

**CS 764/864: Blockchains and Cryptocurrencies: Fundamentals, Technologies, and Economics****Spring 2020 Module 5 Homework 5****Q1 Blind Digital Signatures**

**1.1 Given  $p=5$ ,  $q=11$ , determine the public key  $(e,n)$  and the private key  $(d,n)$ . Assume  $e=7$ .**

$$n = p \cdot q = 55, \text{Phi}(n) = (p-1) \cdot (q-1) = 40$$

$$\text{public key } (e,n) = (7, 55)$$

$$\text{private key } (d,n) = (23, 55)$$

**1.2 Given  $b=6$ , determine the blinding factor using the public key derived in (i)**

$$\text{BF} = (b^e \bmod n) = 41$$

**1.3 Given  $m=11$ , determine the blinded message using the information in (i) and (ii)**

$$\text{BM} = m \cdot \text{BF} \bmod n = 11$$

**1.4 Determine the signature on the blinded message in (iii) using the private key  $(d,n)$**

$$s = \text{BM}^d \bmod n = 11$$

**1.5 Unblind the signature derived in (iv) to determine the signature on the original message  $m=11$**

$$\text{unblinded: } (s/b) \bmod n = (11 + 1 \cdot 55)/6 \bmod 55 = 11$$

**1.6 Verify that the blind signature is the same as the signature on  $m$  using  $(d,n)$ .**

$$\text{signature} = (m^d) \bmod n = 11$$

Therefore, the signatures match.

**Q2. Bit-commitment protocol**

Everyone's hashes

Alice: d4d2aa8f2054da2af4338fb4da561b93  
 Bob : d21fe43f78c846f3a86dd9b63110dc05  
 Carol: 99cc0f19019a963639c8a367f15de477  
 David: 1c39c6b7a55edf8d980b4e64b109e498

The election results have been declared  
 John is the new president

They each send their nonces to the other three

Alice: 1254  
 Bob : 1703  
 Carol: 4106  
 David: 7252

The bet results

Alice: Correct prediction  
 Bob: Correct prediction  
 Carol: Incorrect prediction  
 David: Incorrect prediction

**Q3. Zero-knowledge proofs**

```

C sends its user name to S; it DOES NOT send its password P
C Username:  Alice

S verifies the user name and sends a nonce N to C
Nouce:  2357

C computes a hash  $H = \text{MD5}(N || P || N)$  and sends H to S
The hashed password(H) is  07912fb6e3483d20f92e22307269114b

S independently computes  $G = \text{MD5}(N || P || N)$ . If  $G=H$ , S declares C as an authenticated user
The hashed password(G) is  07912fb6e3483d20f92e22307269114b
C is an  authenticated user

```

**Q4 Discrete Zero-Knowledge Proofs**

Alice and Bob know:  $p=17$ ,  $A=5$ , and  $B=9$ . Alice knows  $x$  is 10 but does not reveal it to Bob.

1. Alice chooses a random number  $r < p$ .
  - a. Let Alice picks 5 as  $r$
2. Alice sends  $q = (A^r \bmod p)$  to Bob.
  - a.  $q = 14$
3. Bob sends a random bit to Alice.
  - a. Random bit = 1
4. Alice computes  $s = (r + i \cdot x) \bmod (p-1)$  and sends to Bob.
  - a.  $S = 15$
5. Bob computes  $C = A^s \bmod p$ . This should be equal to  $D = q^*(B^i) \bmod p$ .
  - a.  $C = 7$
  - b.  $D = 7$
6. Since  $C$  is equal to  $D$ , it means Alice knows the value of  $x$ .

## Readme for Q2 and Q3 codes

### Requirement:

- Python 3.6+
- random and hashlib library installed

### To run the code:

- Simply double click on the .py files or using terminal and use the `python ./hw\_q2.py`  
`python ./hw\_q3.py` commands

### Source Code for Q2

```
import random
import hashlib

# Generate random nonces for the 4 friends
NA = random.randint(10,10000)
NB = random.randint(10,10000)
NC = random.randint(10,10000)
ND = random.randint(10,10000)

# Winner bit
john_bit = 0
jane_bit = 1

# Compute hashes Hi = MD5(Predicted winner bit || Nonce)
# Given that Alice and Bob predicted "John" a
# Carol and David predicted "Jane"
HA = hashlib.md5(str(hex(john_bit + 2*NA)).encode('utf-8'))
HB = hashlib.md5(str(hex(john_bit + 2*NB)).encode('utf-8'))
HC = hashlib.md5(str(hex(jane_bit + 2*NC)).encode('utf-8'))
HD = hashlib.md5(str(hex(jane_bit + 2*ND)).encode('utf-8'))

print("Everyone's hashes")
print("\tAlice: " + HA.hexdigest())
print("\tBob  : " + HB.hexdigest())
print("\tCarol: " + HC.hexdigest())
print("\tDavid: " + HD.hexdigest())

# Randomly select a winner
print("\nThe election results have been declared")
new_president = random.randint(0,1)
if new_president == john_bit :
```

```

    print("John is the new president")
else:
    print("Jane is the new president")

# They each send their nonces to the other three
print("\nThey each send their nonces to the other three")
print("\tAlice: " + str(NA))
print("\tBob  : " + str(NB))
print("\tCarol: " + str(NC))
print("\tDavid: " + str(ND))

# Decide who made the correct predictions
if hashlib.md5(str(hex(new_president + 2 * NA)).encode('utf-8')).hexdigest() == HA.hexdigest():
    A = "Correct prediction"
else:
    A = "Incorrect prediction"
if hashlib.md5(str(hex(new_president + 2 * NB)).encode('utf-8')).hexdigest() == HB.hexdigest():
    B = "Correct prediction"
else:
    B = "Incorrect prediction"
if hashlib.md5(str(hex(new_president + 2 * NC)).encode('utf-8')).hexdigest() == HC.hexdigest():
    C = "Correct prediction"
else:
    C = "Incorrect prediction"
if hashlib.md5(str(hex(new_president + 2 * ND)).encode('utf-8')).hexdigest() == HD.hexdigest():
    D = "Correct prediction"
else:
    D = "Incorrect prediction"

print("\nThe bet results")
print("Alice:\t", A)
print("Bob:\t", B)
print("Carol:\t", C)
print("David:\t", D)

```

## Source Code for Q3

```
import hashlib

# Given
N = 2357
username = "Alice"
password = "alice123"

# (i)
print("C sends its user name to S; it DOES NOT send its password P")
print("C Username: ", username)

# (ii)
print("\nS verifies the user name and sends a nonce N to C")
print("Nouce: ", str(N))

# (iii)
hashed_password = hashlib.md5(str(hex(N) + password + hex(N)).encode('utf-8'))
print("\nC computes a hash H = MD5(N||P||N) and sends H to S")
print("The hashed password(H) is ", hashed_password.hexdigest())

#(iv)
g = hashlib.md5(str(hex(N) + password + hex(N)).encode('utf-8'))
print("\nS independently computes G= MD5(N||P||N). If G=H, S declares C as an authenticated user")
print("The hashed password(G) is ", g.hexdigest())

if g.hexdigest() == hashed_password.hexdigest():
    print("C is an authenticated user")
else:
    print("C is NOT an authenticated user")
```