Overbuilding and Recession: A new Drawback of Housing Market Boom-and-Bust Cycle

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Abstract

In this paper, I unveil a novel mechanism through which a housing market boom leads to a recession following the burst of a housing market bubble. Overbuilding, characterized by increased residential construction driven by optimism or misinformation rather than sound economic foundations, crowds out physical investment during the boom due to the general equilibrium effect. The crowded-out physical investment subsequently induces a recession (or amplifies the losses and prolongs the duration of the recession) through a scarcity of physical capital. The relative intratemporal elasticity of substitution (compared to intertemporal elasticity), financial frictions, and idiosyncratic shocks can exacerbate this crowding-out effect via consumption substitution, liquidity easing, and precautionary saving. Furthermore, wealth distribution plays a crucial role in catalyzing these effects and contributes to the problem of inequality.

JEL classification: E21, E22, E30, E51, E58

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1 Introduction

The Great Recession in US, starting in December 2007, created the largest retrogradation since the Great Depression, which was nearly a central ago. This recession caused a sharp increase in the unemployment rate and a drop in output, consumption, and investment as discussed by Mian and Sufi (2010) and Grusky et al. (2011). The long-lasting recession from the end of 2007 to 2009 came to the end after the central bank and governement introduced unconventional monetary policy and fiscal policy. A lot of scholars have tried to understand the origin of this recession and answer the questions such as where it was born and how it spread throughout the whole economy.

Most of them agree that the housing market boom and bust agitated the financial market's collapse and caused a demand-driven recession after the collapse propagated to the real economy. After this collapse, the Great Recession persisted for a long time and someone[1] argued that the long-lasting drop could result from self-fulling and animal spirit. In addition to the animal spirit, there are other channels which scholars has proposed[2,3] to explain this long-lasting recession. People are focusing more and more on the housing market, as these channels are mainly triggered by the housing market bust and generate real effects through financial friction. The household lost a lot in the wealth of real estate, which previously acted as collateral to borrow money and smoothed their consumption, yet now infected the real economy via the demand side.[4] However, is it true that only the financial market crisis could incur such a large descending in real economy? The answer is no and other aspects of the economy also contribute to the failure in economy.

Earilier's research[5] has argued that a considerable contribution to the Great Recession stems from the investment market, with the supply-side effect accounting for nearly 40% of the recession. This impact is far from negligible and warrants careful investigation. Previous studies have sought to reconcile Hayek's theory, which posits that recessions are caused by a fundamental scarcity of resources such as physical capital or technology, and Keynes's theory, which attributes recessions to economic frictions such as capital misallocation, search-and-match costs, or liquidity traps. These researchers have argued that a lack of capital generated the Great Recession, focusing primarily on the point at which scarcity had already occurred and was

[1] Islam and Verick (2011) and Cochrane (2011) discussed this problem.

[2] Brunnermeier (2009), Ivashina and Scharfstein (2010) and Jermann and Quadrini (2012)argued that the lack of liquidity of financial institution, mostly referring to the commercial bank, helped the crisis diffuse around and induce large recession.

[3] Christiano et al. (2015) and Fisher (2015) did an extension to the liquidity trap happened in great recession and argued that the prolonged trap caused the ZLB later. Recent works such as Guerrieri and Lorenzoni (2017) and Bayer et al. (2019) focused on the heterogeneous agent model and drew the conclusion that idiosyncratic shock and distribution channel are also important to explain the lack of liquidity.

[4] Eggertsson and Krugman (2012), Mian and Sufi (2010), Mian and Sufi (2014) and Qian (2023) discussed this problem. Household extracted their equity via collateral during the boom period which increased the consumption a lot. This constructed a mirage through general equilibrium. When the bust came, people struggled against the rapid constraint tightening and led to the Great Recession.

[5] A lot of people contributed to this direction such as Justiniano et al. (2010) and Justiniano et al. (2011).

treated as an exogenous factor. In contrast, this paper takes a different direction by examining the process through which capital scarcity is created, specifically by overbuilding. Throughout this paper, *overbuilding* is defined as an increase in residential construction not supported by underlying fundamentals, essentially representing a housing market bubble. Few theoretical lenses[6] have been applied to explain how a housing market boom can absorb a significant amount of liquidity. When this boom is a bubble caused by imperfect information and misguided beliefs of households rather than changes in economic fundamentals, the available liquidity, which could otherwise support firms' investments in capital such as factories, equipment, and R&D, is instead directed to the residential sector. This results in inefficiencies when compared to a perfect information scenario. Given a constant amount of liquidity held by financial institutions, a housing market boom attracts these institutions to lean more heavily on the household sector as opposed to the firm sector. They tend to prefer lending money to households as mortgages or subordinated debt rather than lending to firms. More liquidity flowing into the housing market implies less liquidity allocated to the supply side, as long as the supply of liquidity is sticky and cannot be freely expanded. Additionally, a positive correlation between house prices and nondurable consumption suggests that investment in the nondurable production sector will decrease due to general equilibrium effects.[7] This paper first employs a simple model with detailed analytical results to explicitly elaborate on the formation of scarcity, and then uses a comprehensive heterogeneous agent model to quantitatively examine the overbuilding process. My primary contribution lies in uncovering a new mechanism, the crowding-out effect, through which overbuilding exacerbates the scarcity of physical capital and in turn, worsens the recession, whether triggered by an investment hangover (supply-side recession) or demand contraction (demand-side recession). The analysis also reveals that the relative intratemporal elasticity of substitution to intertemporal elasticity of substitution, financial frictions, idiosyncratic shocks, and wealth distribution influence the extent of crowded-out investment.

Academic attention to the relative intratemporal elasticity of substitution and non-separable utility functions has been limited, with most researchers opting for separable utility functions[8] for the sake of simplicity, yet intratemporal and intertemporal elasticity of substitution are interconnected and should not be overlooked. In particular, intratemporal elasticity of substitution plays a critical role in general equilibrium models with flexible housing supply. Suppose a model shuts down the pecuniary effect and assumes there is no collateral constraint. In this case, the

[6]except Beaudry et al. (2018), Rognlie et al. (2018) and J Caballero and Farhi (2018) and Chakraborty et al. (2018) recently.

[7]It is easy to understand this effect as the goods market cleaning condition in non-friction model should be $Y_t = F(L_t, K_t) = C_t + I_t^{residential} + I_t^{nonresidential}$ where K_t is predetermined. For simplicity if labor is fixed such that $L_t = \bar{L}$, higher $I_t^{residential}$, together with its coordinated C_t will return a lower $I_t^{nonresidential}$ which I call *crowd-out effect.*

[8]Iacoviello (2005), Liu et al. (2019) and Greenwald (2018) used the separable utility function to analyze the problem. However because their models lack of intratemporal channel they can only put weight on other elements such as bubbles, self-fulling and multiple credit constraints to generate enough consumption response to house price. On the contrary Berger et al. (2018) and Kaplan et al. (2020) used the nonseparable utility function to discuss the housing problem and they focus on the consumption response more, which requires the intratemporal effect.

only channel through which house prices could affect nondurable consumption would be the intratemporal channel. General equilibrium ensures that all wealth effects are eliminated, as the change in wealth caused by inflation in house prices is offset by rebated profits earned from construction firms. The elimination of the wealth effect implies that the intertemporal substitution effect(for housing service) also vanishes. Concurrently, flexible housing supply guarantees that intratemporal substitution is significant enough to influence nondurable consumption. Otherwise, the housing market would have no effect if the holding of residential estate remained fixed and this finding is supported by recent empirical work.[9] As long as housing services and consumption are weakly complementary within a short period, households will also increase their consumption, in turn crowding out investment. The stronger the intratemporal substitution relative to the intertemporal substitution, the less investment is crowded out by overbuilding, because the complementarity between nondurable goods and housing services weakens as the substitution effect becomes more pronounced.

In addition to the relative intratemporal elasticity of substitution, financial friction also plays a significant role in affecting the crowd-out effect, which has been well-established in the literature.[10] If the housing supply sector does not experience any fundamental cost shocks, the supply function remains unchanged, and house prices will rise, on top of overbuilding which is driven by shifts in demand function. Consequently, the residential property market will boom, easing credit constraints. Households may then allocate a larger portion of their budget to nondurable consumption through equity extraction and this increase in nondurable consumption contributes to the crowding-out of physical capital, which intensifies the subsequent bust and recession. In other words, the greater the financial friction, the more nondurable consumption is stimulated by a housing market boom via the wealth effect. This is because more households are financially constrained in the steady state, exhibiting a larger marginal propensity to consume (MPC), and would increase their nondurable consumption through equity extraction, as demonstrated by Bhutta and Keys (2016).

Additionally, aside from the relative intratemporal substitution and liquidity easing, household heterogeneity is another factor that amplifies the crowd-out effect, working through idiosyncratic income shocks and wealth distribution. If household income cannot be fully insured and everyone must bear idiosyncratic income shocks, households will have a precautionary saving motive, resulting in a larger portion of income(both wage income and asset returns) being saved compared to the representative agent model. When income risk is countercyclical[11], overbuilding often coincides with economic booms and smaller variance in idiosyncratic shocks. Lower risk implies that households are less cautious about accumulating wealth and will spend more on nondurable

[9] Khorunzhina (2021) did this significant work.

[10] Garriga and Hedlund (2020),Hurst et al. (2016),Bailey et al. (2019), Garriga et al. (2017), Gorea and Midrigin (2017) and Chen et al. (2020) contribute a lot on this strand of literature.

[11] Debortoli and Galí (2017), Acharya and Dogra (2020) and Bilbiie and Ragot (2021) analyzed this problem linked with monetary policy theoretically. Storesletten et al. (2004), Schulhofer-Wohl (2011) and Guvenen et al. (2014) analyzed the countercyclical idiosyncratic shock empirically.

consumption. Consequently, overbuilding has a stronger effect on crowding out investment, as households have a lower demand for saving income. Forbye the second-order uncertainty channel, heterogeneous wealth holding is important because wealth distribution is heavily *right-skewed*. Those who have more disposable cash to buy a new house are the ones who hold a larger share of total wealth.[12] Therefore, those who *can* contribute the most to overbuilding are indeed the ones who *contribute* the most to the crowd-out effect at the aggregate level. On the other hand, those with the tightest budget constraints (in the steady state) have a larger marginal propensity to consume (MPC). Despite the right-skewed wealth distribution, the MPC is left-skewed (Orchard et al. (2022)). The poor households, who would spend most of their money (resulting from looser budgets) on nondurable goods, make up a large share of the population and this large inequality also amplifies the crowd-out effect in the pass-through effect (from residential assets to nondurable consumption) and at the aggregate level.

Meanwhile, overbuilding may significantly contribute to a recession through the labor market and general equilibrium. A lack of investment initially can lead to a severe recession, as the total capital available is insufficient to support production in the end. Moreover, the presence of hand-to-mouth households can exacerbate the recession due to their high marginal propensity to consume (MPC) and low labor income (as labor and capital are complementary to each other). Furthermore, since residential property serves not only as a wealth function but also as a source of utility, its durable and irreversible (high transaction cost) characteristics may render a feedback loop from underinvestment to overinvestment during the burst period.[13] As a result, the span of the recession may be extended, and the overall impact of the recession may be magnified.

This paper makes several contributions to the literature. My first contribution is establishing a new connection between the pre-recession housing market boom (overbuilding) and the recession itself. A considerable amount of research has concluded that the boom in the housing market, as well as the nondurable goods market in 2007, was more of a mirage driven by expectation and speculation, as demonstrated by Landvoigt (2017), McQuinn et al. (2021) and Kaplan et al. (2020). Other studies have argued that credit supply also played an important role, as seen in works done by Campbell and Cocco (2007), Favara and Imbs (2015), Favilukis et al. (2017), Justiniano et al. (2019), Mian and Sufi (2022) and Martínez (2023). This expansion was built on sand, without the sustainability provided by investment and R&D, and could easily collapse due to a contractionary demand shock. The panic and pessimistic expectations, or tightened credit constraints, triggered a drop in demand and an increase in precautionary saving, yet the real estate only served as an asset in collateral constraint in these studies. Moreover, lack of investment and the complementarity between capital and labor magnified the demand-driven recession related to self-fulfilling or multiple equilibrium. In other words, the boom in the housing market not

[12] "In 2019, the top 10% of U.S. households controlled more than 70 percent of total household wealth" argued by Batty et al. (2020) and related data can be found in Distributional Financial Accounts in federal reserve web.

[13] McKay and Wieland (2019) refined this channel penetratingly and argued that this channel is important to explain the persistent ZLB and negative real interest rate after the Great Recession. This channel can also explain the low interest rate after the implementing of unconventional monetary policy, as Sterk and Tenreyro (2018) did.

only affected investment in the construction sector (Boldrin et al. (2013)) but also crowded out physical investment in other sectors, when only large companies could make extensive margin investments through self-finance, as discussed in Bachmann et al. (2013) and Winberry (2016). This shortage in investment amplified the recession in general equilibrium and resulted in high unemployment and low production. Some literature also uses the term "crowd-out" to describe the investment trade-off between housing and non-housing sectors (labor, physical asset and intangible asset, etc.), such as Dong et al. (2022) and Dong et al. (2023). However their concept of "crowd-out" is closer to portfolio adjustment in firms' balance sheets and operates in partial equilibrium, which is far from the reality[14], as neither most of the residential asset is held by enterprises nor are residential assets of crucial importance in production activity. They just replaced one type of asset in asset misallocation literature of firms' problem by residential asset cursorily.

In addition to explaining the reasons behind the Great Recession, this mechanism can also account for part of the policy failure, such as the Home Affordable Modification Program (HAMP), which is discussed by Mitman (2016) and Antunes et al. (2020). In this sense, this paper also holds a place in the literature related to long-lasting recessions and the ZLB. Because the recession is fueled by both supply and demand sides, the one-dimensional stimulus in the demand sector is not strong or effective enough to curb the declining economy. On top of that, both of these studies do not consider the supply of housing services, and their models are overly simplistic, even though Khan and Thomas (2008) have shown that a general equilibrium setting would generate entirely different results. My work extends the findings of Chodorow-Reich et al. (2021), Chahrour and Gaballo (2021) and Beaudry et al. (2018), while the former two primarily explained the causes of the Great Recession and real estate issues through over-optimism, the latter focused on labor market frictions and multiple equilibrium. The financial institutions in both sets of studies functioned as brokers who only provided liquidity to households and helped clear the bonds market, thus limiting the role of the housing market to a liquidity trap and demand-driven recession. In contrast, my work focuses on the investment in the nondurable sector and argues that overbuilding exacerbated the crowd-out effect and fueled a deeper recession, which can be treated as a macro-story of Chakraborty et al. (2018) in which they argued that because of housing market boom more liquidity held by commercial bank is lent to household as mortgage debt rather than firms as commercial debt. The closest literature to my paper is Rognlie et al. (2018), who used their partial (in financial market) model to explain the investment hangover via higher real interest rates. They argued that, given overbuilding at time zero, a high real interest rate results in a demand-driven recession due to nominal rigidity and the zero lower bound in monetary policy. However, the real interest rate and demand contraction is not the only reason for the recession, and even in the absence of nominal rigidity, overbuilding can also contribute to a supply-driven recession with significant welfare loss.

[14] Kaplan et al. (2014) shows that "Housing equity forms the majority of illiquid wealth for households in every country with the exception of Germany".

Furthermore, I also contribute to two methodology progresses in literature: an SVAR identification strategy which helps to separately identify true news shock and fake news shock and an improvement in the numerical solution method for tackling the complicated heterogeneous agent model. The new identification result is build on the theoretical result of Wolf and McKay (2022) in which they argue that we can prune the original identification to an ideal one by solving some shocks from underline policy function or self-transition function. Additionally, previous literature either uses a guess-and-verify method, such as Lorenzoni (2009) and Barsky and Sims (2012), or a reconstruction method, such as Baxter et al. (2011), Blanchard et al. (2013) and Hürtgen (2014) to solve the imperfect information DSGE model. The latter requires specific analytical equations regulating the unobserved state variable with other state variables, which is impossible to derive from a heterogeneous agent model that contains too many state variables.

Numerous studies emphasize the importance of household heterogeneity in explaining the housing boom-and-bust cycle, either empirically, such as Etheridge (2019), Mian et al. (2013), Li et al. (2016) and Díaz and Luengo-Prado (2010), or theoretically, such as Kaplan et al. (2020), Favilukis et al. (2017) and Garriga and Hedlund (2020). However, there are hardly any papers that incorporate heterogeneity in capital holding, housing, and income with information, animal spirits, learning, and anticipated shocks. This paper builds a model that demonstrates the distribution of wealth and income is pivotal in determining the strength of overbuilding and supplements the literature on how expectations and animal spirits can fuel a boom. Simultaneously, imperfect information and a slow learning process will inflate the bubble further.

In section 2 I use two identification strategy separately lay out the crowd-out effect generated by a contemporaneous and news shock to housing price. Later in section 3 I analytically demonstrate the crowd-out effect is driven by relative intratemporal elasticity of substitution, financial friction, income inequality and wealth distribution. In section 4 I quantitatively investigate the drawback of crowd-out effect spawned by a fake news shock through the lens of a full fledged heterogeneous agent model. In the last section I conclude the result.

2 Empirical evidence

Firstly we take a glance at the statistic property of the data which conceals the mechanism we want to argue in this paper. Figure 1 shows the amount of nonresidential investment (valued as the share of GDP) from 1960 to 2016. We can see that the total investment gradually increased when economy boomed and dropped when economy busted. Meanwhile the peak has the raised trend not only because the advanced technology required more physical mechanism but also because the increased wage cost required more mechanical equipment to replace physical labor. However, the increased trend starting at 2003 was broken by the great recession happened at the end of 2007. The trend is, even though looks same as before, different with what happened at

1970s and 1990s, because of investment hangover.

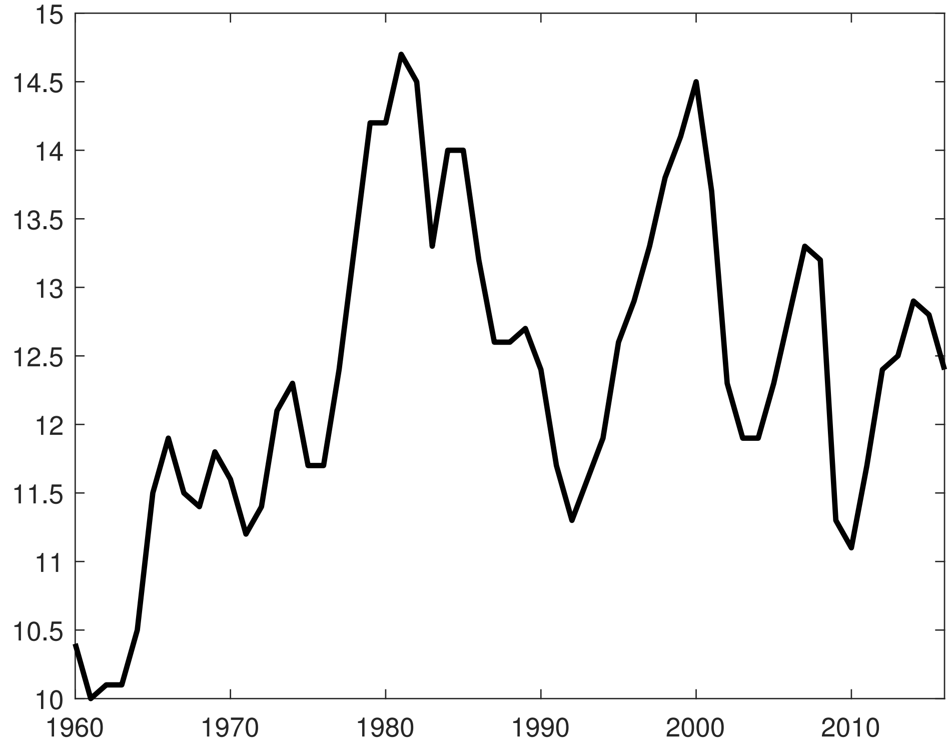


Figure 1: Nonresidential Investment (share of GDP)

The increased trend in 1970s, from 1975 to 1981, created 25.6% increasing from 11.7% of GDP in 1975 to 14.7% of GDP in 1981. It is 4.27% each year on average . The trend in 1990s, from 1992 to 2000 created 3.53% increasing in investment each year on average. The latest trend, from 2010 to 2014, created 4.05% increasing each year on average. While the trend before the great recession, from 2003 to 2007, only created 2.94% increasing each year on average, which is summarized by table 1.

Table 1: Extent of increased investment

Trend range	1975-1981	1992-2000	2003-2007	2010-2014
Increased investment	4.27%	3.53%	2.94%	4.05%

Enlightened by the statistic difference, there must be some reason that caused this distinct drop in investment before great recession and contributed part of the output loss during recession. The house market boom, at least under our investigation, takes some responsible for the investment drop. It is the house market boom that crowed out some investment at the demand sector. Financial institution may put more weight on household and preferred lending money to household for real estates to lending to companies for investment. On the other hand, household may prefer spending more money or liquidity on durable goods to saving at the bank who together with companies can in the end transfer these liquidity to investment and physical capital. Because of the long-lasting property of durable goods and precautionary motivate, household would like to occupy first when the goods price is increasing or has propensity to increase, like what happened from 2005 to 2007. This helps crowed out some part of the investment at the supply side. In summary both the demand and supply side help to elbow out investment and general equilibrium helps amplify this effect. The importance of general equilibrium which helps explain investment activities is widely accepted as Khan and Thomas (2008) proved previous partial analysis such as Caballero et al. (1995) maybe misleading. As given $Y = C_{nd} + I + C_d$, an increased C_{nd} and C_d will have effect on I since Y is concave at predetermined capital and labor which cannot increased too much as it is complementary to capital. After detrending the growth elements in per capita real GDP, real nonresidential investment and new construction housing units the data shows that there is a significant negative correlation between the relative physical investment and residential estate investment.[15] In this sense this paper can also be seen as a complement to Berger and Vavra (2015).

2.1 Contemporaneous real price shock

Figure 14 sheds light on the crowd-out effect crated by a housing market boom. However since the IRF goes back to steady-state so quickly, it may not generate scarcity in physical capital enough and the crowd-out effect may not be important under this simple identification test. Additionally the identification method I used, Sims et al. (1986), has been criticized that it is too strong to identify the underlying shocks and sometimes could be artificially unreliable. Because of these problems I use another canonical workhorse identification method, Cholesky decomposition, to identify the effect of contemporaneous housing price shock. Following Bernanke and Mihov (1998), Cholesky decomposition ensures that the shock can only take its effect on the variable after itself in order, yet the variable before itself in order will not be influenced contemporaneously by this shock. Because throughout this paper I am arguing the drawback of the crowd-out effect spurred by a housing market boom without fundamental support, I leave the housing price in the last to mimic a non-fundamental housing price boom as only the housing price is stimulated at the beginning. Therefore the one unit housing pricing shock evokes other variables' movement following the intrinsic relationship and mechanism (Φ in equation 2). Inspired by the literature I order the economy variables in data vector Y_t as

$$Y_t = [y_t, c_t, i_t, cpi_t, r_t, p_t^a, h_t^s, l_t, r_t^d, d_t, p_t^h]' \quad (1)$$

where y_t is real GDP; c_t is real non-durable consumption; i_t is real investment in non-residential sector; cpi_t is consumer price index without considering residential market; r_t is the nominal interest rate; p_t^a is the real capital price which I use stock market index as a proxy; h_t^s is the house supply; l_t is the labor supply; r_t^d is the mortgage debt rate; d_t is the total real amount of mortgage

[15]The relative correlation between relative physical investment and residential estate investment, $\text{corr}(\frac{I_{t,c}}{y_{t,c}}, \frac{I_{t,c}^H}{y_{t,c}})$ is -0.873 and $\text{corr}(\frac{I_t}{y_t}, \frac{I_t^H}{y_t}) = -0.17764$.(The subscript c denotes the cyclical data detrended from HP filter)

debt; p_t^h is the real housing price. I pick the time interval between 1985Q1 and 2007Q2 when the housing market boom reached its peak before the Great Recession. The left boundary is decided by the dataset I used in section 2.2 and 2.3, NAHB/Wells Fargo Housing Market Index, whose earliest record is on the Q1 of 1985. I choose the right boundary because I want to investigate the overbuilding and crowd-out investment which occurred before the Great Recession and not emerge anymore until now. Moreover, the tremendous drop in economy resulted in persistent abidance in ZLB and let the data after Great Recession hardly unveil the mechanism.[16] All the variable are in logarithm form and are detrended by hybrid specification, a method through which I use all non-stationary variables as growth rate and all the variables in Y_t I finally used in identification pass the unit-root test.

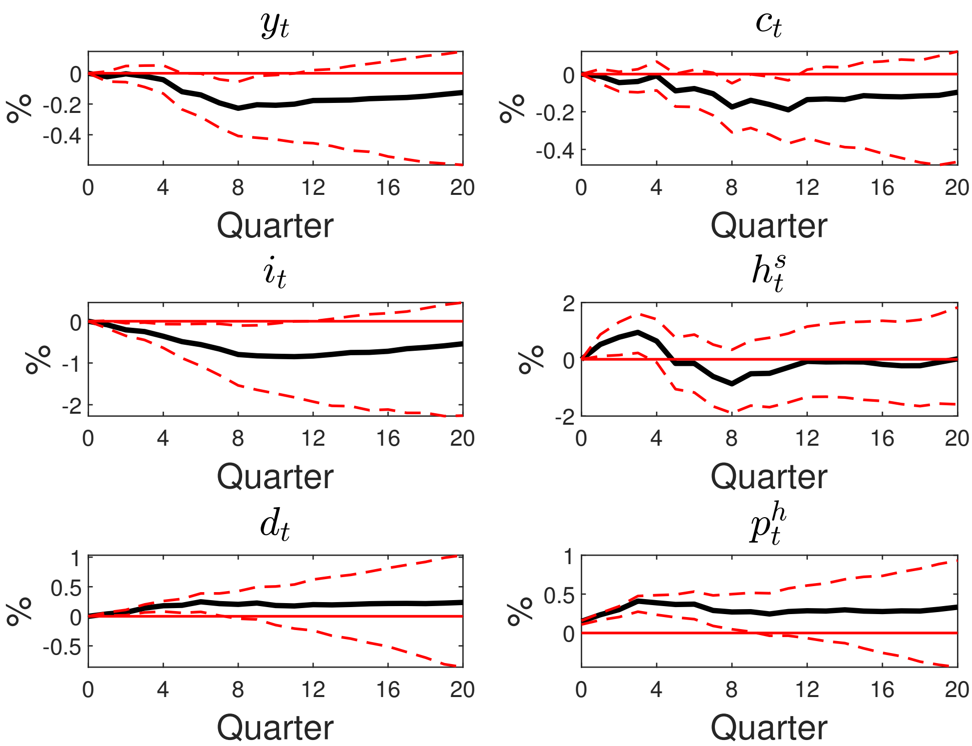


Figure 2: IRF to one unit house price jump

Figure 2 tells us the impulses response to one unit housing price shock with 90% confidence band. We can see that a 10bp jump in housing price p_t^h at period 0, agitates a housing market boom and the housing price climbs to 50bp at the peak 4 quarters later, roughly 5 times larger than the beginning. People without holding residential asset before are optimistic and eager to buy more houses, which shifts the demand curve of residential asset as the price and quantity increase at the same time. However they only pay part of the value of asset as the down payment and the remain part is borrowed from commercial bank as mortgage debt. Additionally people who are already holding the house will use the increased housing price to extract equity and free their

[16] In appendix I do some robustness tests to this span selection by extending the data to 2019Q4 with shadow rate or 1-year treasury bonds rate that is proposed by Gertler and Karadi (2015). The crowd-out effect exists in all these robustness test.

liquidity (if they are financial constrained and want more liquidity to fulfill their consumption demand). These two forces push up the mortgage debt up to 35bp and this rapid expended credit stimulate the economy and builds a prosperity in mirror. However, the consumption of non-durable goods and output at the beginning is insignificant, which may be caused by identification problem or data issue argued by Sims (1998), Christiano et al. (1999) and Romer and Romer (2004). The investment in non-durable sector declines throughout the whole period and becomes stable after 2 years around 1% annualized which uncovers the crowd-out effect clearly. It shows that the crowd-out effect is strong and sensitive to the housing-price stimulation as 10bp ascending in housing price generates 100bp descending in investment. This over-reaction indicates that there is a river underground that passing and magnifying the flow from housing price to physical investment. We can observe that the housing price coordinates with increased housing supply at the same direction and this demand shock verifies two key arguments we discussed before: overbuilding and crowd-out effect spurred by a non-fundamental housing price demand shock. Furthermore, the non-exponential expansion in housing supply shields light on the shape of supply function in housing market which is not fully inelastic, though large amount of literature assumed it is fully inelastic.

2.2 Real price news shock

Although I successfully identify the house market boom, overbuilding and crowd-out effect in previous section, there is still an important question left: where does this "contemporaneous real price shock" come from? Even though I empirically test that an exogenous housing price could move the economy to a housing market boom which the overbuilding and crowd-out effect predicted, the doubt about the reality of this shock arises naturally. It could be the truth that the mechanism I proposed along this paper is correct yet is not the reality what happened during the Great Recession, since the source of the housing market boom before the recession is not simply an exogenous real housing price shock. There is something other than the price that induced the boom such as optimistic expectation, credit supply, secular decline in interest rates. Therefore in this section I identify the news shock in a SVAR model and try to answer the question: given a news about housing price inflation in the future, what would other elements in economy response to this expectation shock? Following the method proposed by Barsky and Sims (2011) (henceforth BS for simpility) with some minor adjustments, I identify the news shock as the component which can explain the variance of forecast error but is orthogonal to contemporaneous effect (to rule out the possibility that any unexpected contemporaneous shock realized in the future may affect the forecast error). Meanwhile, I use the detrend method in last section, hybrid specification, to process the data instead of levels specification that BS used in their identification, because the data I used fail in passing the unit-root test in levels specification.

Firstly I propose the reduce-form VAR equation in companion form as

$$y_t = \Phi y_{t-1} + u_t \tag{2}$$

where the residual follows $u_t = Q\varepsilon_t$, $\varepsilon \sim N(0, I)$ and $\Omega = \text{var}(u_t) = QQ'$. Moreover I assume P is the Cholesky decomposition to the covariance matrix of residual u_t so $P = \text{chol}(\Omega)$. I further define the "news" vector $R = [r_1, r_2, ..., r_{N-1}, r_N]'$ where r_i is the unknown parameters which need to be estimated. It measures the effect of house-price-change news. It is constraint by two condition. The first one is $RR' = 1$ since the identification of shocks(estimation errors) should be orthogonal with each other. The response to the news will be PR and by introducing this "vector shock" R I can directly solve the response to news shock and avoid drawing difference alternative orthogonal matrix persistently.

It is worth to notice that solving the response vector R, instead of solving the response matrix Q, is more convenient and can provide analytical solution argued by Uhlig et al. (2004). As long as the orthogonal assumption 4 holds, we can find an orthogonal matrix $\mathcal{Q}$ which satisfies $\mathcal{Q}'R = e_i$ where $i \in [1, N] \cap \mathbb{N}$. Multiple $\mathcal{Q}$ on LHS to yield $R = \mathcal{Q}e_i$. Therefore R is just the ith column of $\mathcal{Q}$. Throughout this paper I will mix these two definitions 1). Response vector R; 2) A shock R, because they represent the same thing in identification problem.

Meanwhile I define the forecast error decomposition along the horizontal up to time h as

$$\text{fevd}^i_{n,h} = \frac{e'_n \text{var}(y^i_{t+h} - E_{t-1}y^i_{t+h})e_n}{e'_n \text{var}(y_{t+h} - E_{t-1}y_{t+h})e_n}$$

whose economic meaning is that the proposition of variance of variable n's expectation error that can be explained by shock i across time 0 to time h. Respectively the total forecast error from 0 to period H with unit weight should be $\text{fevd}_n = \sum_{h=0}^{H} \text{fevd}_{n,h}$ where $H = 12$.[17]

To identify the news shock, I solve the problem 6 below which find a shock that can explain the variance of expectation error of housing price most.

$$R^* = \text{argmaxfevd}_n = \text{argmax} \sum_{h=0}^{H} \frac{e'_n \left(\sum_{s=0}^{h} \Phi^s PRR'P'\Phi'^s \right) e_n}{e'_n \left(\sum_{s=0}^{h} \Phi^s PP'\Phi'^s \right) e_n} \tag{3}$$

s.t

$$R'R = 1 \tag{4}$$

$$e'_j PR = 0 \tag{5}$$

The first constraint 4 guarantees the orthogonality of response R^* and insures the unit realization

[17] Uhlig et al. (2004) and Barsky and Sims (2011) argued the weight-selection problem and arbitrary maximized horizontal problem. Based on their argument I choose the unit weight and 3 years forecasting as the baseline cases which is reasonable and robust in the range from 5 quarters to 40 quarters.

of news shock which pertains to corresponding column of orthogonal matrix Q. Additionally it renders the existence of maximization problem 3 as the Hessian of the objective function is semi-positive definite where the maximized point is not on the saddle point. The second constraint 5 rules out any future contemporaneous shock that influences the expectation error. Basically there are two type of shocks that can affect the expectation error $y_{t+h} - E_{t-1}y_{t+h}$: one is the news shock that arrives at time t yet realizes at a future time throughout $t+1$ to $t+h$ (based on the type of news and how informative it is); another one is the contemporaneous shock that arrives at any time from t to $t+h$ $\varepsilon_{t+i}, \forall i \in [0,h]$. It is unreasonable to assume that the news shock explains more variation of expectation error than that the contemporaneous shock does, and Sims (2016) shows that most of time it is not true in reality. Therefore I need this second constraint 5 to rule out the effect of contemporaneous shock and the purpose of above problem 3 is to find a shock, except any contemporaneous shock, that can explain expectation error most.

Since the identified news shock R^* is up to sign, I further impose the sign restriction over the impulse response y_t to yield a positive demand shock on housing price.

Proposition 1. *The identification to a news shock R^* through equation 3 is unique to covariance of the residual $\Omega = PP'$ from VAR's DGP 2.*

Proof. Give the covariance matrix of the residual from the DGP 2, the Cholesky P is unique to the covariance matrix Ω. Following Rubio-Ramirez et al. (2010), we know that any identification to the DGP is unique to PQ where Q is an orthogonal matrix. To identify the news shock I solve the maximization problem 3 to get the news shock R^* that maximizes fevd$_n$ subjecting to two constraints 4 and 5 and the rotation Q is identity $Q = I$. However when the rotation Q is not identity, i.e. for any different response matrix $P\widetilde{Q}$, the optimization problem that helps to find $\widetilde{R}^*$ from $g\left(\widetilde{R}\right) = 0$ is equivalent to that helps to find R^* from $g\left(f\left(R\right)\right) = 0$ as long as $f\left(R\right) = \widetilde{R}$ holds. If the mapping $f(\cdot)$ and its inverse $f^{-1}(\cdot)$ are all bijections, for any $\widetilde{R} \in \mathbb{R}^N$ there will exist an unique $R \in \mathbb{R}^N$ which satisfies $f\left(R\right) = \widetilde{R}$. It is easy to set $f^{-1}(\widetilde{R}) = \widetilde{Q}\widetilde{R}$ and $f(R) = \widetilde{Q}'R$. Therefore corresponding identified news shock $\widetilde{R}$ must satisfy $\widetilde{R}^* = \widetilde{Q}'R^*$ because of equation 3 and the impulse response of news shock is same to the Cholesky identification $P\widetilde{Q}\widetilde{Q}'R^* = PR^*$. □

Proposition 1 is intuitive as news or information is neutral to the fundamental and people will response to it according to their own perception or belief about the reliability of the news. Whether the news is fake or true can only be known after the fundamental shock is realized and observed by agents in economy, several periods later. Therefore the response to news at the beginning, time 0, is unique to the covariance matrix and whether the news is fake or true and the corresponding responses cannot be identified.

The last problem of identification 3 is the variable j in constraint 5 that helps to rule out the possibility of contemporaneous shock during the identification. Unlike the standard news literature in which scholars focus on the TFP shock and the beneath exogenous TFP is observable or can be calculated from data, both the demand shock and exogenous demand variation path are unobservablel. Hence I should find a variable which is correlated with the contemporaneous variation of housing demand underneath the demand function, which I call as the direct fundamental impact. The "fundamental impact" represents that it is an index to the fundamental element that drives the demand function of housing demand function, i.e. the preference ϕ_t in Cobb–Douglas utility function $U(c_t, h_t, l_t) = \frac{\left(c_t^{\phi_t} h_t^{1-\phi_t}\right)^{1-\sigma}}{1-\sigma} + \kappa \frac{l_t^{1+\psi}}{1+\psi}$ which follows $\phi_t = (1-\rho_\phi)\overline{\phi} + \rho_\phi \phi_{t-1} + w_{t-\tau} + w_t^\tau$ and $w_{t-\tau}$ is the news shock to housing demand. The "direct" means that variable y_t^j only indicates contemporaneous impact ϕ_t instead of ϕ_{t+i}. Furthermore, if the imperfect information exists and household cannot observe the fundamental precisely, as I argued in section **??** below, the fundamental impact y_t^i should be an indicator to perceived fundamental $\phi_{t|t}$ instead of true fundamental. Therefore the survey data is the closest variable that satisfies requirement to rule out contemporaneous shock via constraint 5 and I use the NAHB/Wells Fargo Housing Market Index (HMI) which is a monthly survey on NAHB members about their perception about the housing market right now him_t, and expectation over the next six month $E_t\text{him}_{t+6}$. However I still need to clean the present perception him_t as it does not only indicate the contemporaneous impact but is mixed with news shock arrives before (yet realized at t) and arrives at time t (yet realized at $t+6$). To extract the contemporaneous impact from present perception to housing market I run following regression

$$\text{him}_t = \alpha_0 + \alpha_1 E_{t-6}\text{him}_t + \alpha_2 E_t \text{him}_{t+6} + u_t^{\text{him}}$$

where the residual u_t^{him} is the variable j in constraint 5. All the news component in current perception about the housing market him_t is eliminated by the lead and lag expectation and the remaining component, residual, is what we need that contains the contemporaneous element.

Figure 3 shows the IRFs of one unit news shock that tells agent about the information of housing price in the future and the read dash lines are the 90% confidence band which shows that the crowd-out effect is significant. The response pattern of housing price is close to that in the contemporaneous shock yet the boom in housing market is almost double. Housing price climbs gradually from 15bp up to 120bp at the peak where roughly the twice higher of that housing price stays under contemporaneous shock. The expansion in the amount of mortgage debt that pertains to expectation and news shock is five times larger than that pertains to contemporaneous shock and reaches to 150bp. Even though there are frictions, household still borrow a lot and push up the mortgage debt, which opens the veil to us that the financial market and financial institution is also important. It is them that may worked as fuel which was ignited by expectation and burned up the boom later. This demonstrates that the identification of news shock works well and all the result is reliable and transparent as the expectation and credit market slackness are the main sources of housing market boom in literature. Furthermore, it demonstrates that house market is sensitive and fragile during the pre-recession period. The house market could be triggered to boom only by the expectation at the beginning and reaches to a high peak in the end, without any

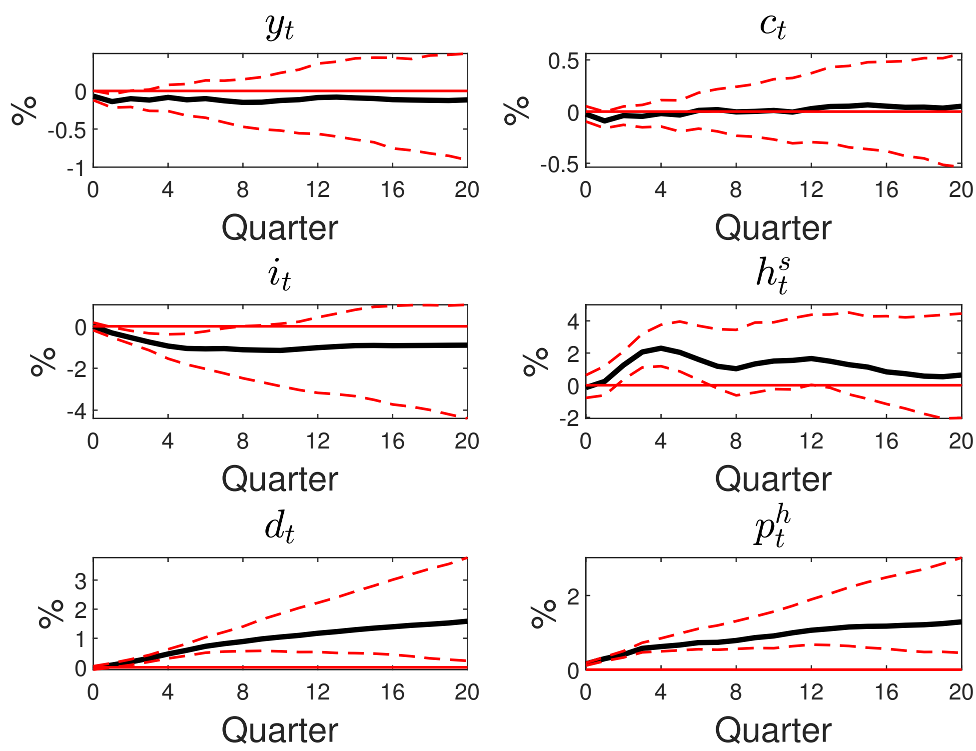


Figure 3: IRF to one unit housing price news shock at 90% confidence band

hesitation or drop. This housing market boom also coordinate with overbuilding and the extent of overbuilding is also twice as large as that under contemporaneous shock, arriving at 200bp. In addition to the housing market boom and overbuilding, the output has a tiny drop because a lot of resources are focus on residential sector. However the contemporaneous response of non-durable consumption is still insignificant which may results from a mixture between illusory-flourish household (they are optimistic to future and because of PIH and wealth effect they feel richer than before and increase consumption) and down-payment household (they need to pay a large down-payment in liquidity asset to exchange for residential asset, and the substitution effect may lead them to hand-to-month household). Same as before, a lot of investment is crowded out during the housing market boom and overbuilding era. Comparing to the response of physical investment to a contemporaneous housing price shock, investment is crowded out at a similar magnitude, up to 100bp. It is reasonable, as the crowd-out effect comes from general equilibrium between investment, output and consumption whose link will not different type of shock on housing price.

2.3 Real price fake news shock

Due to proposition 1, canonical identification strategies such as sign restriction(Uhlig (2005)) and short-run restriction(Sims (1980) and Basu et al. (2006)) are insufficient to separate true news and fake news from the previous identified news shock. In response, I propose a new identification strategy through which the effect of true news about future housing prices is refined by a contemporaneous shock during the realization of news shock so that the effect of fake news is left.

In section 2.2 I introduced the news shock R^*, which is the shock that can most explain the expectation error next H periods from period 0. However it is independent of the shock's status and cannot provide any information about whether the shock is fake or true because it is identified based on expectation error without any proxy to the "fundamental situation" and both the fake news and true news can generate the same response prior to the realization of news' type. In spite of the neutrality of the news shock R^* and our inability to directly identify the fake news before its realization, I introduce a new identification strategy that can be used to distinguish fake news and true news by revising the mixed news with contemporaneous shock and pruning the previous impulse response. Before introducing the identification strategy, which helps me to separately identify fake news, I first add two assumptions with micro foundations as the cornerstones of identification.

Definition 1. Denote the response to fake news realized at time τ as $U^F = \left\{y_0 = \overline{R}_1, y_i\right\}_{i=1}^{i=\infty}$ and the response to true news realized aprior tot time τ as $U^T = \left\{y_0 = \overline{R}_2, y_i\right\}_{i=1}^{i=\infty}$. The response to a news shock we empirically identified through 3 is $U = \{y_0 = R^*, y_i\}_{i=1}^{i=\infty}$.

Assumption 1. *The response to a news shock, either a fake news or a true news, under imperfect information, will be the same before the shock realized. In other words $\overline{R}_1 = \overline{R}_2 = R^*$ and $y_i^F = y_i^T = y_i, \forall y^F \in U^F, y^T \in U^T, y \in U, i \in [0, \tau]$ will hold.*

This assumption is reasonable as under imperfect information the agents cannot distinguish whether the news is true or fake yet they just have the same response to the observation either triggered by true news or triggered by fake news. Therefore only if fully informative was the news , agents would have the different response before the realization of news at time τ. It is well known that certainty equivalence holds in first-order linearized state space model, in which assumption 1 will defiantly hold. In appendix I further provide some numerical examples based on first-order perturbation to show that above assumption hold in a state space model under rational expectation.

Assumption 2. *The empirically identified news shock U lies on the medial of response to fake news U^F and response to true news U^T. In other words, $y_i \in \left[y_i^F, y_i^T\right], \forall y^F \in U^F, y^T \in U^T, y \in U, i \in [\tau+1, \infty]$ will hold. Furthermore, the news shock U is a linear combination of U^F and U^T and $y_i = \alpha y_i^F + \beta y_i^T$ holds.*

This assumption is also reasonable because the identification process 3 is based on expectation error and cannot separately identify U^F and U^T as both of them affect the expectation error. However as long as the DGP 2 is a linear equation what governs the after-realization path is all described in coefficient Φ which is a projection from y_{t-1} to y_t. Therefore the identified path U is just a linear combination of fake news path U^F and true news path U^T which are all mixed together in posterior observation. In appendix I implement the news shock identification strategy 3 on the mock-data generated by a state space model and show that assumption 2 holds.

Now I define the identification to fake news as

$$\widehat{y}_i^F = \begin{cases} y_i & i \leq \tau \\ y_i - \frac{e_j' y_{\tau+1}}{e_j' y_0^{\tau}} y_{i-\tau-1}^{\tau} & i > \tau \end{cases} \tag{6}$$

where $y_i \in U$ and y_i^τ is the response path to a contemporaneous shock on direct fundamental impact, variable j, in equation 5. The basic idea is that the effect of true news realized at time τ can be offset by a contemporaneous negative shock and the remaining part is just the response to a fake news which does not have any impact on variable j and real economy (up to a scalar α which cannot be identified here). This is intuitive because the true news shock has affected the economy since the time of realization τ and it works (real effect) as an contemporaneous shock linearly added into y_τ as long as the shock is iid and the whole system is linear. I provide two examples in appendix to provide some micro foundation to this offset effect.

This identification method uses the same logic that is firstly proposed by Wolf and McKay (2022) in which we can "replace" the underlying state determinant equation (i.e. policy function) by another counterfactual one via solving a system of linear equations. A set of rescaled fundamental shocks can mimic the old identified policy function to a new one through censoring the old impulse response by an extra $\{\Theta_{i,\tau}\}_{\tau=0}^{\tau=\infty}$ generated by fundamental shock. The other endogenous variables such as GDP, investment, labor supply will then be pinned down by the censored path y_i^τ and Wolf and McKay (2022) provides rigorous proof to this argument. Moreover, the similar counterfactual experiment is also used by Hebden and Winkler (2021) and Groot et al. (2021) where they aim to find an optimal policy and achieve their goal through solving some nonlinear problems.

Figure 4 displays the empirical response to a fake news in housing demand. A housing demand shock arrives 4 period ahead, at time 0, but realizes at time 4 with some possibility that the news is just a noisy without any fundamental effect. Before the agents know the true type of the shock, true news or fake news, they response identically to these two shocks as they cannot ascertain the facts. Therefore figure 4 and 3 share the same response preceding period 4 when the agents proceed discerning whether the news is true or fake.[18] The agents finally realize that the news is fake at time 4, so the housing market boom bursts as there is no more fuel supporting the boom. The housing price, mortgage debt and construction of residential asset drop a lot and the drop of housing supply is 300bp which is triple than the peak of that under contemporaneous shock. Thereafter the housing price and mortgage debt go to the negative range which indicates a severe and persistent recession induced by the burst of bubble in hosing market. The physical investment is crowded out at the beginning because of housing market

[18]They may be informed directly at time 4 or gradually learn that whether the news is true or fake, which depends on the information structure and I provide two examples in appendix to illustrate two different information structures.

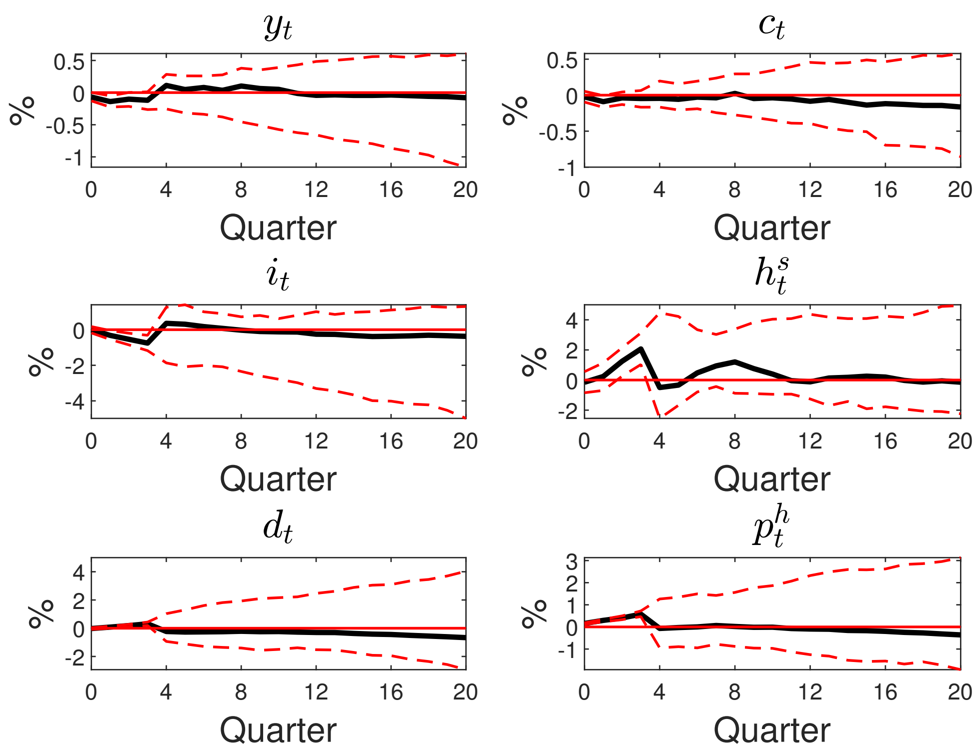


Figure 4: IRF to one unit housing price fake news shock at 90% confidence band

boom yet it raises to positive one to remedy the scarcity in capital when the agents realize that the news is fake. Although the output and consumption revise to the positive range right after the fake news is informed, the scarcity in physical capital still deepen the economy and the recover is mild. This tepid recover ascertains the drawback of housing market boom-burst cycle in which the physical capital is crowded out during boom period yet the scarcity in physical capital results in larger recession during burst period.

3 Crowd-out effect of overbuilding: insight from a simple model

Optimistic expectation about future house price incurs an upward jump of household demand function of real estate. This upward jump induces the housing market boom with inflation in housing price and overbuilding. If there is semi-inelastic supply existing in the economy, any demand-side change will not result in overbuilding problem a lot. On the contrary, if the supply function is elastic enough, a little demand boom could trigger a large overbuilding. The extent of overbuilding, and in turn the extent of crowded physical capital, is decided by the shape of supply function and demand function because the main mechanism through which crowd-out effect works is the general equilibrium in the end, and we need the supply function as well as the demand function to work together. In this section I first introduce a simple Aiyagari-Huggett model beneath an incomplete market. Then I use this model to illustrate that overbuilding leads to crowd-out effect which is influenced by intratemporal substitution, liquidity, precautionary saving and wealth inequality.

3.1 A simple Aiyagari-Huggett model

It is a standard Aiyagari-Huggett model where households use wage income and asset return to fulfill their demand for consumption and real estate. The durable good, house, is produced by real estate companies in complete market with land, capital and labor. Similarly the non-durable good is produced in complete market with capital and labor.

For simplicity I assume that household i provides inelastic labor supply with 1 unit exogenously to solve the problem

$$\max_{c_t^i, h_t^i, a_t^i} \sum_{t=0}^{\infty} \beta^t U^i \left(c_t^i, h_t^i\right) \tag{7}$$

s.t.

$$c_t^i + a_t^i + p_t^H h_t^i = R_t a_{t-1}^i + w_t \varepsilon_t^i + \left(1 - \delta^H\right) p_t^H h_{t-1}^i + T_t \tag{8}$$

$$-a_t^i \leq \gamma p_t^H h_t^i \tag{9}$$

where equation 8 is the budget constraint and equation 9 is the collateral constraint. a_t^i could either be positive or negative but in aggregate level is positive as it is the supply of capital which is used to produce durable and non-durable goods. w_t is wage and household earns productivity-weighted wage income from which ε_t^i is corresponded idiosyncratic shock. p_t^H is the real house price. h_t^i is the house amount hold by household i. T_t is the lump-sum transfer to household. For simplicity I further assume the real interest rate is fixed at $\overline{R}$.[19]

The production sector is a complete market where firms produce non-durable good via $Y_{N,t} = A_{N,t} K_{N,t-1}^{\alpha} L_{N,t}^{1-\alpha}$ and durable good via $Y_{H,t} = A_{H,t} \overline{L}_H^{\theta} K_{H,t-1}^{\nu} L_{H,t}^{1-\nu-\theta}$. The labor market is closed by inelastic labor supply such that $L_{N,t} + L_{H,t} = 1$. The capital is provided by household such that $K_{N,t-1} + K_{H,t-1} = K_{t-1} = \int a_{t-1}^i dG_{t-1}$ where G_{t-1} is the cumulative distribution function of household. The non-durable good is used either to consume or to invest so that $Y_{N,t} = K_t - (1-\delta)K_{t-1} + C_t$. Meanwhile all the increment in house is produced by real estate companies so that $Y_{H,t} = H_t - (1-\delta^H)H_{t-1}$ where $H_{t-1} = \int h_{t-1}^i dG_{t-1}$.

Proposition 2. *Household will adjust their consumption of non-durable goods based on overbuilding and precautionary saving. The extent of adjustment is decided by*

[19] It is not a too strong assumption since this could happen in many scenarios. For instance the nominal interest rate reaches the ZLB and the price is fixed. Or an open economy where the real interest rate is bounded by the international financial market.

$$
\widetilde{c}_t = \underbrace{\Phi_H \widetilde{h}_t}_{\text{substitution effect}} - \underbrace{\Phi_\mu \widetilde{\mu}_t}_{\text{credit effect}} + \underbrace{\Phi_{p^H}\left[\frac{1}{1-(1-\delta^H)\frac{1}{R}} F^H(\widetilde{H}_t) - \frac{(1-\delta^H)\frac{1}{R}}{1-(1-\delta^H)\frac{1}{R}} F^H(\widetilde{H}_{t+1})\right]}_{\text{wealth effect}} \tag{10}
$$

$$
- \underbrace{\Phi_{cov}\widetilde{cov}_t}_{\text{precautionary saving effect}}
$$

where $F^H(\cdot)$ is the inverse supply function,

$$
\Phi_H = \frac{\frac{\lambda}{\lambda-\mu}\eta_{c,p^H} - \eta_{c,p^c}}{\eta_{h,p^c} - \frac{\lambda}{\lambda-\mu}\eta_{h,p^h}} \tag{11}
$$

$$
\Phi_\mu = \frac{\mu}{\lambda-\mu}\frac{\eta_{ch}}{\eta_{h,p^c} - \frac{\lambda}{\lambda-\mu}\eta_{h,p^h}} \tag{12}
$$

$$
\Phi_{p^H} = \frac{\eta_{ch}}{\eta_{h,p^c} - \frac{\lambda}{\lambda-\mu}\eta_{h,p^h}} \tag{13}
$$

$$
\Phi_{cov} = \frac{\eta_c}{\eta_{h,p^c} - \frac{\lambda}{\lambda-\mu}\eta_{h,p^h}} \frac{\beta\left(1-\delta^H\right)\overline{cov}}{h}
$$

and $\eta_{c,p^H} = \frac{u_{ch}u_h}{u_{ch}^2 - u_{cc}u_{hh}}\frac{1}{c}, \eta_{c,p^c} = \frac{u_{hh}u_c}{u_{ch}^2 - u_{cc}u_{hh}}\frac{1}{c}$, $\eta_{h,p^c} = \frac{u_{ch}u_c}{u_{ch}^2 - u_{cc}u_{hh}}\frac{1}{h}$, $\eta_{h,p^h} = \frac{u_{cc}u_h}{u_{ch}^2 - u_{cc}u_{hh}}\frac{1}{h}$, $\eta_{ch} = \frac{u_c u_h}{u_{ch}^2 - u_{cc}u_{hh}}\frac{1}{ch}$, $\eta_c = \frac{u_c}{u_{ch}^2 - u_{cc}u_{hh}}\frac{1}{c}$.

Proposition 2 shows that any perturb occurred in real estate market could be passed to non-durable consumption through 4 channels: substitution effect, wealth effect, credit effect and precautionary saving effect.[20] The directions of these four channels through which the housing market boom affects the consumption of non-durable goods are determined by the relative strength of intertemporal and intratemporal elasticity of substitutions between non-durable and durable goods, as well as the relative feature that housing wealth played in budget constraint and credit constraint. When overbuilding happened, a positive $\widetilde{h}_t$ and $\widetilde{H}_t$ will generate a variation in nondurable consumption through substitution and wealth effect. Meanwhile it may also affect consumption endogenously through the credit effect and precautionary saving effect. This variation in consumption triggered by a boom in house market will influence physical investment ultimately and induce a recession in the future as long as the total effect is positive.

It is worth noticing that η_{x,p^y} denotes the standard Frisch elasticity of variable x with respect to the relative price of y, which is pivotal regulating the clouts of the four effects. If nondurable consumption responses to housing price more than the nondurable goods price, a derivation in the holding of housing service will spark a larger echo in nondurable goods consumption which

[20] Berger et al. (2018) only discussed two of them meticulously but not focused on credit effect and precautionary saving effect. Additionally their goals about decomposition is related to analyze the inequality problem caused by house price inflation.

is unveiled in Φ_H. Contrariwise, if the response of household holding of housing service to nondurable goods price was larger(than to housing price), the elasticity of substitution would attenuate all four channels because now the consumption of durable housing is more stable and household does not variate their consumption a lot, which implies a minor pass through from the consumption of housing servicing to the consumption of nondurable goods.

3.2 Crowd-out effect of overbuilding

I will discuss how intratemporal elasticity of substitution, credit constraint, precautionary saving and wealth inequality amplify the crowd-out effect sparked by overbuilding. Intuitively overbuilding will affect the consumption of non-durable goods and crowd out physical investment as consumption and house are closer linked via complement(in aggregate level). Likely overbuilding will also relax the collateral constraint and this relaxation benefits household as they can borrow more to smooth their consumption demand. Similarly overbuilding will also pass to the consumption response because the inverse supply function of residential assset $F^H(\cdot)$ is monotonic increasing in complete market and more new construction leads to higher housing price in equilibrium. Because the house price entries into the budget constraint of household which alters their income, an increased price makes household feel wealthier as house works not only as utilitarian goods but also as an asset in budget constraint. This increased price derived from monotonic increasing supply function stands that overbuilding will also correspond with house price inflation through the supply side.

By aggregating the consumption decision of household from equation 10 and combining the FOC in supply sectors I can obtain the relationship between overbuilding and physical investment which is summarized in proposition 3.

Proposition 3. *The aggregate investment is driven by overbuilding and precautionary saving following*

$$\begin{aligned} \widetilde{II}_t = -\Bigg\{ & \left(\Phi_H + \frac{\nu}{\alpha} p^H H\right) \int \widetilde{h}_t^i dG_i - \Phi_\mu \int \widetilde{\mu}_t^i dG_i \\ & + \Phi_{p^H} \left[\frac{1}{1-(1-\delta^H)\frac{1}{R}} F^H(\widetilde{H}_t) - \frac{(1-\delta^H)\frac{1}{R}}{1-(1-\delta^H)\frac{1}{R}} \mathbb{E}_t F^H(\widetilde{H}_{t+1}) \right] \\ & - \Phi_{cov}^i \int \widetilde{cov}_t^i dG_i + \frac{\nu}{\alpha} Y_H p^H F^H(\widetilde{H}_t) \Bigg\} \end{aligned} \tag{14}$$

The overbuilding, $\widetilde{H}_t = \int \widetilde{h}_t^i dG_i > 0$, will crowd out physical investment as long as the substitution effect Φ_H and wealth effect Φ_{p^H} are not negative enough.

Equation 14 shows that the overbuilding will lead to a smaller physical investment and a lower physical capital afterwards throguh different story in demand and supply side, at least within a range of parameters. The term Φ_x relates to the contribution of pass-through from housing service to the consumption of nondurable goods and the term $\frac{\nu}{\alpha}$ relates to the supply side effect. Next I am going to discuss detailedly how intratemporal elasticity of substitution, credit constraint, precautionary saving and wealth inequality influence the crowd-out effect of overbuilding.

3.2.1 Intratemporal elasticity of substitution

Intertemporal substitution has been widely studied as it related to the Euler equation and monetary policy. However intratemporal substitution between durable and non-durable goods consumption is still in barren not only theoretically but also empirically. In this section I argue that intratemporal substitution is also important to the decision making of household, at least in analyzing the crowd-out effect created by overbuilding. Empirically in housing market, intratemporal substitution is much more important and powerful than intertemporal substitution[21] as household are mostly myopic or financial constrained so they do not pay much attention or simply cannot weight future consumption on decision today. Focusing on the coefficients of crowd-out effect in proposition 10, it comes to a conclusion in corollary 1 that the intratemporal substitution could theoretically enlarge the crowd-out effect across demand side of housing market.

Firstly I define the intertemporal and intratemporal elasticity of substitution as

Definition 2. The intratemporal elasticity of substitution is

$$\mathrm{ES} = -\frac{\partial \ln \frac{h}{c}}{\partial \ln \frac{U_h}{U_c}} \tag{15}$$

and the intertemporal elasticity of substitution to consumption bundle is

$$\mathrm{EIS} = -\frac{U_{BB}}{U_B}$$

Then based on the definition I obtain following corollary.

Corollary 1. *Ceteris paribus, household with larger intratemporal elasticity of substitution relative to intertemporal elasticity of substitution, as well as the standard CRRA utility function, will crowd out less investment through substitution and wealth effect.*

It is easy to understand corollary 1 that non-durable goods and housing services are both normal goods and if they are substituted more with each other, the crowd-out effect will be further muted because more consumption of housing servicing leads to less consumption of non-durable goods. The intratemporal elasticity of substitution measures the extent to which increased house could be substituted with increased consumption in intraperiod utility level.[22]

[21] Khorunzhina (2021) did this vital work empirically.

[22] It is intuitive to focus on U_{ch} which is closely related to the complementarity between house and non-durable good.

Conversely the intertemporal elasticity of substitution is the metric of propensity to substitute the total consumption bundle over different period. If $\mathrm{ES} > \mathrm{EIS}$ holds, the household will prefer adjusting their consumption between durable and nondurable goods within a period, to adjusting their consumption interperiodicly. The larger the intratemporal elasticity of substitution is relative to intertemporal elasticity of substitution, the less increased consumption responses to the overbuilding within this period, as now they are more substitute rather than complementary. Intratemporal substitution is so powerful enough that decreased relative elasticity will magnify substitution and wealth effect as it directly affects the marginal benefit in utility instead of involving the budget constraint and endowment. However the credit effect derived from financial friction is ambiguous to the decreased relative elasticity[23] because the willingness to substitute nondurable goods for durable goods is constrained by collateral requirement, which will also change the extent to accommodate consumption portfolio.

I solve the model 7 with unit intratemporal elasticity such that $\mathrm{ES} = 1$ but different the intermporal elasticity from 0.67 to 0.5, which is equivalent to increase the relative intratemporal elasticity. Figure 5a shows that the substitution effect shrinks, along with the larger relative intratemporal elasticity. Intuitively if there is a preference shock which increases the relative intratemporal elasticity of substitution relative to intertemporal elasiticity of substitution, the same amount of overbuilding will decrease the consumption response and then crowd out less investment as the complementarity between consumption and housing service is diluted by the stronger substitution. Meanwhile more propensity to substitution will also relax the collateral constraint because the demand to consume nondurable goods is smaller. Yet this higher relative elasticity exacerbates the consumption bundle and forces more household to stay financially constrained in steady state. To illustrate above argument mathematically, we can assume there are two economies a and b whose intertemporal elasticity of substitution satisfy $\frac{\mathrm{ES}_a}{\mathrm{EIS}_{c,a}} < \frac{\mathrm{ES}_b}{\mathrm{EIS}_{c,b}}$ and there are two extra exogenous tax rebate to household which generate the same jump in nondurable consumption $\Delta C_a = \Delta C_b = 0.5$. Because the intratemporal elasticity in economy a is smaller than that in economy b, people in economy a will increase their holding of durable consumption more, for instance, $\Delta H_a = 0.5 > \Delta H_b = 0.3$. These increased holding of residential asset slacks the collateral constraint and the extent of slackness should be proportional to the change of residential asset. Therefore the Karush–Kuhn–Tucker multiplier of equation 9 follow the relationship $\Delta\mu_a < \Delta\mu_b < 0$ which implies $\Phi_\mu^a > \Phi_\mu^b > 0$ in equation 10. This is shown in figure 5b in which the credit constraint grows larger and larger.

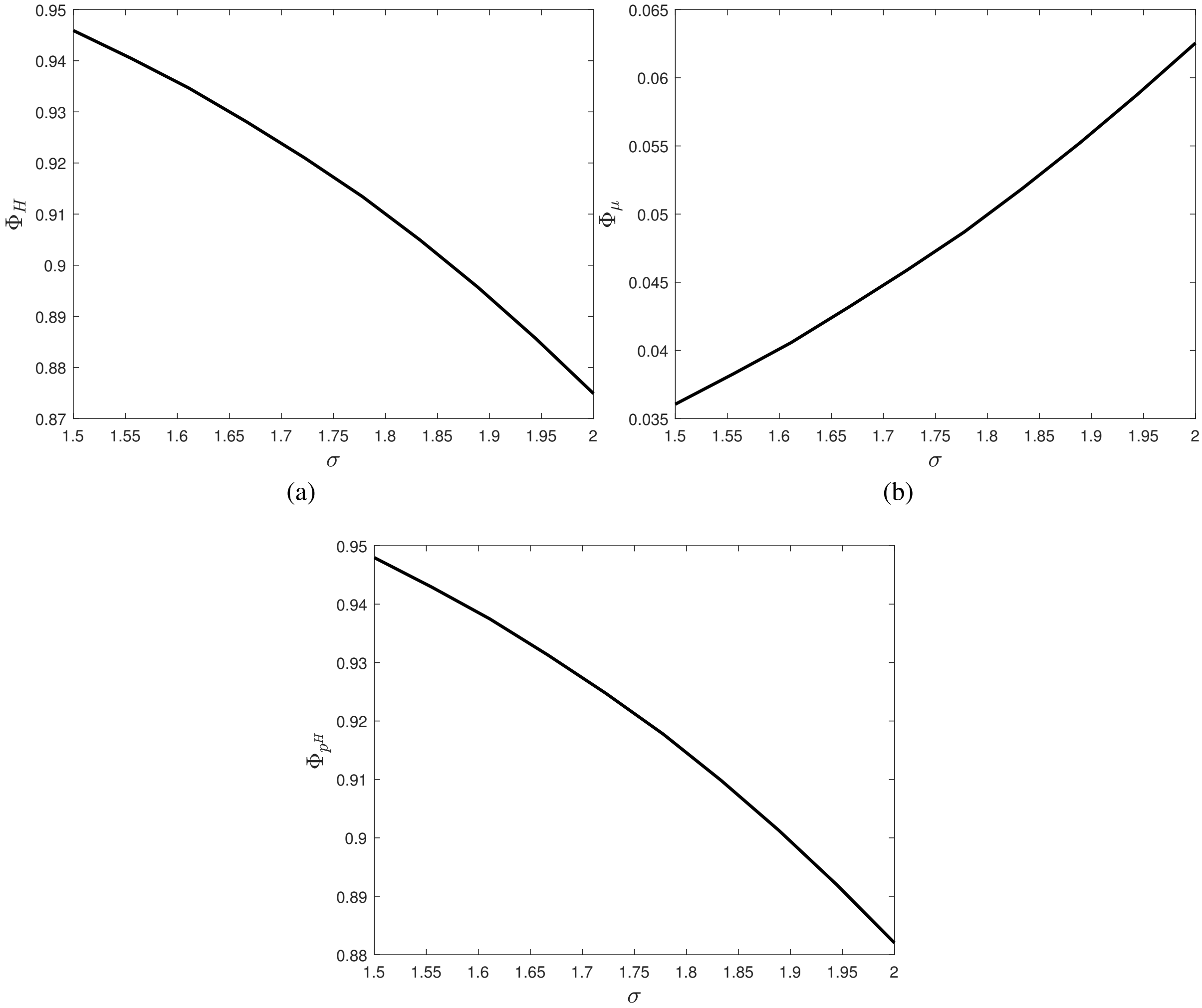
In addition to substitution effect and credit effect, overbuilding will also be passed to the consumption response through the inverse supply function $F^H(\cdot)$ because the residential asset is also a type of asset which enters into the budget constraint, except for acting as consumables in utility function. An inflation(of housing price) in housing market, inspired by overbuilding, also provide liquidity to household as long as they previously hold some amount of house because

[23] It depends on the sign of Φ_μ whose sign cannot be derived analytically. However it is always positive within a range of reasonable parameters.

of the asset's pecuniary character. This wealth effect is amplified as the value, that one unit of housing service provides, now can be transferred to utilitarian value more with a smaller intratemporal elasticity of substitution. The intratemporal consumption decision between durable and nondurable goods, which comes from wealth effect, follows the relative marginal utility equation $\frac{U_{h,t}}{U_{c,t}} = f\left(p_t^+, p_{t+1}^-\right)$. This equation is intuitive and easy to understand. Household can use money to marginally increase one unit of housing servicing at time t and get $U_{h,t}$ unit of extra utility. Alternatively the household can also use the money that affords the housing servicing to buy nondurable consumption and get $U_{c,t}f\left(p_t^+, p_{t+1}^-\right)$ unit of extra utility. The extra unit of nondurable goods is rescaled by the price of housing servicing as the money that affords one unit of housing servicing does not afford the same unit of nondurable goods. If I given a same jump in housing price $\Delta p_{a,t}^H = \Delta p_{b,t}^H > 0$ on RHS and held the housing servicing, there would be an jump in nondurable goods consumption which results in a positive Φ_{p^H} in equation 10. A jump in nondurable goods consumption $\Delta C_t > 0$ will fulfill household's demand for nondurable goods with smaller marginal utility of nondurable goods $\Delta U_{c,t} < 0$ but a higher demand for durable goods(because of complementarity) with larger marginal utility of durable goods $\Delta U_{h,t} > 0$. A larger relative intratemporal elasticity of substitution allows larger variation between marginal utility of housing service and nondurable goods consumption, so a smaller nondurable goods consumption jump can support a given variation($\Delta f\left(p_t, p_{t+1}\right) > 0$) in relative marginal utility. The crowded-out effect is amplified further through the wealth effect and the pass-through from durable goods to nondurable goods. Figure 5c exhibits the decreased strength of wealth channel to crowd-out effect as the relative intratemporal elasticity rises and the one unit housing service becomes less important (can be replaced by nondurable consumption easier). Although in this section I do not quantitatively introduce the aggregate shock into the model and investigate the magnitude change of the precautionary saving effect, it is easy to comprehend that a higher relative intratemporal elasticity of substitution will conduce a smaller precautionary saving effect because the household cherishes the balanced consumption portfolio within a period more than that over periods. To sum up, overbuilding affects crowd-out effect via four channels while three of them are influenced by relative intratemporal elasticity of substitution.

3.2.2 Credit constraint and Liquidity

Overbuilding and house market boom push household to spend more money on nondurable goods through substitution effect as now their holding of real estate jumped. Additionally if the economy is incomplete and household cannot fully insure their idiosyncratic shock through financial market, the consumption of household may be constrained by an incomplete market where they cannot borrow as much as they want to confront the bad shock. This credit constraint generates the liquidity problem and some households may be constrained from time to time and not fulfill their consumption demand even though they could repay the amount they borrow in the future. Overbuilding offers more asset that household could use to borrow as collateral and



(a) (b)

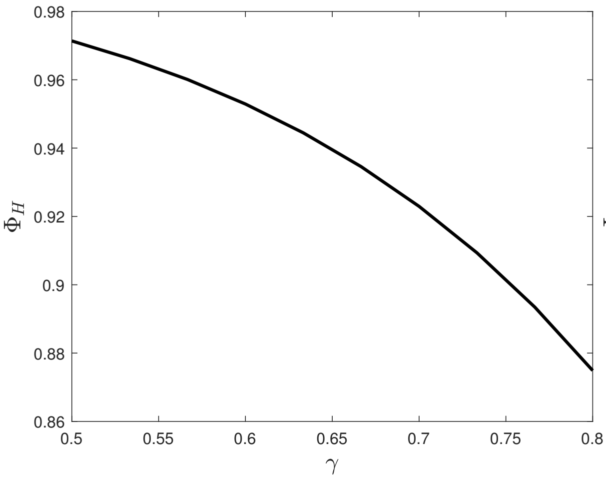
(c)

Figure 5: Elasticity of Substitution

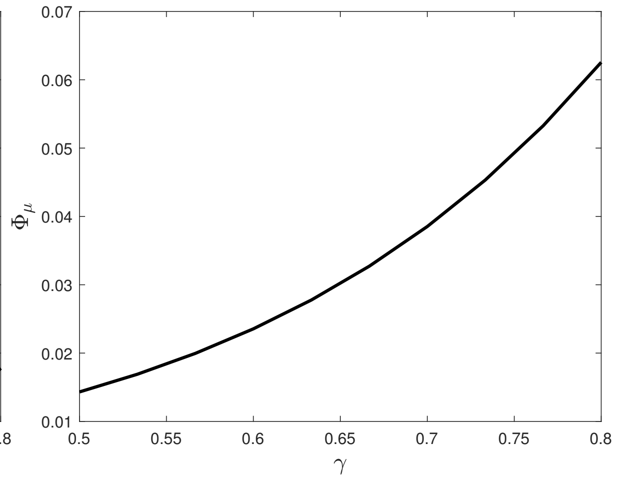
hence it relaxes the previous credit constraint. In figure 6a the extend of financial friction is decreased by increasing the proposition of housing services whose value can be used to borrow money from 0.5 to 0.8. It verifies the argument that a tighter collateral constraint induces a higher substitution effect since one-unit-increased housing servicing becomes more valuable in utility in steady state.

Additionally, a marginal relaxation on the binded collateral constraint represents a smaller K-T multiplier, $\Delta\mu < 0$ in equation 10, and a tighter constraint connects with a smaller nondurable consumption response Φ_μ which ensues a smaller crowded out effect. To understand the credit effect I assume that the unexpected tax rebate spawns the same increased nondurable goods consumption $\Delta C_{a,t} = \Delta C_{b,t}$ and the collateral constraint γ in economy a is tighter than that in economy b and accordingly $\gamma_a < \gamma_b$ will hold in equation 9 as well as in figure 6. A tighter financial constraint reveals a larger K-T multiplier response ergo the absolute change of multiplier in economy a is larger than that in economy $b(\Delta\mu_a < \Delta\mu_b < 0)$. This delineates that a unit change in marginal value of housing servicing in financial constraint performs feebly when the constraint is tight because the unit change in marginal value is now “cheaper” than that in steady state. Figure 5b tells us explicitly the credit crunch (a positive $\widetilde{\mu}_t$) inspired by overbuilding decreases less consumption (or crowd out more investment) when the financial friction is larger.

Comparing to the credit effect, the financial friction works in the opposite direction in wealth effect(but the same in substitution effect). Mathematically a larger financial friction results in a larger K-T multiplier and a larger μ in 13 therefore a larger wealth effect as shown in figure 6c. The mechanism in backdrop is the same as substitution aforementioned since the housing services itself and its price play the same role in collateral constraint 9 and their effect to the pass through should be the same.



(a)



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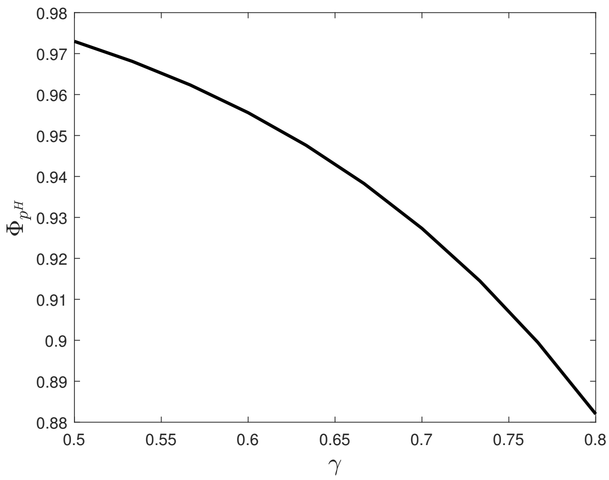


(c)

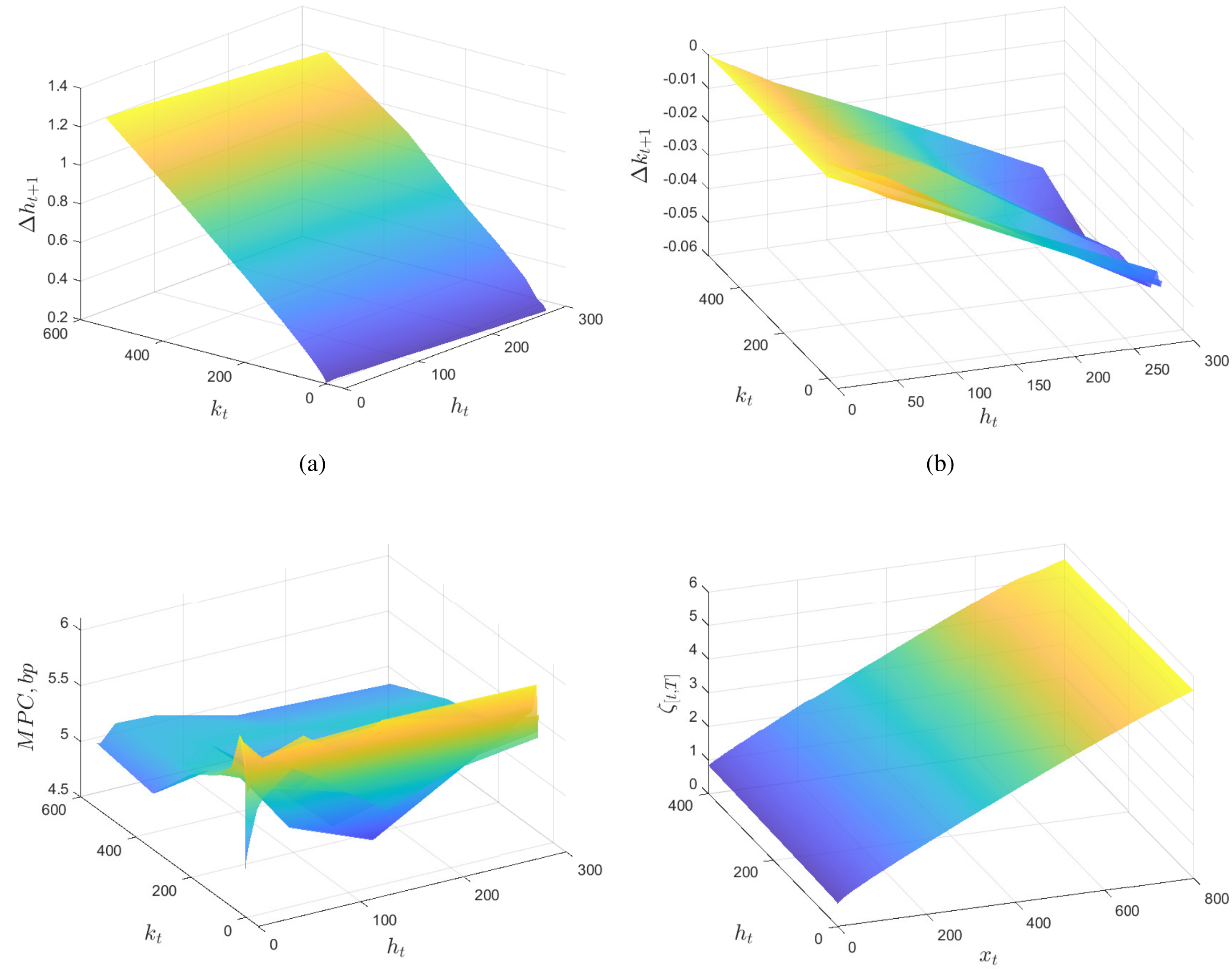
Figure 6: Financial friction

3.2.3 Precautionary saving and Wealth inequality

Household usually will not consume as much as they would do under the scenario without any idiosyncratic shock or they can perfectly insure the idiosyncratic shock. Household have the propensity to put more income into pocket to save for insuring idiosyncratic shock which we call precautionary saving motive. The last term of equation 14 shows that precautionary saving decreases the consumption adjustment as household save extra $\Phi_{cov}\widetilde{cov}_t$ amount instead of spending out when facing the uncertainty in income.

In addition to the four effects discussed above, substitution effect, credit effect, wealth effect and precautionary saving effect, overbuilding can amplify the crowd-out effect through the lens of business cycle. It is well known that idiosyncratic shock is countercyclical while overbuilding is mostly procyclical. Therefore when the overbuilding happens household are less precautionary since aggregate economic conditions are better and there are less large idiosyncratic shock. Boom and lower variation of idiosyncratic shock persuades household that economy is going to be better and they become optimistic to consume more and save less. $\widetilde{cov}_t$ in equation 14 will drop which indicates that household save less and consume more when overbuilding and boom arrival. However this amplification is not covered in my numerical experiment which is left for future study.

Additionally, the wealth distribution may also manipulate the crowd-out effect triggered by overbuilding in aggregate level. Since the increased holding of housing service is funded by liquid asset and wage income, the absolute amount of large jump per capita in holding of housing service comes from those household who hold a lot of liquid asset and earned high wage income at steady state. After aggregating the consumption decision over household which is shown in equation 14, I can conclude that the distribution of wealth is important as it affects the distribution of coefficient and in turn affect the aggregate crowd-out effect. Figure 7a plots the distribution of changing in holding of housing servicing facing a decrease in house price. The wealthier household who hold a lot of liquid asset is the household who buys more unit of housing service and then who decreases the physical investment as shown in figure 7b. The the cohort mass of the wealth people is small whereas the wealth distribution is right-skewed and the skewness is shown in figure 8a for residential asset and figure 8b for effective liquid asset. The most wealth in economy is held by the least people in the top and this right-skewed wealth distribution amplifies the crowd-out effect of overbuilding through the term $\int \widetilde{h_t}^i dG_i$ in equation 14. Furthermore, the distribution of MPC is left-skewed(figure 7c) and the standard general equilibrium effect of hand-to-mouth household will also be effective as it works in the pass through of monetary policy. This left-skewed MPC likewise amplifies the crowd-out effect of overbuilding but through the term $\int \widetilde{\mu}_t^i dG_i$ in equation 14. Figure 7d exhibits the wealth distribution effect of a demand-driven boom, which is triggered by an expectation of housing price inflation as I argued in corollary 2, instead of a supply-driven which I use in figure 7a and 7b. The result does not change the attenuation direction formed by wealth distribution which demonstrates that it is independent with type of housing market boom and direction of the change of house price.



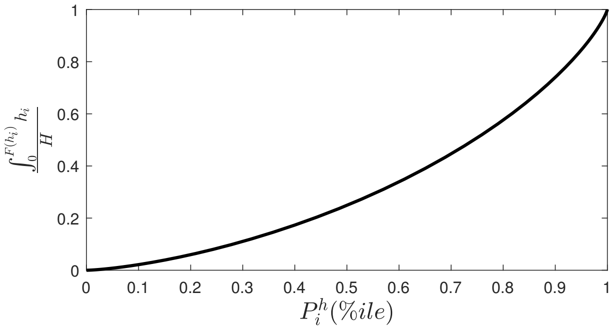
(a) (b)

(c) (d)

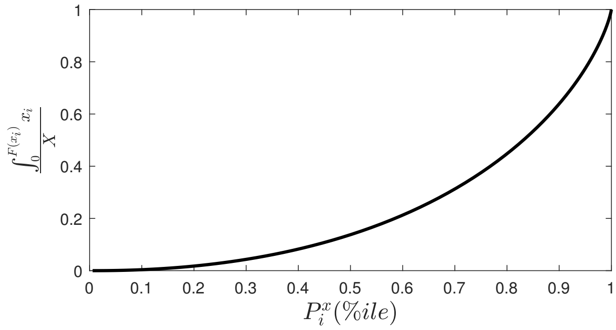
Figure 7: Wealth Distribution

3.2.4 Optimistic expectation and overbuilding

Previous arguments are focus on the crowd-out effect generated by overbuilding and we discussed different mechanisms through which this effect works depending on the assumption that overbuilding is already happened. Here I demonstrate that the existence of overbuilding is not a strong assumption and it can easily be created by an optimistic expectation about housing market in the future. When household have a positive expectation about the change of housing price in the future, they will increase their holding of real estate in this period which is similar to the change in consumption induced by intertemporal new keynesian cross. Corollary 2 shows that an increase in the expectation of the housing price in time $T+1$ will marginally provoke $-\left[\beta\left(1-\delta^{H}\right)\right]^{T}\Pi_{s=1}^{T}\frac{\lambda_{t+s}}{\lambda_{t+s}-\mu_{t+s}}\lambda_{t+T+1}/u''_{h^i}$ unit of increase in demand of housing service. If the expectation is driven by optimism or fake news about future, the increased new buildings will become “over”-building as it is not support by the fundamental change in economy but support



(a) Lorenz curve of $F(h_i)$



(b) Lorenz curve of $F(x_i)$

Figure 8: Lorenz curve

by a mirage. After this mirage vanishes, the crowd-out effect engenders a recession because of the lack of physical capital produced by the illusion in housing market boom.

Corollary 2. *Ceteris paribus, an positive expectation about the housing price change in time* $T+1$ *will induce a jump in demand of housing service in time t. The response extend follows*

$$\widetilde{h}_t^i\Big|_{h_{t+i},\mu_{t+i},\lambda_{t+i},,i\in[1,T]} = \zeta_t^i dp_{t+T+1}^H \tag{16}$$

where $\zeta_t^i = -\frac{1}{u''_{h^i}}\mathbb{E}_t\left[\beta\left(1-\delta^H\right)\right]^T \Pi_{s=1}^T \frac{\lambda_{t+s}}{\lambda_{t+s}-\mu_{t+s}}\lambda_{t+T+1}$

4 Crowd-out effect of overbuilding: Full fledged model

In last section I use a simple model analytically show that an expectation in future housing market boom will inspire household to increase their holding of durable goods' consumption which in turn crowd out the physical investment. This crowd-out effect is influenced by relative intratemporal elasticity of substitution, credit constraint and wealth distribution. In this section I use a full fledged model to analyze the crowd-out effect quantitatively. By linking the model to data I show that news about future can generate a boom-burst cycle in housing market. When the news is fake and the fraud is not realized by household until several periods later, the boom which is supported by a fake news instead of fundamental creates the overbuilding, that induces a large loos in output and consumption during the burst period. I will first introduce the model I used to quantify the drawback of crowd-out effect. Then I use calibration and SMM connect the model with data. In the end I show the large break in economy caused by overbuilding in mirage via some impose response functions.

4.1 Model Setting

4.1.1 Household

Continue household[24] holds housing servicing h and liquid asset b at time t which he takes from last period. He chooses the non-durable consumption c, labor supply l, housing service h' and liquid asset holding b' at time t to solve the optimization problem

$$V(h_{t-1}, b_{t-1}, \varepsilon_{t-1}) = \max_{c,l,b',h'} U(c_t, h_t, l_t) + \beta(1-\theta^d)EV(h_t, b_t, \varepsilon_t)$$

$$\begin{aligned} \text{s.t.} c_t + Q_t b_t + p_t^h \left[h_t - (1-\delta^h)h_{t-1}\right] = R_t Q_{t-1} b_{t-1} + (1-\tau)w_t l_t \varepsilon_t + \Pi_t^h \\ - p_t^h C\left(h_t, h_{t-1}\right) + T_t \end{aligned} \tag{17}$$

$$-Q_t b_t \leq \gamma p_t^h h_t \tag{18}$$

where p_t^h is the relative price of housing unit at time t. R_t is the gross real return of liquid asset which follows $R_t = \frac{Q_t(1-\delta)+r_t}{Q_{t-1}}$. $C\left(h_t, h_{t-1}\right)$ is the adjustment cost function when household want to adjust their holding of housing servicing. γ is the parameter governing the slackness of collateral constraint. δ^h is the depreciation rate. τ is the wage income. Π_t^h is the restitution from construction companies. T is the lump-sum tax transfer payed by government. θ^d is the death rate. ε is the idiosyncratic income shock which follows logarithmic AR1 process with coefficient ρ_ε and standard derivation σ_ε.

The adjustment function follows the canonical form

$$C\left(h_t, h_{t-1}\right) = \frac{\kappa_1}{\kappa_2}\left(h_{t-1} + \kappa_0\right)\left|\frac{h_t - h_{t-1}}{h_{t-1} + \kappa_0}\right|^{\kappa_2}$$

The utility function follows the CRRA form[25]

$$U(c_t, h_t, l_t) = \frac{\left(c_t^\phi h_t^{1-\phi}\right)^{1-\sigma}}{1-\sigma} + \kappa\frac{l_t^{1+\psi}}{1+\psi}$$

4.1.2 Firm

There are two types of firms, construction firms who produce the housing servicing and the non-durable goods producers. All of these two types of producers are staying in complete market but because the construction firms also use exogenous land supply as an input to construct house, they earn non-zero profit which in the end refunded back to their holder, household.

[24] Here for simplicity I omit the index for specific household i.

[25] Piazzesi et al. (2007) use CEX data suggest that intratemporal elasticity of substitution is close to 1. In other words the utility function form of durable and nondurable goods is close to standard Cobb-Douglas case.

Non-durable goods producer use

$$\begin{aligned} Y_{N,t} &= C_t + I_t + C\left(h_t, h_{t-1}\right) \\ &= A_{n,t} K_{n,t}^{\alpha} L_{n,t}^{1-\alpha} \end{aligned} \tag{19}$$

to maximizes profit with the cost from real rental rate of capital used by non-durable goods producer K_n and related wage payment to labor L_n .

Similarly, durable goods (housing services) producer use

$$\begin{aligned} Y_{H,t} &= \left[H_t - (1-\delta^h) H_{t-1}\right] \\ &= A_{h,t} \overline{LD}_t^{\theta} K_{h,t}^{\nu} L_{h,t}^{\iota} \end{aligned} \tag{20}$$

to maximizes profit with the cost from real rental rate of capital used by durable goods producer $K_{h,t}$ and related wage payment to labor $L_{h,t}$. The$\overline{LD}_t$ in production function is the exogenous land supply follows $\overline{LD}_t = \overline{LD} A_{L,t}$ and the new construction is homogeneous to each production factor therefore the share of input satisfies $\theta + \nu + \iota = 1$.

4.1.3 Capital Producer

The capital producer uses final nondurable goods Y_N to produce capital following the maximization problem

$$\begin{aligned} &\max E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \Lambda_{t,t+\tau} \left\{ \left(Q_\tau - 1\right) I_\tau - f\left(I_\tau, I_{\tau-1}\right) I_\tau \right\} \\ &\text{s.t. } f\left(I_\tau, I_{\tau-1}\right) = \frac{\psi_I}{2} \left(\frac{I_\tau}{I_{\tau-1}} - 1 \right)^2 \end{aligned}$$

By solving above optimization problem I could get the capital price as a convex function of investment which is shown below

$$\begin{aligned} Q_t = 1 + \frac{\psi_I}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 + \psi_I \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} - \\ E_t \beta \Lambda_{t,t+1} \psi_I \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \end{aligned} \tag{21}$$

So the goods market clearing condition will become

$$Y_{N,t} = C_t + I_t + f\left(I_t, I_{t-1}\right) I_{t-1} + p^h C(h', h)$$

4.1.4 Market cleaning

Capital is supplied by household with their liquid asset and labor is supplied in effective form

$$K = \left(1-\theta^{d}\right)\int bdG = K_n + K_h$$

$$L = L_h + L_n = \int \varepsilon l dG$$

$$H = \left(1-\theta^{d}\right)\int hdG$$

The goods market cleaning condition is

$$C + I + p^{h}C(h', h) = A_n K_n^{\alpha} L_n^{1-\alpha}$$

where $K' = (1-\delta)K + I$

Similarly, the housing market cleaning condition is

$$\left[H' - (1-\delta^{h})H\right] = A_h \bar{L}^{\theta} K_h^{\nu} L_h^{\iota}$$

The return of gross liquid asset b comes from two component: capital return from firms r and capital gain $\frac{Q'(1-\delta)}{Q}$.

In the end the government close the economy by $T = \tau w L + \theta^{d}\left(K + p^{h}H\right)$ and $\Pi^{h} = p^{h}Y_H - wL_h - (r-1+\delta)K_h$ as all the new born household hold zero liquid asset and housing servicing.

The model contains three types of shock: *contemporaneous unexpected shock, news shock and noise shock* which I introduce detailedly in appendix G.7.1. I introduce the news and noise shock following Chahrour and Jurado (2018) who introduced the news and noise representation to overcome the observational equivalence problem in previous literature such as Schmitt-Grohé and Uribe (2012), Barsky and Sims (2012) and Blanchard et al. (2013).

4.1.5 Shocks

There are two fundamental shocks on the TFP of the two production function 19 and 20 respectively. These two shocks a_t^i follows the standard logarithm AR(1) process $\log(a_t^i) = \rho_a^i \log(a_{t-1}^i) + \varepsilon_t^{a^i}$ where $i \in \{h, n\}$. Thus the TFP of these two production functions follow $A_{n,t} = a_t^n \overline{A}_n$ and $A_{h,t} = a_t^h \overline{A}_h$.

Meanwhile I introduce a preference shock Φ_t^{ϕ} and the shock to its growth rate $\Phi_{g,t}^{\phi}$ to the preference ϕ in utility function in the demand side, cooperating with a land supply shock Φ_t^{L} and the shock to its growth rate $\Phi_{g,t}^{L}$ in the supply side to determinate the housing market.

Meanwhile to incorporate the noise and news into the model I assume that the household can get a news related to the shocks up to 8 periods before they realize and I defined them in companion form in equation 64. However the agents cannot perfectly observe these shocks but mixed with noisy observation shock to $\widetilde{\Phi}_t^i$ and $\widetilde{\Phi}_{g,t}^i$ in equation 66.[26]

4.2 Calibration

4.2.1 Parameter

Most of the parameters I used in production side comes from literature which is standard and robust. I relegate them into appendix G.1 which is summarized in table 5. I use the discount factor, disutility to labor supply, and three parameters in production side to match the gross real interest rate at 1.015 quarterly, labor supply at 1, physical investment over GDP at 0.13 and new construction over GDP at 0.05. The physical investment over GDP is estimated from Private Non-Residential Fixed Investment over Gross Domestic Product and the new construction over GDP is estimated from Private Residential Fixed Investment over Gross Domestic Product. The parameters in adjustment cost function is in line with Kaplan et al. (2018) and Auclert et al. (2021). The intertemporal elasticity of substitution and preference between durable and nondurable goods are borrowed from Kaplan et al. (2020). The AR1 coefficient and standard derivation of idiosyncratic shock follow the estimation by McKay et al. (2016). The death rate is estimated from the Underlying Cause of Death provided by Centers for Disease Control and Prevention from 1999 to 2020. All the value of corresponding parameters I used are summarized in table 2.

Table 2: Key Parameter Values

Parameter	**Value**	**Description**
β	0.9749	Discount factor
τ	0.20	Labor income tax
κ	-1.28	Disutility to supply labor
θ^d	0.21%	Death rate
γ	0.8	Slackness of collateral constraint
κ_0	0.25	Adjustment cost silent set
κ_1	1.3	Adjustment cost slope
κ_2	2	Adjustment cost curvature
σ	2	Inverse of intertemporal elasticity of substitution
ϕ	0.88	Preference between durable and nondurable
ρ_ε	0.966	AR1 coefficient of income shock
σ_ε	0.25	SD of income shock

[26] I define the news and noise shocks following the suggestion made by Chahrour and Jurado (2018) because this form does not suffer from the observational equivalence problem.

4.2.2 Data to Model: Moment Matching

Even though I do not specifically match the moments in distribution, my model generates a lot of merits to replicate the moments extracted from data. Table 3 shows that my model has some nature ability to unveil the reality which I compare the data estimated by Kaplan et al. (2014) and Kaplan et al. (2018) and the moments calculated from model.

Table 3: Distribution Moments

Description	Data	Model
Poor Hand-to-Mouth Household	0.121	0.1102
Wealthy Hand-to-Mouth Household	0.192	0.2059
Top 10 percent share of Liquid asset	0.8	0.5
Top 10 percent share of Iliquid asset	0.7	0.3

To build the bridge between the model and data, I use GMM to estimate the parameters pertaining to the dynamic and business cycle. Particularly I match 34 moments with 31 parameters such as the persistence of shocks, observation matrix and standard derivations of noise shock. For similarity I further assume the covariance matrix of shocks is a diagonal matrix hence all the shocks are independent. The moments in data is calculated by detrending the trend from quarterly time series via hp-filter. I also follow the method proposed by Uhlig et al. (1995) and Ravn and Uhlig (2002) to calculate the moments of model in frequency space so that it is a comparative calculation akin to the filtered data. Table 4 summarizes the primary moments related to the housing market and physical capital investment on which I focus in this paper. The result shows that the model is in line with the reality and can be used to estimate the economic destruction caused by the real estate over-construction. All the details are relegated to the appendix.

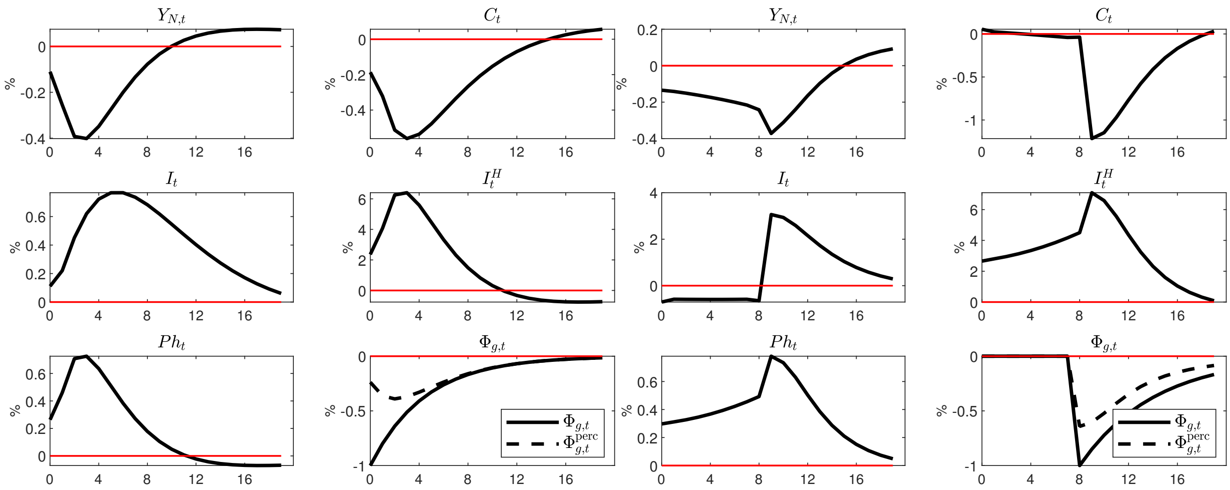
Table 4: Real Business Cycle Moments

Moments	Description	Data	Model
σ_Y	Standard Derivation of output (non-durable goods production)	0.01	0.018
σ_{p^H}	Standard Derivation of real estate price	0.018	0.017
$\frac{\sigma_I}{\sigma_Y}$	Relative Standard Derivation between physical investment and output	4.74	4.28
$\frac{\sigma_{I^H}}{\sigma_Y}$	Relative Standard Derivation between new construction and output	8.66	8.59
$\mathrm{cov}(p^H, I^H)$	Covariance between real estate price and new construction	0.00088	0.00097
$\mathrm{cov}(I, Y)$	Covariance between physical investment and output	0.00023	0.00074

4.3 Quantitative Analysis

4.3.1 Overbuilding and Boom-Burst Cycle: News to the Future and Inefficiency of imperfect information

When a contemporaneous preference shock realized, household will decrease their nondurable goods consumption to exchange for more durable goods consumption, housing servicing because they prefer the real estate to the nondurable goods now. This altered preference draws the housing price up because of a demand curve shift to the right, which in the end generates a housing market boom which is shown in figure 9a. A one unit growth shock to preference is only perceived with 0.5 at the peak by the household. The household increase their consumption to housing servicing and this jump in demand increases the construction to the peak of 3 and house price to the peak of 0.6.



(a) (b)

Figure 9: Contemporaneous and News shock

However if the shock is known by the household ahead of time when it realized, the household will response to this future shock when he known the realization news. They will increase the holding of house right away which pushes up the house price immediately. This will crowd out the physical investment through general equilibrium cycle if the nondurable consumption does not change. Further the household will also increase their consumption either because they are wealthier now fueled by the real estate appreciation or because they can borrow more fund from bank. This will magnify the crowd-out effect as nondurable consumption also entries into the goods market cleaning condition. Figure 9b shows this crowd-out effect triggered by a new shock. After observing a news about preference shock 8 periods later, household increase their holding of housing and there is a boom in housing market. They also increase the nondurable consumption but crowd out the physical investment.

When the housing market boom is a bubble that is blown by a phantasm, the inefficiency of imperfect information could incur a welfare loss. Figure 10 illustrates the welfare loss caused

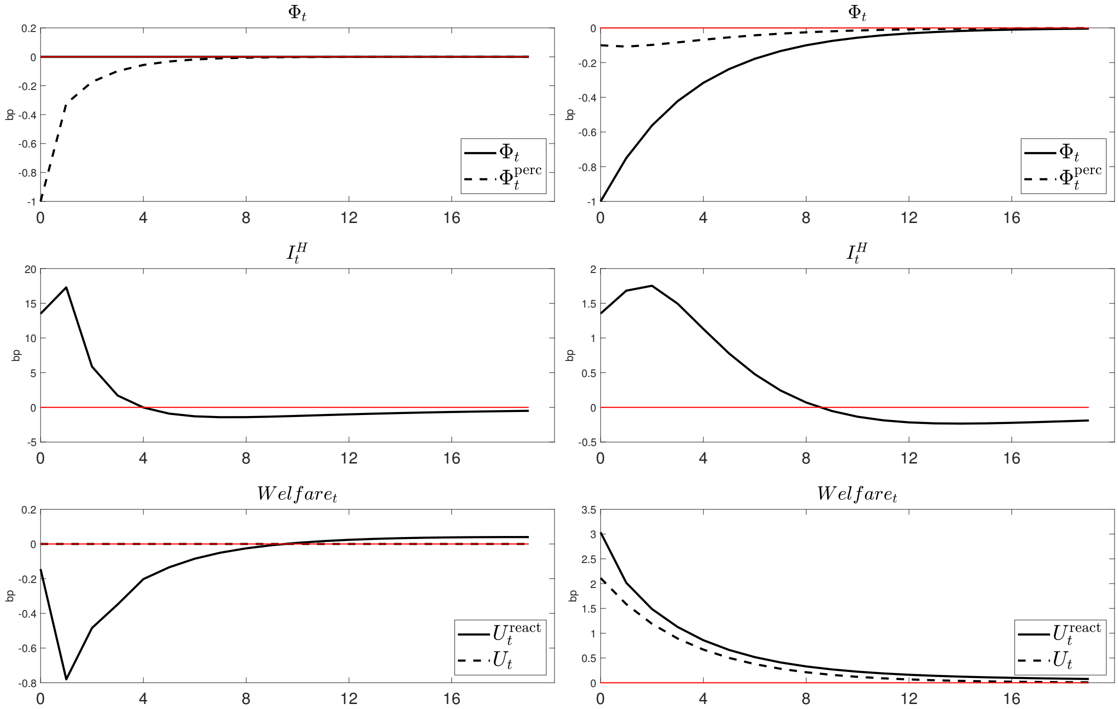


Figure 10: Welfare Loss in Imperfect Information

by imperfect information. RHS is the response of investment and aggregate utility(with unit weight) to a preference shock on nondurable goods. By observing the decrease in contribution of nondurable goods to utility, the household has a perception of this preference shock as which I denotes the dash line in the first row. Because the housing service provides more utility now the household will increase their consumption to housing service and the aggregate welfare jump to 3bp which is shown in the solid line below. If the household does not response to the shock with zero derivation all the time, they will have a relative loss in welfare comparing to the situation that they react because the shock really happened and it is optimal to response to it. Although the household has an absolute increase in welfare because of the distribution effect and existence of hand-to-mouth household. Opposite to the realized preference shock, LHS of figure 10 shows the response of investment and aggregate welfare to a noise shock or observation shock. The household still increase their consumption to housing service because they thought that a preference shock has happened and they loss in welfare from this inappropriate reaction which I denote the solid line in the last row. If they did not react to the noise shock their welfare would have no change at all because nothing had happened which is shown by dash line. The experiment above corroborates the inefficiency of imperfect information as people misleadingly proceed housing market boom and I show that the noise in news, or fake news, can induce a further loss in output and consumption because of crowded-out physical capital.

4.3.2 Overbuilding and Boom-Burst Cycle: Fake News

When a pure noise(observation) shock instead of fundamental shock is informed to household, they would response to this shock as what they did to the fundamental shock because of the existence of information friction. Household cannot know the exact magnitude of the shock but a signal contaminated with noise. They response to what they perceived, or in other words their belief, instead of the fundamental shock. Therefore as long as the household believe there is an housing market boom in the future, they will increase their holding of housing service and crowd out the physical investment, which is a chronic poison to them as long as their belief is incorrect and the housing market boom is built on the Babylon tower. When the household across the manifest they need invest more physical capital because they temerariously exchange the physical capital to real estate just before. This large demand to physical capital results in a huge drop in nondurable consumption which follows a heavy loss in welfare. Additionally because the real estate is also a type of wealth which the household used to borrow money from bank, a housing market burst and a deep deflation in house price break the consumption pattern of low-income household and leave them at financial constrained edge, which leads to a further welfare loss.

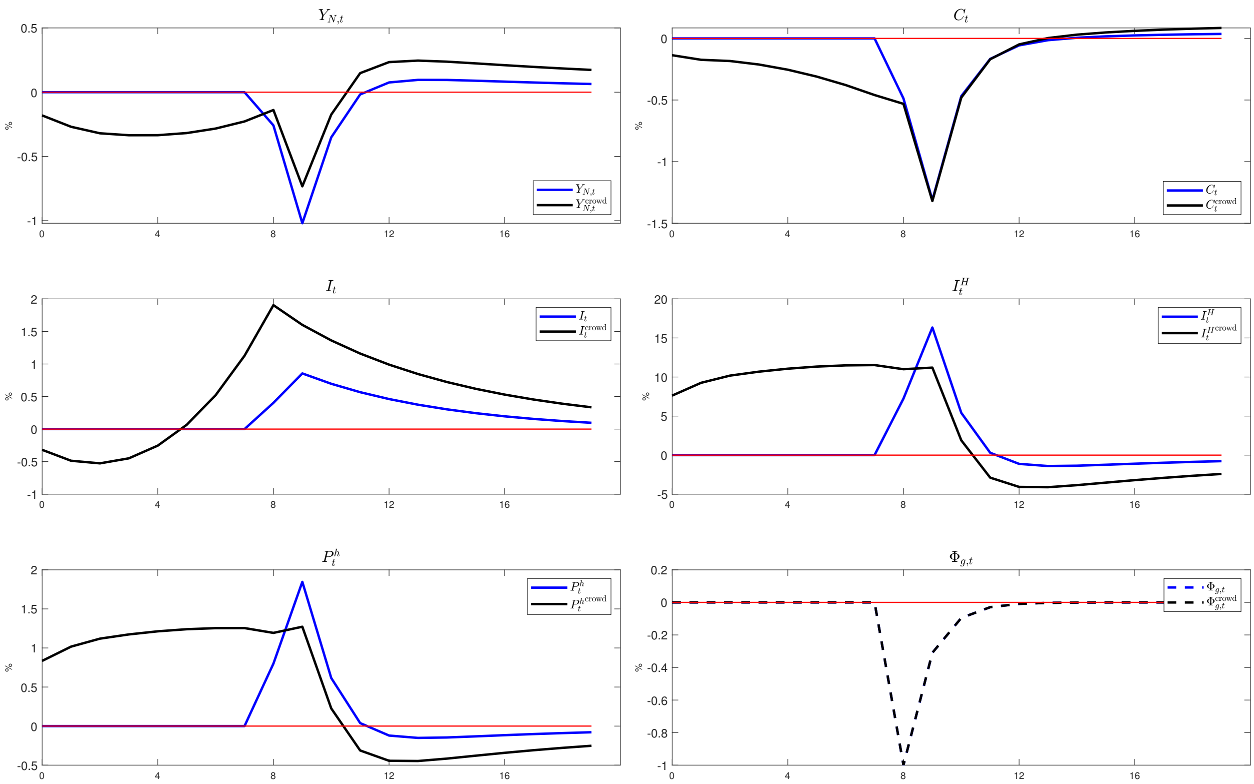


Figure 11: Fake news shock

Figure 11 compares the impulse response to the fake-news preference shock with and without pre-crowded physical capital which demonstrates the large output and welfare loss engendered by crowd-out effect. The blue solid lines are the responses to a contemporaneous noise shock $\widetilde{\Phi}_{g,t}^{\phi}$ of non-durable goods' production, non-durable goods consumption, physical investment,

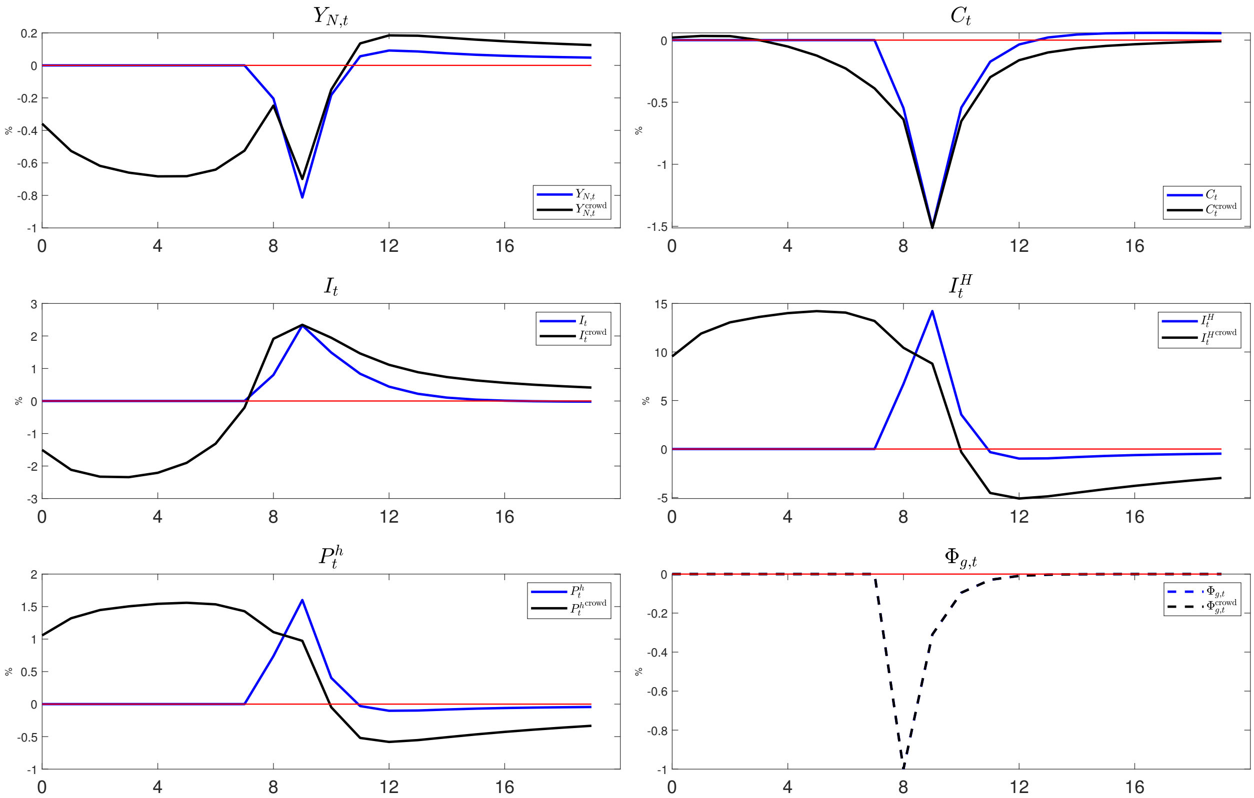


Figure 12: Fake news shock

new construction and real housing price. The black solid lines represent the responses of them to noisy news $\widetilde{\Phi}^{\phi}_{g,t+8}$ which is informed to household 8 period ago. When the household knows that there will be an economic boom in the future, they increase the investment in real estate and induce a housing market boom immediately. Because all the household already hold some amount of real estate, this housing market boom spurs higher non-durable goods consumption because of the wealth effect, which is in line with Mian et al. (2013). This further crowds out the physical investment which is shown by the negative response in 11. After the shock "should" realized two years later, at period 9, household are gradually aware the true and increase the physical capital investment a lot to compensate the scarcity of capital caused by crowd-out effect from negative 2 percentage to positive 6 percentage. The burst in housing market leads to a 2.5 percentage drop in housing price and 2 percentage drop in non-durable goods consumption. On the other hand, if the physical capital is not pre-crowded, the economy response is mild and moderate with smaller output loss, consumption privation and housing market bust. There are only half of the loss in non-crowded scenario relative to crowded scenario. Similarly there are only two third of the boom-burst cycle in housing price and new constructions in the non-crowded situation. The difference in impulse response demonstrates the non-negligible drawback of the crowded-out effect in the housing market boom-and-burst cycle.

4.3.3 Idiosyncratic Income shock, Financial Friction, Relative intratemporal elasticity of substitution

To investigate how the crowded-out effect is influenced by the idiosyncratic income shock, financial friction and relative intratemporal elasticity of substitution, I fix the expected jump in house price and change the relative parameters in model in this section. By decreasing the relative intratempral elasticity of substitution with the same amount in section 3.2.1, the blue dash line in figure 13 illustrates the attenuation caused by the relative intratemporal elasticity of substitution. A smaller relative intratemporal elasticity of substitution, from $\mathrm{ES} - \mathrm{EIS} = \frac{1}{2}$ to $\frac{1}{3}$, results in a huge physical investment drop, roughly 3 times larger than that in baseline model. Given this smaller intratemporal elasticity of substitution, household will care less about the substitution in utility between non-durable goods and housing service (in other words more complementarity) which result in a larger increase non-durable goods consumption in figure 13. These lead to a lower investment in physical capital via general equilibrium.

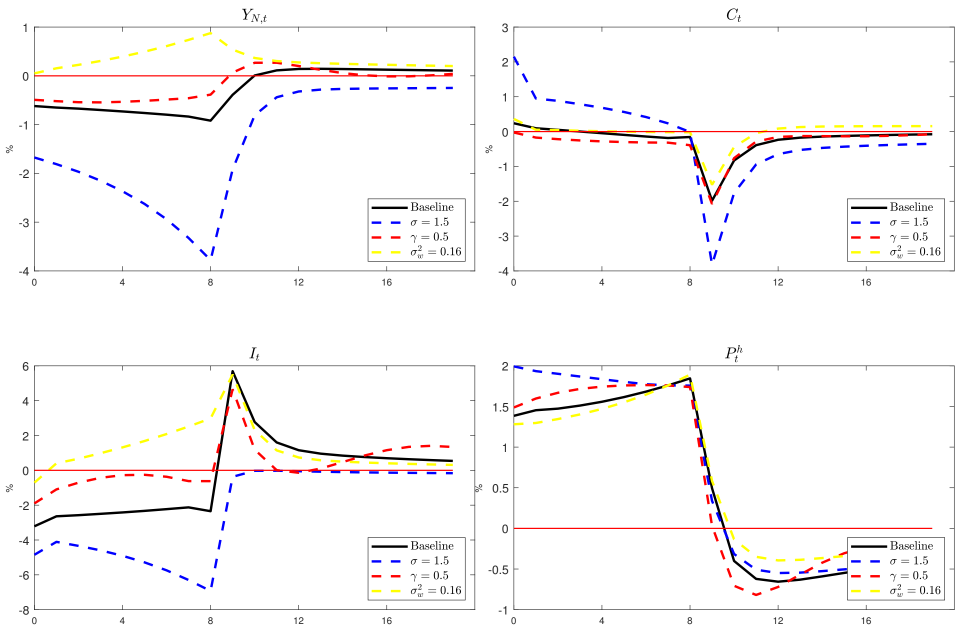


Figure 13: Crowded-out effect comparison

The red dash line in figure 13 depicts the response under a tight credit constraint, which implies an important role of wealth inequality. As shown in section 3.2.2, if we do not consider the wealth distribution (i.e. $\int \widetilde{h_t}^i dG_i$ and $\int \widetilde{\mu}_t^i dG_i$ in equation 14) a tighter financial constraint will result in a severer crowded-out problem because the real estate is more valuable now. However, as shown in section 3.2.3, household cannot increase their non-durable consumption and housing service as much as they want because of financial constraint and wealth inequality. The larger $\widetilde{h_t}^i$ can only be realized in a smaller dG_i and figure 13 shows that this inequality channel dominates other channels. The physical capital is crowded out less than that in baseline model as there are more overwhelmed household who cannot increase their consumption as much as they want.

Additionally I increase the variance of idiosyncratic income shock from $\sigma_w^2 = 0.06$ in baseline model to 0.16 which I characterize with yellow dash line in figure 13. Facing a massive income shock, household will have more precautionary saving motive to hold the asset (to fulfill their consumption demand against potential low income and cash flow state) instead of borrowing money to buy housing services. Even though the household expect a housing market boom they only slightly decrease the physical capital at the first period and then increase until the shock realized. The reason why the physical capital further jumps is that household want to hold more housing services under the effect of expected shock. However they do not want to borrow money and decrease their holding of asset to buy real estate. They can only increase their labor supply to earn more wage income to buy housing services. The complement between labor and physical capital tempts the household to increase their asset instead of decreasing them with a higher asset return, which triggers a positive feedback loop on the boom in physical capital.

5 Conclusion

This paper documents a new mechanism through which the housing market boom magnifies the recession. An unnecessary jump in residential construction arouse by fake news and imperfect information will blow up a bubble in housing market which is a boom without solid inner filler and not supported by economic foundation. This overbuilding in housing market crowds out physical capital which is used to produce both durable and nondurable goods. The crowd-out effect in physical capital market aggravates the decline in output when the housing market bubble bursts because of the deficiency of physical capital. Firms do not have as much as capital they can use to support the optimal production under a specific level of TFP so they will decrease production and labor demand when facing a higher real interest rate and marginal production cost. I use a simple model to argue theoretically that the crowd-out effect of overbuilding is affected by relative intratemporal elasticity of substitution, financial friction, idiosyncratic income shock and wealth distribution. Later the quantitative result from a full-fledged model verifies the argument and demonstrates that the output loss caused by overbuilding is large.

However there are still some problems left for future studies. Even though the imperfect information does not exist the overbuilding and crowd-out effect may still be a significant drawback in the perspective of business cycle as it increases the economic volatility and household leave their first-best equilibrium further. Additionally how can the government introduce an optimal fiscal, monetary, or macroprudential policy to alleviate the crowd-out effect of overbuilding? Is there any complementarity between overbuilding and nominal rigidity in New Keynesian model which will further exacerbate the defect of overbuilding and crowd-out effect?

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A Data Description

Real GDP Y_t is directly downloaded listing "Real Gross Domestic Product" with seasonally adjusted. Real consumption C_t is directly downloaded listing "Real personal consumption expenditures: Nondurable goods" with seasonally adjusted. GDP deflator gdp_{def} is downloaded listing "GDP Implicit Price Deflator in United States" with seasonally adjusted. Nominal nondurable investment I_t^{nom} is downloaded listing "Private Nonresidential Fixed Investment" with seasonally adjusted. I get the real nondurable investment I_t by the formula $I_t = I_t^{\text{nom}}/\text{gdp}_{def} * 100$. The CPI which we take is "Consumer Price Index for All Urban Consumers: All Items Less Shelter in U.S. City Average" since we should consider the correlation between house price and normal CPI. Thus we downloaded the CPI without shelter term. I take the nominal interest rate R_t^{nom} as "Effective Federal Funds Rate". The inflation rate is calculated from the GDP defltor in the form that $\pi_t = \frac{def_t - def_{t-1}}{def_{t-1}}$ (Since we solve the inflation from deflator in quarterly data, the inflation is measured within one quarter instead of annually). Combining the inflation π_t and nominal interest rate R_t^{nom} we can construct the real interest rate $R_t = (\frac{R_t^{\text{nom}}}{100} + 1)/(1 + \pi_t) - 1$ (I divided 100 because the original data is in percentage unit). The house supply H_t is measured by "New Privately-Owned Housing Units Started: Total Units". The nominal mortgage debt MD_t^{nom} comes from "Mortgage Debt Outstanding, All holders (DISCONTINUED)". Since the nominal mortgage debt is in money unit, I can directly get the real mortgage debt value from $MD_t = MD_t^{\text{nom}}/\text{gdp}_{def} * 100$ which is same as we did to get real investment. The real stock price P_t^a is calculated from "NASDAQ Composite Index" and normalized by GDP deflator as I did in constructing real investment and real mortgage debt. The real house price P_t^h is calculated from "All-Transactions Indexes" collected by Federal Housing Finance Agency.

B Identification Step and Robustness Test to VAR Identification

B.1 Identification with sign and zero restricution

Based on the observation and argument, I use a simple SVAR model to decompose the effect of raised house price to investment. Given the model which is similar to Sims et al. (1986)

$$\boldsymbol{y}_t = \boldsymbol{c} + \boldsymbol{\Phi}_1 \boldsymbol{y}_{t-1} + \ldots + \boldsymbol{\Phi}_p \boldsymbol{y}_{t-p} + \boldsymbol{e}_t \tag{22}$$

where

$$\boldsymbol{y_t} = \begin{bmatrix} r_t \\ m_t \\ y_t \\ p_t \\ i_t \\ p_t^h \\ c_t \end{bmatrix} \tag{23}$$

r_t is the nominal interest rate; m_t is the money supply; y_t is the real output; p_t is the price level; i_t is the nominal investment; p_t^h is the nominal price of house; c_t is the real consumption of non-durable goods. Most the data comes from FRED, Federal Reserve Bank of St. Louis. I use treasury bill rate represents the nominal interest and GDP deflator for the price level. The price of house comes from FHFA house price index. The detail about it will be discussed at appendix. Meanwhile I use the short-run restriction as well as corresponding sign restriction to decompose the shock term $\boldsymbol{e_t}$ from $\boldsymbol{v}_t$ that

$$\boldsymbol{Pe_t} = \boldsymbol{v_t} \tag{24}$$

or detailedly

$$Pe_t \equiv \begin{vmatrix} 1 & b_{11} & 0 & 0 & 0 & 0 & 0 \\ b_{21} & 1 & b_{23} & b_{24} & 0 & 0 & 0 \\ b_{31} & 0 & 1 & 0 & b_{35} & 0 & b_{37} \\ b_{41} & b_{42} & b_{43} & 1 & b_{45} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & b_{56} & 0 \\ b_{61} & 0 & b_{63} & b_{64} & 0 & 1 & 0 \\ b_{71} & 0 & b_{73} & 0 & 0 & b_{76} & 0 \end{vmatrix} \begin{bmatrix} e_{rt} \\ e_{mt} \\ e_{yt} \\ e_{pt} \\ e_{it} \\ e_{p^h t} \\ e_{ct} \end{bmatrix} = \begin{bmatrix} v_{rt} \\ v_{mt} \\ v_{yt} \\ v_{pt} \\ v_{it} \\ v_{p^h t} \\ v_{ct} \end{bmatrix}$$

Figure 14 shows the IRF of one unite positive house price shock to output, investment, house price and non-durable goods consumption. The black line is the path of related variable up to 20 period. The read dash line is their related confidence band under 90% calculating by monte-carlo method. We can inspect from IRF that, house price inflation could stimulate the consumption of durable goods as it is long-lasting goods and household could derive out utility by just holding it. The household could feel satisfy and pleased either via living in this house or via owning the house which is valuable every period. Meanwhile the household can obtain utility not only from just holding and enjoying it each period, but also from financial market. The house is a goods that could be consumed. While at the same time it is also a asset that could be collateral and offers more liquidity to household. Household would use this liquidity to smooth their non-durable consumption leisurely, which provide extra benefit to household.

Therefore after observing one unit positive shock in house price, household snap up the house as house it not only a goods but also an asset which we discuss before. This increased demand

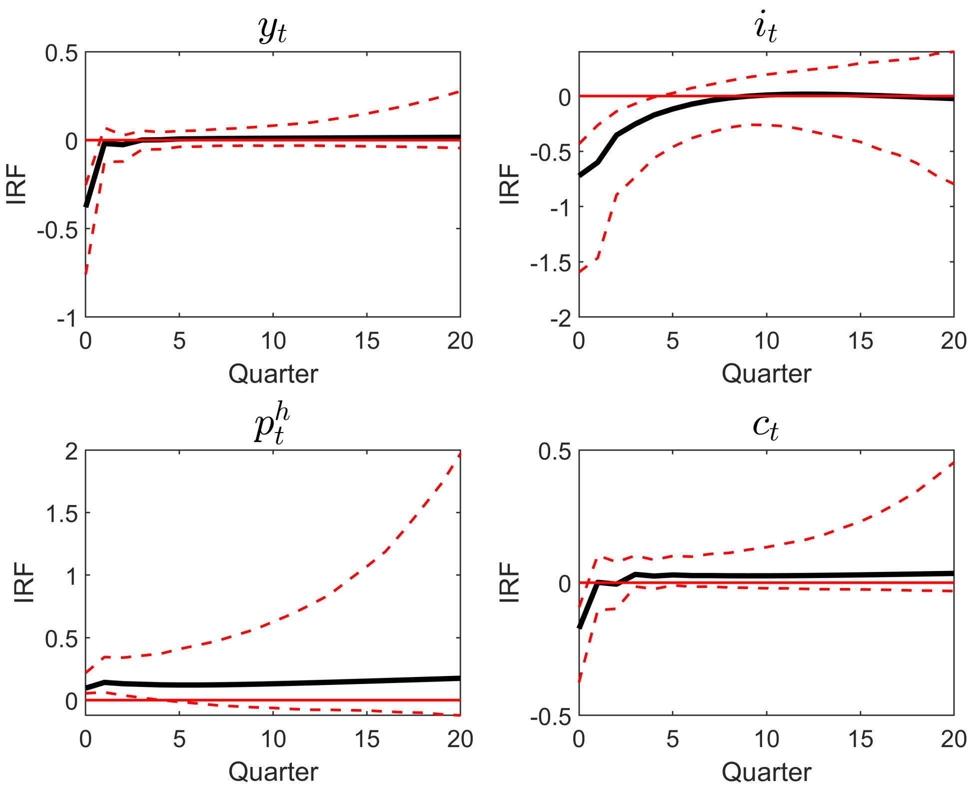


Figure 14: IRF of house price inflation

draw up the house price even more which we can see the house price is raising not only lat the beginning but also later. The house price in the end permanently increased because of increased household demand. This increased house price stimulates household who would borrow more from bank to buy house (the house supply discontinuity will aggravate this channel) or borrow more to help them share the risk as collateral is more expensive. Firms will be more difficult to borrow money to invest and the decreased demand in non-durable goods will also weaken firms' propensity to invest or R&D. Investment is crowded out by this two effects and this is what we can observe from the IRF. Investment drops the most and also spends longest time to recover. Output and non-durable consumption stands behind it. However both of them go back to steady state quickly which indicates that only the first jump in house price affects them. Later household use their more valuable collateral to smooth the consumption as well as output. Thus these two variable converge back quickly while because of strong and amplified effect both in demand and supply side, investment converges much slower than other two variables. This portends that there would be much larger drop in output if recession occurs because the accumulated decreased investment will pass its influence through the capital, a long-lasting things, later.

B.2 Contemporaneous real price shock

B.2.1 Process of estimation and identification

I detrend the main variable by taking logarithm first and first-order difference later. Then I get the detrended real GDP, real consumption, real investment, cpi, house supply, real mortgage debt, stock price and house price in lower-case letter. Then I ordered them in the vector

$$Y_t = [y_t, c_t, i_t, cpi_t, r_t, p_t^a, hs_t, md_t, p_t^h]'$$

I use the data period between 1987Q2 and 2006Q4. Then I add lagged term into the model up to 4 quarter and estimate the model

$$Y = [Y_5, Y_6...]$$

$$X_{t-1} = [y_{t-1}, c_{t-1}, i_{t-1}, cpi_{t-1}, r_{t-1}, p_{t-1}^a, hs_{t-1}, md_{t-1}, p_{t-1}^h, y_{t-2}, c_{t-2}, ..., p_{t-4}^h]'$$

$$X = [\mathbf{1}, X_4, X_5, ...]$$

Then use the projection matrix we can solve the factor that

$$\hat{\Phi} = YX'(XX')^{-1}$$

The residue is

$$\hat{e} = Y - \Phi X$$

and the variance of estimation error would be

$$\hat{\Omega} = cov(\hat{e}')$$

To simulate the model we can rewrite the variables into companion form such that

$$\boldsymbol{Y_t} = [y_t, c_t, i_t, cpi_t, r_t, p_t^a, hs_t, md_t, p_t^h, y_{t-1}, c_{t-1}, ..., p_{t-3}^h]'$$

Denote $\hat{P} = \text{chol}(\hat{\Omega})$ and

$$\hat{\boldsymbol{\Phi}} = \begin{bmatrix} \Phi_1 & \Phi_2 & \Phi_3 & \dots & \Phi_p \\ I_n & 0 & 0 & \dots & 0 \\ 0 & I_n & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & I_n & 0 \end{bmatrix}$$

where $\Phi(:,2:\text{end}) = [\Phi_1\Phi_2\Phi_3 \dots \Phi_p]$ since I have intercept coefficient term with $\mathbf{1}$ in X.

Meanwhile we define

$$\hat{\boldsymbol{P}} = \begin{bmatrix} \hat{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$$

The shock term is

$$\nu_{n\times 1} = [0, 0, ..., 1]'$$

which means there is only one unit shock happened at house price row.

Similarly I should write it in companion form such that

$$\boldsymbol{\nu} = [\nu, \mathbf{0}]$$

Then we can get the IRF that

$$\text{IRF}_t = \hat{\mathbf{\Phi}}^t \hat{\boldsymbol{P}} \boldsymbol{\nu}$$

where $t = 0, 1, 2, ..., 20$.

Finally we only take first 1 to n items in IRF$_t$. Since I take first-order difference to most of the data, at this stage I also calculate the cumsum of IRF to return the accumulated response.

B.2.2 Contemporaneous shock under larger confidence band

B.2.3 News shock under larger confidence band

B.2.4 Alternative detrend Method

Alternatively I also use another method to deal with the data which we call Vector Error Correction Method (VECM) in literature. I add the year number into the model to try to detrend the data. I marked the year with its "number" and add 0.1 to 0.4 on it as the label of quarter. Then I divided these "number" by 1000 to get a comfortable scalar. Specifically we take

$$Y_t = [t, t^2, t^3, y_t, c_t, i_t, cpi_t, r_t, p_t^a, hs_t, md_t, p_t^h]'$$

B.2.5 Confidence Band-MC Method

Here I explain the detailed steps that I used to calculate the confidence band of t he estimation using Monte Carlo method. Since there is no difference in steps between I estimate the confidence band in method I and method II, I only show the first part for simplicity.

I can calculate the estimated variance of the coefficient by

$$\hat{\sigma}^2_{\hat{\Phi}} = \frac{\hat{\Omega} \bigotimes \left(\frac{XX'}{T}\right)^{-1}}{T}$$

Then I draw the coefficient simple $\tilde{\Phi}^{(b)}$ from the distribution

$$vec(\hat{\Phi}) \sim N\left(\text{vec}\left(\hat{\Phi}'\right), \hat{\sigma}^2_{\hat{\Phi}}\right)$$

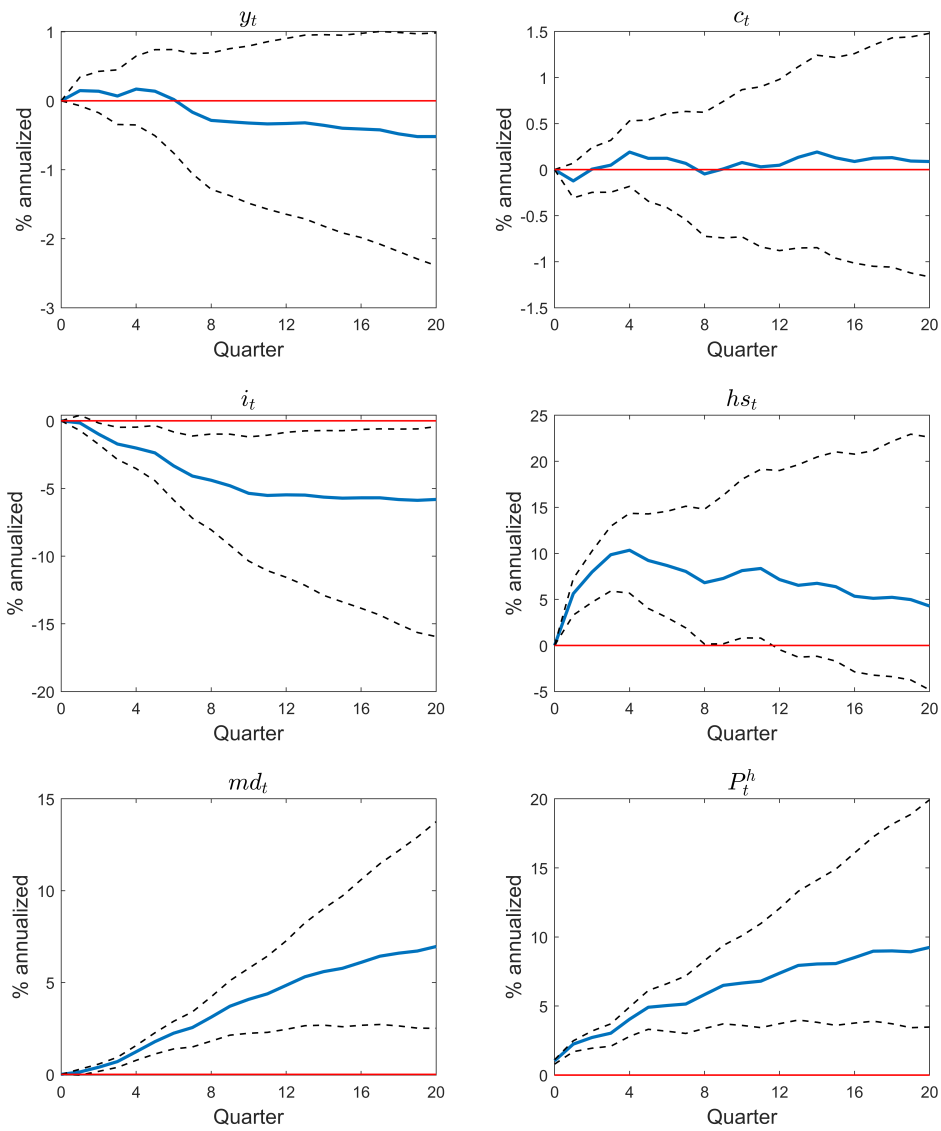


Figure 15: IRF with 90% confidence band

At the same time the estimated variance of the coefficient variance would be

$$\hat{\sigma}^2_{\hat{\Omega}} = \frac{2D_n^+ \left(\hat{\Omega} \bigotimes \hat{\Omega}\right) D_n^{+\prime}}{T}$$

where $D_n^+ = \left(D_n' D_n\right)^{-1} D_n$ is the Moore-Penrose generalized inverse of duplication matrix D_n

I generate the variance simple $\tilde{\Omega}^{(b)}$ from the distribution

$$\text{vech}(\hat{\Omega}) \sim N\left(\text{vech}(\hat{\Omega}), \hat{\sigma}^2_{\hat{\Omega}}\right)$$

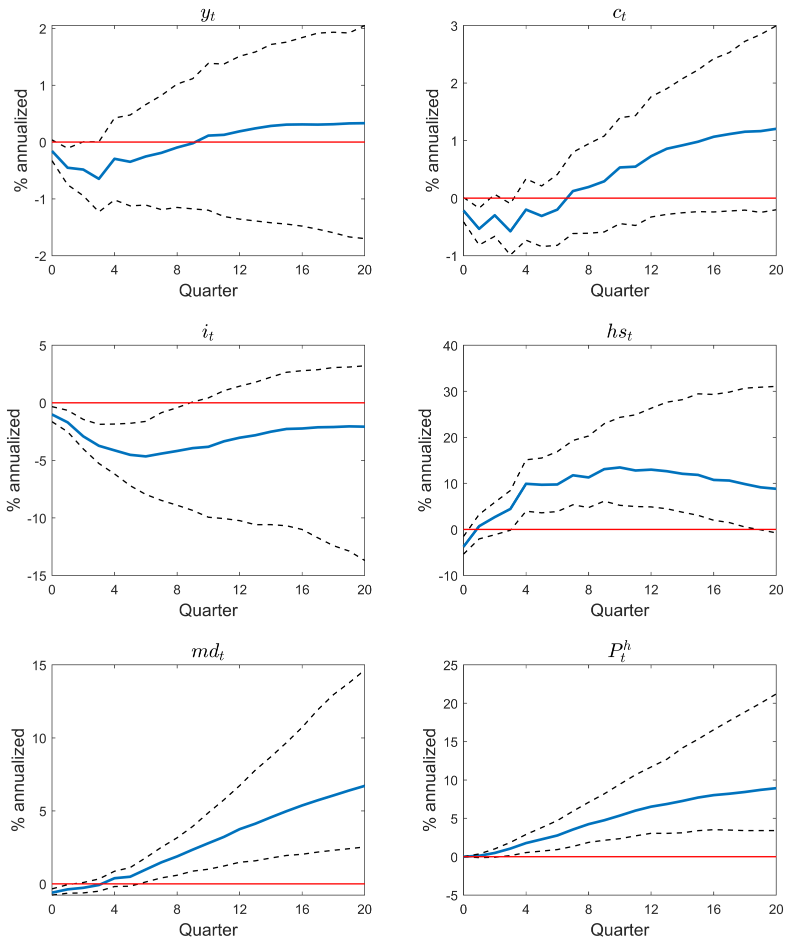


Figure 16: IRF with 90% confidence band

Then use the duplication matrix to transfer back to

$$\text{vec}(\tilde{\Omega}^{(b)}) = D_n \text{vech}(\tilde{\Omega}^{(b)})$$

C Micro Foundation to Identification and Tests

In this section I provide some micro foundation related to fake-news identification in section 2.3 and some tests to my identification as proof to the reliability. I first provide several different setting about news and fake news in the literature. Then I describe the standard rbc model that I used to provide some numerical examples and micro foundation to the identification in main page.

C.1 Literature in modeling the news and fake news

C.1.1 Perfect News

This type of "fake news" is the setting following Christiano et al. (2008), Schmitt-Grohé and Uribe (2012), Barsky et al. (2015) and Sims (2016) in which household gets a news about a shock ν_τ realized at time τ which is true for sure. However after the household reaches at time τ there is an identical negative unexpected shock $-\nu$ just offsetting the effect of positive shock ν_τ. Comparing to the setting in equation 26, in which household gets a news about ν_τ via ϵ (and totally believe it) but is misled because the observation ϵ is generated by noise w, Anderson and Moore (2012) and Chahrour and Jurado (2018) shows that this type of "fake news" shock is *observational equivalent.*[27] To theoretically formulate this type of fake news shock, we can consider the shock series

$$\phi_t = \nu_{0,t} + \nu_{1,t-\tau} \tag{25}$$

where $\nu_{0,t}$ and $\nu_{1,t-\tau}$ are iid over time and follow

$$\begin{bmatrix} \nu_{0,t} \\ \nu_{1,t} \end{bmatrix} \overset{iid}{\sim} \mathcal{N}\left(0, \begin{bmatrix} \sigma^2_{\nu,0} & 0 \\ 0 & \sigma^2_{\nu,1} \end{bmatrix}\right)$$

C.1.2 Noisy News

This type of news is used by Lorenzoni (2009), Baxter et al. (2011), Barsky and Sims (2012), Blanchard et al. (2013), et al. The most intuitive one.

$$\epsilon_t = \nu_{t+\tau} + w_t \tag{26}$$

where ν is the true news shock observed by agents τ periods ahead and w is the noise or fake news shock. These two shocks are independent with each other and follow

$$\begin{bmatrix} \nu_t \\ w_t \end{bmatrix} \overset{iid}{\sim} \mathcal{N}\left(0, \begin{bmatrix} \sigma^2_{\nu} & 0 \\ 0 & \sigma^2_{w} \end{bmatrix}\right)$$

C.1.3 Fake News

It is worth to notice that when we consider the dynamic cases of equation 25 and 26, everything and every realization of $\nu_{0,t}$, $\nu_{1,t-\tau}$, $\nu_{t+\tau}$ and w_t could happen. Given $\phi_t = 1$, different combination such as $(\nu_{0,t} = 0.5, \nu_{1,t-\tau} = 0.5)$ or $(\nu_{0,t} = 1.5, \nu_{1,t-\tau} = -0.5)$ may all hold. Similarly given $\epsilon_t = 1$, $(\nu_{t+\tau} = 0.5, w_t = 0.5)$ or $(\nu_{t+\tau} = -0.5, w_t = 1.5)$ may all hold.

[27] They call this representation to fundamental and belief as *news representation* and the representation in equation 26 as a *noise representation.*

In this section what I am considering is the "pure shock" scenario or the impulse response to a single shock. In other words, for instance, one unit realization of noisy news $\epsilon_t = 1$ can only come from $\nu_{t+\tau} = 1$ or $w_t = 1$. It does not mean I have an implicit restriction on the shock $\nu_{t+\tau}$ and w_t that $\nu_{t+\tau} w_t = 0$. They are iid shocks. Similarly, given one unit realization of perfect news $\nu_{1,t-\tau} = 1$, it can be true news $\nu_{0,t} = 0$ or fake news $\nu_{0,t} = -1$. It does not mean I have an implicit restriction on the shock $\nu_{0,t}$ and $\nu_{1,t-\tau}$ that $\text{corr}(\nu_{0,t}, \nu_{1,t-\tau}) = -1$. They are iid shocks.

C.1.4 Fake News in Perfect News

To model a fake news in perfect news model, there is a realization of perfect news $\nu_{1,t-\tau} = 1$ at time $t - \tau$ and known by household, though this shock would have fundamental effect later, at time t. Then at time t there is an unexpected contemporaneous shock $\nu_{0,t} = -1$ to "neutralize" or "offset" the perfect news effect to make the fundamental stay at the beginning. The VAR identification to this type of fake news is easy. Because all the news in this model is true or perfectly foreseen by household, we just need to find a news shock first. Then at time τ there is a same shock but an opposite direction. We only need to identify the response to shock once.

Sims (2016) did this identification.

C.1.5 Fake News in Noisy News

To model a fake news in noisy news model, there is a realization of observation $\epsilon_t = 1$ at time t which can either be a signal to a fundamental shock in the future, time $t + \tau$, $\nu_{t+\tau} = 1$, or be a noisy $w_t = 1$, which does not have any fundamental effect to the economy. In noisy news model given an observation $\epsilon_t = 1$ household will response to their perception to the true news $\nu_{t+\tau|t}$ which is smaller than ϵ_t under rational expectation and we can write it as $\nu_{t+\tau|t} = \alpha \epsilon_t$ where $\alpha < 1$. There exist learning and belief updating in this type of modeling and theoretically their is no point when household "realizes" that the news is fake. For fake news their perception converge to zero faster than that in true news. In other words, $\underset{i \to \infty}{lim} \nu_{t+\tau|t+\tau+i} = 0$ will be faster for fake news than true news.

To model the "awareness" of fake news, we now consider a scenario in which no more information about shock $\nu_{t+\tau}$ is delivered to household throughout time $t + 1$ and time $t + \tau - 1$. Therefore the belief to $\nu_{t+\tau}$ of household will not be updated and $\nu_{t+\tau|t} = \nu_{t+\tau|t+1} = \cdots = \nu_{t+\tau|t+\tau-1}$. However when the news realize at time $t + \tau$, household gets a further signal, or information to it. In other words household can also observe $\epsilon^{\tau}_{t+\tau} = \nu_{t+\tau} + w^{\tau}_{t+\tau}$ and this new observation $\epsilon^{\tau}_{t+\tau}$ will update or twist the household's belief to shock $\nu_{t+\tau}$. Therefore their exists a value of $w^{\tau}_{t+\tau}$ which can "correct" the belief of household. Thus, $\nu_{t+\tau|t+\tau} = 0$ and household at time $t + \tau$ realize that the news $\nu_{t+\tau}$ which they known at time t is a fake news.

C.2 Numerical test to identification: A simple RBC model

C.2.1 Equations used to solve the state space model

In this subsection I describe a simple 8 variables RBC model to test my identification strategy and show that it can successfully recover the impulse response to news and fake news shocks. I will first introduce the DSGE model briefly and then show that my identification process works well by comparing the identified empirical impulse response with the theoretical one.

The 8 variables RBC model is a standard one in which household provides labor and earns labor income. Given the labor income and capital return, which is paid by firms with real rental rate as they rent capital to produce goods, the household decides their investment and consumption level. In additional to these endogenous variation there is an exogenous government spending shock following equation 27 and other 4 standard shocks such as TFP shock and preference shock.

Household

$$c_t^{-\sigma} = \beta R_{t+1} c_{t+1}^{-\sigma}$$

$$h_t^{\varphi} = w c_t^{-\sigma}$$

Firm

$$R_t = \alpha \frac{y_t}{k_{t-1}} + \delta - 1$$

$$w_t = (1-\alpha)\frac{y_t}{h_t}$$

$$y_t = A_t k_{t-t}^{\alpha} h_t^{1-\alpha}$$

Market Cleaning

$$y_t = c_t + I_t + \log(G_t)$$

$$I_t = k_t - (1-\delta)k_{t-1}$$

$$g_t = \rho_g g_{t-1} + w_{t-\tau} + w_t^{\tau} \tag{27}$$

The household cannot know the value of G_t and w_t but a signal to then

$$\widetilde{g}_t = g_t + \nu_t^{\tau}$$

$$\widetilde{w}_{t-\tau} = w_{t-\tau} + \nu_{t-\tau}$$

Household at time $t-\tau$ will have a perception of $w_{t-\tau}$ given the observation $\widetilde{w}_{t-\tau}$ and I denote it as $w_{t-\tau|t-\tau} = \theta \widetilde{w}_{t-\tau}$

Denote $\widetilde{w}_t^i$ as an observation to shock w_{t-i}. For example, a news shock w_t will have effect on G at $t+\tau$. At time $t+1$ household gets a new observation related to w_t, $\widetilde{w}_{t+1}^1$, in addition to the old observation of w_t at time t $\widetilde{w}_t$. I further assume

$$\widetilde{w}^1_{t-\tau+1} = \widetilde{w}^2_{t-\tau+2} = \cdots = \widetilde{w}^{\tau-1}_{t-1} = 0$$

holds. Therefore

$$w_{t-\tau|t-\tau} = w_{t-\tau|t-\tau+1} = w_{t-\tau|t-\tau+2} = \cdots = w_{t-\tau|t-1}$$

C.2.2 Quantitative Exercise

C.2.2.1 Same perception: $g^{\nu}_{t|t} = g^{w}_{t|t} = g^{\nu+\nu^{\tau}}_{t|t}$

Notation: Throughout exercise 1 to 3, imperfect information holds.

1) Only noisy shock $\nu_{t-\tau}$;

2) Fake news shock. A noisy shock on $w_{t-\tau}$ at time $t-\tau$, $\nu_{t-\tau}$, as well as a negative noisy shock on g_t at time t, ν^{τ}_t;

3) A news shock $w_{t-\tau}$.

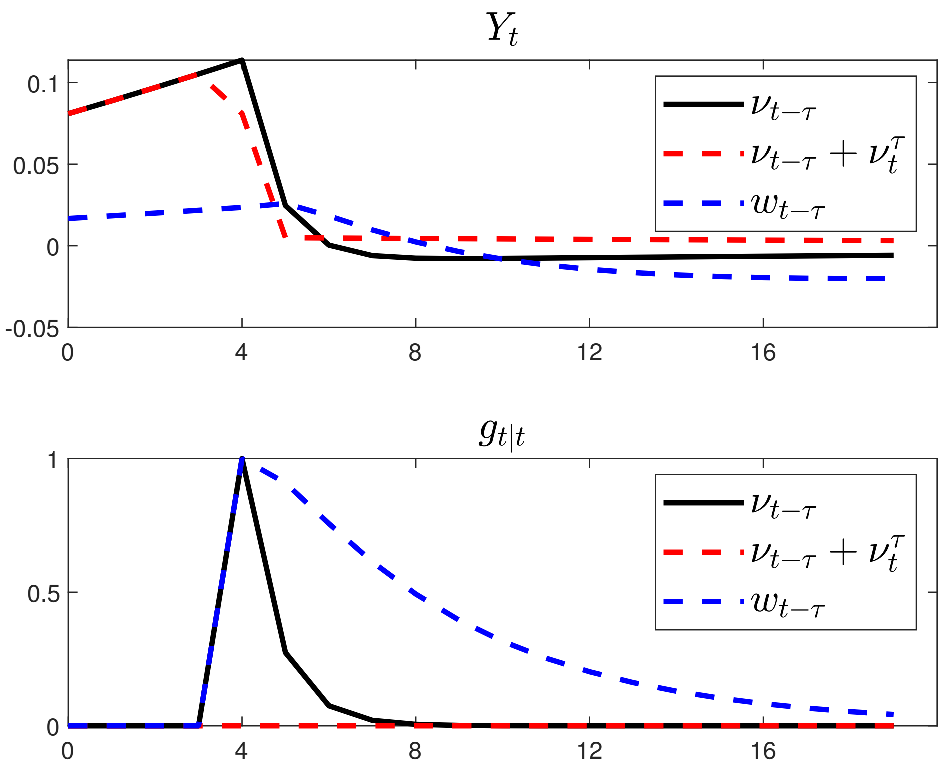


Figure 17: Same Perception $g^{w}_{t|t} = g^{w^{\tau}}_{t|t} = g^{w+\nu^{\tau}}_{t|t}$

C.2.2.2 Same observation at time $t-\tau$: $\widetilde{w}_{t-\tau}$

Notation: Throughout exercise 1 to 2, imperfect information holds. In exercise 3, it is the type of perfect news.

1) Only noisy shock $\nu_{t-\tau}$;

2) Fake news shock. A noisy shock on $w_{t-\tau}$ at time $t-\tau$, $\nu_{t-\tau}$, as well as a negative noisy shock on g_t at time t, ν_t^τ;

3) A perfect news shock $w_{t-\tau}$.

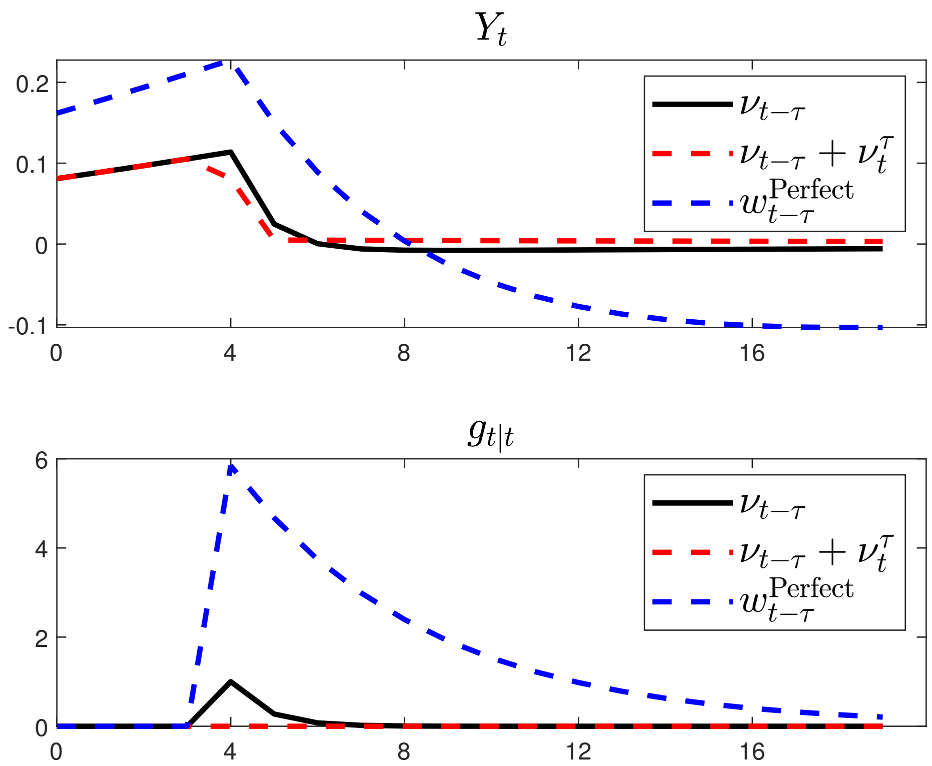


Figure 18: Same observation $\widetilde{w}_{t-\tau}$

C.2.2.3 Same observation at time $t-\tau$: $\widetilde{w}_{t-\tau}$

Notation: Throughout exercise 1 to 3, imperfect information holds.

1) Only noisy shock $\nu_{t-\tau}$;

2) Fake news shock. A noisy shock on $w_{t-\tau}$ at time $t-\tau$, $\nu_{t-\tau}$, as well as a negative noisy shock on g_t at time t, ν_t^τ;

3) A news shock $w_{t-\tau}$.

C.3 Two examples of "offset" identification

Denote the fundamental impact (i.e. housing demand variation, TFP) g_t follows an AR1 process

$$g_t = \rho_g g_{t-1} + w_{t-\tau} + w_t^\tau \tag{28}$$

where $w_{t-\tau}$ is the news shock known by household at time $t-\tau$ yet has real effect at time t, w_t^τ is the contemporaneous shock. Because of the imperfect information, household cannot know

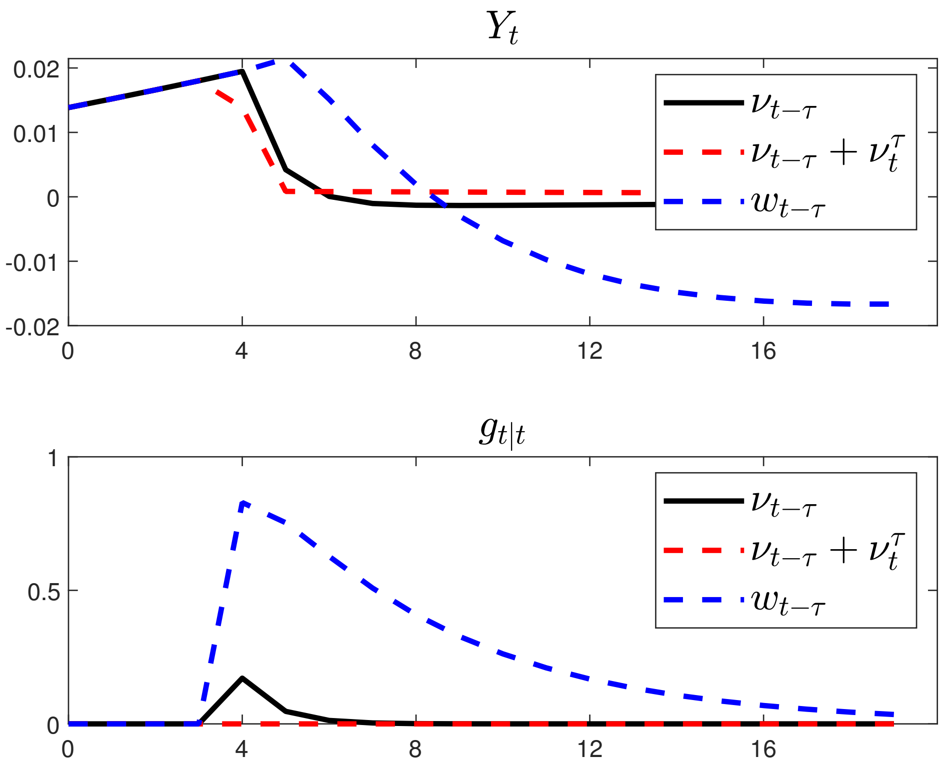


Figure 19: Same observation $\widetilde{w}_{t-\tau}$

the exact value of news shock $w_{t-\tau}$ but an observation to it with noisy shock

$$\widetilde{w}_{t-\tau} = w_{t-\tau} + \nu_{t-\tau}$$

where $\widetilde{w}_{t-\tau}$ is the observation to $w_{t-\tau}$ but may be contaminated by a noisy $\nu_{t-\tau}$ which does not have any real effect to economy. There are two scenarios that household comprehend whether the jump in observation $\widetilde{w}_{t-\tau}$ comes from news $w_{t-\tau}$ or noisy $\nu_{t-\tau}$ which I call 1). suddenly realization and 2). realization by learning.

C.3.1 The fundamental impact g_t is observable.

When the fundamental impact g_t is observable, whether the news $\widetilde{w}_{t-\tau}$ is true or fake is informed to household via g_t at time t without any delay. Since it is the impact g_t that affects the economy through which the shock $w_{t-\tau}$ and w_t^τ affect the economy, the household only care about the impact value g_t is $w_{t-\tau}$ (true news) or 0 (fake news). Therefore $y_{i-\tau-1}^\tau$ in equation 6 works as a contemporaneous shock w_t^τ offsets the true shock realized at t, $w_{t-\tau}$ and generates $g_t = 0$ which is what the fake news $\nu_{t-\tau}$ would do. This scenario is a standard one in literature and Christiano et al. (2008), Schmitt-Grohé and Uribe (2012), Barsky et al. (2015) and Sims (2016) did the similar process to generate fake news.

C.3.2 The fundamental impact g_t is unobservable.

When the fundamental impact g_t is unobservable, there is no other signal that household can use to infer whether $\widetilde{w}_{t-\tau}$ comes from $w_{t-\tau}$ or $\nu_{t-\tau}$ but learn through observation gradually. In this scenario household cannot know g_t but an observation to it $\widetilde{g}_t$ following

$$\widetilde{g}_t = g_t + \nu_t^\tau$$

I can show that the perception to the fundamental impact at time t, $g_{t|t}$ follows

$$g_{t|t} = \gamma_1 g_{t-1|t-1} + \gamma_2 w_{t-\tau|t-\tau} + \gamma_3 g_{t-1} + \gamma_4 w_{t-\tau} + \gamma_5 \nu_t^\tau + \gamma_6 w_t^\tau \tag{29}$$

where $\gamma_1 = \rho \left[1 - \frac{z_{11}}{z_{11}+\sigma_{\nu^\tau}^2}\right]$, $\gamma_2 = 1 - \frac{z_{11}}{z_{11}+\sigma_{\nu^\tau}^2}$, $\gamma_3 = \gamma_7 \rho$ and $\gamma_4 = \gamma_5 = \gamma_6 = \gamma_7 = \frac{z_{11}}{z_{11}+\sigma_{\nu^\tau}^2}$. z_{11} can be solved from the positive root of quadratic equation

$$z_{11}^2 + \left(\sigma_{\nu^\tau}^2 - \rho^2\sigma_{\nu^\tau}^2 - \sigma_w^2 + \sigma_{w^\tau}^2 - \frac{\sigma_w^4}{\sigma_w^2+\sigma_\nu^2}\right) z_{11} - \sigma_{\nu^\tau}^2 \left(\sigma_w^2 + \sigma_{w^\tau}^2 - \frac{\sigma_w^4}{\sigma_w^2+\sigma_\nu^2}\right) = 0$$

Therefore the only difference between fake news and true news at time t is the term $\gamma_4 w_{t-\tau}$ which comes from the observation $\widetilde{g}_t$ as it truly spur a jump in g_t, though the household cannot distinguish whether this jump is caused by realized news $w_{t-\tau}$ or contemporaneous shock w_t^τ and ν_t^τ. That is the reason why these three terms share the same coefficient $\gamma_4 = \gamma_5 = \gamma_6$, and similarly $y_{i-\tau-1}^\tau$ in equation 6 works as a contemporaneous shock w_t^τ which offsets the effect of true shock $w_{t-\tau}$ at time t.

C.3.3 Proof to equation 29

Firstly I assume the law of motion of the shock g_t follows

$$g_t = \rho g_{t-1} + w_{t-\tau} + w_t^\tau$$

where $w_{t-\tau}$ is a shock realized at $t-\tau$ yet has effect on t. w_t^τ is a contemporaneous unexpected shock realized at time t.

The household cannot know the value of the value of shock underneath g_t and w_t but a signal to then

$$\widetilde{g}_t = g_t + \nu_t^\tau$$

$$\widetilde{w}_{t-\tau} = w_{t-\tau} + \nu_{t-\tau}$$

Household at time $t-\tau$ will have a perception of $w_{t-\tau}$ given the observation $\widetilde{w}_{t-\tau}$ and I denote it as $w_{t-\tau|t-\tau} = \theta \widetilde{w}_{t-\tau}$

Denote $\widetilde{w}_t^i$ as an observation to shock w_{t-i}. For example, a news shock w_t will have effect on G at $t+\tau$. At time $t+1$ household gets a new observation related to w_t, $\widetilde{w}_{t+1}^1$, in addition to the old observation of w_t at time t $\widetilde{w}_t$. I further assume

$$\widetilde{w}^1_{t-\tau+1} = \widetilde{w}^2_{t-\tau+2} = \cdots = \widetilde{w}^{\tau-1}_{t-1} = 0$$

holds. Therefore

$$w_{t-\tau|t-\tau} = w_{t-\tau|t-\tau+1} = w_{t-\tau|t-\tau+2} = \cdots = w_{t-\tau|t-1}$$

Above system of equation can be written as a state equation

$$\begin{bmatrix} g_t \\ w_{t-\tau+1} \end{bmatrix} = \begin{bmatrix} \rho & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} g_{t-1} \\ w_{t-\tau} \end{bmatrix} + \begin{bmatrix} w^\tau_t \\ w_{t-\tau+1} \end{bmatrix}$$

and observation(moment) equation

$$\begin{bmatrix} \widetilde{g}_t \\ \widetilde{w}_{t-\tau+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} g_t \\ w_{t-\tau+1} \end{bmatrix} + \begin{bmatrix} \nu^\tau_t \\ \nu_{t-\tau+1} \end{bmatrix}$$

For simplicity I denote $y_t = \begin{bmatrix} g_t \\ w_{t-\tau+1} \end{bmatrix}$, $\widetilde{y}_t = \begin{bmatrix} \widetilde{g}_t \\ \widetilde{w}_{t-\tau+1} \end{bmatrix}$, $B = \begin{bmatrix} \rho & 1 \\ 0 & 0 \end{bmatrix}$, $H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\omega_t = \begin{bmatrix} w^\tau_t \\ w_{t-\tau+1} \end{bmatrix}$ and $v_t = \begin{bmatrix} \nu^\tau_t \\ \nu_{t-\tau+1} \end{bmatrix}$.

Following Hamilton (2020) we can solve the conditional expectation of the variance of $Z = \Sigma_y(t|t)$ follows

$$B\left[Z - Z\left(Z + \Sigma_\nu\right)^{-1} Z\right] B' + \Sigma_\omega = Z \tag{30}$$

where I omit the observation matrix H as it is an identity matrix.

Since the second row of B is zero, the matrix $D = BXB'$ must follow $D = \begin{bmatrix} d & 0 \\ 0 & 0 \end{bmatrix}$. Plugging the matrix D back to equation 30 yidelds $D + \Sigma_\omega = Z$. Therefore we must have

$$Z = \begin{bmatrix} d + \sigma^2_{w^\tau} & 0 \\ 0 & \sigma^2_w \end{bmatrix} = \begin{bmatrix} z_{11} & 0 \\ 0 & \sigma^2_w \end{bmatrix}$$

By solving the equation

$$\begin{bmatrix} z_{11} - \sigma^2_{w^\tau} & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \rho & 1 \\ 0 & 0 \end{bmatrix} \left\{ \begin{bmatrix} z_{11} & 0 \\ 0 & \sigma^2_w \end{bmatrix} - \begin{bmatrix} z_{11} & 0 \\ 0 & \sigma^2_w \end{bmatrix} \begin{bmatrix} (z_{11} + \sigma^2_{\nu^\tau})^{-1} & 0 \\ 0 & \sigma^2_w \end{bmatrix} \begin{bmatrix} z_{11} & 0 \\ 0 & \sigma^2_w \end{bmatrix} \right\} \begin{bmatrix} \rho & 0 \\ 1 & 0 \end{bmatrix}$$

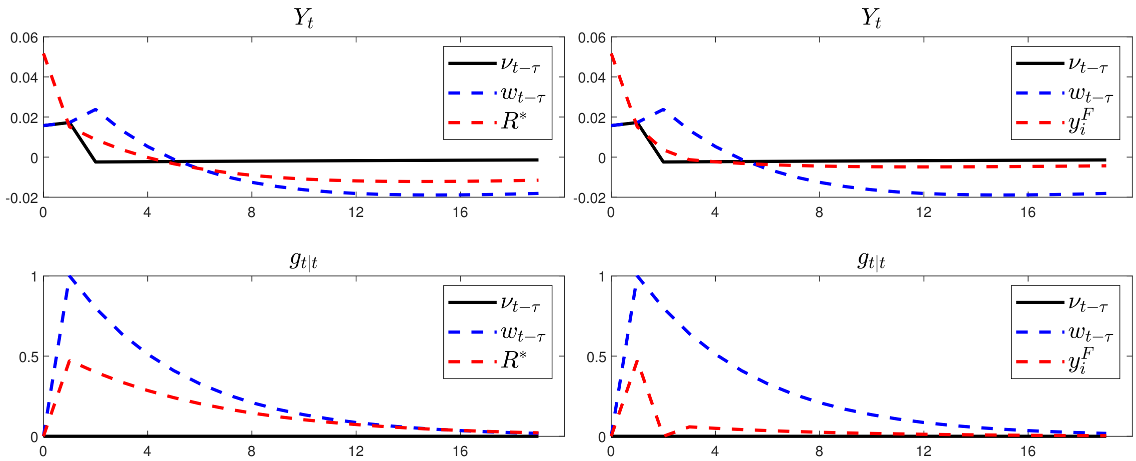
we can solve out z_{11} as the positive root of quadratic equation

$$z^2_{11} + \left(\sigma^2_{\nu^\tau} - \rho^2\sigma^2_{\nu^\tau} - \sigma^2_w + \sigma^2_{w^\tau} - \frac{\sigma^4_w}{\sigma^2_w + \sigma^2_\nu}\right) z_{11} - \sigma^2_{\nu^\tau}\left(\sigma^2_w + \sigma^2_{w^\tau} - \frac{\sigma^4_w}{\sigma^2_w + \sigma^2_\nu}\right) = 0$$

Then we can solve the law of motion of perception(conditional expectation) of y_t as $y_{t|t} = (I - PH) B y_{t-1|t-1} + P\widetilde{y}_t$ where P is the Kalman gain following $P = ZH' (HZH' + \Sigma_v)^{-1}$.

C.4 Identification Test

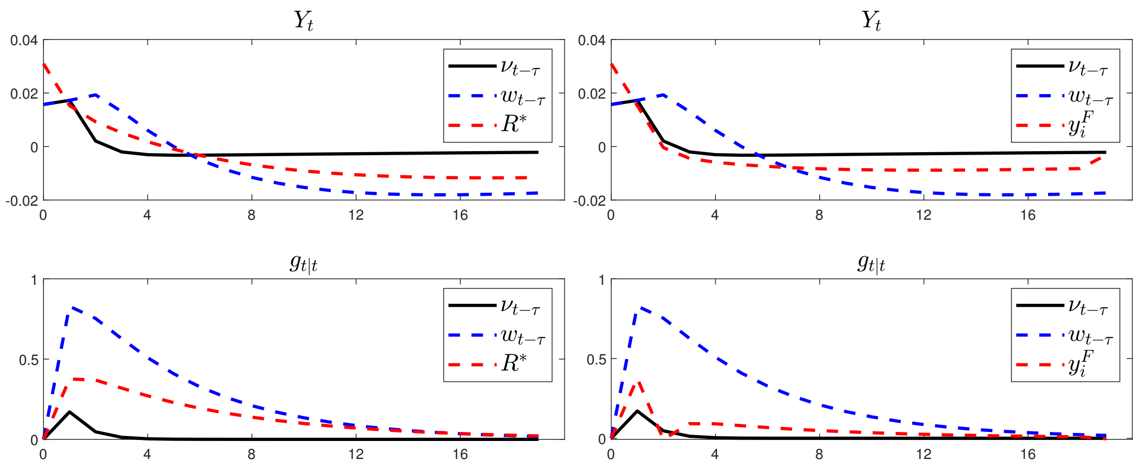
C.4.1 The fundamental impact g_t is observable.



(a) Identification to news shock R^* (b) Identification to fake news shock R_F^*

Figure 20: Identification Test to observable fundamental impact

C.4.2 The fundamental impact g_t is unobservable.



(a) Identification to news shock R^* (b) Identification to fake news shock R_F^*

Figure 21: Identification Test to unobservable fundamental impact

Figure 21 shows the result of the identification test.

D Perturbation result around the Simple Model

D.1 Proof of Proposition 2

The Lagrangian of the problem 7 could be written as

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t U^i\left(c_t^i, h_t^i\right) + \sum_{t=0}^{\infty} \lambda_t^i \left[R_t a_{t-1}^i + w_t \varepsilon_t^i + \left(1-\delta^H\right) p_t^H h_{t-1}^i + \pi_t^i + \pi_t^{H,i} - c_t^i - a_t^i - p_t^H h_t^i\right]$$
$$+ \sum_{t=0}^{\infty} \mu_t^i \left(p_t^H h_t^i + a_t^i\right)$$

I omit the superscript i henceforth for convenience. Then the first order condition would be

$$U_{c_t} = \lambda_t \tag{31}$$

$$-\lambda_t + \mu_t + \beta E_t R_{t+1} \lambda_{t+1} = 0 \tag{32}$$

$$U_{h_t} - \lambda_t p_t^H + \mu_t p_t^H + \beta\left(1-\delta^H\right) E_t \lambda_{t+1} p_{t+1}^H = 0 \tag{33}$$

To break the expectation I can rearrange the equation 33 as

$$U_{h_t} = \left(\lambda_t - \mu_t\right) p_t^H - \left(1-\delta^H\right)\left(\lambda_t - \mu_t\right) \frac{1}{E_t R_{t+1}} E_t p_{t+1}^H + \beta\left(1-\delta^H\right) \frac{cov\left(\lambda_{t+1}, R_{t+1}\right)}{E_t R_{t+1}} E_t p_{t+1}^H$$
$$- \beta\left(1-\delta^H\right) cov\left(\lambda_{t+1}, p_{t+1}^H\right) \tag{34}$$

Since the interest rate here is not related to the issue we want to solve, I further assume the exogenous TFP of non-durable goods production function is constant. Together with some assumption on the production function of durable and non-durable goods[28], $R_{t+1} = R_t = \overline{R}$ and $cov\left(\lambda_{t+1}, R_{t+1}\right) = 0$ will hold. Combining this assumption I log linearize equation 34 to get

$$\widetilde{U}_{h_t} = \frac{(\lambda-\mu)\left[p^H - \left(1-\delta^H\right) p^H \frac{1}{R}\right]}{U_h} \left\{ \frac{\lambda}{\lambda-\mu}\widetilde{\lambda}_t - \frac{\mu}{\lambda-\mu}\widetilde{\mu}_t + \frac{p^H}{p^H - \left(1-\delta^H\right) p^H \frac{1}{R}} \widetilde{p}_t^H - \right.$$
$$\left. \frac{\left(1-\delta^H\right) p^H \frac{1}{R}}{p^H - \left(1-\delta^H\right) p^H \frac{1}{R}} \widetilde{p}_{t+1}^H \right\} - \frac{\beta\left(1-\delta^H\right) \overline{cov}}{U_h} \widetilde{cov}_t \tag{35}$$

where $\widetilde{cov}_t$ is the percentage derivation from steady state of $cov\left(\lambda_t, p_t^H\right)$

[28] The related assumptions are described at appendix F.1.1.

Then following Etheridge (2019) I expand U_{c_t} around its steady-state value U_c to get

$$U_{c_t} \approx U_c + U_{cc} c \widetilde{c}_t + U_{ch} h \widetilde{h}_t$$

I rearrange above equation to get

$$\frac{U_{c_t} - U_c}{U_c} = d \ln u_{c_t} = \widetilde{U}_{c_t} = \frac{U_{cc} c}{U_c} \widetilde{c}_t + \frac{U_{ch} h}{U_c} \widetilde{h}_t \tag{36}$$

Similarly expanding U_{h_t}around its steady-state value U_h gives

$$\frac{U_{h_t} - U_h}{U_h} = d \ln u_{h_t} = \widetilde{U}_{h_t} = \frac{U_{hc} c}{U_h} \widetilde{c}_t + \frac{U_{hh} h}{U_h} \widetilde{h}_t \tag{37}$$

Perturbing around its steady state for equation 31 returns

$$\widetilde{U}_{c_t} = \widetilde{\lambda}_t \tag{38}$$

Combining equation 35, 36, 37 and 38 I can solve out

$$\widetilde{c}_t = \left(\frac{\lambda}{\lambda - \mu} \eta_{c,p^H} - \eta_{c,p^c} \right) \widetilde{\lambda}_t - \frac{\mu}{\lambda - \mu} \eta_{c,p^H} \widetilde{\mu}_t + \eta_{c,p^H} \left[\frac{1}{1 - (1 - \delta^H) \frac{1}{R}} \widetilde{p}_t^H - \right.$$
$$\left. \frac{(1 - \delta^H) \frac{1}{R}}{1 - (1 - \delta^H) \frac{1}{R}} \widetilde{p}_{t+1}^H \right] - \frac{U_{ch}}{U_{ch}^2 - U_{cc} U_{hh}} \frac{\beta (1 - \delta^H) \overline{cov}}{c} \widetilde{cov}_t$$

Then plugging back equation 31 gives

$$\widetilde{c}_t = \frac{\frac{\lambda}{\lambda-\mu} \eta_{c,p^H} - \eta_{c,p^c}}{\eta_{h,p^c} - \frac{\lambda}{\lambda-\mu} \eta_{h,p^h}} \widetilde{h}_t - \frac{\mu}{\lambda - \mu} \frac{\eta_{ch}}{\eta_{h,p^c} - \frac{\lambda}{\lambda-\mu} \eta_{h,p^h}} \widetilde{\mu}_t + \frac{\eta_{ch}}{\eta_{h,p^c} - \frac{\lambda}{\lambda-\mu} \eta_{h,p^h}} \left[\frac{1}{1 - (1 - \delta^H) \frac{1}{R}} \widetilde{p}_t^H - \right.$$
$$\left. \frac{(1 - \delta^H) \frac{1}{R}}{1 - (1 - \delta^H) \frac{1}{R}} \widetilde{p}_{t+1}^H \right] - \frac{\eta_c}{\eta_{h,p^c} - \frac{\lambda}{\lambda-\mu} \eta_{h,p^h}} \frac{\beta (1 - \delta^H) \overline{cov}}{h} \widetilde{cov}_t$$

where η_{h,p^c}, η_{h,p^h}, η_{c,p^H}, η_{c,p^c}, η_{ch} and η_c are

$$\eta_{c,p^H} = \frac{u_{ch} u_h}{u_{ch}^2 - u_{cc} u_{hh}} \frac{1}{c}$$

$$\eta_{c,p^c} = \frac{u_{hh} u_c}{u_{ch}^2 - u_{cc} u_{hh}} \frac{1}{c}$$

$$\eta_{h,p^c} = \frac{u_{ch} u_c}{u_{ch}^2 - u_{cc} u_{hh}} \frac{1}{h}$$

$$\eta_{h,p^h} = \frac{u_{cc} u_h}{u_{ch}^2 - u_{cc} u_{hh}} \frac{1}{h}$$

$$\eta_{ch} = \frac{u_c u_h}{u_{ch}^2 - u_{cc} u_{hh}} \frac{1}{ch}$$

$$\eta_c = \frac{u_c}{u_{ch}^2 - u_{cc} u_{hh}} \frac{1}{c}$$

D.2 Derivation of the Definition of Intratemporal Elasticity of substitution 15

Firstly, following the standard procedure I first define the optimization problem

$$\underset{c,h}{max} u(c,h)$$

$$\text{s.t. } c + p^h h = y$$

where c is the consumption,p^h is the relative price of housing services and y is the exogenous income. The interior solution implies

$$p^h = \frac{u_h}{u_c}$$

which is used to define the intratemporal elasticity of substitution

$$\begin{aligned} \text{ES} &= -\frac{d\ln\left(\frac{c}{h}\right)}{d\ln\left(p^h\right)} \\ &= -\frac{d\ln\left(\frac{c}{h}\right)}{d\ln\left(\frac{U_c}{U_h}\right)} \end{aligned}$$

D.3 Proof of Proposition 3

I first use the same production function 19 and 20 which I defined at section 4. Since the sample model in section 3 is frictionless in adjusting housing and physical capital, the goods market clearing condition should be

$$\begin{aligned} Y &= Y_H + Y_N \\ &= C + I_N + I_H \end{aligned}$$

where $Y_H = I_H$ and $Y_N = C + I_N$

Combining equation 47 and the market clearing condition of capital I can get

$$\alpha Y_{N,t} + \nu P_t^H Y_{H,t} = (r_t + \delta) K_{t-1}$$

Taking differential on both side of above equation around their steady state will yield

$$\alpha dY_{N,t} + \nu Y_H dP_t^H + \nu P^H dY_{H,t} = 0$$

because the total capital K_{t-1} is predetermined and r_t is fixed by assumption. Further because the amount of total housing service at time $t-1$, H_{t-1} is predetermined, above equation can be rewritten to

$$\alpha dY_{N,t} + \nu Y_H dP_t^H + \nu P^H dH_t = 0$$

Plugging this back to goods market clearing condition will return the general equilibrium condition of crowd-out effect

$$-I_N \widetilde{I}_{N,t} = C\widetilde{C}_t + \frac{\nu}{\alpha} Y_H P^H \widetilde{P}_t^H + \frac{\nu}{\alpha} P^H H \widetilde{H}_t$$

Finally the equation 14 can be obtained by plugging equation 10 into above equation.

D.4 Proof of Corollary 1

If the household utility function follows the standard CRRA form

$$u_t = \frac{\left(\phi c_t^{\gamma} + (1-\phi) s_t^{1-\gamma}\right)^{\frac{1-\sigma}{1-\gamma}}}{1-\sigma}$$

Therefore the intratempral elasticity of substitution will be $\text{ES} = \frac{1}{\gamma}$ and the intertemporal elasticity of substitution will be $\text{EIS} = \frac{1}{\sigma}$ and $u_{ch} = \phi(1-\phi)(\gamma-\sigma)c^{\gamma-\sigma-1}h^{-\gamma}\left[\phi + (1-\phi)(\frac{h}{c})^{1-\gamma}\right]^{\frac{\gamma-\sigma}{1-\gamma}}$. Then based on the definition of relative force of substitution effect Φ_H and wealth effect Φ_{p^H} the prove process is straightforward.

D.5 Proof of Corollary 2

Iterating equation 33 forward with expectation at t on both side, I can eliminate the intra-price term until time $T+1$ with the chain rule of expectation

$$\begin{aligned}
U_{h_t} + (\mu_t - \lambda_t) p_t^H + \beta\left(1-\delta^H\right) E_t \lambda_{t+1} p_{t+1}^H &= 0 \\
U_{h_{t+1}} + (\mu_{t+1} - \lambda_{t+1}) p_{t+1}^H + \beta\left(1-\delta^H\right) E_{t+1} \lambda_{t+2} p_{t+2}^H &= 0 \\
U_{h_{t+2}} + (\mu_{t+2} - \lambda_{t+2}) p_{t+2}^H + \beta\left(1-\delta^H\right) E_{t+2} \lambda_{t+3} p_{t+3}^H &= 0 \\
&\vdots \\
U_{h_{t+T}} + (\mu_{t+T} - \lambda_{t+T}) p_{t+T}^H + \beta\left(1-\delta^H\right) E_{t+T} \lambda_{t+T+1} p_{t+T+1}^H &= 0
\end{aligned} \tag{39}$$

Multiple $\frac{\beta\left(1-\delta^H\right)\lambda_{t+i}}{\lambda_{t+i}-\mu_{t+i}}$ on both side of above equation will yield (here I only take equation 39 as an example)

$$\frac{\beta\left(1-\delta^H\right)\lambda_{t+1}}{\lambda_{t+1}-\mu_{t+1}} U_{h_{t+1}} - \beta\left(1-\delta^H\right)\lambda_{t+1} p_{t+1}^H + \beta\left(1-\delta^H\right)\frac{\beta\left(1-\delta^H\right)\lambda_{t+1}}{\lambda_{t+1}-\mu_{t+1}} E_{t+1}\lambda_{t+2} p_{t+2}^H = 0$$

The last term can be rearranged to $\left[\beta\left(1-\delta^H\right)\right]^2 E_{t+1}\frac{\lambda_{t+1}}{\lambda_{t+1}-\mu_{t+1}}\lambda_{t+2}p^H_{t+2}$ because the term $\frac{\lambda_{t+1}}{\lambda_{t+1}-\mu_{t+1}}$ only contains the term at time $t+1$ which is known at time $t+1$. Then take expectation with the information at time t on both side of this equation to aggregate as

$$U_{h_t}+\mathbb{E}_t\sum_{i=1}^{T}\left[\beta\left(1-\delta^H\right)\right]^i\left[\prod_{s=1}^{i}\frac{\lambda_{t+s}}{\lambda_{t+s}-\mu_{t+s}}\right]U_{h_{t+i}}+\mathbb{E}_t\left[\beta\left(1-\delta^H\right)\right]^T\prod_{s=1}^{T}\frac{\lambda_{t+s}}{\lambda_{t+s}-\mu_{t+s}}\lambda_{t+T+1}p^H_{t+T+1}=0$$

Equation 16 can be derived by take total differential on both side to above equation.

E Toy model with global solution

Given the budget constraint of household

$$c_0+a_1+p_0\left[s_1-(1-\delta^h)s_0\right]=(1+R_0)a_0+w_0+\pi_0^h+\pi_0$$

$$c_1+a_2+p_1\left[s_2-(1-\delta^h)s_1\right]=(1+R_1)a_1+w_1+\pi_1^h+\pi_1$$
$$c_2=(1+R_2)a_2+p_2(1-\delta^h)s_2+w_2+\pi_2^h+\pi_2$$

From utility function and FOC of household we can get the key equation

$$u_{c_0}\left[p_0-\frac{1}{1+R_1}(1-\delta^h)p_1\right]=u_{s_1} \tag{40}$$

Then if we assume the utility function is non-separable such that

$$u_t=\frac{\left(c_t^{\nu}s_t^{1-\nu}\right)^{1-\sigma}}{1-\sigma}$$

By using the Euler equation of consumption as well as housing we can simplify equation 40 to

$$\left[p_0-\frac{1}{1+R_1}(1-\delta^h)p_1\right]=\frac{c_1}{s_1^{\Phi}s_0^{\Psi}}$$

E.1 General equilibrium is important

A perturb happened at p_1 will decrease c_1 which in tern decrease c_2, If p_0, s_1 and R_1 not change.(This is the total effect of substitution and income as we derive from max utility which means from Marshallian demand function. This is pseudo-effect as we assume s_1 fixed)

However this analysis is based on the assumption that p_0, s_1 and R_1 will not change. Now we assume s_1 is not changed. Meanwhile the production is $Y_t=Aa_t$ so that $R_t=MPK=A$ which means R_1 will also be fixed. Which direction of p_0 changed?

The answer is that any small perturb increased happened in p_1 which returns $\tilde{p_1} = p_1 + \varepsilon$, p_0 will increase relative amount to make sure $p_0 - (1-\delta^h)p_1$ is fixed. This tells us that c_1 will in fact not change at all.[29]

Later we can also proof that given the decreasing return to scale production function such as $Y_t = Aa_t^{\alpha}$ will not change the result.

Intuition: Given p_1 increased, the household want to buy more s_1 at period 0. The fixed s_1 will caused p_0 increases a lot to even offset the wealth effect. If we assume s_1 increases and p_0 not change ($s_1$supply increased to the level that just fulfill the demand and p_0 does not change) the direction of c_1 will depends on the extent of increased s_1 and intratemperal substitution and intertemperal substitution). Another condition, p_0 increases more than related to $\frac{1}{1+R_1}(1-\delta^h)p_1$ is somehow less likely as an expectation causes a much higher inflation this period.

E.2 House supply is the key to determine non-durable consumption

Now we losse the assumption that s_1 does not change. From last section we know that under general equilibrium as long as the house supply does not increase, then no matter how large changed in p_1, c_1 will not change anymore because p_0 will adjusted one-to-one with it.

This give us the argument that the house supply or elasticity of house supply is much more important than scholar's focusing, as most of time we just take it as an IV in empirical research.

A right-hand shift in period 0 house demand(caused by a perturb in p_1) happened, the elasticity of house supply then determine the equilibrium changed in s. We have prove at previous section that when $e_1 = 0$, the increased p_0 will caused c_0 not change. In other words, under the most increased p_0, c_0 not changed. Then assume $e_1 > 0$, Δp_0 will decrease. LHS of equation 40 decrease. But because the intratemporal effect is larger than intertemporal effect, c_1 and c_0 will increase. In other words, the degree of elasticity of house supply determinate the non-durable consumption.

E.3 Unseparable utility function

E.3.1 partial effect

If the utility function is

$$u_t = \frac{\left(c_t^{\nu} s_t^{1-\nu}\right)^{1-\sigma}}{1-\sigma}$$

then we will have

$$s_0^{(1-\nu)(1-\sigma)} c_0^{\nu(1-\sigma)-1} = \beta R_1 s_1^{(1-\sigma)(1-\nu)} c_1^{\nu(1-\sigma)-1}$$

[29]The proof process is simple using induction. Given p_0 increases little but not enough to offset total decreased c_1. Then c_1 and c_0 will decreases little. Then using budget constraint, a_1 and $a_2$will relatively changed. Then to the final period we can get a contradiction. Inversely given p_0 increases a lot to result in c_1 increassing, we can get similar contradiction.

$$s_1^{(1-\nu)(1-\sigma)} c_1^{\nu(1-\sigma)-1} = \beta R_2 s_2^{(1-\sigma)(1-\nu)} c_2^{\nu(1-\sigma)-1}$$

$$\nu s_0^{(1-\nu)(1-\sigma)} c_0^{\nu(1-\sigma)-1} p_0 = \beta\nu s_1^{(1-\sigma)(1-\nu)} c_1^{\nu(1-\sigma)-1} p_1(1-\delta^h) + \beta(1-\nu) c_1^{\nu(1-\sigma)} s_1^{\nu(\sigma-1)-\sigma}$$

$$\nu s_1^{(1-\nu)(1-\sigma)} c_1^{\nu(1-\sigma)-1} p_1 = \beta\nu s_2^{(1-\sigma)(1-\nu)} c_2^{\nu(1-\sigma)-1} p_2(1-\delta^h) + \beta(1-\nu) c_2^{\nu(1-\sigma)} s_2^{\nu(\sigma-1)-\sigma}$$

Then we will solve out c_1, c_2, s_1, s_2 by these four equations

$$c_1 = \left[\frac{1}{\beta R_1}\right]^{\frac{(1-\nu)(1-\sigma)-1}{\sigma}} \left\{\frac{\nu}{1-\nu}\frac{1}{\beta}\left[p_0 - \frac{1}{R_1}p_1\left(1-\delta^h\right)\right]\right\}^{-\frac{(1-\nu)(1-\sigma)}{\sigma}} \left[s_0^{(1-\nu)(1-\sigma)} c_0^{\nu(1-\sigma)-1}\right]^{-\frac{1}{\sigma}}$$

$$\begin{aligned} s_1 &= \left\{\frac{\nu}{1-\nu}\frac{1}{\beta}\frac{s_0^{(1-\nu)(1-\sigma)} c_0^{\nu(1-\sigma)-1}}{c_1^{\nu(1-\sigma)}}\left[p_0 - \frac{1}{R_1}p_1\left(1-\delta^h\right)\right]\right\}^{\frac{1}{(1-\nu)(1-\sigma)-1}} \\ &= \left[s_0^{(1-\nu)(1-\sigma)} c_0^{\nu(1-\sigma)-1}\right]^{-\frac{1}{\sigma}} \\ &\left\{\frac{\nu}{1-\nu}\frac{1}{\beta}\left[p_0 - \frac{1}{R_1}p_1\left(1-\delta^h\right)\right]\right\}^{\frac{(1-\nu)(1-\sigma)}{(1-\nu)(1-\sigma)-1}\frac{\nu(1-\sigma)}{\sigma}+\frac{1}{(1-\nu)(1-\sigma)-1}} \left[\frac{1}{\beta R_1}\right]^{\frac{\nu(1-\sigma)}{\sigma}} \end{aligned}$$

$$\begin{aligned} c_2 &= \left[\frac{1}{\beta^2 R_1 R_2}\right]^{\frac{(1-\nu)(1-\sigma)-1}{\sigma}} \\ &\left\{\frac{\nu}{1-\nu}\frac{1}{\beta^2 R_1}\left[p_1 - \frac{1}{R_2}p_2\left(1-\delta^h\right)\right]\right\}^{-\frac{(1-\nu)(1-\sigma)}{\sigma}} \left[s_0^{(1-\nu)(1-\sigma)} c_0^{\nu(1-\sigma)-1}\right]^{-\frac{1}{\sigma}} \end{aligned}$$

$$\begin{aligned} s_2 &= \left\{\frac{\nu}{1-\nu}\frac{1}{\beta^2 R_1}\frac{s_0^{(1-\nu)(1-\sigma)} c_0^{\nu(1-\sigma)-1}}{c_2^{\nu(1-\sigma)}}\left[p_1 - \frac{1}{R_2}p_2\left(1-\delta^h\right)\right]\right\}^{\frac{1}{(1-\nu)(1-\sigma)-1}} \\ &= \left[s_0^{(1-\nu)(1-\sigma)} c_0^{\nu(1-\sigma)-1}\right]^{-\frac{1}{\sigma}} \\ &\left\{\frac{\nu}{1-\nu}\frac{1}{\beta^2 R_1}\left[p_1 - \frac{1}{R_2}p_2\left(1-\delta^h\right)\right]\right\}^{\frac{(1-\nu)(1-\sigma)}{(1-\nu)(1-\sigma)-1}\frac{\nu(1-\sigma)}{\sigma}+\frac{1}{(1-\nu)(1-\sigma)-1}} \left[\frac{1}{\beta^2 R_1 R_2}\right]^{\frac{\nu(1-\sigma)}{\sigma}} \end{aligned}$$

Under infinite horizon we will have

$$\begin{aligned} c_t &= \left[\frac{1}{\beta^t \prod_{i=1}^t R_i}\right]^{\frac{(1-\nu)(1-\sigma)-1}{\sigma}} \\ &\left\{\frac{\nu}{1-\nu}\frac{1}{\beta^t \prod_{i=1}^{t-1} R_i}\left[p_{t-1} - \frac{1}{R_t}p_t\left(1-\delta^h\right)\right]\right\}^{-\frac{(1-\nu)(1-\sigma)}{\sigma}} \left[s_0^{(1-\nu)(1-\sigma)} c_0^{\nu(1-\sigma)-1}\right]^{-\frac{1}{\sigma}} \end{aligned}$$

$$s_t = \left[\frac{1}{\beta^t \prod_{i=1}^{t} R_i}\right]^{\frac{\nu(1-\sigma)}{\sigma}}$$

$$\left\{\frac{\nu}{1-\nu}\frac{1}{\beta^t \prod_{i=1}^{t-1} R_i}\left[p_{t-1} - \frac{1}{R_t}p_t\left(1-\delta^h\right)\right]\right\}^{\frac{(1-\nu)(1-\sigma)}{(1-\nu)(1-\sigma)-1}\frac{\nu(1-\sigma)}{\sigma}+\frac{1}{(1-\nu)(1-\sigma)-1}} \left[s_0^{(1-\nu)(1-\sigma)} c_0^{\nu(1-\sigma)-1}\right]^{-\frac{1}{\sigma}}$$

E.3.2 Other utility function

If the utility function is

$$u_t = log\left(c_t^{\nu} s_t^{1-\nu}\right)$$

then no GE effect

If the utility function is

$$u_t = log\left(c_t^{\nu} + s_t^{1-\nu}\right)$$

still unsolvable.

E.4 Standard utility function

E.4.1 general effect

No we assume that the utility function is no longer logarithmic such that

$$u_t = \frac{\left(c_t^{\nu} s_t^{1-\nu}\right)^{1-\sigma}}{1-\sigma}$$

Then we have two key market cleaning condition that

$$a_2 = A_1 a_1^{\alpha} - c_1 + (1-\delta)a_1 = A_1 a_1^{\alpha} - c_0\left(\beta R_1\right)^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{s_1}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}} + (1-\delta)a_1$$

$$(1-\delta)a_2 + A_2 a_2^{\alpha} = c_2 = c_0\left(\beta^2 R_1 R_2\right)^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{s_2}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}}$$

Based on these two equations we can rewrite equation as

$$(1-\delta)\left[A_1\left(A_0a_0^\alpha+(1-\delta)a_0-c_0\right)^\alpha-c_0\left(\beta\alpha A_1\left(A_0a_0^\alpha+(1-\delta)a_0-c_0\right)^{\alpha-1}\right)^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{s_1}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}}+\right.$$

$$\left.(1-\delta)\left(A_0a_0^\alpha+(1-\delta)a_0-c_0\right)\right]+$$

$$A_2\left[A_1\left(A_0a_0^\alpha+(1-\delta)a_0-c_0\right)^\alpha-c_0\left(\beta\alpha A_1\left(A_0a_0^\alpha+(1-\delta)a_0-c_0\right)^{\alpha-1}\right)^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{s_1}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}\alpha}+\right. \tag{41}$$

$$\left.(1-\delta)\left(A_0a_0^\alpha+(1-\delta)a_0-c_0\right)\right]=$$

$$c_0\left\{\beta^2\alpha^2A_1A_2\left(A_0a_0^\alpha+(1-\delta)a_0-c_0\right)^{\alpha-1}\right.$$

$$\left[A_1\left(A_0a_0^\alpha+(1-\delta)a_0-c_0\right)^\alpha-c_0\left(\beta\alpha A_1\left(A_0a_0^\alpha+(1-\delta)a_0-c_0\right)^{\alpha-1}\right)^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{s_1}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}}\right. \tag{42}$$

$$\left.\left.+(1-\delta)\left(A_0a_0^\alpha+(1-\delta)a_0-c_0\right)\right]^{\alpha-1}\right\}^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{s_2}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}}$$

Similarly we set $\alpha=1$, equation 41 becomes

$$(1-\delta)\left[A_1\left(A_0a_0+(1-\delta)a_0-c_0\right)-c_0\left(\beta\alpha A_1\right)^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{s_1}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}}+\right.$$

$$\left.(1-\delta)\left(A_0a_0+(1-\delta)a_0-c_0\right)\right]+$$

$$A_2\left[A_1\left(A_0a_0+(1-\delta)a_0-c_0\right)-c_0\left(\beta\alpha A_1\right)^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{s_1}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}\alpha}+\right.$$

$$\left.(1-\delta)\left(A_0a_0+(1-\delta)a_0-c_0\right)\right]=$$

$$c_0\left(\beta^2\alpha^2A_1A_2\right)^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{s_2}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}}$$

Now we can solve the c_0 as

$$c_0=\frac{\left(A_2+1-\delta\right)\left(A_1+1-\delta\right)\left(A_0a_0+(1-\delta)a_0\right)}{\left(A_2+1-\delta\right)\left[A_1+1-\delta+\left(\beta\alpha A_1\right)^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{s_1}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}}\right]+\left(\beta^2\alpha^2A_1A_2\right)^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{s_2}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}}}$$

$$=\frac{\left(A_2+1-\delta\right)\left(A_1+1-\delta\right)\left(A_0a_0+(1-\delta)a_0\right)}{\left(A_2+1-\delta\right)\left[A_1+1-\delta+\left(\beta\alpha A_1\right)^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{(1-\delta^h)s_0+\bar{s}_1}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}}\right]+\left(\beta^2\alpha^2A_1A_2\right)^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{\bar{s}_2+(1-\delta^h)\bar{s}_1+(1-\delta^h)^2s_0}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}}}$$

Under the GE and determined economy, c_0 can only be decided by the equalized house stock. It is intuitive as in the end because all excess profit are payback by construction companies and consumption is mainly determined by IES & market cleaning condition. If we assume that good market clean does not involve construction industry, the house market can only affect the consumption via the Euler equation of asset. Here $\bar{s}_2$ decreases will lead p_2 increase, but it increase c_0 at the same time.

E.4.2 Infinite horizon condition

The market cleaning condition will be

$$a_1 = A_0 a_0^{\alpha} + (1-\delta)a_0 - c_0$$

$$a_2 = A_1 a_1^{\alpha} - c_1 + (1-\delta)a_1$$

$$a_3 = A_2 a_2^{\alpha} - c_2 + (1-\delta)a_2$$

$$(1-\delta)a_{\infty} + A_{\infty}a_{\infty}^{\alpha} = c_{\infty} = c_0\left(\beta^3 R_1 R_2 R_3\right)^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{s_3}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}}$$

$$\begin{aligned} c_0 &= \frac{\left(A_0 a_0 + (1-\delta)a_0\right)\prod_{t=1}^{\infty}\left(A_t + 1 - \delta\right)}{\sum_{t=1}^{T}\left[\prod_{i=t}^{T}\left(A_i + 1 - \delta\right)\right]\left(\beta^{t-1}\alpha^{t-1}\prod_{i=0}^{t-1}A_i\right)^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{s_{t-1}}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}} + \left(\beta^T\alpha^T\prod_{t=0}^{T}A_t\right)^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{s_T}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}}} \\ &= \frac{\left(A_0 a_0 + (1-\delta)a_0\right)\prod_{t=1}^{T}\left(A_t + 1 - \delta\right)}{\sum_{t=1}^{T}\left[\prod_{i=t}^{T}\left(A_i + 1 - \delta\right)\right]\left(\beta^{t-1}\alpha^{t-1}\prod_{i=0}^{t-1}A_i\right)^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{\sum_{i=0}^{t-1}(1-\delta^h)^i\bar{s}_i}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}} + \left(\beta^T\alpha^T\prod_{t=0}^{T}A_t\right)^{\frac{1}{1-\nu(1-\sigma)}}\left(\frac{s_0}{\sum_{i=0}^{T}(1-\delta^h)^i\bar{s}_i}\right)^{\frac{(1-\nu)(1-\sigma)}{\nu(1-\sigma)-1}}} \end{aligned}$$

when normalizes $A_0 = 1$

E.5 Separable utility function

E.5.1 partial effect

$$c_1 = c_0\left(\beta R_1\right)^{\frac{1}{\sigma}}$$

$$c_2 = c_0\left(\beta^2 R_1 R_2\right)^{\frac{1}{\sigma}}$$

$$s_1 = \left[p_0 R_1 - p_1(1-\delta^h)\right]^{-\frac{1}{\nu}}$$

$$s_2 = \left[p_1 R_2 - p_2(1-\delta^h)\right]^{-\frac{1}{\nu}}$$

$$\begin{aligned} c_0\left(\beta^2 R_1 R_2\right)^{\frac{1}{\sigma}} + R_2 c_0\left(\beta R_1\right)^{\frac{1}{\sigma}} + R_1 R_2 c_0 + \\ R_2 p_1\left\{\left[p_1 R_2 - p_2(1-\delta^h)\right]^{-\frac{1}{\nu}} - (1-\delta^h)\left[p_0 R_1 - p_1(1-\delta^h)\right]^{-\frac{1}{\nu}}\right\} + \\ R_1 R_2 p_0\left\{\left[p_0 R_1 - p_1(1-\delta^h)\right]^{-\frac{1}{\nu}} - p_0(1-\delta^h)\right\} = \\ R_0 R_1 R_2 a_0 + R_1 R_2\left(w_0 + \pi_0\right) + R_2\left(w_1 + \pi_1\right) + w_2 + \pi_2 \\ + p_2(1-\delta^h)\left[p_1 R_2 - p_2(1-\delta^h)\right]^{-\frac{1}{\nu}} \end{aligned}$$

$$
\begin{aligned}
F_{p_1} &= R_2 \left\{ \left[p_1 R_2 - p_2(1-\delta^h)\right]^{-\frac{1}{\nu}} - (1-\delta^h)\left[p_0 R_1 - p_1(1-\delta^h)\right]^{-\frac{1}{\nu}} \right\} \\
&+ R_2 p_1 \left\{ -\frac{1}{\nu} R_2 \left[p_1 R_2 - p_2(1-\delta^h)\right]^{-\frac{1+\nu}{\nu}} - \frac{1}{\nu}(1-\delta^h)^2 \left[p_0 R_1 - p_1(1-\delta^h)\right]^{-\frac{1+\nu}{\nu}} \right\} \\
&+ \frac{(1-\delta^h)}{\nu} R_1 R_2 p_0 \left[p_0 R_1 - p_1(1-\delta^h)\right]^{-\frac{1+\nu}{\nu}} + \frac{1}{\nu} p_2 R_2 (1-\delta^h)\left[p_1 R_2 - p_2(1-\delta^h)\right]^{-\frac{1+\nu}{\nu}}
\end{aligned}
$$

$$
F_{c_0} = \left(\beta^2 R_1 R_2\right)^{\frac{1}{\sigma}} + R_2 \left(\beta R_1\right)^{\frac{1}{\sigma}} + R_1 R_2
$$

E.5.2 general effect

$$
a_1 = A_0 a_0^\alpha + (1-\delta)a_0 - c_0
$$

$$
\begin{aligned}
a_2 &= A_1 \left[A_0 a_0^\alpha + (1-\delta)a_0 - c_0\right]^\alpha - c_0 \left[\beta\alpha A_1 \left(A_0 a_0^\alpha + (1-\delta)a_0 - c_0\right)^{\alpha-1}\right]^{\frac{1}{\sigma}} \\
&+ (1-\delta)\left[A_0 a_0^\alpha + (1-\delta)a_0 - c_0\right]
\end{aligned}
$$

we can solve c_0 by

$$
(1-\delta)a_2 + A_2 a_2^\alpha = c_0 \left(\beta^2 \alpha^2 A_1 A_2 \left(a_1 a_2\right)^{\alpha-1}\right)^{\frac{1}{\sigma}}
$$

which means it is predetermined.

F Equilibrium condition of the full fledged model

F.1 Focs

F.1.1 Focs in production sector

The non-durable goods producer solve the problem

$$
\underset{K_n, L_n}{max} A_n K_{n,t}^\alpha L_{n,t}^{1-\alpha} - (r_t + \delta)K_{n,t} - w L_{n,t}
$$

to yield the Foc

$$
(1-\alpha) A_n K_{n,t}^\alpha L_{n,t}^{-\alpha} = w_t \tag{43}
$$

and

$$
\alpha A_n K_{n,t-1}^{\alpha-1} L_{n,t}^{1-\alpha} = r_t + \delta \tag{44}
$$

Similarly the durable goods producer solve the problem

$$
\underset{K_h, L_h}{max} \Pi^h = p_t^h A_h \overline{L}_t^\theta K_{h,t}^\nu L_{h,t}^\iota - (r_t + \delta)K_{h,t} - w L_h
$$

to yield the Foc

$$\iota A_h p_t^h \overline{L}_t^{\theta} K_{h,t}^{\nu} L_{h,t}^{\iota-1} = w_t \tag{45}$$

and

$$\nu A_h p_t^h \overline{L}_t^{\theta} K_{h,t}^{\nu-1} L_{h,t}^{\iota} = r_t + \delta \tag{46}$$

Combine equation 44 and 46 will yield

$$\frac{\nu p_t^h Y_{H,t}}{K_{h,t}} = r_t + \delta = \frac{\alpha Y_{N,t}}{K_{n,t}} \tag{47}$$

It is easy to check that when $\frac{\iota}{\nu} = \frac{1-\alpha}{\alpha}$ the real rental rate and wage at time t is fixed, as long as the total capital used at time t, K_{t-1} and labor L_t is fixed. I attach the proof process below.

By dividing equation 43, 44, 45 and 46 with each other I can get the relative input sharing condition

$$\frac{\iota\alpha}{\nu\left(1-\alpha\right)} \frac{K_{h,t}}{K_{n,t}} \frac{L_{n,t}}{L_{h,t}} = 1$$

when $\frac{\iota}{\nu} = \frac{1-\alpha}{\alpha}$ holds, above equation will change to $\frac{K_{h,t}}{K_{n,t}} = \frac{L_{n,t}}{L_{h,t}}$.

Furthermore, the relative value of $K_{n,t}$ and $L_{n,t}$ can be pinned down with the market clearing condition $K_{H,t-1} = K_{h,t} + K_{n,t}$ and $L_t = L_{h,t} + L_{n,t}$. In section 3 I assume that the labor supply is exogenous which will help to demonstrate that the relative value of $K_{n,t}$ and $L_{n,t}$ follows

$$\frac{K_{n,t}}{L_{n,t}} = \frac{K_{H,t-1}}{L} \frac{1 + \frac{K_{n,t}}{L_{n,t}}}{1 + \frac{K_{h,t}}{L_{h,t}}}$$

Because $K_{H,t-1}$ is predetermined and $\frac{K_{h,t}}{K_{n,t}} = \frac{L_{n,t}}{L_{h,t}}$, the $\frac{K_{n,t}}{L_{n,t}}$ is fixed. Therefore r_t is fixed from equation 47.

F.1.2 Focs in consumer sector

The household solve the problem

$$V(h_{t-1}, x_{t-1}, \varepsilon_{t-1}) = \max_{h_t, x_t, l_t, c_t} U\left(c_t, h_t, l_t\right) + \beta E V(h_t, x_t, \varepsilon_t)$$

$$\begin{aligned} \text{s.t.} c_t + x_t + \left(1-\gamma\right) p_t^h h_t = & \left[\left(1-\delta^h\right) p_t^h - \gamma R_t p_{t-1}^h\right] h_{t-1} + R_t x_{t-1} \\ & + (1-\tau) w_t l_t \varepsilon_{t-1} - p_t^h C\left(h_t, h_{t-1}\right) + T_t \end{aligned} \tag{48}$$

and

$$x_t \geq 0$$

The related Lagrange is

$$
\begin{aligned}
\mathcal{L} = & U\left(c_t, h_t, l_t\right) + \beta E_t V(h_t, x_t, \varepsilon_t) \\
& + \lambda_t \left[c_t + x_t + (1-\gamma)\, p_t^h h_t - \left[\left(1-\delta^h\right) p_t^h - \gamma R_t p_{t-1}^h\right] h_{t-1}\right. \\
& \left. - R_t x_{t-1} - (1-\tau) w_t l_t \varepsilon_{t-1} + p_t^h C\left(h_t, h_{t-1}\right) - T_t\right] \\
& + \mu_t x_t
\end{aligned}
$$

Then the FOCs related to consumer's problem will be

$$U_{c,t} + \lambda_t = 0 \tag{49}$$

$$U_{h,t} + \beta E_t V_{h,t} + \lambda_t \left(1 - \gamma + C_{h,t}\right) p_t^h = 0 \tag{50}$$

$$\beta E_t V_{x,t} + \lambda_t + \mu_t = 0 \tag{51}$$

$$U_{l,t} - \lambda_t (1-\tau) w_t \varepsilon_{t-1} = 0 \tag{52}$$

The envelop conditions are

$$V_h(h_{t-1}, x_{t-1}, \varepsilon_{t-1}) = -\lambda_t \left[\left(1-\delta^h\right) p_t^h - \gamma R_t p_{t-1}^h - C_{h_{t-1}}\left(h_t, h_{t-1}\right) p_t^h\right] \tag{53}$$

$$V_x(h_{t-1}, x_{t-1}, \varepsilon_{t-1}) = -\lambda_t R_t \tag{54}$$

F.2 Alternative Setting to Capital Producer

F.2.1 Capital Producer(Setting I)

The capital producer uses final nondurable goods Y_N to produce capital following the maximization problem

$$
\begin{aligned}
& \max\left(Q_t - 1\right) I_t - f\left(I_t, K_{t-1}\right) K_{t-1} \\
& \text{s.t. } f\left(I_t, K_{t-1}\right) = \frac{\psi_{I,1}}{\psi_{I,2}} \left(\frac{I_t}{K_{t-1}} - \overline{\delta}\right)^{\psi_{I,2}}
\end{aligned}
$$

where $\overline{\delta}$ is the steady-state investment rate.

By solving above optimization problem I could get the capital price as a convex function of investment which is shown below

$$Q_t = 1 + \psi_{I,1} \left(\frac{I_t}{K_{t-1}} - \overline{\delta}\right)^{\psi_{I,2}-1}$$

So the goods market clearing condition will become

$$Y_{N,t} = C_t + I_t + f\left(I_t, K_{t-1}\right) K_{t-1} + p^h C(h', h)$$

F.2.2 Capital Producer(Setting II)

The capital producer uses final nondurable goods Y_N to produce capital following the maximization problem

$$\begin{aligned} &\max Q_t I_t - f\left(I_t, K_{t-1}\right) K_{t-1} \\ \text{s.t. } f\left(I_t, K_{t-1}\right) = &\frac{\overline{\delta}^{-1/\phi}}{1+1/\phi}\left(\frac{I_t}{K_{t-1}}\right)^{1+1/\phi} + \frac{\overline{\delta}}{\phi+1} \end{aligned}$$

where $\overline{\delta}$ is the steady-state investment rate following $\overline{\delta} = \frac{\overline{I}}{\overline{K}}$

By solving above optimization problem I could get the capital price as a convex function of investment which is shown below

$$Q_t = \left(\frac{I_t}{K_{t-1}\overline{\delta}}\right)^{1+1/\phi}$$

So the goods market clearing condition will become

$$Y_{N,t} = C_t + f\left(I_t, K_{t-1}\right) K_{t-1} + p^h C\left(h', h\right)$$

F.2.3 Capital Producer(Setting III)

The capital producer uses final nondurable goods Y_N to produce capital following the maximization problem

$$\begin{aligned} &\max Q_t f\left(I_t, K_{t-1}\right) K_{t-1} - I_t \\ \text{s.t. } f\left(I_t, K_{t-1}\right) = &\frac{\overline{\delta}^{1/\phi}}{1-1/\phi}\left(\frac{I_t}{K_{t-1}}\right)^{1-1/\phi} - \frac{\overline{\delta}}{\phi+1} \end{aligned}$$

where $\overline{\delta}$ is the steady-state investment rate.

By solving above optimization problem I could get the capital price as a convex function of investment which is shown below

$$Q_t = \left(\frac{I_t}{K_{t-1}\overline{\delta}}\right)^{1-1/\phi}$$

and the law of motion of capital will become

$$K_t = (1 - \delta)K_{t-1} + f\left(I_t, K_{t-1}\right) K_{t-1}$$

The goods market clearing condition will become

$$Y_{N,t} = C_t + I_t + p^h C\left(h', h\right)$$

F.2.4 Capital Producer(Setting IV)

The capital producer uses final nondurable goods Y_N to produce capital following the maximization problem

$$\begin{aligned} &\max E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \Lambda_{t,t+\tau} \left\{ \left(Q_\tau - 1\right) I_\tau - f\left(I_\tau, I_{\tau-1}\right) I_\tau \right\} \\ &\text{s.t. } f\left(I_\tau, I_{\tau-1}\right) = \frac{\psi_{I,1}}{\psi_{I,2}} \left(\frac{I_\tau}{I_{\tau-1}} - 1 \right)^{\psi_{I,2}} \end{aligned}$$

By solving above optimization problem I could get the capital price as a convex function of investment which is shown below

$$\begin{aligned} Q_t = 1 + \frac{\psi_{I,1}}{\psi_{I,2}} \left(\frac{I_t}{I_{t-1}} - 1 \right)^{\psi_{I,2}} + \psi_{I,1} \left(\frac{I_t}{I_{t-1}} - 1 \right)^{\psi_{I,2}-1} \frac{I_t}{I_{t-1}} - \\ E_t \beta \Lambda_{t,t+1} \psi_{I,1} \left(\frac{I_{t+1}}{I_t} - 1 \right)^{\psi_{I,2}-1} \left(\frac{I_{t+1}}{I_t} \right)^2 \end{aligned}$$

So the goods market clearing condition will become

$$Y_{N,t} = C_t + I_t + f\left(I_t, I_{t-1}\right) I_{t-1} + p^h C(h', h)$$

G Numerical solution

G.1 Calibration to full fledged model

All the parameters related to production sector are selected from literature. The depreciation rate of physical capital is 0.03 which implies 12% annually. The depreciation rate of housing service is estimated from data which is constructed by Rognlie et al. (2018) as my model in supply side is too simple to use the gross GDP in NIPA. Therefore I use the GDP constructed by Rognlie et al. (2018) which is more suitable to this simple supply side. The depreciation rate of housing service is roughly 1.9% quarterly which is in line with Kaplan et al. (2020). The relative share of production factors in construction function ν, θ and ι comes from Favilukis et al. (2017). The last three parameters, exogenous land supply, TFP in production function and TFP in construction function, together with other parameters in household problem, are selected to match the real gross rate, labor demand, liquid asset over GDP and iliquid asset over GDP.

Table 5: Parameter Values

Parameter	**Value**	**Description**
δ	0.03	Depreciation rate of physical capital
δ^h	0.01873	Depreciation rate of housing service
α	0.36	Capital share in production function
ν	0.27	Capital share in construction function
ι	0.36	Labor share in construction function
θ	0.1	Land share in construction function
$\overline{LD}$	4.95	Land supply
A_n	0.67	TFP in production function
A_h	2.75	TFP in construction function

G.2 Bayesian estimation to full fledged model

I use Bayesian method to estimate the parameters that control the impulse response and transition path such as the AR1 coefficients ρ_a^i, the observation matrix H and related covariance matrix $\eta\eta'$ and $\epsilon\epsilon'$. Since the data process itself is not stationary it is not appropriate to use the full-information Bayesian and if we used the statistic method to detrend such as first-order difference and hp filter, the Bayesian update rule would not be further used and the posterior $p\left(\theta|Y^T\right) \propto p\left(Y^T|\theta\right) p\left(\theta\right)$ would be unsolvable as $p\left(Y^T|\theta\right)$ was unknown. Therefore I use GMM to match the moments in data and model to proceed the estimation. In this subsection I first introduce the moments I used to match the data and then explain the Bayesian estimation strategy in detail.

G.2.1 Moments Selection and Theoretical moments after filter

I impose hp filter on the data and calculate moments from the cyclical elements such as the autocovariance of output, standard derivation of output, physical investment, new constructed residential estate, relative housing price and their related covariance. The covariance between output and physical investment $\mathrm{cov}(y_t, I_t)$ captures the general equilibrium $Y = C + I$. Similarly the covariance between residential investment and physical investment $\mathrm{cov}(I_t^H, I_t)$ captures the crowded-out effect. The covariance between new constructed residential estate and relative housing price capture the demand and supply equilibrium in the housing market. All these eight moments are summarized in vector $g(\cdot) = \Psi$ following

$$\Psi = \left[\begin{array}{ccc} \varrho_m' & \sigma_{m,m}' & \sigma_{m,n}' \end{array} \right]'$$

where ϱ_m is the vector that contains the autocovariance moments (ρ_m^i represents the $\mathrm{AR}(i)$'s coefficient of variable m)

$$\varrho_m = \left[\begin{array}{cccccc} \rho_y^1 & \rho_c^1 & \rho_I^1 & \rho_{I_H}^1 & \rho_{pH}^1 & \rho_Q^1 \end{array} \right]'$$

$\sigma_{m,m}$ is the vector that contains the standard derivation moments

$$\sigma_{m,m} = \left[\begin{array}{ccccc} \sigma_y & \sigma_c & \sigma_I & \sigma_{pH} & \sigma_Q \end{array} \right]'$$

$\sigma_{m,n}$ is the vector that contains the covariance moments of variables $\phi_v = \left[\begin{array}{ccccccc} y & c & I & I_H & p_H & Q & R \end{array} \right]'$

$$\sigma_{m,n} = \left[\begin{array}{ccccccccc} \sigma_{y,c} & \sigma_{y,I} & \sigma_{y,I_H} & \sigma_{y,pH} & \sigma_{y,Q} & \sigma_{y,R} & \sigma_{c,I_H} & \cdots & \sigma_{Q,R} \end{array} \right]'$$

Moreover I solve the theoretical moments from model after hp filter by switching to frequency domain and the spectrum. After some algebra I can solve the covariance matrix

$$\mathbb{E}\left[\widetilde{Y}_t \widetilde{Y}_{t-1}\right] = \int_{-\pi}^{\pi} g^{\mathrm{HP}}(\omega) e^{i\omega k} d\omega$$

where $\widetilde{Y}_t = \left[\begin{array}{ccc} s_t' & s_{t|t}' & Ec_{t+1}' \end{array} \right]'$ in equation 73. The spectral density of HP filter $g^{\mathrm{HP}}(\omega)$ follows $g^{\mathrm{HP}}(\omega) = h^2(\omega) g(\omega)$. $h(\omega) = \frac{4\lambda(1-\cos(\omega))^2}{1+4\lambda(1-\cos(\omega))^2}$ is the transfer function of HP derived from King and Rebelo (1993). The spectral density of state and control variables Y_t is solved by

$$g(\omega) = \left[\begin{array}{cc} I_{ns} & 0_{ns,nq} \\ M_{21}\mathrm{e}^{-i\omega} & D_2 \\ 0_{nq,ns} & I_{nq} \end{array} \right] f(\omega) \left[\begin{array}{ccc} I_{ns} & M_{21}' e^{i\omega} & 0_{ns,nq} \\ 0_{nq,ns} & D_2' & I_{nq} \end{array} \right] = W f(\omega) W' \tag{55}$$

$$f(\omega) = \frac{1}{2\pi} \left[\begin{array}{c} \left(I_{ns} - M_{11}\mathrm{e}^{-i\omega}\right)^{-1} D_1 \\ I_{nq} \end{array} \right] \Sigma \left[D_1' \left(I_{ns} - M_{11}' e^{i\omega}\right)^{-1}, I_{nq} \right] \tag{56}$$

where ns is the number of state variables and nq is the number of shocks. M and D come from the policy function 79 and Σ is the covariance matrix of shocks. Because I assume the shock term Ξ_t in system 73 follows standard normal distribution and all the covariance terms are absorbed in η and ϵ, Σ in equation 56 is an identity matrix.

W.L.O.G, I assume the shock Ξ_t in equation 79 is independent with each other and all the covariance term is stored in response D. Therefore the covariance term Σ in equation 56 is an identity matrix and the equation can be further simplified as

$$f(\omega) = \frac{1}{2\pi}\begin{bmatrix} \left(I_{ns} - M_{11}\mathrm{e}^{-i\omega}\right)^{-1} D_1 D_1' \left(I_{ns} - M_{11}' e^{i\omega}\right)^{-1} & \left(I_{ns} - M_{11}\mathrm{e}^{-i\omega}\right)^{-1} D_1 \\ D_1' \left(I_{ns} - M_{11}' e^{i\omega}\right)^{-1} & I_{nq} \end{bmatrix}$$

Then equation 55 becomes

$$\begin{aligned} g(\omega) &= \frac{1}{2\pi}\begin{bmatrix} \left(I_{ns} - M_{11}\mathrm{e}^{-i\omega}\right)^{-1} D_1 D_1' \left(I_{ns} - M_{11}' e^{i\omega}\right)^{-1} & \left(I_{ns} - M_{11}\mathrm{e}^{-i\omega}\right)^{-1} D_1 \\ M_{21}\mathrm{e}^{-i\omega}\left(I_{ns} - M_{11}\mathrm{e}^{-i\omega}\right)^{-1} D_1 D_1' \left(I_{ns} - M_{11}' e^{i\omega}\right)^{-1} & M_{21}\mathrm{e}^{-i\omega}\left(I_{ns} - M_{11}\mathrm{e}^{-i\omega}\right)^{-1} D_1 \\ D_1' \left(I_{ns} - M_{11}' e^{i\omega}\right)^{-1} & I_{nq} \end{bmatrix} W' \\ &+ \frac{1}{2\pi}\begin{bmatrix} 0 & 0 \\ D_2 D_1' \left(I_{ns} - M_{11}' e^{i\omega}\right)^{-1} & D_2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} I_{ns} & M_{21}' e^{i\omega} & 0_{ns,nq} \\ 0_{nq,ns} & D_2' & I_{nq} \end{bmatrix} W' \\ &= \frac{1}{2\pi}\left(\Upsilon_1 + \Upsilon_2 + \Upsilon_3 + \Upsilon_4\right) \end{aligned}$$

where

$$\Upsilon_1 = \begin{bmatrix} \left(I_{ns} - M_{11}\mathrm{e}^{-i\omega}\right)^{-1} D_1 D_1' \left(I_{ns} - M_{11}' e^{i\omega}\right)^{-1} & \left(I_{ns} - M_{11}\mathrm{e}^{-i\omega}\right)^{-1} D_1 D_1' \left(I_{ns} - M_{11}' e^{i\omega}\right)^{-1} M_{21}' e^{i\omega} & \left(I_{ns} - M_{11}\mathrm{e}^{-i\omega}\right)^{-1} D_1 \\ M_{21}\mathrm{e}^{-i\omega}\left(I_{ns} - M_{11}\mathrm{e}^{-i\omega}\right)^{-1} D_1 D_1' \left(I_{ns} - M_{11}' e^{i\omega}\right)^{-1} & M_{21}\left(I_{ns} - M_{11}\mathrm{e}^{-i\omega}\right)^{-1} D_1 D_1' \left(I_{ns} - M_{11}' e^{i\omega}\right)^{-1} M_{21}' & M_{21}\mathrm{e}^{-i\omega}\left(I_{ns} - M_{11}\mathrm{e}^{-i\omega}\right)^{-1} D_1 \\ D_1' \left(I_{ns} - M_{11}' e^{i\omega}\right)^{-1} & D_1' \left(I_{ns} - M_{11}' e^{i\omega}\right)^{-1} M_{21}' e^{i\omega} & I_{nq} \end{bmatrix}$$

$$\Upsilon_2 = \begin{bmatrix} 0 & \left(I_{ns} - M_{11}\mathrm{e}^{-i\omega}\right)^{-1} D_1 D_2' & 0 \\ 0 & M_{21}\mathrm{e}^{-i\omega}\left(I_{ns} - M_{11}\mathrm{e}^{-i\omega}\right)^{-1} D_1 D_2' & 0 \\ 0 & D_2' & 0 \end{bmatrix}$$

$$\Upsilon_3 = \begin{bmatrix} 0 & 0 & 0 \\ D_2 D_1' \left(I_{ns} - M_{11}' e^{i\omega}\right)^{-1} & D_2 D_1' \left(I_{ns} - M_{11}' e^{i\omega}\right)^{-1} M_{21}' e^{i\omega} & D_2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Upsilon_4 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & D_2 D_2' & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

To further decrease the computation burden it is easy to show that $M_{21}\left(I_{ns} - M_{11}\mathrm{e}^{-i\omega}\right)^{-1} = \mathrm{e}^{i\omega} M_{21} U_M \left(\mathrm{e}^{i\omega} I_{ns} - T_M\right)^{-1} U_M'$ where $M_{11} = U_M T_M U_M'$ is prederived from Schur decomposition.

G.2.2 Bayesian GMM

Following Rotemberg and Woodford (1997), Christiano et al. (2005) and Barsky and Sims (2012), to construct the asymptotic properties of the moments which I select to conduct the

Bayesian GMM, I first construct the auxiliary variable ψ_t

$$\psi_t = \begin{bmatrix} y_t & c_t & I_t & I_{t,H} & p_{t,H} & Q_t & R_t & y_t y_{t-1} & c_t c_{t-1} & \cdots & y_t^2 & c_t^2 & \cdots & p_{t,H}^2 & y_t c_t & y_t I_t & \cdots & Q_t R_t \end{bmatrix}'$$

Additionally I define the moment function as $g(\cdot)$ which yields the moments

$$g(\psi_t) = \Psi$$

If the sample estimation of ψ_t is $\widehat{\psi}$ the moment function is well defined as

$$g(\widehat{\psi}) = \begin{bmatrix} \hat{\psi}_{y_t y_{t-1}} - \hat{\psi}_y^2 \\ \hat{\psi}_{c_t c_{t-1}} - \hat{\psi}_c^2 \\ \vdots \\ \sqrt{\hat{\psi}_{y^2} - \hat{\psi}_y^2} \\ \sqrt{\hat{\psi}_{c^2} - \hat{\psi}_c^2} \\ \vdots \\ \hat{\psi}_{yc} - \hat{\psi}_y \hat{\psi}_c \\ \hat{\psi}_{yI} - \hat{\psi}_y \hat{\psi}_I \\ \vdots \\ \hat{\psi}_{QR} - \hat{\psi}_Q \hat{\psi}_R \end{bmatrix}$$

Therefore the Jacobian of moment function $\Gamma_g(\cdot)$ should be

$$\Gamma_g(\widehat{\psi}) = \frac{\partial g}{\partial \psi} = \begin{bmatrix}
-2\mu_y & 0 & \cdots & 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -2\mu_c & 0 & \cdots & 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
-\frac{\mu_y}{\sigma_y} & 0 & \cdots & 0 & 0 & \frac{1}{2\sigma_y} & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{\mu_c}{\sigma_c} & 0 & \cdots & 0 & 0 & \frac{1}{2\sigma_c} & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & -\frac{\mu_Q}{\sigma_Q} & 0 & \cdots & 0 & 0 & \frac{1}{2\sigma_Q} & 0 & \cdots & 0 & 0 & 0 \\
-\mu_c & -\mu_y & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\
-\mu_I & 0 & -\mu_y & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 1 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & -\mu_R & -\mu_Q & 0 & \cdots & 0 & 0 & 0 & \cdots & 1
\end{bmatrix}$$

By applying the Delta Method the sample estimation of moments $\widehat{\Psi}$ has the following asymptotic properities

$$\sqrt{T}\left(\widehat{\Psi} - \Psi\right) \xrightarrow{d} N\left(0, \Gamma_g \Sigma \Gamma_g'\right)$$

where Σ is the LRV of ψ_t.

G.3 Solution method to simple model

G.3.1 Reconstruction

Similar to the section G.7.1, I replace the saving a_t by the effective asset holding x_t which follows $x_t = \gamma p_t^H h_t + a_t$. Then the problem 3 change to

$$\max_{c_t, h_t, x_t} \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) \tag{57}$$

s.t.

$$c_t + x_t + (1-\gamma) p_t^H h_t = R_t x_{t-1} + w_t \varepsilon_t + \left[(1-\delta^H) p_t^H - \gamma R_t p_{t-1}^H\right] h_{t-1} + T_t \tag{58}$$

$$x_t \geq 0$$

The related FOCs 31, 32 and 33 will become

$$U_{c_t} = \lambda_t \tag{59}$$

$$-\lambda_t + \mu_t + \beta E_t R_{t+1} \lambda_{t+1} = 0 \tag{60}$$

$$U_{h_t} - (1-\gamma) \lambda_t p_t^H + \beta E_t \lambda_{t+1} \left[(1-\delta^H) p_{t+1}^H - \gamma R_{t+1} p_t^H\right] = 0 \tag{61}$$

Similar to the full fledged model, I assume the utility function $U(c_t, h_t)$ follows the Cobb-Douglas formula

$$U(c_t, h_t) = \frac{\left(c_t^{\phi} h_t^{1-\phi}\right)^{1-\sigma}}{1-\sigma} \tag{62}$$

Since I assume there is no aggregate shock existing in the simple model, R_{t+1}, p_{t+1}^H and p_t^H can be perfectly expected. Therefore for non-constrained household there exists a static relationship between c_t and h_t from the combining of equation 59, 60 and 61

$$c_t = \frac{\phi}{1-\phi} h_t \left[p_t^H - (1-\delta^H) \frac{p_{t+1}^H}{R_{t+1}}\right] \tag{63}$$

When the collateral constraint is binding, it is worth to notice that the two FOC 32 and 60 have the same form. Therefore the Khun-Tucker multiplier is the same between the two model, the original one and the reconstructed one. To sum up, the problem 57 degenerates to a one state x_t problem which can be solved easily by value function iteration.

G.3.2 Solution Steps

Since in this simple problem I use Cobb-Douglas utility function where intratemporal elasticity of substitution between housing service and non-durable consumption is constant at 1, the consumption and housing servicing is homogeneous in degree 1 (linear) in the frictionless scenario. Therefore it is solvable to use value function iteration method.

1. Take an initial guess about value function $V(h_{-1}, x_{-1}, \varepsilon_{-1}) = \sum_{t=0}^{\infty} \beta^t U(c_t, h_t)$. If h_0, x_0 is still on grid I can remove the expectation with $\widetilde{V}(h_0, x_0, \varepsilon_{-1}) = E_0 V(h_0, x_0, \varepsilon_0) = \Pi V(h_0, x_0, \varepsilon_0)$ as h_0, x_0 is determined at time 0.

2. If the budget constraint is not binding, equation 63 will always hold. Therefore given an initial guess of $h_0(h_{-1}, x_{-1}, \varepsilon_{-1})$, I can get the unique mapping $x_0(h_0, h_{-1}, x_{-1}, \varepsilon_{-1})$ and $c_0(h_0, h_{-1}, x_{-1}, \varepsilon_{-1})$ through budget constraint 58 and equation 63. Then it is easy to find

$$h_0^{uc}(h_{-1}, x_{-1}, \varepsilon_{-1}) = \underset{h_0}{argmax} U\left[c_0(h_0, h_{-1}, x_{-1}, \varepsilon_{-1}), h_0\right] + \beta \widetilde{V}\left[h_0, x_0(h_0, h_{-1}, x_{-1}, \varepsilon_{-1}), \varepsilon_{-1}\right]$$

   where $\widetilde{V}\left[h_0, x_0(h_0, h_{-1}, x_{-1}, \varepsilon_{-1}), \varepsilon_{-1}\right]$ can be solved from linear interpolation on the on-grid value $\widetilde{V}(h_0, x_0, \varepsilon_{-1})$ in last step. I also define and save the value

$$\text{RHS}^{UC} = \max U\left[c_0(h_0, h_{-1}, x_{-1}, \varepsilon_{-1}), h_0\right] + \beta \widetilde{V}\left[h_0, x_0(h_0, h_{-1}, x_{-1}, \varepsilon_{-1}), \varepsilon_{-1}\right]$$

   .

3. If the budget constraint is binding, the Euler equation does not hold anymore. Therefore the mapping between h_0 and c_0 is no longer useful. However the effective wealth is known as now the household is constrained so $x_0(h_{-1}, x_{-1}, \varepsilon_{-1}) = 0$. Given any guess of $h_0(h_{-1}, x_{-1}, \varepsilon_{-1})$ the consumption $c_0(h_0, h_{-1}, x_{-1}, \varepsilon_{-1})$ can be solved from budget constraint 58. Then it is easy to find

$$h_0^{c}(h_{-1}, x_{-1}, \varepsilon_{-1}) = \underset{h_0}{argmax} U\left[c_0(h_0, h_{-1}, x_{-1}, \varepsilon_{-1}), h_0\right] + \beta \widetilde{V}\left[h_0, 0, \varepsilon_{-1}\right]$$

   where $\widetilde{V}\left[h_0, 0, \varepsilon_{-1}\right]$ can be solved from linear interpolation on the on-grid value $\widetilde{V}(h_0, 0, \varepsilon_{-1})$ in step 1. I also define and save the value

$$\text{RHS}^{C} = \max U\left[c_0(h_0, h_{-1}, x_{-1}, \varepsilon_{-1}), h_0\right] + \beta \widetilde{V}\left[h_0, 0, \varepsilon_{-1}\right]$$

   .

4. Because the result of constrained optimization in convex function optimization problem is always inferior than that of unconstrained optimization, the updated value function

$V(h_{-1}, x_{-1}, \varepsilon_{-1})$ will follows

$$V(h_{-1}, x_{-1}, \varepsilon_{-1}) = \begin{cases} \text{RHS}^{UC} & x_0^{uc} \geq 0 \\ \text{RHS}^{C} & x_0^{c} < 0 \end{cases}$$

Update the value function and go back to step 1.

G.4 Solution method to simple model with separable utility function

G.4.1 Reconstruction and new FOCs

Change the utility function from 62 to the separable utility function

$$U(c_t, h_t) = \frac{\phi c_t^{1-\sigma} + (1-\phi) h_t^{1-\sigma}}{1-\sigma}$$

Then the mapping from c_t to h_t under the frictionless scenario changes to

$$c_t = \left(\frac{\phi}{1-\phi}\right)^{\frac{1}{\sigma}} \left[p_t^H - (1-\delta^H)\frac{p_{t+1}^H}{R_{t+1}}\right]^{\frac{1}{\sigma}} h_t$$

G.5 Expected news shock

Then denote the "fundamental" variable X_t as

$$X_t = \begin{bmatrix} \log \Phi_t^i & \log \Phi_{g,t}^i & \varepsilon_t^8 & \varepsilon_{t-1}^8 & \varepsilon_{t-2}^8 & \varepsilon_{t-3}^8 & \varepsilon_{t-4}^8 & \varepsilon_{t-5}^8 & \varepsilon_{t-6}^8 & \varepsilon_{t-7}^8 \end{bmatrix}' \tag{64}$$

Then X_t follows

$$X_t = B^s X_{t-1} + \eta w_t \tag{65}$$

where

$$B^s = \begin{bmatrix} \rho_a & 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & \rho_g & 0 & 0 & 0 & \cdots & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 0 \end{bmatrix}_{10\times 10}$$

$$\eta = \begin{bmatrix} \sigma_a & 0 & 0 \\ 0 & \sigma_g & 0 \\ 0 & 0 & \sigma_g^8 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix}_{10\times 3}$$

$$\boldsymbol{w}_t = \begin{bmatrix} w_t^a \\ w_t^g \\ w_t^8 \end{bmatrix}$$

However household can only observe the variable $\widetilde{X}_t$ such that

$$\widetilde{X}_t = \begin{bmatrix} \log \widetilde{\Phi}_t & \log \widetilde{\Phi}_{g,t} & \widetilde{\varepsilon}_t^8 & \widetilde{\varepsilon}_{t-1}^8 & \widetilde{\varepsilon}_{t-2}^8 & \widetilde{\varepsilon}_{t-3}^8 & \widetilde{\varepsilon}_{t-4}^8 & \widetilde{\varepsilon}_{t-5}^8 & \widetilde{\varepsilon}_{t-6}^8 & \widetilde{\varepsilon}_{t-7}^8 \end{bmatrix}' \tag{66}$$

which follows

$$\widetilde{X}_t = HX_t + \epsilon v \tag{67}$$

where

$$H = \begin{bmatrix} H_{3\times 3}^{11} & 0_{3\times 5} \\ 0_{5\times 3} & m_4 I_{5\times 5} \end{bmatrix}$$

$$H^{11} = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}$$

$$m \in \mathbb{R}^+$$

$$\epsilon = \begin{bmatrix} \sigma_a^s & 0 & 0 & \cdots & 0 \\ 0 & \sigma_g^s & 0 & \cdots & 0 \\ 0 & 0 & \sigma_{g1}^s & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \sigma_{g8}^s \end{bmatrix}_{10\times 10}$$

$$v_t = \begin{bmatrix} v_t^a \\ v_t^g \\ v_t^{g1} \\ \vdots \\ v_t^{g8} \end{bmatrix}$$

G.6 Kalman Filter

Even though the household can successfully observe A_t at time t, he cannot observe g_t at time t. This make the household harder to estimate the A_{t+1} as $E_t \text{log}(A_{t+1}) = \rho_a \log A_t + E_t \log g_t$.

Thus we need get $g_{t|t}$ to get the expectation of A_{t+1}. Based on the Kalman filter and equation 65 and 67, we can solve out the perception of g_t by household as[30]

$$X_{t+1|t+1} = A^s X_{t|t} + P^s \widetilde{X}_{t+1} \tag{68}$$

where P^s is the Kalman gain and $A^s = (I - P^s H)B^s$

G.7 Model Reconstruction and Solution Process

The computation process follows the augmented endogenous gird method which is proposed by Auclert et al. (2021).

G.7.1 Preliminaries

I define the risk-adjusted expected value function as

$$\widetilde{V}(h_t, b_t, \varepsilon_{t-1}) = \beta EV(h_t, b_t, \varepsilon_t)$$

Therefore the marginal risk-adjusted expected value should be

$$\widetilde{V}_h(h_t, b_t, \varepsilon_{t-1}) = \beta EV_h(h_t, b_t, \varepsilon_t)$$

and

$$\widetilde{V}_b(h_t, b_t, \varepsilon_{t-1}) = \beta EV_b(h_t, b_t, \varepsilon_t)$$

To simplify the computation process, I further define the auxiliary variable x_t as t he effective asset holding which follows $x_t = \gamma p_t^h h_t + b_t$. Therefore the budget constraint 8 becomes

$$\begin{aligned} c_t + x_t + (1-\gamma)\, p_t^h h_t = {} & \left[\left(1-\delta^h\right) p_t^h - \gamma R_t p_{t-1}^h\right] h_{t-1} + R_t x_{t-1} \\ & + (1-\tau) w_t l_t \varepsilon_{t-1} - p_t^h C\left(h_t, h_{t-1}\right) + T_t \end{aligned} \tag{69}$$

Correspondingly collateral constraint becomes

$$x_t \geq 0$$

G.7.2 Decision Problems

The household solve the problem

$$V(h_{t-1}, x_{t-1}, \varepsilon_{t-1}) = \max_{h_t, x_t, l_t, c_t} U\left(c_t, h_t, l_t\right) + \beta EV(h_t, x_t, \varepsilon_t)$$

[30]For the reference Hamilton (2020) provides rigorous proof to this equation.

$$\text{s.t.} c_t + x_t + (1-\gamma)\, p_t^h h_t = \left[\left(1-\delta^h\right) p_t^h - \gamma R_t p_{t-1}^h\right] h_{t-1} + R_t x_{t-1}$$
$$+ (1-\tau) w_t l_t \varepsilon_{t-1} - p_t^h C\left(h_t, h_{t-1}\right) + T_t$$

and

$$x_t \geq 0$$

G.7.3 Solve step

1. Take the initial guess to marginal value function at time $t+1$ as $V_h(h_t, x_t, \varepsilon_t)$ and $V_x(h_t, x_t, \varepsilon_t)$

2. Solve the expectation problem on marginal value function to get risk-adjusted expected value function

$$\widetilde{V}_h(h_t, x_t, \varepsilon_{t-1}) = \beta \Pi V_h(h_t, x_t, \varepsilon_t)$$

and

$$\widetilde{V}_x(h_t, x_t, \varepsilon_{t-1}) = \beta \Pi V_x(h_t, x_t, \varepsilon_t)$$

3. Assuming the collateral constraint is unconstrained, I can combine equation 49, 50 and 51 to get

$$F\left(h_t, x_t, \varepsilon_{t-1}, h_{t-1}\right) = \frac{U_{h,t} + \widetilde{V}_h}{p_t^h \widetilde{V}_x} - \left(1 - \gamma + C_{h,t}\right) = 0$$

Further because the unseparable utility function $U\left(c_t, h_t, l_t\right)$ is homogeneous between c_t and h_t, $U_{h,t}$ can be written as a function of $\widetilde{V}_x$

$$U_{h,t} = (1-\phi)\left(\frac{\widetilde{V}_x}{\phi}\right)^{\frac{\phi(1-\sigma)}{\phi(1-\sigma)-1}} h_t^{\frac{\phi(1-\phi)(1-\sigma)^2}{1-\phi(1-\sigma)} + (1-\phi)(1-\sigma) - 1} \tag{70}$$

This can be used to solve $h_t\left(h_{t-1}, x_t, \varepsilon_{t-1}\right)$. The related mapping weight can also be used to map $\widetilde{V}_x(h_t, x_t, \varepsilon_{t-1})$ into $\widetilde{V}_x(h_{t-1}, x_t, \varepsilon_{t-1})$. Then $c\left(h_{t-1}, x_t, \varepsilon_{t-1}\right)$ and $l\left(h_{t-1}, x_t, \varepsilon_{t-1}\right)$ can be solved straightforward from

$$c\left(h_{t-1}, x_t, \varepsilon_{t-1}\right) = \left(\frac{\widetilde{V}_x(h_{t-1}, x_t, \varepsilon_{t-1})}{\phi}\right)^{\frac{1}{\phi(1-\sigma)-1}} h_t\left(h_{t-1}, x_t, \varepsilon_{t-1}\right)^{\frac{(1-\phi)(1-\sigma)}{1-\phi(1-\sigma)}} \tag{71}$$

and

$$l\left(h_{t-1}, x_t, \varepsilon_{t-1}\right) = \left(-\phi \frac{(1-\tau) w_t \varepsilon_{t-1}}{\kappa}\right)^{\frac{1}{\psi}} c\left(h_{t-1}, x_t, \varepsilon_{t-1}\right)^{\frac{\phi(1-\sigma)-1}{\psi}} h_t\left(h_{t-1}, x_t, \varepsilon_{t-1}\right)^{\frac{(1-\phi)(1-\sigma)}{\psi}} \tag{72}$$

4. Then the effective asset holding can be solved from budget constraint

$$\begin{aligned} x_{t-1}\left(h_{t-1}, x_t, \varepsilon_{t-1}\right) = & \frac{c\left(h_{t-1}, x_t, \varepsilon_{t-1}\right) + x_t + (1-\gamma)\, p_t^h h_t\left(h_{t-1}, x_t, \varepsilon_{t-1}\right)}{R_t} \\ & - \frac{\left[\left(1-\delta^h\right) p_t^h - \gamma R_t p_{t-1}^h\right] h_{t-1} + (1-\tau)\varepsilon_{t-1} w_t l\left(h_{t-1}, x_t, \varepsilon_{t-1}\right) + T_t}{R_t} \\ & + \frac{p_t^h C\left(h_t\left(h_{t-1}, x_t, \varepsilon_{t-1}\right), h_{t-1}\right)}{R_t} \end{aligned}$$

   Now invert above function $x_{t-1}\left(h_{t-1}, x_t, \varepsilon_{t-1}\right)$ to $x_t\left(h_{t-1}, x_{t-1}, \varepsilon_{t-1}\right)$. After this invert process the function $h_t\left(h_{t-1}, x_t, \varepsilon_{t-1}\right)$ can be mapped to $h_t\left(h_{t-1}, x_{t-1}, \varepsilon_{t-1}\right)$ by the function $x_t\left(h_{t-1}, x_{t-1}, \varepsilon_{t-1}\right)$.

5. Assuming the collateral constraint is constrained, I further define the relative Khun-Tucker multiplier as $\widetilde{\mu}_t(h_t, 0, \varepsilon_{t-1}) = \frac{\mu_t}{\widetilde{V}_x(h_t, 0, \varepsilon_{t-1})}$ so that equation 51 becomes

$$U_{c,t} = \left(1 + \widetilde{\mu}_t\right) \widetilde{V}_x$$

   Therefore the equation 70 changes to

$$U_{h,t} = (1-\phi)\left(\frac{\left(1+\widetilde{\mu}_t\right)\widetilde{V}_x}{\phi}\right)^{\frac{\phi(1-\sigma)}{\phi(1-\sigma)-1}} h_t^{\frac{\phi(1-\phi)(1-\sigma)^2}{1-\phi(1-\sigma)}+(1-\phi)(1-\sigma)-1}$$

   Similar to the process in step 3 this can be used to solve $h_t\left(h_{t-1}, \widetilde{\mu}_t, \varepsilon_{t-1}\right)$ from

$$F\left(h_t, \widetilde{\mu}_t, \varepsilon_{t-1}, h_{t-1}\right) = \frac{1}{1+\widetilde{\mu}_t} \frac{U_{h,t} + \widetilde{V}_h}{p_t^h \widetilde{V}_x} - \left(1 - \gamma + C_{h,t}\right) = 0$$

   and equation 71 changes to

$$c\left(h_{t-1}, \widetilde{\mu}_t, \varepsilon_{t-1}\right) = \left(\frac{\left(1+\widetilde{\mu}_t\right)\widetilde{V}_x(h_t, 0, \varepsilon_{t-1})}{\phi h_t\left(h_{t-1}, \widetilde{\mu}_t, \varepsilon_{t-1}\right)^{(1-\phi)(1-\sigma)}}\right)^{\frac{1}{\phi(1-\sigma)-1}}$$

   and corresponded optimal labor supply $l\left(h_{t-1}, \widetilde{\mu}_t, \varepsilon_{t-1}\right)$ from equation 72.

6. The effective asset holding under the constraint scenario can be solved from budget constraint

$$\begin{aligned} x_{t-1}\left(h_{t-1}, \widetilde{\mu}_t, \varepsilon_{t-1}\right) = & \frac{c\left(h_{t-1}, \widetilde{\mu}_t, \varepsilon_{t-1}\right) + (1-\gamma)\, p_t^h h_t\left(h_{t-1}, \widetilde{\mu}_t, \varepsilon_{t-1}\right)}{R_t} \\ & - \frac{\left[\left(1-\delta^h\right) p_t^h - \gamma R_t p_{t-1}^h\right] h_{t-1} + (1-\tau)\varepsilon_{t-1} w_t l\left(h_{t-1}, \widetilde{\mu}_t, \varepsilon_{t-1}\right) + T_t}{R_t} \\ & + \frac{p_t^h C\left(h_t\left(h_{t-1}, \widetilde{\mu}_t, \varepsilon_{t-1}\right), h_{t-1}\right)}{R_t} \end{aligned}$$

Now invert above function $x_{t-1}(h_{t-1}, \widetilde{\mu}_t, \varepsilon_{t-1})$ to $\widetilde{\mu}_t(h_{t-1}, x_{t-1}, \varepsilon_{t-1})$. After this invert process the function $h_t(h_{t-1}, \widetilde{\mu}_t, \varepsilon_{t-1})$ can be mapped to $h_t^c(h_{t-1}, x_{t-1}, \varepsilon_{t-1})$.[31] It is worth to notice that $x_t^c(h_{t-1}, x_{t-1}, \varepsilon_{t-1})$ is already known such that $x_t^c(h_{t-1}, x_{t-1}, \varepsilon_{t-1}) = 0$.

7. Compare $x_t(h_{t-1}, x_{t-1}, \varepsilon_{t-1})$ and $x_t^c(h_{t-1}, x_{t-1}, \varepsilon_{t-1})$ to select the largest elemental value. Then replace the unconstrained optimal housing service choice $h_t(h_{t-1}, x_{t-1}, \varepsilon_{t-1})$ with $h_t^c(h_{t-1}, x_{t-1}, \varepsilon_{t-1})$. Then for each grid point solve the nonlinear equation

$$\begin{aligned} c(h_{t-1}, x_{t-1}, \varepsilon_{t-1}) = &\left[\left(1-\delta^h\right) p_t^h - \gamma R_t p_{t-1}^h\right] h_{t-1} + R_t x_{t-1} \\ &+ (1-\tau) w_t \varepsilon_{t-1} \left(-\phi \frac{(1-\tau) w_t \varepsilon_{t-1}}{\kappa}\right)^{\frac{1}{\psi}} \\ &c(h_{t-1}, x_{t-1}, \varepsilon_{t-1})^{\frac{\phi(1-\sigma)-1}{\psi}} h_t(h_{t-1}, x_{t-1}, \varepsilon_{t-1})^{\frac{(1-\phi)(1-\sigma)}{\psi}} \\ &- p_t^h C(h_t(h_{t-1}, x_{t-1}, \varepsilon_{t-1}), h_{t-1}) + T_t \\ &- x_t(h_{t-1}, x_{t-1}, \varepsilon_{t-1}) - (1-\gamma) p_t^h h_t(h_{t-1}, x_{t-1}, \varepsilon_{t-1}) \end{aligned}$$

Then update the marginal value function through the envelop condition 53 and 54

$$V_h(h_{t-1}, x_{t-1}, \varepsilon_{t-1}) = U_{c,t}\left[\left(1-\delta^h\right) p_t^h - \gamma R_t p_{t-1}^h - C_{h_{t-1}}(h_t, h_{t-1}) p_t^h\right]$$

$$V_x(h_{t-1}, x_{t-1}, \varepsilon_{t-1}) = U_{c,t} R_t$$

G.8 Solve Rational Expectation model with imperfect information

Following Baxter et al. (2011) and Hürtgen (2014), I first solve perfect information model

$$AY_t = BY_{t-1} + C^{\text{pseo}}\Xi_t \tag{73}$$

where $Y_t = \begin{bmatrix} s_t' & Ec_{t+1}' \end{bmatrix}'$ where s_t is the vector of state variable and c_t is the vector of control variable. Ξ_t is the vector of pseudo-shock and composed with fundamental shock w_t and noisy shock v_t such that $\Xi_t = \begin{bmatrix} w_t' & v_t' \end{bmatrix}'$. The effect of shock C^{pseo} naturally becomes $C^{\text{pseo}} = \begin{bmatrix} P^s H\eta \\ P^s\epsilon \end{bmatrix}$ where P^s is the Kalman gain from equation 68. This linear model can be easily solved by Klein (2000) to yield $Y_t = PY_{t-1} + Q\Xi_t$. Take partition on P as

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$$

It is widely known that to solve the linear rational expectation model we pre-impose the restriction that $P_{12} = 0$ and $P_{22} = 0$. Further because of the holding of CEQ under first-order perturbation

[31] Here I use c in superscript as the notation to "constrained".

method, the policy function of control variables c_t will follow

$$c_t = P_{21} s_{t-1|t-1} + Q_2^w w_t + Q_2^v v_t \tag{74}$$

where Q_2^w and Q_2^v are subset of Q^w and Q^v which comes from Q such that $Q = \left[\begin{array}{cc} Q^w & Q^s \end{array} \right]$. Plug equation 74 into partition of equation 73 but replace $C^{\text{pseo}}\Xi_t$ with true fundamental shock process ηw_t such that

$$A_{11} s_t + A_{12} E c_{t+1} = B_{11} s_{t-1} + B_{12} c_t + \eta w_t$$

$$A_{11} s_t + A_{12} P_{21} s_{t|t} = B_{11} s_{t-1} + B_{12} \left(P_{21} s_{t-1|t-1} + Q_2^w w_t + Q_2^v v_t \right) + \eta w_t \tag{75}$$

It is worth to notice that here I use the first ns linear equations of equation 73 which is not free of choice yet a simplification in notation. The basic purpose now is to solve the law of motion of perceived state variable $s_{t|t}$ therefore we need ns "core" linear equations related to state variables to pin down ns state variable $s_{t|t}$. The word "core" refers to those equations that affect state variables directly, or more specifically, the law of motion of state variables. For instance, if we want to select one out of two linear equations in 73, 1) Euler equation $-\sigma\widetilde{c}_t = \widetilde{R}_t - \sigma\widetilde{c}_{t+1}$ and 2) Law of Motion of Capital $K\widetilde{k}_t = I\widetilde{I}_t + K\widetilde{k}_{t-1}$, which is used in equation 75, we should select the equation 2 because the equation 1 is implicitly comprised in the mapping from $s_{t-1|t-1}$ to c_t in equation 74. Otherwise we redundantly use the linear constraints and the matrix $A_{11} + A_{12}P_{21}G$ in equation 78 will not be well-defined.

Furthermore, the law of motion of perception of unobservable variables could be derived through plugging equation 67 into equation 68 to yield

$$X_{t|t} = A^s X_{t-1|t-1} + P^s H X_t + P^s \epsilon v_t \tag{76}$$

However, It is not all the state variables s_t that is unobservable, so I rewrite the law of motion of perceived state variable $s_{t|t}$ below. Without loss of generality, I assume the unobservable state variables lay on the last nx row (in this paper $nx = 10$ as equation 64 shows).

$$s_{t|t} = F s_{t-1|t-1} + G s_t + G_{P^s} \epsilon v_t \tag{77}$$

where $F = \begin{bmatrix} 0 & 0 \\ 0 & A^s \end{bmatrix}$, $G = \begin{bmatrix} I & 0 \\ 0 & P^s H \end{bmatrix}$ and $G_{P^s} = \begin{bmatrix} 0 \\ P^s \end{bmatrix}$.

And then plug equation 77 back to above equation 75

$$A_{11} s_t + A_{12} P_{21} \left(F s_{t-1|t-1} + G s_t + G_{P^s} \epsilon v_t \right) = B_{11} s_{t-1} + B_{12} \left(P_{21} s_{t-1|t-1} + Q_2^w w_t + Q_2^v v_t \right) + \eta w_t$$

$$
\begin{aligned}
\left(A_{11}+A_{12}P_{21}G\right)s_t &= B_{11}s_{t-1}+\left(B_{12}P_{21}-A_{12}P_{21}F\right)s_{t-1|t-1}+\left(B_{12}Q_2^w+\eta\right)w_t \\
&\quad+\left(B_{12}Q_2^v-A_{12}P_{21}G_{P^s}\epsilon\right)v_t
\end{aligned} \tag{78}
$$

Simplify above equation to

$$
\widetilde{Y}_t = M\widetilde{Y}_{t-1}+D\Xi_t \tag{79}
$$

where

$$
\widetilde{Y}_t = \begin{bmatrix} s_t \\ s_{t|t} \\ c_t \end{bmatrix}
$$

$$
A_L = \begin{bmatrix} I & 0 & 0 \\ -G & I & 0 \\ 0 & 0 & I \end{bmatrix}
$$

$$
B_L = \begin{bmatrix} \widetilde{P}_{11} & \widetilde{P}_{12} & 0 \\ 0 & F & 0 \\ 0 & P_{21} & 0 \end{bmatrix}
$$

$$
C_L = \begin{bmatrix} \widetilde{Q}_{11} & \widetilde{Q}_{12} \\ 0 & P^s\epsilon \\ Q_2^w & Q_2^v \end{bmatrix}
$$

$M = A_L^{-1}B_L$, $D = A_L^{-1}C_L$, $\widetilde{P}_{11} = \left(A_{11}+A_{12}P_{21}G\right)^{-1}B_{11}$,
$\widetilde{P}_{12} = \left(A_{11}+A_{12}P_{21}G\right)^{-1}\left(B_{12}P_{21}-A_{12}P_{21}F\right)$, $\widetilde{Q}_{11} = \left(A_{11}+A_{12}P_{21}G\right)^{-1}\left(B_{12}Q_2^w+\eta\right)$
and
$\widetilde{Q}_{12} = \left(A_{11}+A_{12}P_{21}G\right)^{-1}\left(B_{12}Q_2^v-A_{12}P_{21}G_{P^s}\epsilon\right)$.

G.9 Arguments to fake news and inefficiency

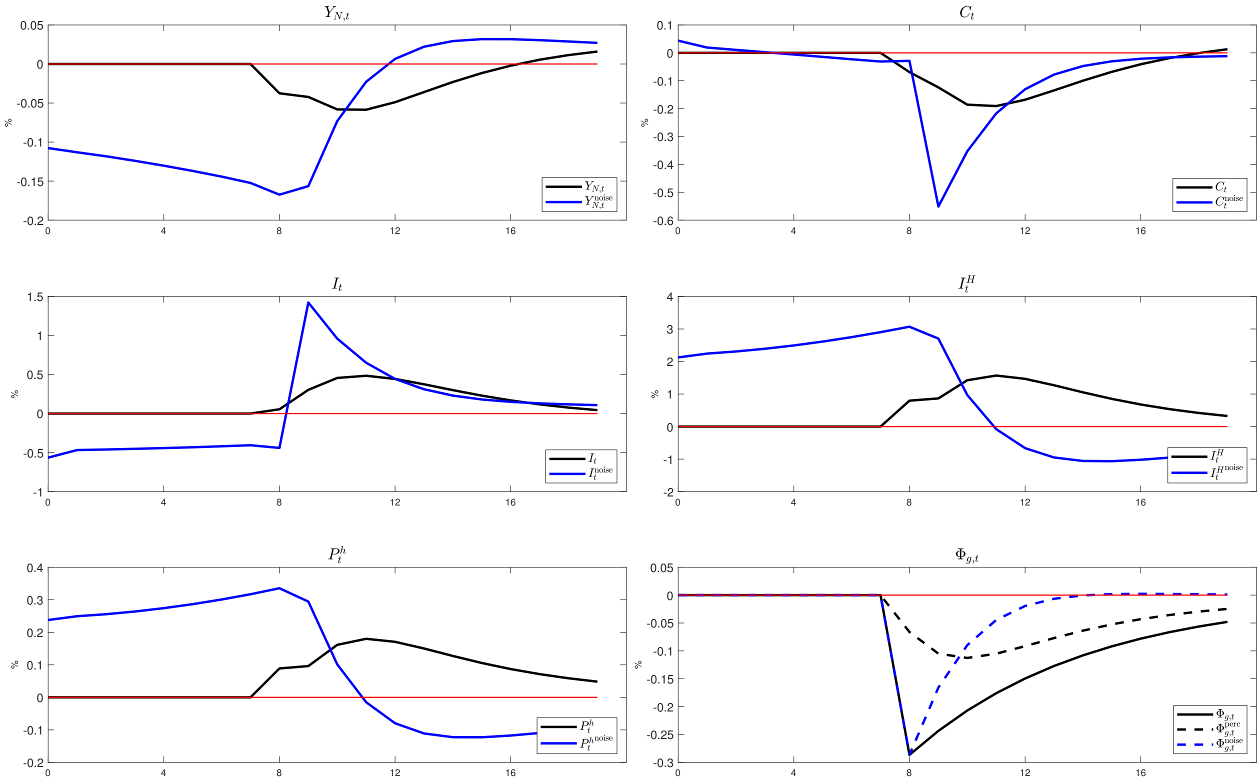


Figure 22: Fake news and True shock

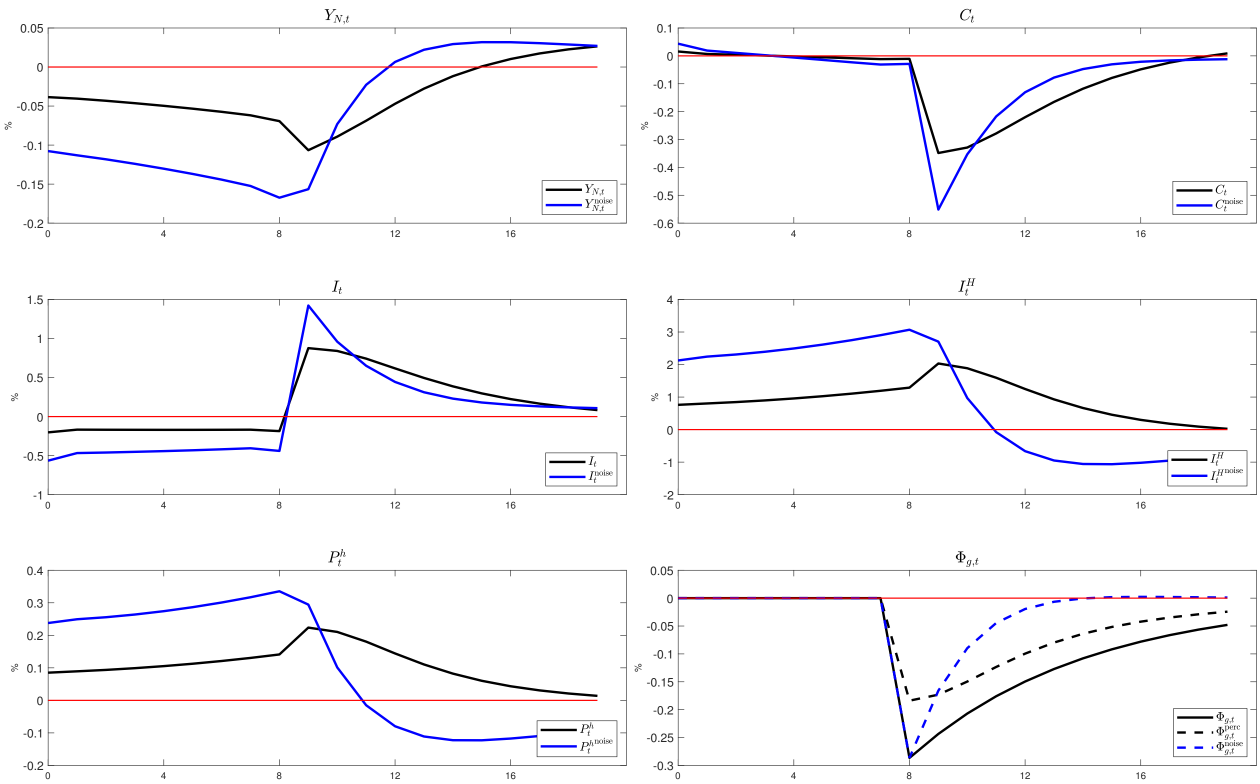


Figure 23: Fake news and True News