE}$ *such that*

$$\frac{\partial \widehat{R}_t^a}{\partial \widehat{q}_t^B}\Bigg|_{\widehat{q}_t^B+\widehat{B}_t^m=q^B+B^m}^{PE} = \frac{(1-\theta^m)\, R^B B^m q^B}{RN^h}\frac{\rho^B q^B}{1+\rho^B q^B} > 0$$

Proposition 1 shows that the partial equilibrium effect of unconventional policy to illiquid asset return via liquidity channel is positive within first order linear system. Meanwhile from equation 71 we can know that if we also consider the capital and goods market, above effect will be larger. The financial accelerator further expands this effect through the endogenous leverage and feedback loop. This inflated illiquid asset return may work positively or negatively at the demand side which is determined by the illiquid asset substitution effect of household. A larger $\widehat{R}_t^a$ will affect neither wealthy nor poor hand-to-mouth household since the consumption of the former one is only governed by real interest rate and the latter one is only governed by budget constraint (but they do not hold any illiquid asset). Conversely a larger $\widehat{R}_t^a$ will affect the wealthy hand-to-mouth household as they indeed hold the illiquid asset and their consumption is governed by the budget constraint. However whether this effect is positive or negative is regulated by the substitution effect of wealthy hand-to-mouth household. They can consume most of the increased asset return and save a small amount of it; alternatively they can also save most of the increased asset return or even decrease their consumption to further invest in the illiquid asset because now the illiquid asset return is raised[24]. I assume that the illiquid asset withdrawing $X_t^i$ is fixed in baseline model because of tractability so that the PE effect of unconventional monetary policy at demand side is muted. Nonetheless the general equilibrium effect of unconventional monetary policy at demand side is always survived.

## 3.2 Transmission magnitude and Complementary effect

In the previous two subsections I argued that the unconventional monetary policy worked through liquidity and interest rate channels at supply and demand sides separately, yet they do not only influence the economy separately. They also have a positive cross effect through which the unconventional monetary policy stimulates the economy. This complementary effect is not unique to unconventional monetary policy and Bilbiie et al. (2022) first proposed this complementary effect for conventional monetary policy which they called *multiplier of multiplier* effect. The proposition below decomposes the stimulation power of unconventional monetary into liquidity channel and interest channel.

**Proposition 2.** *When the price and depreciation rate is fixed, the contemporaneous effect of unconventional monetary policy on output can be decomposed to liquidity and interest rate channel such that*

[24] This substitution effect is propelled by the marginal propensity to take risk which was first argued by Kekre and Lenel (2021).

$$\left.\frac{\partial \widehat{Y}_t}{\partial\left(\widehat{q}_t^B+\widehat{B}_t^m\right)}\right|_{\widehat{q}_t^B=q^B}=-\frac{\frac{1}{h^n}-1+\frac{\lambda^b}{\phi}+\varphi_1^m\frac{\varphi_1^h}{Th^n}}{C^n\frac{\psi}{\sigma}+\frac{(1-\tau)WL}{h^n}+\varphi_1^m\varphi_4^h}q^B B^m=-\varphi_L q^B B^m \tag{20}$$

$$\left.\frac{\partial \widehat{Y}_t}{\partial \widehat{q}_t^B}\right|_{\widehat{q}_t^B+\widehat{B}_t^m=q^B+B^m}=\frac{\rho}{C^n\frac{\psi}{\sigma}+\frac{(1-\tau)WL}{h^n}+\varphi_1^m\varphi_4^h}q^B B^m=\varphi_R q^B B^m \tag{21}$$

where $\varphi_1^h=h_C^w T_{C^w}+h_C^p T_{C^p}$ , $\varphi_2^h=h_C^w\Theta_{C^w}^T+h_C^p\Theta_{C^p}^T$, $\varphi_3^h=h_C^w\frac{C^n}{C^w}+h_C^p\frac{C^n}{C^p}$, $\varphi_4^h=Y_C-\delta K_C-\varphi_2^h+\frac{\varphi_1^h(1-\tau)WL}{h^nT}+\left(h_C^n+\varphi_3^h\right)\frac{\psi}{\sigma}$ and $\varphi_1^m=\left(N^h\frac{1-\theta^m+\theta^m\eta}{\theta^m}\lambda\frac{R^k}{R^k-R}\frac{1}{KQ+\Pi^f}-1+\frac{1}{\phi}\right)\frac{\varphi_I}{\delta}(1-\beta\Lambda)-\left(1-\frac{1}{\phi}\right)Q$.

Firstly let us focus on the non-cross effect at supply and demand sides. The heterogeneous household on demand side involves in denominator for either liquidity channel or interest rate channel. The existence of idiosyncratic shock and precautionary saving augments the power of unconventional monetary policy through hand-to-mouth household as $C^n$ is now smaller than its value in represented agent model. Nevertheless the income effect is attenuated now as $\frac{(1-\tau)WL}{h^n}$ is larger ($h^n<1$) because only the wealthy household is earning the wage income and the GE effect cannot work through this[25]. The phenomenon that these two effects offset each other is just the difference between average consumption and cross-sectional consumption dispersion which is argued by Debortoli and Galí (2022). As argued by them, if we allow the hand-to-mouth household work and the hand-to-mouth community is generated endogenously by financial friction like Kaplan et al. (2018) did, instead of purely assumption, the attenuation effect $\frac{(1-\tau)WL}{h^n}$ would vanish. This GE effect will emerge on two channels as the stimulation starts from the production sector where financial institutions invest and passes to household sector via income effect.

In addition to the GE effect on denominator (of equation 20 and 21) through which either the liquidity channel or interest rate channel works, there is another redistribution effect on the numerator for liquidity channel which I call *redistribution credit* effect. The money that central bank uses to buy long-term treasury bonds from financial institutions is funded by borrowing from households, specifically, wealthy non-hand-to-mouth household. The non-hand-to-mouth household pays 1 unit of money to central bank who passes it to financial institutions and exchanges for 1 unit market-value-based long-term treasury bonds. However there are two tunnels that this 1 unit of money goes back to household: one is the net worth of the financial insitutions and another one is the liability of the financial institiutions which in the end flows into lump-sum tax transfer because they fund their liability by borrowing from central bank. Hence there is a redistribution effect such that $\frac{1}{h^n}-1$ amount of wealth is redistributed from

[25] In my model there is no cyclical redistribution problem proposed by Broer et al. (2020). The offset problem they argued is based on the assumption that wealthy household does not supply labor and grape all the equity return from firms. In this type of setting the effect of countercyclical wage income to hand-to-mouth household will offset the effect of procyclical markup to non-hand-to-mouth household. Therefore the total effect will become tiny small but only the redistribution effect exists.

non-hand-to-mouth household to hand-to-mouth households. Along this redistribution credit effect there is an extra term $\frac{\lambda^b}{\phi}$ comes from the portfolio adjustment effect through which the financial institutions' portfolios are adjusted because long-term treasury bonds they previously hold are crowded out by central bank.

Except for the non-cross effect at supply and demand sides, the unconventional monetary policy works complementarily at both sides. Proposition 2 shows that $\varphi_1^m \varphi_4^h$ is one of the complementary effect at supply and demand side. $\varphi_4^h$ is the component related to demand side, and takes effect through general equilibrium $Y_C - \delta K_C$. This effect is amplified by heterogeneous household through lump-sum tax transfer $\varphi_1^h$, unemployment insurance $\varphi_2^h$ and consumption dispersion $\varphi_3^h$. $\varphi_1^m$ is the component related to supply side which determines the direction of complementarity between heterogeneous household and financial friction. The proposition below decomposes the component of complementary in financial market into two effects: redistribution return effect and redistribution wealth effect.

**Proposition 3.** *The complementary component of stimulation effect at supply side can be further decomposed as*

$$\varphi_1^m = \underbrace{\left( N^h \frac{1 - \theta^m + \theta^m \eta}{\theta^m} \lambda \frac{R^k}{R^k - R} \frac{1}{KQ + \Pi^f} - 1 + \frac{1}{\phi} \right) \frac{\varphi_I}{\delta} (1 - \beta \Lambda)}_{\text{redistribution return}} - \underbrace{\left( 1 - \frac{1}{\phi} \right) Q}_{\text{redistribution wealth}}$$

Redistribution return effect comes from contemporaneous asset return which is expended by the unconventional monetary policy since more demand for output entails a larger real rental rate. However this inflated return is all grabbed by wealthy non-hand-to-mouth household which does not contribute to the GE stimulation at demand side as their consumption is driven by real interest rate in Euler equation. Redistribution wealth effect is similar to the redistribution credit effect in liquidity channel but at a different position of asset. Redistribution credit effect is related to the credit of central bank to wealthy household from who central bank borrows 1 unit of money (to buy long-term bonds) but only refunds $\frac{1}{h^n}$ back to them which I use the notation $D^h$ in model part. Redistribution wealth effect is related to the asset and net worth of the financial institutions whose capital holding increases 1 unit of money based value, because of the stimulation from central bank, but only $\frac{1}{\phi}$ is owned by wealthy household. $1 - \frac{1}{\phi}$ portion is owned by hand-to-mouth household in the end.

It is obvious that these two effects are offsetting each other. Redistribution return effect abates the amplification ability of consumption inequality as a large proportion of excess return is earned by the wealthy household. While the redistribution wealth effect contributes in another direction as not all the stimulated boom of assets belongs to wealthy household despite there are still $\frac{1}{\phi}$ amount. The complementary component at supply side works as the *multiplier of multiplier* and the financial friction here works as *multiplier of the multiplier of multiplier* because the leverage ratio $\phi$ is larger than 1.

The corollary below implies that only when the redistribution wealth effect overwhelmed the redistribution return effect the complementary effect between financial accelerator and heterogeneous household would further magnified the effect of unconventional monetary policy.

**Corollary 1.** *The contemporaneous effect of unconventional monetary policy to output is magnified by complementarity between demand side and supply side as long as capital price at steady state is not too small.*

*Proof.* It is straightforward to prove that as long as $Q$ is not too small $\varphi_1^m$ will be negative. The effect through interest rate channel will be larger from equation 21. I can also get the relationship that $\frac{\varphi_1^h}{Th^n} < \varphi_4^h$ from their definition so that negative $\varphi_1^m$ will also increase the effect through liquidity channel from equation 20. $\square$

Given the calibration in next section figure 1 reveals the coefficient value of liquidity channel $\varphi_L$ and interest rate channel $\varphi_R$, after varying different parameters. 1a shows that complementary effect grows stronger along the process through which redistribution wealth effect surpassing redistribution return effect. 1b shows that the stimulation effect via liquidity channel expends, accompanying the severity of consumption inequality. Nevertheless the effect through interest rate channel becomes more and more silent because the complementary effect is negative to the inequality at steady state where I pin down the capital price to 1. 1c illustrates the amplification power of financial accelerator through the random exit and entry. When the possibility to exit is higher ($\theta^m$ is smaller), the financial institutions will respond to unconventional monetary policy more intensively, ergo larger stimulation power. At steady state the ratio of stimulation power between these two channels, $\frac{\varphi_L}{\varphi_R}$, is 1.26 , a number which is close to the empirical result. The empirical stimulation power ratio $\frac{\Delta \widehat{Y}_L}{\Delta \widehat{Y}_R}$ to the media impulse response at figure 5 is 1.46, averaging along the time line.

# 4 Quantitative experiment

The argument in last section about two tunnels through which the unconventional monetary policy works is restricted and conservative because of unrealistic assumptions such as fixed price and depreciation rate. Therefore I solve the model quantitatively in this section to clarify the channels through that unconventional monetary policy works. Firstly I discuss how to calibrate the model and then compare the quantitative results with estimated empirical impulse response. Finally I conduct some counterfactual experiments to quantitatively exhibit the power of unconventional monetary policy on different channels.

## 4.1 Calibration

The calibration process is standard and supported by literature composed of four main parts: household, financial institutions, central bank and production sector. I only elaborate the first

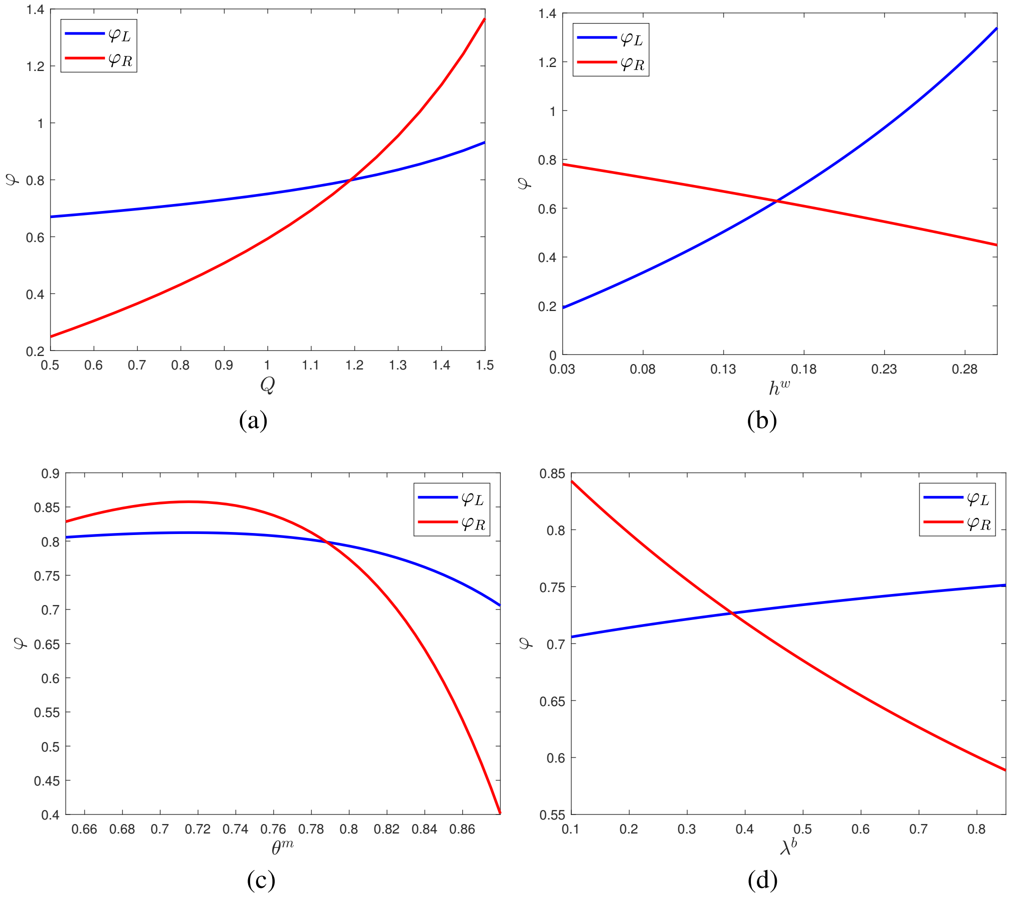


Figure 1: Complementary effect

three parts and relegate production sector into appendix in which parameters come from literature. All of the key parameters in these three sectors are summarized at table 2.

### 4.1.1 Household

The parameters in utility function are standard such that I set the intertemporal elasticity of substitution $\sigma$ to 2, as well as the inverse Frisch elasticity $\psi$. The disutility of labour $\kappa$ is set to 1 for convenience and conventional. The discount factor is set to $0.98$ which is a common value to match precautionary saving motive in heterogeneous agent literature like Auclert et al. (2021) did. The real interest rate of liquid bonds is $2\%$ at annual rate which is close to the interest rate before the Great Recession happened. The shares of different types of household come from the literature and are identified by Kaplan et al. (2014). The share of poor hand-to-mouth household $h^{\text{pHtM}}$, wealthy hand-to-mouth household $h^{\text{wHtM}}$ and non-hand-to-mouth household $h^{\text{nHtM}}$ are $0.121$, $0.192$ and $0.687$ relatively. The possibility of household that entries into hand-to-mouth state from non-hand-to-mouth state $p^{EU}$ is $0.044$ targeted by the monthly inflow rate of unemployment $1.5\%$ which Current Population Survey collected. Meanwhile I assume that conditional on becoming a hand-to-mouth household, non hand-to-mouth household uniformly becomes poor hand-to-mouth and wealthy hand-to-mouth household following related group size so that $p^{\mathrm{pHtM}} = p^{EU}\frac{h^{\mathrm{pHtM}}}{h^{\mathrm{pHtM}}+h^{\mathrm{wHtM}}} = 0.017$ and $p^{\mathrm{wHtM}} = 0.027$. Although the total illiquid asset withdrawing is determined at steady state by the market clearing condition, the distribution of the withdrawing is not. To calibrate the distribution of illiquid asset withdrawing, I adjust the withdrawing ratio $\frac{X_t^{\mathrm{wHtM}}}{X_t^{\mathrm{nHtM}}}$ to match the income over output ratio of wealthy hand-to-mouth household[26].

### 4.1.2 Financial Institutions and Central Bank

The relative collateral constraint between equity and long-term bonds $\lambda^b$ is $0.83$ which is proposed by Gertler and Karadi (2018) and Karadi and Nakov (2021). The gross collateral constraint $\lambda^\nu$ targets the leverage ratio at 6. The geometric decay rate of long-term treasury bonds is set to target the duration of long-term bonds as $10$ years. I calibrate the proportion of start-up companies $\varphi$ to match the public debt over gdp ratio. Further I calibrate the possibility of mutual fund surviving at $0.85$ to match the quarterly equity return $R^k = 1.0256$ which I derive from stock market as operating income after depreciation (Compustat item OIADPQ). This surviving rate is a little bit smaller than previous literature which does not match the equity return and takes excess return as zero. However I target the return because my focus is on the financial market's contribution to the power of QE in which the asset return and non-arbitrage condition between different markets are important.

The parameters that inhibit monetary policy are directly followed by Cui and Sterk (2021) and Sims et al. (2022) where the central bank does not adjust the holding of long-term bonds endogenously and $\theta_\pi^{QE} = \theta_y^{QE} = 0$, because it helps to decompose the different mechanisms through which QE works[27].

Table 2: Key Parameter Values

| **Parameter** | **Value** | **Description** |
|---|---|---|
| $\beta$ | 0.98 | Discount factor |
| $\tau$ | 0.25 | Labor income tax |
| $\rho$ | 0.995 | Geometric decay rate of long-term bonds |
| $\theta^m$ | 0.85 | Exist rate of mutual funds |
| $\lambda^b$ | 0.83 | Relative financial friction slackness |
| $\lambda^\nu$ | 0.36 | Absolute financial friction |
| $h^{\mathrm{HtM}}$ | 0.313 | Share of hand-to-mouth household |
| $h^{\mathrm{nHtM}}$ | 0.687 | Share of non hand-to-mouth household |
| $h^{\mathrm{wHtM}}$ | 0.192 | Share of wealthy hand-to-mouth household |

[26]The data of $\frac{c_t^{\mathrm{wHtM}}}{Y_t}$ is calculated based on the average annual income of wealthy hand-to-mouth household which is roughly 40000 dollars in US at 2010 price and estimated by Kaplan et al. (2014). Then based on the population and nominal GDP at 2010 we can get $\frac{c^{\mathrm{wHtM}}}{Y} = 0.82$.

[27]The goals of this paper is not to find the optimal monetary policy rule.

| Table 2 – Continued | | |
|---|---|---|
| **Parameter** | **Value** | **Description** |
| $h^{\mathrm{pHtM}}$ | 0.121 | Share of poor hand-to-mouth household |
| $p^{\mathrm{EU}}$ | 0.044 | Possibility go from nHtM to HtM |
| $p^{\mathrm{UE}}$ | 0.097 | Possibility go from HtM to nHtM |
| $h^{\mathrm{wHtM\|HtM}}$ | 0.613 | Share of wealthy hand-to-mouth conditional on HtM |
| $h^{\mathrm{pHtM\|HtM}}$ | 0.387 | Share of poor hand-to-mouth conditional on HtM |
| $X$ | 0.55 | Total illiquid asset withdrawing |

## 4.2 Quantitative result

I first conduct a stimulation based on my baseline model to show that the model successfully replicates some critical facts of macroeconomics during the implement of large scale asset purchasing policy. I fixed nominal interest rate at the steady state to mimic the ZLB condition and consider a $1$ percentage unexpected unconventional monetary policy shock $\gamma_t^{QE}$. Figure 2 shows the impulse response to the unconventional monetary policy shock. Central bank spends real money borrowed from households to buy long-term treasury bonds from financial institutions, which expands the demand for long-term treasury bond and increases the bond price to $0.64$ percent. This crowds out the long-term treasury bonds' holding of financial institutions and inspires them to invest more in stock market, which triggers an enormous physical investment. The expansion in capital demand elevates the capital price and spurs the real economy through partial and general equilibrium. The crowded long-term treasury bonds are replaced by investment, generating a stimulation in output through partial equilibrium. In addition to the physical investment, the labour market is also activated and the households increase their consumption as their labour income jump. This jump, compared with that in physical capital, generates the stimulation in output through general equilibrium. Combining the partial and general equilibrium effects the output has a maximized $0.656$ percent jump corresponding to a $0.485$ percent jump at consumption. The more demand for the final good encourages the retailer to set a higher price, pushing up the inflation and generating a boom.

In addition to the inflation in capital market and goods market, the unconventional monetary policy decreases the interest rate on long-term bonds $R_{t+1}^B$ and it is the shadow rate of the economy. The model shows that a $0.38$ percent decline in shadow rate stimulates $0.64$ percent in real output, which is supported by the empirical findings of Wu and Xia (2016) in which output effect is $0.59$ percent with the same decline in shadow rate. Figure 3a displays a comparison of impulse response between the empirical identification and model's stimulation and reveals that the model indeed can help us to unveil the stimulation mechanisms of unconventional monetary

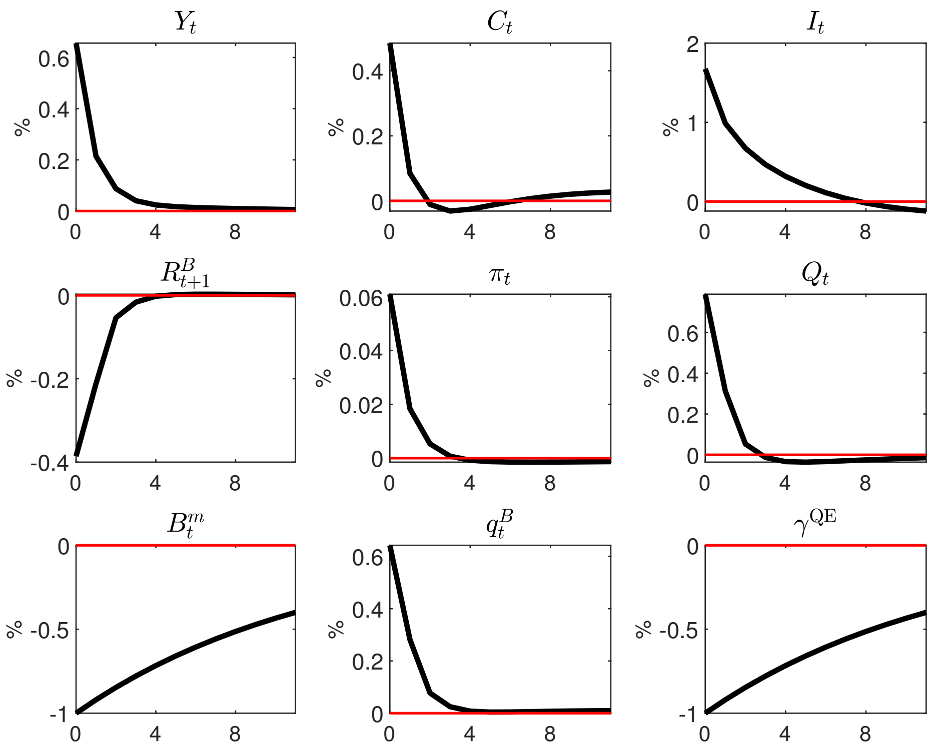



Figure 2: IRF to an unexpected expansionary unconventional monetary policy shock

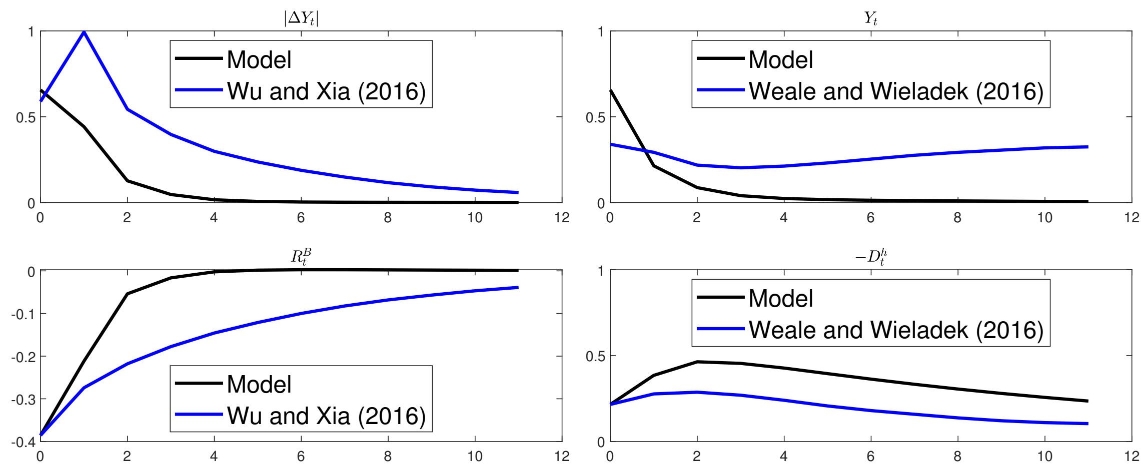
policy[28]. In addition to the empirical identification focusing on shadow rate, other scholars tried to use the accumulated asset purchasing announcement to identify the power of unconventional monetary policy[29]. The model yields a $0.76$ percent jump in GDP at the peak given a spending on monetary policy with $1$ percentage GDP at annual rate, a number which is practically the same as Weale and Wieladek (2016) did empirically[30]. Figure 3b shows the association between the model and the empirical result of Weale and Wieladek (2016) by resealing the money that central bank used to buy long-term treasury bonds[31]. The comparison between empirical identification result and the stimulation result generated by model provides the rationality of the model such that it indeed represents what happened in reality quantitatively. Thus it is credible to use this model to do several counterfactual experiments to disclose how unconventional monetary policy works via interest rate channel and liquidity channel.

[28]Here I compare the absolute value of first-order difference in output after the first period. The reason is that the VAR result of Wu and Xia (2016) generates a permanent shock in output which implies all the $\Delta Y_t$ in their model are positive. However in my model the QE shock is transitory so that all the $\Delta Y_t$ is negative as long as there is no hump shape or over shooting. Therefore I plot the absolute value in figure 3a otherwise the line will be symmetric along the x axis.

[29]Gambacorta et al. (2014); Panizza et al. (2016); Weale and Wieladek (2016); Hesse et al. (2018) did these work and used the accumulated asset purchasing announcement as the unconventional monetary policy shock to identify the QE effect to macroeconomic.

[30]By the identification scheme 2 they yield a $0.77$ percent jump in GDP at the peak.

[31]The reason why relative effect of GDP is smaller, comparing to 0.76 vs 0.77 at the peak, is that the result of Weale and Wieladek (2016) is based on monthly frequency but the model is based on quarterly frequency (though the peak point is comparable as their meaning is percentage deviation from steady sate where two results have different steady state). To convert their result to quarterly frequency I need to take sum of GDP but take mean of the accumulated asset purchasing. This mutated the stimulation effect of their work.




(a) Comparing to Wu and Xia (2016). (b) Comparing to Weale and Wieladek (2016)

Figure 3: Model vs Empirical. IRFs are in percentage level.

Proposition 2 shows that pecuniary effect is pivotal in interest rate channel which is regulated by the geometric decay rate $\rho$. Accordingly, the QE effect via liquidity channel could be isolated as long as $\rho = 0$, a condition where the interest rate channel is silent and only liquidity channel is left[32]. Additionally the effect of unconventional monetary policy on supply and demand sides can be transparently revealed by replacing the heterogeneous households with a representative household[33] or weakening the capacity of financial accelerator[34]. Figure 4 illustrates the stimulation power of unconventional monetary policy with different channels given the same influence on shadow rate[35] at time $0$. The solid black line represents the combination of all mechanisms through which the unconventional monetary policy operates and it is the largest one in output because all the four effects in table 1 are positive. After shutting down the interest rate channel, we can observe that the stimulation effect in output becomes smaller, as now the stimulation only acts via liquidity channel despite the extend of drop is not large. This is shown by the dashed blue line which lies below the baseline model almost throughout the timeline. The real GDP increases $0.4$ percent at the peak through liquidity channel which is roughly $60$ percent of the effect in baseline model. The gap in output between these two lines implies the ratio of two stimulation channels $\frac{\varphi_L}{\varphi_R}$ is approximately[36] $1.5$ which is slightly larger than analytical result

[32]In experiment I set $\rho$ to a infinit small number, $10^{-8}$, instead of exactly at $0$ for computational convenience.

[33]The interest rate at steady state changes to $\frac{1}{\beta}$ at RA scenario because the precautionary saving motive disappears under representative-household setting but the Euler equation still holds.

[34]To be consistent with the argument in last section I undermine the effect of financial accelerator by setting the possibility of surviving $\theta_m$ close to $1$.

[35]Since at different situations the amount of long-term bonds and price are different at steady state, it is inappropriate to compare with the same influence on the amount of crowded long-term treasury bonds or the money spent by central bank. Therefore shadow rate is the best one to connect the power of different channels compared to other state variables, response of which is determined endogenously and influenced by their steady states. Moreover, matching shadow rate can also tie the model to the VAR identification in next section.

[36]I approximate the effect of interest rate channel $\varphi_R = 0.6562 - 0.3944 = 0.2618$ by assuming the effects of these two channels are separable and additive. Therefore the ratio of the effect of two channels $\frac{\varphi_L}{\varphi_R}$ is approximated with $\frac{0.3944}{0.2618} = 1.5$.

1.3 (derived under ultra assumptions) and exactly the same as the empirical result in next section. The difference between baseline and liquidity models suggests that unconventional monetary policy stimulates the economy across these two pipes which have the same direction yet different diameters. Liquidity channel helps to engender an immense stimulation relative to interest rate channel because the central bank injects liquidity into the market and blows up more demand for physical capital when the financial institutions are constrained and lack liquidity.

Furthermore, the two tubes, interest rate and liquidity, are all connected with supply and demand side in economy, into which financial institutions flow fund. Figure 4 shows that neither supply side nor demand side plays an important and focal role through which the unconventional monetary policy undertakes the stimulation. It is the complementarity between supply (financial friction) and demand (heterogeneous household) sides that make monetary policy effective again during ZLB period[37]. The dashed pink line delineates the refined demand side effect through the HANK model where only heterogeneous household exists yet no financial accelerator. Similarly the dashed green line highlights the immaculate supply side effect through the RANK model, where only financial friction prevails yet no heterogeneous household regarding the baseline model[38]. All of these two models generate a slight stimulation on output and pertain to a higher shadow rate, compared to the baseline model which combines them. The vast disparity ascertains a new discrepancy between conventional and unconventional monetary policy. Even though both heterogeneous household and financial friction can magnify the effect of conventional monetary policy separately, they hardly take effect on unconventional monetary policy alone. However they can considerably amplify the power of unconventional monetary policy as long as they are connected and work together.

Additionally the stimulation direction of two channels, interest rate and liquidity, are stable and will not become opposite in a subset of the economy, neither supply sector nor demand sector. The effect of unconventional monetary policy in the model with representative household without interest rate channel is depicted by the dashed red line in figure 4. The stimulation effect of the model with liquidity channel is slightly smaller than that with both channels in output, though all of them are small at the peak as there is no financial accelerator here. This discrepancy verifies the argument in last section that financial friction and recession generate a scarcity of liquidity. The two channels become effective by providing liquidity through direct injection or pecuniary effect, despite not being augmented by heterogeneous household and

[37]The "complementarity" here is different with the "complementarity" in proposition 2. In previous section the complementarity $\varphi_1^m \varphi_4^h$ is the effect conditional on the existence of heterogeneous household and financial accelerator. The complementarity in figure 4 is the difference between "together" effect and "separate" effect. In other words to identify the complementarity in figure 4 I also change the HA effect and financial accelerator effect in proposition 2. $C^n \frac{\psi}{\sigma} + \frac{(1-\tau)WL}{h^n}$, $\frac{1}{h^n} - 1$, $\varphi_1^h$, $\varphi_2^h$ and other factors changed when I identified the complementarity in figure 4.

[38]The RANK model is modified by endogenous illiquid asset withdrawing following the extension experiments in appendix of Cui and Sterk (2021) because RANK model with exogenous withdrawing cannot be solved as Blanchard-Kahn condition is not satisfied. The extension in appendix B.1 implies that the response of RANK model in figure 4 is a conservative result as endogeneity amplifies the power of unconventional monetary policy.

general equilibrium. However the heterogeneous household and redistribution effect still play a role here by spawning a positive effect of interest rate channel on consumption, regarding the negative effect in RANK model in figure 4. Both of these channels require the financial institutions to borrow money from central bank to invest, which results in a higher leverage ratio. The debts borrowed by financial institutions are funded by lump-sum tax and come from household in the end. Therefore the larger stimulation effect on output is, the smaller stimulation on consumption will be, since now redistribution effect is closed.

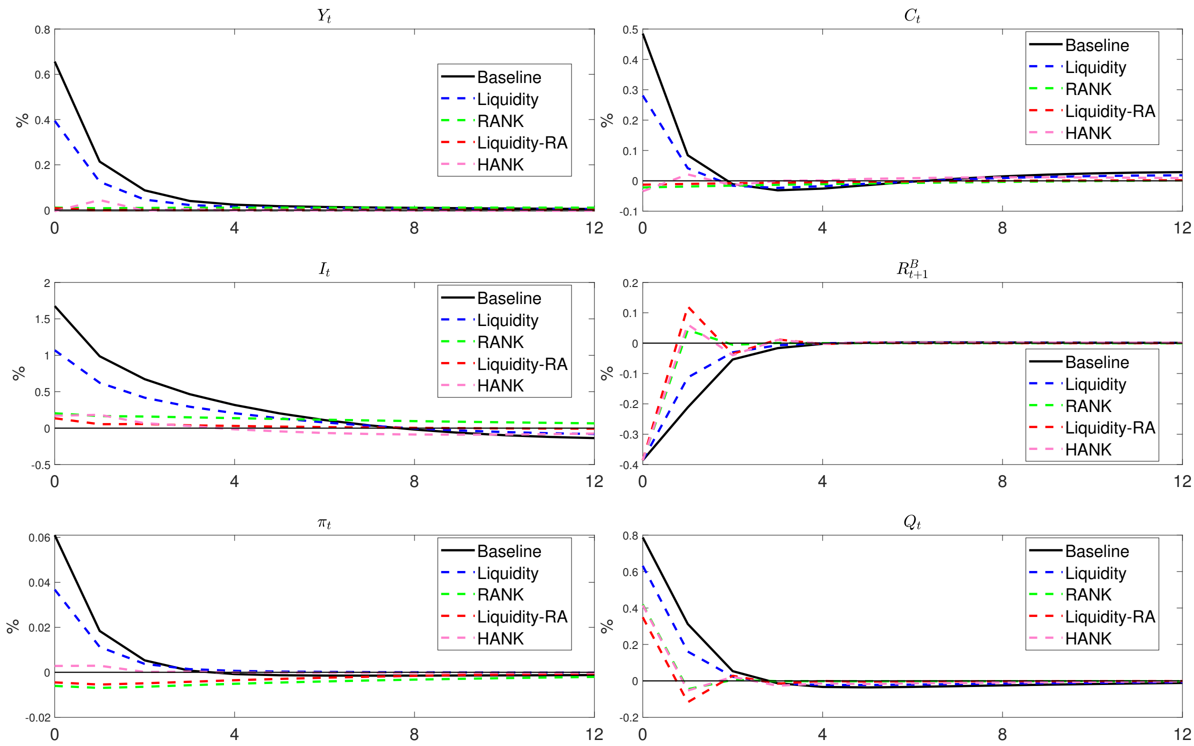



Figure 4: Decomposition of unconventional monetary policy in different channels

# 5 Empirical Evidence

In last two sections I analytically discussed how the unconventional monetary policy affects stimulating economy through the two channels and quantitatively illustrated their relative effect. In this section I provide the empirical evidence to show that the analytical discussion and quantitative result are tractable and reasonable. I first propose a new method and a new type of intuitive inequality constraint to conduct Bayesian estimation on VAR model with multiple instrument variables. Then I propose a new instrument variable, together with an existing instrument variable to disentangle the effect of unconventional monetary policy from staggering to isolating the two channels.

## 5.1 Methodology

I use a new IV estimation process to estimate a VAR to identify the liquidity supply effect and interest rate expectation effect of monetary policy. The formal one links to liquidity channel and the later one links to interest rate channel. To estimate the monetary policy effect to the economy, I need to estimate the following reduce-form var model

$$Y_t = \sum_{j=1}^{p} A_j Y_{t-j} + B\boldsymbol{\varepsilon}_t$$

where $\boldsymbol{\varepsilon}_t$ is iid shock and I can normalize its covariance matrix to identity such that $E\left[\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t'\right] = I$.

It is easy to estimate $A_j$ while we can only get $E\left[u_t u_t'\right] = BB'$ where

$$u_t = B\boldsymbol{\varepsilon}_t$$

It is impossible to identify $B$ from $BB'$ and we need further $\frac{n(n+1)}{2}$ restrictions to identify it.

Stock and Watson (2012) and Mertens and Ravn (2013) proposed a new method that introduced proxies variable to help identifying the shock effect on economy. They introduce k new variable which satisfies

$$E\left[m_t \boldsymbol{\varepsilon'}_{1t}\right] = \Phi \tag{22}$$

$$E\left[m_t \boldsymbol{\varepsilon'}_{2t}\right] = 0 \tag{23}$$

where $m_t$ is a k-by-1 vector and I take partition on $\boldsymbol{\varepsilon}_t$ such that $\boldsymbol{\varepsilon}_t = \begin{bmatrix} \boldsymbol{\varepsilon}_{1t}' & \boldsymbol{\varepsilon}_{2t}' \end{bmatrix}'$.

Introducing $m_t$ helps us to identify the $\boldsymbol{\varepsilon}_{1t}$ effect to economy because $\Phi$ provide more information about $\boldsymbol{\varepsilon}_{1t}$ given $m_t$. Unfortunately identifying $\boldsymbol{\varepsilon}_{2t}$'s effect cannot receive any help as $E\left[m_t \boldsymbol{\varepsilon}_{\mathbf{2t}}'\right] = 0$ and I need other restrictions on $B$ if fully identifying $B$ is the objective. However I am only interested in the monetary policy effect and only need to identify first k column of $\boldsymbol{B}$, like Gertler and Karadi (2015) did. While I still need more restrictions on $B$ to identify $\varepsilon_{1t}$ effect since identifying $\Phi$ waists some degree of freedom. As long as $k > 1$ held and $\Phi$ was fully identified, the first $k$ column of $B$ cannot be fully identified. Mertens and Ravn (2013) added several linear restrictions helping to identify the effect of $\varepsilon_{1t}$. In this paper one of my contribution is that conversely I propose a modified method that imposes restriction on $\Phi$ and leaves the first $k$ column of $B$ free to identify.

Write $B$ into partition

$$B = \begin{bmatrix} b_{11} & b_{12} & \boldsymbol{b}_{13} \\ b_{21} & b_{22} & \boldsymbol{b}_{23} \\ \boldsymbol{b}_{31} & \boldsymbol{b}_{32} & \boldsymbol{b}_{33} \end{bmatrix}_{n \times n} = \begin{bmatrix} \beta_1 & \beta_2 \end{bmatrix}$$

$$\boldsymbol{\beta}_1 = \begin{bmatrix} \beta_{11} & \boldsymbol{\beta}'_{21} \\ \beta_{12} & \boldsymbol{\beta}'_{22} \end{bmatrix}'_{n\times 2}$$

Combining with equation 22 and 23, it is easy to yield

$$\Phi\boldsymbol{\beta}'_1 = \Sigma_{mu'} \tag{24}$$

Furthermore denote $s_{11} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ the covariance matrix of coefficient $\Phi$ can be written as

$$\Phi\Phi' = \Sigma_{mu'_1}\left(\Sigma'^{-1}_{mu'_1} s_{11} s'_{11}\right)^{-1} \tag{25}$$

where $\Sigma_{mu'_1}$ is the first $k \times k$ elements of $\Sigma_{mu'}$; $s_{11}^{-1} s_{11}'^{-1}$ can be estimated from equation 81. I demote related derivative process of equation 25 to appendix.

Then taking the Cholesky decomposition on $\Phi\Phi'$ yields the lower triangle matrix $\Phi_{tr}$. $\Phi$ will be identified up to the rotation matrix $Q$ such that $\Phi = \Phi_{tr}Q$. Note that $Q$ is an orthogonal matrix such that $Q \in \mathcal{O}(n)$. Following Caldara and Herbst (2019) and Arias et al. (2018) I develop an algorithm using Metropolis-Hastings sampler across Haar measurement space to draw $Q$ from $Q|Y, X, M, B, u, \Sigma$. Since Baumeister and Hamilton (2015) argued that the standard method that uses uniform distribution on reduced-form parameters causes a flat likelihood problem, I use a bayesian method to estimate the reduced-form parameter $\Phi$ and $\boldsymbol{\beta}'_1$ which is developed by Caldara and Herbst (2019), Braun et al. (2020), Arias et al. (2021) and Giacomini et al. (2021).

Any $\widehat{\Phi} = \mathit{\Omega}_{tr}Q$ that satisfies restrictions will give us a fully identified $\boldsymbol{\beta}'_1$ through equation 24 in which is what I am interested.

Alternatively I also do some robustness check to the estimation. I tried to impose off-diagonal zero restriction and lower triangle restriction on $\Phi$. Meanwhile I also tried to use uniform prior and standard Bayesian estimation to estimate $\Phi$. To estimate $\boldsymbol{\beta}'_1$ I use frequentist method. All the detailed explanation and result of robustness check are shown at Appendix D.

Considering the possibility of plausibly exogenous[39] I also impose inequality constraint which idea is analogous to the $S(\phi, Q)$ inequality proposed by Giacomini et al. (2021). Here I assume that

$$\rho\left(m_{it}\varepsilon_{it}\right)^2 > \rho\left(m_{it}\varepsilon_{jt}\right)^2 ,\forall i \neq j \tag{26}$$

This inequality is intuitive and meaningful that implies the instrument explain much of the information of structural shock at the same row relative to the structural shock at other row. Given the definition of $\rho\left(m_{it}\varepsilon_{jt}\right)$

$$\rho\left(m_{it}\varepsilon_{jt}\right)^2 = \frac{\operatorname{cov}\left(m_{it}\varepsilon_{jt}\right)^2}{\sigma^2_{m_{it}}\sigma^2_{\varepsilon_{jt}}}$$

[39] Conley et al. (2012) detailedly discussed this type of proxy variables.

and I normalize the structural shock $\varepsilon_t$ to standard normal distribution, equation 26 can be simplified to

$$\operatorname{cov}\left(m_{it}\varepsilon_{it}\right)^2 > \operatorname{cov}\left(m_{it}\varepsilon_{jt}\right)^2$$

Furthermore, because the equation 24, write this inequality to

$$F\left(\Phi_{tr}, Q; \gamma\right) \equiv \operatorname{diag}\left\{\left(\Phi_{tr}Q\right) \circ \left(\Phi_{tr}Q\right)\begin{bmatrix} 1 & -\gamma_2 \\ -\gamma_1 & 1 \end{bmatrix}\right\} > 0$$

where $\gamma_i$ represents the weakness[40] of instrument variables $m_i$. A valid instrument variable $m_i$ should satisfy $\gamma_i \geq 1$ which I assume will be satisfied across this paper. Based on the assumption of $\gamma_i \geq 1$ the domain of $\gamma$ should be $[1, \infty]$. $\gamma_i = 1$ denotes the slackest restriction on the power of instrument variable $m_i$ that explains shock $\varepsilon_i$. It only needs to correlate with $\varepsilon_i$ slightly more than that with $\varepsilon_j$. $\gamma_i = \infty$ denotes the strongest restriction which requires that instrument variable $m_i$ is not correlated with $\varepsilon_j$ at all and only correlated with $\varepsilon_i$. It is straightforward to write $\gamma_i$ as $\kappa_i = 1 - \frac{1}{\gamma_i}$, $\kappa_i \in [0, 1]$ which is better to understand. $\kappa = 0$ represents no restriction[41] and $\kappa = 1$ represents the strongest restriction. Therefore above restriction can be write as

$$F\left(\Phi_{tr}, Q; \rho\right) \equiv \operatorname{diag}\left\{\left(\Phi_{tr}Q\right) \circ \left(\Phi_{tr}Q\right)\begin{bmatrix} 1 & -\frac{1}{1-\kappa_2} \\ -\frac{1}{1-\kappa_1} & 1 \end{bmatrix}\right\} > 0 \tag{27}$$

## 5.2 Bayesian estimation and result

Following Gertler and Karadi (2015) I use four variables to estimate the monetary policy effect to economy such that $Y_t = \begin{bmatrix} r_t & cpi_t & y_t & \varDelta_t \end{bmatrix}$. $r_t$ is the market yield on 2-Year U.S. Treasury Securities. $cpi_t$ is logarithmic consumer price index. $y_t$ is logarithmic industrial production. $\varDelta_t$ is the excess bonds premium which is estimated through the methods proposed by Gilchrist and Zakrajšek (2012). Write the two instrument variables as $m_t = \begin{bmatrix} m_{1t} & m_{2t} \end{bmatrix}'$ and I use conventional high-frequency identification to construct related instrument variables w.r.t monetary policy shock. $m_{1t}$ is the surprises in current month's fed funds future (FF1) contract price during the day when treasury department announces a new issue of treasury bonds/notes. $m_{2t}$ is the surprises in FF1 contract price during the day when FOMC meeting is held and FOMC releases announcement. It is widely explored by scholar who uses the changed future contract price during FOMC announcement day to analyze monetary policy[42]. Similar to FOMC announcement, US department of the treasury will disclose their intended amount of treasury bond which they plan to issue at related announcement day. In contrast to the FOMC announcement which shows the targeted short-term interest rate or federal fund rate, announcement issued by treasury department

[40] The larger $\gamma_i$ is, the stronger related instrument is that can be used to explain $\varepsilon_i$.

[41] Here the word "no" means as long as the instruments is valid the estimation of $\mathbf{\Phi}$ is acceptable.

[42] A large series of literature discuss monetary policy through this high-frequency identification such as Kuttner (2001); Gürkaynak et al. (2004); Bernanke and Kuttner (2005); Hamilton (2008); Campbell et al. (2012); Hanson and Stein (2015); Nakamura and Steinsson (2018)

only provide the information related to the nominal face value of the bond that they want to issue. The interest rate or yield to maturity is determined by auction which will be held several days later. Given the central bank imposes monetary policy through liquidity channel by open-market operation and buying long-term treasury bonds, the pure increased treasury bonds supply will be *isomorphic* to decreased treasury bonds holding of central bank, as long as general equilibrium always been reached. Therefore the surprises in future federal fund rate contract price is a valid and appropriate instrument variable to correlate with the pure liquidity effect of monetary policy since the only extra information treasury bonds announcement provides is just the liquidity amount[43]. More detailed illustration about data can be referred at Appendix C.

Figure 5 shows the estimation result without imposing any constraint such that $\kappa_1 = \kappa_2 = 0$. Left column(5a) is the IRF to the liquidity shock and right column(5b) is the IRF the interest rate shock. Both the shocks are normalized to generate 1 percentage interest rate deviation at 0 period.

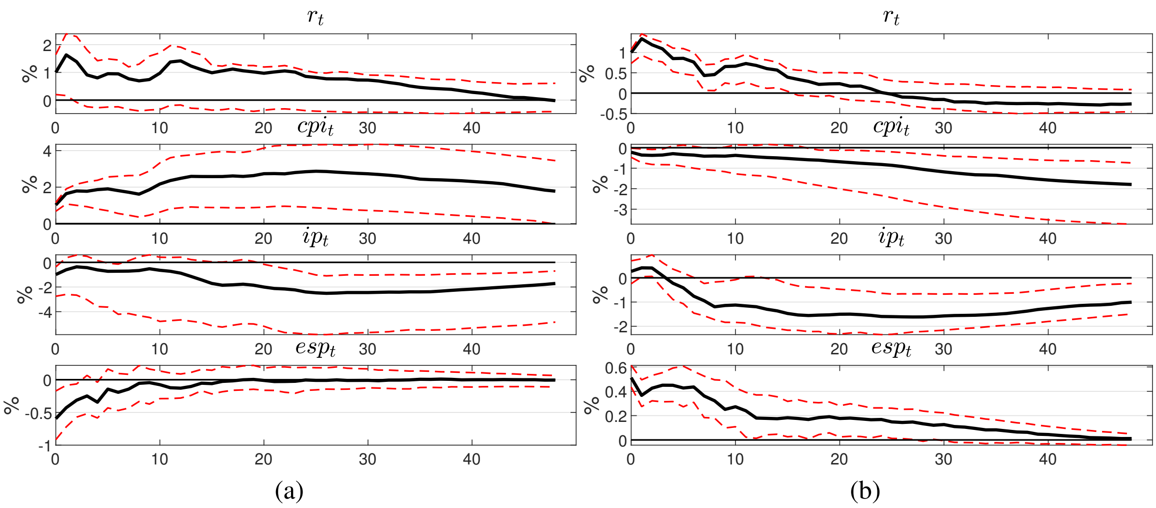



Figure 5: IRF to unconventional monetary policy under liquidity and interest rate channel with 90% confidence band.

Figure 5 sheds light on the power of unconventional monetary policy in liquidity and interest rate channels empirically. A 1 percent increase in 2-years treasury bond rate, triggered by a contractionary unconventional monetary policy, will suppress the output $2.5\%$ at the peak via liquidity channel and $1.6\%$ at the peak via interest rate channel. The empirical result justifies the argument that liquidity channel is more powerful than interest channel to stimulate the output. To associate the empirical result to the quantitative result in last section, I can hardly use above result since the long-term rate in figure 5 is the change of yield-to-maturity rate while the shadow rate in model is the current-yield rate. Therefore I use the same identification method but 3-Month U.S. Treasury Securities instead of 2-Year to conduct the estimation again

[43]Respectively the only information provided by FOMC is the targeted interest rate (federal fund rate) instead of the liquidity amount. Though during the QE period central bank indeed provided liquidity information in some statements, I do the robustness check which excludes QE period and it yields the same result.

in appendix. The consistence between my empirical and quantitative results substantiates my main contribution in this paper that the effect of liquidity channel to stimulation power of QE on output is approximately 1.5 times larger than that of interest rate channel.

# 6 Conclusion

I introduce household heterogeneity and financial accelerator into a general equilibrium model to analyze how the unconventional monetary policy plays a part in economy through liquidity and interest rate channel. I first decompose the two channels and discuss that the effect of the two channels are governed by supply and demand sides through financial friction, pecuniary easing, redistribution in wealth and income, and the complementarity, under a knife-edge condition where price and depreciation rate are fixed. After the discussion I carefully calibrate the model and show that the liquidity channel is 1.5 times larger than the interest rate channel quantitatively. Meanwhile I also show that the complementary effect between household heterogeneity and financial friction is the pivot to determine the power of unconventional monetary through the two channels. Finally I use an IV-VAR to demonstrate that the discussion and argument is reasonable both qualitatively and quantitatively.

However my research still remains some drawbacks waiting for future exploration. The model is a simple DSGE with only three types of household so that the mass and consumption decisions of hand-to-mouth households are all determined by exogenous assumption. Moreover, the setting of financial friction and endogenous leverage ratio is relatively too easy to reality where there exists heterogeneity within financial institutions and their exit and entry are endogenously regulated.

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# A Derivation steps and supplements to Model

## A.1 Supplements to Model

In this section I provide some supplements to the model part with more detailed explanation and definition related to the model setting, as well as some important first-order conditions. The order of subsections are akin to section 2 from household to central bank and government.

### A.1.1 Household

Since the hand-to-mouth household will consume all their income each period, the consumption of poor and wealth hand-to-mouth household is static such that

$$c_t^{pHtM} = \Theta_t^{\mathrm{HtM}} + T_t$$

and

$$c_t^{\mathrm{wHtM}} = X^{\mathrm{wHtM}} + \Theta_t^{\mathrm{HtM}} + T_t$$

Meanwhile as only the non hand-to-mouth household can supply the labour to firm, the supply function of labour will be pinned down by the first-order condition of non hand-to-mouth household

$$\frac{L_t}{h^{\mathrm{nHtM}}} = \left(-\frac{(1-\tau_l)\, w_t}{\kappa}\right)^{\frac{1}{\psi}} \left(c_t^{\mathrm{nHtM}}\right)^{-\frac{\sigma}{\psi}}$$

It is worth to notice that the distributional cyclicity problem between labour income and dividend income[44] in HANK model does not emerge here because only the non hand-to-mouth household provides labour into market.

### A.1.2 Mutual funds

The budget constraint of the mutual fund 5 can be written as

$$n_t = R_t^k Q_{t-1} s_{t-1} - Q_t s_t + \frac{\left(1+\rho q_t^B\right)}{\Pi_t} b_{t-1}^m - q_t^B b_t^m - R_{t-1} d_{t-1}^m$$

Therefore I define the return of long-term treasury bonds earned by mutual fund as

$$R_t^B = \frac{\left(1+\rho^B q_t^B\right)}{q_{t-1}^B \Pi_t}$$

Meanwhile the value function of mutual fund $W_t$ and $V_t$ can be solved via guess and verify such that

$$W_t = \eta_t n_t$$

[44]This is firstly proposed by Broer et al. (2020) and further discussed by Cantore and Freund (2021).

$$V_t = \mu_t^s Q_t s_t + \mu_t^b q_t^B b_t^m + \zeta_t n_t$$

$$\zeta_t = E_t \beta \Omega_{t,t+1} R_t$$

$$\mu_t^s = E_t \beta \Omega_{t,t+1} \left( R_{t+1}^k - R_t \right)$$

$$\mu_t^b = E_t \beta \Omega_{t,t+1} \left( R_{t+1}^B - R_t \right)$$

### A.1.3 Intermediate goods producer

The first-order conditions to maximization problem 9 are

$$P_t^m \alpha \frac{Y_t^m}{U_t} = \delta' \left( U_t \right) \xi_t K_{t-1}$$

and

$$P_t^m (1-\alpha) \frac{Y_t^m}{L_t} = W_t$$

Notice that the intermediate goods market is a complete market where producer earns zero profit and this pins down the real price of intermediate goods $P_t^m$.

### A.1.4 Retailer and Final goods producer

The price setting problem 10 can be solved and rewritten into recursive formula

$$\Pi_t^* = \frac{\sigma_p}{\sigma_p - 1} \frac{F_t^p}{Z_t^p} \Pi_t$$

$$\Pi_t^* = \frac{P_t^*}{P_{t-1}}$$

$$F_t^p = P_t^m Y_t + \beta \theta E_t \Lambda_{t,t+1} \Pi_t^{-\gamma_p \sigma_p} \Pi_{t+1}^{\sigma_p} F_{t+1}^p$$

$$Z_t^p = Y_t + \beta \theta E_t \Lambda_{t,t+1} \Pi_t^{\gamma_p (1-\sigma_p)} \Pi_{t+1}^{\sigma_p - 1} Z_{t+1}^p$$

$$\Pi_t^{1-\sigma_p} = \theta \left( \Pi_{t-1}^{\gamma_p} \right)^{1-\sigma_p} + (1-\theta) \Pi_t^{*1-\sigma_p}$$

where $\Pi_t^*$ is the inflation rate for those retailers who adjust their price at period $t$.

## A.2 Log-linearization of Baseline Model

## Household

$$\widehat{C}_t^w = \frac{T}{C^w} \widehat{T}_t + \frac{\Theta^T}{C^w} \widehat{\Theta}_t^T \tag{28}$$

$$\widehat{C}_t^p = \frac{T}{C^p} \widehat{T}_t + \frac{\Theta^T}{C^p} \widehat{\Theta}_t^T \tag{29}$$

$$-\sigma\widehat{C}_t^n = \widehat{R}_t - \sigma\left[\frac{p^E C^n}{\Sigma p^i C^{i(-\sigma)}}\widehat{C}_{t+1}^n + \frac{p^{EUw} C^w}{\Sigma p^i C^{i(-\sigma)}}\widehat{C}_{t+1}^w + \frac{p^{EUp} C^p}{\Sigma p^i C^{i(-\sigma)}}\widehat{C}_{t+1}^p\right] \tag{30}$$

$$\widehat{L}_t = \frac{1}{\psi}\widehat{W}_t - \frac{\sigma}{\psi}\widehat{C}_t^n \tag{31}$$

$$\widehat{C}_t = \frac{h^n}{C}\widehat{C}_t^n + \frac{h^w}{C}\widehat{C}_t^w + \frac{h^p}{C}\widehat{C}_t^p \tag{32}$$

# Financial market

$$\widehat{\phi}_t = \frac{QK}{\phi N^h}\left(\widehat{Q}_t + \widehat{K}_t\right) + \frac{\lambda^b q^B B^m}{\phi N^h}\left(\widehat{q}_t^B + \widehat{B}_t^m\right) - \widehat{N}_t^h \tag{33}$$

$$\widehat{\Omega}_t = \widehat{\Lambda}_t + \frac{\theta^m}{1-\theta^m+\theta^m\eta}\widehat{\eta}_t \tag{34}$$

$$\widetilde{\lambda}_t = \frac{\zeta}{\phi\gamma^\lambda\lambda^\nu}\left(\widehat{\phi}_t + \widehat{\gamma}_t^\lambda - \widehat{\zeta}_t\right) \tag{35}$$

$$\widehat{\zeta}_t = \widehat{\Omega}_{t+1} + \widehat{R}_t \tag{36}$$

$$\widehat{\eta}_t = \widehat{\zeta}_t + \frac{1}{1-\lambda}\widetilde{\lambda}_t \tag{37}$$

$$\widehat{R}_t^k = \widehat{\xi}_t + \frac{Q}{Q+\frac{\Pi^f}{\xi K}}\widehat{Q}_t - \widehat{Q}_{t-1} + \frac{\Pi^f}{\xi KQ+\Pi^f}\left(\widehat{\Pi}_t^f - \widehat{\xi}_t - \widehat{K}_{t-1}\right) \tag{38}$$

$$\widehat{R}_t^B = \frac{\rho q^B}{1+\rho q^B}\widehat{q}_t^B - \widehat{\Pi}_t - \widehat{q}_{t-1}^B \tag{39}$$

$$\widehat{\Omega}_{t+1} + \frac{R^k}{R^k-R}\widehat{R}_{t+1}^k - \frac{R}{R^k-R}\widehat{R}_t = \frac{1}{\lambda}\widetilde{\lambda}_t + \widehat{\gamma}_t^\lambda \tag{40}$$

$$\widehat{\Omega}_{t+1} + \frac{R^B}{R^B-R}\widehat{R}_{t+1}^B - \frac{R}{R^B-R}\widehat{R}_t = \frac{1}{\lambda}\widetilde{\lambda}_t + \widehat{\gamma}_t^\lambda \tag{41}$$

$$\frac{QK}{N^h}\left(\widehat{Q}_t + \widehat{K}_t\right) + \frac{q^B B^m}{N^h}\left(\widehat{q}_t^B + \widehat{B}_t^m\right) - \frac{D^m}{N^h}\widehat{D}_t^m = \widehat{N}_t^h \tag{42}$$

$$\widehat{R}_t^a = \frac{1}{R}\widehat{N}_t^h + \frac{\Pi^r}{RN^h}\widehat{\Pi}_t^r + \frac{1}{RN^h}\widetilde{\Pi}_t^I + \frac{\Pi^m}{RN^h}\widehat{\Pi}_t^m - \widehat{N}_{t-1}^h \tag{43}$$

# Production Sector

$$\widehat{Y}_t^m = \widehat{\gamma}_t^{\text{TFP}} + \alpha\left(\widehat{U}_t + \widehat{\xi}_t + \widehat{K}_{t-1}\right) + (1-\alpha)\,\widehat{L}_t \tag{44}$$

$$\widehat{K}_t = \xi\left(\widehat{\xi}_t + \widehat{K}_{t-1}\right) + \frac{I_{ss}}{K}\widehat{I}_t \tag{45}$$

$$\widehat{Y}_t^m + \widehat{P}_t^m = \widehat{K}_{t-1} + \widehat{\xi}_t + (1+\nu)\,\widehat{U}_t \tag{46}$$

$$\widehat{W}_t = \widehat{Y}_t^m + \widehat{P}_t^m - \widehat{L}_t \tag{47}$$

$$\widehat{\delta}_t = (1+\nu)\,\widehat{U}_t \tag{48}$$

$$Q\widehat{Q}_t = \varphi_I\left(1-\beta\Lambda\right)\left(\widehat{I}_t - \widehat{I}_{t-1}\right) \tag{49}$$

$$\widehat{F}^p_t = \frac{P^m Y}{F^p}\left(\widehat{P}^m_t + \widehat{Y}_t\right) + \beta\Lambda\theta\Pi^{\sigma_p(1-\gamma_p)}\left(\widehat{\Lambda}_{t+1} + \sigma_p\widehat{\Pi}_{t+1} - \sigma_p\gamma_p\widehat{\Pi}_t + \widehat{F}^p_{t+1}\right) \tag{50}$$

$$\widehat{Z}^p_t = \frac{Y}{Z^p}\widehat{Y}_t + \beta\Lambda\theta\Pi^{(\sigma_p-1)(1-\gamma_p)}\left[\widehat{\Lambda}_{t+1} + \left(\sigma_p-1\right)\widehat{\Pi}_{t+1} - \gamma_p\left(\sigma_p-1\right)\widehat{\Pi}_t + \widehat{Z}^p_{t+1}\right] \tag{51}$$

$$\widehat{\Pi}_t = \theta\gamma_p\Pi^{(1-\sigma_p)(\gamma_p-1)}\widehat{\Pi}_{t-1} + \left(1-\theta\right)\frac{\Pi^{*(1-\sigma_p)}}{\Pi^{1-\sigma_p}}\widehat{\Pi}^*_t \tag{52}$$

$$\widehat{\Pi}^*_t = \widehat{\Pi}_t + \widehat{F}^p_t - \widehat{Z}^p_t \tag{53}$$

$$\widehat{\mu}_t = -\sigma_p\left(1-\theta\right)\frac{\Pi^{*(-\sigma_p)}}{\mu\Pi^{-\sigma_p}}\left(\widehat{\Pi}^*_t - \widehat{\Pi}_t\right) + \theta\frac{\Pi^{(-\gamma_p\sigma_p)}}{\mu\Pi^{-\sigma_p}}\left[\widehat{\mu}_{t-1} - \sigma_p\left(\gamma_p\widehat{\Pi}_{t-1} - \widehat{\Pi}_t\right)\right] \tag{54}$$

$$\widehat{Y}_t = \widehat{Y}^m_t - \widehat{\mu}_t \tag{55}$$

## Central Bank

$$T\widehat{T}_t = RD^h\left(\widehat{R}_{t-1} + \widehat{D}^h_{t-1}\right) + RD^m\left(\widehat{R}_{t-1} + \widehat{D}^m_{t-1}\right) - D^m\widehat{D}^m_t - \frac{\left(1+\rho q^B\right)B^m}{\Pi}\left(\frac{\rho q^B}{1+\rho q^B}\widehat{q}^B_t + \widehat{B}^m_{t-1} - \widehat{\Pi}_t\right) \tag{56}$$

$$D^h\widehat{D}^h_t = q^B B^m\left(\widehat{q}^B_t + \widehat{B}^m_t\right) \tag{57}$$

$$\widehat{R}^i_t = \widehat{\gamma}^{\mathrm{MP}}_t \tag{58}$$

$$\widehat{B}^m_t = \widehat{\gamma}^{\mathrm{QE}}_t \tag{59}$$

$$\widehat{R}^i_t = \widehat{R}_t - \widehat{\Pi}_{t+1} \tag{60}$$

## Market Clearing

$$\widetilde{\Pi}^I_t = \left(Q-1\right)I\widehat{I}_t + \left(I - I_{ss}\right)Q\widehat{Q}_t \tag{61}$$

$$\widehat{\Pi}^r_t = \widehat{Y}_t - \frac{P^m\mu}{1-P^m\mu}\left(\widehat{P}^m_t + \widehat{\mu}_t\right) \tag{62}$$

$$\widetilde{\Pi}^I_t + \Pi^r\widehat{\Pi}^r_t + \Pi^m\widehat{\Pi}^m_t = 0 \tag{63}$$

$$\widehat{\Pi}^f_t = \frac{Y^m P^m}{\Pi^f}\left(\widehat{Y}^m_t + \widehat{P}^m_t\right) - \frac{LW}{\Pi^f}\left(\widehat{L}_t + \widehat{W}_t\right) - \frac{K\xi\delta}{\Pi^f}\left(\widehat{K}_{t-1} + \widehat{\xi}_t + \widehat{\delta}_t\right) \tag{64}$$

$$\begin{aligned}\widehat{\Pi}^m_t = \left(1-\theta^m\right)\Bigg[&\frac{N^h}{\Pi^m}\widehat{N}^h_{t-1} + \frac{R}{\Pi^m}\widehat{R}_{t-1} + \frac{KQ\left(R^k-R\right)}{\Pi^m}\left(\widehat{Q}_{t-1} + \widehat{K}_{t-1} + \frac{R^k}{R^k-R}\widehat{R}^k_t - \frac{R}{R^k-R}\widehat{R}_{t-1}\right) + \\ &\frac{B^m q^B\left(R^B-R\right)}{\Pi^m}\left(\widehat{q}^B_{t-1} + \widehat{B}^m_{t-1} + \frac{R^B}{R^B-R}\widehat{R}^B_t - \frac{R}{R^B-R}\widehat{R}_{t-1}\right)\Bigg] - \frac{\Phi\phi N^h}{\Pi^m}\left(\widehat{N}^h_{t-1} + \widehat{\phi}_{t-1}\right)\end{aligned} \tag{65}$$

$$\widehat{\Lambda}_t = -\sigma \widehat{C}_t + \sigma \widehat{C}_{t-1} \tag{66}$$

$$\widehat{\Theta}_t^T = \widehat{L}_t + \widehat{W}_t \tag{67}$$

$$\widehat{C}_t = \frac{Y}{C}\widehat{Y}_t - \frac{I}{C}\widehat{I}_t - \frac{\xi K \delta}{C}\left(\widehat{K}_{t-1} + \widehat{\xi}_t + \widehat{\delta}_t\right) \tag{68}$$

where$\widehat{I}_t = \widehat{I_{n,t} + I_{ss}}$

## A.3 Derivative of Key Equations

### A.3.1 Financial market

Firstly I can use equation 64, 45, 46, 47, 48 and 38 to get the return of firm equities given the assumption $\xi = 1$

$$\widehat{R}_t^k = \widehat{\xi}_t + \frac{KQ}{KQ + \Pi^f}\widehat{Q}_t - \widehat{Q}_{t-1} + \frac{\Pi^f}{KQ + \Pi^f}\widehat{\delta}_t \tag{69}$$

which I will use it later to link the investment to the leverage ratio.

Using equation 35, 36 and 40, together with $\gamma^\nu = 1$, yields$\left(1 + \frac{\zeta}{\lambda\phi\lambda^\nu}\right)\widehat{\Omega}_{t+1} + \frac{R^B}{R^B - R}\widehat{R}_{t+1}^k - \frac{R}{R^B - R}\widehat{R}_t = \frac{\zeta}{\lambda\phi\lambda^\nu}\left(\widehat{\phi}_t + \widehat{\gamma}_t^\lambda - \widehat{R}_t\right) + \widehat{\gamma}_t^\lambda$. Then the law of motion w.r.t stochastic discount factor can be found by combining equation 34, 35, 36 and 37 as $\widehat{\Omega}_t = \widehat{\Lambda}_t + \frac{\theta^m}{1-\theta^m+\theta^m\eta}\left[\left(1 - \frac{\zeta}{(1-\lambda)\phi\lambda^\nu}\right)\left(\widehat{\Omega}_{t+1} + \widehat{R}_t\right) + \right.$
Then plug the previous equation into this law of motion will yield

$$\widehat{\phi}_t + \widehat{\gamma}_t^\lambda = \frac{1 - \theta^m + \theta^m\eta}{\theta^m}\left[\frac{\zeta}{\phi\lambda^\nu}\left(\widehat{\phi}_{t-1} + \widehat{\gamma}_{t-1}^\lambda - \widehat{R}_{t-1}\right) + \lambda\widehat{\gamma}_{t-1}^\lambda - \lambda\left(\frac{R^k}{R^k - R}\widehat{R}_t^k - \frac{R}{R^B - R}\widehat{R}_{t-1}\right) - \widehat{\Lambda}_t\right] \tag{70}$$

with the condition that in steady state $(1 - \lambda)\phi\lambda^\nu = \zeta$.

### A.3.2 Illiquid asset return

Plugging equation 61, 62, 63, 65, 33 and 39 into equation 43 yields

$$\begin{aligned}
\widehat{R}_t^a = {} & \frac{1}{R}\left[\frac{QK}{\phi N^h}\left(\widehat{Q}_t + \widehat{K}_t\right) - \widehat{\phi}_t\right] + \frac{\lambda^b q^B B^m}{\phi N^h}\left(\widehat{q}_t^B + \widehat{B}_t^m\right) + \frac{(1-\theta^m)R^B B^m q^B}{RN^h}\frac{\rho q^B}{1+\rho q^B}\widehat{q}_t^B + \frac{\Pi^r}{RN^h}\widehat{\Pi}_t^r + \\
& + \frac{\Pi^m}{RN^h}(1-\theta^m)\left[\frac{N^h}{\Pi^m}\widehat{N}_{t-1}^h + \frac{R}{\Pi^m}\widehat{R}_{t-1} + \frac{KQ\left(R^k - R\right)}{\Pi^m}\left(\widehat{Q}_{t-1} + \widehat{K}_{t-1} + \frac{R^k}{R^k - R}\widehat{R}_t^k - \frac{R}{R^k - R}\widehat{R}_{t-1}\right)\right. \\
& \left.\frac{B^m q^B\left(R^B - R\right)}{\Pi^m}\left(\widehat{q}_{t-1}^B + \widehat{B}_{t-1}^m - \frac{R}{R^B - R}\widehat{R}_{t-1}\right)\right] - \frac{\Phi\phi N^h}{RN^h}\left(\widehat{N}_{t-1}^h + \widehat{\phi}_{t-1}\right) \\
& - \frac{(1-\theta^m)R^B B^m q^B}{RN^h}\left(\widehat{\Pi}_t + \widehat{q}_{t-1}^B\right)
\end{aligned} \tag{71}$$

### A.3.3 General equilibrium

The redistribution effect can be identified by plugging the consumption of hand-to-mouth household 28 and 29 into the aggregate goods market clearing condition 68 and 32robustness

$$Y_C\widehat{Y}_t - I_C\widehat{I}_t - K_C\delta\left(\widehat{K}_{t-1} + \widehat{\xi}_t + \widehat{\delta}_t\right) = h_C^n\widehat{C}_t^n + \varphi_1^h\widehat{T}_t + \varphi_2^h\widehat{\Theta}_t^T$$

where $\varphi_1^h = h_C^w T_{C^w} + h_C^p T_{C^p}$ and $\varphi_2^h = h_C^w \Theta_{C^w}^T + h_C^p \Theta_{C^p}^T$

This equation can be further simplified by equation 46, 47, 48 and 67

$$\left(Y_C - \delta K_C - \varphi_2^h\right)\widehat{Y}_t - I\hat{I}_t - \left(K_C\delta + \varphi_2^h\right)\left(\widehat{P}_t^m + \widehat{\mu}_t\right) = h_C^n\widehat{C}_t^n + \varphi_1^h\widehat{T}_t$$

Further I use the budget constraint of the non hand-to-mouth household $C^n\widehat{C}_t^n - \frac{D^h}{h^n}\widehat{D}_t^h = \frac{(1-\tau)WL}{h^n}\left(\widehat{W}_t + \widehat{L}_t\right) - \frac{RD^h}{h^n}\left(\widehat{R}_{t-1} + \widehat{D}_{t-1}^h\right) + T\widehat{T}_t$ and long-term bonds clearing condition 57 to get

$$\left(Y_C - \delta K_C - \varphi_2^h + \frac{\varphi_1^h(1-\tau)WL}{h^nT}\right)\widehat{Y}_t = I\widehat{I}_t + \left(K_C\delta + \varphi_2^h - \frac{\varphi_1^h}{h^nT}(1-\tau)WL\right)\left(\widehat{P}_t^m + \widehat{\mu}_t\right) +$$
$$\left(h_C^n + \varphi_3^h\right)\widehat{C}_t^n + \frac{\varphi_1^h}{h^nT}\left[-q^BB^m\left(\widehat{q}_t^B + \widehat{B}_t^m\right) + RD^h\widehat{R}_{t-1} + Rq^BB^m\left(\widehat{q}_{t-1}^B + \widehat{B}_{t-1}^m\right)\right] \qquad (72)$$

where $\varphi_3^h = h_C^w\frac{C^n}{C^w} + h_C^p\frac{C^n}{C^p}$

Furthermore I can combine equation 31, 44, 46 and 47 to get the consumption of non hand-to-mouth household is

$$\widehat{C}_t^n = -\frac{\psi}{\sigma}\widehat{Y}_t + \left(\frac{1}{\sigma} + \frac{(1+\psi)\alpha}{(1-\alpha)\sigma}\right)\widehat{P}_t^m - \frac{\psi}{\sigma}\widehat{\mu}_t + \frac{1+\psi}{(1-\alpha)\sigma}\widehat{\gamma}_t^{\mathrm{TFP}} - \frac{(1+\psi)\alpha\nu}{(1-\alpha)\sigma}\widehat{U}_t \qquad (73)$$

Then plug above equation back into equation 72

$$\left(Y_C - \delta K_C - \varphi_2^h + \frac{\varphi_1^h(1-\tau)WL}{h^nT} + \left(h_C^n + \varphi_3^h\right)\frac{\psi}{\sigma}\right)\widehat{Y}_t = I\widehat{I}_t + \left(K_C\delta + \varphi_2^h - \frac{\varphi_1^h}{h^nT}(1-\tau)WL\right)$$
$$\left(\widehat{P}_t^m + \widehat{\mu}_t\right) + \frac{\varphi_1^h}{h^nT}\left[-q^BB^m\left(\widehat{q}_t^B + \widehat{B}_t^m\right) + RD^h\widehat{R}_{t-1} + Rq^BB^m\left(\widehat{q}_{t-1}^B + \widehat{B}_{t-1}^m\right)\right] + \left(h_C^n + \varphi_3^h\right)$$
$$\left[\left(\frac{1}{\sigma} + \frac{(1+\psi)\alpha}{(1-\alpha)\sigma}\right)\widehat{P}_t^m - \frac{\psi}{\sigma}\widehat{\mu}_t + \frac{1+\psi}{(1-\alpha)\sigma}\widehat{\gamma}_t^{\mathrm{TFP}} - \frac{(1+\psi)\alpha\nu}{(1-\alpha)\sigma}\widehat{U}_t\right] \qquad (74)$$

If I assume that the investment is a constant fraction of output and fixed price, then there will exist $\widehat{R}_{t-1} = \widehat{P}_t^m = \widehat{\mu}_t = 0$. From equation 74 you can see the coefficient $\frac{Y_C - \delta K_C - \varphi_2^h + \frac{\varphi_1^h(1-\tau)WL}{h^nT} + \left(h_C^n + \varphi_3^h\right)\frac{\psi}{\sigma}}{\frac{\varphi_1^h}{h^nT}q^BB^m}$ is just the redistribution effect of the unconventional monetary policy. The hand-to-mouth household generate a multiplier of the monetary policy through additionally terms $-\varphi_2^h + \frac{\varphi_1^h(1-\tau)WL}{h^nT} + \left(h_C^n + \varphi_3^h\right)\frac{\psi}{\sigma}$ and $\frac{\varphi_1^h}{h^nT}$ which is firstly mentioned by

Bilbiie (2020).

By the same logic I can also combine the non hand-to-mouth household's budget constraint and central bank's budget constraint 56 and 57 which yields

$$
\begin{aligned}
C^n \widehat{C}^n_t - \frac{q^B B^m}{h^n}\left(\widehat{q}^B_t + \widehat{B}^m_t\right) = \frac{(1-\tau)WL}{h^n}\left(\widehat{Y}_t + \widehat{P}^m_t + \widehat{\mu}_t\right) - \left(\frac{1}{h^n} - 1\right) RD^h \widehat{R}_{t-1} \\
- \left(\frac{1}{h^n} - 1\right) Rq^B B^m \left(\widehat{q}^B_{t-1} + \widehat{B}^m_{t-1}\right) + RD^m \left(\widehat{R}_{t-1} + \widehat{D}^m_{t-1}\right) \\
- D^m \widehat{D}^m_t - \left(1 + \rho q^B\right) B^m \left(\frac{\rho q^B}{1 + \rho q^B}\widehat{q}^B_t + \widehat{B}^m_{t-1} - \widehat{\Pi}_t\right)
\end{aligned}
$$

To eliminate the money that financial institutions borrow from t he central bank $\widehat{D}^m_t$ I plug equation 33 and 42 into above equation to get

$$
\begin{aligned}
C^n \widehat{C}^n_t = &\frac{(1-\tau)WL}{h^n}\left(\widehat{Y}_t + \widehat{P}^m_t + \widehat{\mu}_t\right) - \left(\frac{1}{h^n} - 1\right) RD^h \widehat{R}_{t-1} \\
&- \left(\frac{1}{h^n} - 1\right) Rq^B B^m \left(\widehat{q}^B_{t-1} + \widehat{B}^m_{t-1}\right) + RD^m \left(\widehat{R}_{t-1} + \widehat{D}^m_{t-1}\right) - \left(1 - \frac{1}{\phi}\right) QK\left(\widehat{Q}_t + \widehat{K}_t\right) \\
&- N^h \widehat{\phi}_t + \left(\frac{1}{h^n} - 1 + \frac{\lambda^b}{\phi}\right) q^B B^m \left(\widehat{q}^B_t + \widehat{B}^m_t\right) - \left(1 + \rho q^B\right) B^m \left(\frac{\rho q^B}{1 + \rho q^B}\widehat{q}^B_t + \widehat{B}^m_{t-1} - \widehat{\Pi}_t\right)
\end{aligned}
$$

Plugging equation 70 and 74 into equation above equation yields

$$
\begin{aligned}
-C^n \frac{\psi}{\sigma}\widehat{Y}_t = &-C^n \left(\frac{1}{\sigma} + \frac{(1+\psi)\alpha}{(1-\alpha)\sigma}\right)\widehat{P}^m_t + C^n \frac{\psi}{\sigma}\widehat{\mu}_t - C^n \frac{1+\psi}{(1-\alpha)\sigma}\widehat{\gamma}^{\text{TFP}}_t + C^n \frac{(1+\psi)\alpha\nu}{(1-\alpha)\sigma}\widehat{U}_t \\
&+ \frac{(1-\tau)WL}{h^n}\left(\widehat{Y}_t + \widehat{P}^m_t + \widehat{\mu}_t\right) - \left(\frac{1}{h^n} - 1\right) RD^h \widehat{R}_{t-1} \\
&- \left(\frac{1}{h^n} - 1\right) Rq^B B^m \left(\widehat{q}^B_{t-1} + \widehat{B}^m_{t-1}\right) + RD^m \left(\widehat{R}_{t-1} + \widehat{D}^m_{t-1}\right) - \left(1 - \frac{1}{\phi}\right) QK\left(\widehat{Q}_t + \widehat{K}_t\right) \\
&+ N^h \frac{1 - \theta^m + \theta^m \eta}{\theta^m}\lambda \frac{R^B}{R^B - R}\widehat{R}^k_t + N^h \widehat{\gamma}^\lambda_t \\
&- N^h \frac{1 - \theta^m + \theta^m \eta}{\theta^m}\left[\frac{\zeta}{\phi\lambda^\nu}\left(\widehat{\phi}_{t-1} + \widehat{\gamma}^\lambda_{t-1} - \widehat{R}_{t-1}\right) + \lambda\widehat{\gamma}^\lambda_{t-1} + \lambda\frac{R}{R^B - R}\widehat{R}_{t-1} - \widehat{\Lambda}_t\right] \\
&+ \left(\frac{1}{h^n} - 1 + \frac{\lambda^b}{\phi}\right) q^B B^m \left(\widehat{q}^B_t + \widehat{B}^m_t\right) - \left(1 + \rho q^B\right) B^m \left(\frac{\rho q^B}{1 + \rho q^B}\widehat{q}^B_t + \widehat{B}^m_{t-1} - \widehat{\Pi}_t\right)
\end{aligned}
$$

This equation can set the relationship between unconventional monetary policy and real output by combining equation 45, 49, 69 and 72

$$
\begin{aligned}
0 = & -C^n\left(\frac{1}{\sigma}+\frac{(1+\psi)\alpha}{(1-\alpha)\sigma}\right)\widehat{P}_t^m + C^n\frac{\psi}{\sigma}\widehat{\mu}_t - C^n\frac{1+\psi}{(1-\alpha)\sigma}\widehat{\gamma}_t^{\mathrm{TFP}} + C^n\frac{(1+\psi)\alpha\nu}{(1-\alpha)\sigma}\widehat{U}_t \\
& + \left(C^n\frac{\psi}{\sigma}+\frac{(1-\tau)WL}{h^n}+\varphi_1^m\varphi_4^h\right)\widehat{Y}_t + \frac{(1-\tau)WL}{h^n}\left(+\widehat{P}_t^m+\widehat{\mu}_t\right) - \left(\frac{1}{h^n}-1\right)RD^h\widehat{R}_{t-1} \\
& - \left(\frac{1}{h^n}-1\right)Rq^BB^m\left(\widehat{q}_{t-1}^B+\widehat{B}_{t-1}^m\right) + RD^m\left(\widehat{R}_{t-1}+\widehat{D}_{t-1}^m\right) - \left(1-\frac{1}{\phi}\right)QK\left(\widehat{\xi}_t+\widehat{K}_{t-1}\right) \\
& - \left(N^h\frac{1-\theta^m+\theta^m\eta}{\theta^m}\lambda\frac{R^k}{R^k-R}\frac{1}{KQ+\Pi^f}-1+\frac{1}{\phi}\right)K\varphi_I\left(1-\beta\Lambda\right)\widehat{I}_{t-1} \\
& - \varphi_1^m\left[\left(K_C\delta+\varphi_2^h-\frac{\varphi_1^h}{h^nT}\left(1-\tau\right)WL\right)\left(\widehat{P}_t^m+\widehat{\mu}_t\right)+\frac{\varphi_1^h}{h^nT}\left[RD^h\widehat{R}_{t-1}+Rq^BB^m\left(\widehat{q}_{t-1}^B+\widehat{B}_{t-1}^m\right)\right]\right. \\
& + \left(h_C^n+\varphi_3^h\right)\left.\left[\left(\frac{1}{\sigma}+\frac{(1+\psi)\alpha}{(1-\alpha)\sigma}\right)\widehat{P}_t^m-\frac{\psi}{\sigma}\widehat{\mu}_t+\frac{1+\psi}{(1-\alpha)\sigma}\widehat{\gamma}_t^{\mathrm{TFP}}-\frac{(1+\psi)\alpha\nu}{(1-\alpha)\sigma}\widehat{U}_t\right]\right] \\
& + N^h\frac{1-\theta^m+\theta^m\eta}{\theta^m}\lambda\frac{R^k}{R^k-R}\left(\widehat{\xi}_t-\widehat{Q}_{t-1}+\frac{\Pi^f}{KQ+\Pi^f}\widehat{\delta}_t\right)+N^h\widehat{\gamma}_t^\lambda \\
& - N^h\frac{1-\theta^m+\theta^m\eta}{\theta^m}\left[\frac{\zeta}{\phi\lambda^\nu}\left(\widehat{\phi}_{t-1}+\widehat{\gamma}_{t-1}^\lambda-\widehat{R}_{t-1}\right)+\lambda\widehat{\gamma}_{t-1}^\lambda+\lambda\frac{R}{R^B-R}\widehat{R}_{t-1}-\widehat{\Lambda}_t\right] \\
& + \left(\frac{1}{h^n}-1+\frac{\lambda^b}{\phi}+\varphi_1^m\frac{\varphi_1^h}{Th^n}\right)q^BB^m\left(\widehat{q}_t^B+\widehat{B}_t^m\right)-\left(1+\rho q^B\right)B^m\left(\frac{\rho q^B}{1+\rho q^B}\widehat{q}_t^B+\widehat{B}_{t-1}^m-\widehat{\Pi}_t\right)
\end{aligned}
$$

where $\varphi_1^m = \left(N^h\frac{1-\theta^m+\theta^m\eta}{\theta^m}\lambda\frac{R^k}{R^k-R}\frac{1}{KQ+\Pi^f}-1+\frac{1}{\phi}\right)\frac{\varphi_I}{\delta}\left(1-\beta\Lambda\right)-\left(1-\frac{1}{\phi}\right)Q$ and $\varphi_4^h = Y_C - \delta K_C - \varphi_2^h + \frac{\varphi_1^h(1-\tau)WL}{h^nT} + \left(h_C^n+\varphi_3^h\right)\frac{\psi}{\sigma}$.

When the price and depreciation are fixed I will get the unconventional monetary policy effect to the output as$-\frac{\frac{1}{h^n}-1+\frac{\lambda^b}{\phi}+\varphi_1^m\frac{\varphi_1^h}{Th^n}}{C^n\frac{\psi}{\sigma}+\frac{(1-\tau)WL}{h^n}+\varphi_1^m\varphi_4^h}q^BB^m$ and $\frac{\rho}{C^n\frac{\psi}{\sigma}+\frac{(1-\tau)WL}{h^n}+\varphi_1^m\varphi_4^h}q^BB^m$.

## A.4 Calibration for production sector

All the parameters related to production sector are selected from literature. The possibility of retailer to retain price is $0.85$ which is a balance within $0.75$ of Sims et al. (2022), 0.779 of Gertler and Karadi (2011) and $0.92$ of Karadi and Nakov (2021). The elasticity of substitution and capital production cost are directly borrowed from Gertler and Karadi (2011). The utilization elasticity of marginal depreciation rate is estimated by Christiano et al. (2014).

Table 3: Parameter Values

| Parameter | Value | Description |
|---|---|---|
| $\theta$ | 0.85 | Possibility to retain price |
| $\sigma_p$ | 4.167 | Inverse elasiticity of substitution between different intermediate goods |
| $\gamma_p$ | 0 | Parameter for retained price that grow as aggregate inflation |
| $\alpha$ | 0.36 | Capital share in production function |

| Table 3 – Continued | | |
|---|---|---|
| **Parameter** | **Value** | **Description** |
| $\nu$ | 2.54 | utilization elasticity of marginal depreciation rate |
| $\overline{\delta}$ | 0.025 | Depreciation rate in ss |
| $\psi_I$ | 1.728 | Capital production cost |
| $\overline{\xi}$ | 1.000 | Capital effectiveness in ss |
| $\theta_r^{QE}$ | 0.92 | AR1 coefficient of QE policy rule |
| $\overline{u}$ | 1.000 | utilizebar |

# B Extensions

## B.1 Endogenous illiquid asset withdrawing

In baseline model the illquid asset withdrawing $X_t^{\mathrm{nHtM}}$ and $X_t^{\mathrm{wHtM}}$ are exogenous and fixed at steady state for conventional purpose as it shuts down the substitution effect between liquid and illiquid asset in liquidity and interest rate channel. To illustrate that the baseline model is good enough to unveil the two channels of unconventional monetary policy, I change the exogenous illiquid asset withdrawing to endogenous following the extension that Cui and Sterk (2021) did. Either wealthy hand-to-mouth household or non-hand-to-mouth household now will select the their optimal illiquid asset withdrawing $X_t^i$ because of an adjustment friction in utility base. Therefore the marginal cost of altering the portfolio of investment should be equal to the marginal benefit (which is the marginal consumption in this setting) as equation 75 shows.

$$\gamma_1 + \frac{X_t^{\gamma_3}}{\gamma_2^{1+\gamma_3}} = C_t^{-\sigma} \tag{75}$$

The adjustment cost function is standard and proposed by Kaplan et al. (2018) where $\gamma_1$ is the extend of friction in linear part and $\gamma_3$ governs the curvature of the cost function which in fact is the nonlinear part. $\gamma_2$ works as a resealing factor. Following Cui and Sterk (2021) I set $\gamma_2 = 1.5$ and $\gamma_3 = 5$ which are not unique to the result and other value will also work. Then the linear part $\gamma_1$, which is the least important and does not display in linearization system, is pinned down by matching the same steady state of consumption and illquid asset withdrawing in baseline model.

Figure 6 shows that the endogeneity of illiquid asset withdrawing amplifies the power of unconventional monetary policy yet the magnitude is not large. In addition to that all the patterns and directions between the exogenous and endogenous model are same which demonstrates that the baseline model works well at uncovering the two channels.

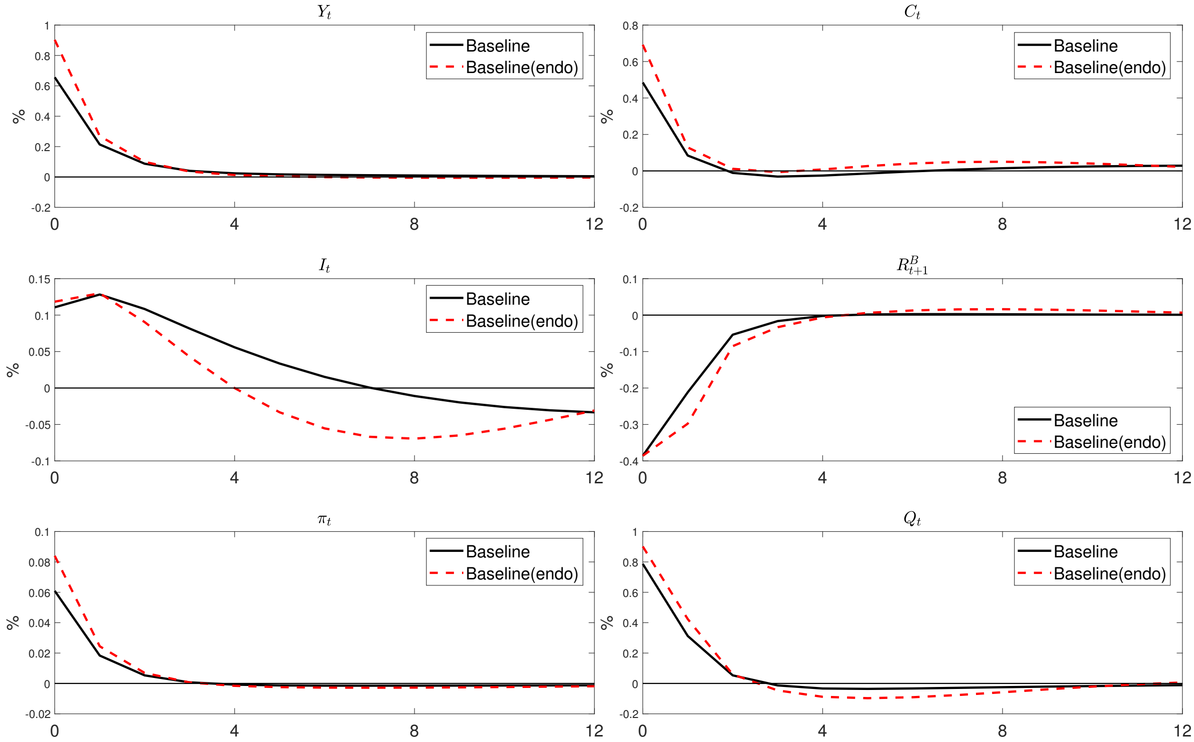


Figure 6: Exogenous vs Endogenous Illiquid asset withdrawing

### B.2 Model vs Empirical result

## C Data

### C.1 Data used in VAR estimation

### C.2 Data used in Model Calibration

## D Robustness check

### D.1 Off-diagonal zero restriction

Since I want to identify the liquidity channel and interest rate channel of monetary policy, I can write $\boldsymbol{\varepsilon}'_{1t}$ as $\boldsymbol{\varepsilon}'_{1t} = \left[ \begin{array}{cc} \varepsilon^s_{1t} & \varepsilon^d_{1t} \end{array} \right]'$ where $\varepsilon^s_{1t}$ and $\varepsilon^d_{1t}$ denote the liquidity shock and demand shock.

I made a restriction on $\boldsymbol{\Phi}$ and assume it is a diagonal matrix such that

$$\boldsymbol{\Phi} = \left[ \begin{array}{cc} \alpha^s & 0 \\ 0 & \alpha^d \end{array} \right] \tag{76}$$

The meaning of this restriction is that the instrument variables do not have cross effect on different shock.

For

$$\mathbf{m}_t = \left[ \begin{array}{cc} m^s_t & m^d_t \end{array} \right]'$$

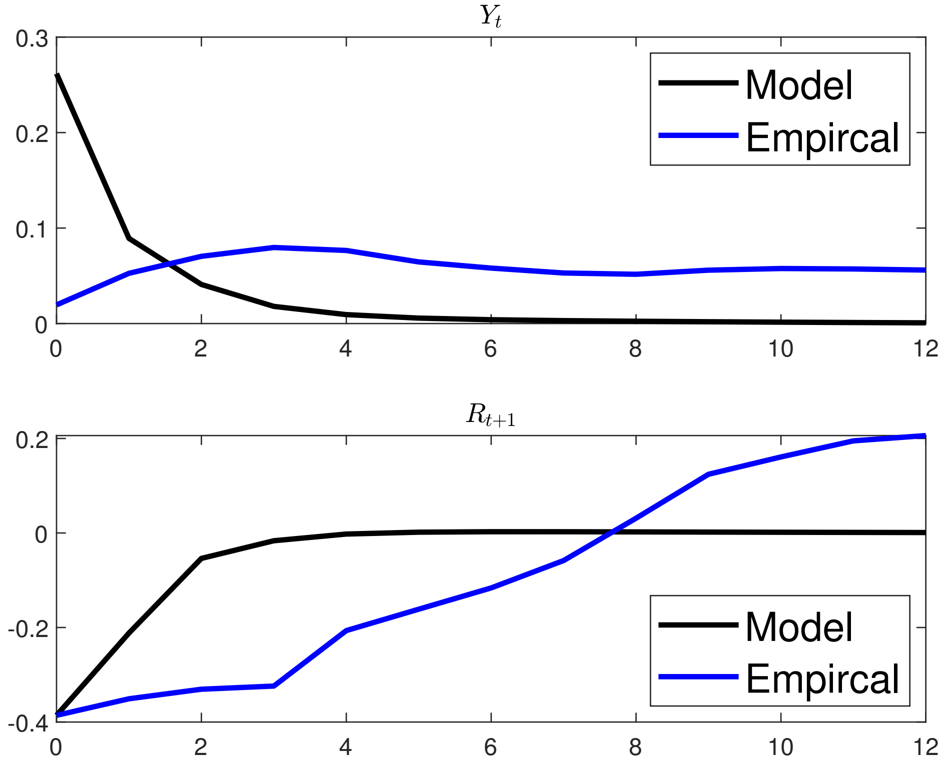


Figure 7

we would have the assumption $\operatorname{cov}\left(m_t^s, \varepsilon_{1t}^d\right) = 0$ and $\operatorname{cov}\left(m_t^d, \varepsilon_{1t}^s\right) = 0$. This assumption can be true as long as I find appropriate instrument variables. Similar to the high frequency identification literature, the instrument helps to identify the effect of liquidity channel of monetary policy is the change of future contract price during the treasury bonds issuing announcement day. The instrument helps to identify the effect of interest rate channel of monetary policy is the change of future contract price during the FOMC announcement day.

Write the covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{mu'}}$ into partition

$$\boldsymbol{\Sigma}_{\boldsymbol{mu'}} = \begin{bmatrix} \Sigma_{mu'_{11}} & \boldsymbol{\Sigma}_{\boldsymbol{mu'}_{12}} \\ \Sigma_{mu'_{21}} & \boldsymbol{\Sigma}_{\boldsymbol{mu'}_{22}} \end{bmatrix}$$

and plug into assumption 76 to get

$$\boldsymbol{\beta}_{21} = \left(\Sigma_{mu'_{11}}^{-1} \boldsymbol{\Sigma}_{\boldsymbol{mu'}_{21}}\right) \beta_{11}$$

and

$$\boldsymbol{\beta}_{22} = \left(\Sigma_{mu'_{12}}^{-1} \boldsymbol{\Sigma}_{\boldsymbol{mu'}_{22}}\right) \beta_{12}$$

because $\boldsymbol{\Phi}$ is a diagonal matrix and I can easily subtract $\alpha^s$ and $\alpha^d$ in each row of $\boldsymbol{\Phi}\boldsymbol{\beta}_1'$ out separately trough each row of $\boldsymbol{\Sigma}_{\boldsymbol{mu'}}$

Use the method proposed by Gertler and Karadi (2015) I can easily identify $\boldsymbol{\beta}_{21}$ and $\beta_{11}$. Additionally it is easy to prove that for $\boldsymbol{\beta}_{22}$ and $\beta_{12}$, $\boldsymbol{\beta}_{22}\beta_{12}^{-1}$ can be estimated by estimating

$\left(\Sigma_{mu'_{21}}^{-1}\boldsymbol{\Sigma}_{\boldsymbol{mu}'_{22}}\right)$. Then we can solve $\beta_{12}$ up to a sign convention that[45]

$$\beta_{12}^2 = b_{12}^2 = \Sigma_{11} - \left(b_{11}^2 + \boldsymbol{b}_{13}\boldsymbol{b}'_{13}\right) \tag{77}$$

where

$$\boldsymbol{b}_{13}\boldsymbol{b}'_{13} = \left(\boldsymbol{\Sigma}_{31} - \frac{\boldsymbol{b}_{32}}{b_{12}}\Sigma_{11} - \boldsymbol{b}_{31}b_{11} + \frac{\boldsymbol{b}_{32}}{b_{12}}b_{11}^2\right)' D^{-1}\left(\boldsymbol{\Sigma}_{31} - \frac{\boldsymbol{b}_{32}}{b_{12}}\Sigma_{11} - \boldsymbol{b}_{31}b_{11} + \frac{\boldsymbol{b}_{32}}{b_{12}}b_{11}^2\right)$$

and

$$\begin{aligned} D = \boldsymbol{\Sigma}_{33} + \frac{\boldsymbol{b}_{32}}{b_{12}}\Sigma_{11}\frac{\boldsymbol{b}'_{32}}{b_{12}} - \boldsymbol{b}_{31}\boldsymbol{b}'_{31} - \frac{\boldsymbol{b}_{32}}{b_{12}}b_{11}^2\frac{\boldsymbol{b}'_{32}}{b_{12}} - \\ \left(\frac{\boldsymbol{b}_{32}}{b_{12}}\boldsymbol{\Sigma}'_{31} + \boldsymbol{\Sigma}_{31}\frac{\boldsymbol{b}'_{32}}{b_{12}} - \frac{\boldsymbol{b}_{32}}{b_{12}}\boldsymbol{b}'_{31}b_{11} - \boldsymbol{b}_{31}b_{11}\frac{\boldsymbol{b}'_{32}}{b_{12}}\right) \end{aligned}$$

The estimation result is shown below.

[45]Because of space limit, I degraded related proof process to appendix.

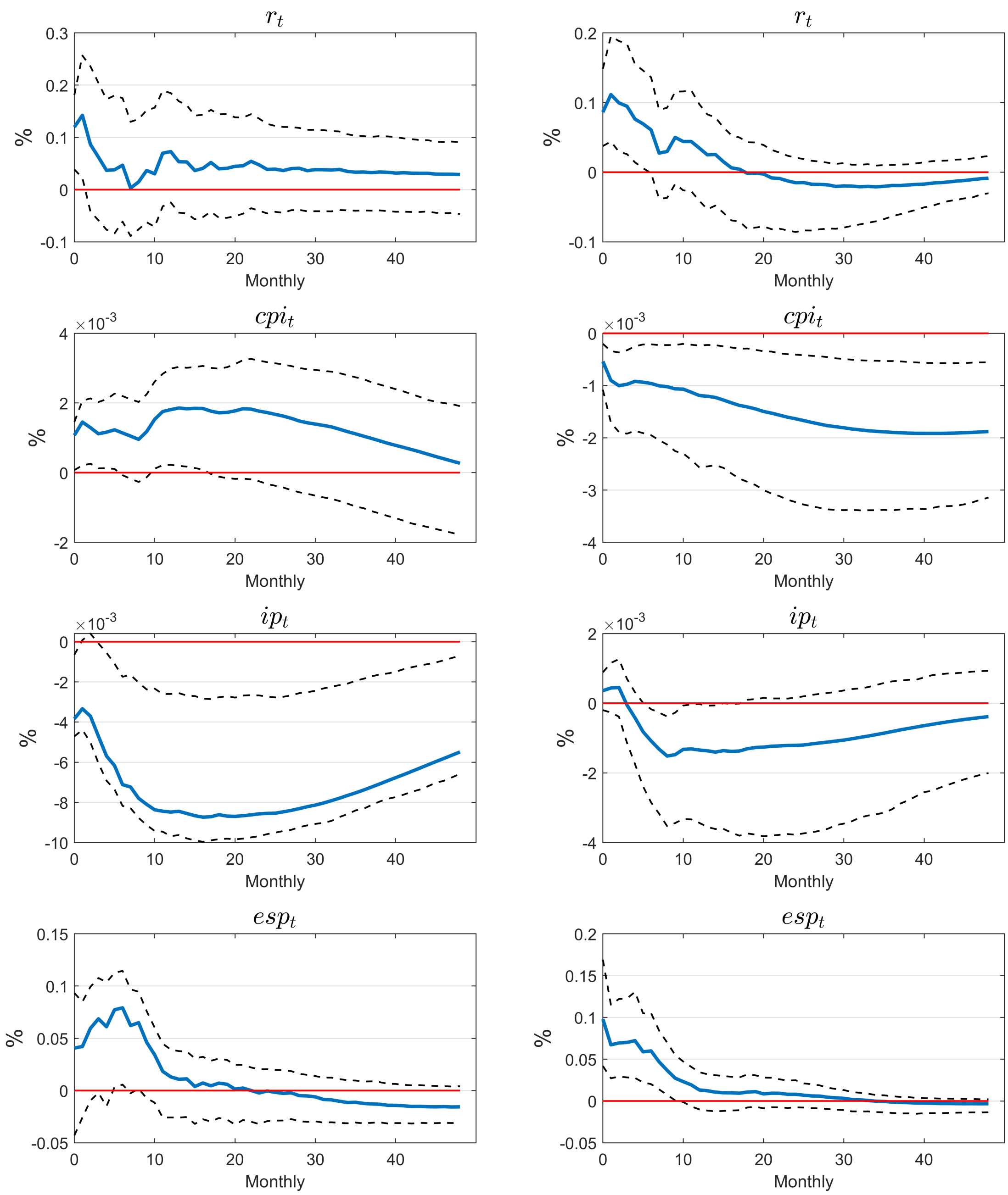


Figure 8: Liquidity and interest rate channel

The left column is related to liquidity channel of QE. The monetary authority stimulates the economy through changing the long-term bonds rate via providing liquidity and buying long-term bonds. The right column is related to interest rate channel of QE. The monetary authority stimulates the economy through changing the long-term bonds rate(people's expectation about future interest rate) directly via announcement but does not provide any liquidity to the market.

## D.2 Lower triangle restriction

Alternatively I can only impose the triangle restriction on matrix $\mathbf{\Phi}$[46] such that the matrix $\mathbf{\Phi}$ is in the form

$$\mathbf{\Phi} = \begin{bmatrix} \alpha^s & 0 \\ \alpha^p & \alpha^d \end{bmatrix}$$

Similar to use the restriction 76, I can estimate $b_{11}$, $b_{21}$ and $\boldsymbol{b}_{31}$ by ruling out $\alpha^s$ using instrument $m_t^s$. While it is a little bit trivial to estimate $b_{12}$, $b_{22}$ and $\boldsymbol{b}_{32}$ and I provide the detailed process in another appendix section.

[46]It is not freely to impose this restriction relative to the zero on off-diagonal element as I add one more unknown. This cause the sign $b_{22}$ undetermined. Therefore I put this restriction on the robustness check.

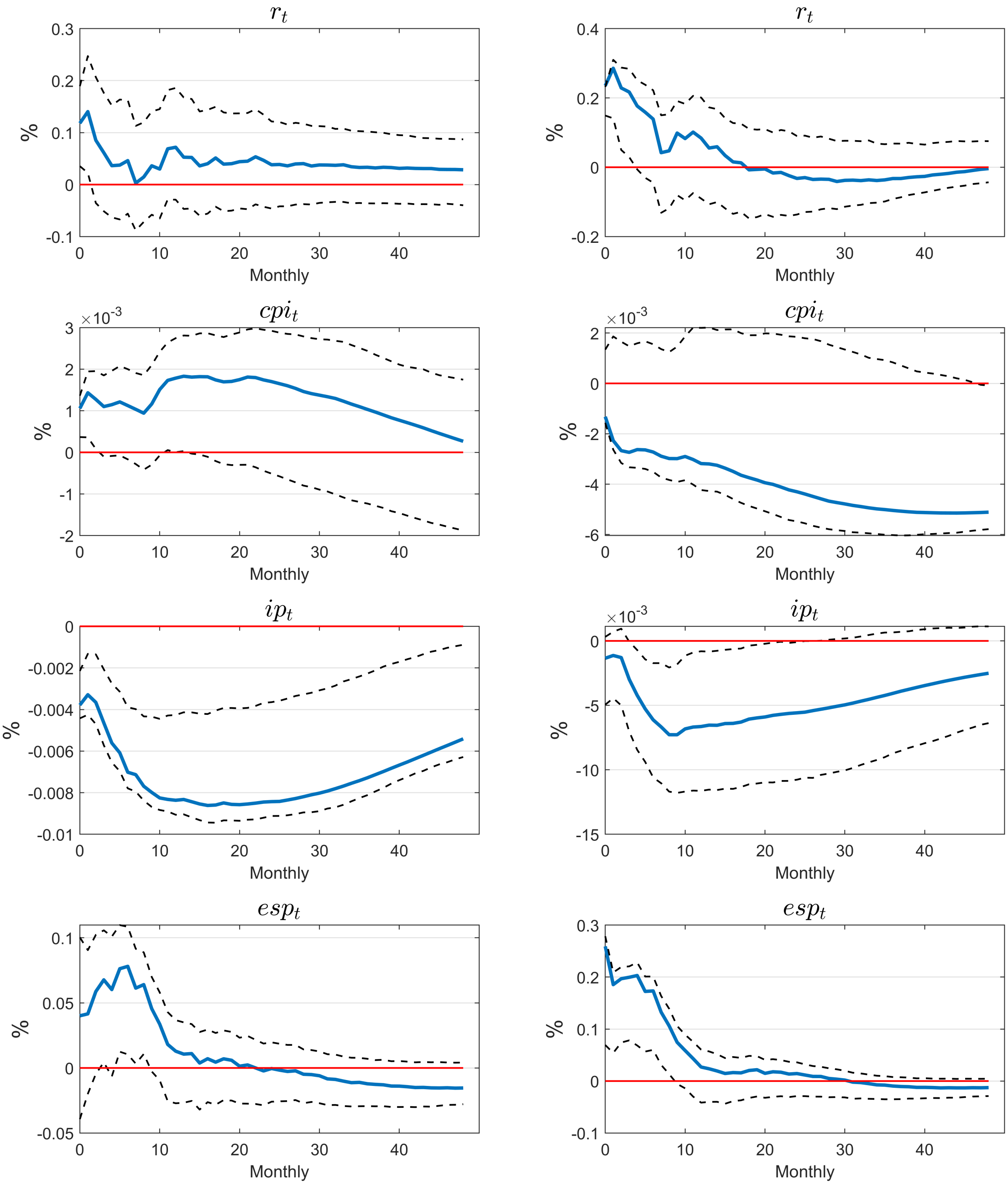


Figure 9: Liquidity and interest rate channel

## D.3 Uniform prior

Alternative I can identify $\beta_1$ through inequality restriction and bayesian estimation. Because of equation 24, when I only want to identify $\beta_1$ and do not care about $\beta_2$, it could be fully identified as long as $\Phi$ is know. Because of the constraint of freedom, when $k = 2$ only half of $\Phi$

can be identified as long as I do not want impose any restriction more on $\boldsymbol{\beta}_1$. Inspired by the half-information identification problem, I can use the standard and canonical method that are used to identify the $\boldsymbol{B}$ matrix[47] which is already widely developed. Firstly it is easy and valuable to notice that

$$\boldsymbol{\Phi}\boldsymbol{\Phi}' = \boldsymbol{\Sigma}_{\boldsymbol{mu}_1'} \left( \boldsymbol{\Sigma}_{\boldsymbol{mu}_1'}'^{-1} \boldsymbol{s}_{11} \boldsymbol{s}_{11}' \right)^{-1}$$

Then taking the Cholesky decomposition on $\boldsymbol{\Phi}\boldsymbol{\Phi}'$ yields the lower triangle matrix $\boldsymbol{\Phi}_{tr}$. $\boldsymbol{\Phi}$ will be identified up to the rotation matrix $Q$ such that $\boldsymbol{\Phi} = \boldsymbol{\Phi}_{tr} Q$.

Therefore I can impose inequality constraint to identify $\boldsymbol{\Phi}$(then the $\boldsymbol{\beta}_1$ is identified through equation 24), similar to the sign restriction proposed by Uhlig (2005). Here I use the method proposed by Rubio-Ramirez et al. (2010) with uniform prior of $Q$ to identify[48].

Draw 1000 times.

- $\kappa_1 = \kappa_2 = 0$

[47] Since in general we only have half information of matrix $B$ which is the covariance matrix $BB'$.

[48] Baumeister and Hamilton (2015) argued that using the uniform prior distribution will result in a Cauchy distribution.

$\gamma_1 = \gamma_2 = 1$

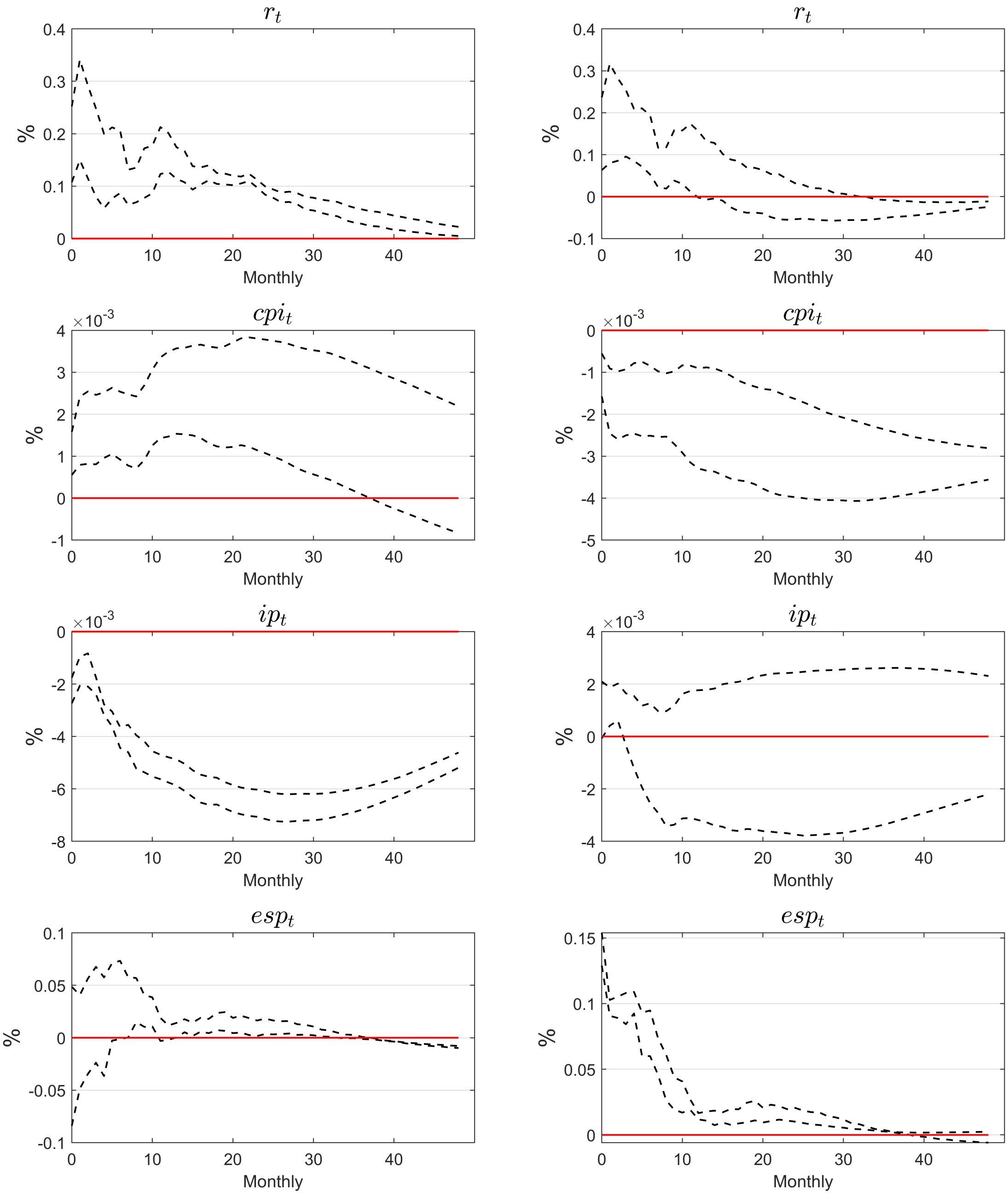


Figure 10: Liquidity and interest rate channel

- $\kappa_1 = 1$ but $\kappa_2 = 0.9$

$\gamma_1 = 1, \gamma_2 = 10$

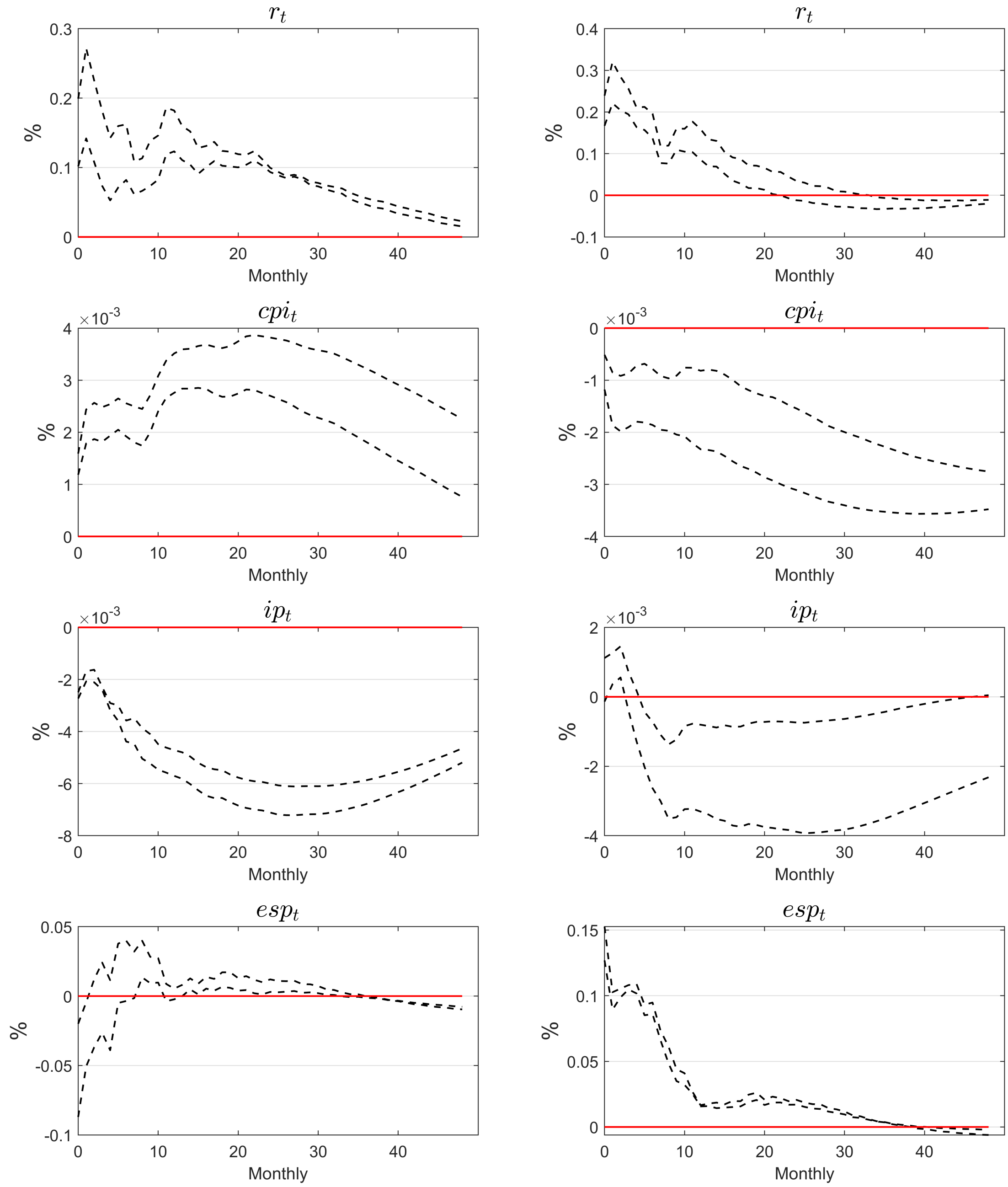


Figure 11: Liquidity and interest rate channel

## D.4 Bayesian estimation using different drawing algorithm

This section presents the Bayesian estimation of VAR under more conservative algorithm **??** and **??**.

Draw 1000 times with 7000 burn-in try. 90% confidence band.

Algorithm **??**:

- $\kappa_1 = \kappa_2 = 0$

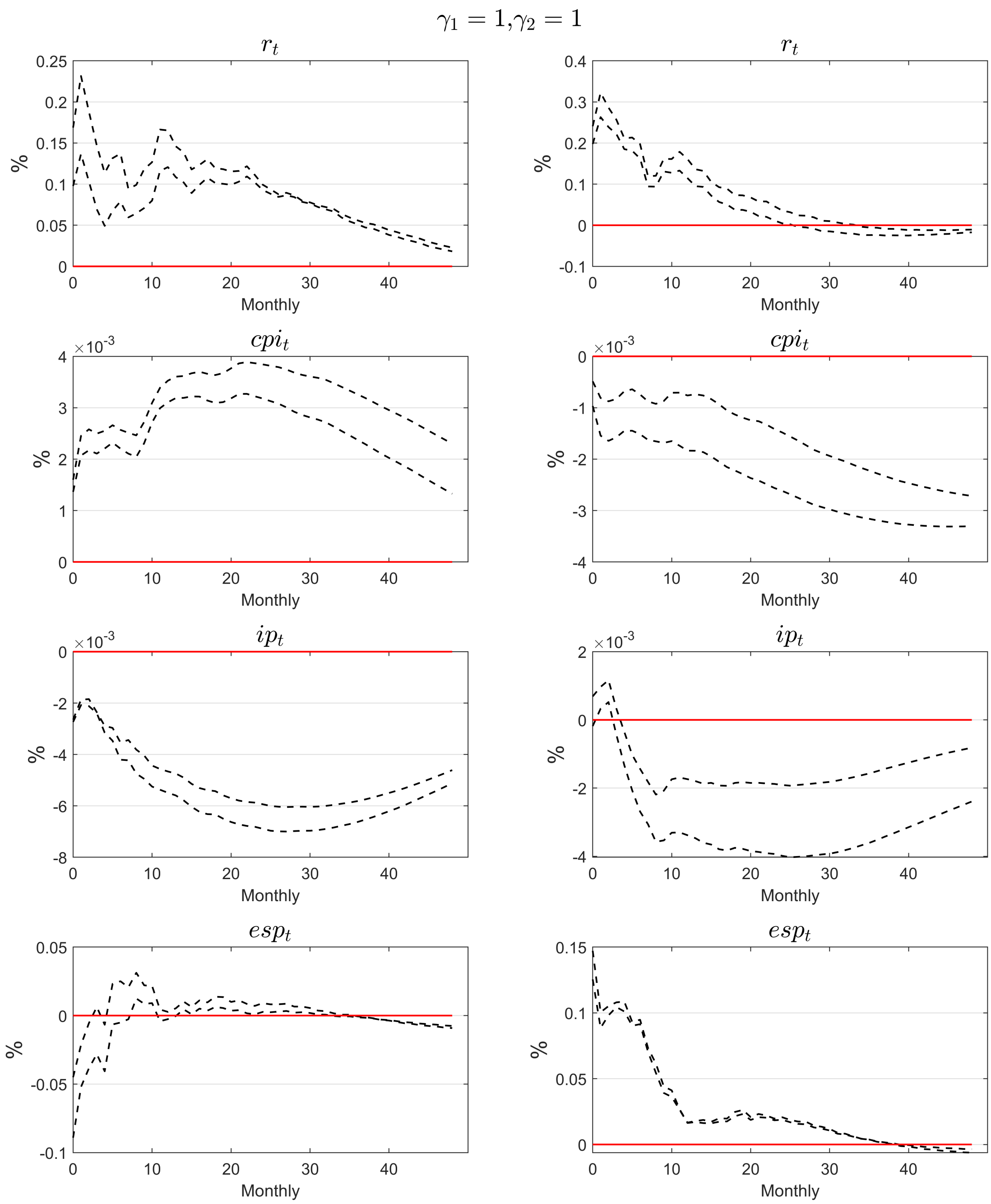


Figure 12: Liquidity and interest rate channel

Algorithm **??**:

- $\kappa_1 = \kappa_2 = 0$

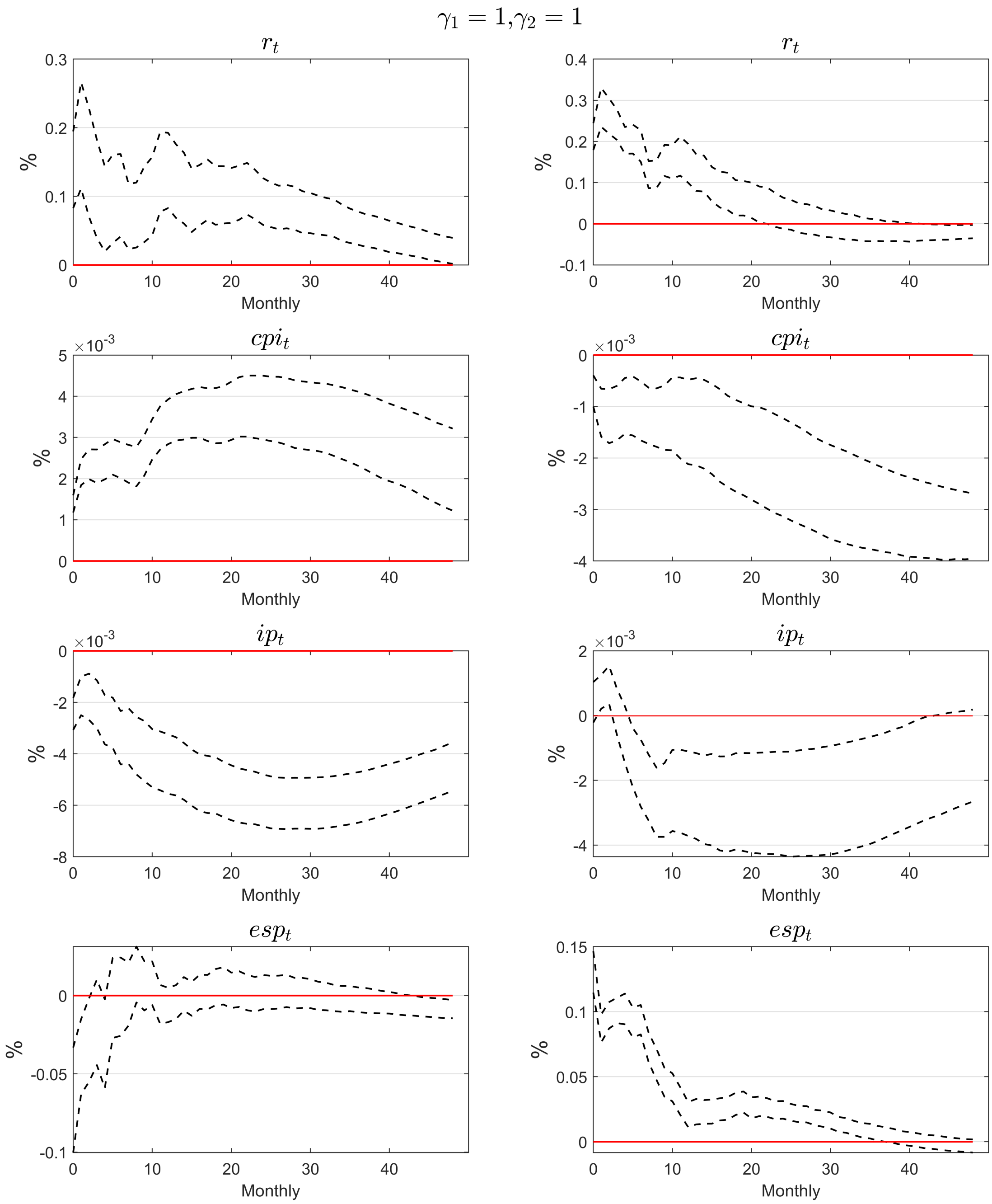


Figure 13: Liquidity and interest rate channel

# E Derivative process related to IV-VAR

## E.1 Estimation of coefficient under off-diagonal zero restriction

To the coefficient parameters $\beta_{11}$ and $\beta_{12}$ they can be directly estimated by the method used in literature because I assume the coefficient matrix of instrument variables is off-diagonal zero. Therefore the first instrument will only be correlated with the first shock, a scenario where Gertler and Karadi (2015) considered. Therefore $\beta_{11}$ can be estimated up to sign convention such that

$$\beta_{11}^2 = \Sigma_{11} - \beta_{12}\mathbf{fi}_{12}'$$

where

$$\beta_{12}\mathbf{fi}_{12}' = \left(\Sigma_{21} - \frac{\mathbf{fi}_{21}}{\beta_{11}}\Sigma_{11}\right)' \mathbf{Q}^{-1} \left(\Sigma_{21} - \frac{\mathbf{fi}_{21}}{\beta_{11}}\Sigma_{11}\right)$$

and

$$\mathbf{Q} = \frac{\mathbf{fi}_{21}}{\beta_{11}}\Sigma_{11}\frac{\mathbf{fi}_{21}'}{\beta_{11}} - \left(\Sigma_{21}\frac{\mathbf{fi}_{21}'}{\beta_{11}} + \frac{\mathbf{fi}_{21}}{\beta_{11}}\Sigma_{21}'\right) + \Sigma_{22}.$$

and $\widehat{\frac{\mathbf{fi}_{21}}{\beta_{11}}}$ comes from the coefficient of IV regression.

Similarly another instrument can be used to estimate $\beta_{12}$ and $\beta_{22}$ though the process becomes a little bit more complicated.

Firstly note that because $E\left(\boldsymbol{u}_t\boldsymbol{u}_t'\right) = E\left(\boldsymbol{B}\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'\boldsymbol{B}'\right) = \boldsymbol{B}E\left(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'\right)\boldsymbol{B}' = \boldsymbol{B}\boldsymbol{B}' = \boldsymbol{\Sigma}$ and $\boldsymbol{\beta}_{22}\beta_{12}^{-1}$ can be easily estimated, we can know that as long as $\boldsymbol{b}_{13}\boldsymbol{b}_{13}'$ is revealed, $\boldsymbol{\beta}_{22}$ and $\beta_{12}$ will be uncovered from equation 77. The steps below I show how to estimate $\boldsymbol{b}_{13}\boldsymbol{b}_{13}'$.

Now I construct a matrix such that

$$\boldsymbol{V} = \boldsymbol{b}_{33} - \frac{\boldsymbol{b}_{32}\boldsymbol{b}_{13}}{b_{11}}$$

It is easy to verify that

$$\boldsymbol{V}\boldsymbol{b}_{13}' = \boldsymbol{\Sigma}_{31} - \frac{\boldsymbol{b}_{32}}{b_{12}}\Sigma_{11} - \boldsymbol{b}_{31}b_{11} + \frac{\boldsymbol{b}_{32}}{b_{12}}b_{11}^2 \tag{78}$$

**Lemma 1.** *Matrix $\boldsymbol{V}$ is full rank.*

*Proof.* It is easy to construct a transformation matrix $\boldsymbol{A} = \begin{bmatrix} I & 0 & \mathbf{0} \\ 0 & I & \mathbf{0} \\ -\frac{\boldsymbol{b}_{32}}{b_{12}} & \mathbf{0} & \boldsymbol{I} \end{bmatrix}$.

Note that var $(\boldsymbol{A}\boldsymbol{u}_t) = \boldsymbol{A}\boldsymbol{\Sigma}\boldsymbol{A}'$ is full rank as $\boldsymbol{A}$ and $\boldsymbol{\Sigma}$are full rank. Further notice that

$$\boldsymbol{A}\boldsymbol{u}_t = \boldsymbol{A}\boldsymbol{B}\boldsymbol{\varepsilon}_t = \begin{bmatrix} b_{11} & b_{12} & \boldsymbol{b}_{13} \\ b_{21} & b_{22} & \boldsymbol{b}_{23} \\ -\frac{\boldsymbol{b}_{32}}{b_{12}}b_{11} + \boldsymbol{b}_{31} & \mathbf{0} & \boldsymbol{V} \end{bmatrix} \boldsymbol{\varepsilon}_t$$

Since var $(\boldsymbol{\varepsilon}_t) = \boldsymbol{I}$ is full rank, $\boldsymbol{AB}$ must be full rank. Therefore $\boldsymbol{V}$ must be full rank. $\square$

From above equation 78 we can know that

$$\boldsymbol{b}_{13}\boldsymbol{V}'\boldsymbol{V}\boldsymbol{b}_{13}' = \left(\boldsymbol{\Sigma}_{31} - \frac{\boldsymbol{b}_{32}}{b_{12}}\Sigma_{11} - \boldsymbol{b}_{31}b_{11} + \frac{\boldsymbol{b}_{32}}{b_{12}}b_{11}^2\right)'\left(\boldsymbol{\Sigma}_{31} - \frac{\boldsymbol{b}_{32}}{b_{12}}\Sigma_{11} - \boldsymbol{b}_{31}b_{11} + \frac{\boldsymbol{b}_{32}}{b_{12}}b_{11}^2\right)$$

Then we define $\boldsymbol{Q} = \boldsymbol{V}\boldsymbol{V}'$.

**Lemma 2.** *$\boldsymbol{Q}$ is full rank and $\boldsymbol{Q}^{-1}$ exists.*

*Proof.* From Lemma 1 we know that matrix $\boldsymbol{V}$ is a square matrix and is full rank, too. Therefore $\boldsymbol{Q}$ is a square matrix and full rank since demote$\boldsymbol{Q} = \boldsymbol{V}\boldsymbol{V}'$. Then we can yield the conclusion that $\boldsymbol{Q}^{-1}$ exists. $\square$

**Lemma 3.** *Given the $\boldsymbol{Q}$ we would have the relationship that $\boldsymbol{V}'\boldsymbol{Q}^{-1}\boldsymbol{V} = \boldsymbol{I}$.*

*Proof.* This is easy to prove. It is worth to notice that $\boldsymbol{V}'\boldsymbol{Q}^{-1}\boldsymbol{V}$ is just a projection matrix on the column space of $\boldsymbol{V}'$. Since $\boldsymbol{Q}$ and $\boldsymbol{V}$ are full rank, we must have a complete mapping which means $\boldsymbol{V}'\boldsymbol{Q}^{-1}\boldsymbol{V} = \boldsymbol{I}$. $\square$

Because of $\boldsymbol{V}'\boldsymbol{Q}^{-1}\boldsymbol{V} = \boldsymbol{I}$, we would have

$$\boldsymbol{b}_{13}\boldsymbol{b}_{13}' = \boldsymbol{b}_{13}\boldsymbol{V}'\boldsymbol{Q}^{-1}\boldsymbol{V}\boldsymbol{b}_{13}' = \left(\boldsymbol{\Sigma}_{31} - \frac{\boldsymbol{b}_{32}}{b_{12}}\Sigma_{11} - \boldsymbol{b}_{31}b_{11} + \frac{\boldsymbol{b}_{32}}{b_{12}}b_{11}^2\right)'\boldsymbol{Q}^{-1}\left(\boldsymbol{\Sigma}_{31} - \frac{\boldsymbol{b}_{32}}{b_{12}}\Sigma_{11} - \boldsymbol{b}_{31}b_{11} + \frac{\boldsymbol{b}_{32}}{b_{12}}b_{11}^2\right)$$

## E.2 Estimation of coefficient vectors under lower triangle assumption.

Write the partition of covariance $\boldsymbol{\Sigma}_{\boldsymbol{mu}'}$

$$\boldsymbol{\Sigma}_{\boldsymbol{mu}'} = \begin{bmatrix} \Sigma_{mu'_{11}} & \Sigma_{mu'_{12}} & \boldsymbol{\Sigma}_{\boldsymbol{mu}'_{13}} \\ \Sigma_{mu'_{21}} & \Sigma_{mu'_{22}} & \boldsymbol{\Sigma}_{\boldsymbol{mu}'_{23}} \end{bmatrix}$$

Based on $\boldsymbol{\Phi}\boldsymbol{\beta}_1' = \boldsymbol{\Sigma}_{\boldsymbol{mu}'}$ and $\boldsymbol{\Phi} = \begin{bmatrix} \alpha^s & 0 \\ \alpha^p & \alpha^d \end{bmatrix}$ I can write

$$\alpha^p \begin{bmatrix} b_{11} & b_{21} & \boldsymbol{b}_{31}' \end{bmatrix} + \alpha^d \begin{bmatrix} b_{12} & b_{22} & \boldsymbol{b}_{32}' \end{bmatrix} = \begin{bmatrix} \Sigma_{mu'_{21}} & \Sigma_{mu'_{22}} & \boldsymbol{\Sigma}_{\boldsymbol{mu}'_{23}} \end{bmatrix} \tag{79}$$

Using the first linear relationship I can rule out $\alpha^p$ as

$$\alpha^p = \Sigma_{mu'_{21}}\frac{1}{b_{11}} - \alpha^d\frac{b_{12}}{b_{11}} \tag{80}$$

Plugging above equation into equation 79 yields

$$\Sigma_{mu'_{21}}\frac{1}{b_{11}}\begin{bmatrix} b_{11} & b_{21} & \boldsymbol{b}_{31}' \end{bmatrix} + \alpha^d \begin{bmatrix} b_{12} - b_{12} & b_{22} - b_{12}\frac{b_{21}}{b_{11}} & \boldsymbol{b}_{32}' - b_{12}\frac{\boldsymbol{b}_{31}'}{b_{11}} \end{bmatrix} = \begin{bmatrix} \Sigma_{mu'_{21}} & \Sigma_{mu'_{22}} & \boldsymbol{\Sigma}_{\boldsymbol{mu}'_{23}} \end{bmatrix}$$

Then use the linear restriction at the second column to eliminate $\alpha^d$ as

$$\alpha^d = \frac{\Sigma_{mu'_{22}} - \Sigma_{mu'_{21}} \frac{b_{21}}{b_{11}}}{b_{22} - b_{12} \frac{b_{21}}{b_{11}}}$$

which will yield the final valuable restriction

$$\left( \Sigma_{mu'_{22}} - \Sigma_{mu'_{21}} \frac{b_{21}}{b_{11}} \right) \left[ \frac{\boldsymbol{b}'_{32} - b_{12} \frac{\boldsymbol{b}'_{31}}{b_{11}}}{b_{22} - b_{12} \frac{b_{21}}{b_{11}}} \right] = \left[ \boldsymbol{\Sigma}_{\boldsymbol{mu}'_{23}} - \Sigma_{mu'_{21}} \frac{\boldsymbol{b}'_{31}}{b_{11}} \right]$$

Since $\frac{\left[ \boldsymbol{\Sigma}_{\boldsymbol{mu}'_{23}} - \Sigma_{mu'_{21}} \frac{\boldsymbol{b}'_{31}}{b_{11}} \right]}{\Sigma_{mu'_{22}} - \Sigma_{mu'_{21}} \frac{b_{21}}{b_{11}}}$ is estimable, I can estimate $\left[ \begin{array}{ccc} b_{12} & b_{22} & \boldsymbol{b}'_{32} \end{array} \right]$ as long as given $b_{22}$ and $b_{12}$.

To estimate $\left[ \frac{\boldsymbol{b}'_{32} - b_{12} \frac{\boldsymbol{b}'_{31}}{b_{11}}}{b_{22} - b_{12} \frac{b_{21}}{b_{11}}} \right]$ I can run the IV regression such that

$$\boldsymbol{u}_{3t} - \frac{\boldsymbol{b}_{31}}{b_{11}} u_{1t} = \gamma + \theta \left( u_{2t} - \widehat{\frac{b_{21}}{b_{11}}} u_{1t} \right) + \zeta_t$$

where $\left( u_{2t} - \widehat{\frac{b_{21}}{b_{11}}} u_{1t} \right)$ comes from the instrument estimation

$$u_{2t} - \frac{b_{21}}{b_{11}} u_{1t} = \tau + \psi m_{2t} + \eta_t$$

To estimate $b_{22}$ and $b_{12}$ I use the covariance matrix $\boldsymbol{\Sigma}$ under a convention to sign. Write the $\boldsymbol{B}$ into partition

$$\boldsymbol{B} = \left[ \begin{array}{cc} \boldsymbol{s}_{11} & \boldsymbol{s}_{12} \\ \boldsymbol{s}_{21} & \boldsymbol{s}_{22} \end{array} \right]$$

$$\boldsymbol{\Sigma} = \left[ \begin{array}{cc} \boldsymbol{\sigma}_{11} & \boldsymbol{\sigma}_{12} \\ \boldsymbol{\sigma}_{21} & \boldsymbol{\sigma}_{22} \end{array} \right]$$

where $\boldsymbol{s}_{11}$ is the 2-by-2 matrix. Then I can use the relationship

$$\boldsymbol{s}_{11} \boldsymbol{s}'_{11} = \boldsymbol{\sigma}_{11} - \boldsymbol{s}_{12} \boldsymbol{s}'_{12} \tag{81}$$

$$\boldsymbol{s}_{12} \boldsymbol{s}'_{12} = \left( \boldsymbol{\sigma}_{21} - \boldsymbol{s}_{21} \boldsymbol{s}_{11}^{-1} \boldsymbol{\sigma}_{11} \right)' \boldsymbol{Q}^{-1} \left( \boldsymbol{\sigma}_{21} - \boldsymbol{s}_{21} \boldsymbol{s}_{11}^{-1} \boldsymbol{\sigma}_{11} \right)$$

$$\boldsymbol{Q} = \boldsymbol{s}_{21} \boldsymbol{s}_{11}^{-1} \boldsymbol{\sigma}_{11} \left( \boldsymbol{s}_{21} \boldsymbol{s}_{11}^{-1} \right)' - \left( \boldsymbol{\sigma}_{21} \left( \boldsymbol{s}_{21} \boldsymbol{s}_{11}^{-1} \right)' + \boldsymbol{s}_{21} \boldsymbol{s}_{11}^{-1} \boldsymbol{\sigma}'_{21} \right) + \boldsymbol{\sigma}_{22}$$

where $\boldsymbol{s}_{21} \boldsymbol{s}_{11}^{-1}$ is estimated by iv regression.

## E.3 Bayesian estimation of coefficient in IV

Because of equation 24 I can write it as

$$\boldsymbol{\Phi}\boldsymbol{\beta}_1'\boldsymbol{\beta}_1\boldsymbol{\Phi}' = \boldsymbol{\Sigma}_{\boldsymbol{mu}'}\boldsymbol{\Sigma}_{\boldsymbol{mu}'}'$$

Taking Cholesky decomposition of $\Sigma$ yields

$$\boldsymbol{B}\boldsymbol{B}' = \Sigma = \boldsymbol{\Sigma}_{tr}\Gamma\Gamma'\boldsymbol{\Sigma}_{tr}' = \boldsymbol{\Sigma}_{tr}\boldsymbol{\Sigma}_{tr}'$$

where $\boldsymbol{\Sigma}_{tr}$ is a lower triangle matrix and $\Gamma$ is an orthogonal matrix such that $\Gamma \in \mathcal{O}(n)$ and $\boldsymbol{B} = \boldsymbol{\Sigma}_{tr}\Gamma$

Since $\boldsymbol{\beta}_1'\boldsymbol{\beta}_1 = e_1'\boldsymbol{B}'\boldsymbol{B}e_1 = e_1'\Gamma'\boldsymbol{\Sigma}_{tr}'\boldsymbol{\Sigma}_{tr}\Gamma e_1$ and I denote $D = \boldsymbol{\Sigma}_{tr}'\boldsymbol{\Sigma}_{tr}$

Then above equation can be write as

$$(\boldsymbol{\Phi}e_1'\Gamma' \otimes \boldsymbol{\Phi}e_1'\Gamma')\,\mathrm{vec}\,(D) = \mathrm{vec}\,(\boldsymbol{\Sigma}_{\boldsymbol{mu}'}\boldsymbol{\Sigma}_{\boldsymbol{mu}'}')$$

Therefore this can yield

$$(\boldsymbol{\Phi}e_1'\Gamma' \otimes \boldsymbol{\Phi}e_1'\Gamma') = \mathrm{vec}\,(\boldsymbol{\Sigma}_{\boldsymbol{mu}'}\boldsymbol{\Sigma}_{\boldsymbol{mu}'}')\,[\mathrm{vec}\,(D)]'\,\varXi^{-1}$$

where

$$\varXi = \mathrm{vec}\,(D)\,[\mathrm{vec}\,(D)]'$$

Then by right multiplying $\mathrm{vec}\,(I)$ on both side I can rearrange this equation to

$$(\boldsymbol{\Phi}e_1'\Gamma' \otimes \boldsymbol{\Phi}e_1'\Gamma')\,\mathrm{vec}\,(I) = \mathrm{vec}\,(\boldsymbol{\Sigma}_{\boldsymbol{mu}'}\boldsymbol{\Sigma}_{\boldsymbol{mu}'}')\,[\mathrm{vec}\,(D)]'\,\varXi^{-1}\mathrm{vec}\,(I)$$

$$\mathrm{vec}\,(\boldsymbol{\Phi}e_1'\Gamma'\Gamma e_1\boldsymbol{\Phi}') = \mathrm{vec}\,(\boldsymbol{\Phi}\boldsymbol{\Phi}') = \mathrm{vec}\,(\boldsymbol{\Sigma}_{\boldsymbol{mu}'}\boldsymbol{\Sigma}_{\boldsymbol{mu}'}')\,[\mathrm{vec}\,(D)]'\,\varXi^{-1}\mathrm{vec}\,(I)$$

As long as $\varXi$ is invertable.

Otherwise write equation 24 as

$$\boldsymbol{\Phi}\boldsymbol{s}_{11}' = \boldsymbol{\Sigma}_{\boldsymbol{mu}_1'}$$

Because $\boldsymbol{s}_{11}'$, $\boldsymbol{\Sigma}_{\boldsymbol{mu}'1}$ and $\boldsymbol{\Phi}$ are full rank,

$$\boldsymbol{\Phi} = \boldsymbol{\Sigma}_{\boldsymbol{mu}_1'}\boldsymbol{s}_{11}'^{-1}$$

$$\boldsymbol{\Phi}\boldsymbol{\Phi}' = \boldsymbol{\Sigma}_{\boldsymbol{mu}_1'}\boldsymbol{s}_{11}'^{-1}\boldsymbol{s}_{11}^{-1}\boldsymbol{\Sigma}_{\boldsymbol{mu}_1'}'$$

$$\boldsymbol{\Phi}\boldsymbol{\Phi}'\boldsymbol{\Sigma}_{\boldsymbol{mu}_1'}'^{-1} = \boldsymbol{\Sigma}_{\boldsymbol{mu}_1'}\boldsymbol{s}_{11}'^{-1}\boldsymbol{s}_{11}^{-1}$$

$$\boldsymbol{\Phi}\boldsymbol{\Phi}'\boldsymbol{\Sigma}_{\boldsymbol{mu}_1'}'^{-1}\boldsymbol{s}_{11}\boldsymbol{s}_{11}' = \boldsymbol{\Sigma}_{\boldsymbol{mu}_1'}$$

$$\boldsymbol{\Phi}\boldsymbol{\Phi}' = \boldsymbol{\Sigma}_{\boldsymbol{m}\boldsymbol{u}_1'} \left( \boldsymbol{\Sigma}_{\boldsymbol{m}\boldsymbol{u}_1'}'^{-1} \boldsymbol{s}_{11} \boldsymbol{s}_{11}' \right)^{-1}$$

where $\boldsymbol{s}_{11}^{-1} \boldsymbol{s}_{11}'^{-1}$ can be estimated from equation 81

Firstly I show how to derive the conditional likelihood $p\left(M|Y,X,Q,B,u,\sigma_m\right)$

Following Giacomini et al. (2021) I can write the stochastic variable jointly such that

$$\begin{bmatrix} Y_t - \Psi X_t \\ m_t - \nu \end{bmatrix} = \begin{bmatrix} B & 0 \\ \boldsymbol{\Phi}^0 & \sigma_m \end{bmatrix} \begin{bmatrix} \boldsymbol{\varepsilon}_t \\ \zeta_t \end{bmatrix} \tag{82}$$

where

$$\boldsymbol{\Phi}^0 = \begin{bmatrix} \boldsymbol{\Phi} & \mathbf{0} \end{bmatrix}$$

that is based on the assumption of instrument variable, the last $n-k$ element in $\boldsymbol{\varepsilon}_t$ do not have any cross effect with $m_t$[49]

Therefore we would have

$$M|Y,X,Q,B,u,\sigma_m \sim N\left(\mu_{m|u}, \Sigma_{m|u}\right) \tag{83}$$

where

$$\begin{aligned} \mu_{m|u} &= E\left(\boldsymbol{\Phi}^0 \boldsymbol{\varepsilon}_t + \sigma_m \zeta_t + \nu\right) + \boldsymbol{\Phi}^0 B' \left(\boldsymbol{\Sigma}\right)^{-1} \left(u_t - E\left(B\boldsymbol{\varepsilon}_t\right)\right) \\ &= \boldsymbol{\Sigma}_{\boldsymbol{m}\boldsymbol{u}'} \boldsymbol{\Sigma}^{-1} u_t + \nu \end{aligned}$$

and

$$\begin{aligned} \Sigma_{m|u} &= \boldsymbol{\Phi}^0 \boldsymbol{\Phi}^{0\prime} + \sigma_m \sigma_m' - \boldsymbol{\Phi}^0 B' \left(BB'\right)^{-1} B \boldsymbol{\Phi}^0 \\ &= \sigma_m \sigma_m' \end{aligned}$$

since $B'\left(BB'\right)^{-1}B$ is a projection matrix on the column space of $B$ and it is a full rank matrix, this term will be an identity matrix.

To calculate the prior distribution of $\sigma_m$ I use the regression

$$m_t = \nu + P u_t + \zeta_t$$

where it is straightforward to prove that

$$P = \boldsymbol{\Phi}^0 B$$

$\zeta_t$ and $\nu$ are the same variables comparing to equation 82.

[49]In fact this step is necessary otherwise the $\boldsymbol{\beta}_2$ effect will eliminate by $\mathbf{0}$ in $\boldsymbol{\Phi}^0$ and I need more degree of freedom to identify. For instance, $\Sigma_{m_t|u_t} = BB' - B\boldsymbol{\Phi}^{0\prime}\left(\boldsymbol{\Phi}^0\boldsymbol{\Phi}^{0\prime} + \sigma_m\sigma_m'\right)^{-1}\boldsymbol{\Phi}^0 B'$, $\mathbf{0}$ in $\boldsymbol{\Phi}^0$ will cause $B\boldsymbol{\Phi}^{0\prime} = \boldsymbol{\beta}_1\boldsymbol{\Phi}$ such that $\boldsymbol{\beta}_2$ has no effect.

Then because the posterior distribution of the parameter can be write as

$$\begin{aligned} p\left(B, \Sigma, Q, \sigma_m \sigma_m' | Y, M\right) &\propto p\left(Y, M, B, \Sigma, Q, \sigma_m \sigma_m'\right) p\left(B, \Sigma, Q, \sigma_m \sigma_m'\right) \\ &\propto p\left(M | Y, Q^i, B, \Sigma, \sigma_m^i\right) p\left(B, \Sigma | Y\right) \end{aligned}$$

The first term $p\left(M|Y, Q^i, B, \Sigma, \sigma_m^i\right)$ can be calculated through the distribution 83. The second term $p\left(B, \Sigma|Y\right)$ can also easily be calculated through the normal-inverse-Wishart family of distributions which is proposed by Arias et al. (2018).

Then draw rotation matrix $Q$ based on the algorithm discussed below. After drawing a set of $Q$, $\Omega^Q$, I can solve the posterior distribution $p\left(Q|Y, X, M, B, u, \Sigma\right)$where $\boldsymbol{\varepsilon}_t$ and $\zeta_t$ follow two independent standard normal distribution.

#### E.3.1 Algorithm used to produce figure 5

**Algorithm 1** Draw $Q$ from $Q|Y, X, M, B, u, \sigma_m$

1. Draw $B^i$ and $\Sigma^i$ from the NIW $(\nu, \Phi, \Psi, \Omega)$.

2. Accept $B^i$ and $\Sigma^i$ based on the probability

$$\rho = \min\left\{\frac{\mathcal{L}\left(M|Y, Q^{i-1}, B^i, \Sigma^i, \sigma_m^{i-1}\right)}{\mathcal{L}\left(M|Y, Q^{i-1}, B^{i-1}, \Sigma^{i-1}, \sigma_m^{i-1}\right)}, 1\right\}$$

3. Based on $B^i$ and $\Sigma^i$ to draw new residual and $\boldsymbol{\Sigma_{mu'}}$

4. Draw $Q^i$ based on the Theorem 9 in Rubio-Ramirez et al. (2010).

5. Draw $\sigma_m^i \sigma_m'^i$ from $IW_k(S, T+\tau)$ where $\tau$ is the prior degree of freedom. Denote $V$ the prior variance of $\boldsymbol{\Phi}$,

$$S = (M - PU)(M - PU)' + S_0 + \hat{A}UU'\hat{A}' + A^* V^{-1} A^{*\prime} - \bar{A}\left(V^{-1} + UU'\right)\bar{A}'$$

where $A^*$ is the prior mean of $\boldsymbol{\Phi}$;$V$ is the prior covariance matrix of $\boldsymbol{\Phi}$

$$\hat{A} = MU'\left(UU'\right)^{-1}$$

$$A^* = \left[\begin{array}{cc} \nu & \boldsymbol{\Phi}_{tr} Q^{i-1} \end{array}\right]$$
$$V = \alpha^* I_{k+1}$$
$$\bar{A} = \left(A^* V^{-1} + MU'\right)\left(V^{-1} + UU'\right)^{-1}$$

6. calculate

$$\rho = \min\left\{\frac{\mathcal{L}\left(M|Y, Q^{i}, B^i, \Sigma^i, \sigma_m^{i}\right)}{\mathcal{L}\left(M|Y, Q^{i-1}, B^{i}, \Sigma^{i}, \sigma_m^{i-1}\right)}, 1\right\}$$

and I take $\boldsymbol{\rho}$ as the probability that retains new $Q^i$, otherwise $Q^i = Q^{i-1}$.

$$\begin{aligned}\mathcal{L}\left(M|Y, Q^i, B, \Sigma, \sigma_m^i\right) \propto & \left\{-\frac{kT}{2}\log(2\pi) - \frac{1}{2}\log\left(\left|I_T \otimes \sigma_m \sigma_m'\right|\right)\right. \\ & - \frac{1}{2}\left[\mathrm{vec}(M) - \left(U' \otimes I_{k+1}\right)\mathrm{vec}(Z)\right]'\left[I_T \otimes \left(\sigma_m \sigma_m'\right)^{-1}\right] \\ & \left.\left[\mathrm{vec}(M) - \left(U' \otimes I_{k+1}\right)\mathrm{vec}(Z)\right]\right\}\end{aligned}$$

where

$$M \equiv \left[\begin{array}{ccccc} m_1 & m_2 & ... & m_{T-1} & m_T \end{array}\right]$$
$$U \equiv \left[\begin{array}{ccccc} u_1^0 & u_2^0 & ... & u_{T-1}^0 & u_T^0 \end{array}\right]$$
$$u_t^0 = \left[1, u_t'\right]'$$
$$Z = \left[\begin{array}{cc} \nu & \boldsymbol{\Phi}_{tr} Q^i \left(\boldsymbol{\Phi}_{tr} Q^{i-1}\right)^{-1} \boldsymbol{\Sigma_{mu'}} \boldsymbol{\Sigma}^{-1} \end{array}\right]$$

$\nu$ can be calculated by run regression of $m_t$ on $u_t^0$ as

$$M = PU + \zeta$$