ENV Set

In [373…

```
import numpy as np
from scipy.io import loadmat
data = loadmat('astro_data.mat')
xx, vv = data['xx'], data['vv']

def norm(x, mean, std):
    return np.exp(-0.5 * ((x - mean) ** 2) / (std** 2)) / std

def log_pstar(state):
    log_omega, mm, pie, mu1, mu2, log_sigma1, log_sigma2 = state
    N = xx.shape[0]

    # exp process
    sigma1 = np.exp(log_sigma1)
    sigma2 = np.exp(log_sigma2)
    omega = np.exp(log_omega)

    x_mu = xx.mean()
    x_std = xx.std()
    ext = xx.max() - xx.min()

    log_ext = np.log(ext)

    forbidden_conditions = [pie < 0,
                            pie > 1,
                            np.abs(mm - x_mu) > 10 * x_std,
                            np.abs(mu1 - log_ext) > 20,
                            np.abs(mu2 - log_ext) > 20,
                            np.abs(log_sigma1) > np.log(20),
                            np.abs(log_sigma2) > np.log(20),
                            np.abs(log_omega) > 20,
                            ]

    if any(forbidden_conditions):
        return -np.inf

    log_A = 0.5 * np.log((xx - mm) ** 2 + (vv / omega) ** 2)

    log_prior = np.sum(np.log(pie * norm(log_A, mu1, sigma1)
                          + (1 - pie) * norm(log_A, mu2, sigma2)))
    log_like = -2 * log_A.sum() - N * np.log(omega)
    logp = log_like + log_prior

    return logp


def dumb_metropolis(init, log_ptilde, iters, sigma, xx=xx, vv=vv):
    D = init.shape[0]


    accept = 0
    # init state
    state = init
    Logp_state = log_ptilde(state)
```

```
    # init all the samples
    samples = np.zeros((D, iters))

    for iter in range(0, iters):
        propose = state + sigma * np.random.randn(len(state))
        Logp_prop = log_ptilde(propose)
        if (np.log(np.random.rand()) < (Logp_prop - Logp_state)).all():
            accept += 1
            state = propose        # accept propose param
            Logp_state = Logp_prop # update state
        samples[:, iter] = state.squeeze()
    accept_rate = accept / iters
    return (samples, accept_rate)
```

In [371…

```
from scipy.io import loadmat
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

In [58]:

```
log_omega = 1
mm = 1
pie = 1
mu1 = 1
mu2 = 1
log_sigma1 = 1
log_sigma2 = 1
# logp = log_pstar(log_omega, mm, pie, mu1, mu2, log_sigma1, log_sigma2, xx, vv)
params = np.array([log_omega, mm, pie ,mu1, mu2, log_sigma1, log_sigma2,])
```

4.1

What is the effect of Metropolis's step-size parameter?

step_size = 1

In [49]:

```
def plot_history(samples, acceptance_rate):
    params_name = ['log_omega', 'mm', 'pie', 'mu1', 'mu2', 'log_sigma1', 'log_si

    df = pd.DataFrame(samples.T, columns=params_name)

    df['iter'] = df.index
    lines = df.plot.line(x='iter', y=params_name)
```

In [50]:

```
samples, acceptance_rate = dumb_metropolis(params, log_pstar, 1000, sigma=1)
```

In [51]:

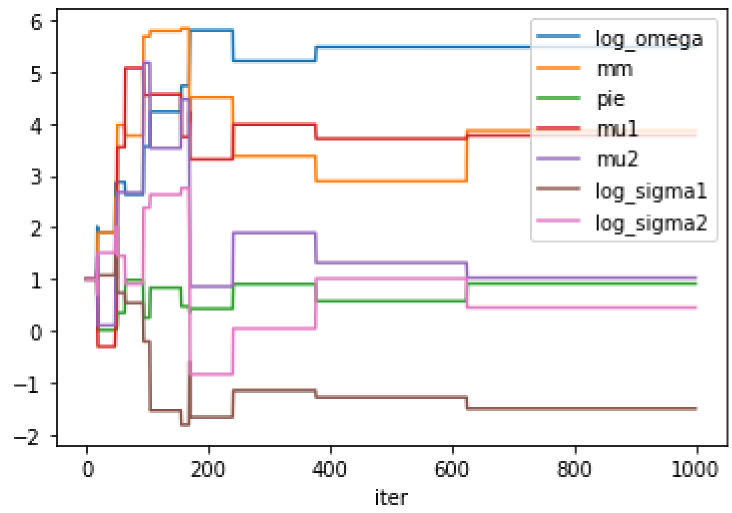
```
acceptance_rate
```

Out[51]:

```
0.013
```

In [52]:

```
plot_history(samples, acceptance_rate)
```



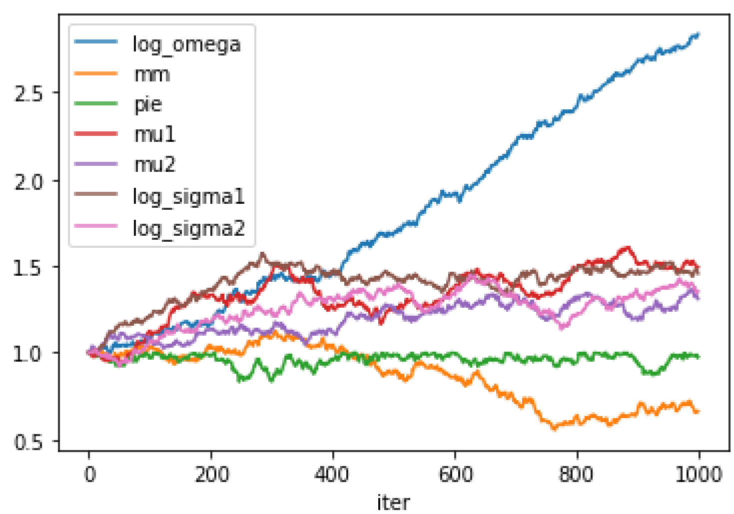
step_size = 0.01

In [53]:
```
samples, acceptance_rate = dumb_metropolis(params, log_pstar, 1000, sigma=0.01)
```

In [54]:
```
acceptance_rate
```

Out[54]: 0.777

In [55]:
```
plot_history(samples, acceptance_rate)
```



step_size = 10

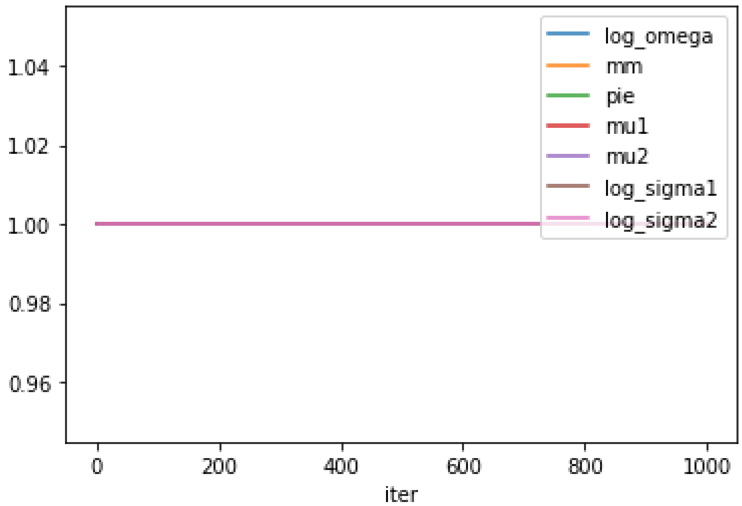
In [46]:
```
samples, acceptance_rate = dumb_metropolis(params, log_pstar, 1000, sigma=10)
```

In [47]:
```
acceptance_rate
```

Out[47]: 0.0

In [48]:

```
plot_history(samples, acceptance_rate)
```



- Little step size will lead to small exploration range
- Too Large step size will crash the sampling, no sample accepted
- We need a suitable step size and burn-in time

4.2

Is the way you initialize the chain critical for Metropolis and/or slice sampling?

We just change our init

In [63]:

```
log_omega = 1
mm = 1
pie = 0.1
mu1 = 1
mu2 = 10
log_sigma1 = 1
log_sigma2 = 1
# logp = log_pstar(log_omega, mm, pie, mu1, mu2, log_sigma1, log_sigma2, xx, vv)
params = np.array([log_omega, mm, pie ,mu1, mu2, log_sigma1, log_sigma2,])
```

In [64]:

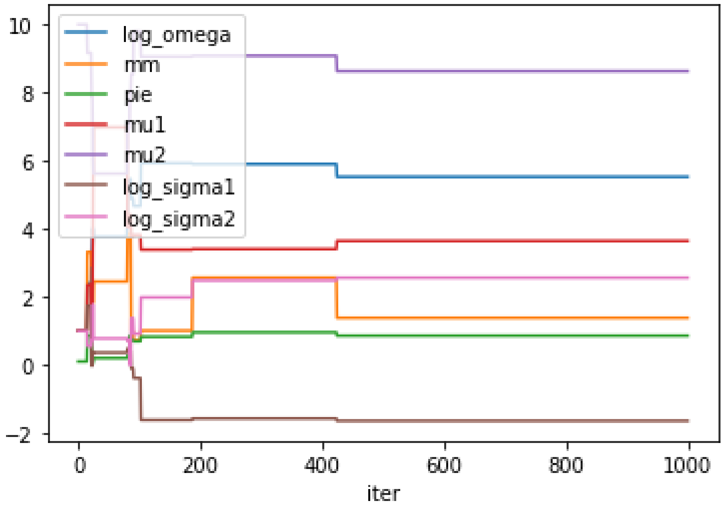
```
samples, acceptance_rate = dumb_metropolis(params, log_pstar, 1000, sigma=1)
acceptance_rate
```

Out[64]:

```
0.011
```

In [65]:

```
plot_history(samples, acceptance_rate)
```



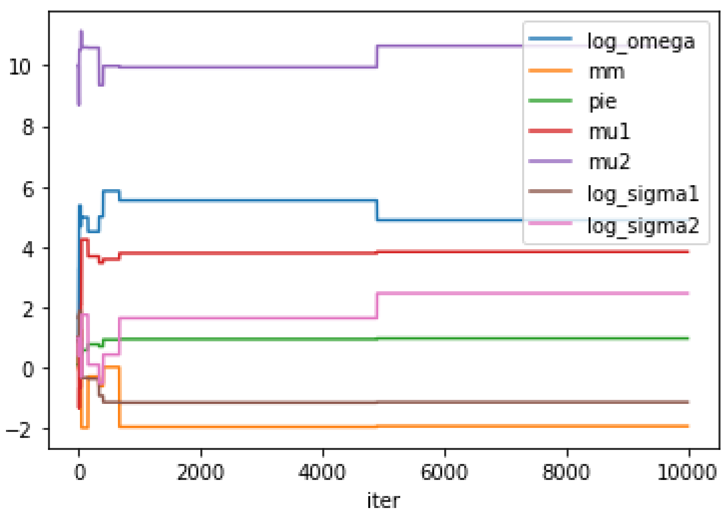
In [66]:

```
samples, acceptance_rate = dumb_metropolis(params, log_pstar, 10000, sigma=1)
acceptance_rate
```

Out[66]: 0.0012

In [67]:

```
plot_history(samples, acceptance_rate)
```



We may need more burn-in time to make the sampling consistent.

4.3

What are the relative advantages of slice-sampling and Metropolis? Can you say good and bad things about both of them?

slice-sampling

pros: slice-sampling can be more efficient for not rejecting the samples, which is suitable for the case that we do not have much information about the distribution.

cons: slice-sampling may suffer from locality for those complicated distribuion. It is hard for slice-sampling to explore those far away high probability area across some low probability area.

Metropolis

pros: Metropolis is quite simple and can generally working in most cases.

cons: As we showed above, Metropolis may rely on its hyperparameters(step-size) and have burn-in period.

4.4

Why have I taken logs of quantities like ω and A? Need I have bothered? Does taking logs affect the model and/or the sampler?

Log can convert the multiply to addition, which in some sense can prevent the precision problem. The final result is not affected. (just like log likelihood)

4.5

The true values are ω = 875.2 and m= 31.79. Are your posterior beliefs consistent with this? If not, what do you think went wrong with the sampling, modelling or both?

In [316…

```
log_omega = 10
mm = 1
pie = 1
mu1 = 1
mu2 = 1
log_sigma1 = 0.1
log_sigma2 = 0.1
# logp = log_pstar(log_omega, mm, pie, mu1, mu2, log_sigma1, log_sigma2, xx, vv)
params = np.array([log_omega, mm, pie ,mu1, mu2, log_sigma1, log_sigma2,])
```

In [324…

```
samples, acceptance_rate = dumb_metropolis(params, log_pstar, 100000, sigma=0.08
acceptance_rate
```

Out[324…

```
0.12078
```

In [319…

```
ori_samples = samples
```

In [320…

```
samples = ori_samples
```

In [366…

```
# ori_samples = samples
print(samples.shape)
samples = samples.T
print(samples.shape)
# samples = samples.T
samples = samples[5000:]
```

```
samples = samples.T
print(samples.shape)
```

```
(7, 55000)
(55000, 7)
(7, 50000)
```

In [367…

```
params_name = ['log_omega', 'mm', 'pie', 'mu1', 'mu2', 'log_sigma1', 'log_sigma2
df = pd.DataFrame(samples.T, columns=params_name)
```

In [368…

```
df['mm'].mean()
```

Out[368…

```
31.874691194919823
```

In [369…

```
np.exp(df['log_omega'].mean())
```

Out[369…

```
873.151930476839
```

We have a burn-in = 50000, and total iter = 100000, we get similar posterior